# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.
- What are the eigenvalues of A?

(A) 1 and -3

(B) -1 and 3

(C) 1 and 3

(D) -1 and -3

(E) Explain, please.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.
- What are the eigenvalues of A?

(A) 1 and -3

(B) -1 and 3

(C) 1 and 3

(D) -1 and -3

(E) Explain, please.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

$$\det \begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda^2 - 2\lambda - 3 = 0)$$

$$\lambda = 1 \pm 2 = -1, 3$$

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$
$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$
$$\det(A - \lambda I) = 0$$
$$\det \begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$
$$(1-\lambda)^2 - 4 = 0$$
$$(\lambda^2 - 2\lambda - 3 = 0)$$
$$\lambda = 1 \pm 2 = -1, 3$$

- What are the eigenvectors associated with $\lambda_1$=-1?

(A) $\mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(B) $\mathbf{v_1} = c\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(C) $\mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(D) $\mathbf{v_1} = c\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(E) Explain, please.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

$$\det\begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda^2 - 2\lambda - 3 = 0)$$

$$\lambda = 1 \pm 2 = -1, 3$$

- What are the eigenvectors associated with $\lambda_1$=-1?

(A) $\mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

★(B) $\mathbf{v_1} = c\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(C) $\mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(D) $\mathbf{v_1} = c\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(E) Explain, please.

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

$$\det\begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda^2 - 2\lambda - 3 = 0)$$

$$\lambda = 1 \pm 2 = -1, 3$$

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$

$$\lambda_1 = -1$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

$$\det \begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda^2 - 2\lambda - 3 = 0)$$

$$\lambda = 1 \pm 2 = -1, 3$$

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

$$\det\begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda^2 - 2\lambda - 3 = 0)$$

$$\lambda = 1 \pm 2 = -1, 3$$

$$\lambda_1 = -1$$

$$(A + I)\mathbf{v_1} = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\mathbf{v_1} = 0$$

$$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$$

$$2v_1 + v_2 = 0$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

(and any scalar multiple of it)

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$

$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$

$$\det(A - \lambda I) = 0$$

$$\det \begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$

$$(1-\lambda)^2 - 4 = 0$$

$$(\lambda^2 - 2\lambda - 3 = 0)$$

$$\lambda = 1 \pm 2 = -1, 3$$

$$\lambda_1 = -1$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\lambda_2 = 3$$

$$\mathbf{v_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

# Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values $\lambda$ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

$$A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$$
$$(A - \lambda I)\mathbf{v} = \mathbf{0}$$
$$\det(A - \lambda I) = 0$$
$$\det\begin{pmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{pmatrix} = 0$$
$$(1-\lambda)^2 - 4 = 0$$
$$(\lambda^2 - 2\lambda - 3 = 0)$$
$$\lambda = 1 \pm 2 = -1, 3$$

$$\lambda_1 = -1$$
$$\mathbf{v_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\lambda_2 = 3$$
$$\mathbf{v_2} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

- How do we use eigenvalues and eigenvectors to construct a general solution?