Today

- Method of Undetermined Coefficients for any periodic function.
- Fourier Series and method of undetermined coefficients
- Fourier series calculations

Fourier series

- Recall Method of Undetermined Coefficients for equations of the form

$$ay'' + by' + cy = f(t)$$

https://www.desmos.com/calculator/tqaia8bt7n

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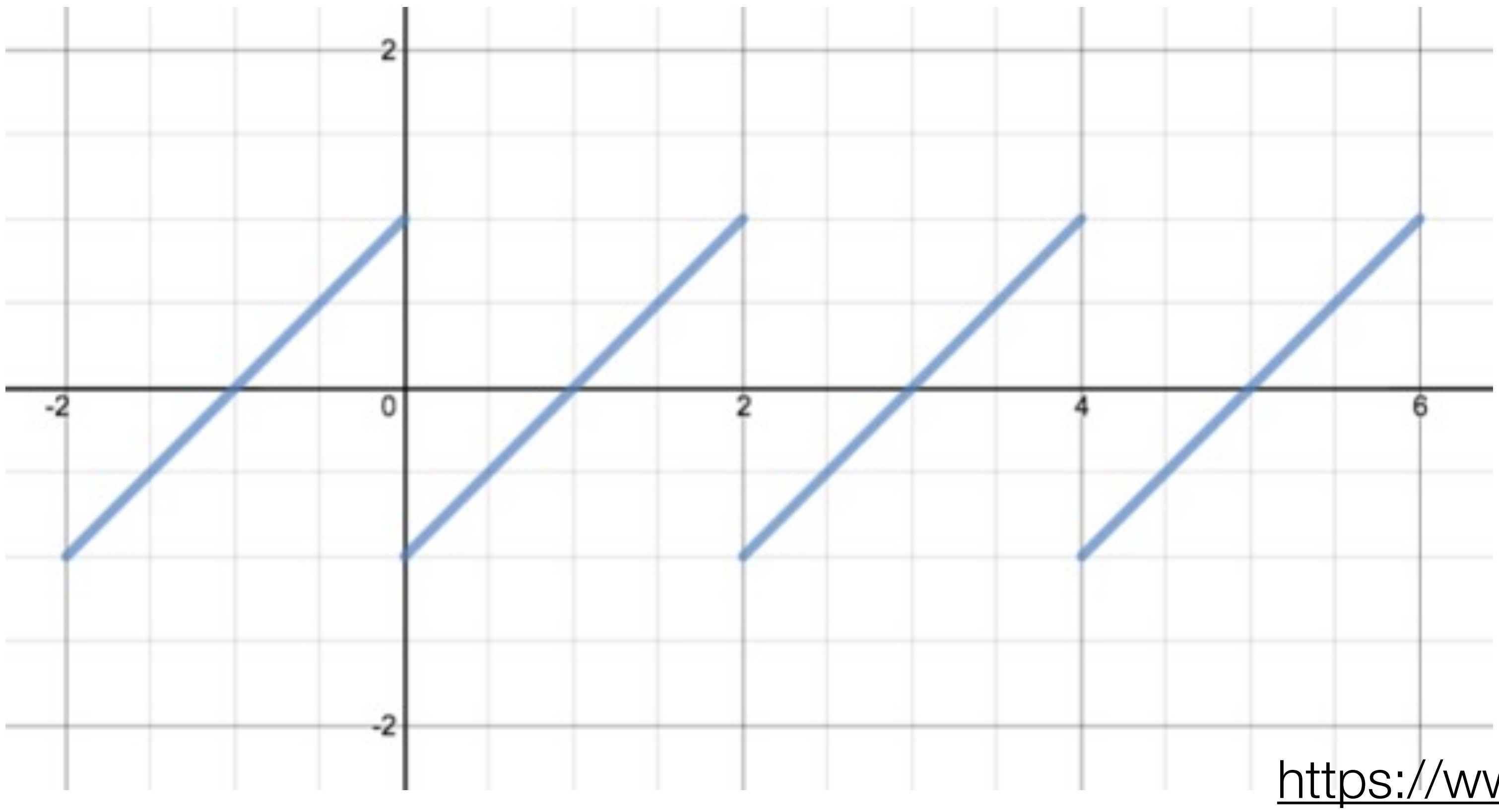

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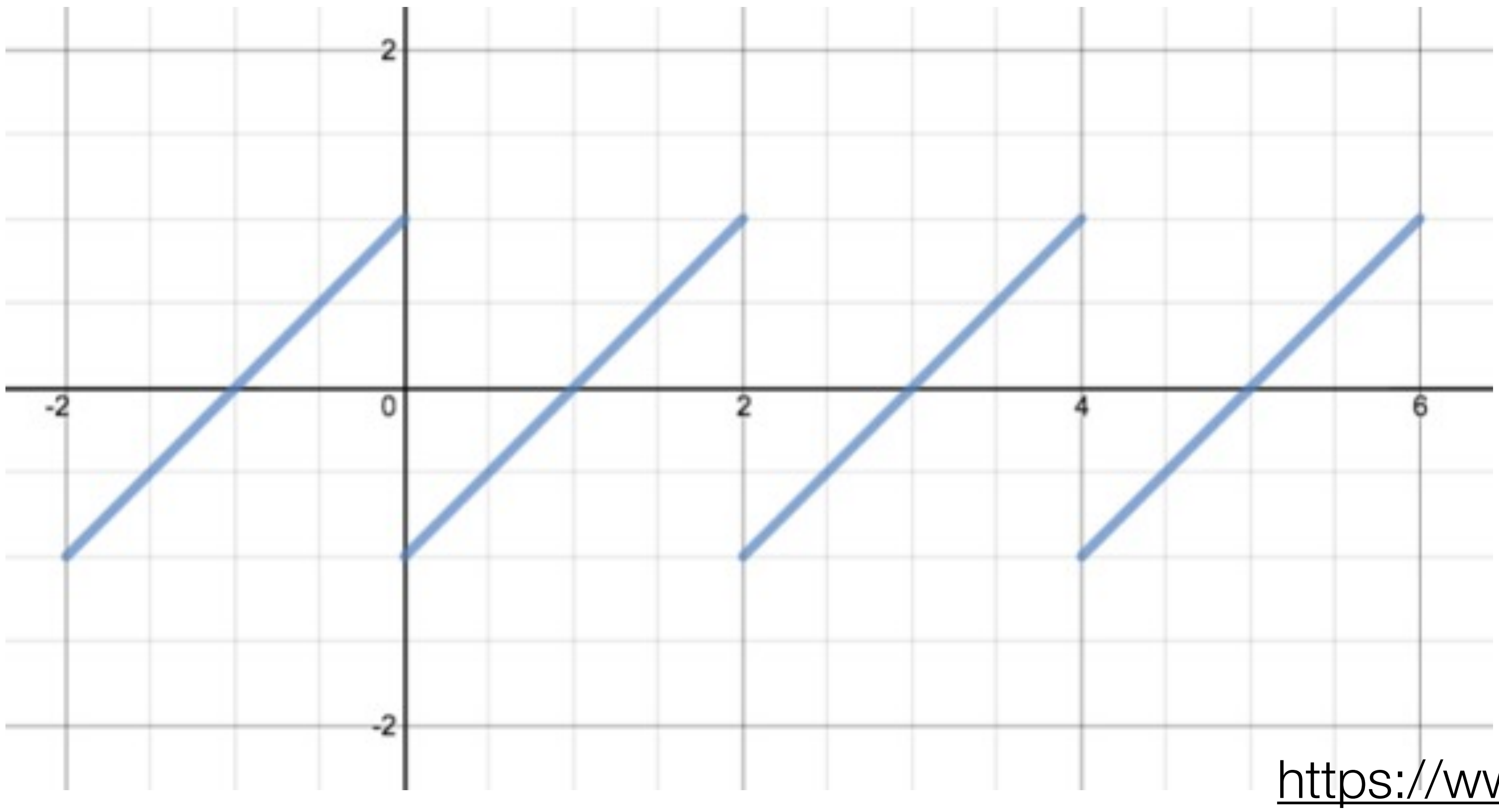

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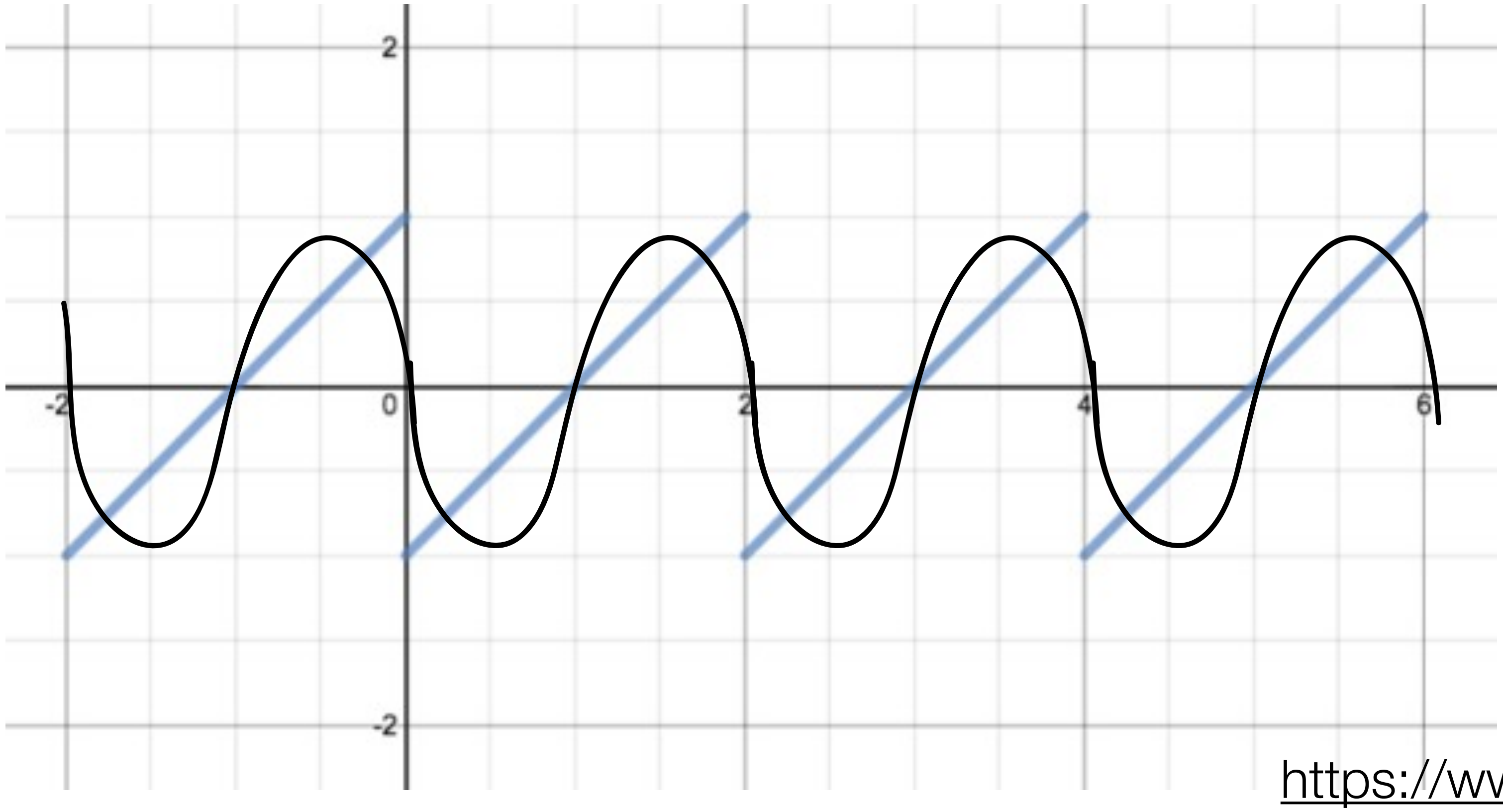

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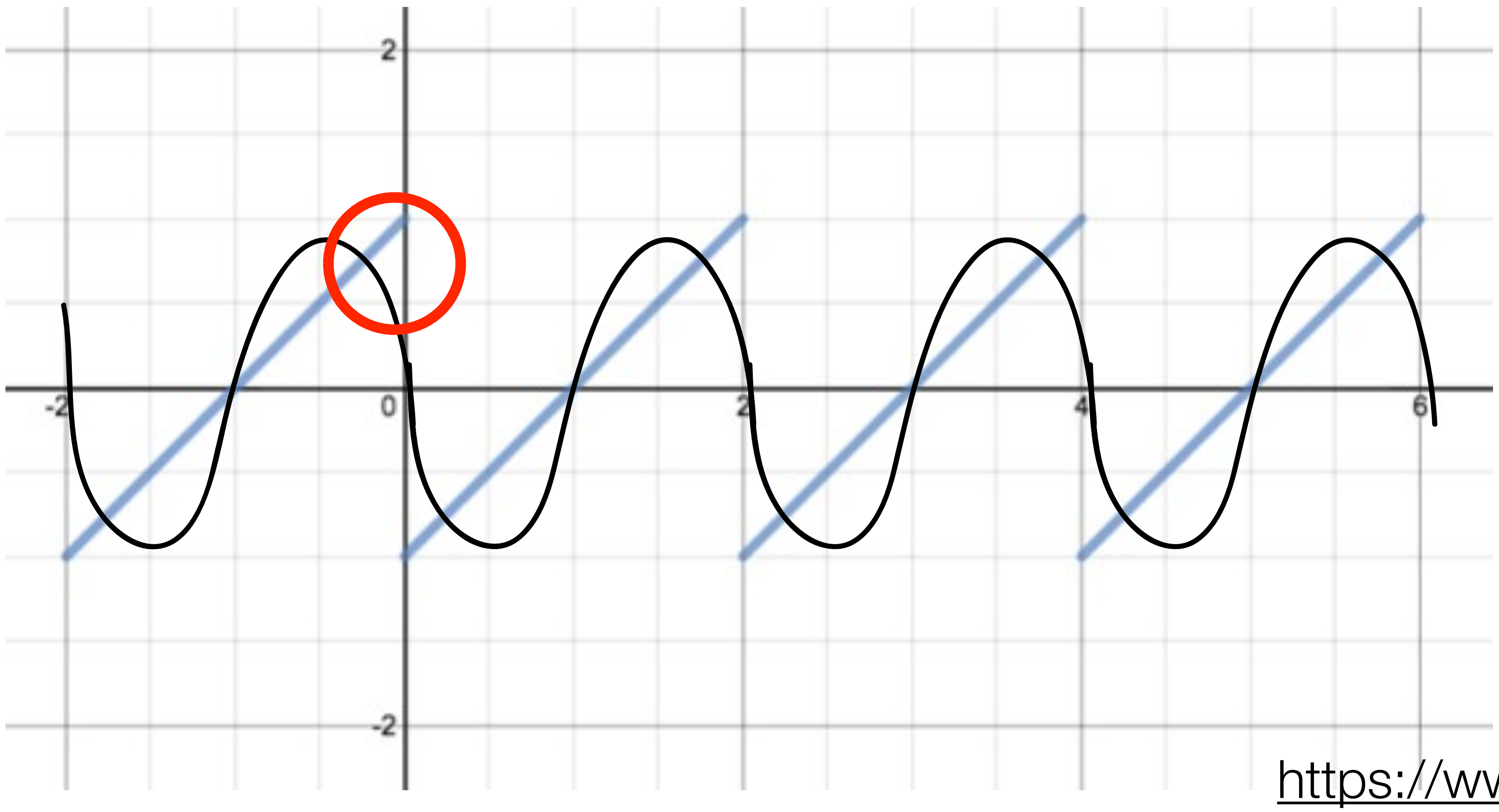

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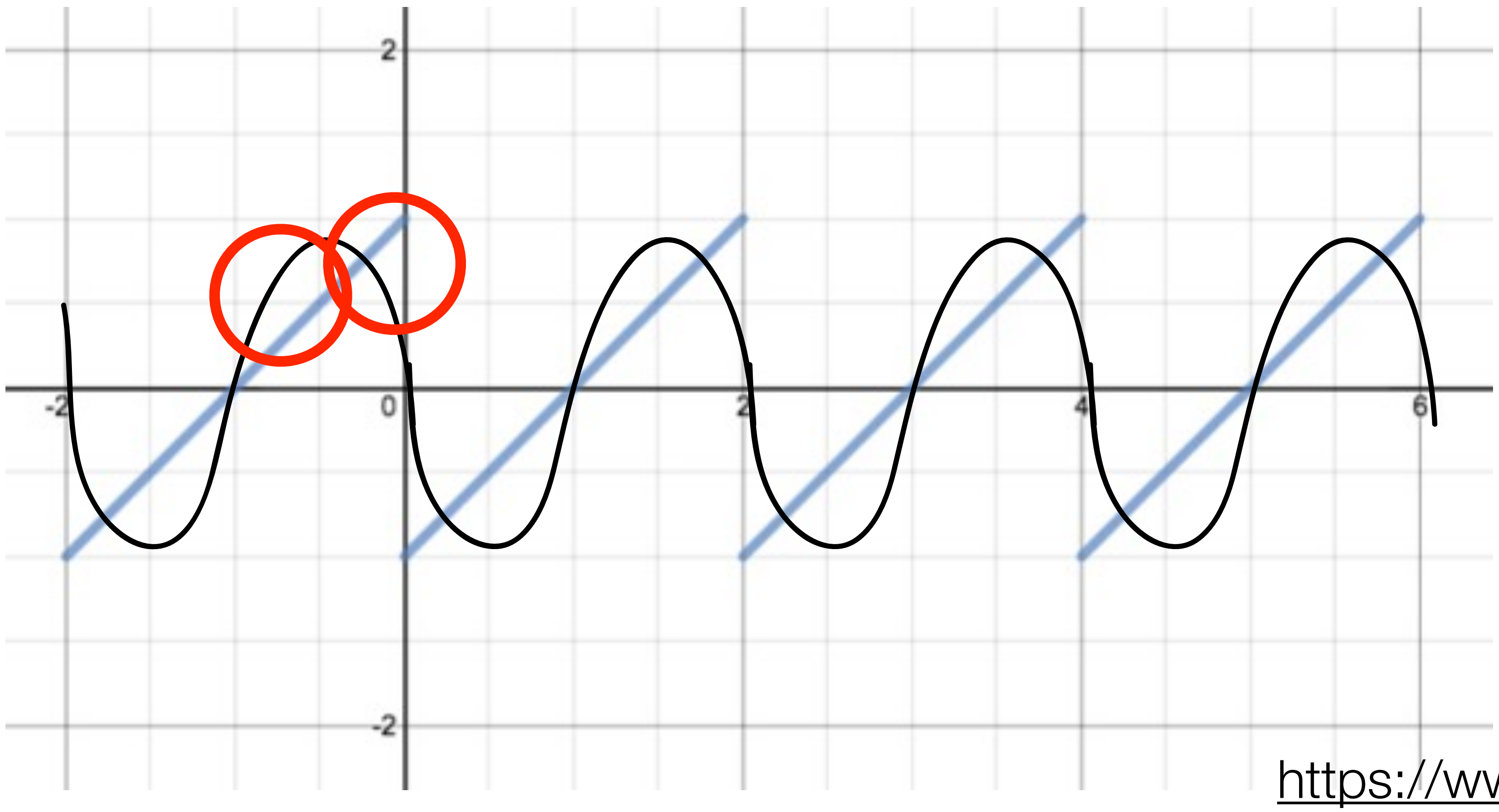

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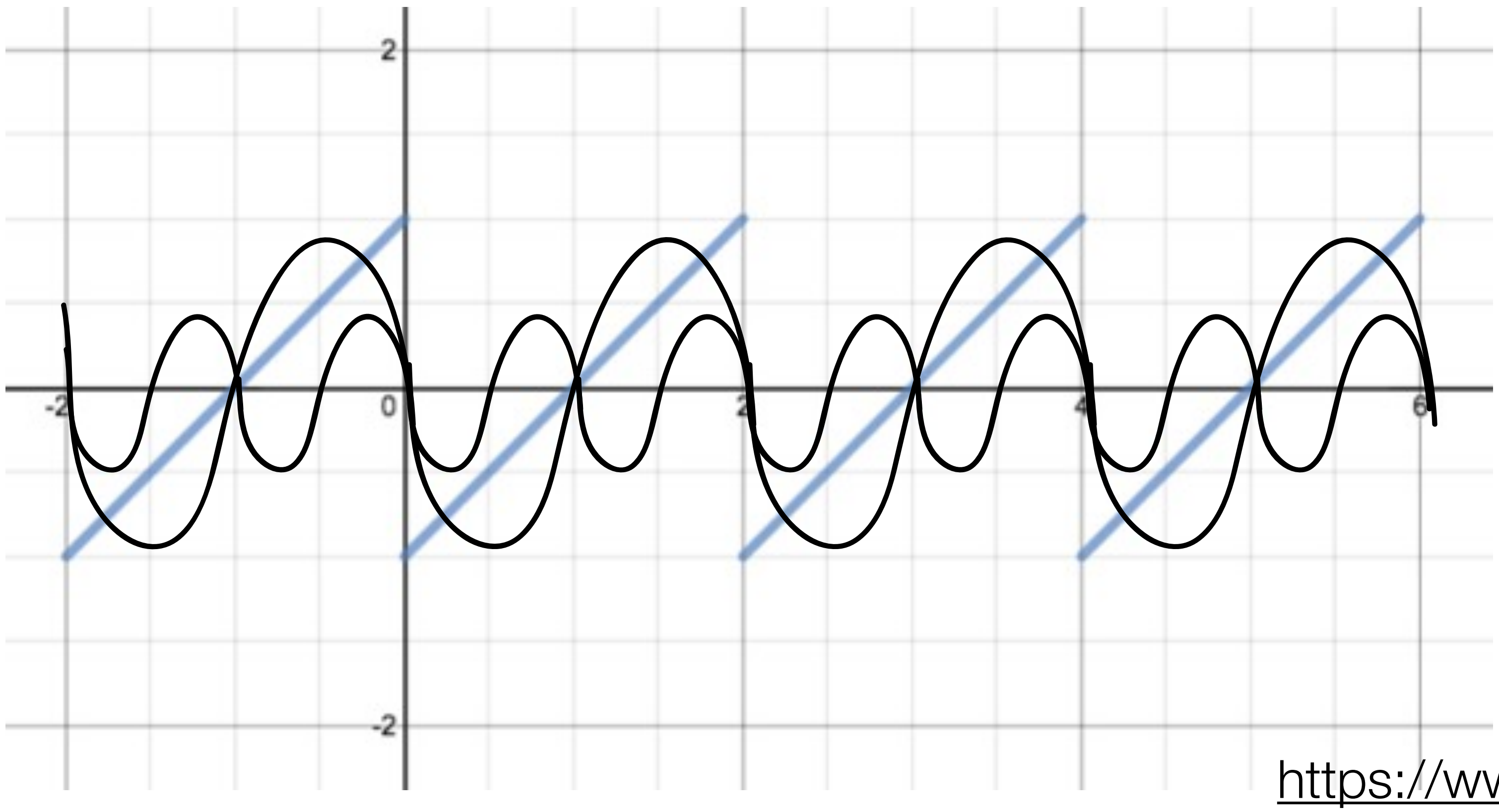

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Fourier series

- For the equation

$$y'' + 10y = \cos(t) + \frac{1}{2}\cos(2t) + \frac{1}{3}\cos(3t) + \frac{1}{4}\cos(4t) + \cdots$$

- what will be the dominant frequency (largest coefficient) in the solution?

(A) w = 1

(B) w = 2

(C) w = 3

(D) w = 4

(E) Don’t know. Explain please.

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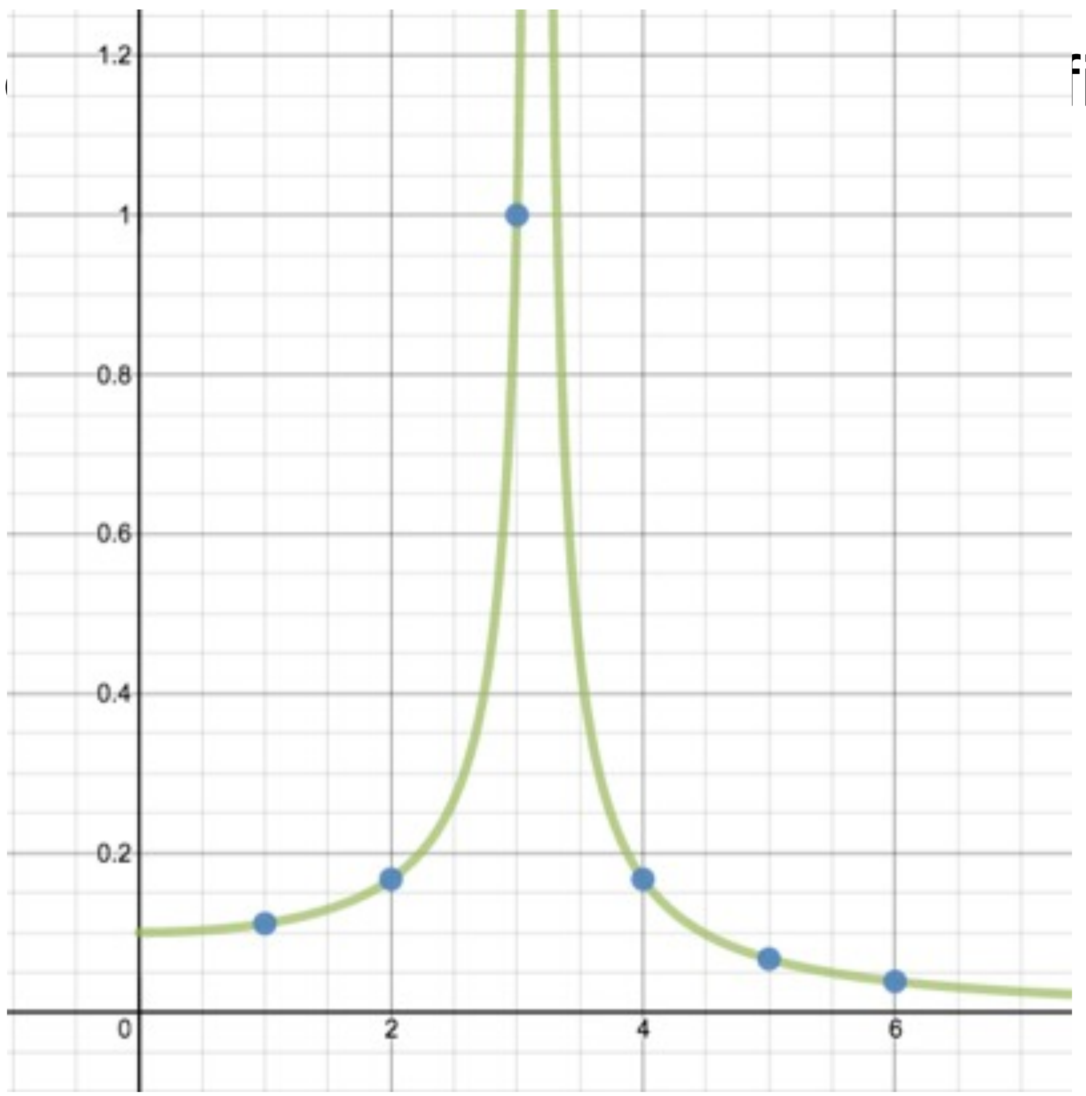

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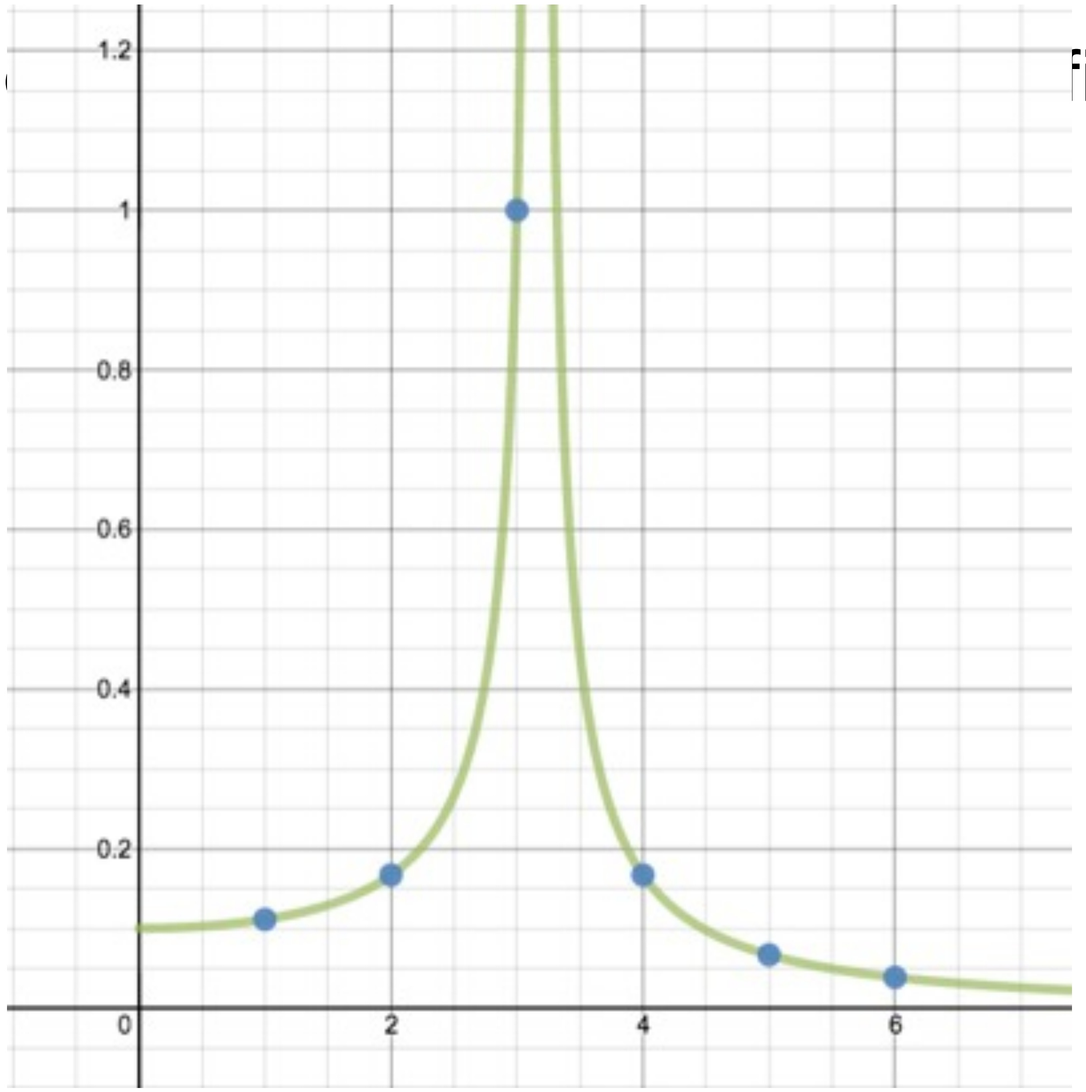

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- Replace f(t) by a sum of trig functions, if possible:

$$ay'' + by' + cy = f(t)$$

- For any f(t), how do we find the best choice of A_0, a_n, b_n?
- This problem is closely related to an analogous vector problem: how do you choose c_1, c_2 so that $w = c_1 v_1 + c_2 v_2$?
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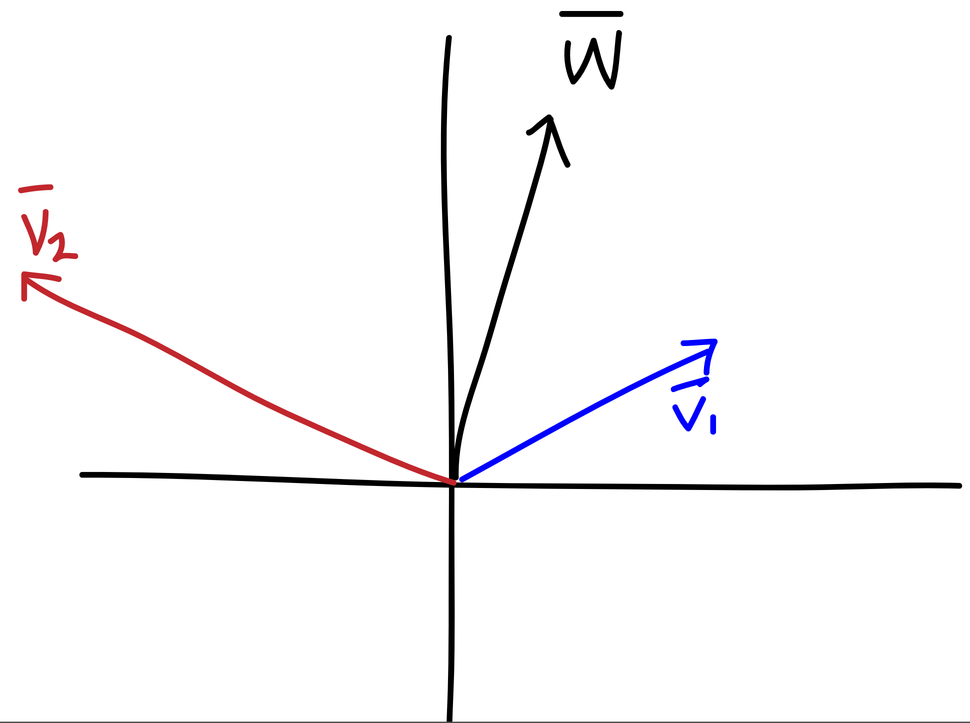

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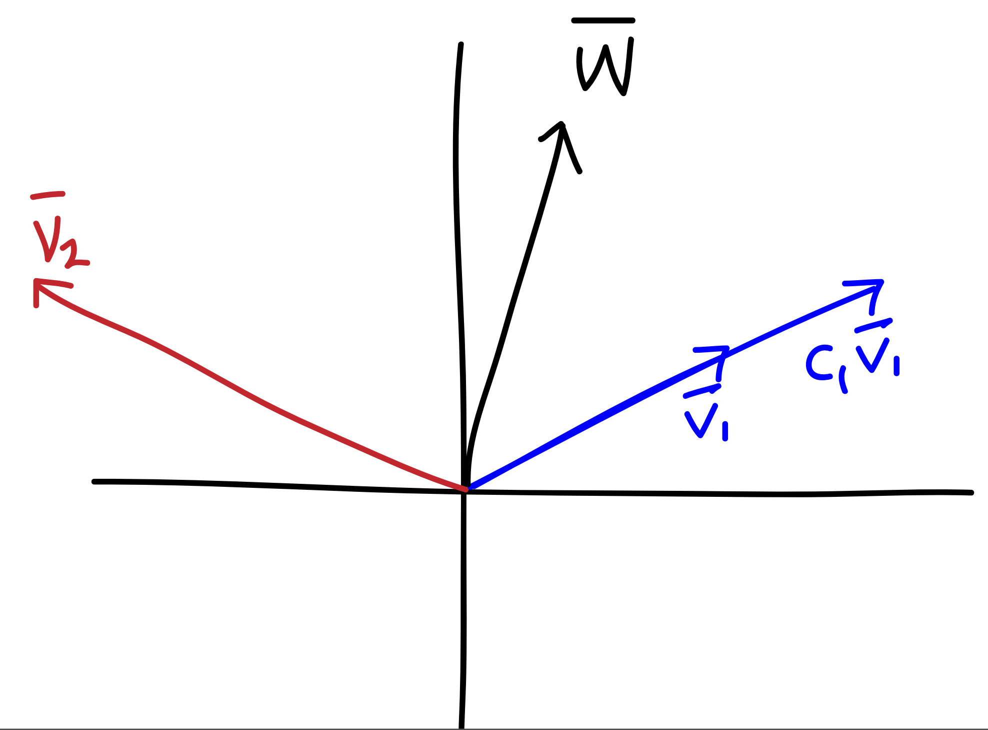

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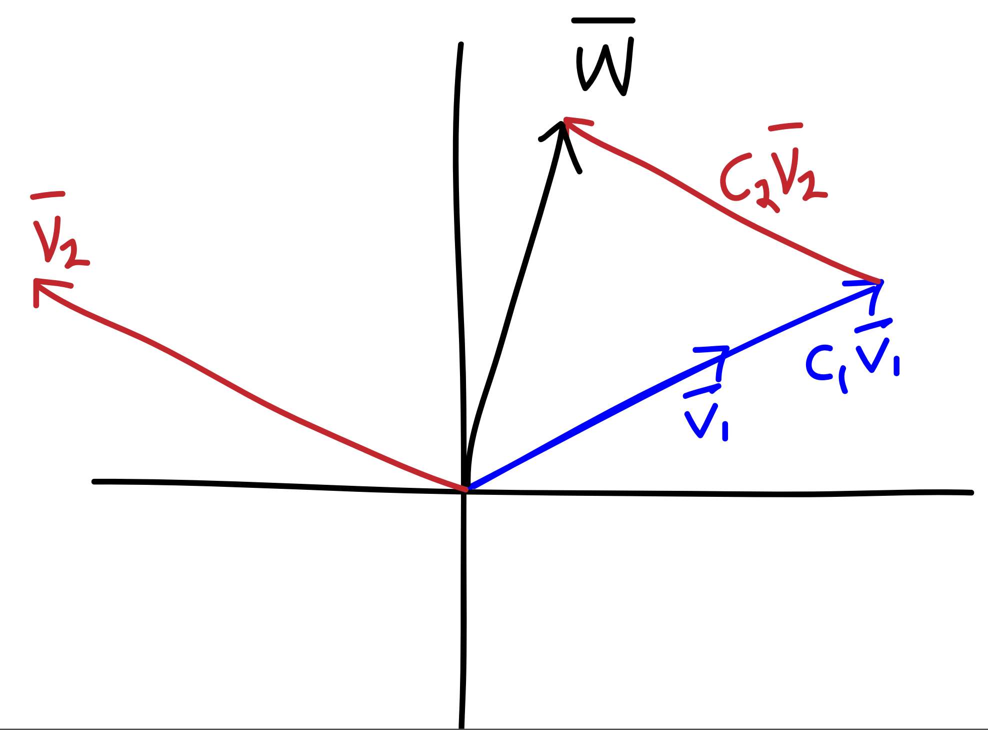

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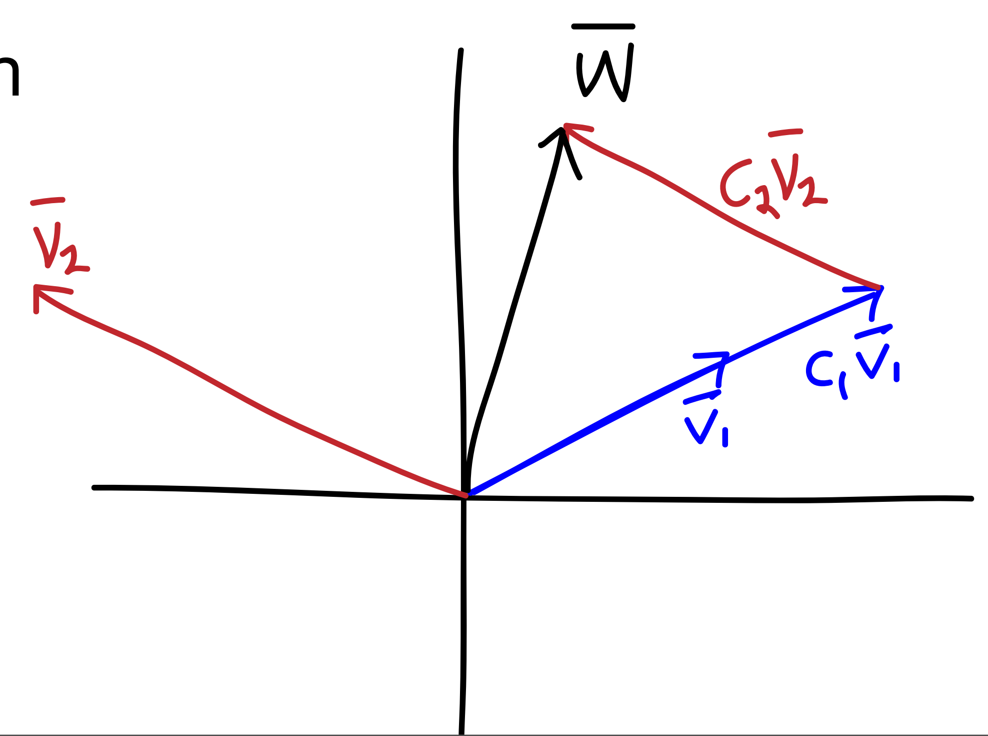

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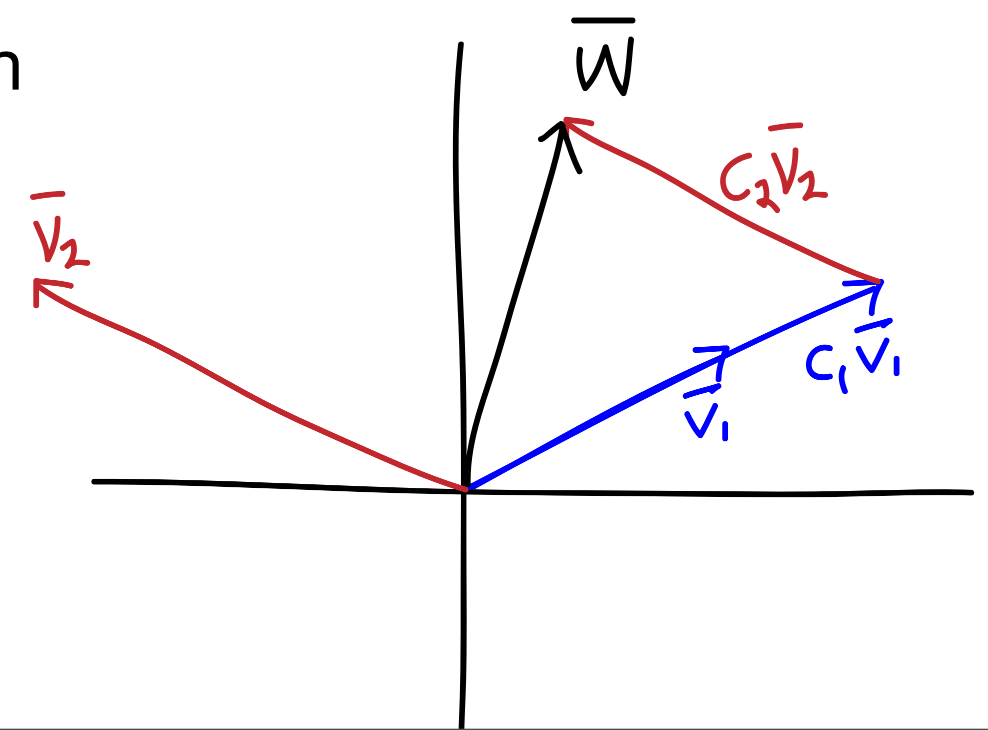

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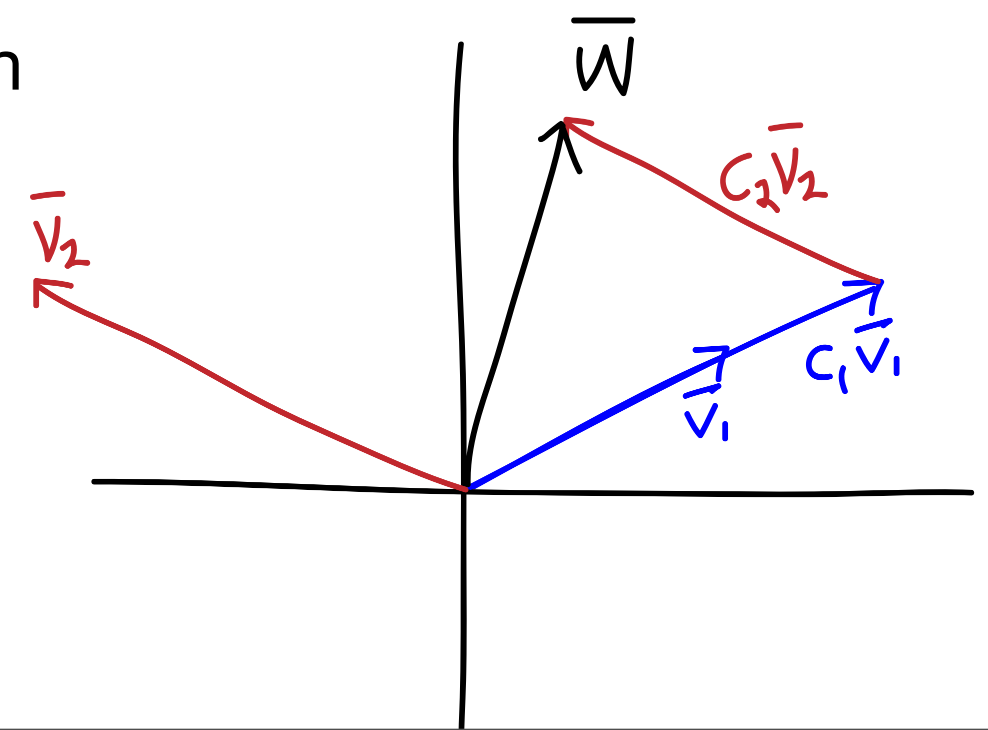

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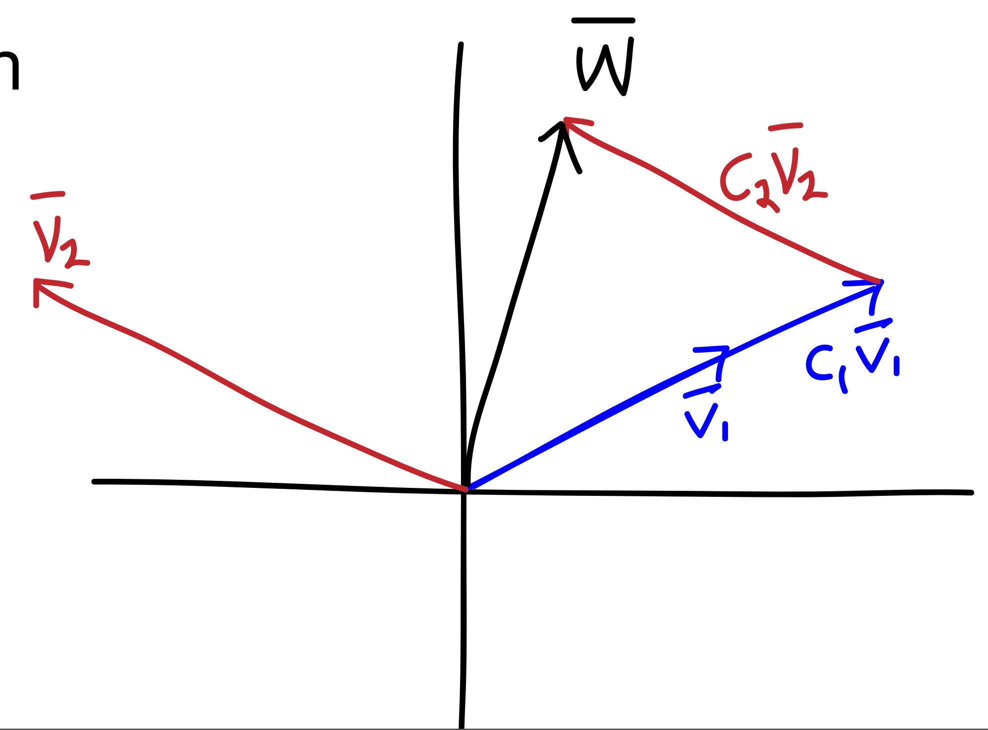

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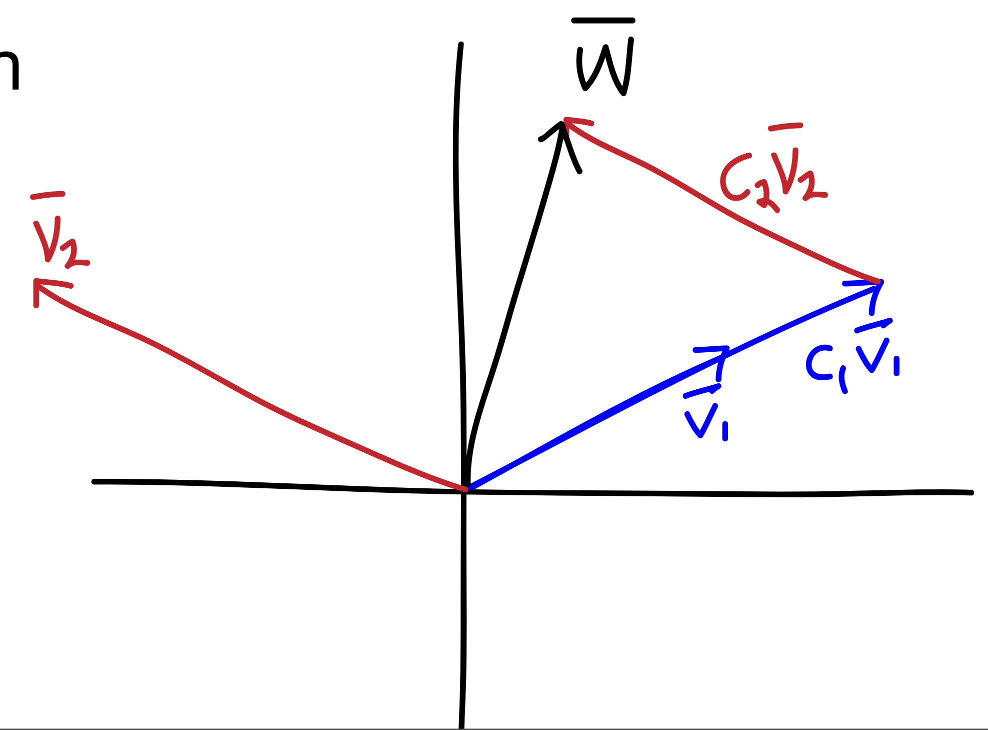

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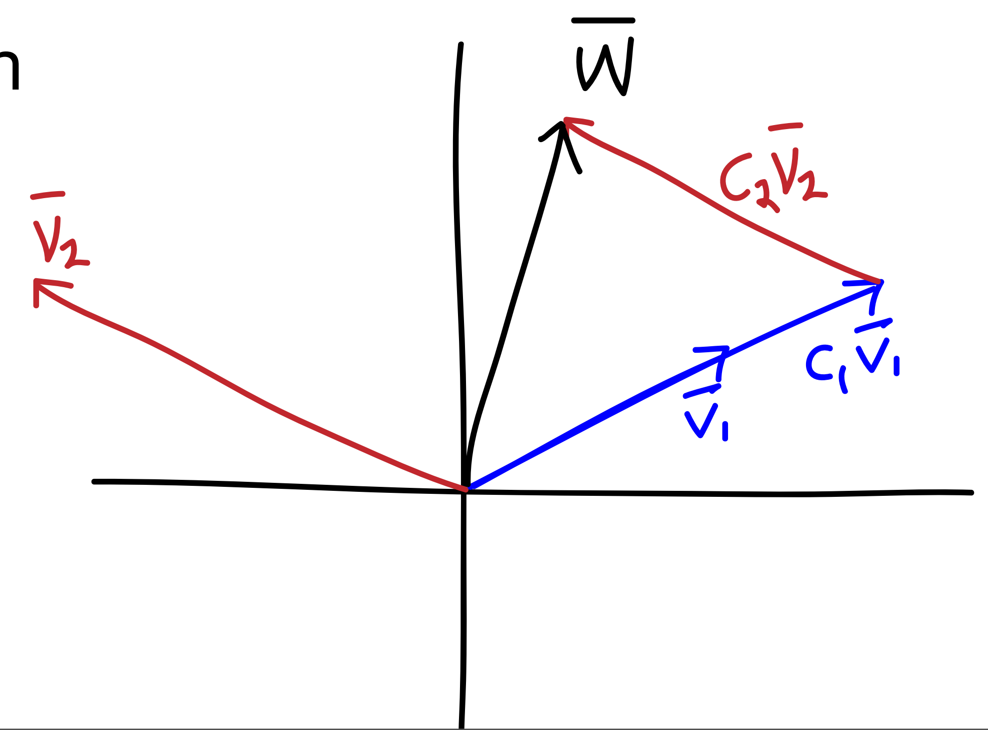

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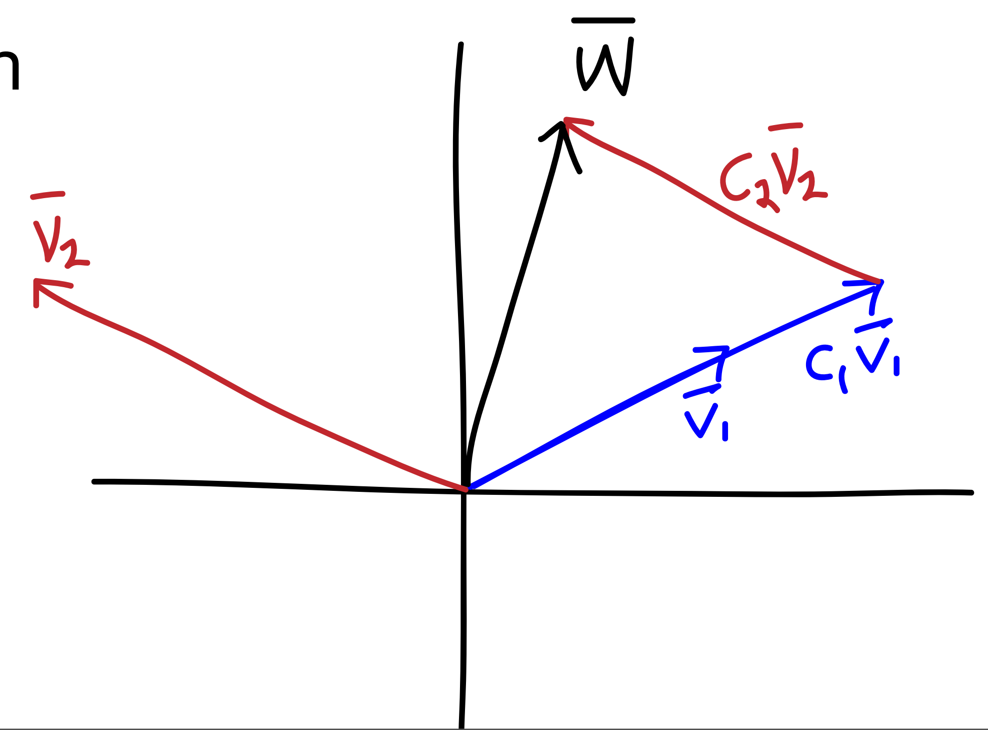

Fourier series

- For functions, define dot product as

$$g(x) \circ h(x) = \int_{\text{one period}} g(x)h(x)\ dx$$

- just like for vectors but indexed over all x instead of 1, 2, 3:

$$\mathbf{v} \circ \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

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- Back to our ODE, what do we choose for the w_n if f(t) has period T? Keep in mind that we want all the functions involved to have period T.

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Draw graphs on doc cam.

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For FS in general, people use $w_n = \pi n / T$ for reasons that will make more sense once we cover PDEs.

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Finding the Fourier series coefficients

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★(A) 0

(B) π

(C) π/2

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Integral of a trig function over one period = 0

$$v_0 \circ w_n = 0$$

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Integral of an odd function over a symmetric interval = 0

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2

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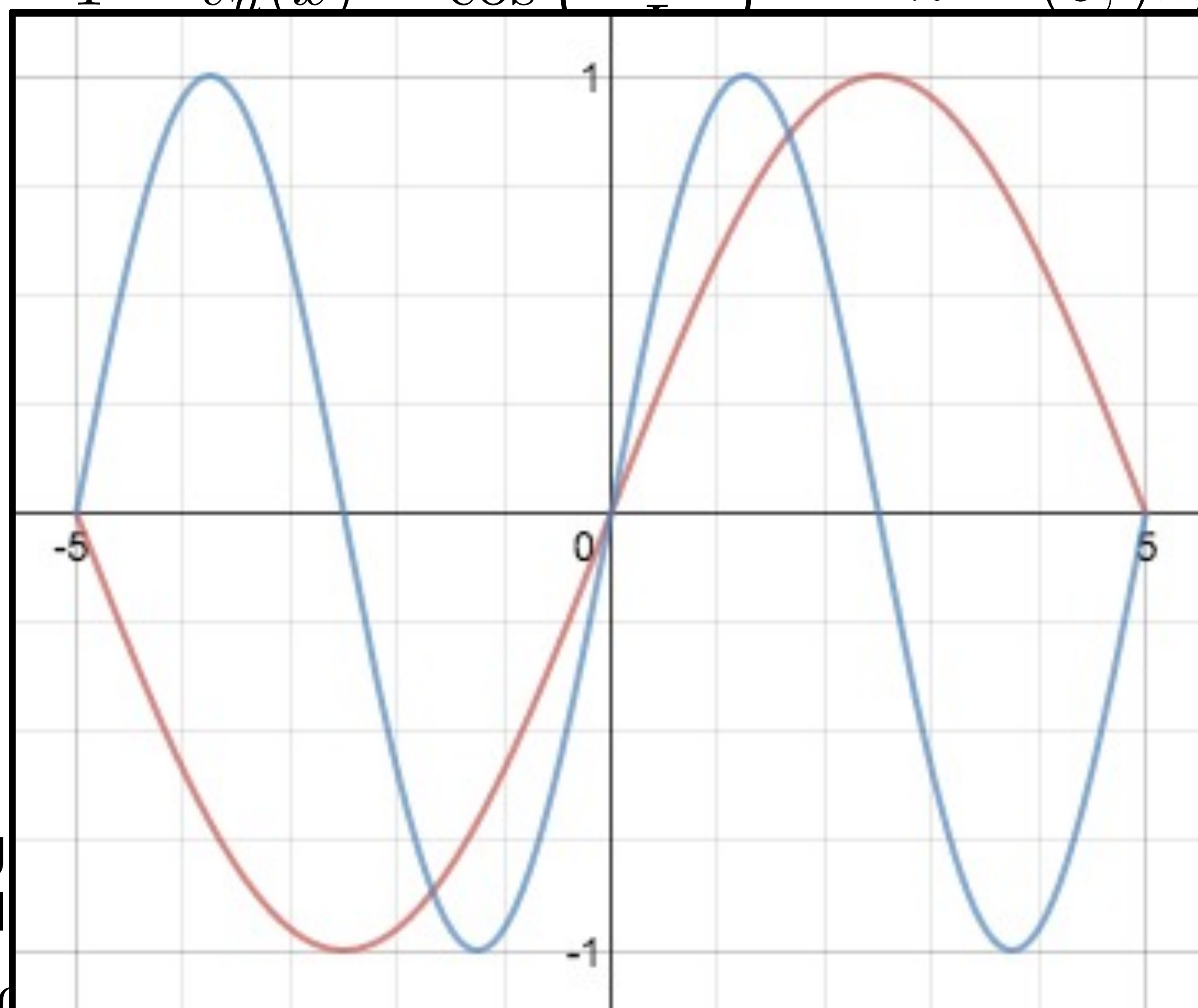

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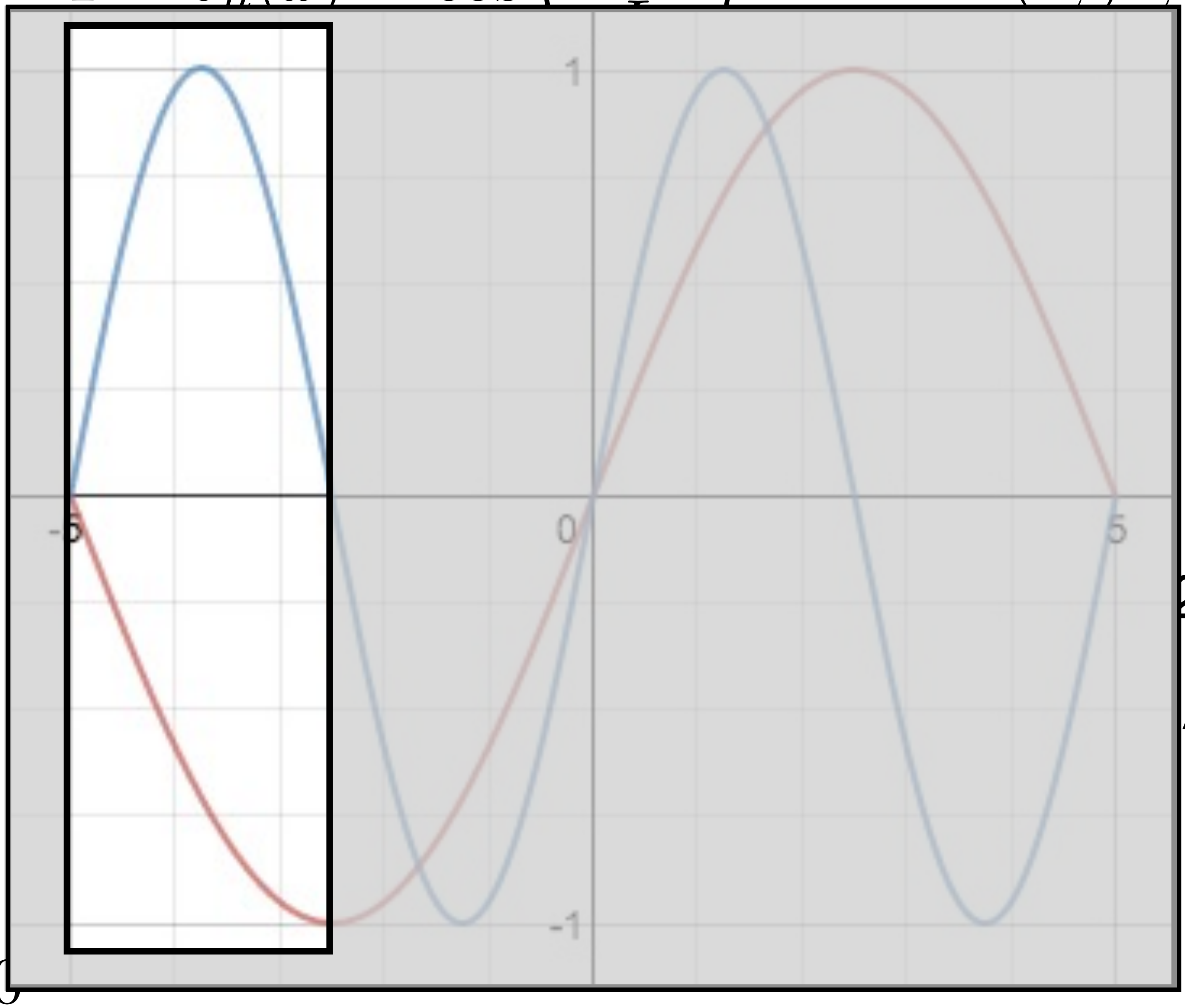

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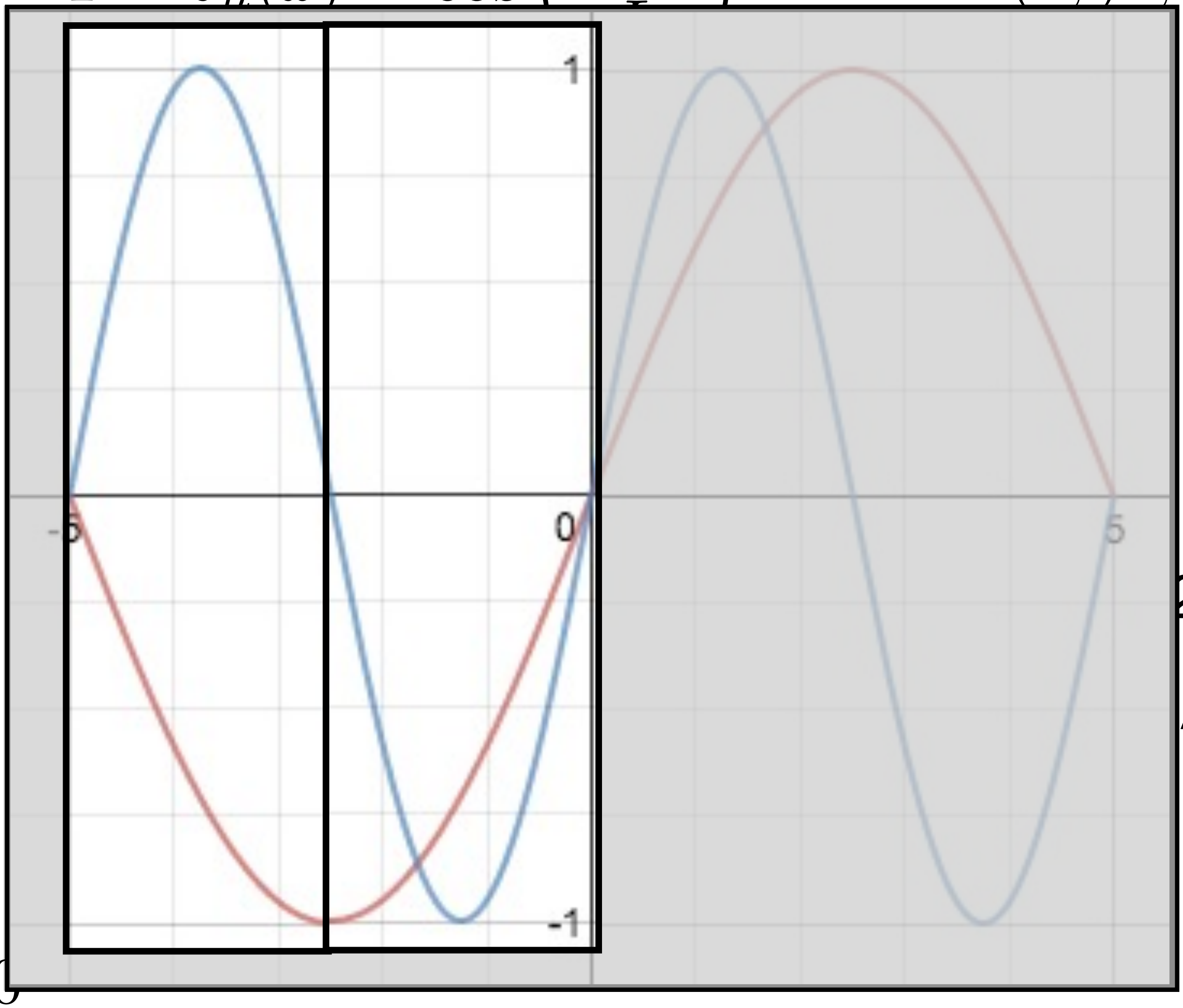

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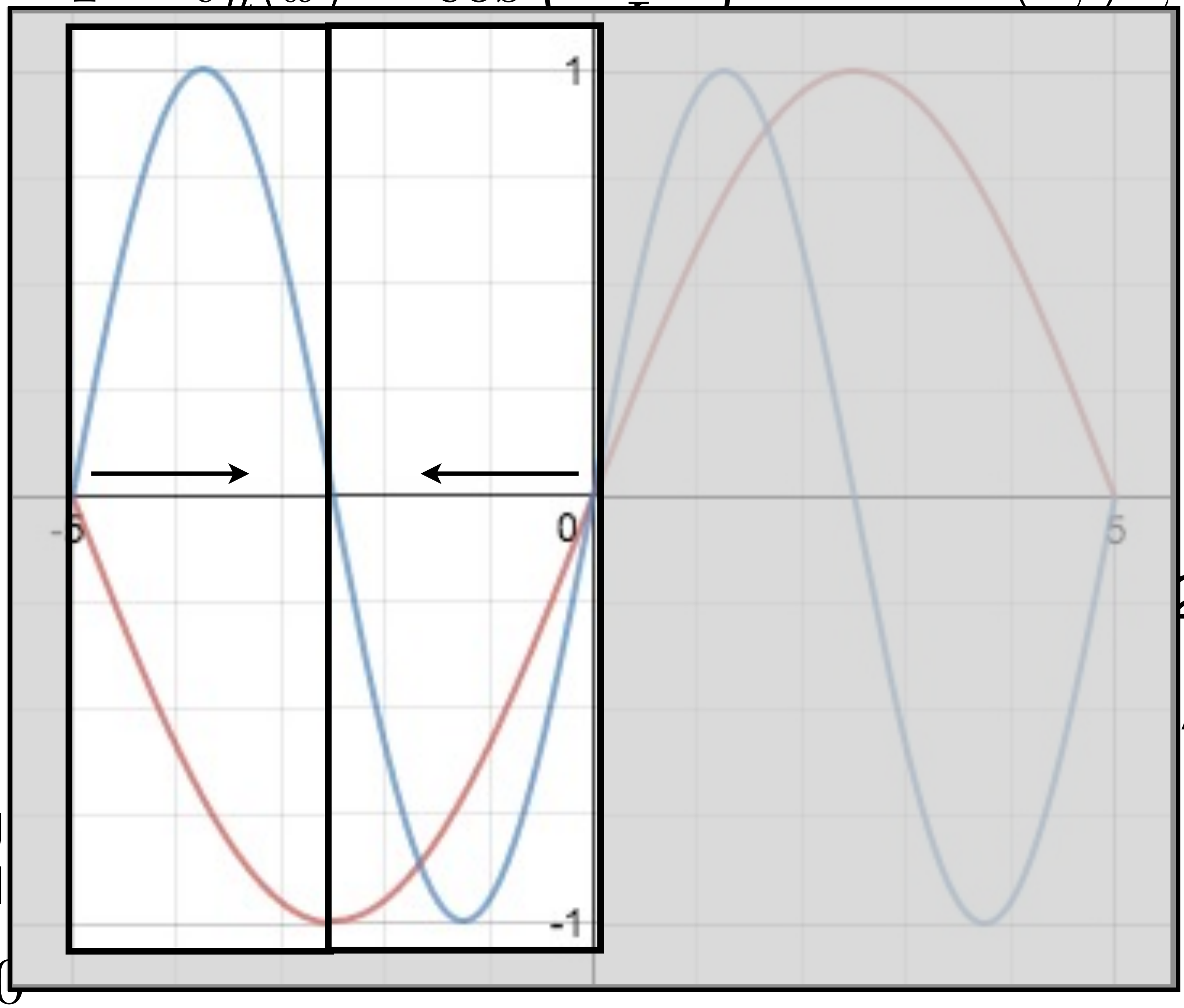

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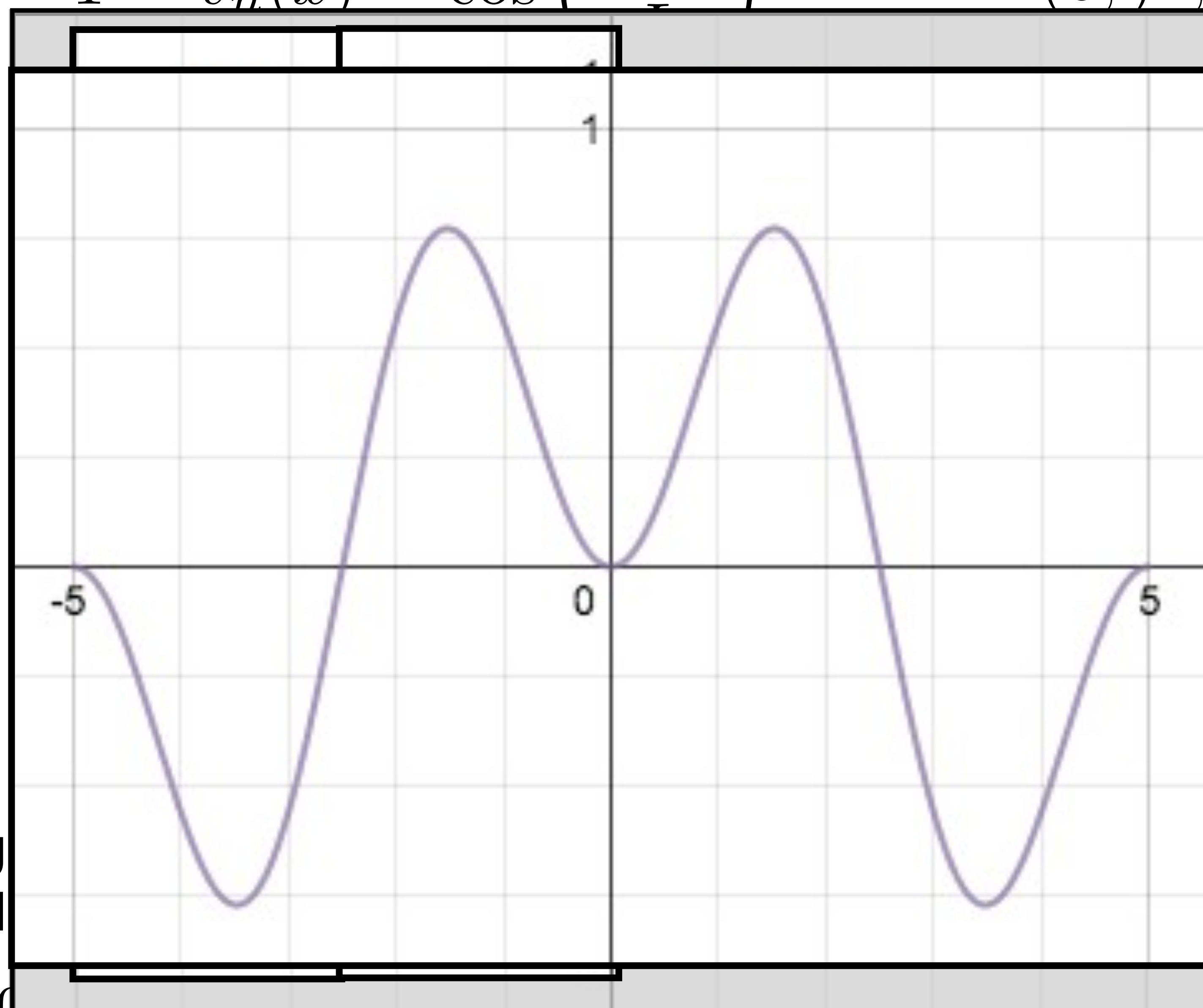

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$$v_n \circ v_n = \int_{-L}^{L} \cos^2\left(\frac{n\pi x}{L}\right)\ dx = L$$

$$g(x) \circ h(x) = \int_{-L}^{L} g(x)h(x)\ dx$$

Fourier series

- Defining Fourier series:

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- Define a function $f_{FS}(x)$ on the interval [-L,L] by choosing coefficients A_0, a_n and b_n and setting

$$f_{FS}(x) = A_0 + a_1 \cos\left(\frac{\pi x}{L}\right) + a_2 \cos\left(\frac{2\pi x}{L}\right) + \cdots$$
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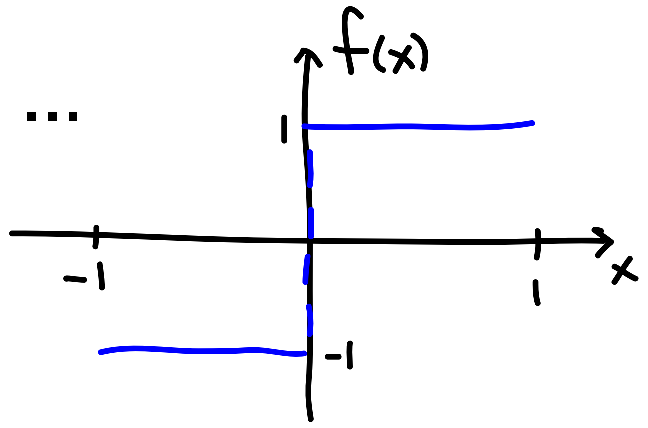
- Let's check for $f(x) = 2u_0(x)-1$ on the interval [-1,1] ...



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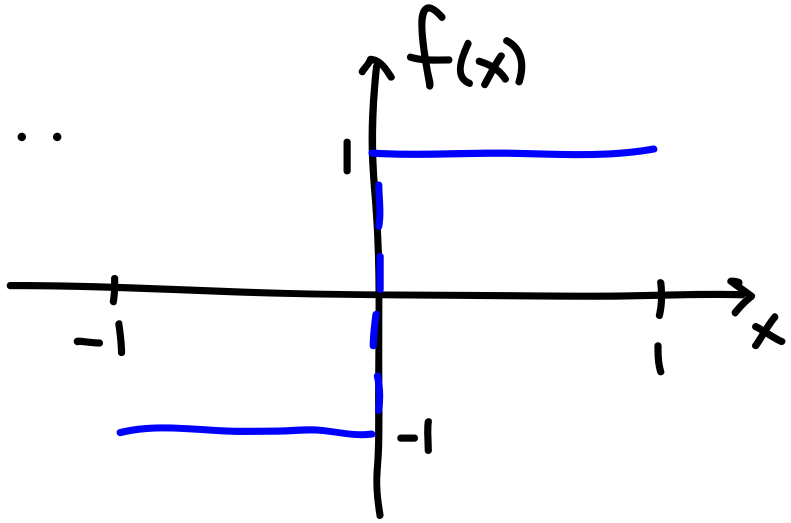

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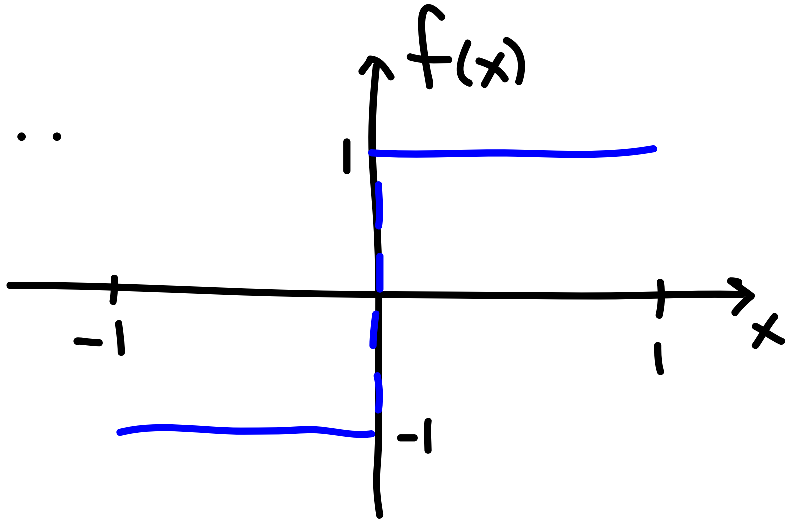

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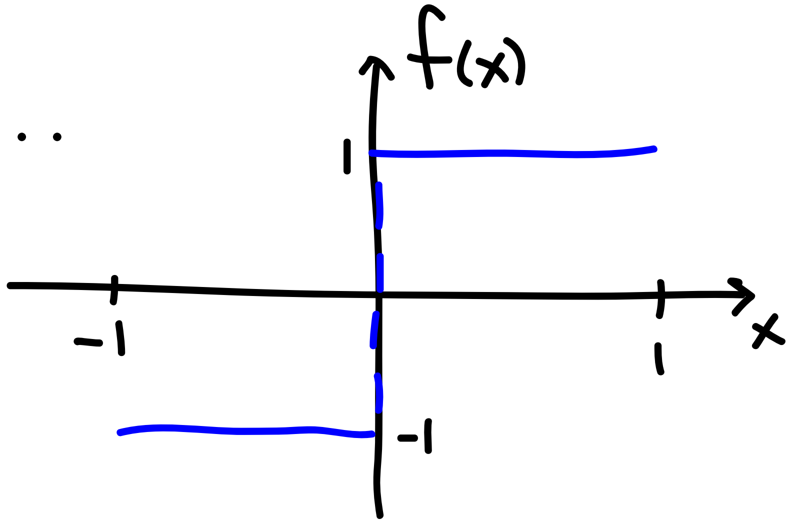

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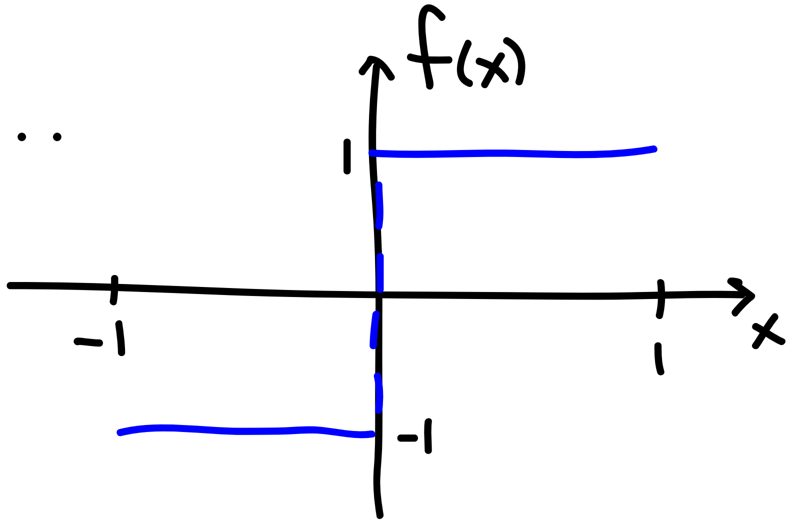

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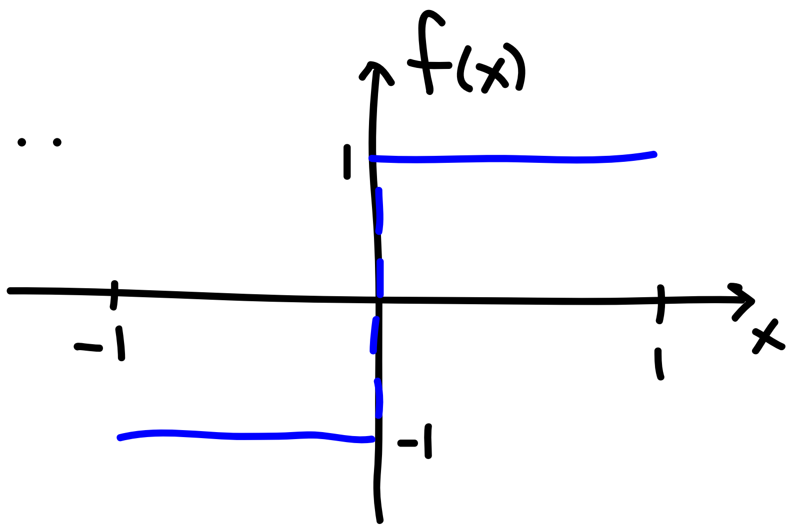

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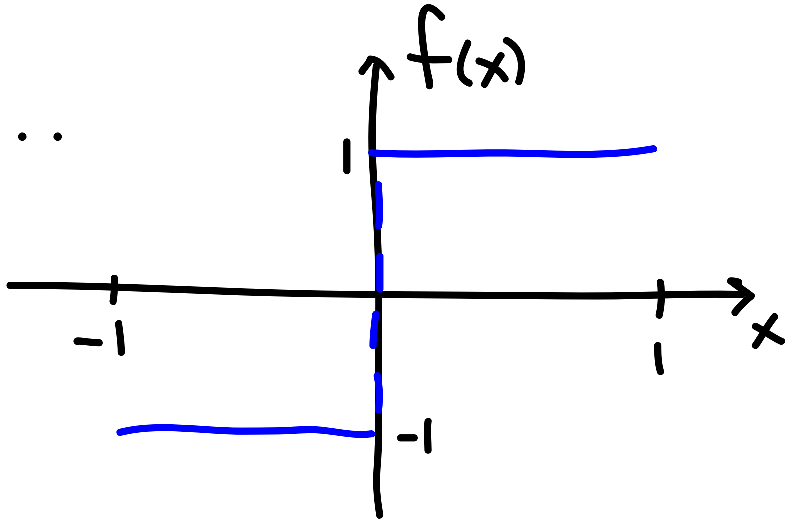

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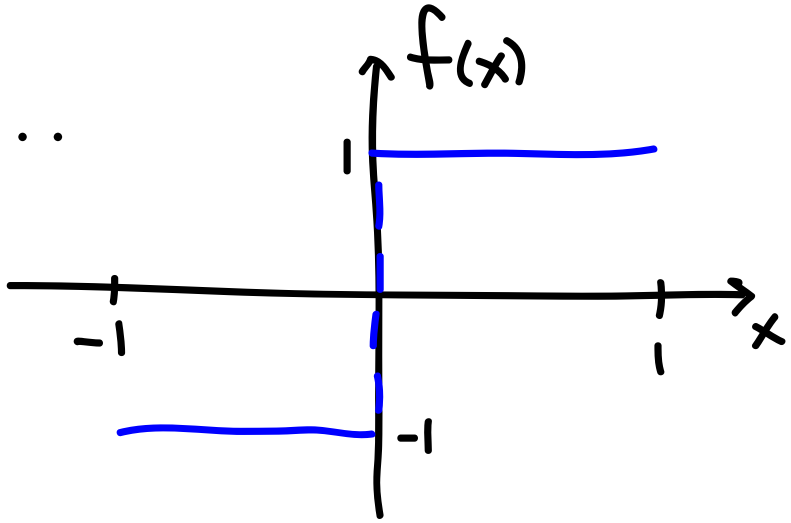

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A_0 is the average value of f(x)!

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