Today

- Shapes of solutions for distinct eigenvalues case.

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to $\mathbf{x}'=A\mathbf{x}$ is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**'=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?
- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$
$$x_2' = 4x_1 + x_2 \qquad x_2(0) = -12$$

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**'=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?

- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$

$$x_2' = 4x_1 + x_2 \qquad x_2(0) = -12$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**'=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t} \mathbf{v_1} + C_2 e^{\lambda_2 t} \mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?

- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$

$$x_2' = 4x_1 + x_2 \qquad x_2(0) = -12$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**’=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?
- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$

$$x_2' = 4x_1 + x_2 \qquad x_2(0) = -12$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**'=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?

- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$

$$x_2' = 4x_1 + x_2 \qquad x_2(0) = -12$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \ C_2 = 0$$

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**'=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?
- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$

$$x_2' = 4x_1 + x_2 \qquad x_2(0) = -12$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

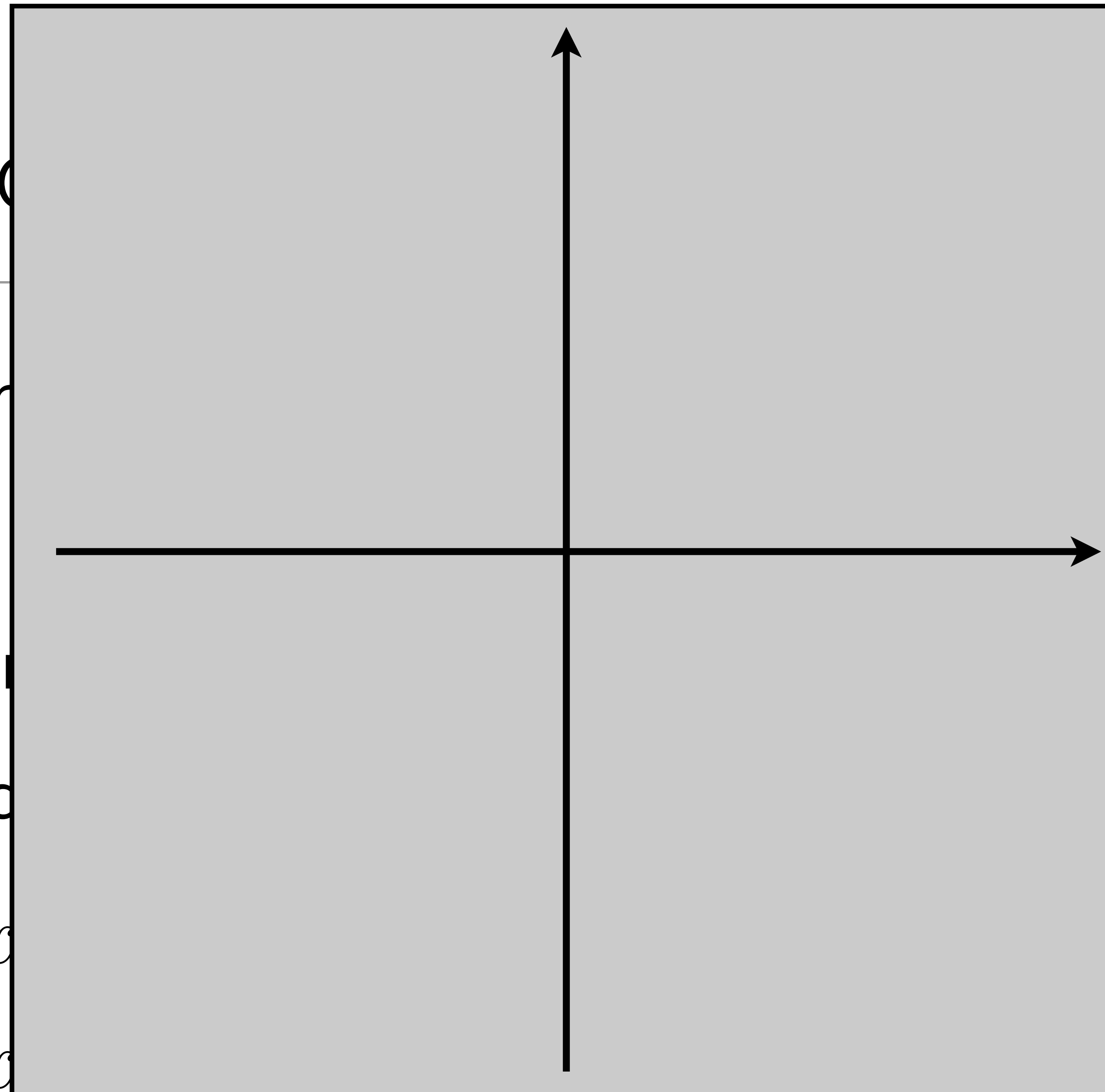
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t}\begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so … plane



- When matrix A h … olution to **x**’=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

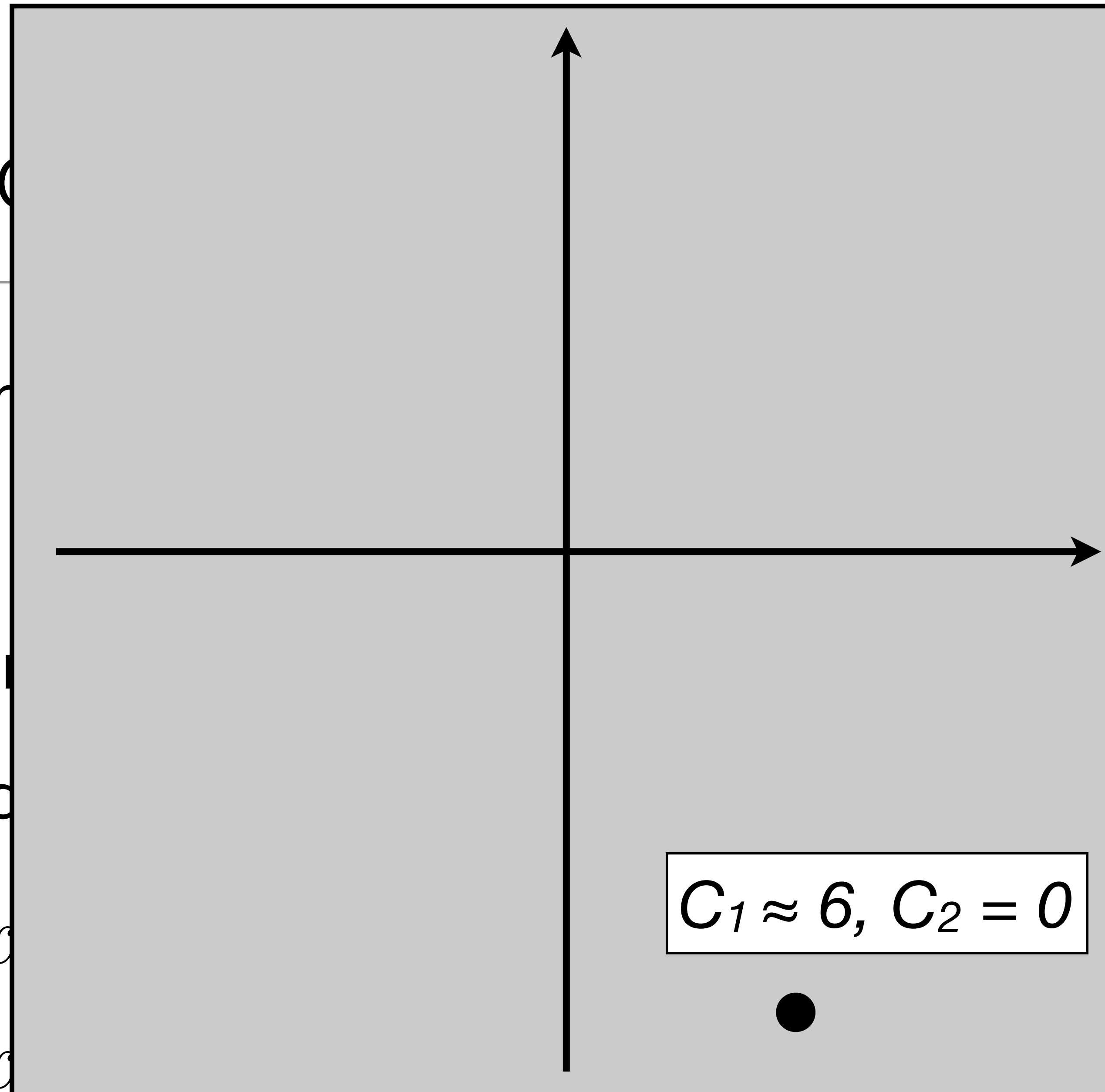
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of sc … plane



- When matrix A h … olution to **x**'=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

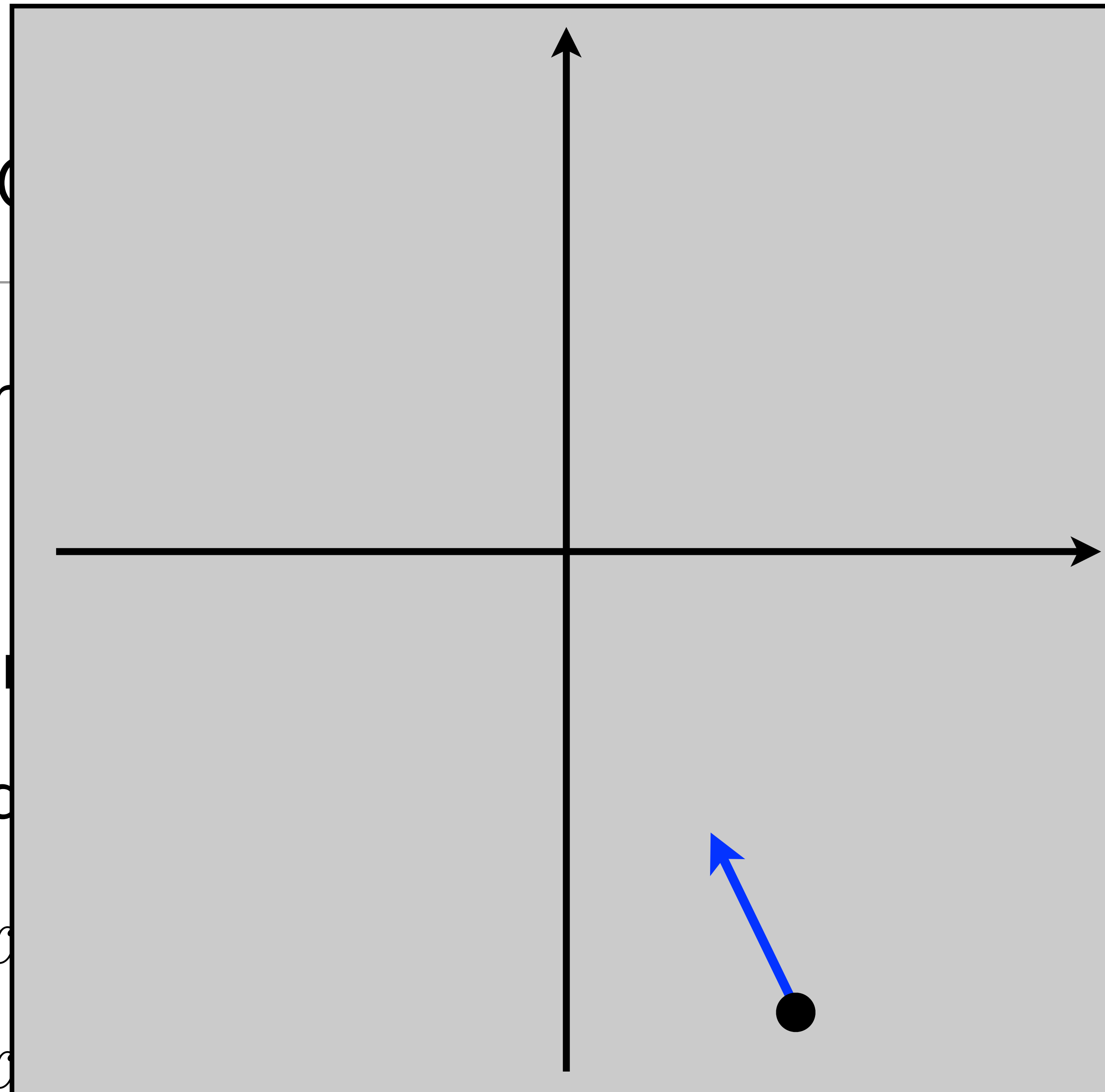
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6,\ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of s plane



- When matrix A h olution to **x**'=A**x** is
- What do solutio e phase plane)?
- If the initial cond on is a straight line.
  Example:

x 6

x -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

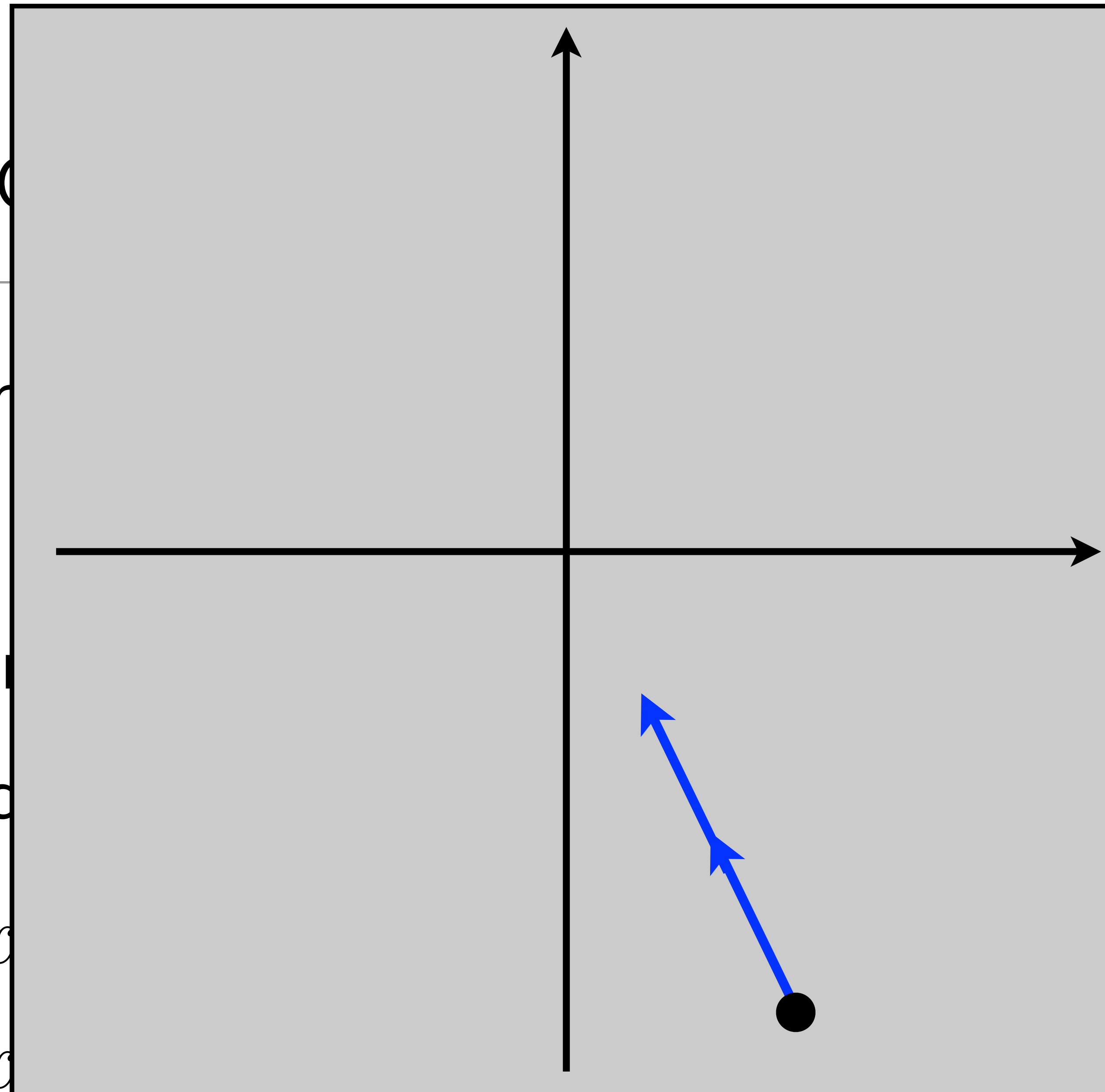
$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of s … plane

- When matrix A h … olution to **x**'=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line.
  Example:



x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

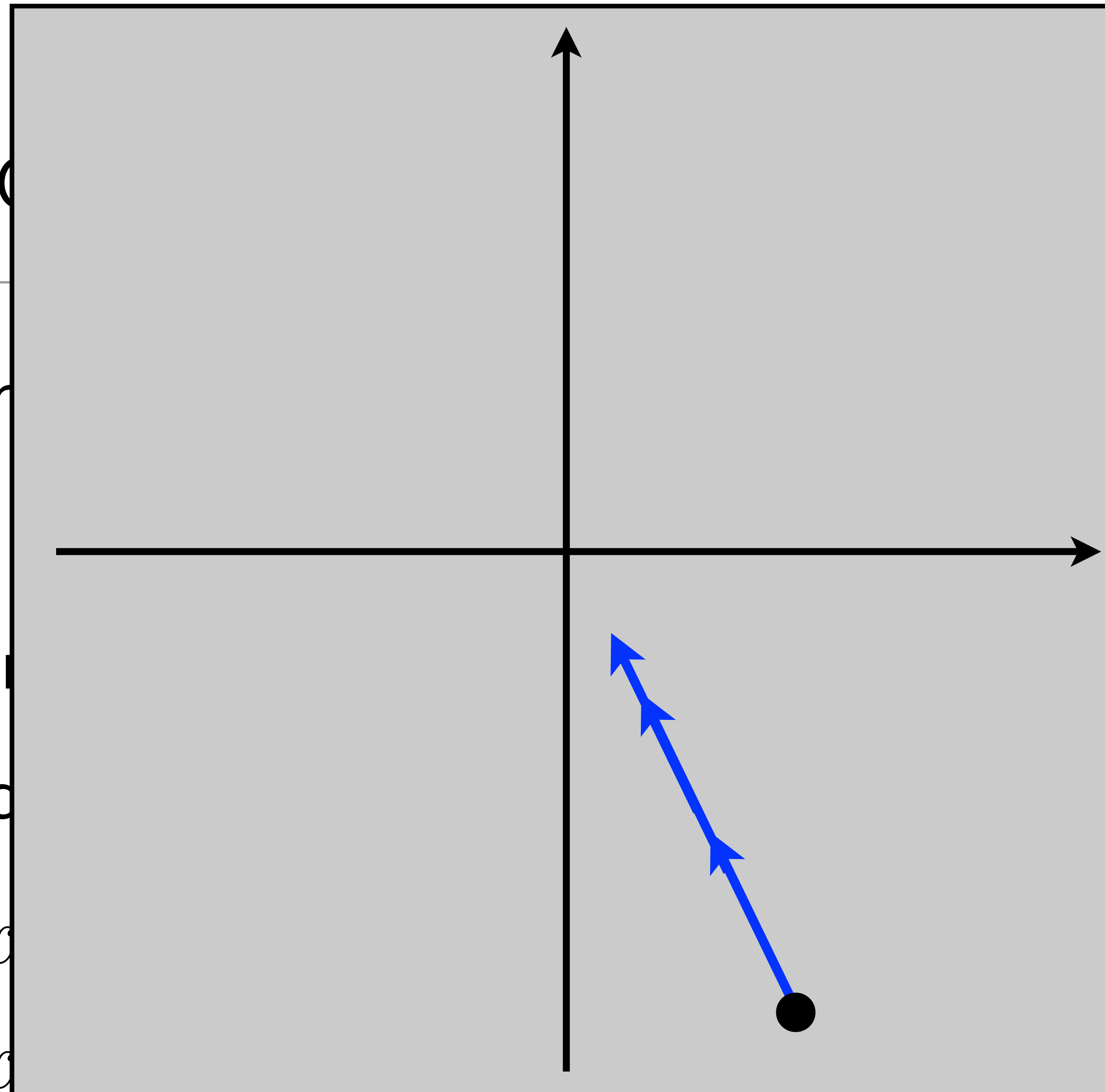
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of s... plane



- When matrix A h... olution to **x**'=A**x** is
- What do solutio... e phase plane)?
- If the initial cond... on is a straight line. Example:

x ... 6

x_2 ... -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

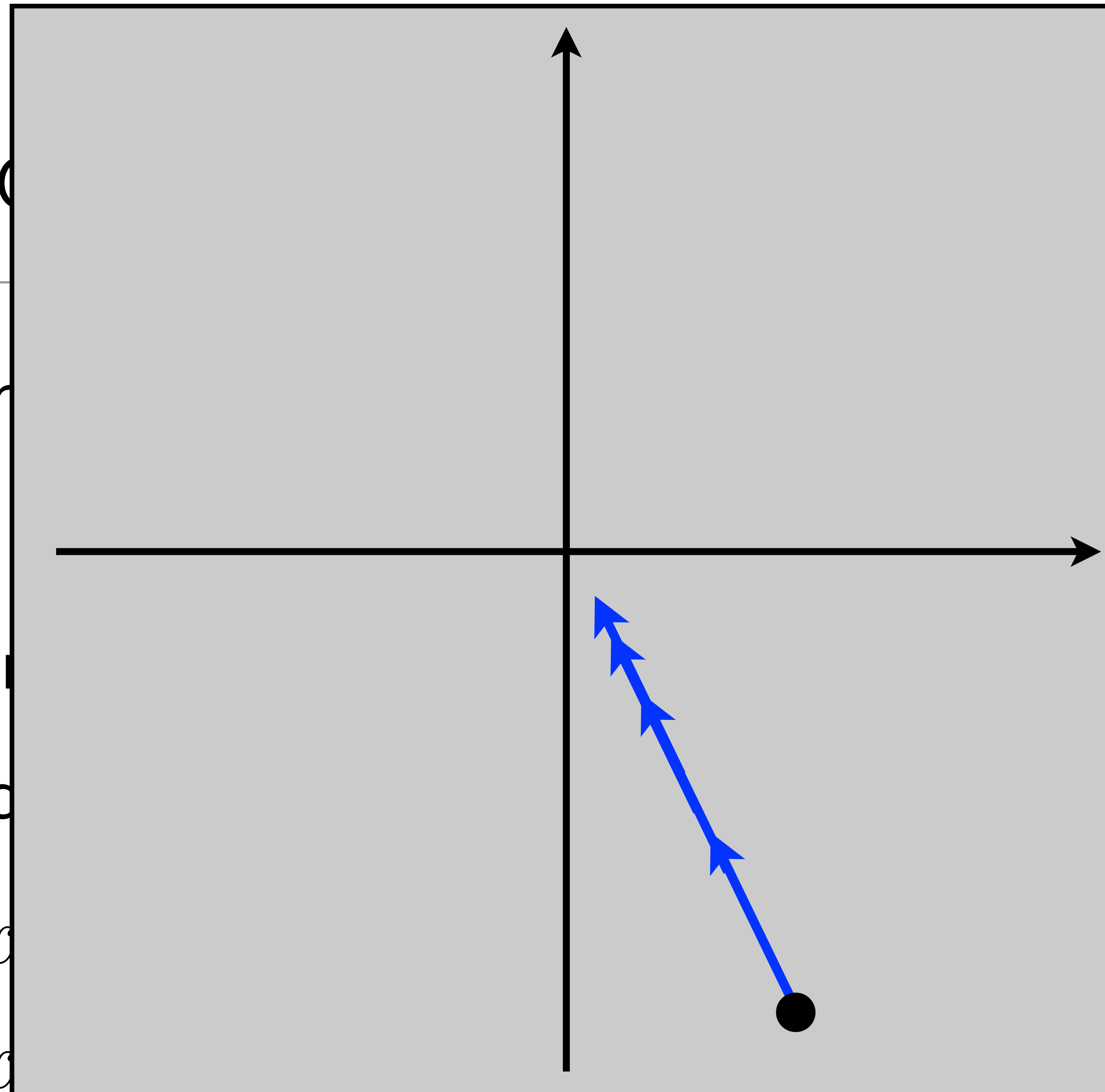
$$C_1 = 6, \;\; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so … plane

- When matrix A h … olution to **x**'=A**x** is



- What do solutio … e phase plane)?
- If the initial cond … on is a straight line.
  Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}} + C_2 e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

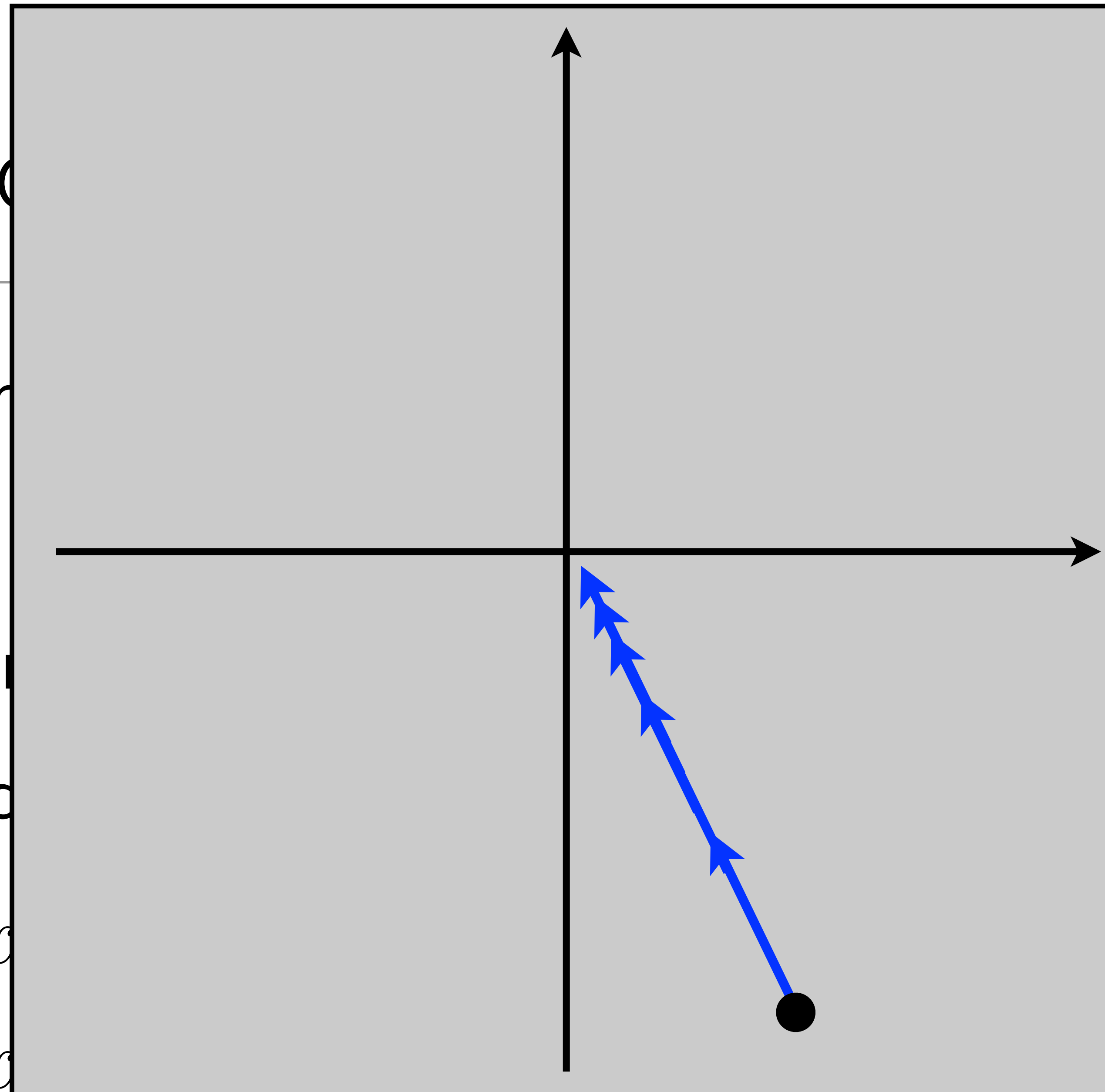
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6,\ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t}\begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of sc … plane



- When matrix A h … olution to **x**'=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

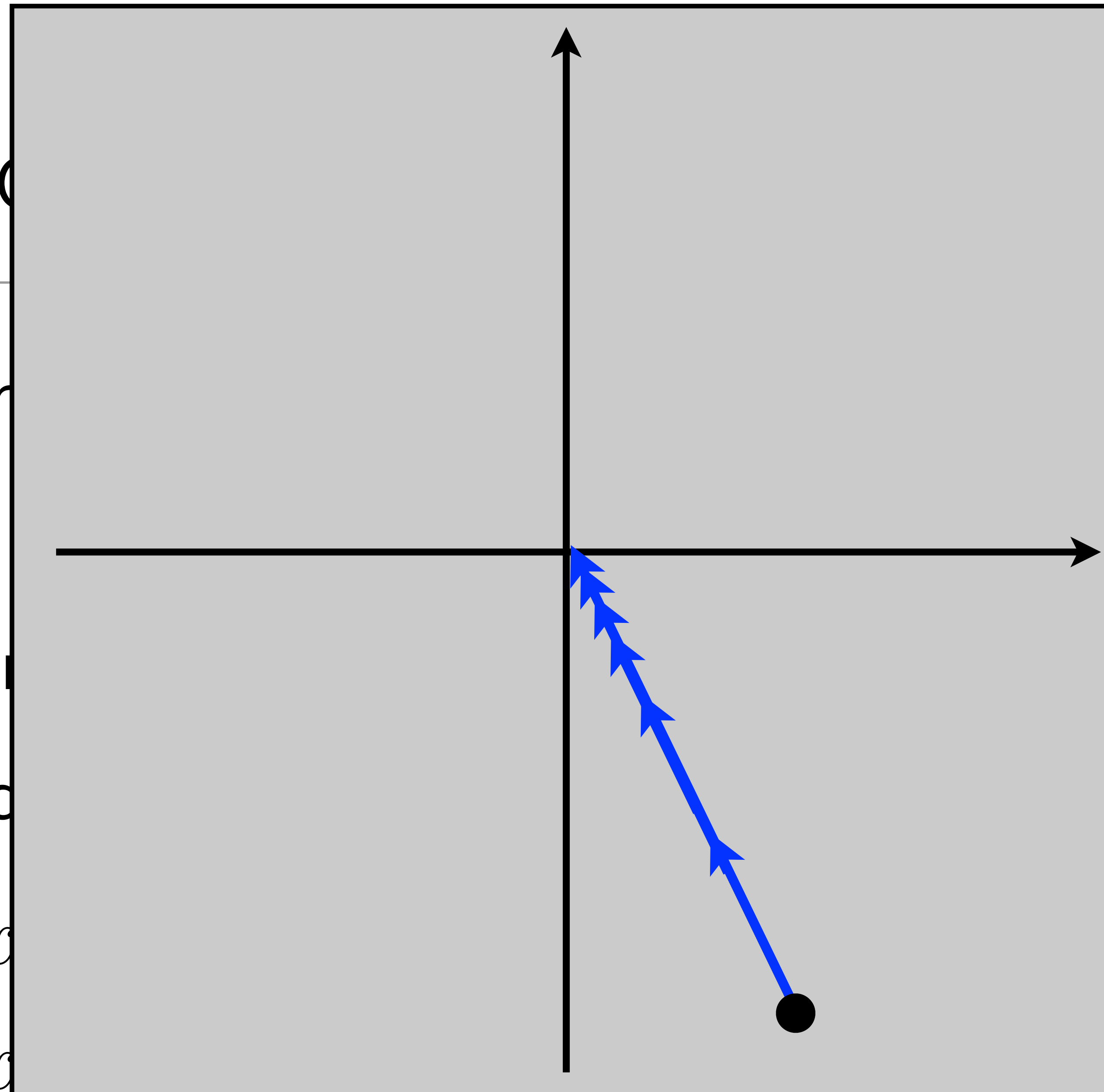
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so... plane

- When matrix A h... olution to **x**'=A**x** is
- What do solutio... e phase plane)?
- If the initial cond... on is a straight line. Example:

x ... 6

x_2 ... -12



$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

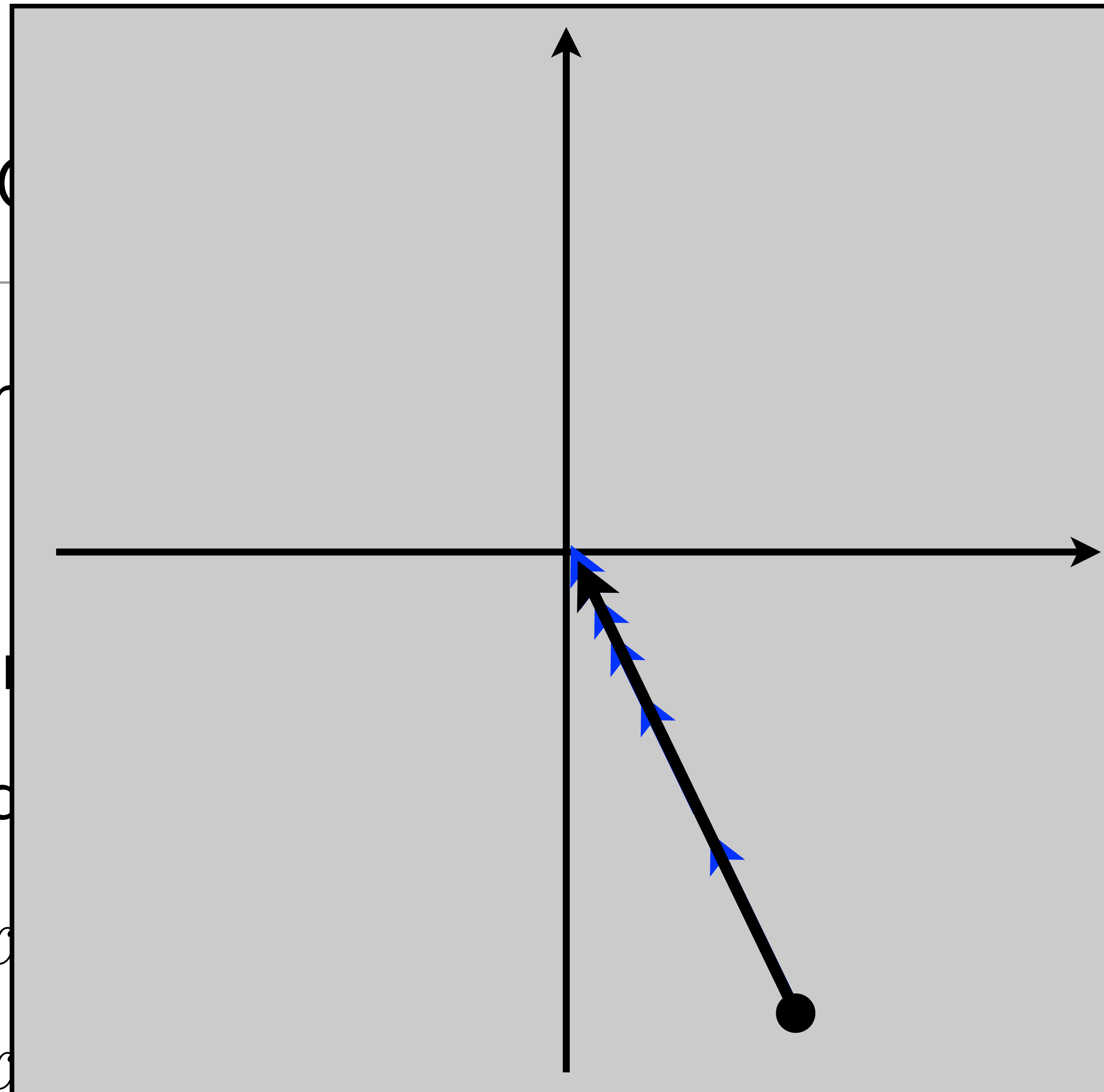
$$C_1 = 6, \;\; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane

- When matrix A h… olution to **x**'=A**x** is



- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:

x … 6

x … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

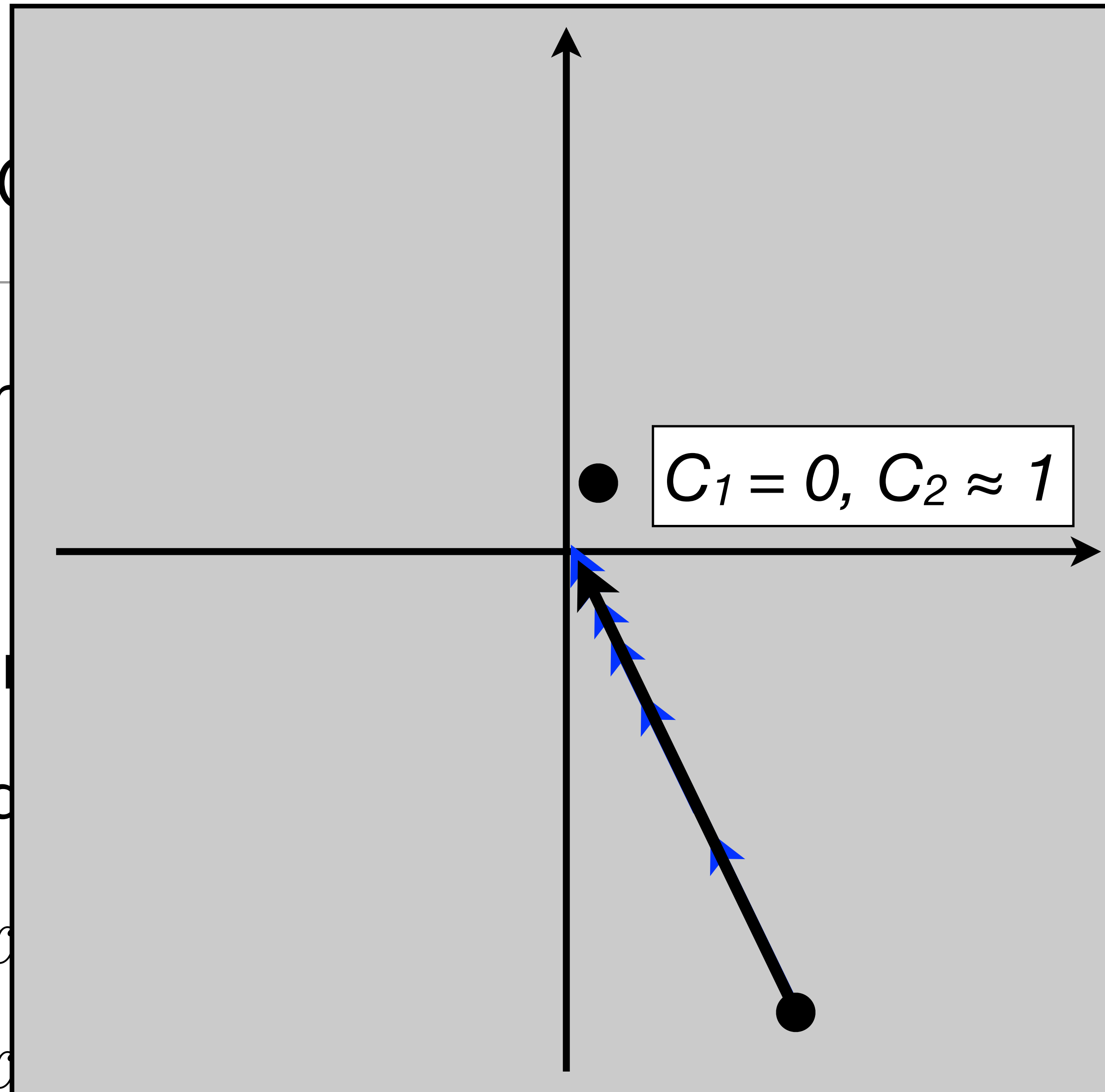
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so … plane



- When matrix A h … olution to **x**'=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

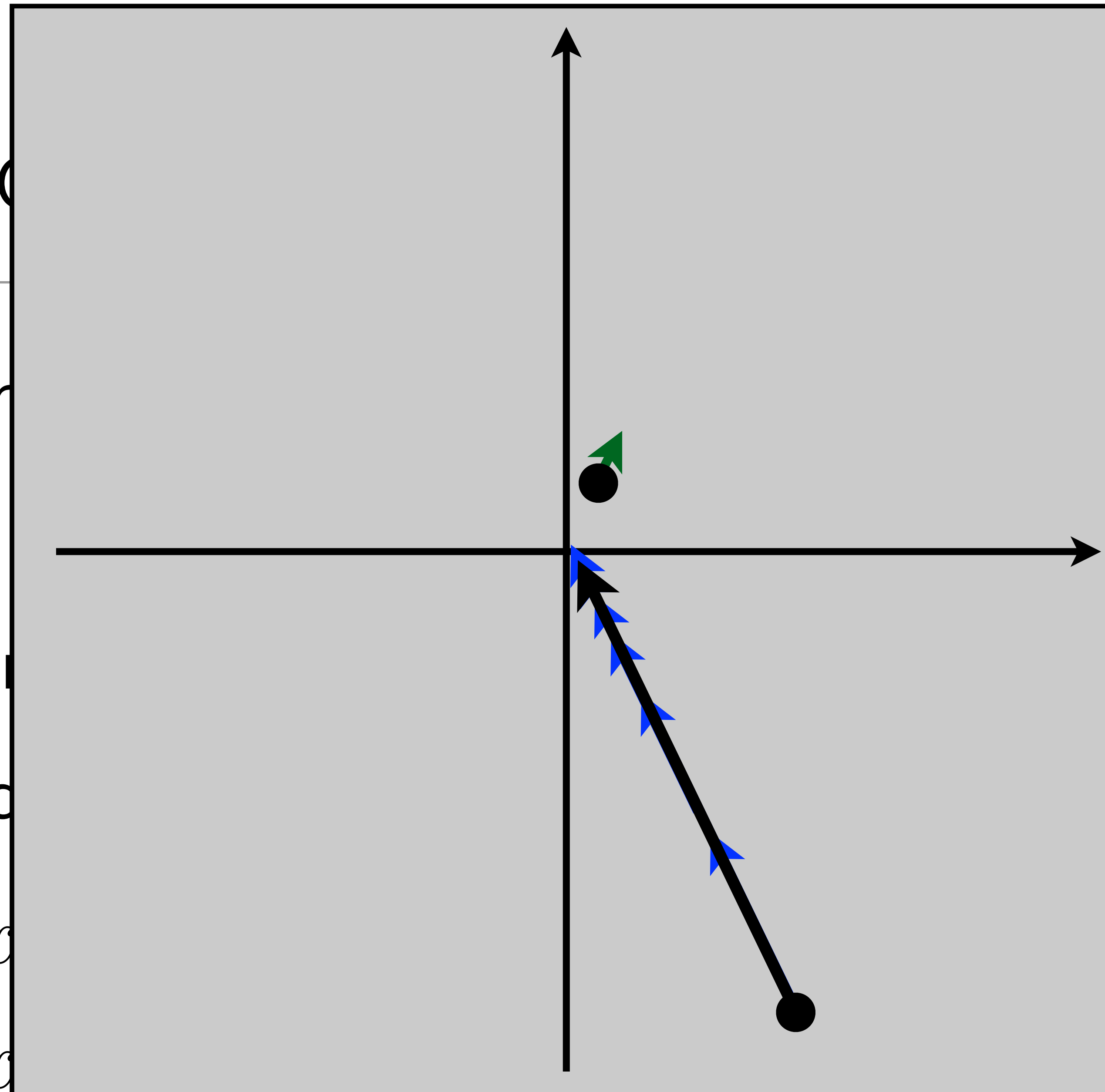
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6,\ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane



- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

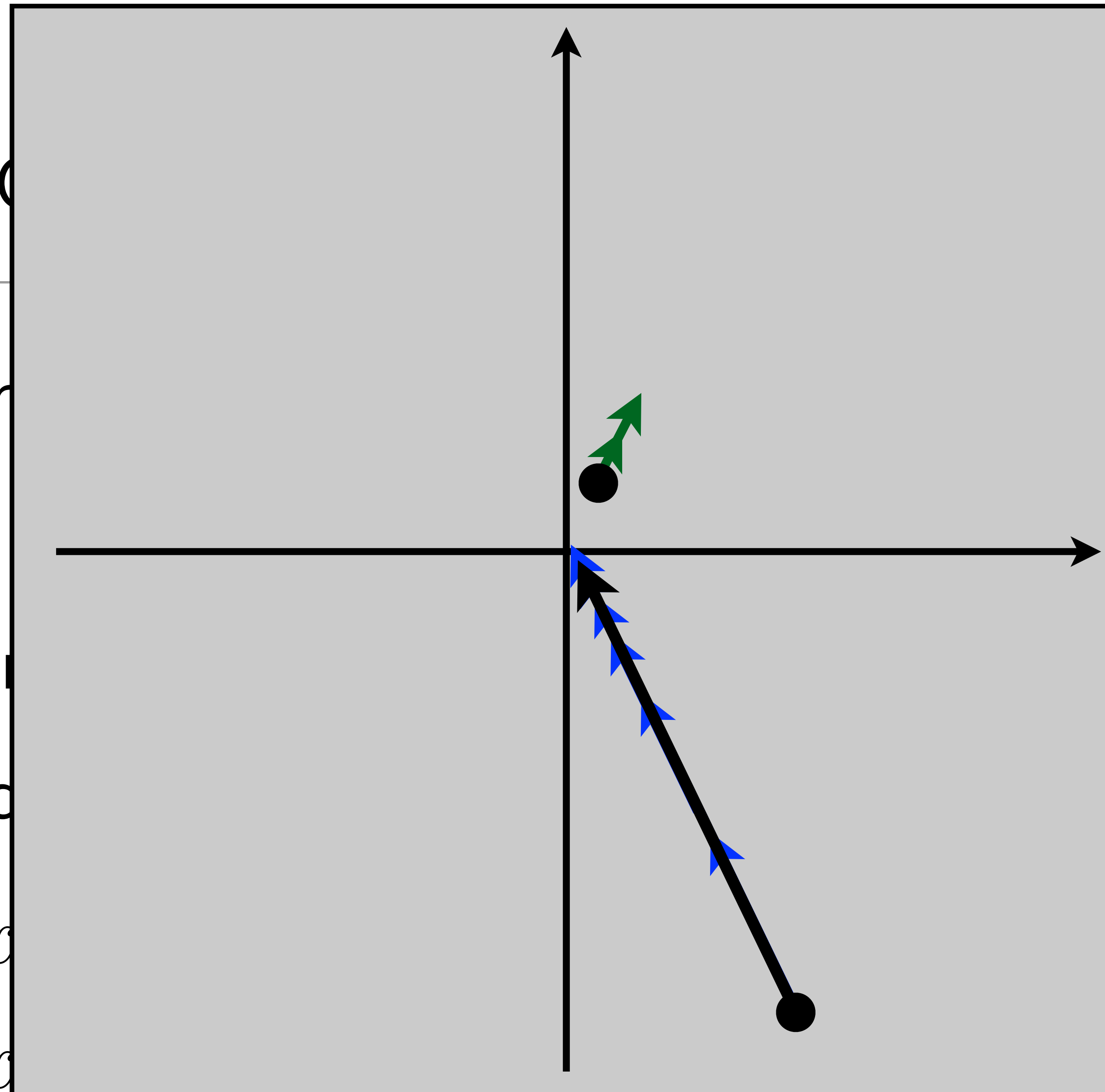
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \;\; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so … plane



- When matrix A h … olution to **x**'=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

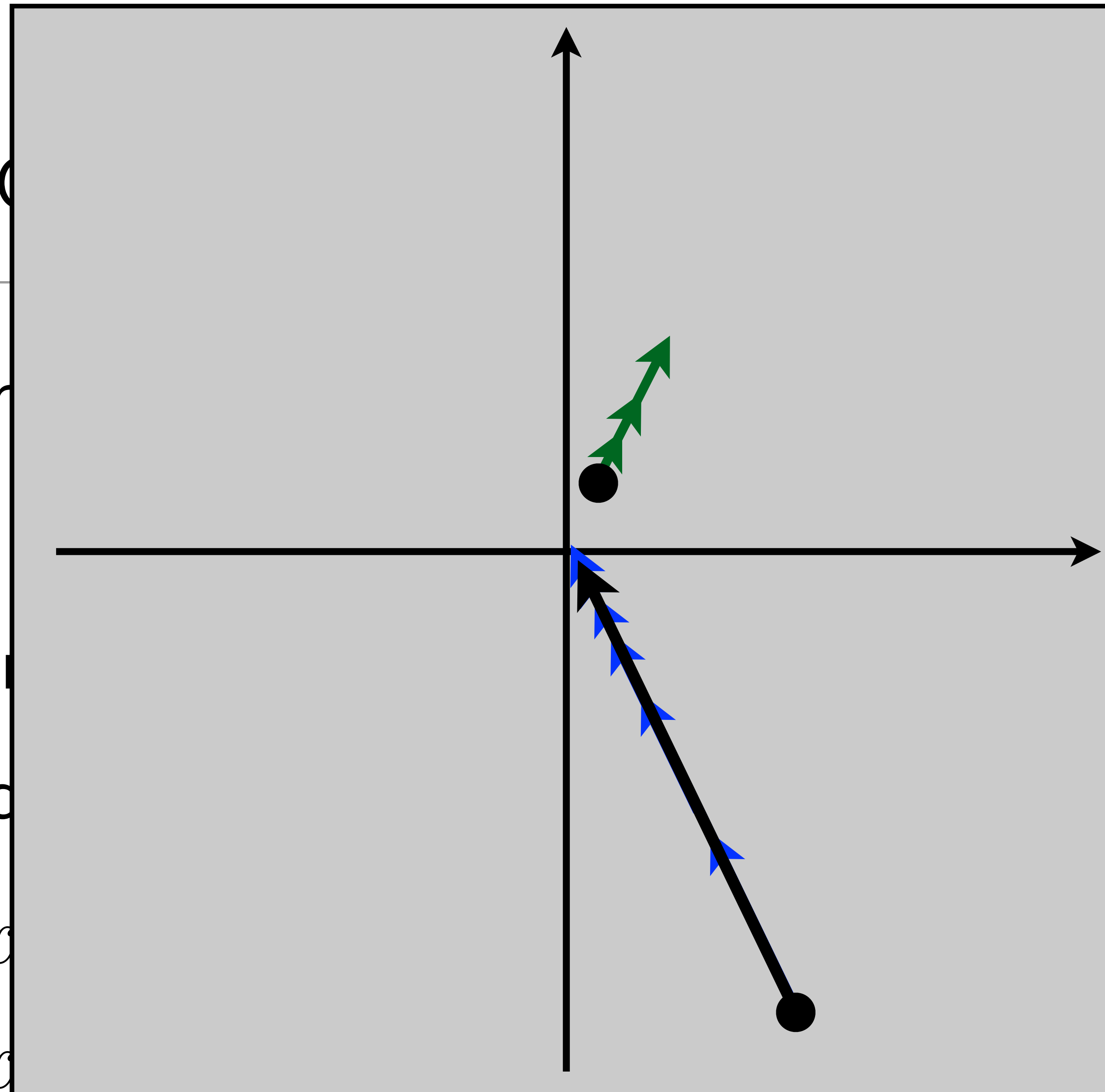
$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so ... plane

- When matrix A h ... olution to **x**'=A**x** is



- What do solutio ... e phase plane)?
- If the initial cond ... on is a straight line. Example:

x ... 6

x_2 ... -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

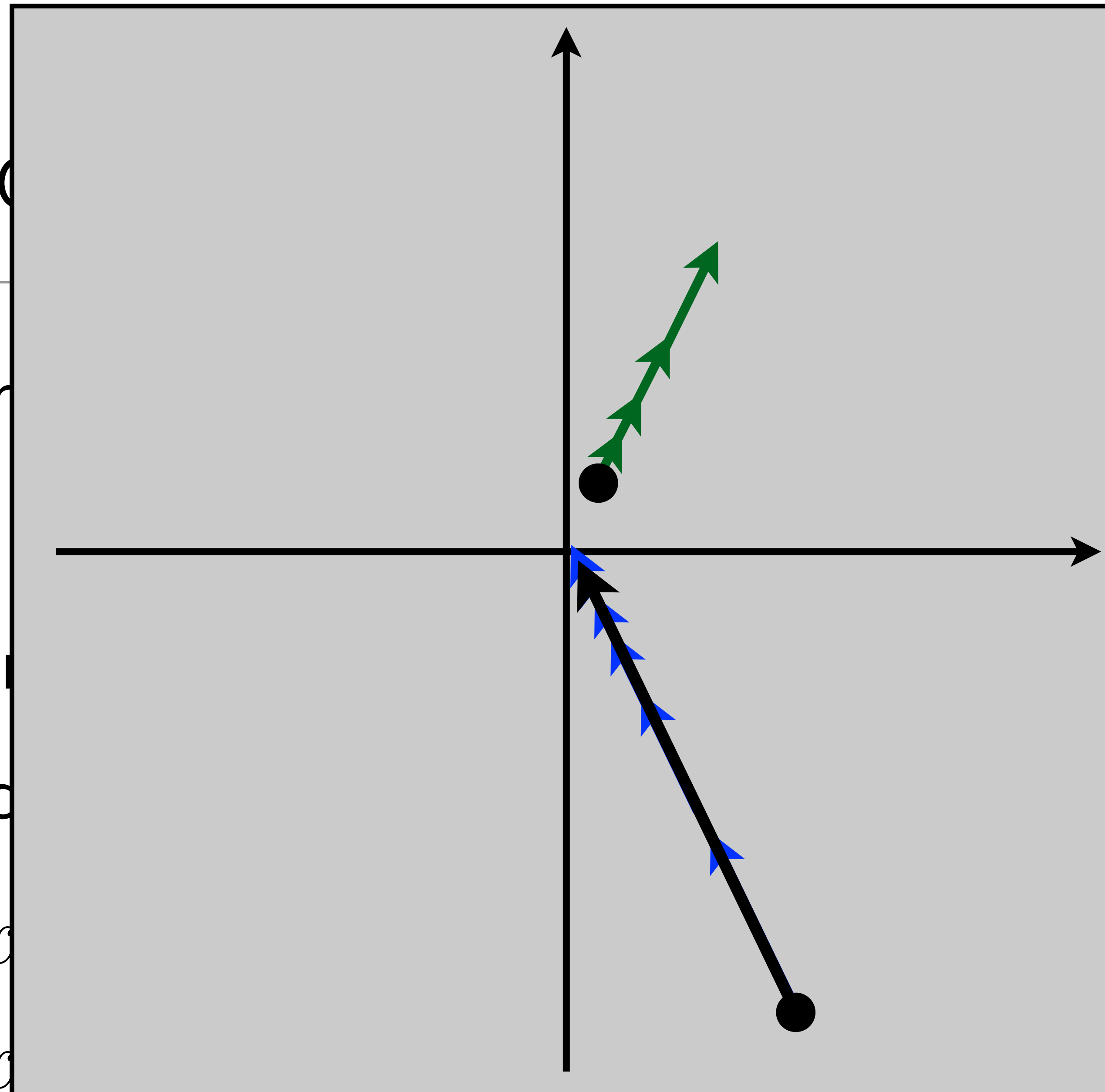
$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane

- When matrix A h… olution to **x**’=A**x** is



- What do solution… e phase plane)?
- If the initial cond… on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

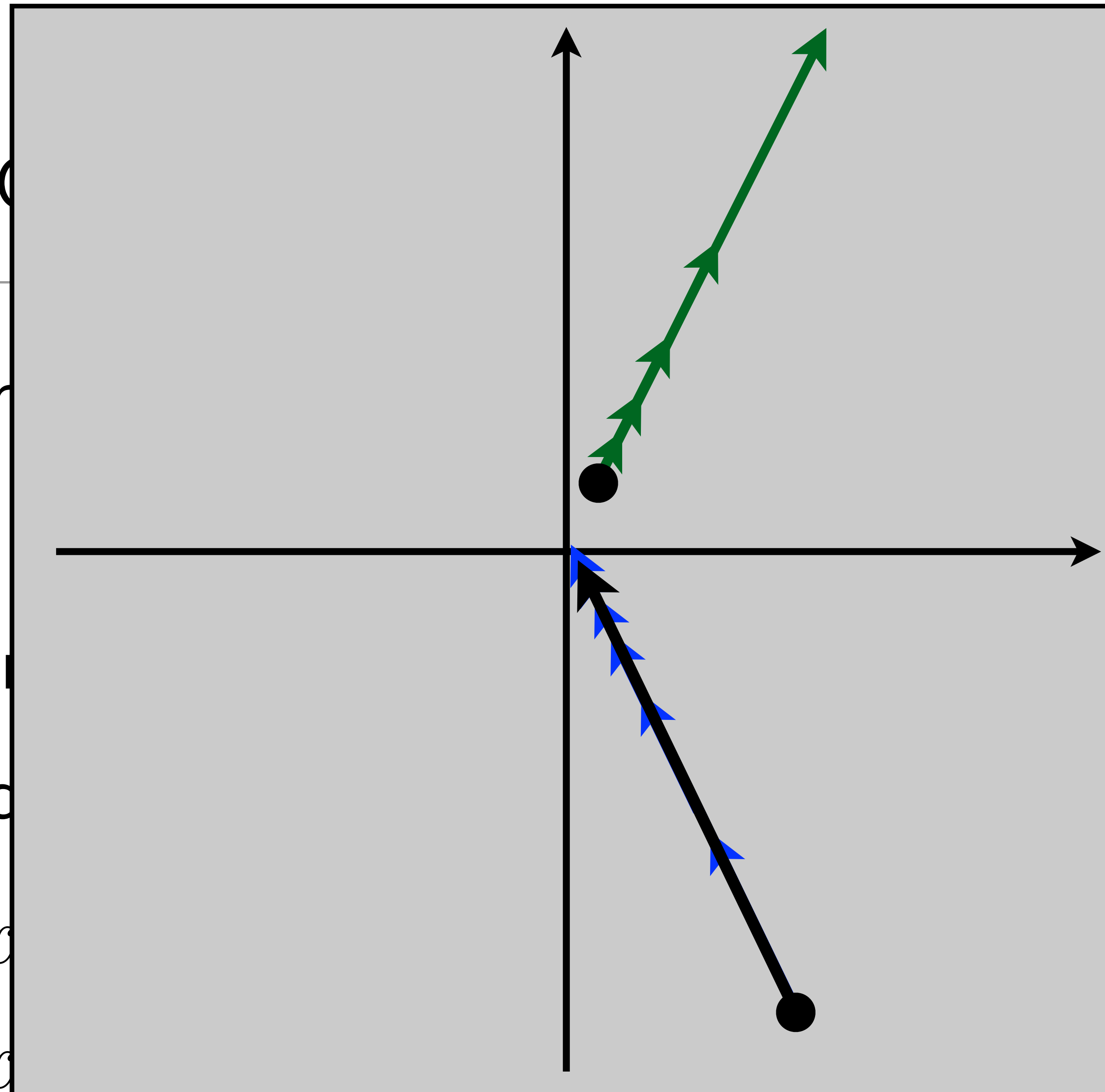
$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of s… plane

- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:



x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

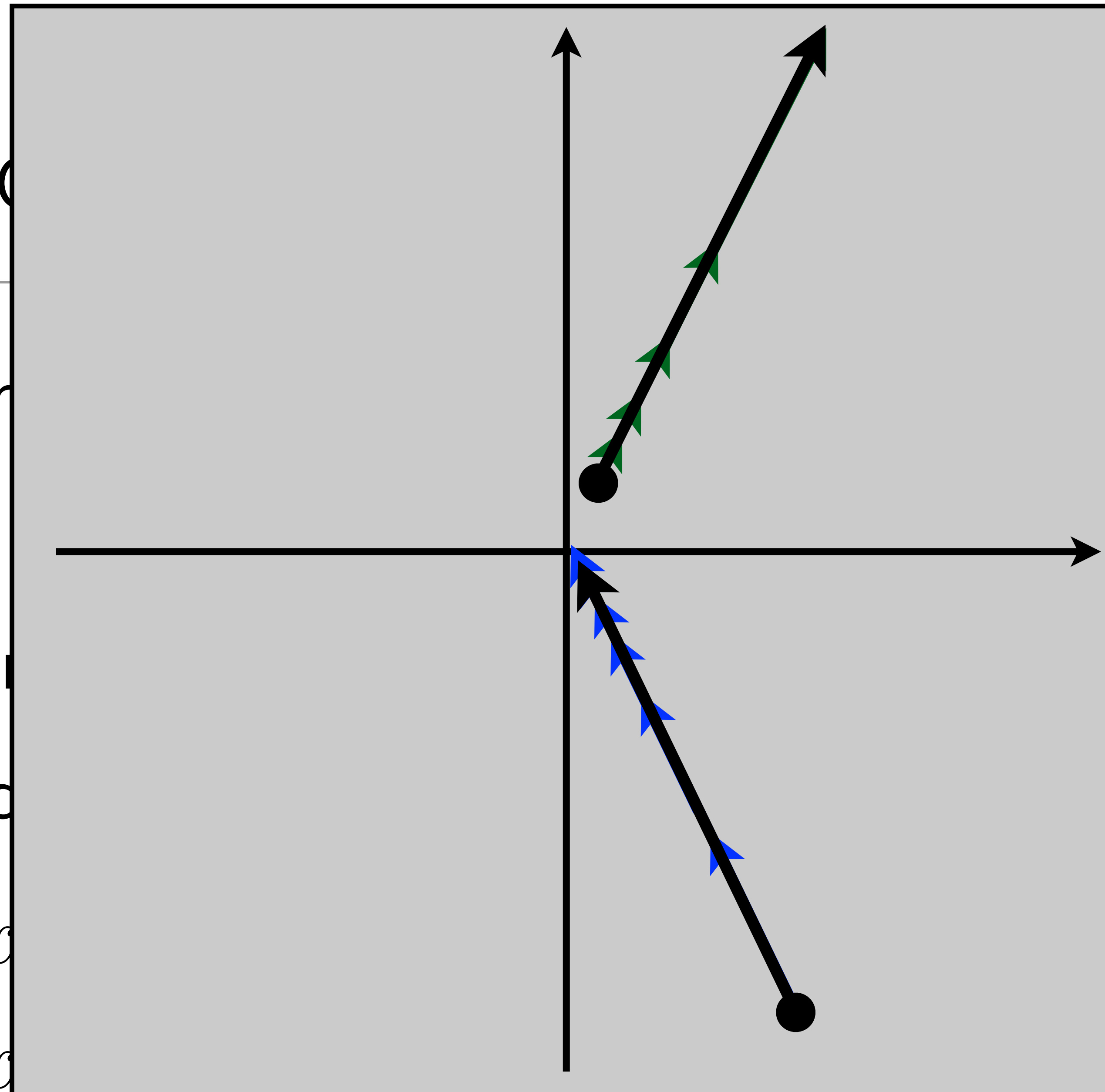
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane



- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

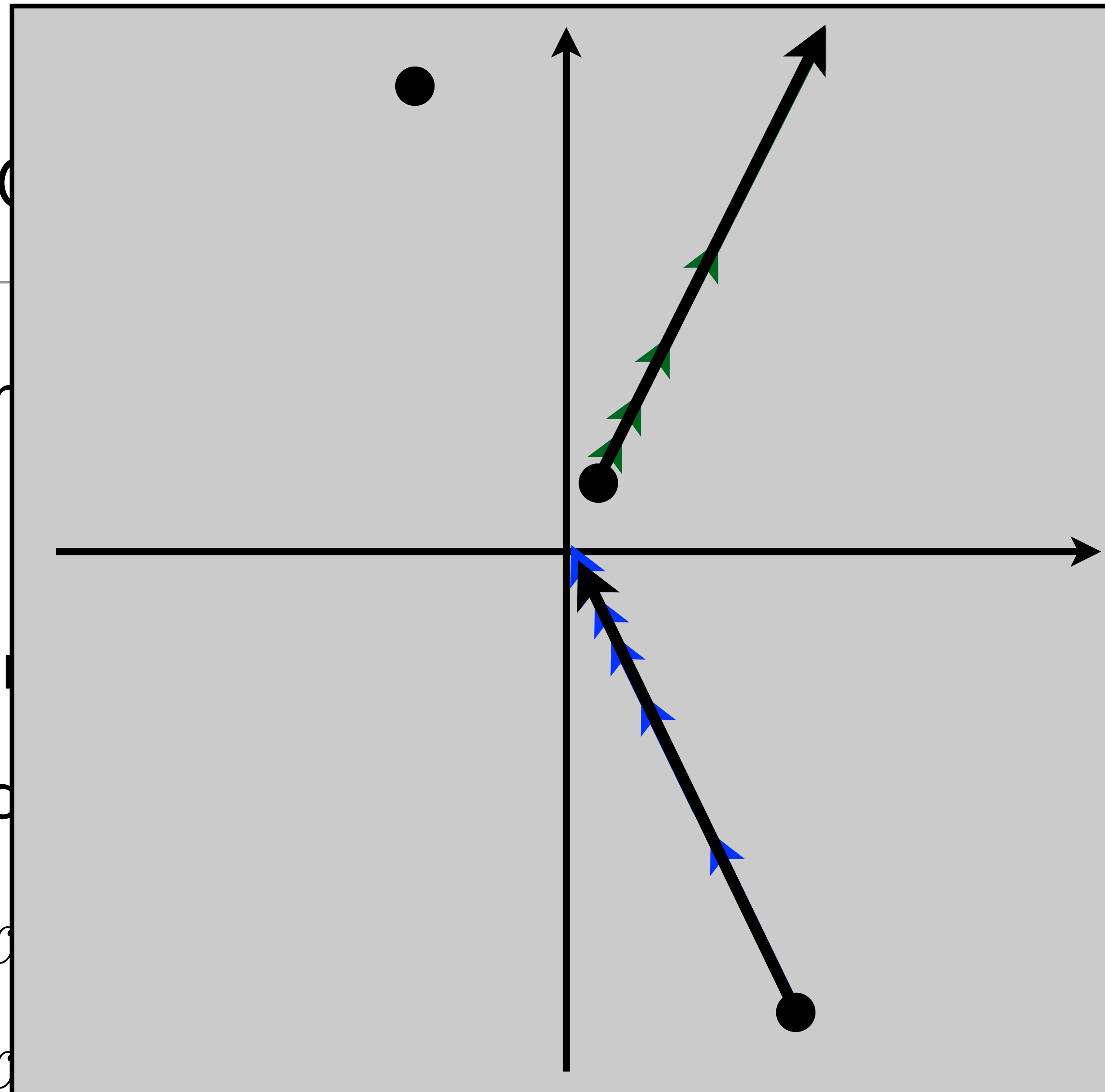
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so … plane



- When matrix A h … olution to **x**'=A**x** is
- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

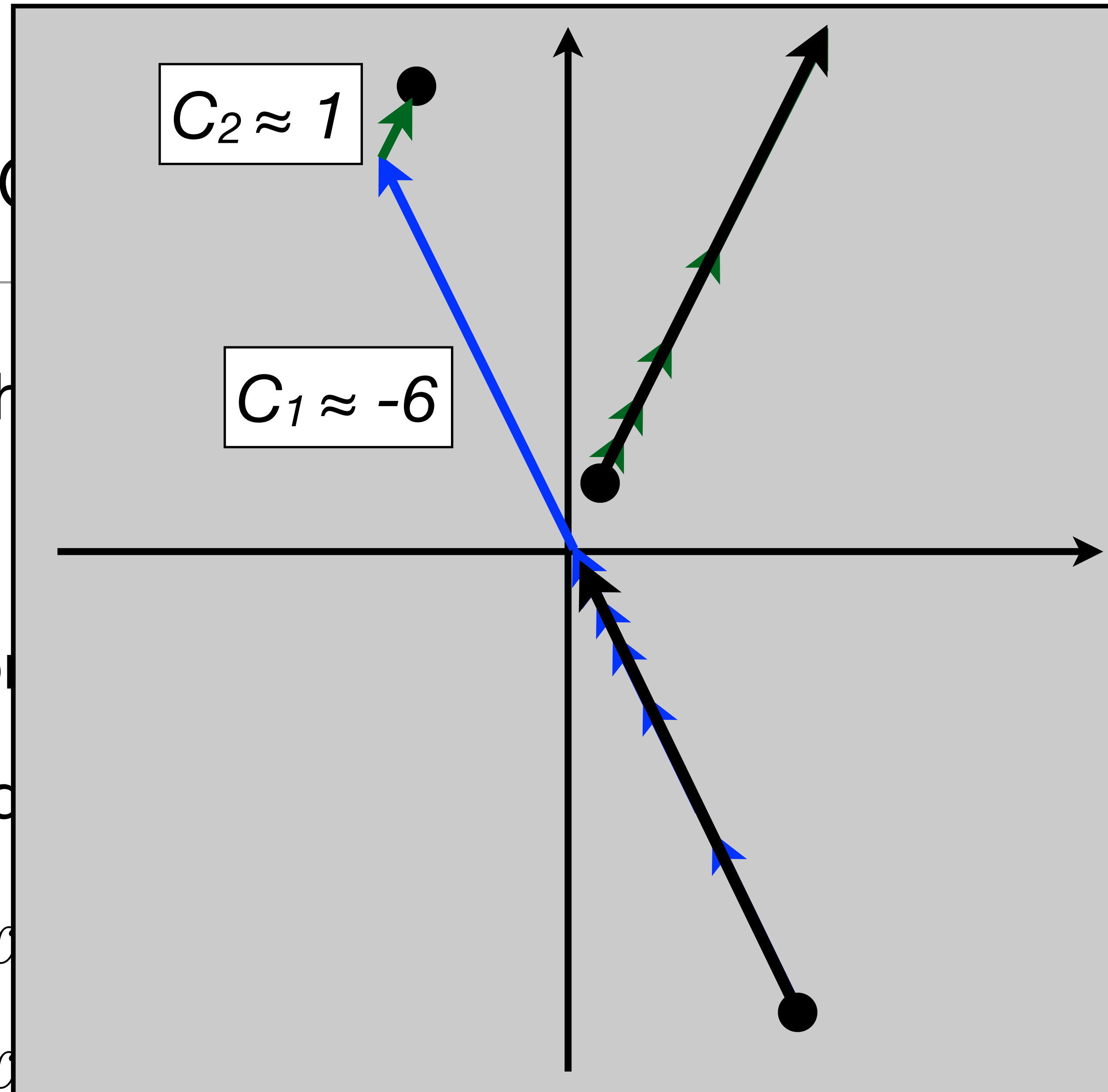
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6,\ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane



- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line.
  Example:

x … 6

x … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

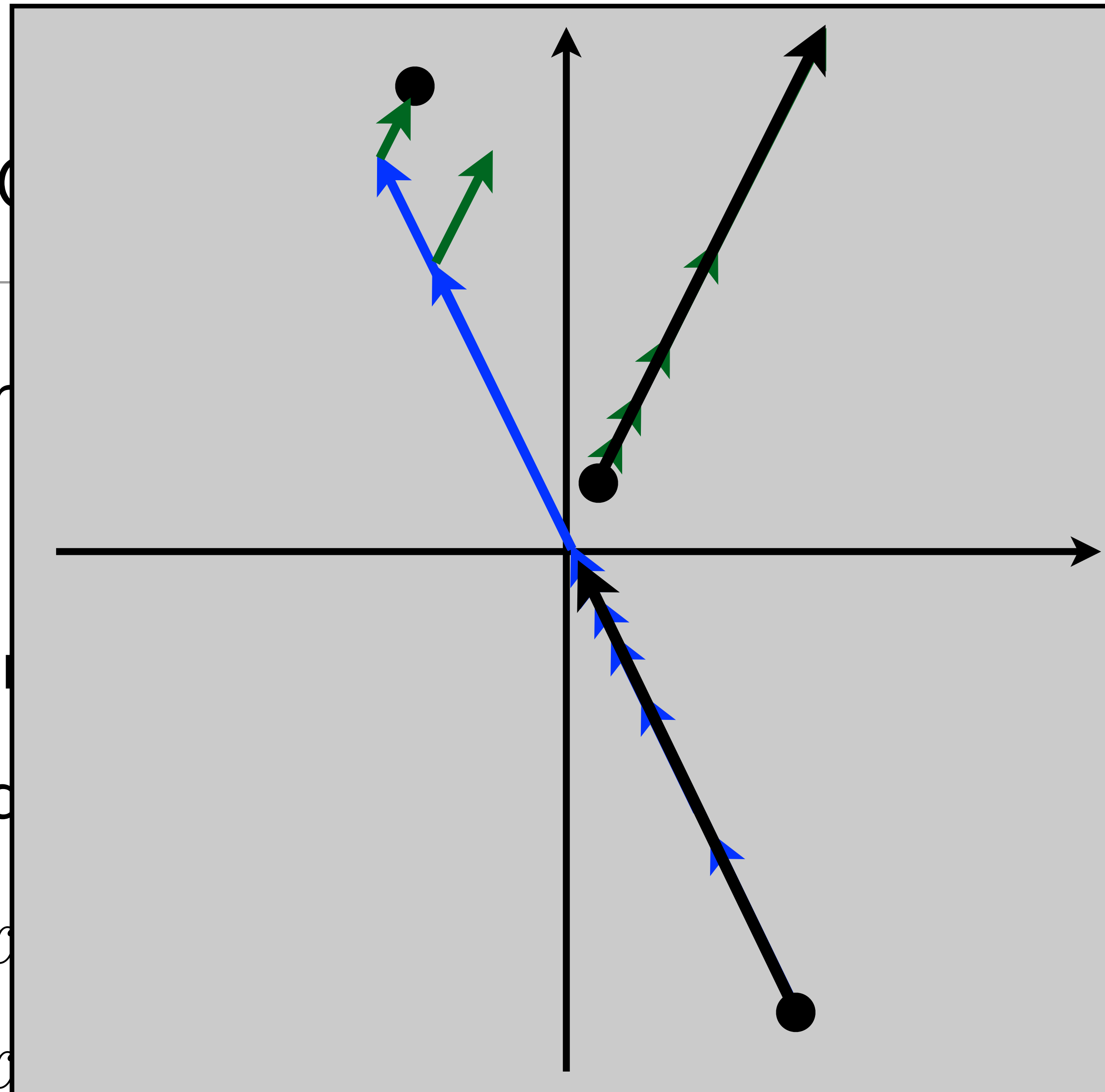
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of sc plane

- When matrix A h olution to **x**'=A**x** is

- What do solutio e phase plane)?

- If the initial cond on is a straight line. Example:



$$x \quad 6$$

$$x_2 \quad -12$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

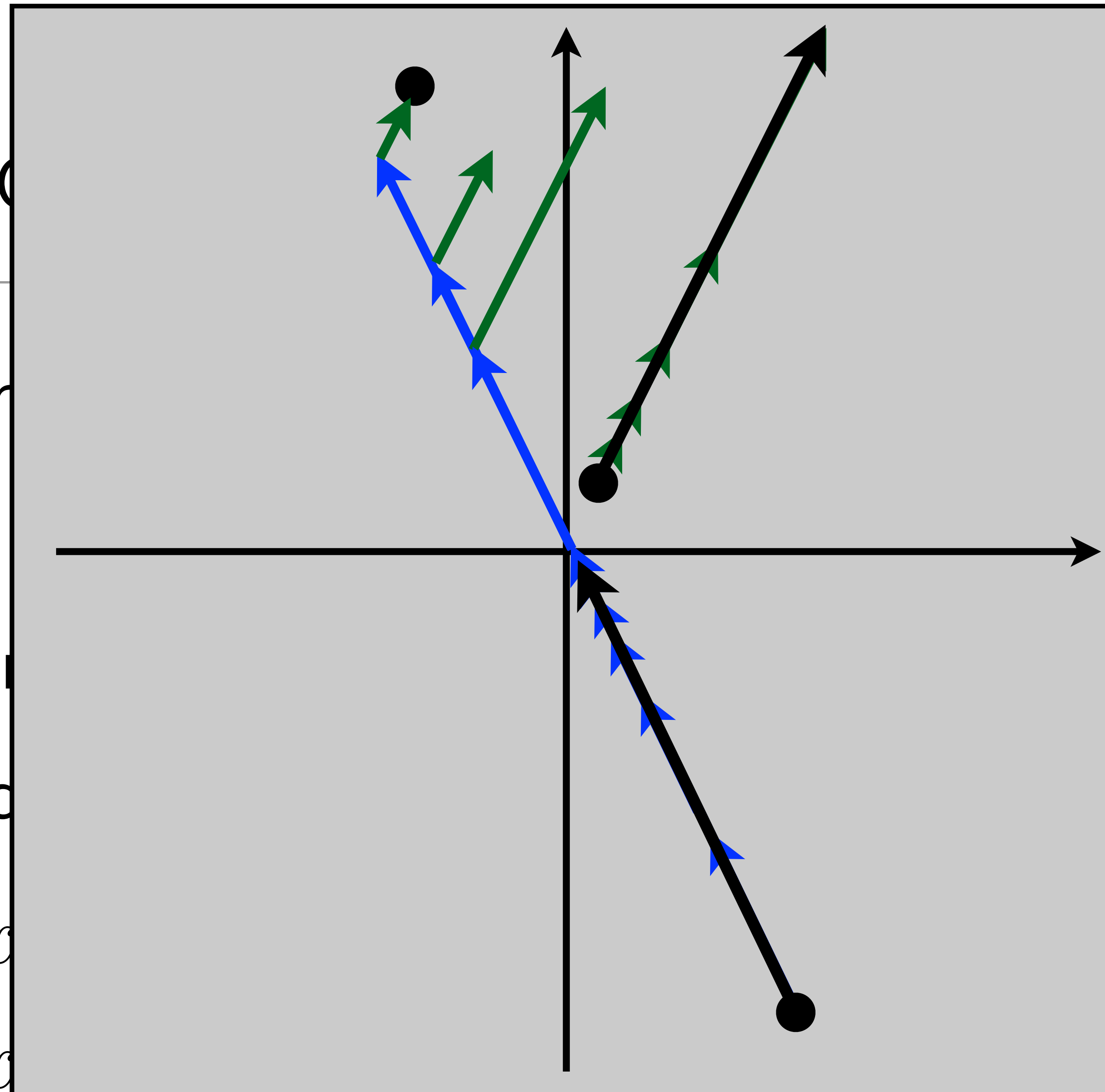
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of sc plane



- When matrix A h olution to **x**'=A**x** is
- What do solutior e phase plane)?
- If the initial conc on is a straight line. Example:

x 6

x -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

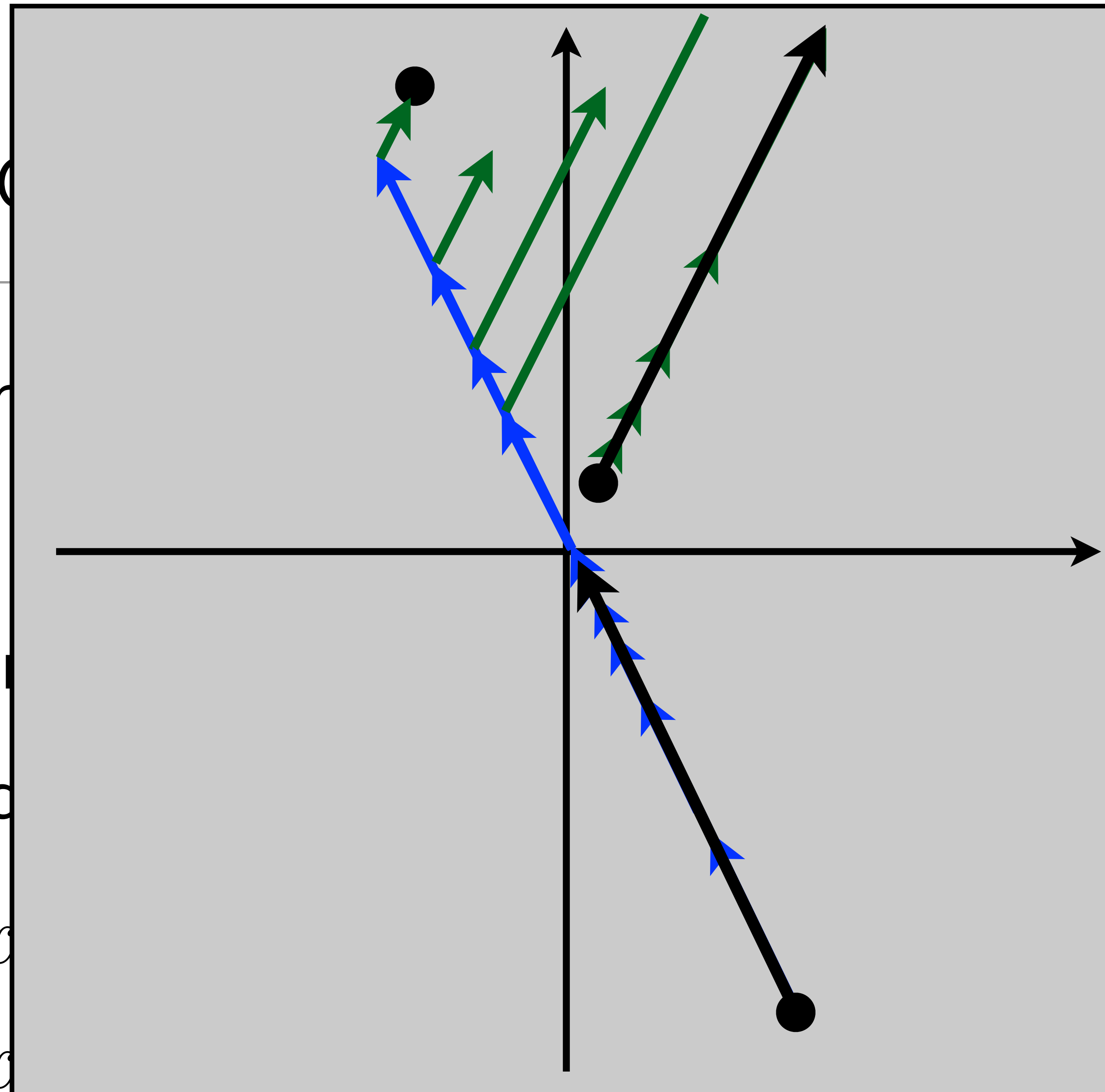
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of s… plane



- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

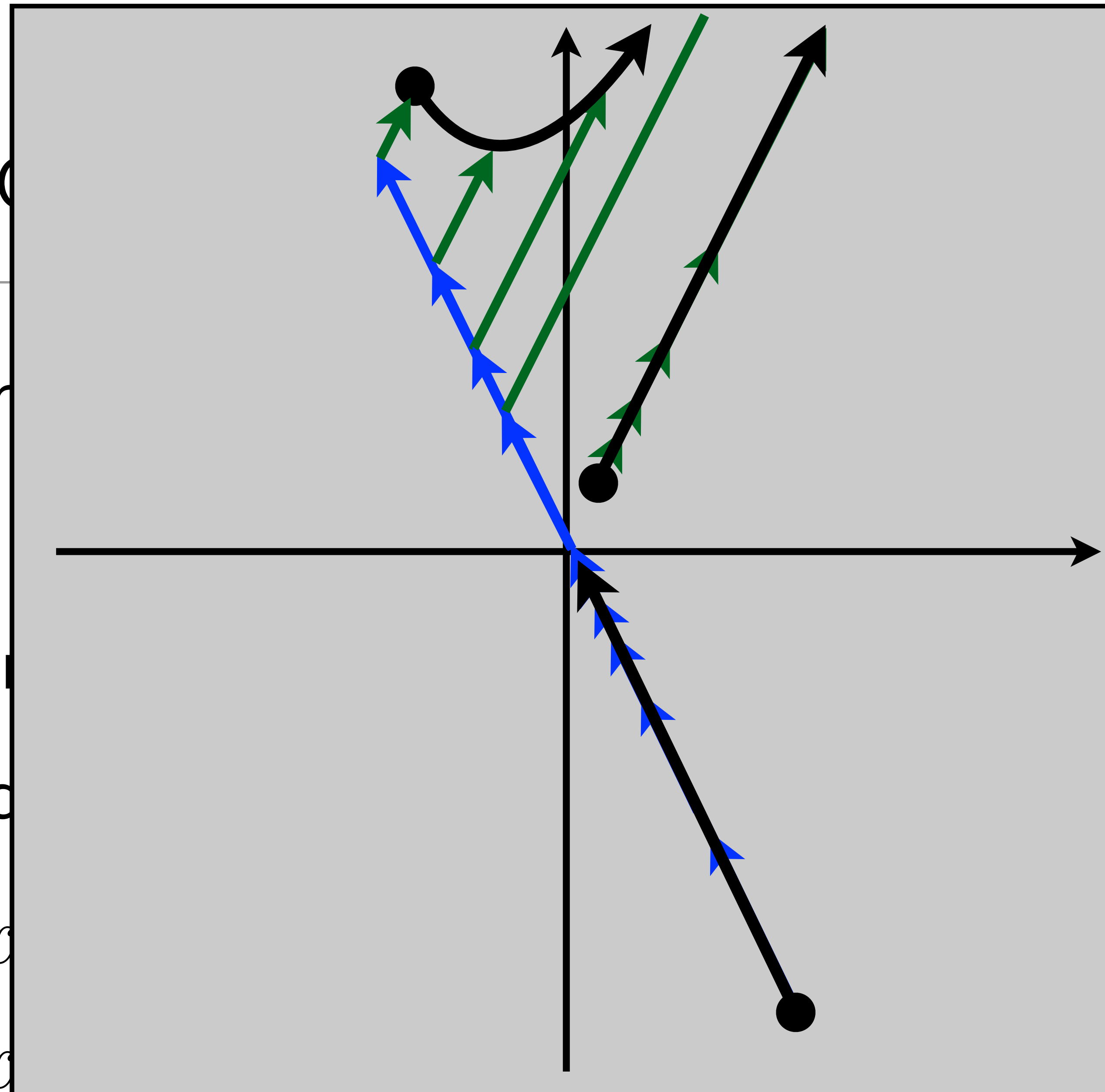
$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane

- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:



x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

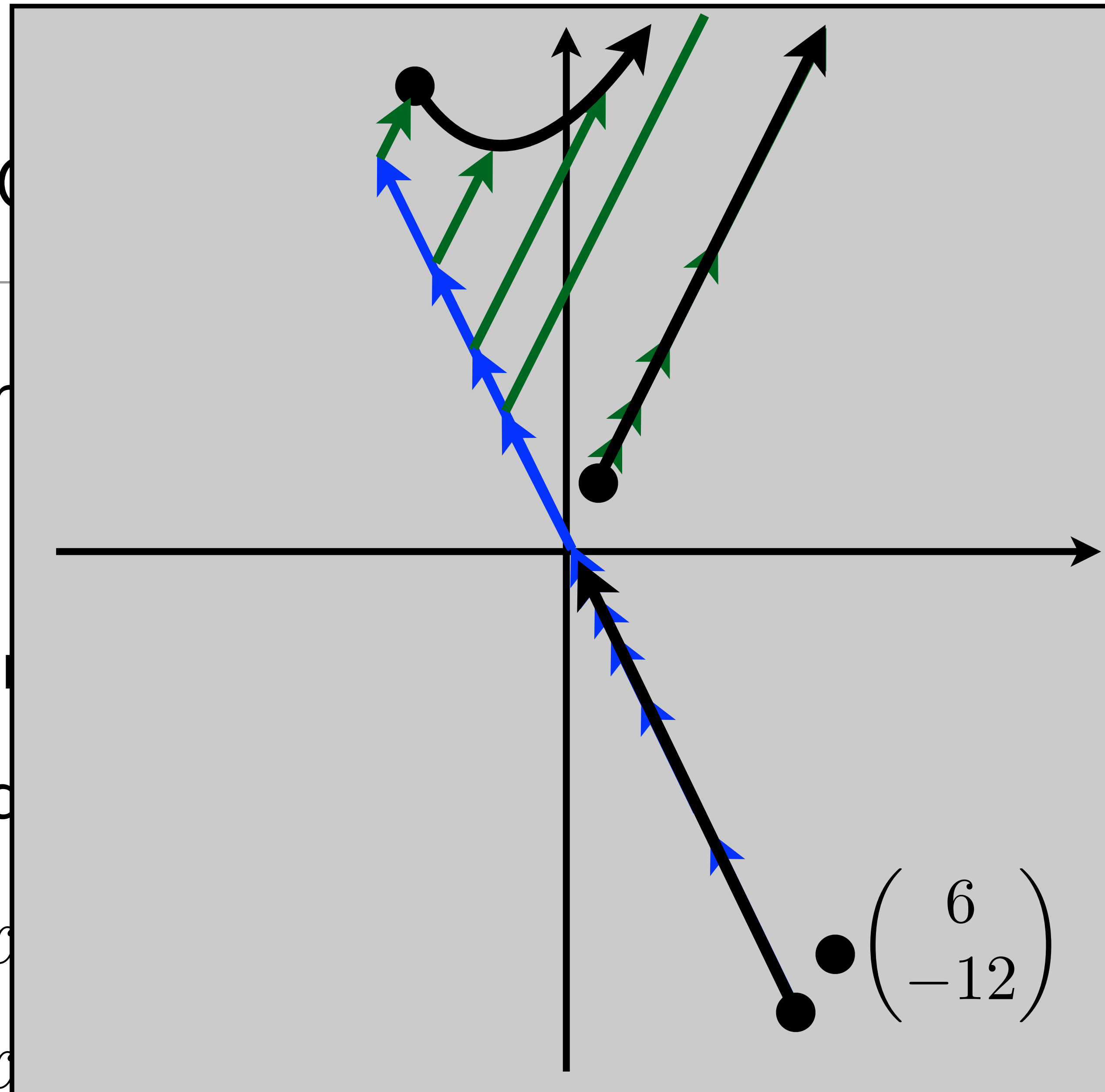
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so... plane

- When matrix A h... olution to **x**'=A**x** is

- What do solutio... e phase plane)?

- If the initial cond... on is a straight line. Example:



x ... 6

x_2 ... -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6,\ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of sc plane

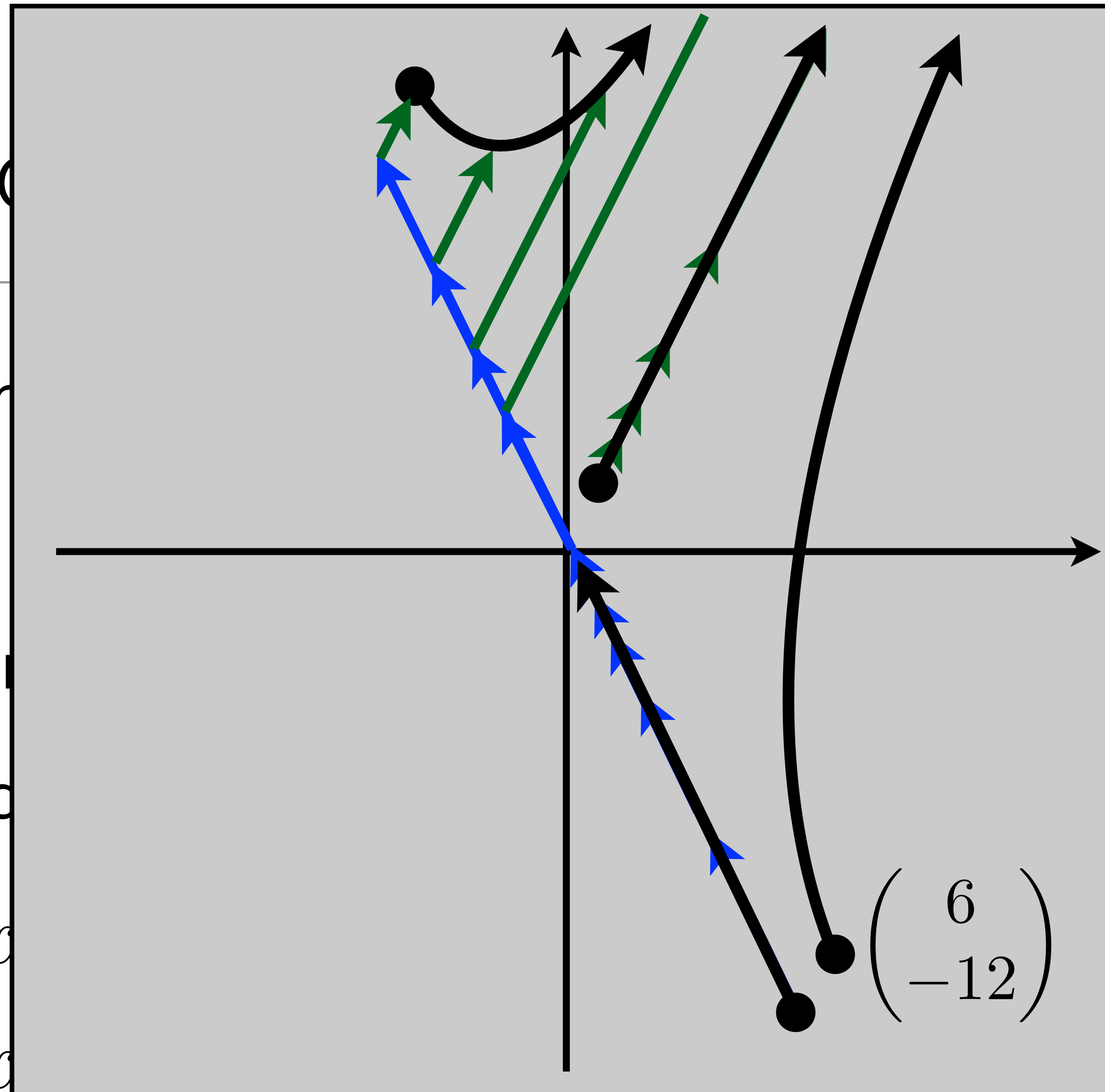
- When matrix A h olution to **x**'=A**x** is

- What do solutio e phase plane)?

- If the initial cond on is a straight line. Example:

x 6

x_2 -12



$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

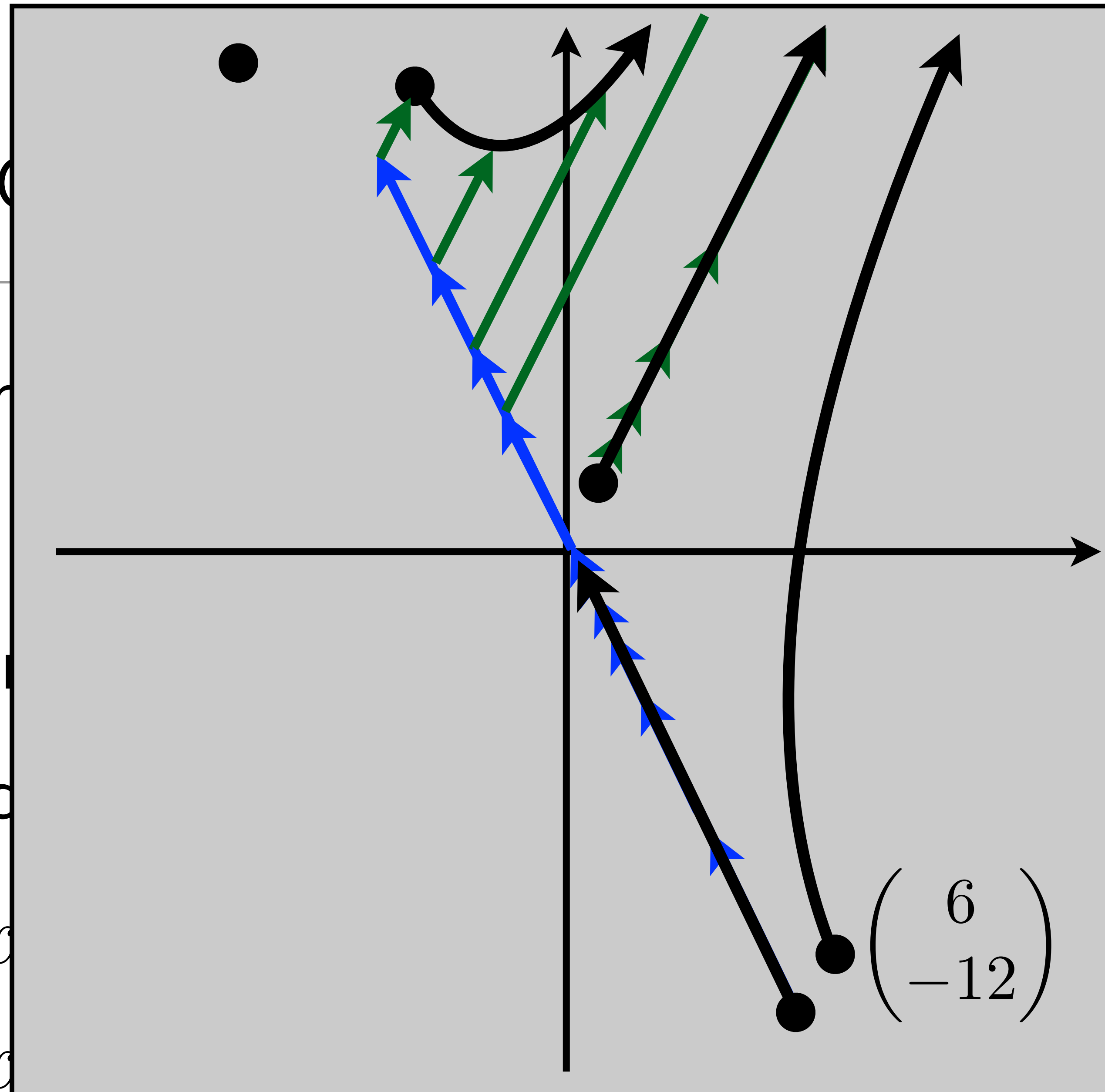
$$C_1 = 6,\ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so … plane

- When matrix A h … olution to **x**’=A**x** is



- What do solution … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

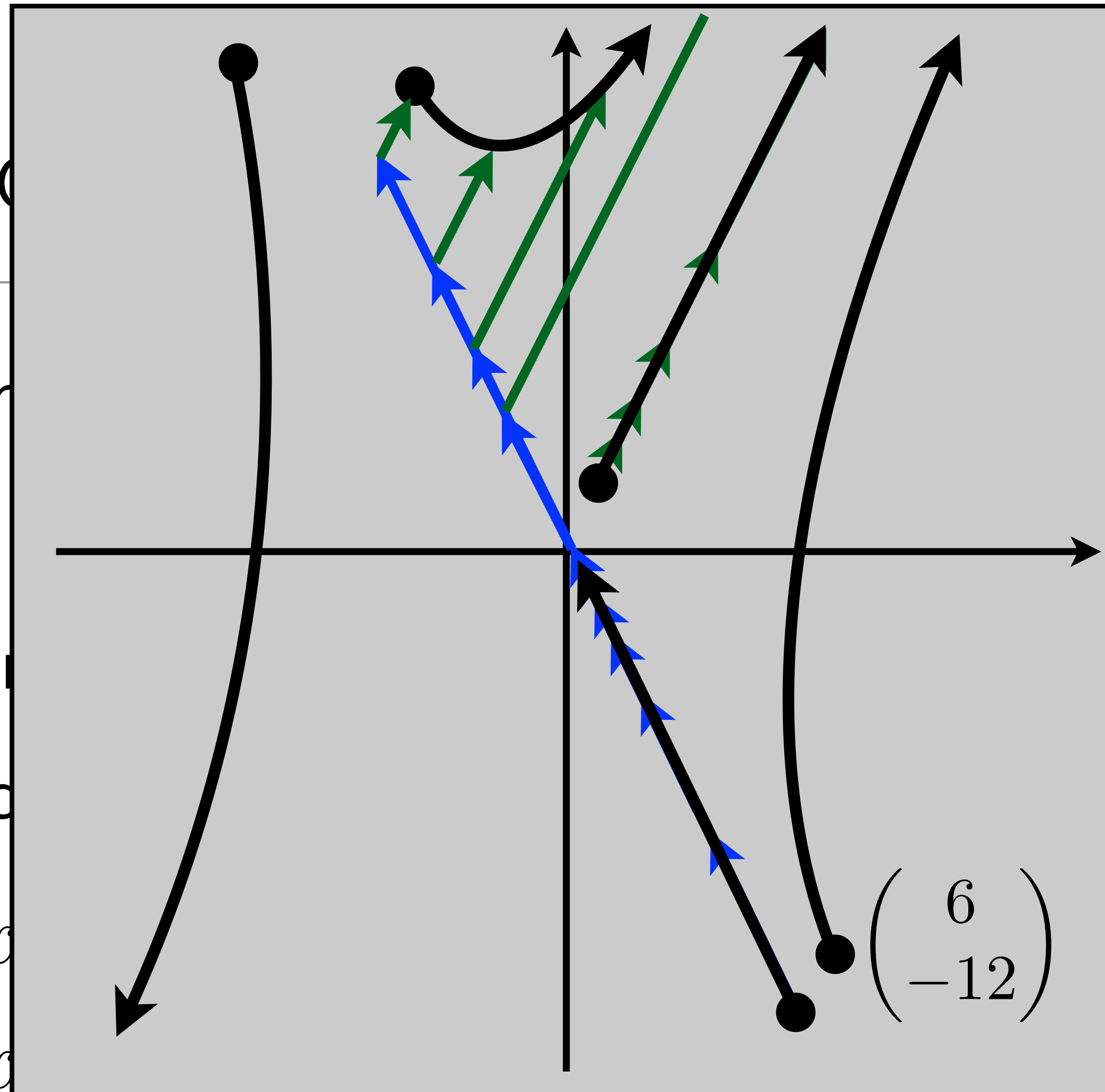
$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of sc plane

- When matrix A h olution to **x**'=A**x** is
- What do solutio e phase plane)?
- If the initial cond on is a straight line. Example:



x 6

x_2 -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

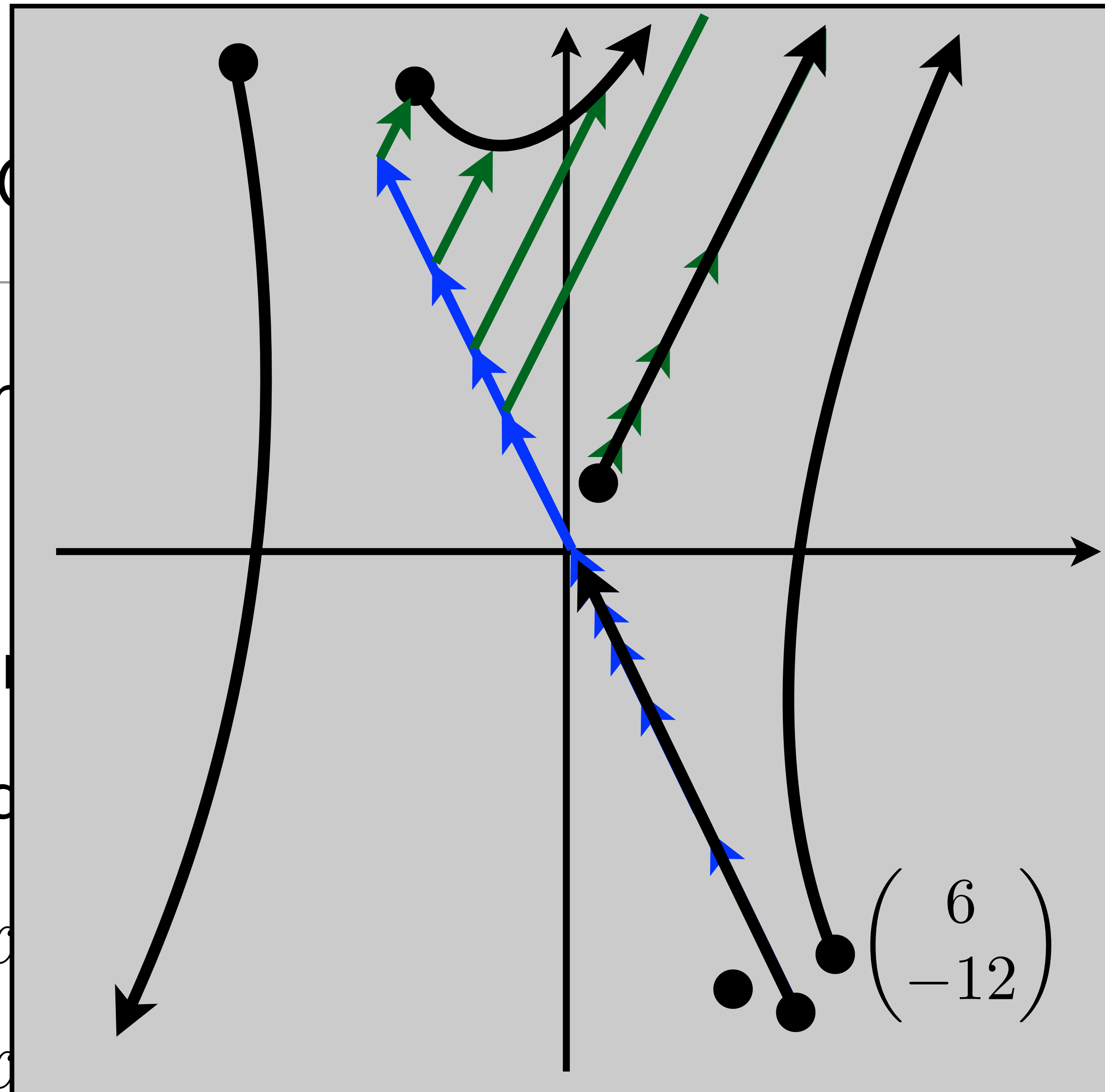
$$C_1 = 6, \ C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane

- When matrix A h… olution to **x**'=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:



x … 6

x … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \;\; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

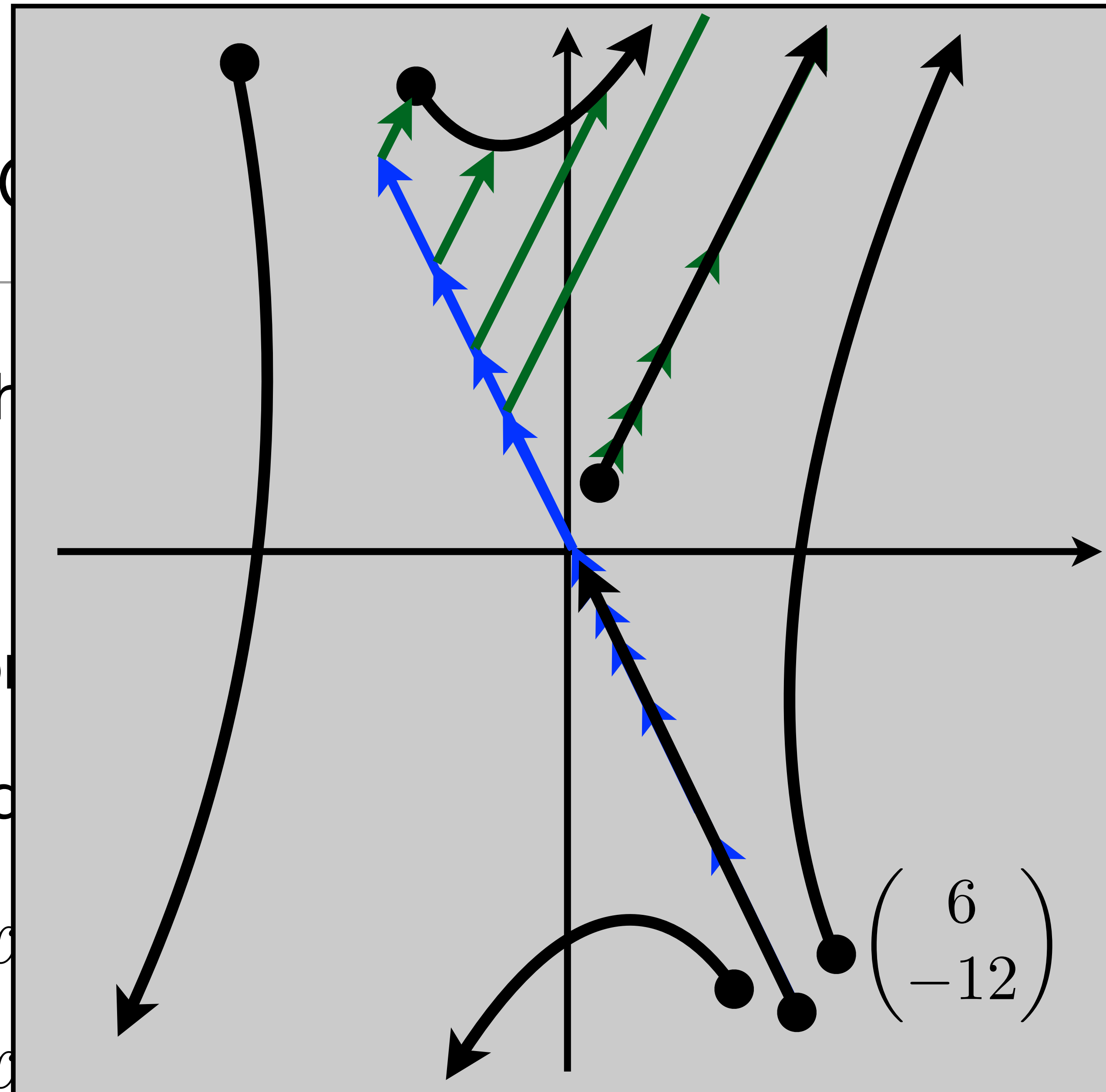
Shapes of so … plane

- When matrix A h … olution to **x**'=A**x** is

- What do solutio … e phase plane)?
- If the initial cond … on is a straight line. Example:

x … 6

x_2 … -12



$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

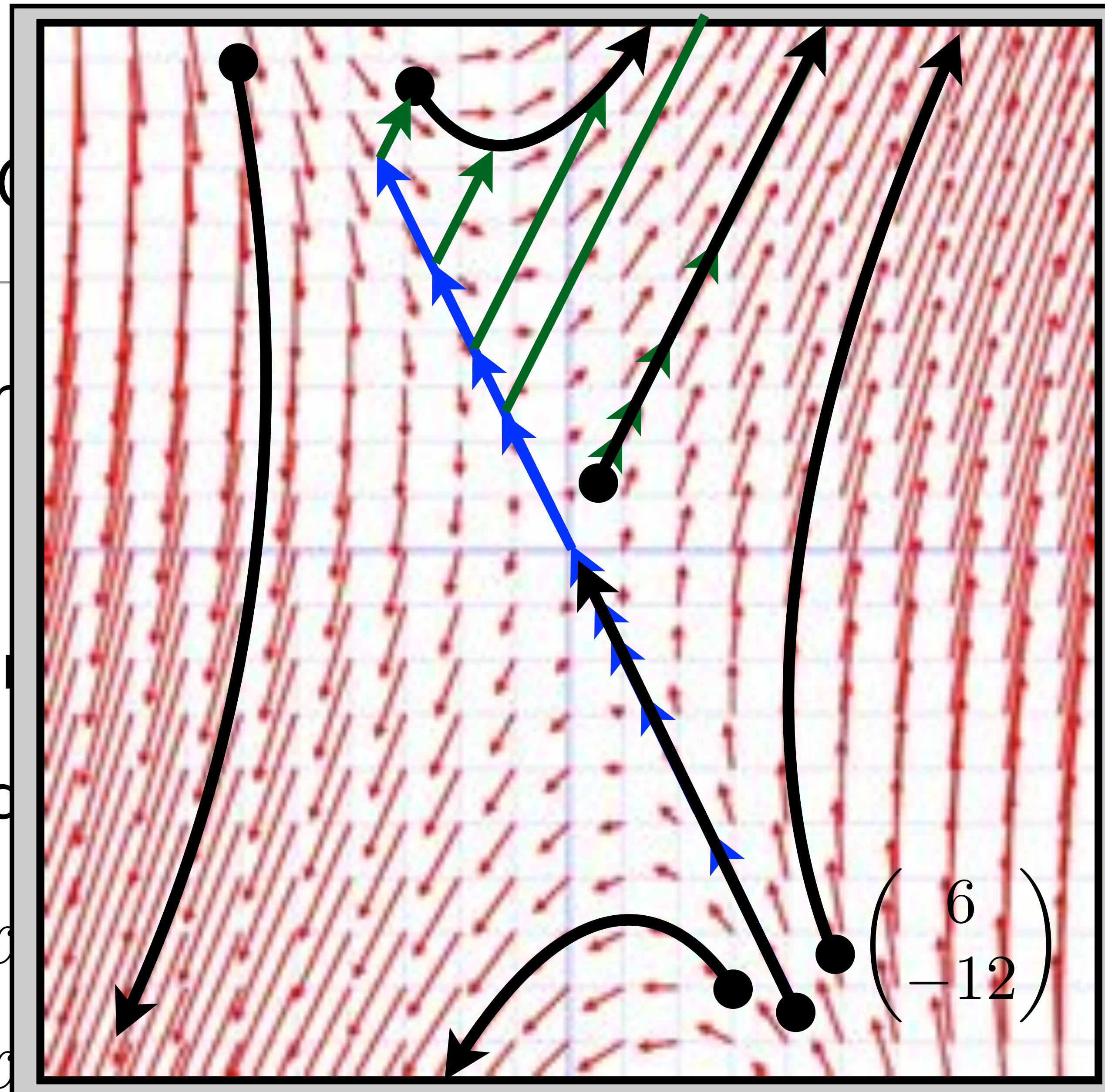
$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Shapes of so… plane

- When matrix A h… olution to **x**’=A**x** is
- What do solutio… e phase plane)?
- If the initial cond… on is a straight line. Example:



x … 6

x … -12

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

$$C_1 = 6, \; C_2 = 0$$

- IVP solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{-t} \begin{pmatrix} 6 \\ -12 \end{pmatrix}$$

Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \qquad C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

Plotting x(t) vs y(t) compared to t vs x(t)

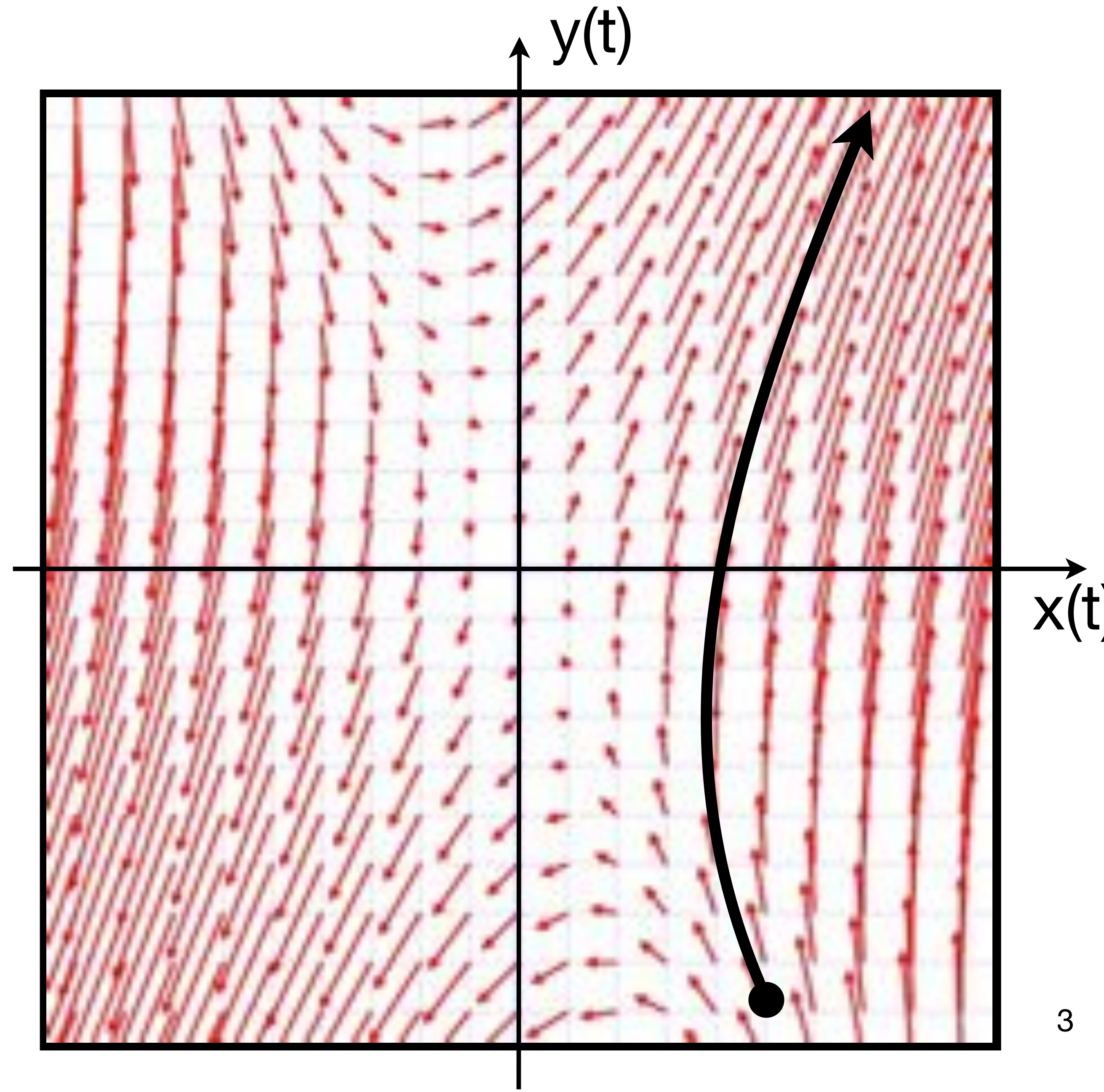
$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \qquad C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{7}{2} e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2} e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \qquad C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{7}{2} e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2} e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

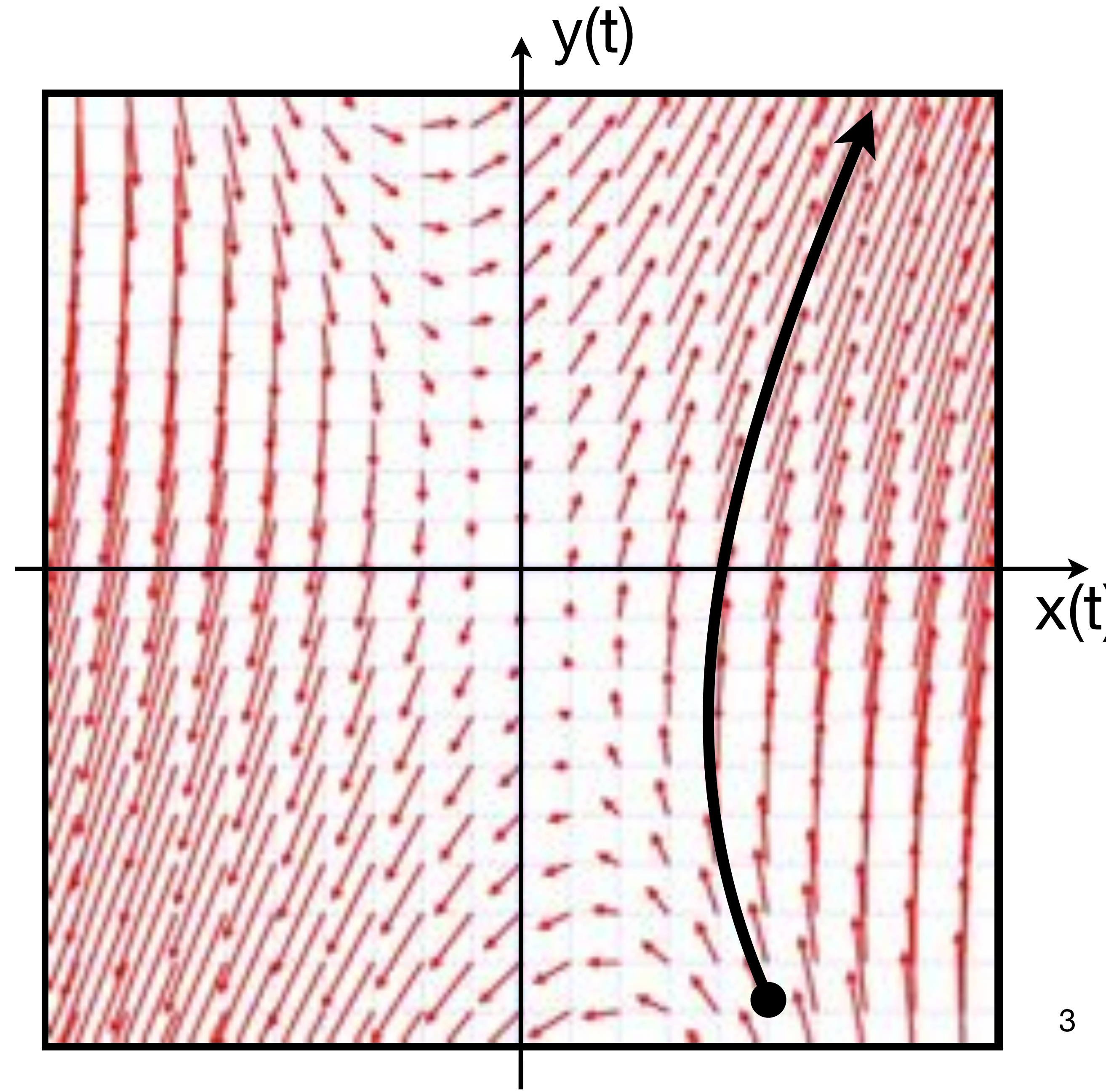


Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$$

$$C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

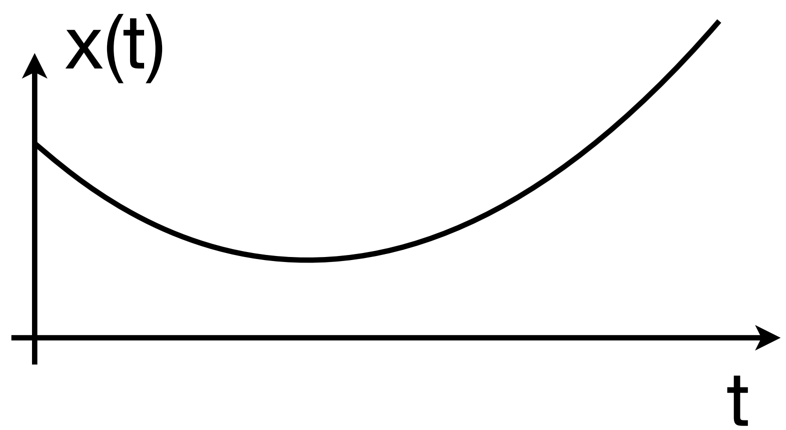
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{7}{2} e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2} e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

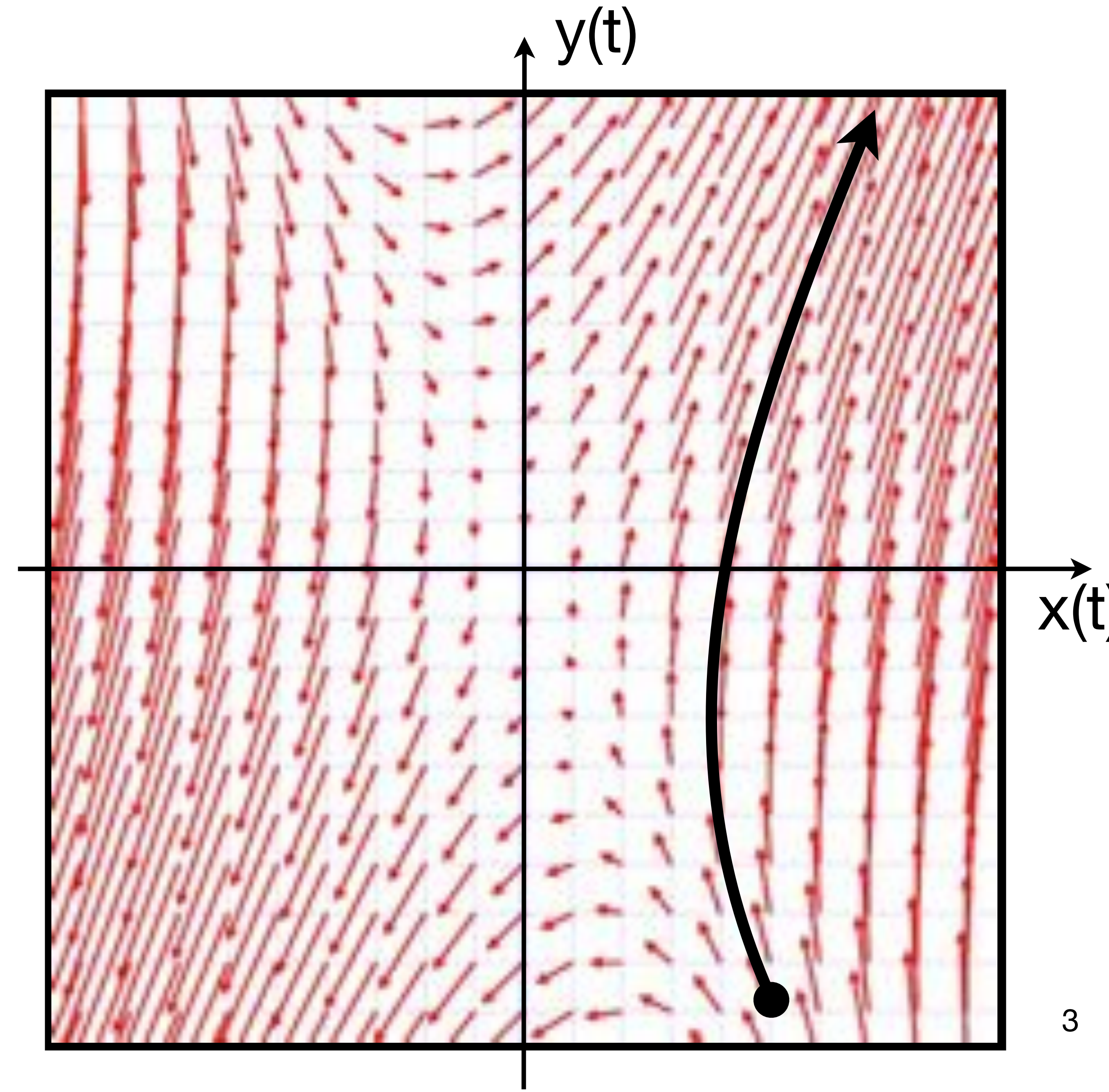


Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \qquad C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{7}{2} e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2} e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

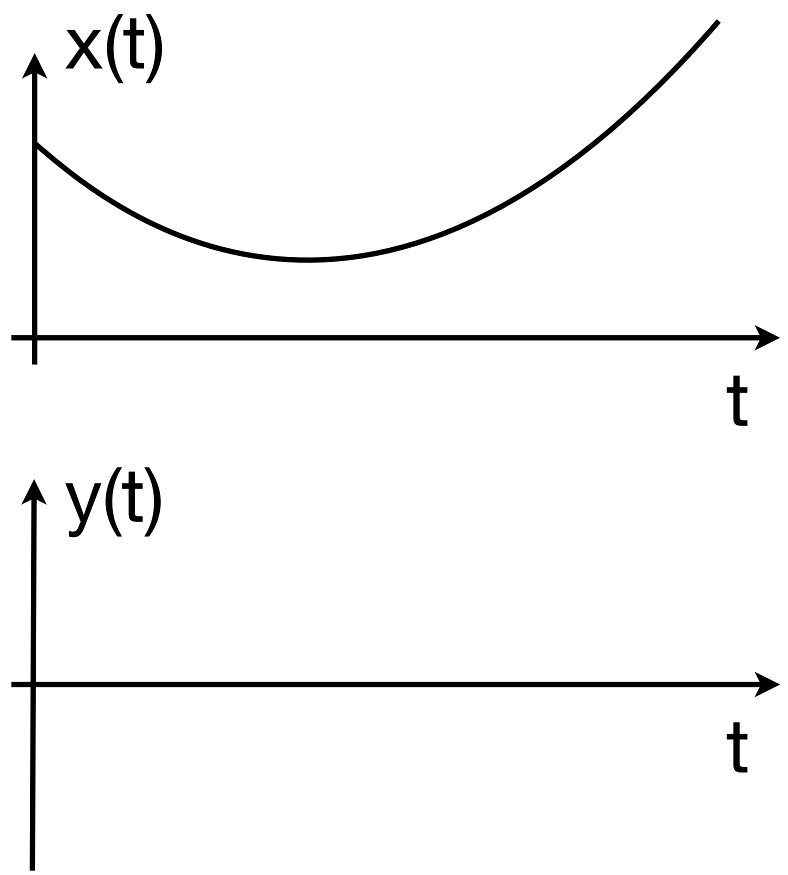


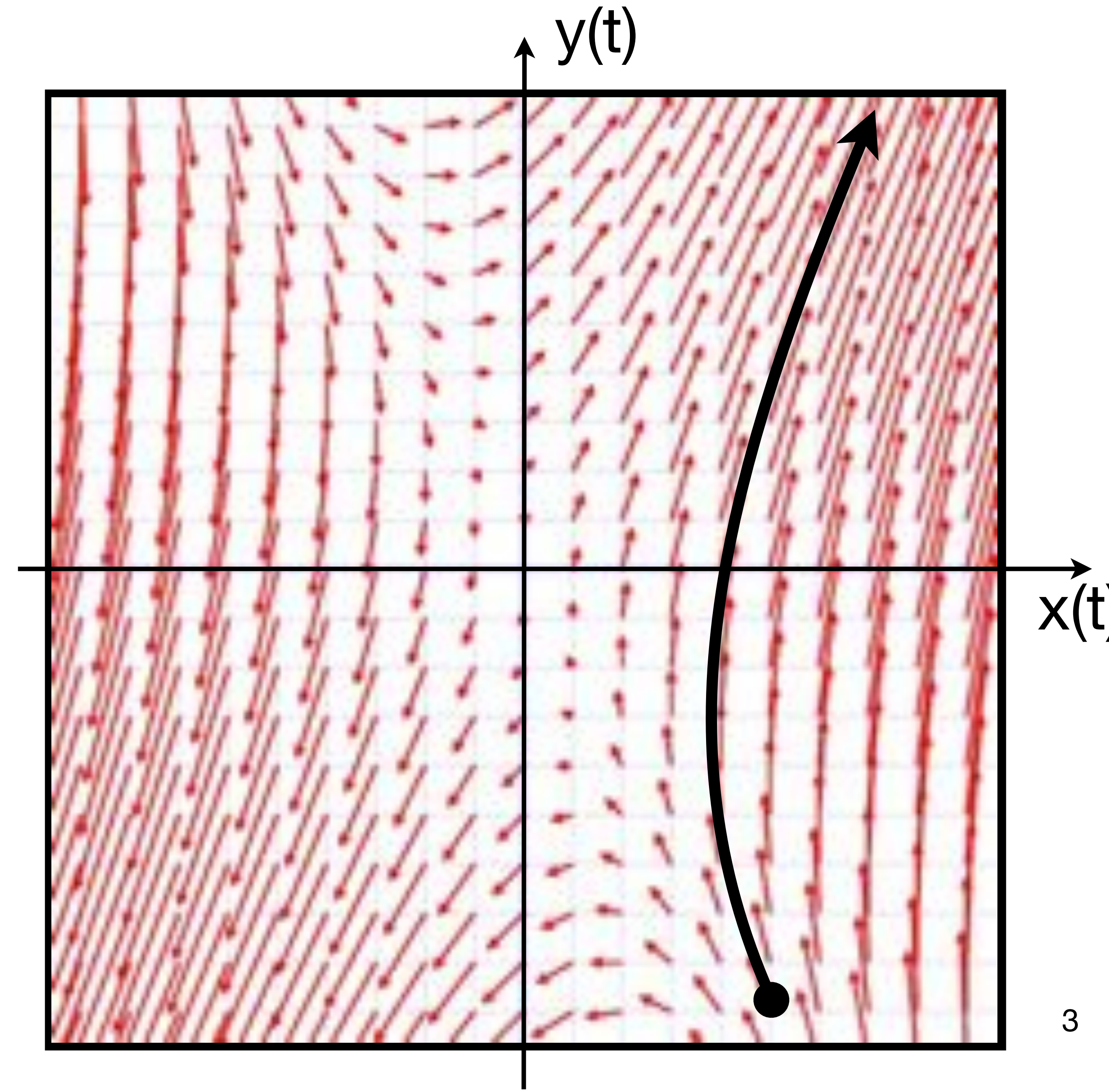


Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \qquad C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{7}{2}e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2}e^{3t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

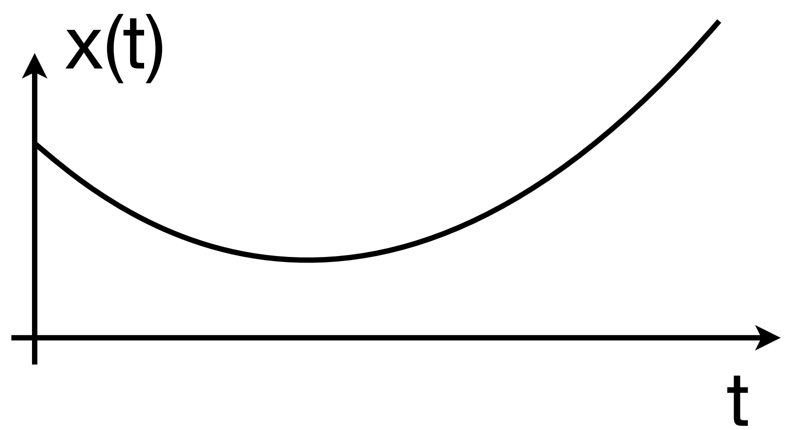


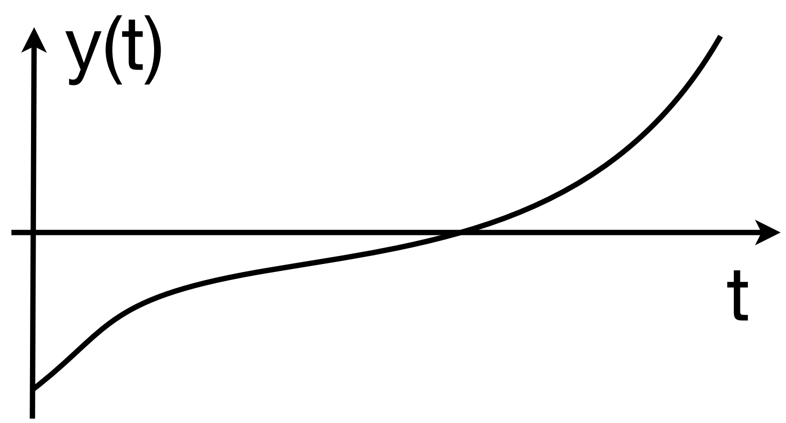


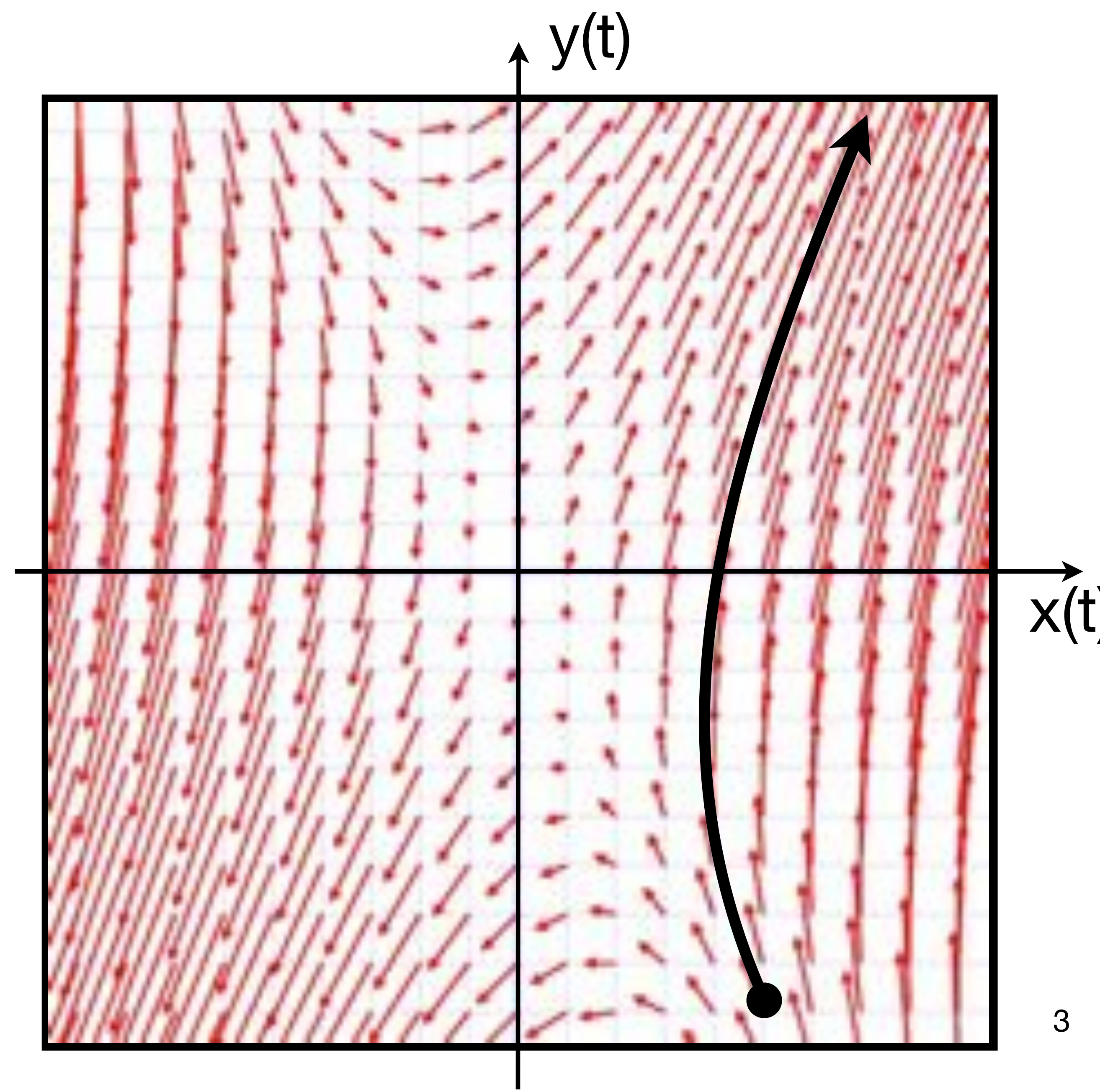
Plotting x(t) vs y(t) compared to t vs x(t)

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \qquad C_1 = \frac{7}{2},\ C_2 = \frac{1}{2}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{7}{2} e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2} e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$







Shapes of solution curves in the phase plane

- Simple example to show general idea. $$\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $$\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = C_1 e^{\lambda_1 t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{\lambda_2 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = C_1 e^{\lambda_1 t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{\lambda_2 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 e^{\lambda_1 t}$$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = C_1 e^{\lambda_1 t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{\lambda_2 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 e^{\lambda_1 t}$$

$$x_2(t) = C_2 e^{\lambda_2 t}$$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = C_1 e^{\lambda_1 t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{\lambda_2 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 e^{\lambda_1 t}$$

$$x_2(t) = C_2 e^{\lambda_2 t}$$

- Can we plot solutions in x_1-x_2 plane by graphing x_2 versus x_1?

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = C_1 e^{\lambda_1 t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{\lambda_2 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 e^{\lambda_1 t}$$

$$x_2(t) = C_2 e^{\lambda_2 t}$$

- Can we plot solutions in x_1-x_2 plane by graphing x_2 versus x_1?

Shapes of solution curves in the phase plane

- Simple example to show general idea.

$$\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x} = C_1 e^{\lambda_1 t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{\lambda_2 t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 e^{\lambda_1 t} \qquad t = \frac{1}{\lambda_1} \ln\left(\frac{x_1}{C_1}\right)$$

$$x_2(t) = C_2 e^{\lambda_2 t} \qquad t = \frac{1}{\lambda_2} \ln\left(\frac{x_2}{C_2}\right)$$

$$\frac{1}{\lambda_2} \ln\left(\frac{x_2}{C_2}\right) = \frac{1}{\lambda_1} \ln\left(\frac{x_1}{C_1}\right)$$

$$\ln\left(\frac{x_2}{C_2}\right) = \frac{\lambda_2}{\lambda_1} \ln\left(\frac{x_1}{C_1}\right)$$

$$\ln\left(\frac{x_2}{C_2}\right) = \ln\left(\frac{x_1}{C_1}\right)^{\frac{\lambda_2}{\lambda_1}}$$

$$x_2 = C_2 \left(\frac{x_1}{C_1}\right)^{\frac{\lambda_2}{\lambda_1}}$$

- Can we plot solutions in x_1-x_2 plane by graphing x_2 versus x_1?

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

$\lambda_2 = -3\lambda_1$

Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$

Shapes of solution curves in the phase plane

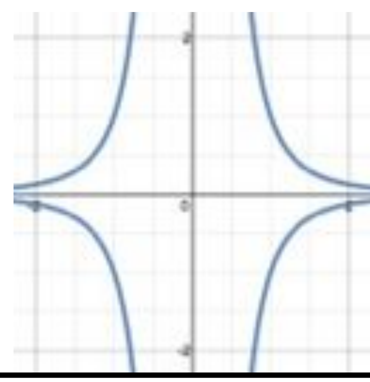
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



Shapes of solution curves in the phase plane

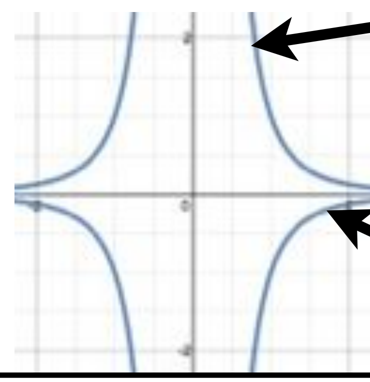
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, w … know the sign and size of $\frac{\lambda_2}{\lambda_1}$.

$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



Shapes of solution curves in the phase plane

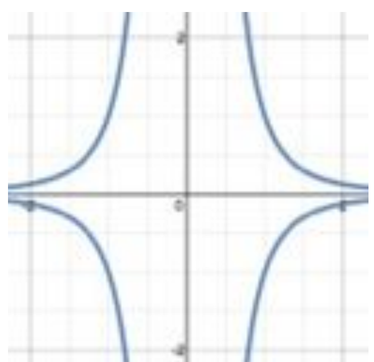
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

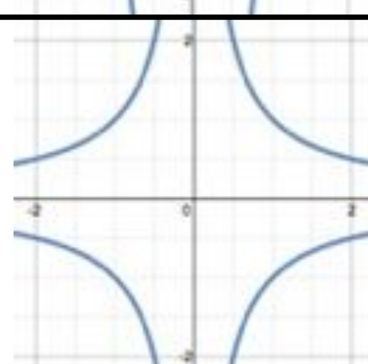
$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$



Shapes of solution curves in the phase plane

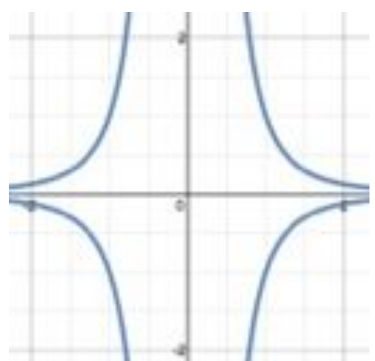
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

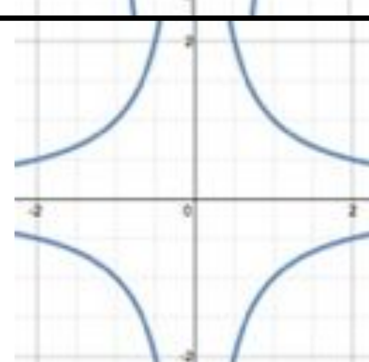
$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



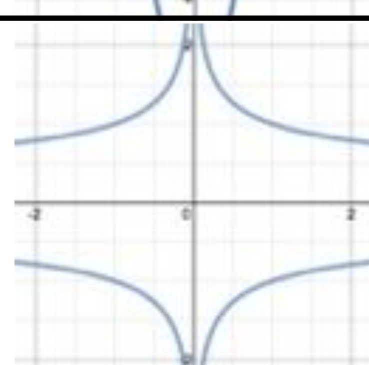
$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$



$$\lambda_2 = -\frac{1}{3}\lambda_1$$

$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$



Shapes of solution curves in the phase plane

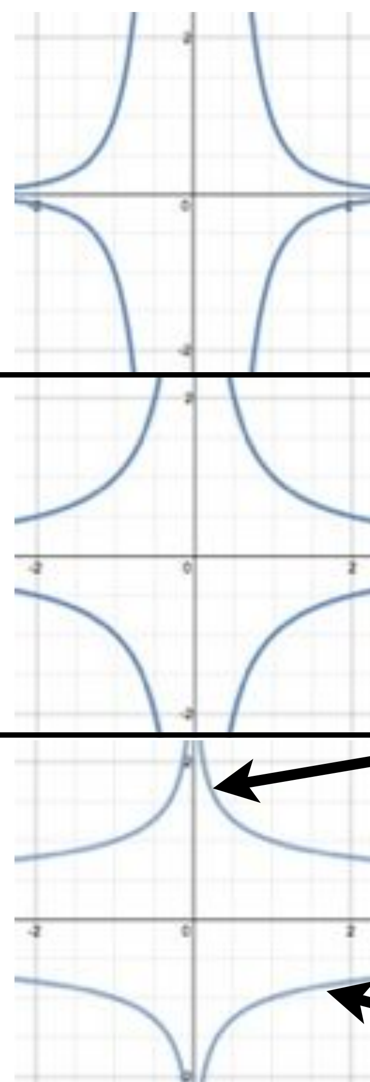
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$

$$\lambda_2 = -\frac{1}{3}\lambda_1$$

$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$

close to x_2 axis

far from x_1 axis

Shapes of solution curves in the phase plane

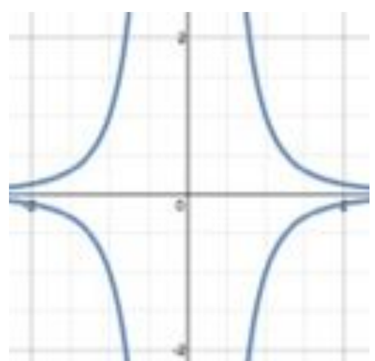
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

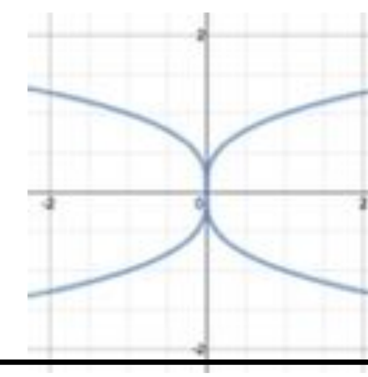
$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



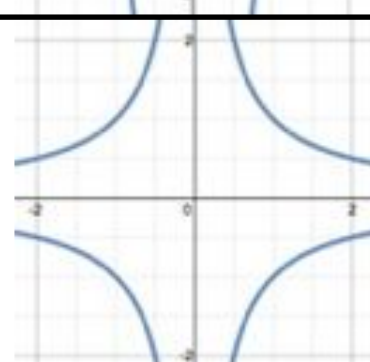
$$\lambda_2 = \frac{1}{3}\lambda_1$$

$$x_2 = C\sqrt[3]{x_1}$$



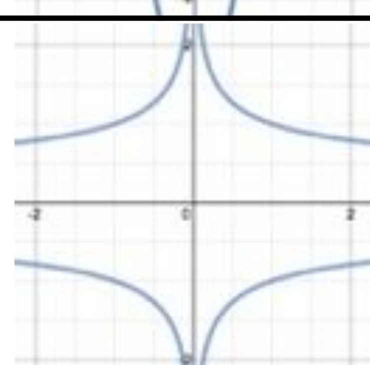
$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$



$$\lambda_2 = -\frac{1}{3}\lambda_1$$

$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$



Shapes of solution curves in the phase plane

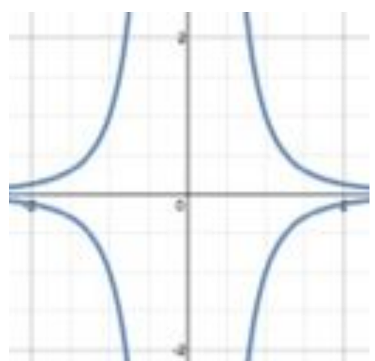
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

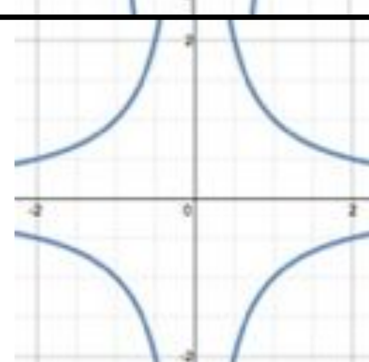
$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



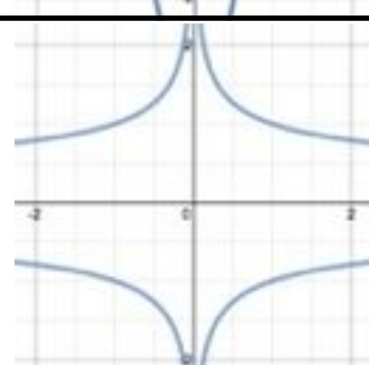
$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$



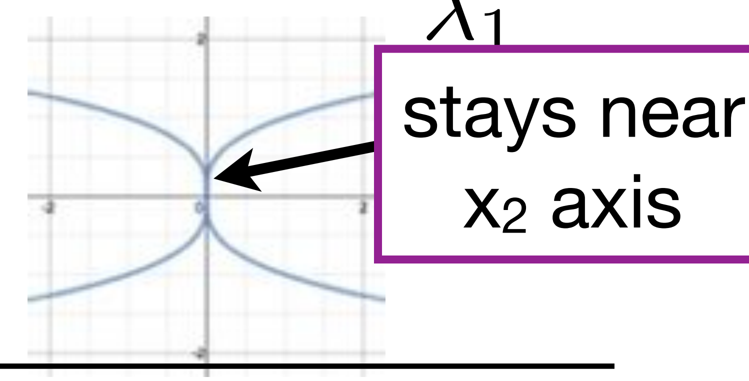
$$\lambda_2 = -\frac{1}{3}\lambda_1$$

$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$



$$\lambda_2 = \frac{1}{3}\lambda_1$$

$$x_2 = C\sqrt[3]{x_1}$$



Shapes of solution curves in the phase plane

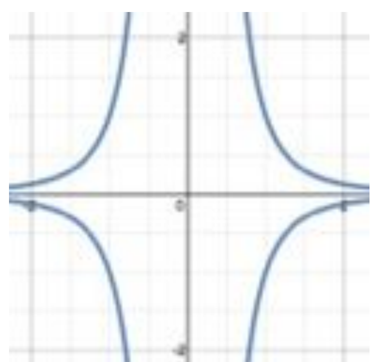
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

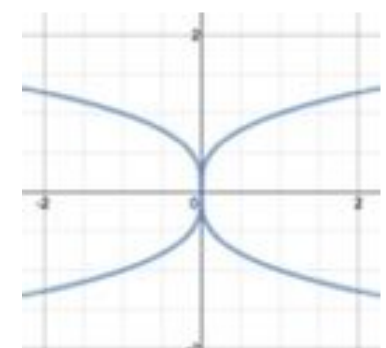
$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



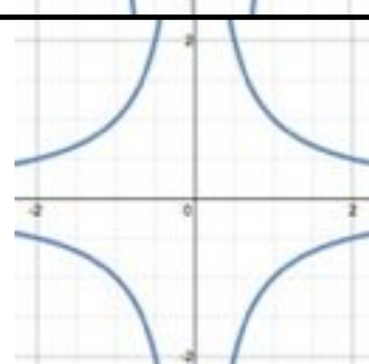
$$\lambda_2 = \frac{1}{3}\lambda_1$$

$$x_2 = C\sqrt[3]{x_1}$$



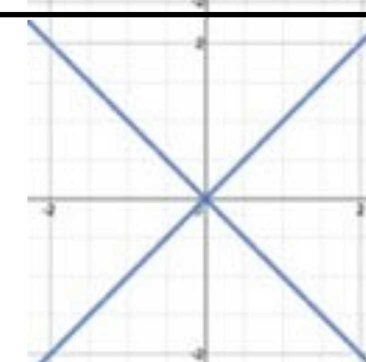
$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$



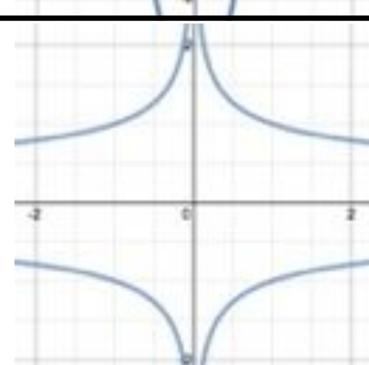
$$\lambda_2 = \lambda_1$$

$$x_2 = Cx_1$$



$$\lambda_2 = -\frac{1}{3}\lambda_1$$

$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$



Shapes of solution curves in the phase plane

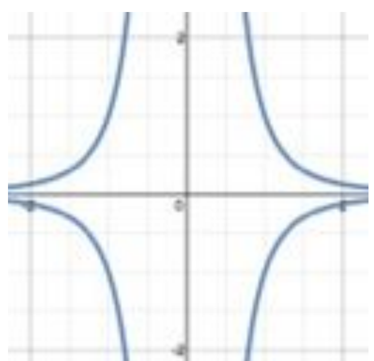
- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\frac{\lambda_2}{\lambda_1}$.

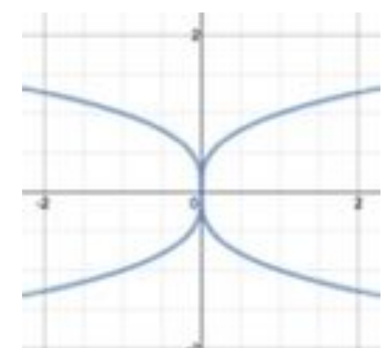
$$\lambda_2 = -3\lambda_1$$

$$x_2 = \frac{C}{x_1^3}$$



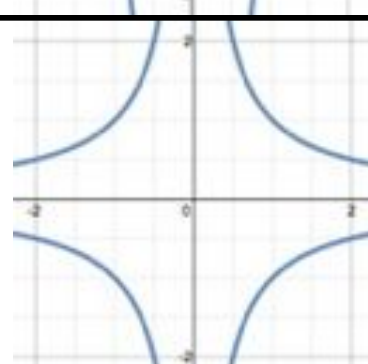
$$\lambda_2 = \frac{1}{3}\lambda_1$$

$$x_2 = C\sqrt[3]{x_1}$$



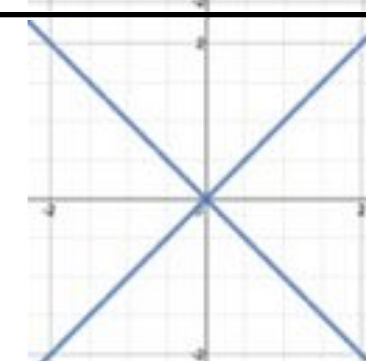
$$\lambda_2 = -\lambda_1$$

$$x_2 = \frac{C}{x_1}$$



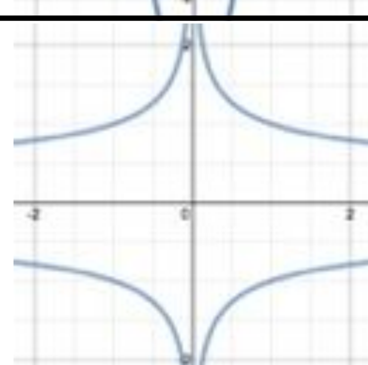
$$\lambda_2 = \lambda_1$$

$$x_2 = Cx_1$$



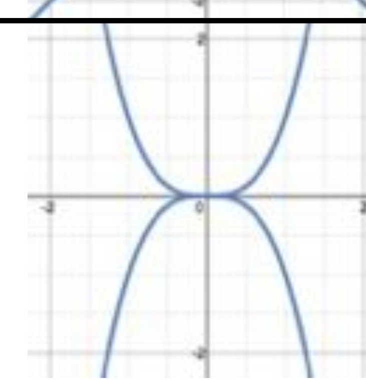
$$\lambda_2 = -\frac{1}{3}\lambda_1$$

$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$



$$\lambda_2 = 3\lambda_1$$

$$x_2 = Cx_1^3$$



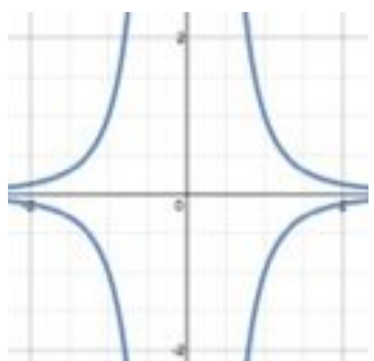
Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

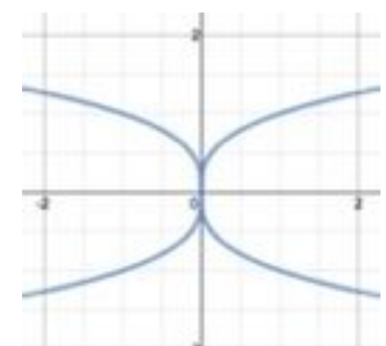
$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

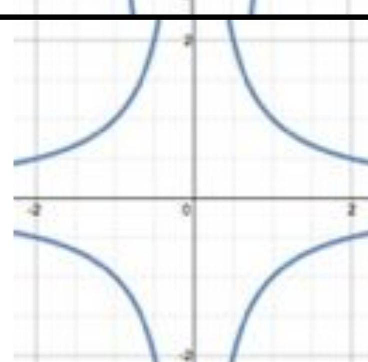
$$\lambda_2 = -3\lambda_1$$
$$x_2 = \frac{C}{x_1^3}$$



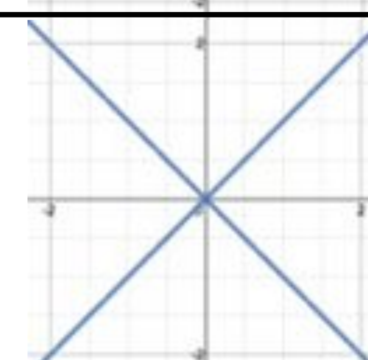
$$\lambda_2 = \frac{1}{3}\lambda_1$$
$$x_2 = C\sqrt[3]{x_1}$$



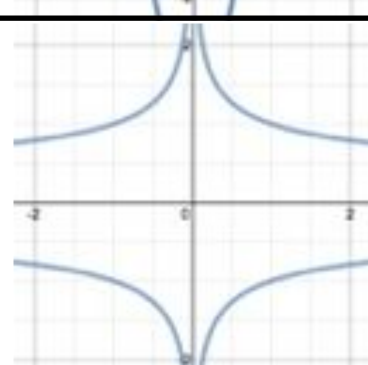
$$\lambda_2 = -\lambda_1$$
$$x_2 = \frac{C}{x_1}$$



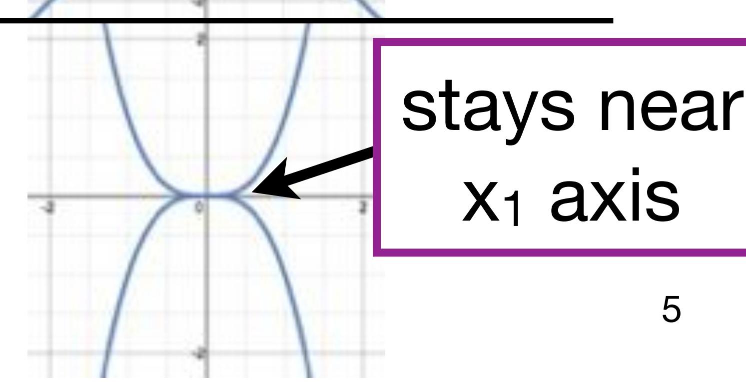
$$\lambda_2 = \lambda_1$$
$$x_2 = Cx_1$$



$$\lambda_2 = -\frac{1}{3}\lambda_1$$
$$x_2 = \frac{C}{\sqrt[3]{x_1}}$$



$$\lambda_2 = 3\lambda_1$$
$$x_2 = Cx_1^3$$



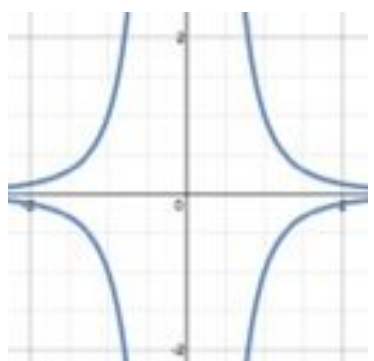
Shapes of solution curves in the phase plane

- Simple example to show general idea. $\mathbf{x}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \mathbf{x}$

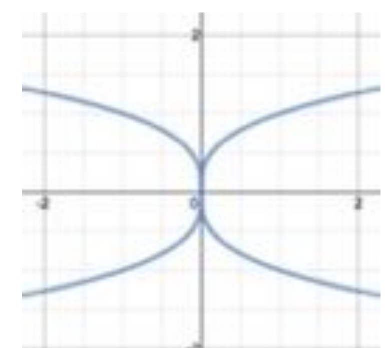
$$x_2 = C_2 \left(\frac{x_1}{C_1} \right)^{\frac{\lambda_2}{\lambda_1}}$$

- For the shape of solutions, we need to know the sign and size of $\dfrac{\lambda_2}{\lambda_1}$.

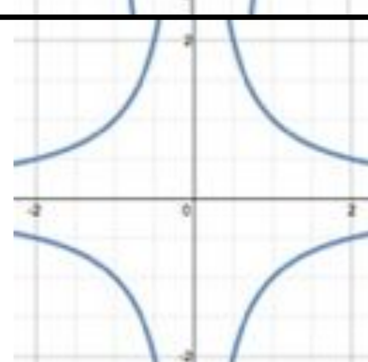
$$\lambda_2 = -3\lambda_1 \qquad x_2 = \frac{C}{x_1^3}$$



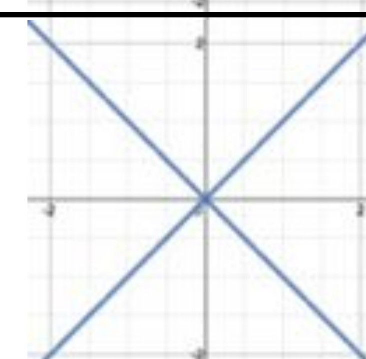
$$\lambda_2 = \frac{1}{3}\lambda_1 \qquad x_2 = C\sqrt[3]{x_1}$$



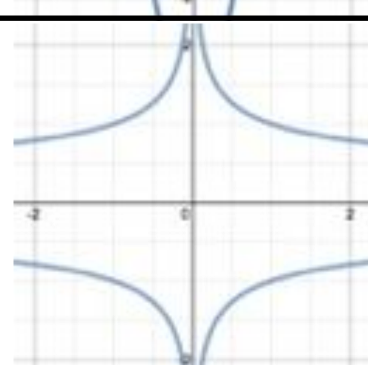
$$\lambda_2 = -\lambda_1 \qquad x_2 = \frac{C}{x_1}$$



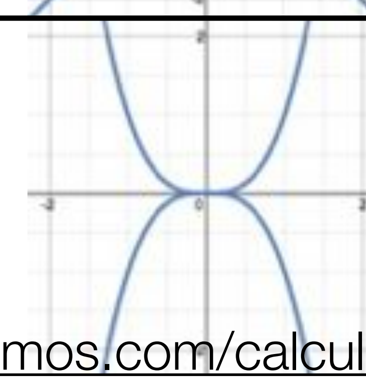
$$\lambda_2 = \lambda_1 \qquad x_2 = Cx_1$$



$$\lambda_2 = -\frac{1}{3}\lambda_1 \qquad x_2 = \frac{C}{\sqrt[3]{x_1}}$$



$$\lambda_2 = 3\lambda_1 \qquad x_2 = Cx_1^3$$



https://www.desmos.com/calculator/c4rhrgotmo

Shapes of solution curves in the phase plane

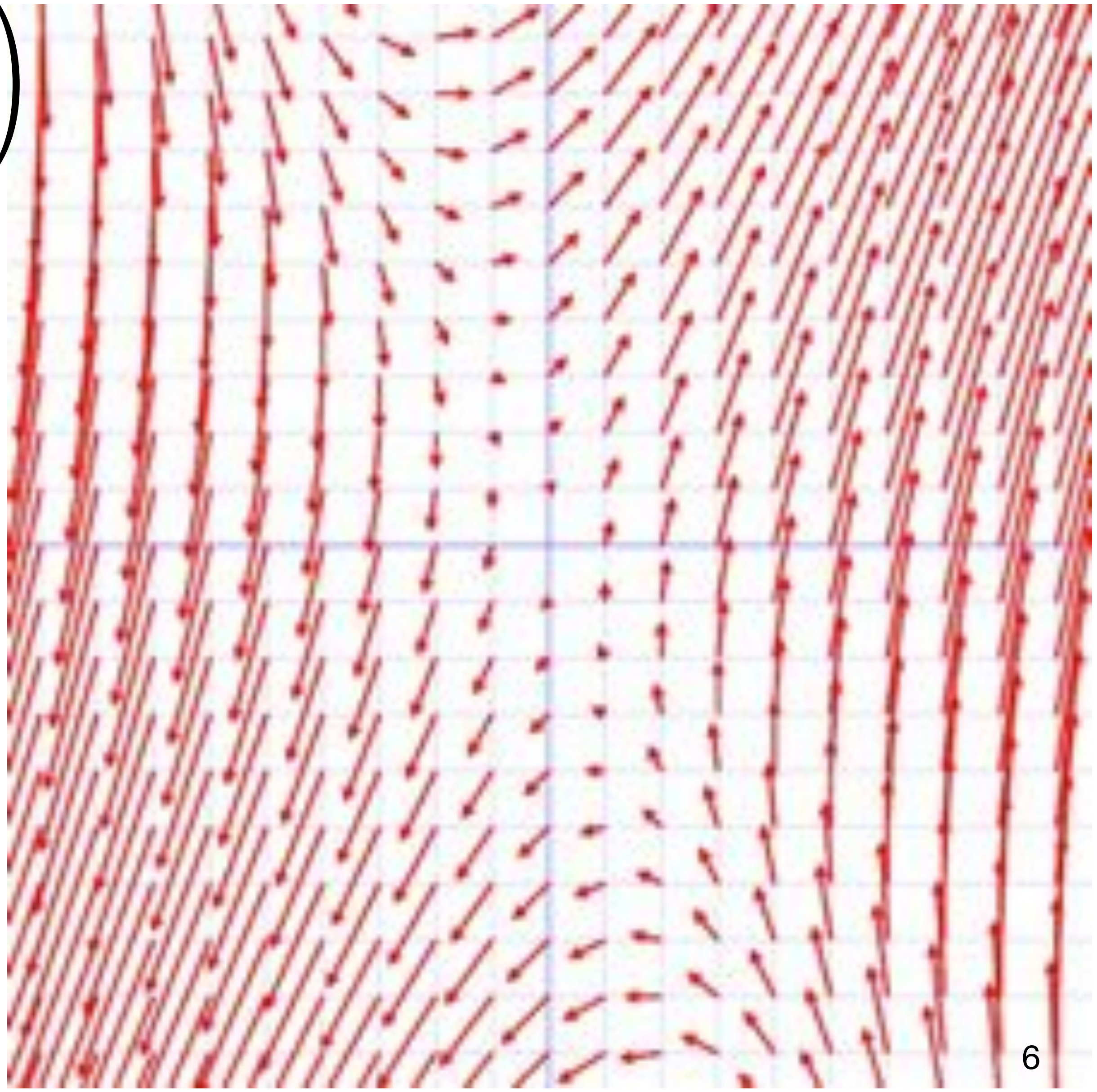
- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

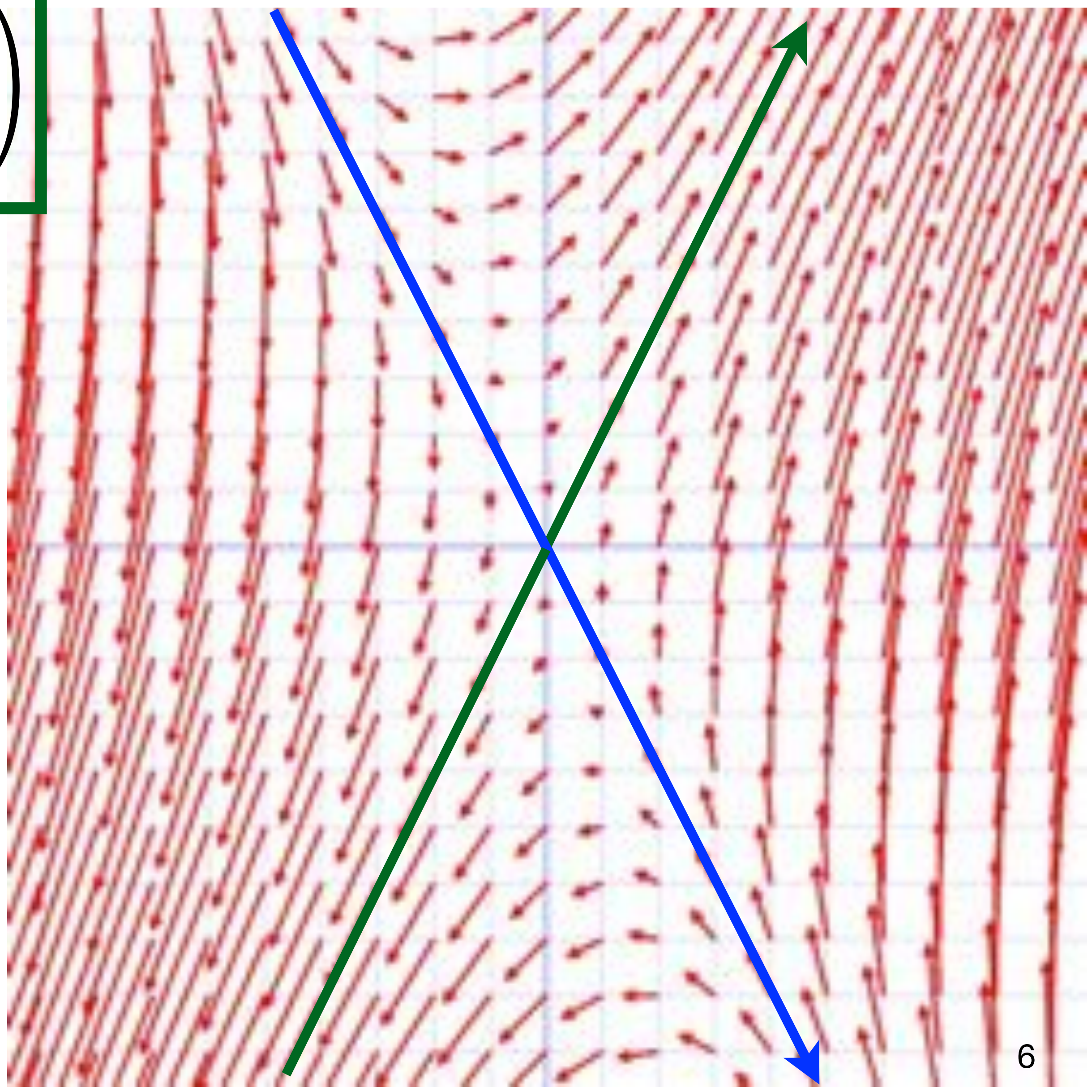
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

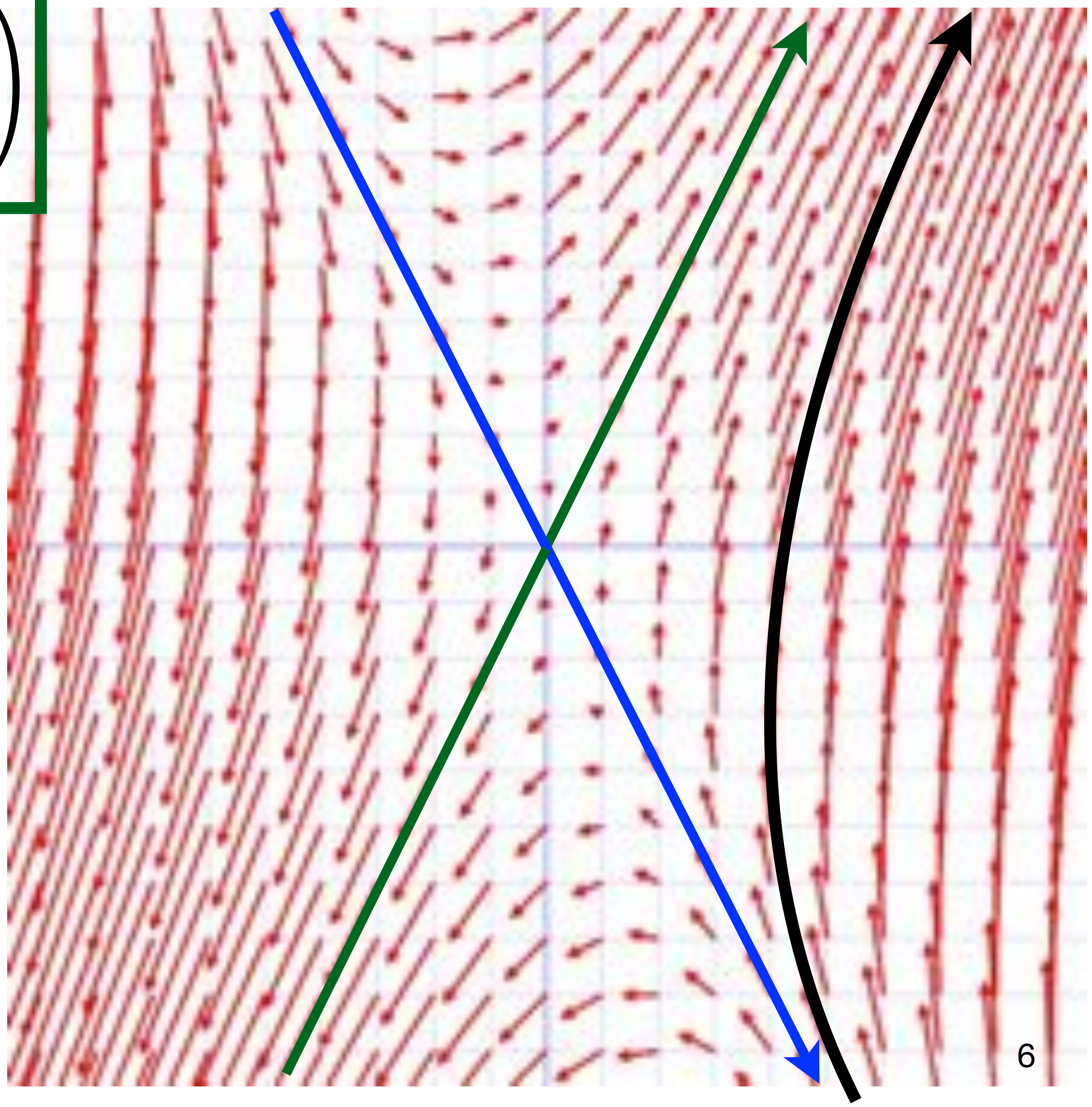
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

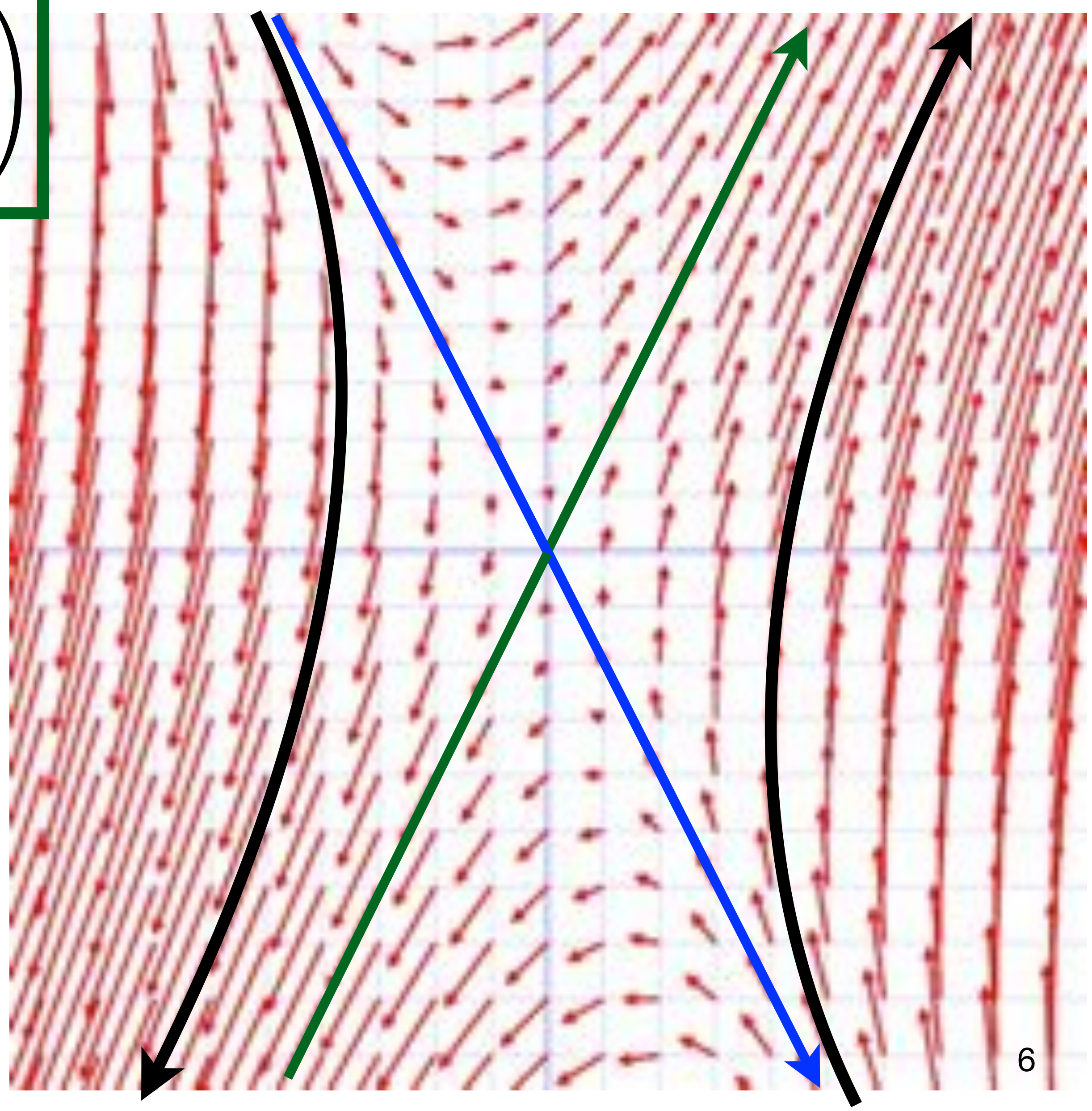
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$



Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

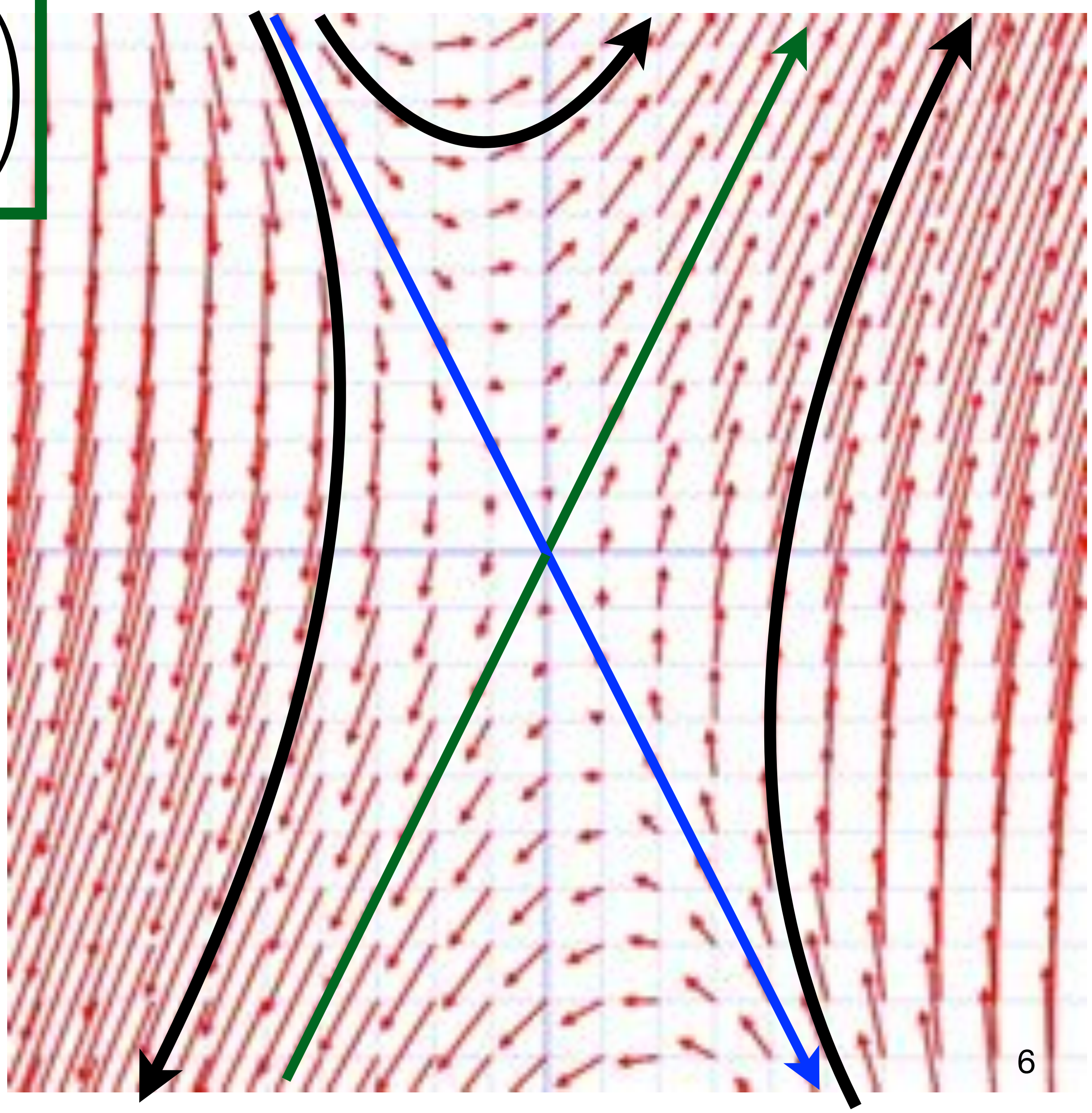
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$



Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

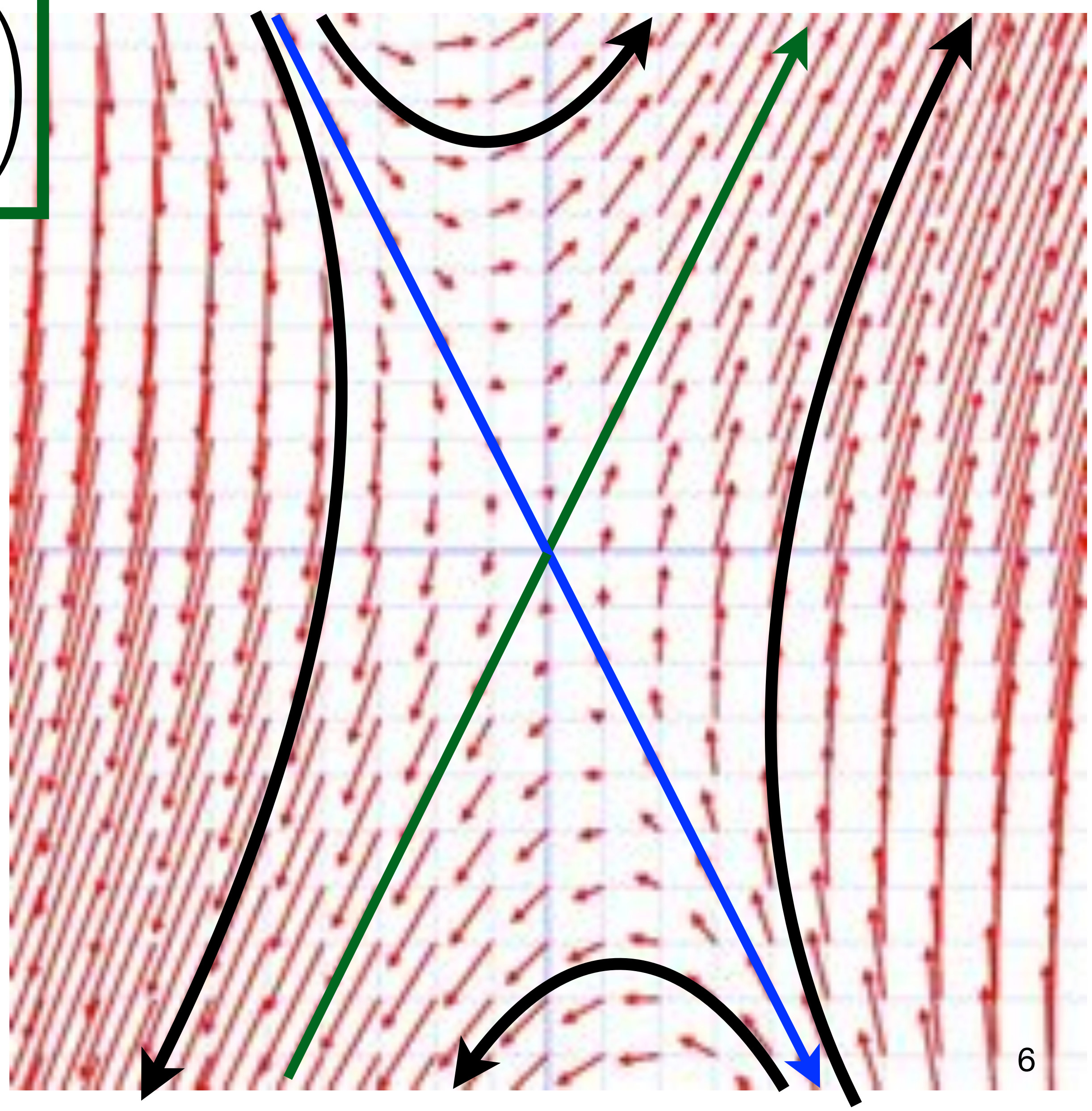
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



Shapes of solution curves in the phase plane

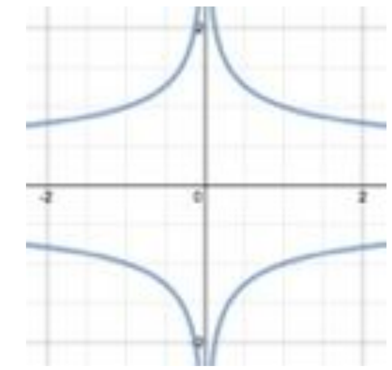
- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

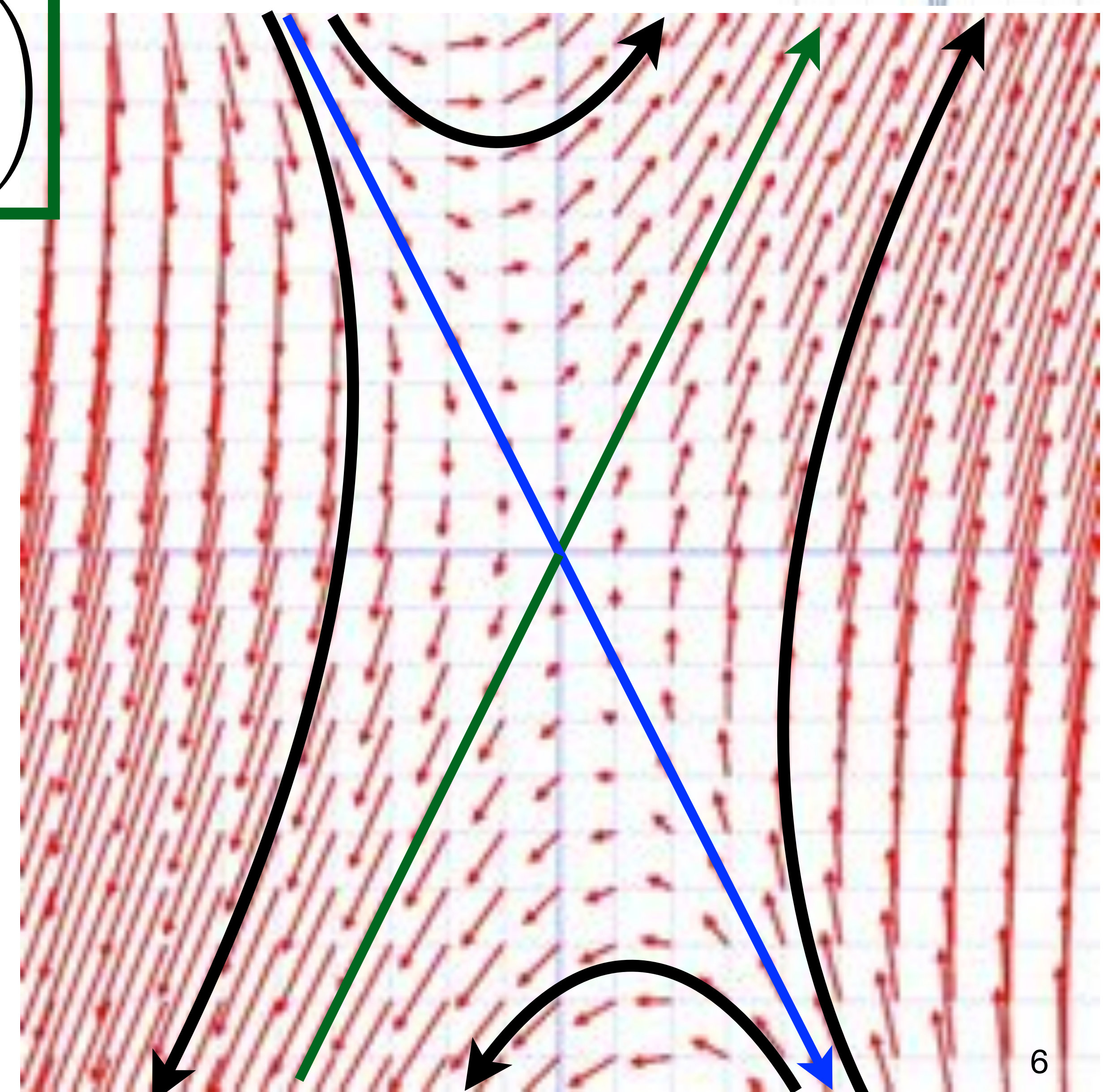


Shapes of solution curves in the phase plane

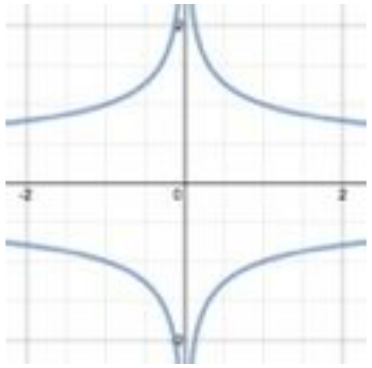
- With more complicated solutions (evectors off-axis), tilt shapes accordingly.



$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$



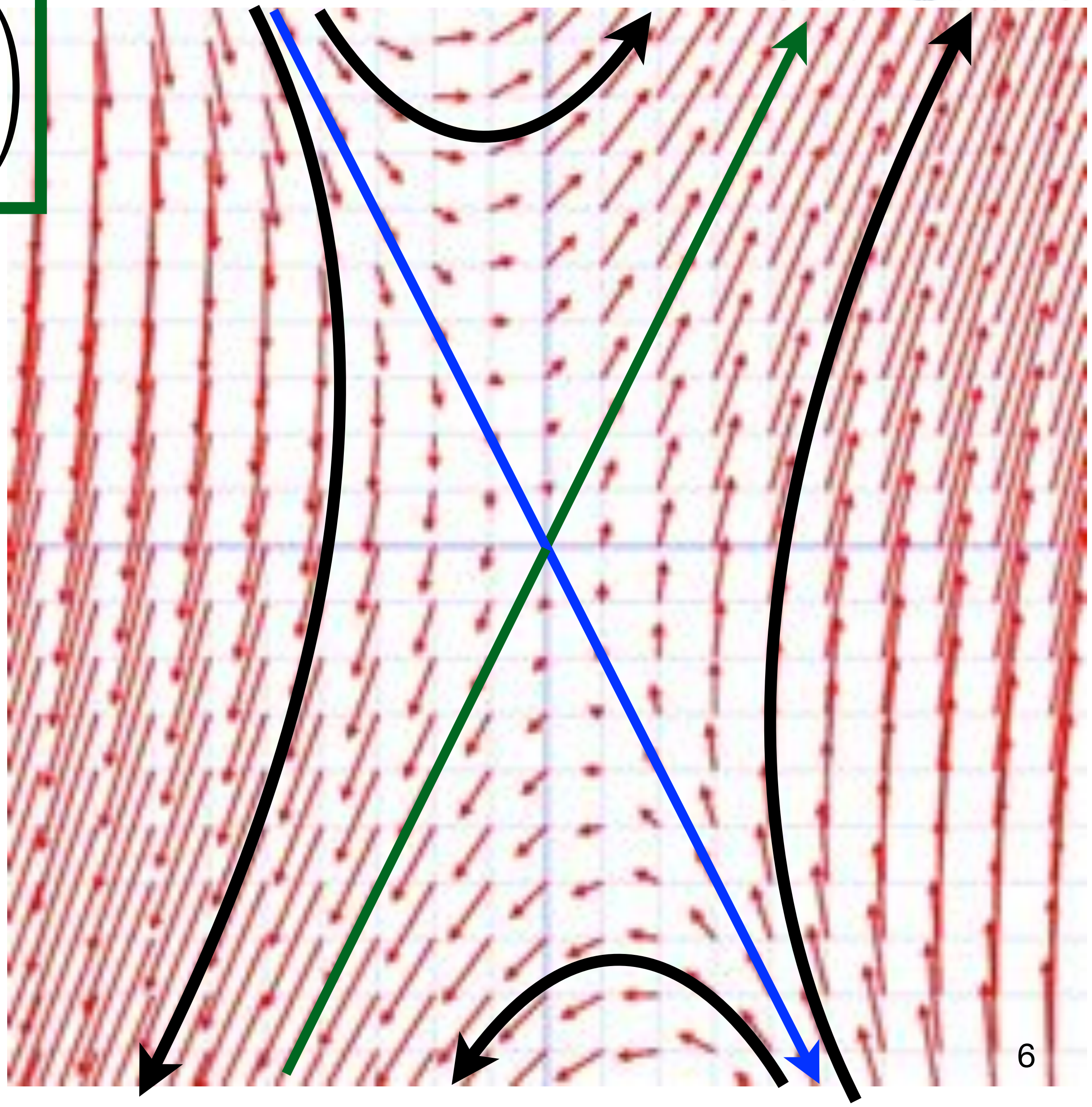
Shapes of solution curves in the phase plane



- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \boxed{C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}} + \boxed{C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

- Going forward in time, the blue component shrinks slower than the green component grows so solutions appear closer to blue "axis" than to green "axis"

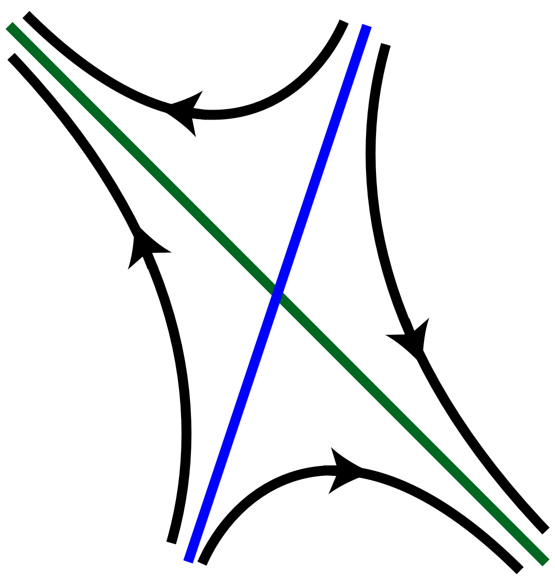


Shapes of solution curves in the phase plane

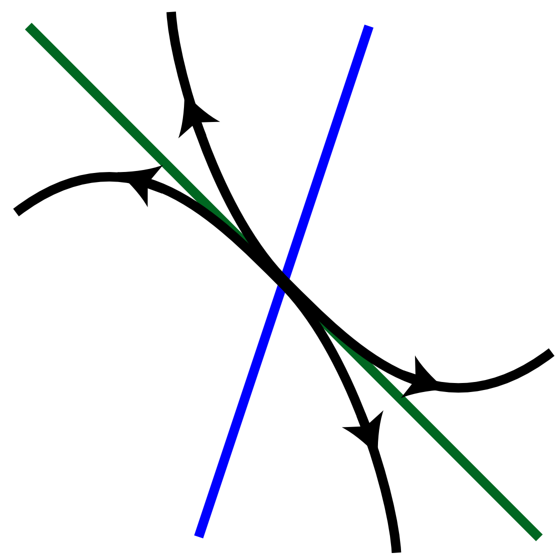
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{3t}\begin{pmatrix}1\\3\end{pmatrix} + C_2 e^{-t}\begin{pmatrix}1\\-1\end{pmatrix}\ ?$$

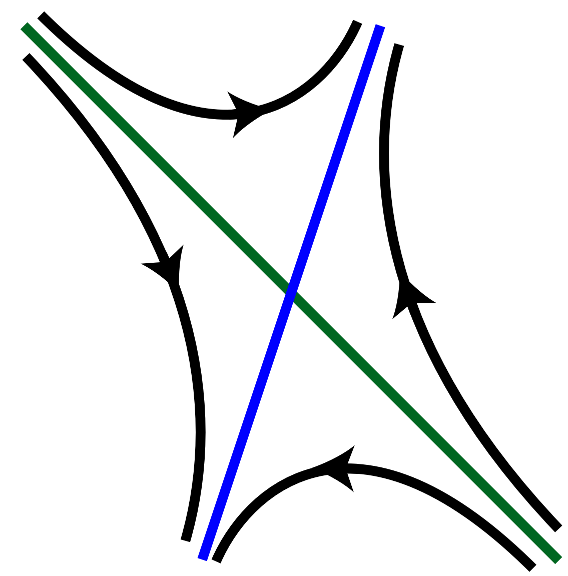
(A)



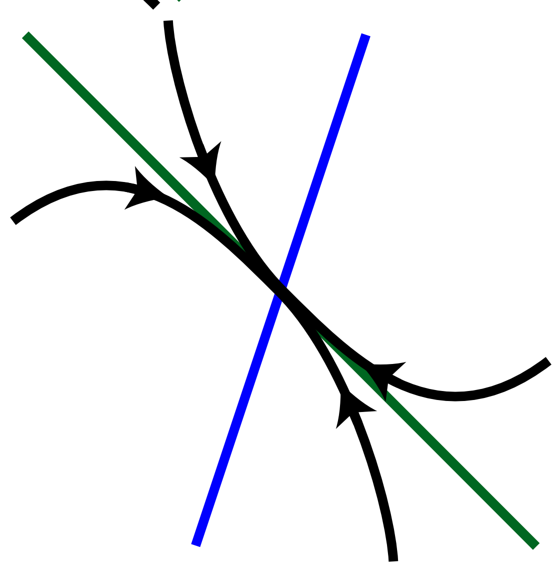
(B)



(C)



(D)



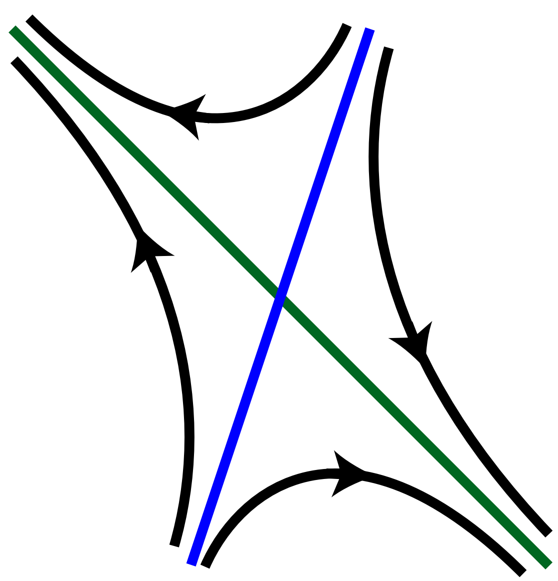
(E) Explain, please.

Shapes of solution curves in the phase plane

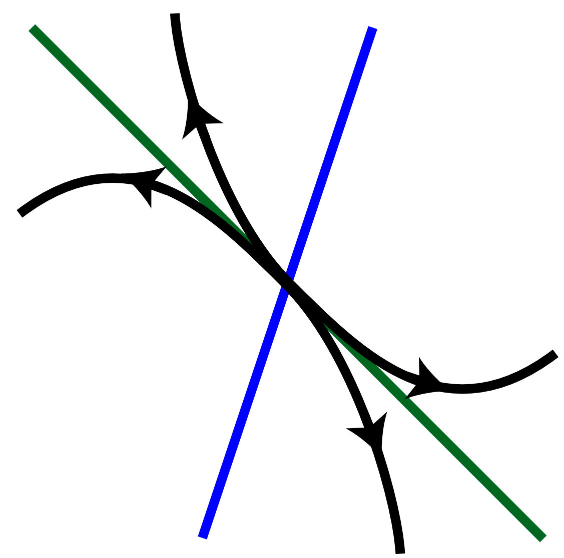
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \ ?$$

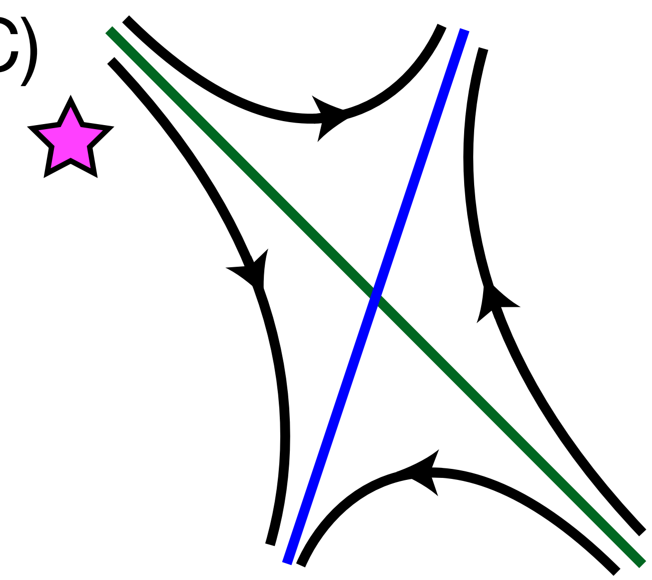
(A)



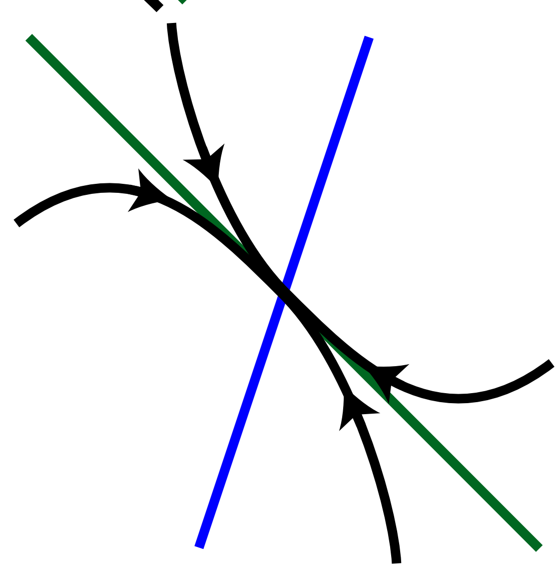
(B)



(C)



(D)



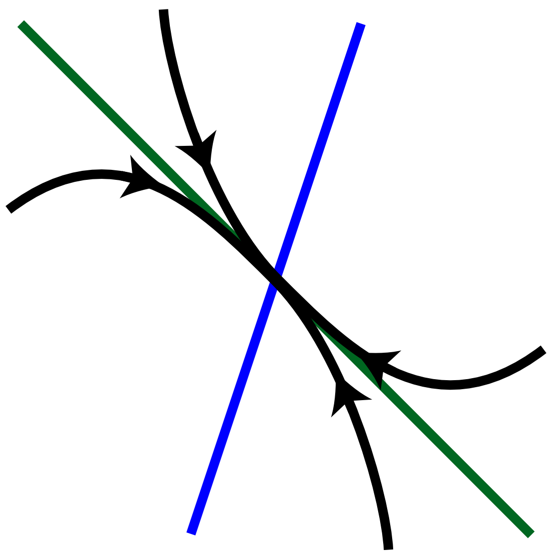
(E) Explain, please.

Shapes of solution curves in the phase plane

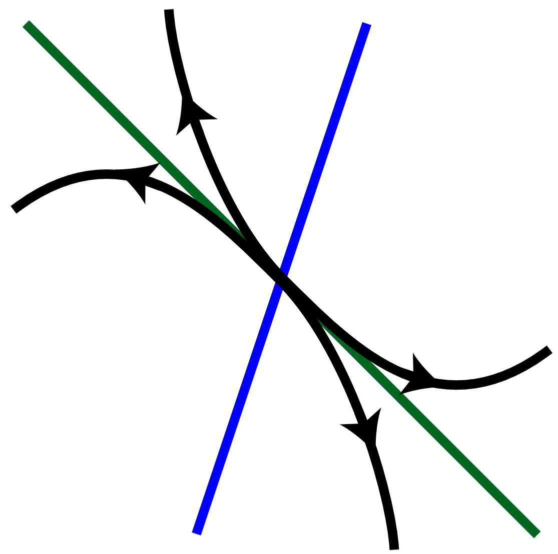
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \; ?$$

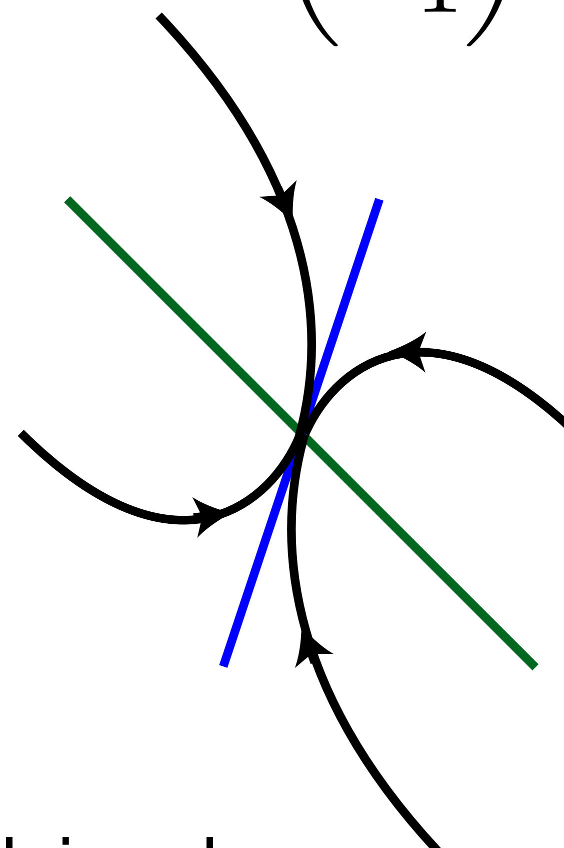
(A)



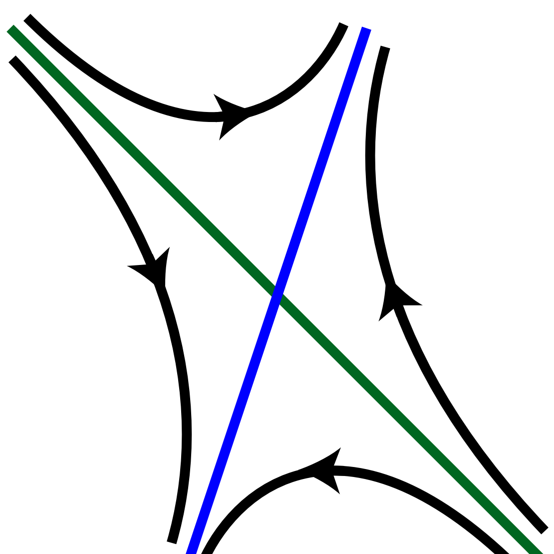
(B)



(C)



(D)



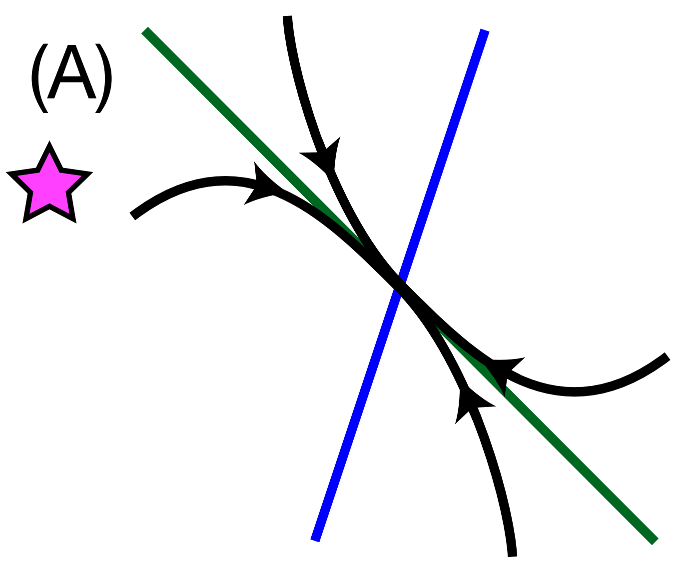
(E) Explain, please.

Shapes of solution curves in the phase plane

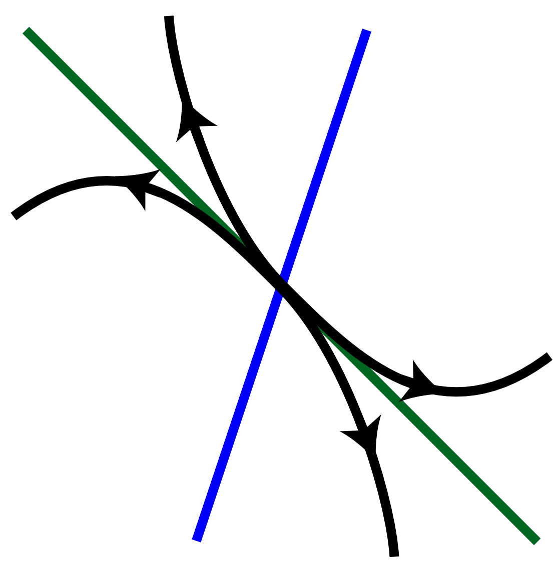
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{-3t}\begin{pmatrix}1\\3\end{pmatrix} + C_2 e^{-t}\begin{pmatrix}1\\-1\end{pmatrix} \ ?$$

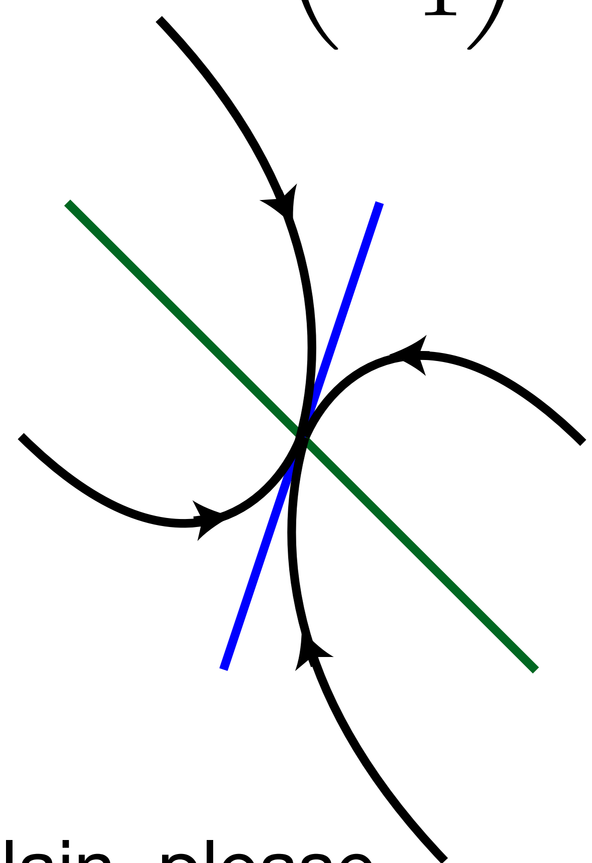
(A)



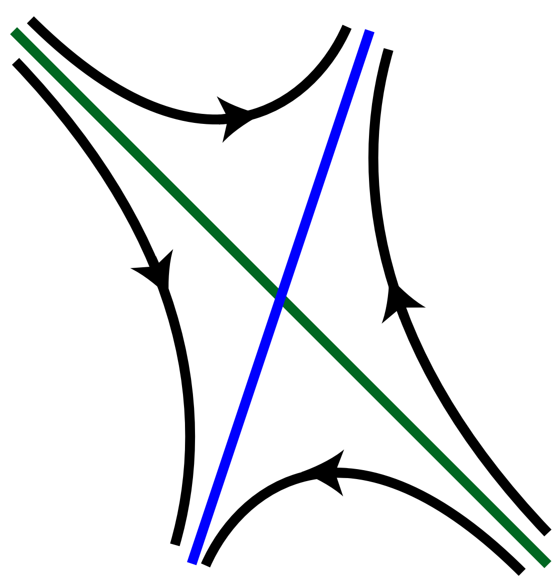
(B)



(C)



(D)



(E) Explain, please.