Quiz 1

Math 105 - Section 209

SOLUTION

Name: ____________________

Student Nr.: ____________________

Question 1 *(2 points)*
Find an equation for the plane parallel to the plane $2x + y - z = 1$ which passes through the point $P_0(0, 2, -2)$.

The normal vector is $\underline{n} = (2, 1, -1)$

$\Rightarrow$ the equation is $2x + y - z = d$ for some d (1pt)

P_0 is on the plane if $2 \cdot 0 + 2 - (-2) = d$

$4 = d$

$\Rightarrow$ The equation of the plane is

$$2x + y - z = 4$$ (1pt)

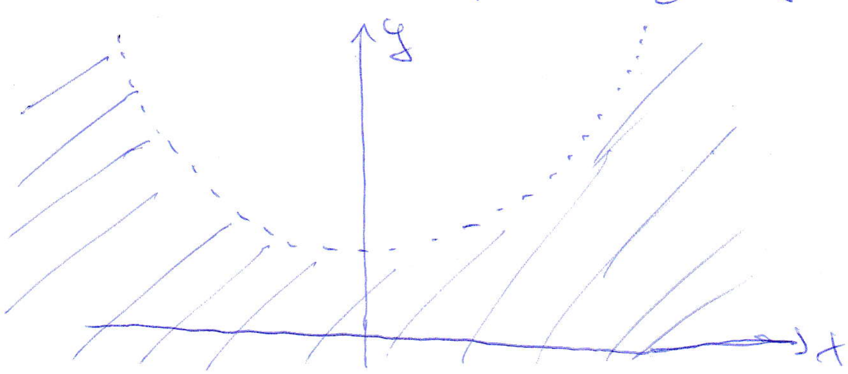
Question 2 *(3 points)*
Find and sketch the *domain* of the function $f(x, y) = \ln(x^2 - y + 1)$.

Recall that the argument z for $\ln(z)$ must be strictly positive. (1pt)

So we want $x^2 - y + 1 > 0$

$x^2 + 1 > y$

$\Rightarrow$ The domain of f is $\{(x, y) : y < x^2 + 1\}$ (1pt)



(1pt)

Question 3 *(5 points)*
Compute the second order partial derivatives for the following function and verify Clairaut's theorem:
$f(x, y) = e^{-(x^2+y^2)}$.

$f_x(x,y) = e^{-(x^2+y^2)}(-2x)$ $\quad$ $f_y(x,y) = e^{-(x^2+y^2)}(-2y)$

$f_{xx}(x,y) = e^{-(x^2+y^2)}4x^2 - 2e^{-(x^2+y^2)}$

$f_{xy}(x,y) = e^{-(x^2+y^2)}(-2x)(-2y)$

(3pt)

$f_{yx}(x,y) = e^{-(x^2+y^2)}(-2y)(-2x)$

$f_{yy}(x,y) = e^{-(x^2+y^2)}4y^2 - 2e^{-(x^2+y^2)}$

Clairaut's theorem: $f_{xy} = f_{yx}$

$f_{xy} = 4xy\, e^{-(x^2+y^2)} = f_{yx}$

(2pt)