Question 1 (5 points)
Find the critical points of the following function and use the second derivative test to classify them as local minima, local maxima, or saddle points: \( f(x, y) = e^{-(x^2 + y^2)} \).

\[
\begin{align*}
f_x(x, y) &= -2xe^{-(x^2 + y^2)} = 0 \quad (1 \text{ pt}) \\
f_y(x, y) &= -2ye^{-(x^2 + y^2)} = 0 \Rightarrow x = 0, y = 0 \\
e^{-(x^2 + y^2)} \text{ can never be 0}
\end{align*}
\]

\( \Rightarrow \) There is only one critical point: \( (x, y) = (0, 0) \) (1 pt)

\[
\begin{align*}
f_{xx}(x, y) &= -2e^{-(x^2 + y^2)}(1 - 2x^2) \\
f_{yy}(x, y) &= -2e^{-(x^2 + y^2)}(1 - 2y^2) \\
f_{xy}(x, y) &= 4xye^{-(x^2 + y^2)} \quad (1 \text{ pt})
\end{align*}
\]

\[
D(0, 0) = f_{xx}(0, 0) \cdot f_{yy}(0, 0) - f_{xy}^2(0, 0) = (-2)(-2) - 0^2 = 4 > 0 \quad (1 \text{ pt})
\]

\( f_{xx}(0, 0) = -2 \)

\( \Rightarrow (0, 0) \) is a local maximum (1 pt)