Today

- I’m out of town Tuesday (Jan 28)
    - no office hours, no lecture,
    - read Variations of Parameters (3.6) - for interest, not on the exam.
- The geometry of homogeneous and nonhomogeneous matrix equations
- Solving nonhomogeneous equations
    - Method of undetermined coefficients

Second order, linear, constant coeff, **non**homogeneous (3.5)

- Our next goal is to figure out how to find solutions to nonhomogeneous equations like this one:

$$y'' - 6y' + 8y = \sin(2t)$$

- But first, a bit more on the connections between matrix algebra and differential equations . . .

Some connections to linear (matrix) algebra

- A homogeneous matrix equation has the form

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$$L[y] = 0$$

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- A non-homogeneous differential equation has the form

$$L[y] = g(t)$$

Solutions to homogeneous matrix equations

- The matrix equation $A\overline{x} = \overline{0}$ could have (depending on A)

(A) no solutions.

(B) exactly one solution.

(C) a one-parameter family of solutions.

(D) an n-parameter family of solutions.

Choose the answer that is incorrect.

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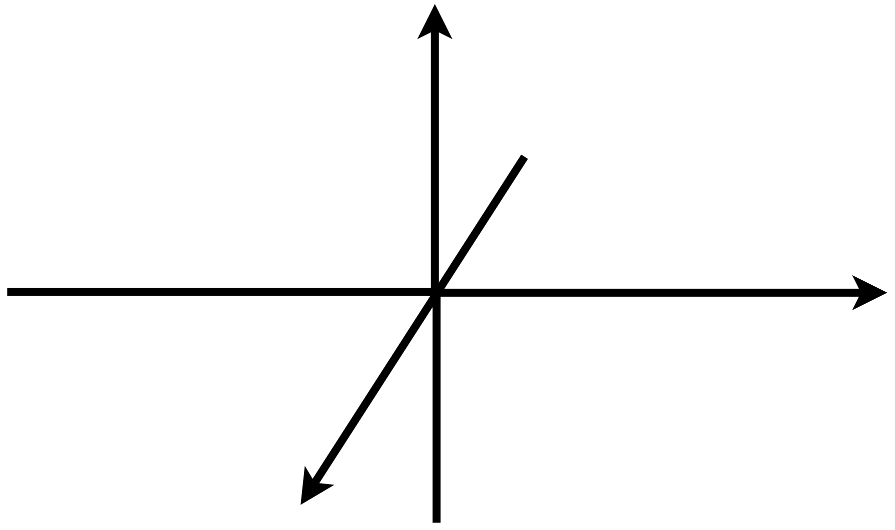

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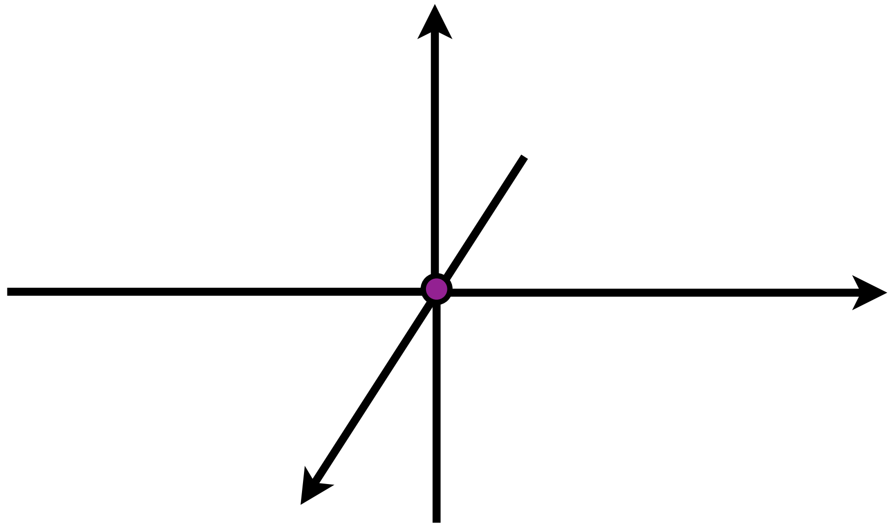

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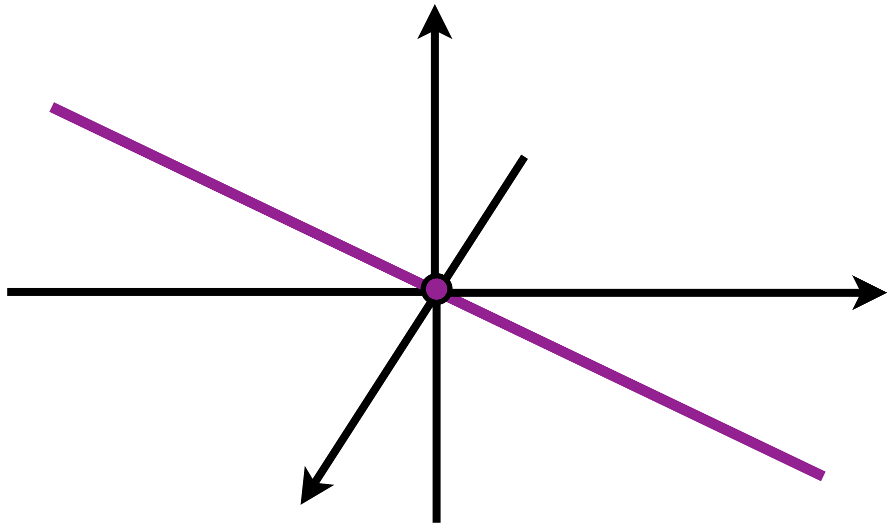

Possibilities:

$$\overline{x} = C\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

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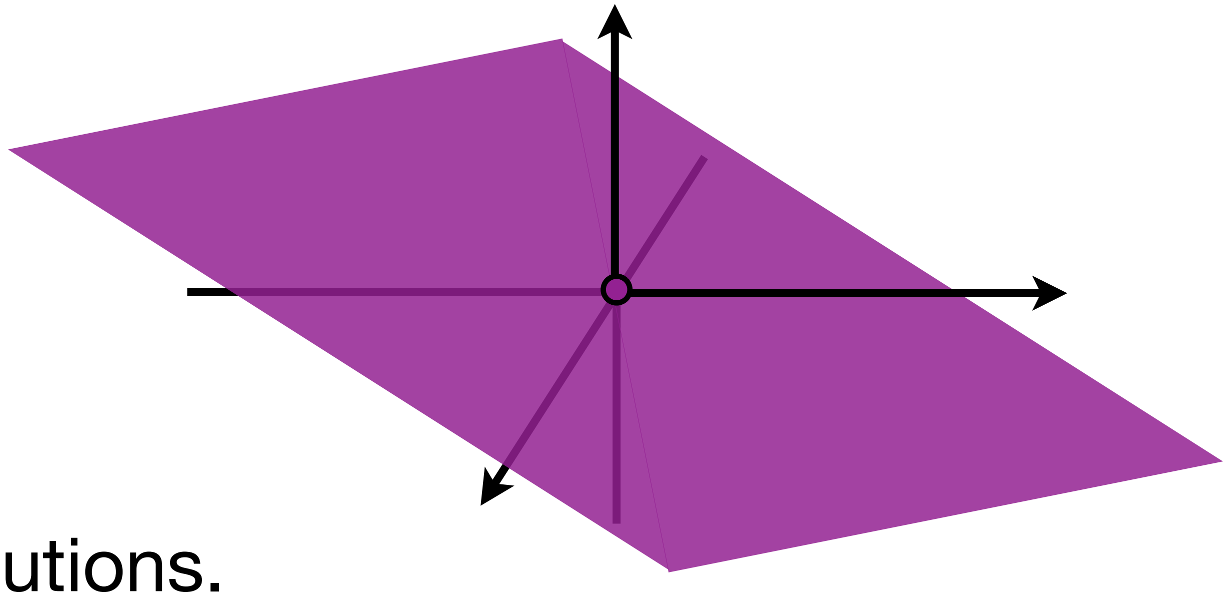

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Possibilities:

$$\overline{x} = C_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Choose the answer that is incorrect.

Solutions to homogeneous matrix equations

- Example 1. Solve the equation $A\overline{x} = \overline{0}$ where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & -2 \\ 2 & 1 & 1 \end{pmatrix}$$

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In this case, only two of them really matter.

- so $x_1 - \frac{1}{3}x_3 = 0$ and $x_2 + \frac{5}{3}x_3 = 0$ and x_3 can be whatever (because it doesn't have a leading one).

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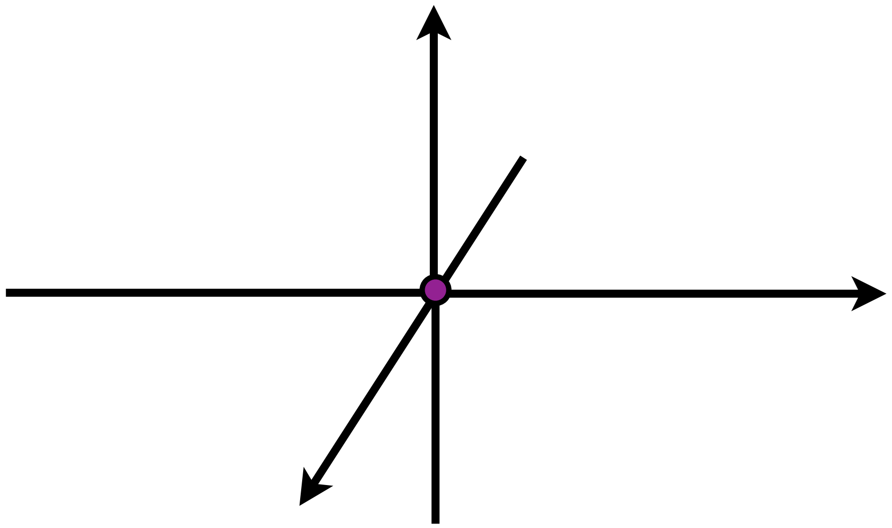

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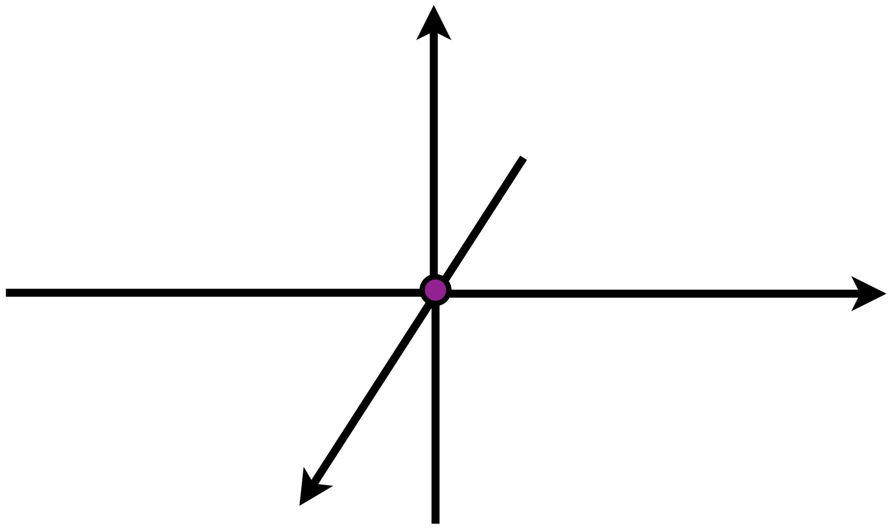

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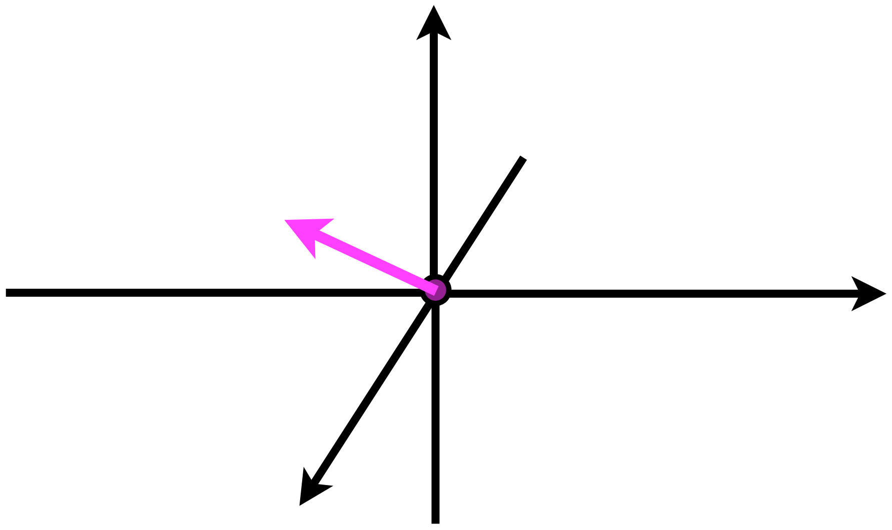

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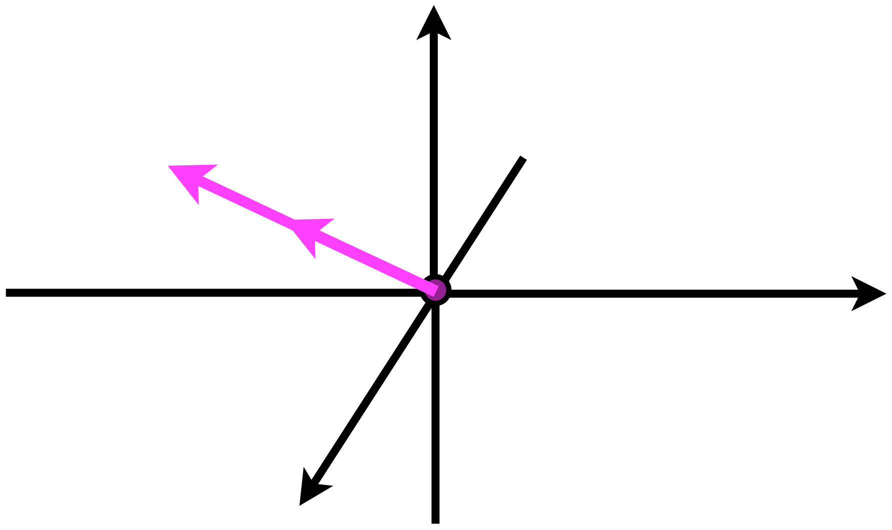

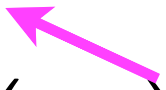

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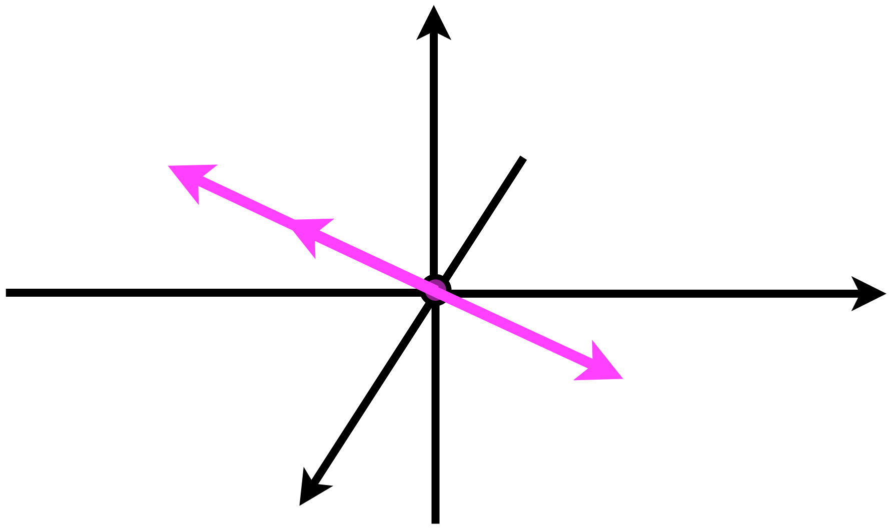

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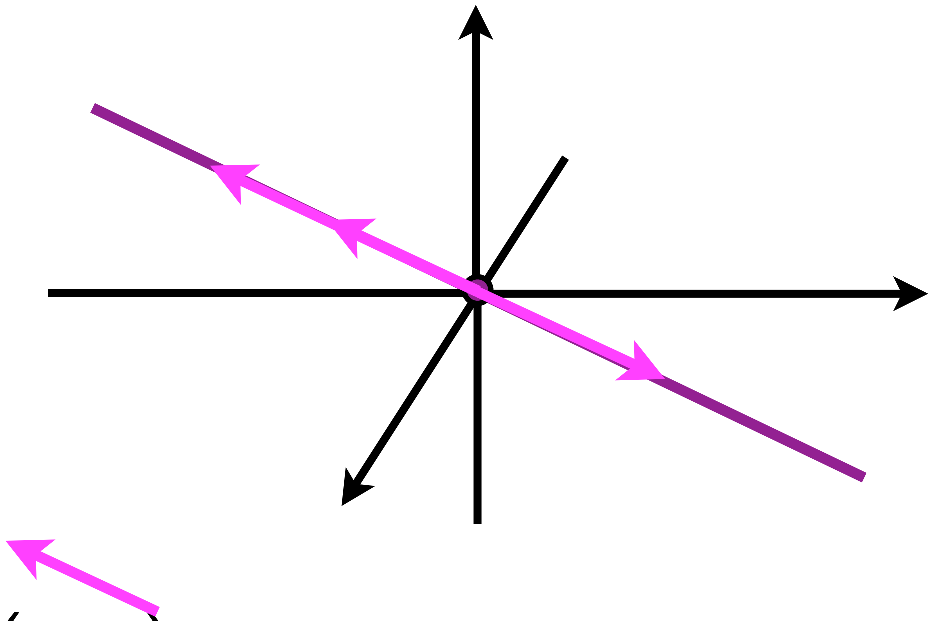

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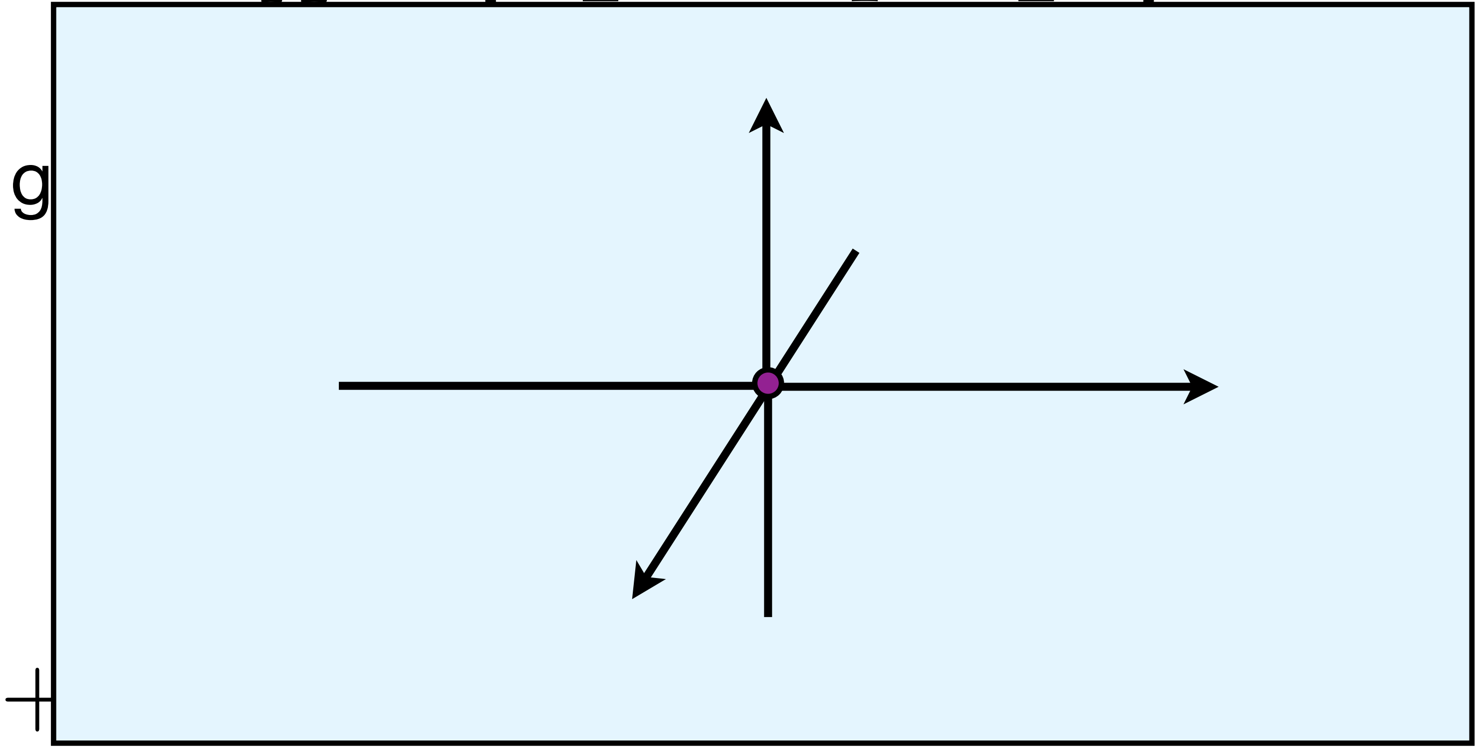

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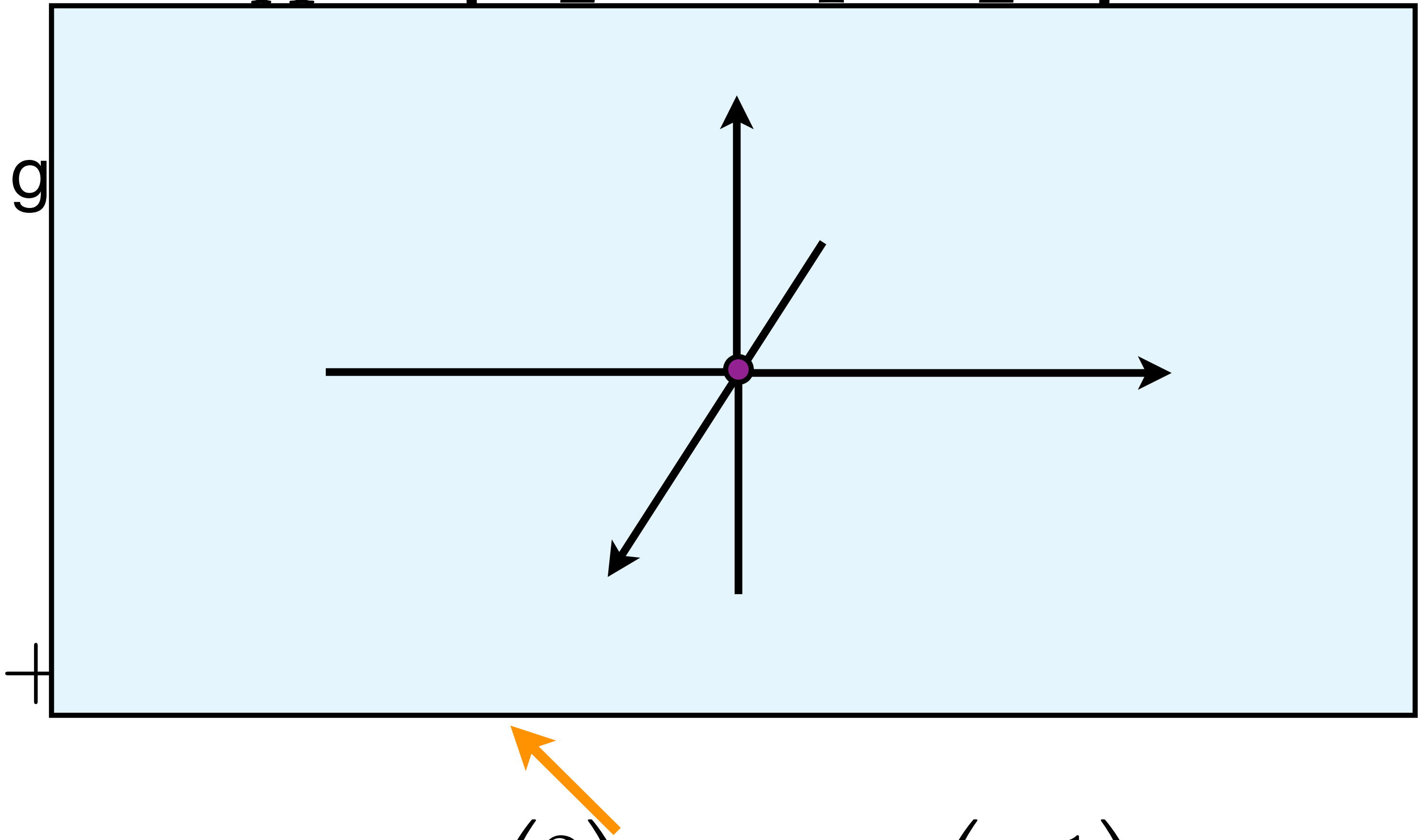

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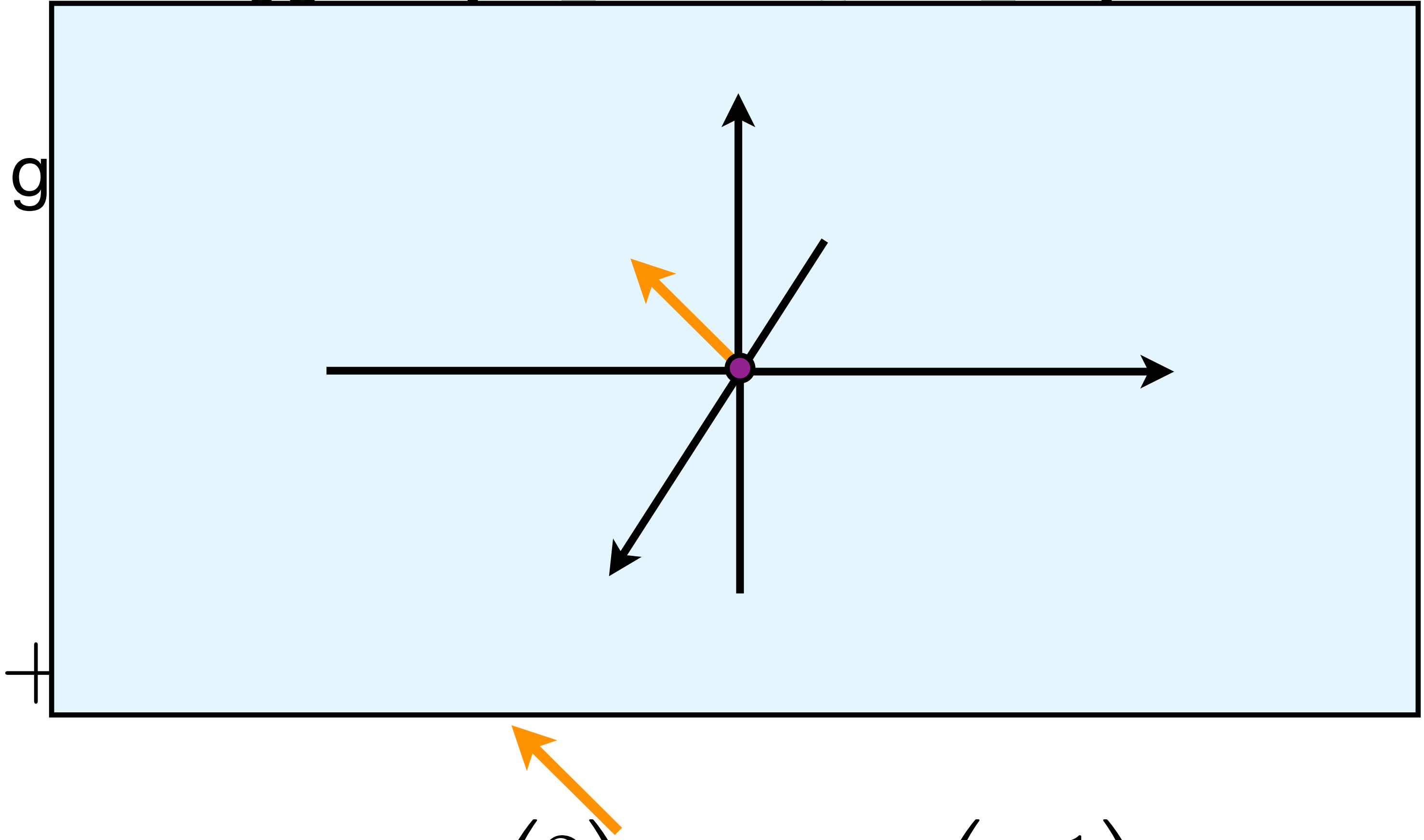

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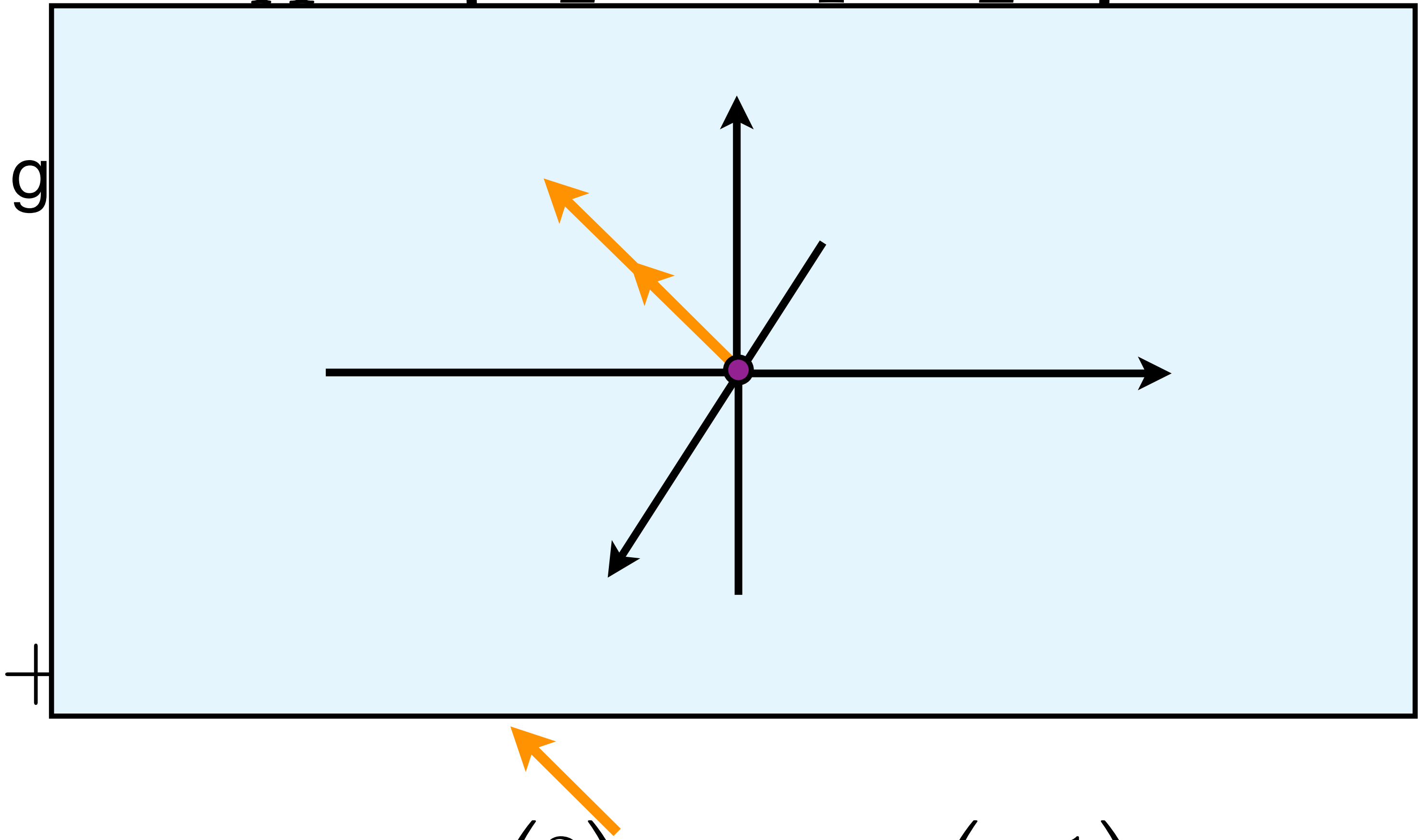

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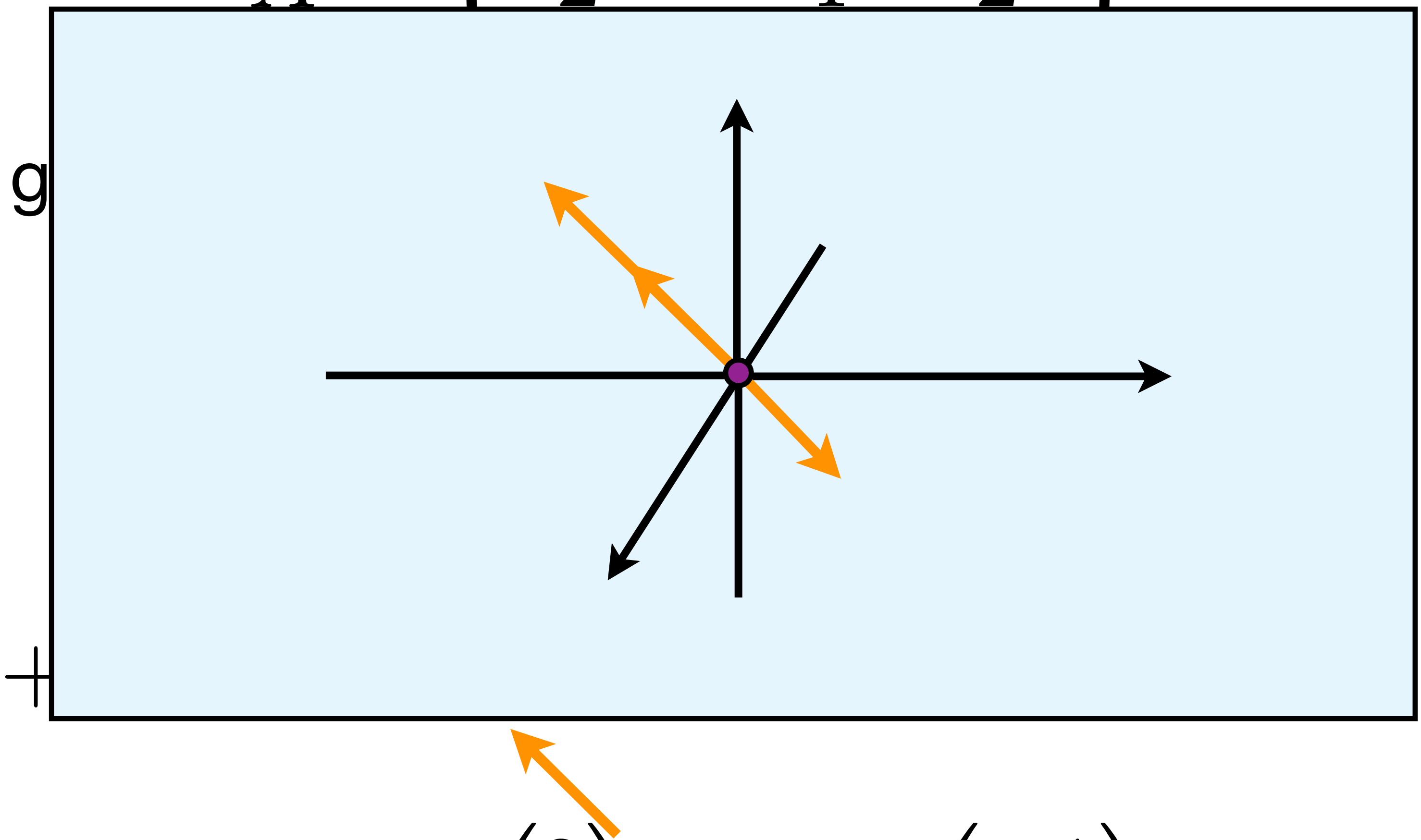

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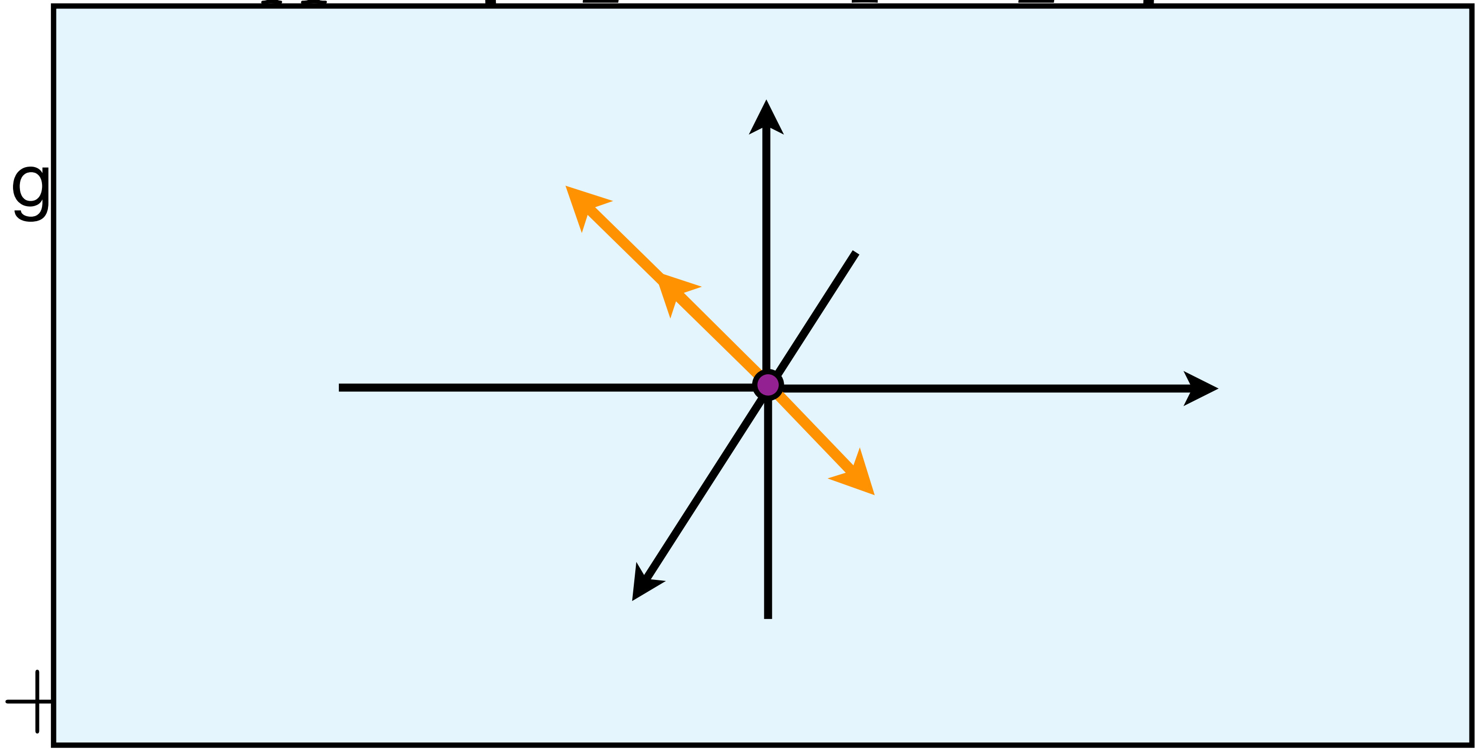

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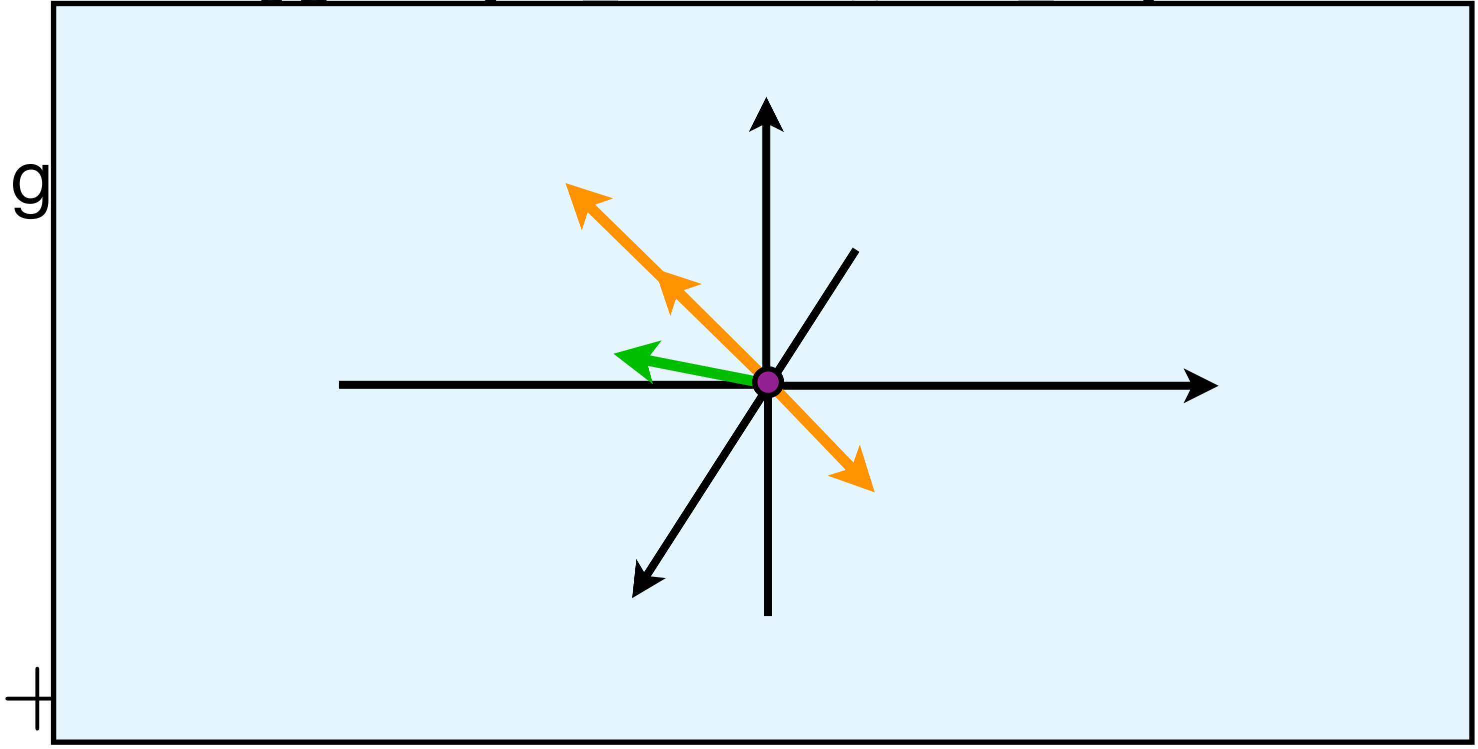

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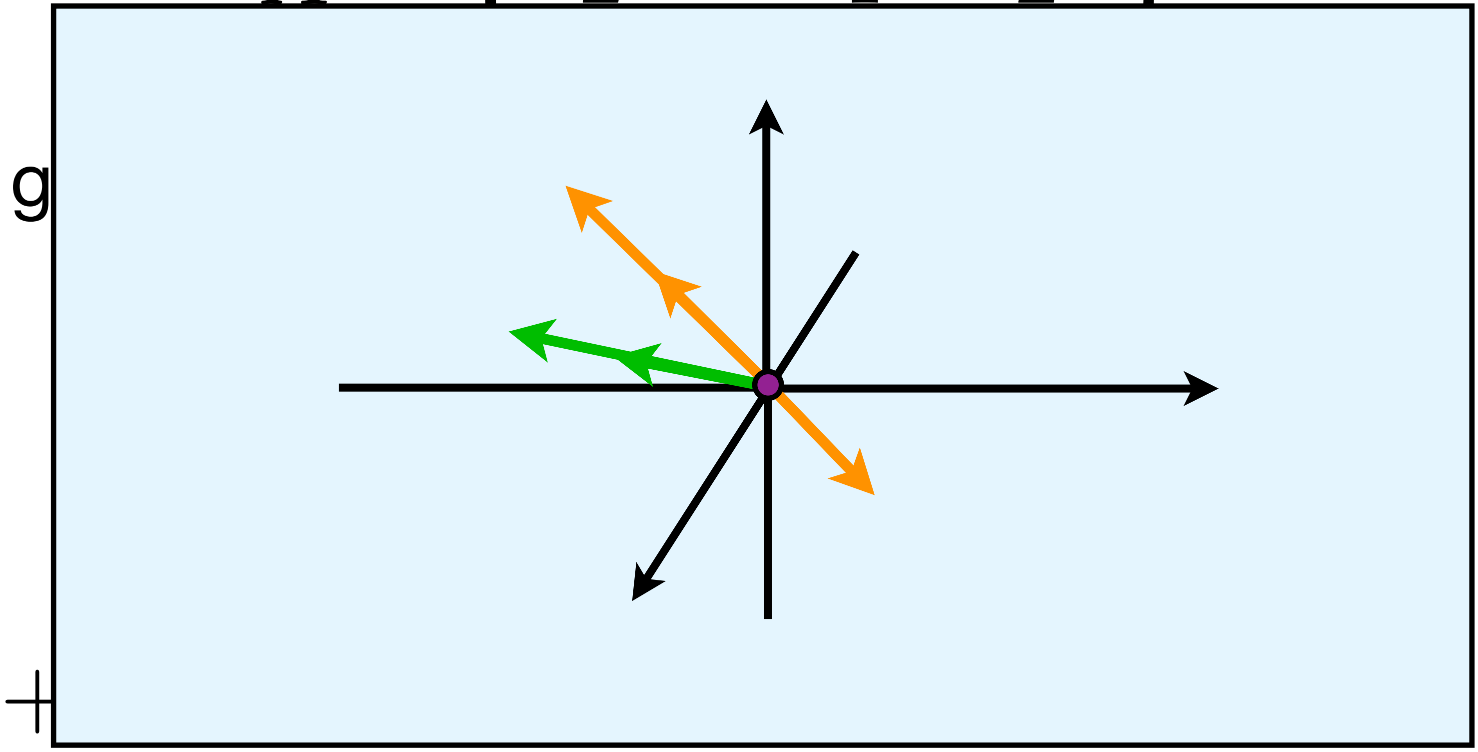

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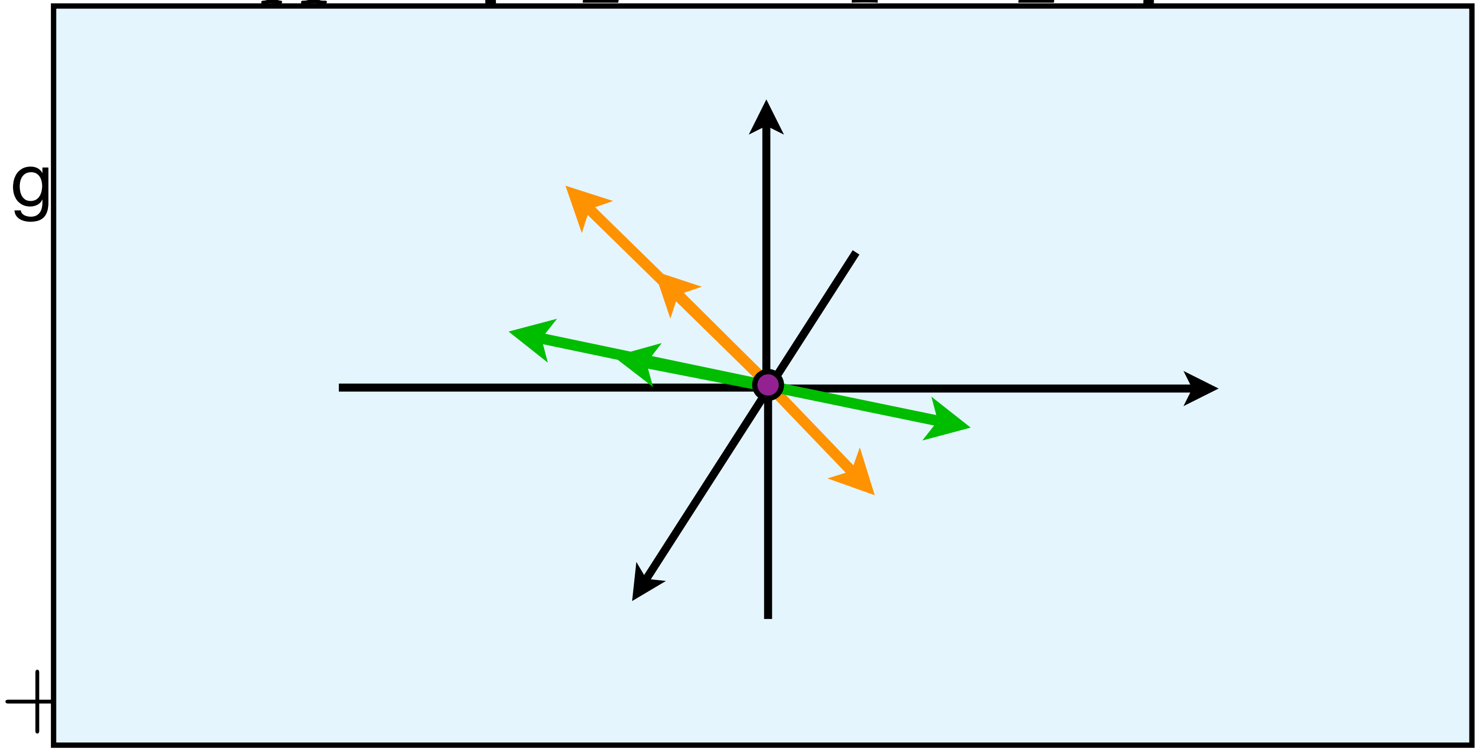

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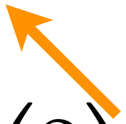

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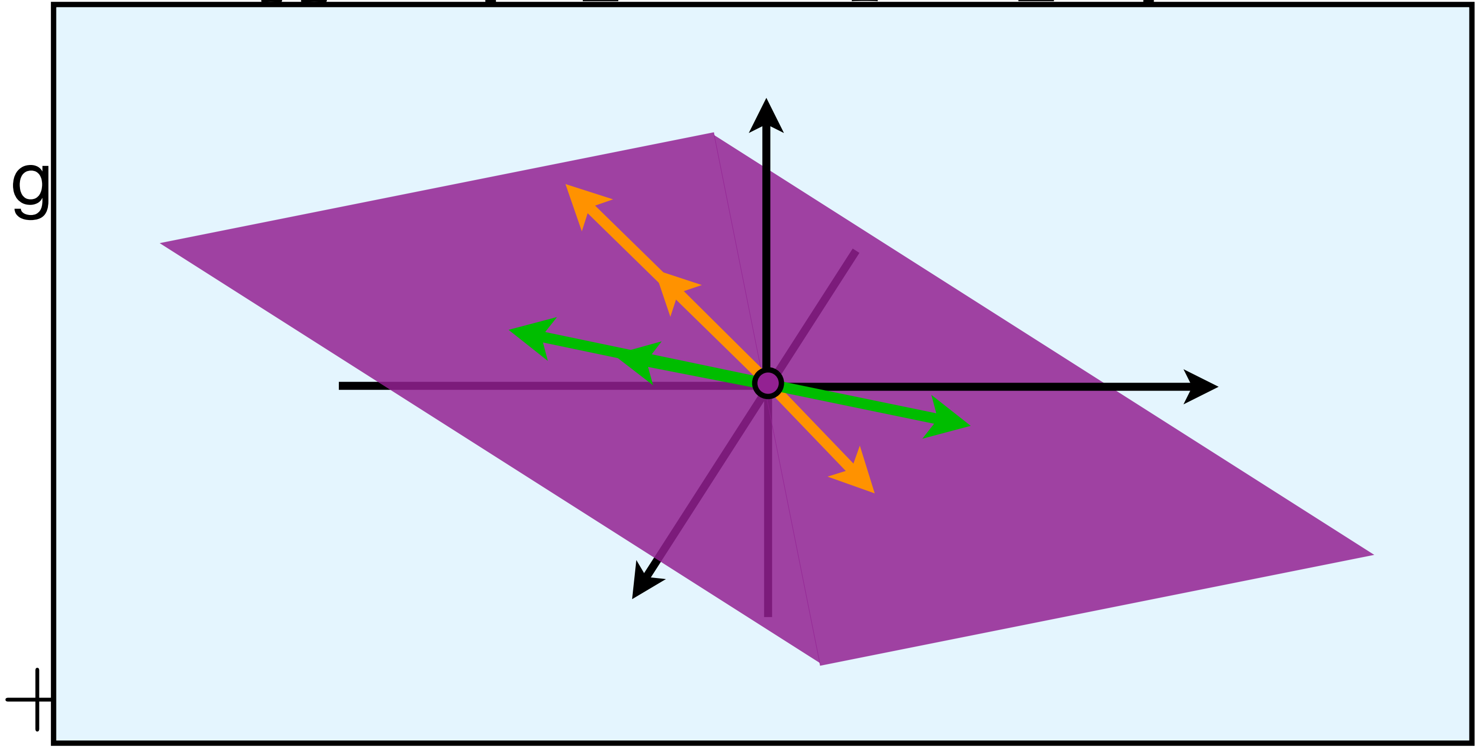

- Row reduction g

- so $x_1 - 2x_2 +$ atever.

$$\overline{x} = C_1 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Solutions to non-homogeneous matrix equations

- Example 3. Solve the equation $A\overline{x} = \overline{b}$ where

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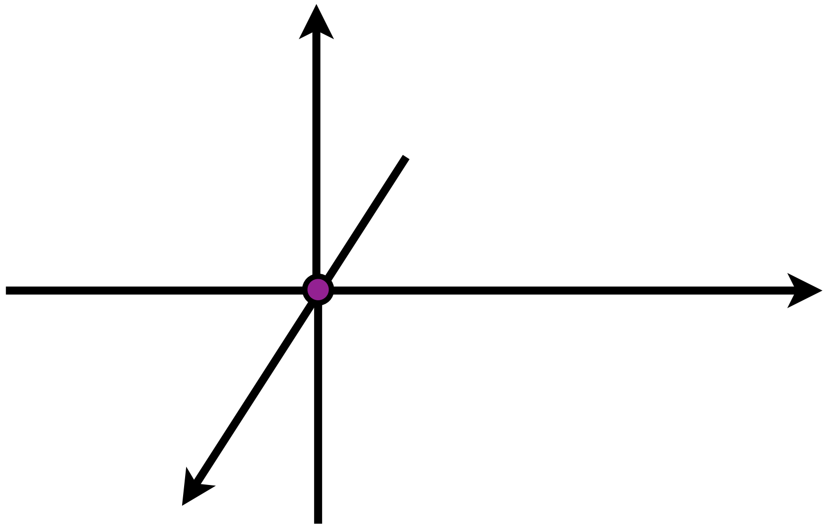

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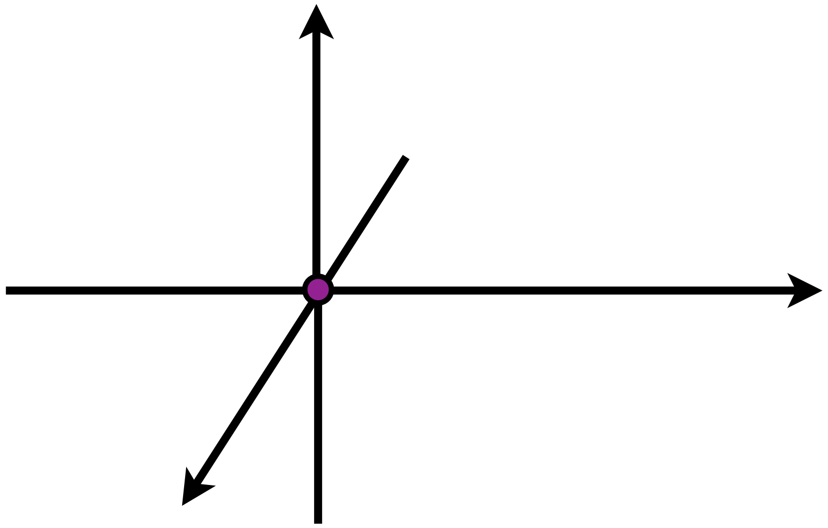

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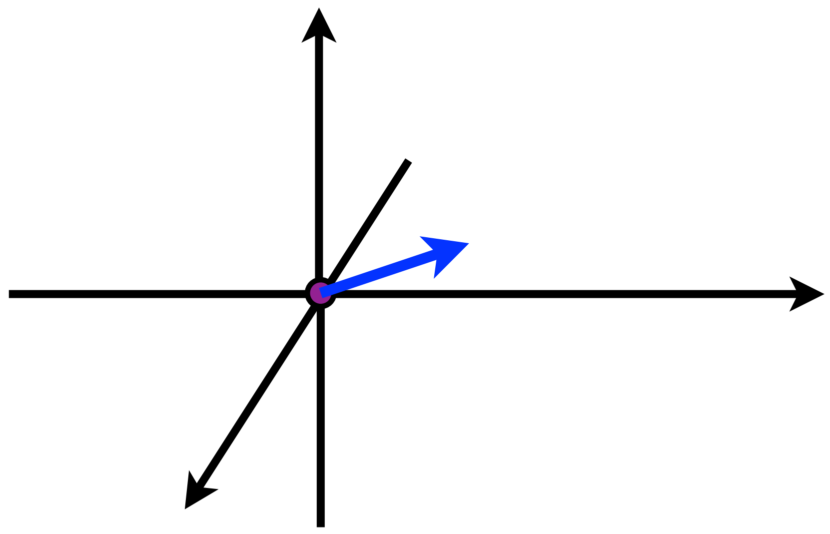

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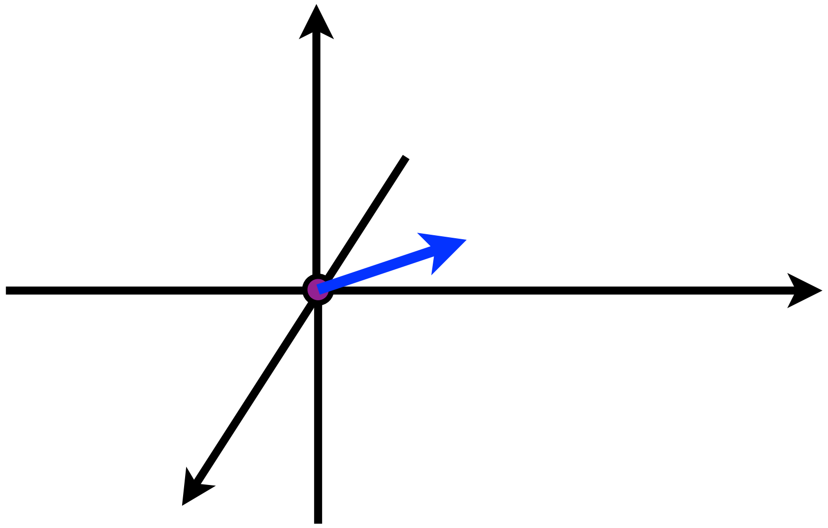

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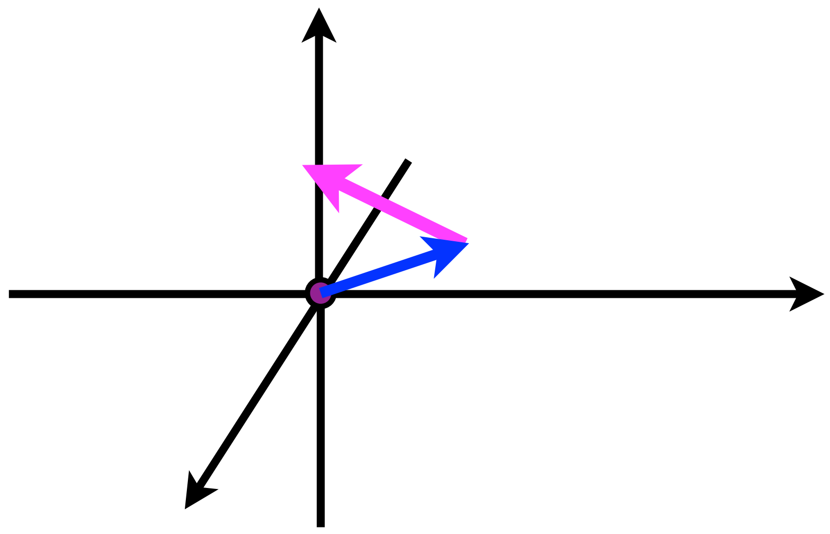

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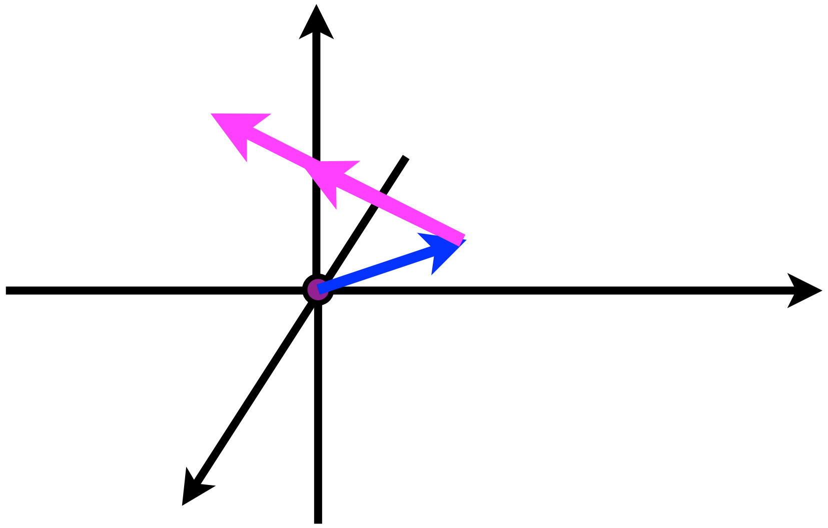

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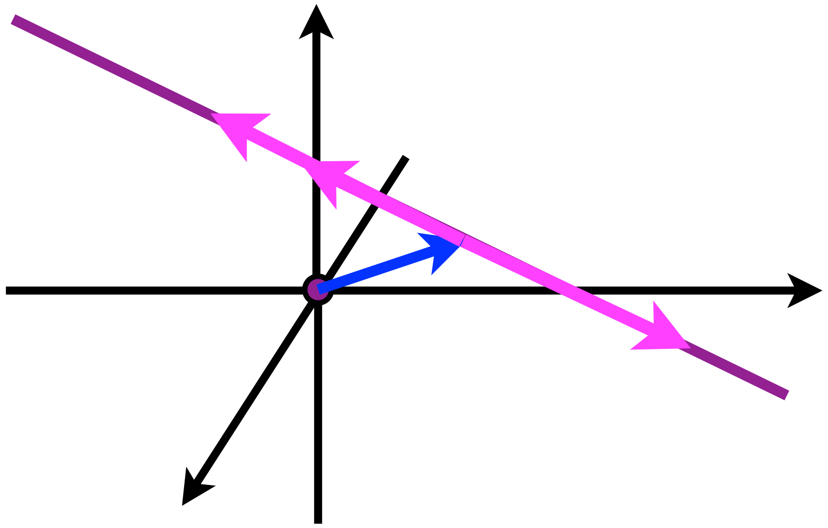

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Solutions to nonhomogeneous differential equations

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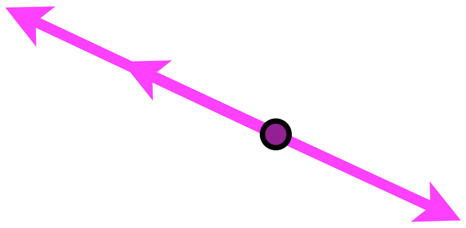

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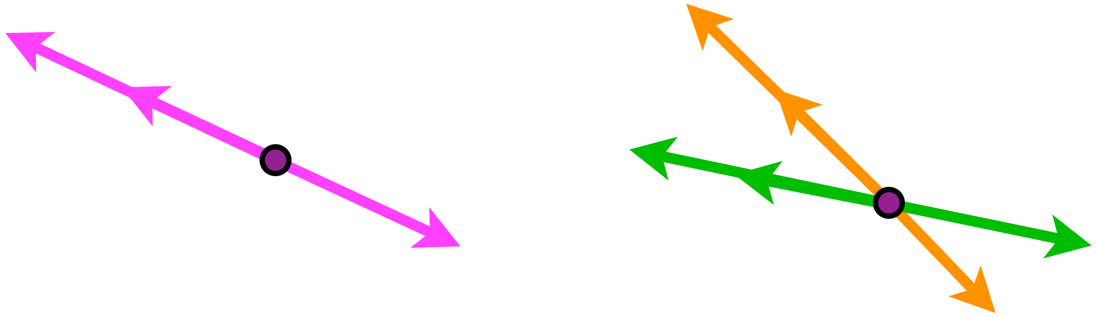

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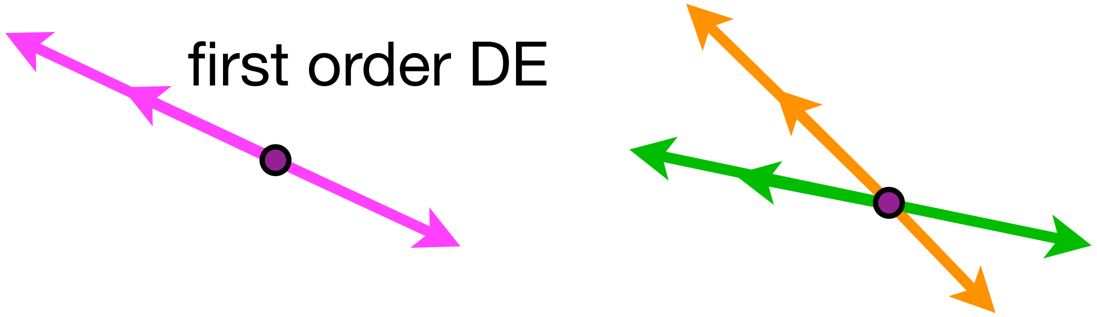

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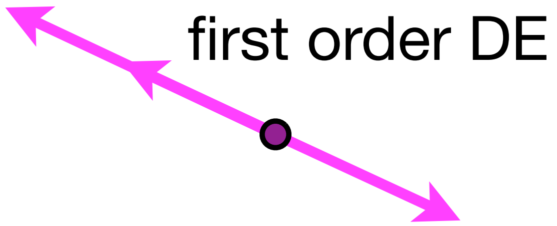

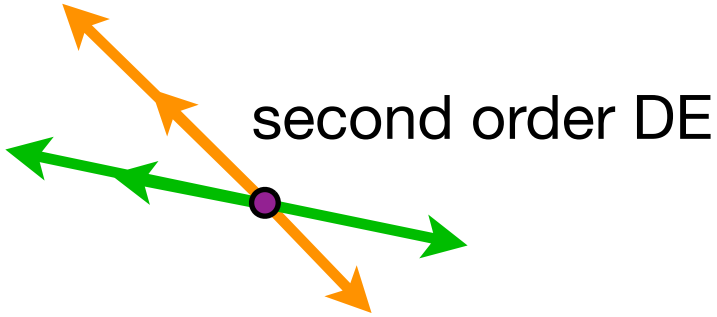

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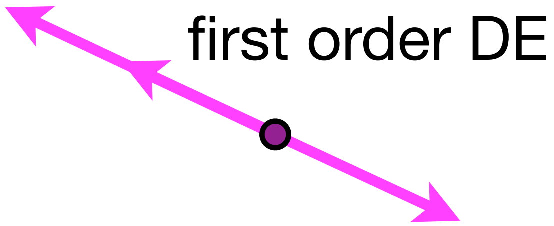

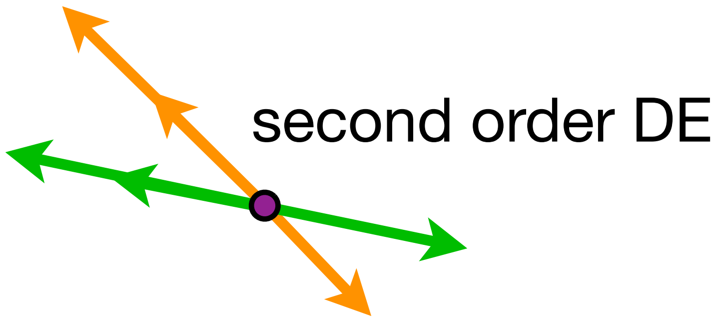

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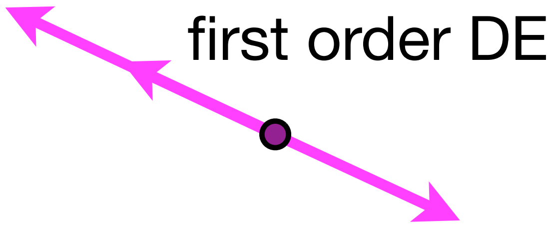

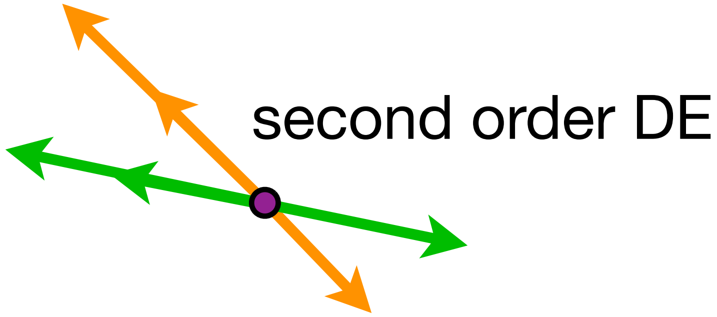

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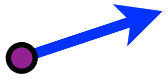

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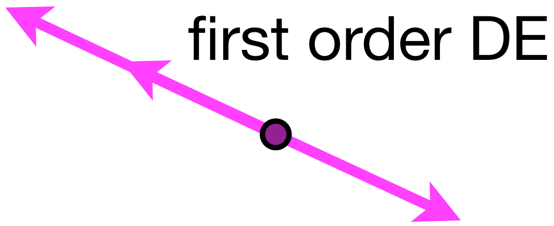

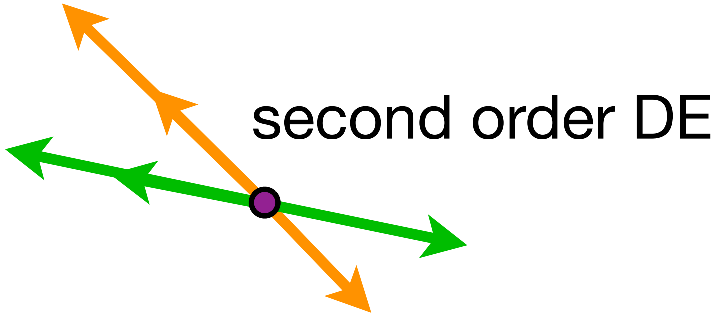

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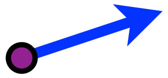

  3. The general solution to the nonhomogeneous problem is their sum:

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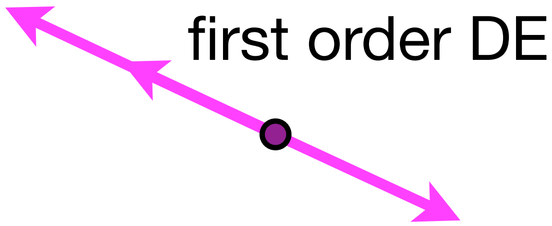

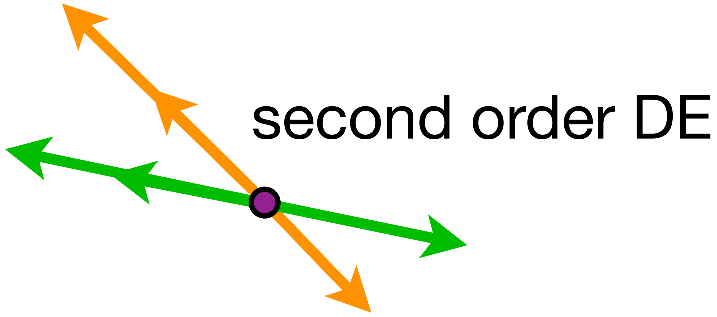

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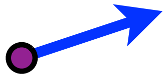

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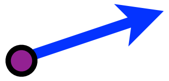

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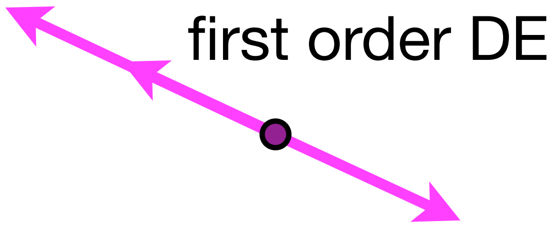

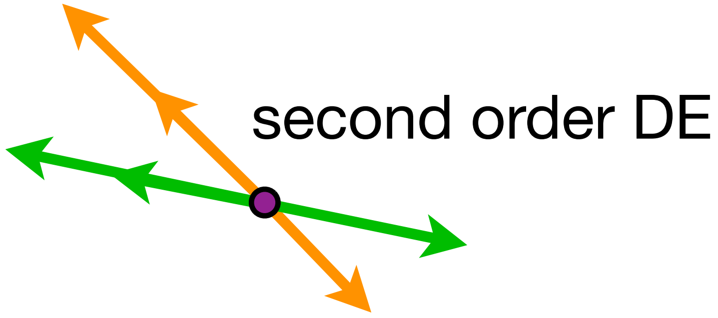

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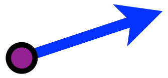

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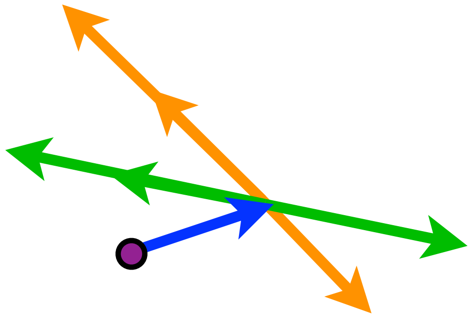

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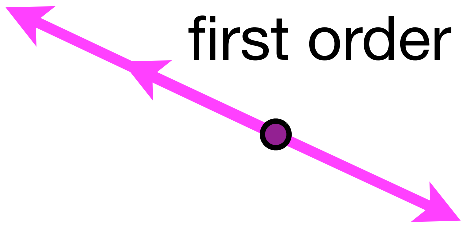

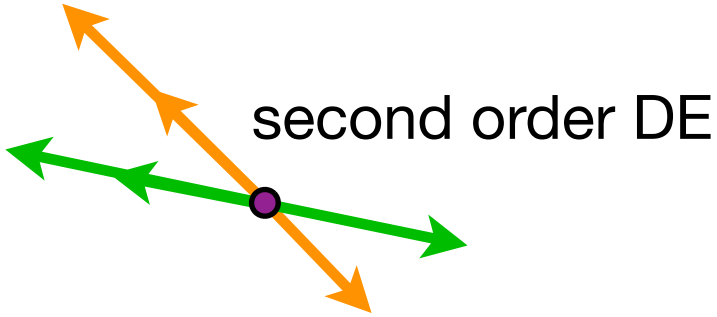

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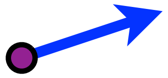

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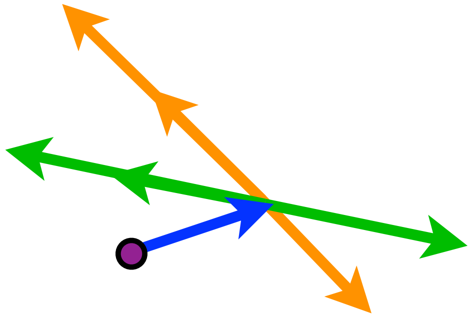

- For step 2, try “Method of undetermined coefficients”...

Method of undetermined coefficients (3.5)

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  - Step 2: What do you have to plug in to $L[\,\cdot\,]$ to get e^{2t} out?
    - Try $y_p(t) = Ae^{2t}$.

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y.$ Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
  - Step 1: Solve the associated homogeneous equation

$$y'' + 2y' - 3y = 0.$$

$$y_h(t) = C_1 e^t + C_2 e^{-3t}$$

  - Step 2: What do you have to plug in to $L[\ \cdot\ ]$ to get e^{2t} out?
    - Try $y_p(t) = Ae^{2t}$.
    - $L[y_p(t)] = L[Ae^{2t}] =$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
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$$y_h(t) = C_1 e^t + C_2 e^{-3t}$$

  - Step 2: What do you have to plug in to $L[\ \cdot\ ]$ to get e^{2t} out?
    - Try $y_p(t) = Ae^{2t}$.
    - $L[y_p(t)] = L[Ae^{2t}] = \left\{ \begin{array}{ll} \text{(A) } 5e^{2t} & \text{(C) } 4e^{2t} \\ \text{(B) } 5Ae^{2t} & \text{(D) } 4Ae^{2t} \end{array} \right.$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
  - Step 1: Solve the associated homogeneous equation

$$y'' + 2y' - 3y = 0.$$

$$y_h(t) = C_1 e^{t} + C_2 e^{-3t}$$

  - Step 2: What do you have to plug in to $L[\ \cdot\ ]$ to get e^{2t} out?
    - Try $y_p(t) = Ae^{2t}$.
    - $L[y_p(t)] = L[Ae^{2t}] =$

(A) $5e^{2t}$ (C) $4e^{2t}$

[✓ (B) $5Ae^{2t}$] (D) $4Ae^{2t}$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
  - Step 1: Solve the associated homogeneous equation
$$y'' + 2y' - 3y = 0.$$
$$y_h(t) = C_1 e^t + C_2 e^{-3t}$$
  - Step 2: What do you have to plug in to $L[\ \cdot\ ]$ to get e^{2t} out?
    - Try $y_p(t) = Ae^{2t}$.
    - $L[y_p(t)] = L[Ae^{2t}] =$
      - (A) $5e^{2t}$
      - [✓ (B) $5Ae^{2t}$]
      - (C) $4e^{2t}$
      - (D) $4Ae^{2t}$
    - A is an undetermined coefficient (until you determine it).

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.

$$L[C_1e^t + C_2e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
  - Summarizing:

$$L[C_1e^t + C_2e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y.$ Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}.$
  - Summarizing:
    - We know that, for any C_1 and C_2,

$$L[C_1 e^t + C_2 e^{-3t}] = 0$$

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients (3.5)

- **Example 4.** Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
  - Summarizing:
    - We know that, for any C_1 and C_2,

$$L[C_1 e^t + C_2 e^{-3t}] = 0$$

    - We also know that

$$L[Ae^{2t}] = 5Ae^{2t}$$

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
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    - We know that, for any C_1 and C_2,

$$L[C_1 e^t + C_2 e^{-3t}] = 0$$

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    - Finally, by linearity, we know that

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients (3.5)

- **Example 4.** Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
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    - We know that, for any C_1 and C_2,

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    - Finally, by linearity, we know that

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

    - So what's left to do to find our general solution? Pick A =?

Method of undetermined coefficients (3.5)

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  - Summarizing:
    - We know that, for any C_1 and C_2,

$$L[C_1e^t + C_2e^{-3t}] = 0$$

    - We also know that

$$L[Ae^{2t}] = 5Ae^{2t}$$

    - Finally, by linearity, we know that

$$L[C_1e^t + C_2e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

    - So what's left to do to find our general solution? Pick A = 1/5.

Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
    - What is the solution to the associated homogeneous equation?

(A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

(B) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t)$

(C) $y_h(t) = C_1 e^{2t} + C_2 t e^{2t}$

(D) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t} + e^t$

(E) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t) + e^t$

Method of undetermined coefficients (3.5)

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(D) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t} + e^t$

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Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

(B) $y_p(t) = Ae^{-2t}$

(C) $y_p(t) = Ate^{-2t}$

(D) $y_p(t) = Ae^t$

(E) $y_p(t) = Ate^t$

Method of undetermined coefficients (3.5)

- **Example 5.** Find the general solution to the equation $y'' - 4y = e^t$.
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(A) $y_p(t) = Ae^{2t}$

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(C) $y_p(t) = Ate^{-2t}$

★ (D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the value of A that gives the particular solution (Ae^t) ?

(A) A = 1

(B) A = 3

(C) A = -3

(D) A = 1/3

(E) A = -1/3

Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the value of A that gives the particular solution (Ae^t) ?

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Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
    - What is the solution to the associated homogeneous equation?

(A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

(B) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t)$

(C) $y_h(t) = C_1 e^{2t} + C_2 t e^{2t}$

(D) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t} + e^t$

(E) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t) + e^t$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the solution to the associated homogeneous equation?

★ (A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

(B) $y_h(t) = C_1 \cos(2t) + C$

(C) $y_h(t) = C$

$C_1 e^{2t} + C_2 e^{-2t} + e^t$

(E) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t) + e^t$

Same as the last example

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

(B) $y_p(t) = Ae^{-2t}$

(C) $y_p(t) = Ate^{2t}$

(D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

Method of undetermined coefficients (3.5)

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(C) $y_p(t) = Ate^{2t}$

(D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \; !$$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

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(D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \;!$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^t$$

Method of undetermined coefficients (3.5)

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$$(Ae^{2t})'' - 4Ae^{2t} = 0 \ !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^t$$

$$e^{-t}y' - e^{-t}y = 1$$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

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(C) $y_p(t) = Ate^{2t}$

(D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \; !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^t$$

$$e^{-t}y' - e^{-t}y = 1$$

$$y = te^t + Ce^t$$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

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(C) $y_p(t) = Ate^{2t}$

(D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \;!$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^{t}$$

$$e^{-t}y' - e^{-t}y = 1$$

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Method of undetermined coefficients (3.5)

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★ (C) $y_p(t) = Ate^{2t}$

(D) $y_p(t) = Ae^{t}$

(E) $y_p(t) = Ate^{t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \; !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^{t}$$

$$e^{-t}y' - e^{-t}y = 1$$

$$y = \boxed{te^{t}} + Ce^{t}$$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the value of A that gives the particular solution (Ate^{2t})?

(A) A = 1

(B) A = 4

(C) A = -4

(D) A = 1/4

(E) A = -1/4

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
    - What is the value of A that gives the particular solution $\left(Ate^{2t}\right)$?

(A) A = 1

(B) A = 4

$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) =$$

(C) A = -4

(D) A = 1/4

(E) A = -1/4

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the value of A that gives the particular solution $\left(Ate^{2t}\right)$?

(A) A = 1

(B) A = 4

$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) = 4Ae^{2t}$$

(C) A = -4

(D) A = 1/4

(E) A = -1/4

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
    - What is the value of A that gives the particular solution $\left(Ate^{2t}\right)$?

(A) A = 1

(B) A = 4

$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) = \;4Ae^{2t}$$

(C) A = -4

★ (D) A = 1/4

(E) A = -1/4

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = \cos(2t)$.
  - What is the form of the particular solution?

(A) $y_p(t) = A\cos(2t)$

(B) $y_p(t) = A\sin(2t)$

(C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = \cos(2t)$.
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☆ (A) $y_p(t) = A\cos(2t)$

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☆ (C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = \cos(2t)$.
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★ (A) $y_p(t) = A\cos(2t)$

(B) $y_p(t) = A\sin(2t)$

★ (C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

What small change to the DE makes (D) correct?

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' + y' - 4y = \cos(2t)$.
  - What is the form of the particular solution?

(A) $y_p(t) = A\cos(2t)$

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(C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

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Method of undetermined coefficients (3.5)

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[✓ (C) $y_p(t) = A\cos(2t) + B\sin(2t)$]

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = At^3$

(B) $y_p(t) = At^3 + Bt^2 + Ct$

(C) $y_p(t) = At^3 + Bt^2 + Ct + D$

(D) $y_p(t) = At^3 + Be^{2t} + Ce^{-2t}$

(E) $y_p(t) = At^3 + Bt^2 + Ct + D + Ee^{2t} + Fe^{-2t}$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = At^3$

(B) $y_p(t) = At^3 + Bt^2 + Ct$

★ (C) $y_p(t) = At^3 + Bt^2 + Ct + D$

(D) $y_p(t) = At^3 + Be^{2t} + Ce^{-2t}$

(E) $y_p(t) = At^3 + Bt^2 + Ct + D + Ee^{2t} + Fe^{-2t}$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

(C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

(E) $y_p(t) = Ae^{2t} + Bte^{2t} + Ct^3 + Dt^2 + Et + F$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

★ (C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

(E) $y_p(t) = Ae^{2t} + Bte^{2t} + Ct^3 + Dt^2 + Et + F$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
    - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

☆ (C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$
$y_p(t) = Ae^{2t} + t(Bt^3 + Ct^2 + Dt + E)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

(E) $y_p(t) = Ae^{2t} + Bte^{2t} + Ct^3 + Dt^2 + Et + F$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
    - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

★ (C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$
$y_p(t) = Ae^{2t} + t(Bt^3 + Ct^2 + Dt + E)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

(E) $y_p(t) = Ae^{2t} + Bte^{2t} + Ct^3 + Dt^2 + Et + F$

For each wrong answer, for what DE is it the correct form?

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = t^3 e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = (At^3 + Bt^2 + Ct + D)e^{2t}$

(B) $y_p(t) = (At^3 + Bt^2 + Ct)e^{2t}$

(C) $y_p(t) = (At^3 + Bt^2 + Ct)e^{2t}$
$+(Dt^3 + Et^2 + Ft)e^{-2t}$

(D) $y_p(t) = (At^4 + Bt^3 + Ct^2 + Dt)e^{2t}$

(E) $y_p(t) = (At^4 + Bt^3 + Ct^2 + Dt + E)e^{2t}$

Method of undetermined coefficients (3.5)

- Example 6. Find the general solution to $y'' - 4y = t^3 e^{2t}$.
  - What is the form of the particular solution?

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  - If your guess includes a solution to the h-problem, you may as well remove it as it won’t survive L[ ] so you won’t be able to determine its undetermined coefficient.