# Today

- Introduction to systems of equations
- Direction fields
- Eigenvalues and eigenvectors
- Finding the general solution (distinct e-value case)

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$$\begin{pmatrix} x \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\gamma}{m} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}$$

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    - populations of two species (e.g. predator and prey).

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- As with single equations, we have linear and nonlinear systems:

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$$\frac{dy}{dt} = x + 4\sin(t)y + t^3 \qquad \frac{dy}{dt} = \sqrt{x} - y$$

- And we also have nonhomogeneous and homogeneous systems.

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- We'll focus on the case in which the matrix has constant entries. And homogeneous. For example,

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

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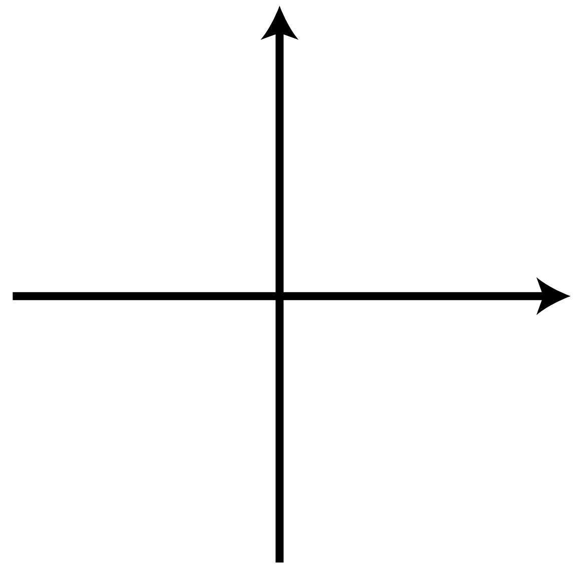

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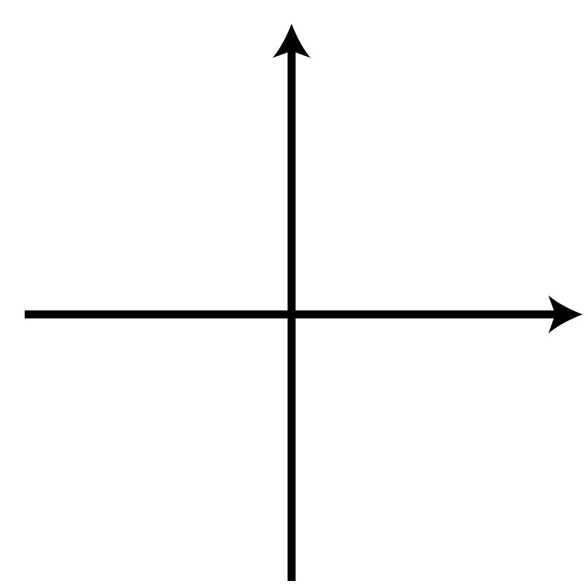

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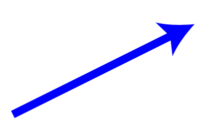

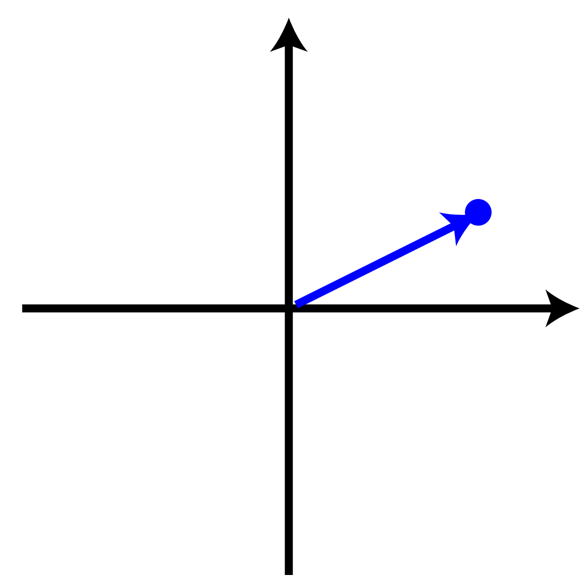

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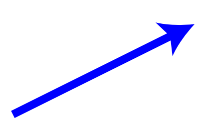

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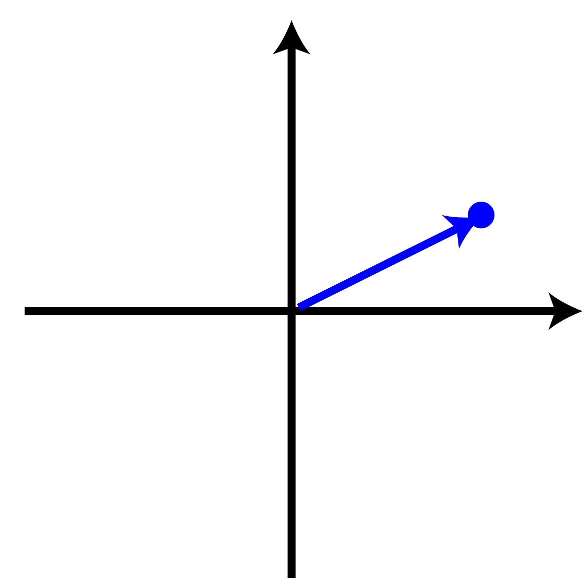

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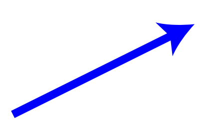

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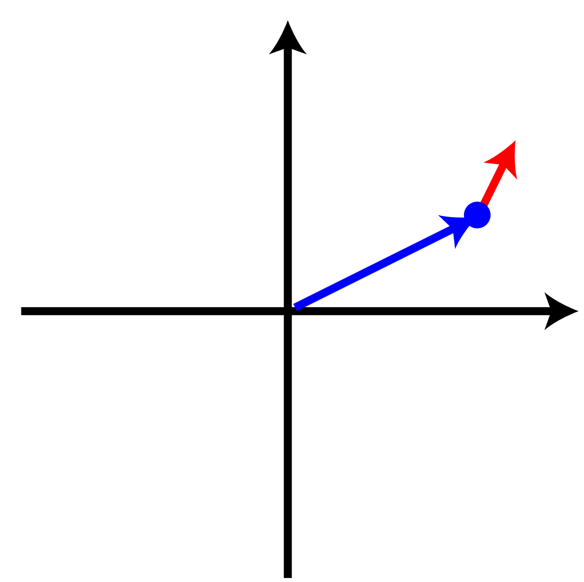

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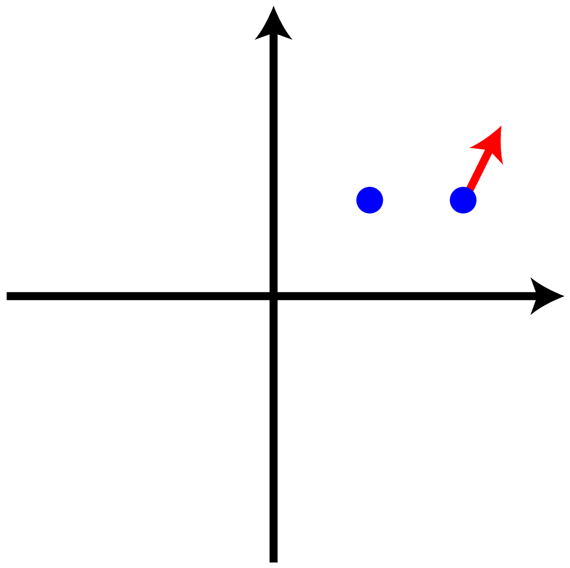

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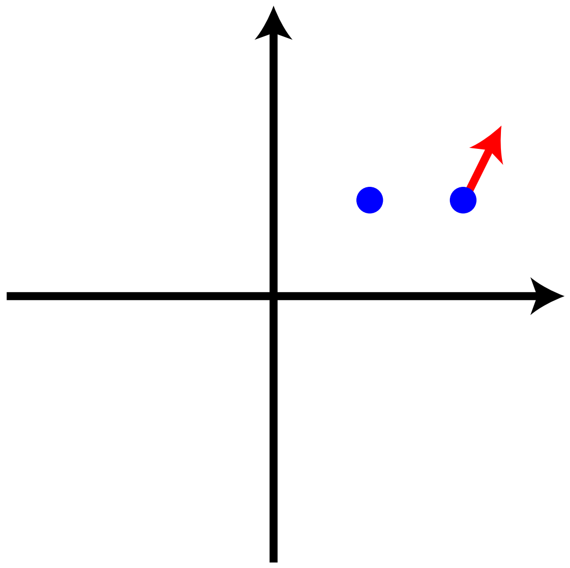

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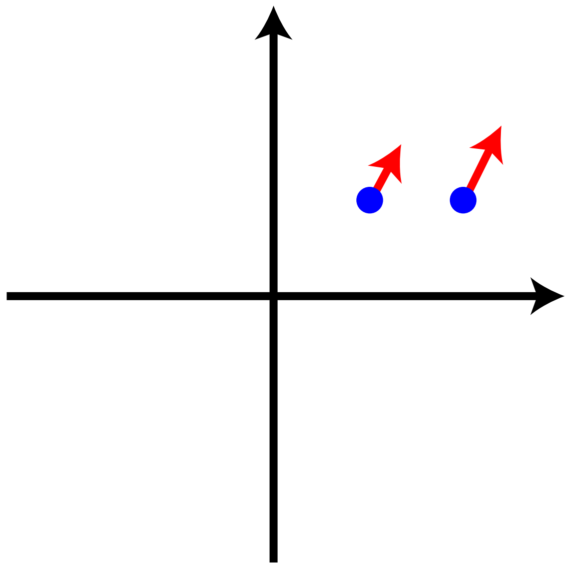

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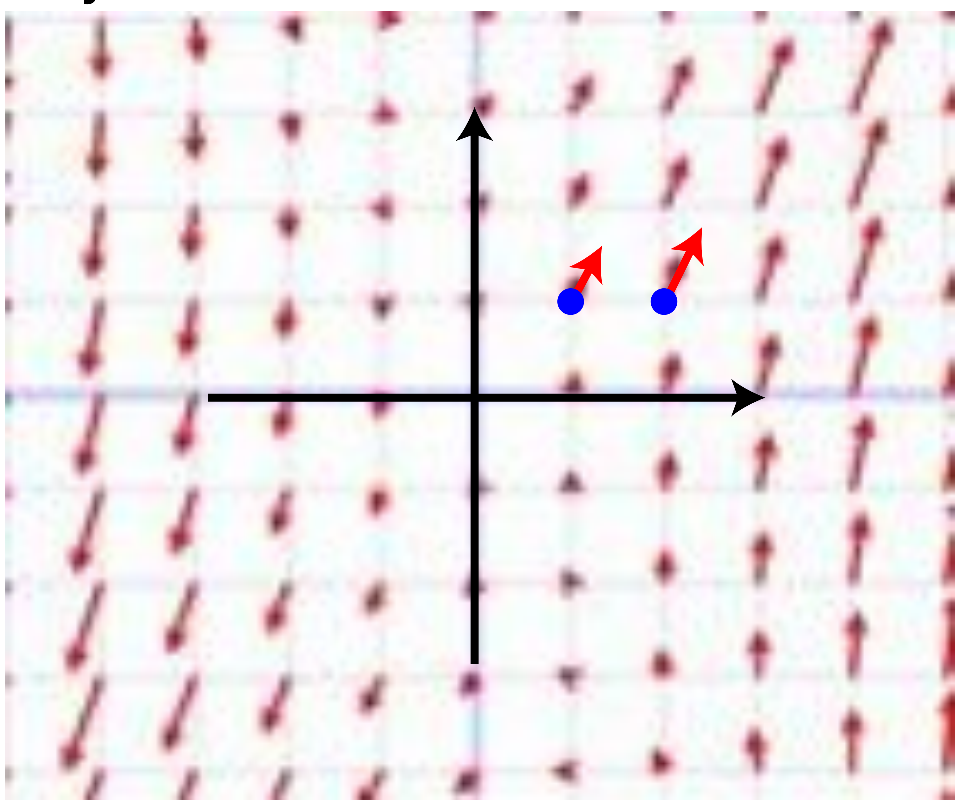

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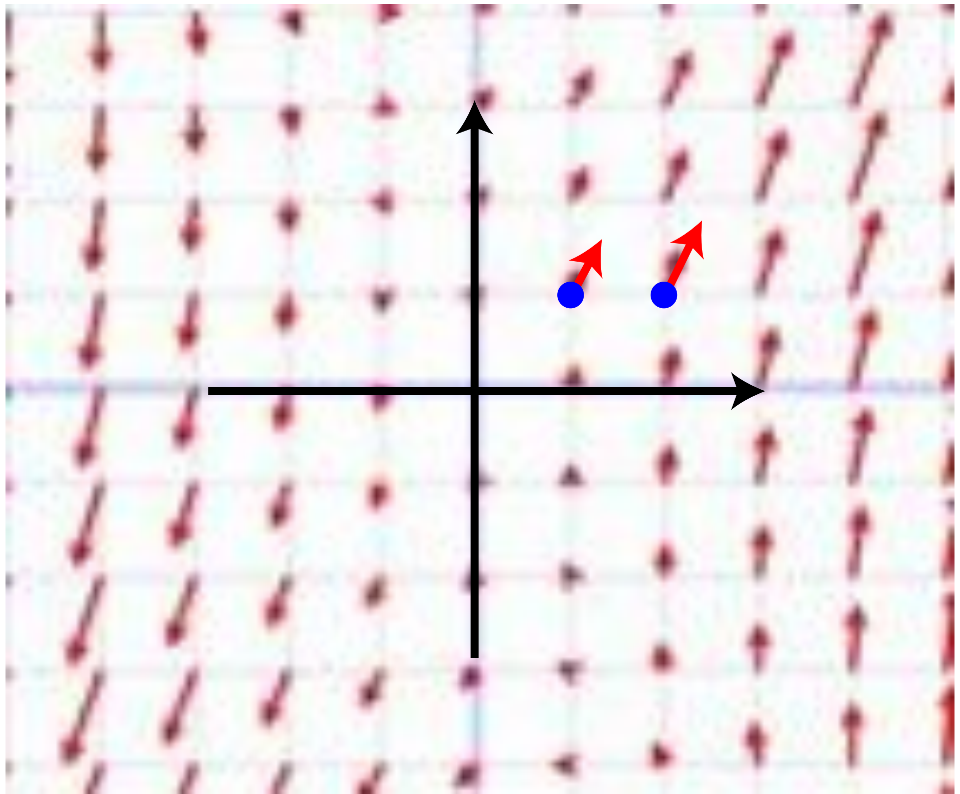

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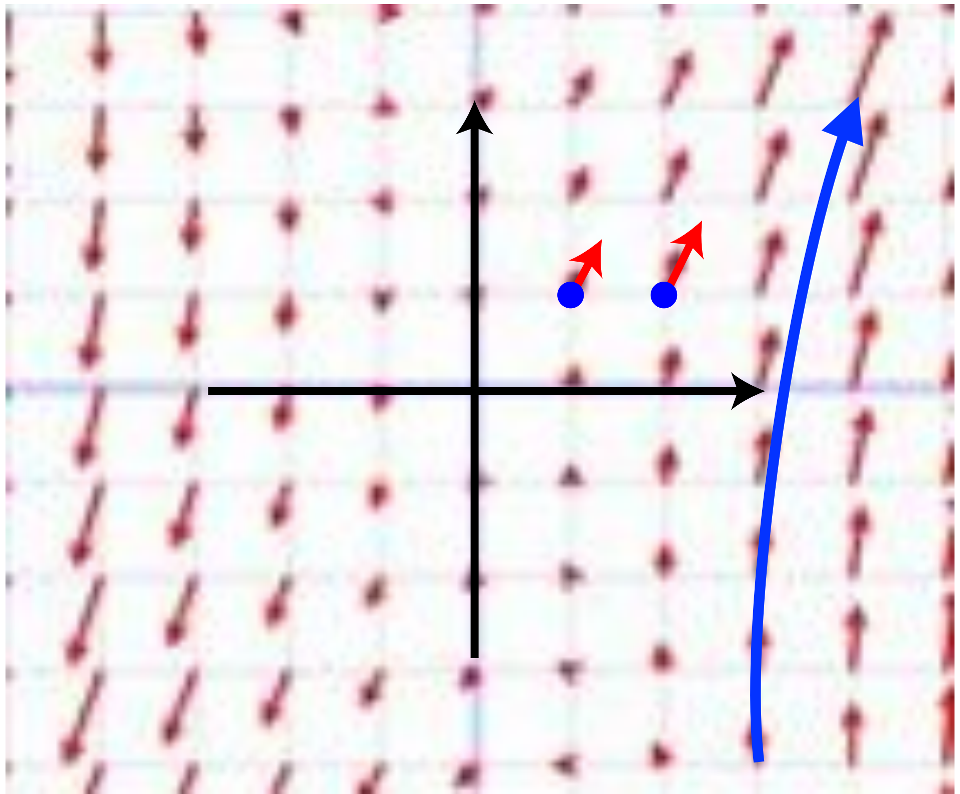

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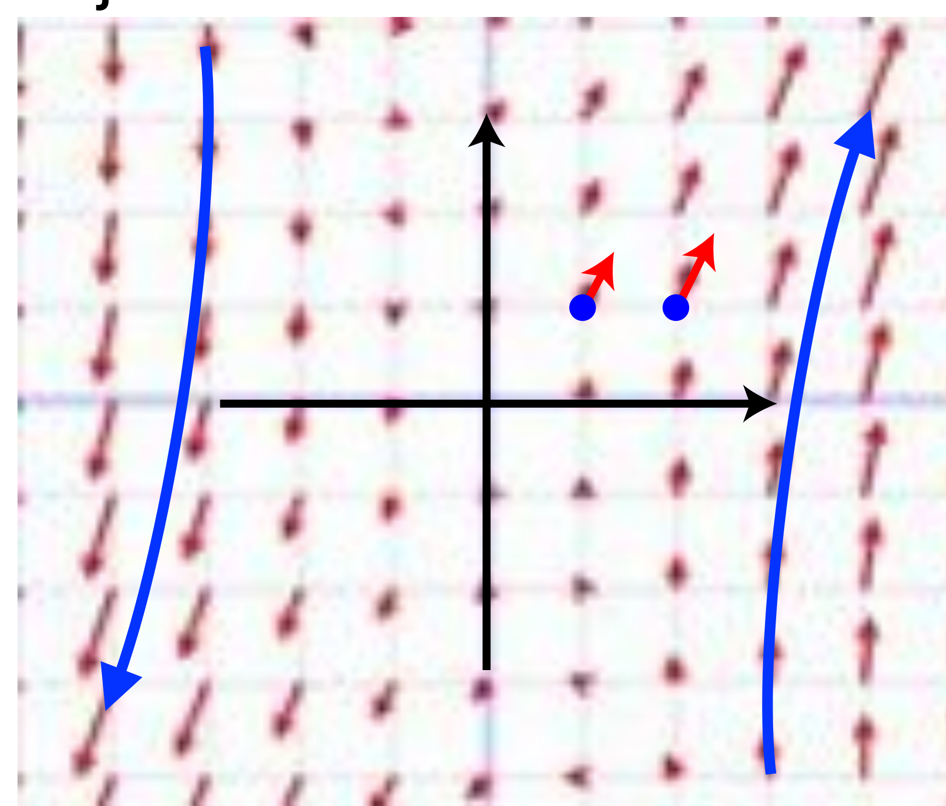

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$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \quad \text{or} \quad \mathbf{x}' = A\mathbf{x}$$

- Think of the unknown functions as coordinates $(x(t), y(t))$ of an object in the plane.

- $A\mathbf{x}$ gives the velocity vector of the object located at $\mathbf{x}$.

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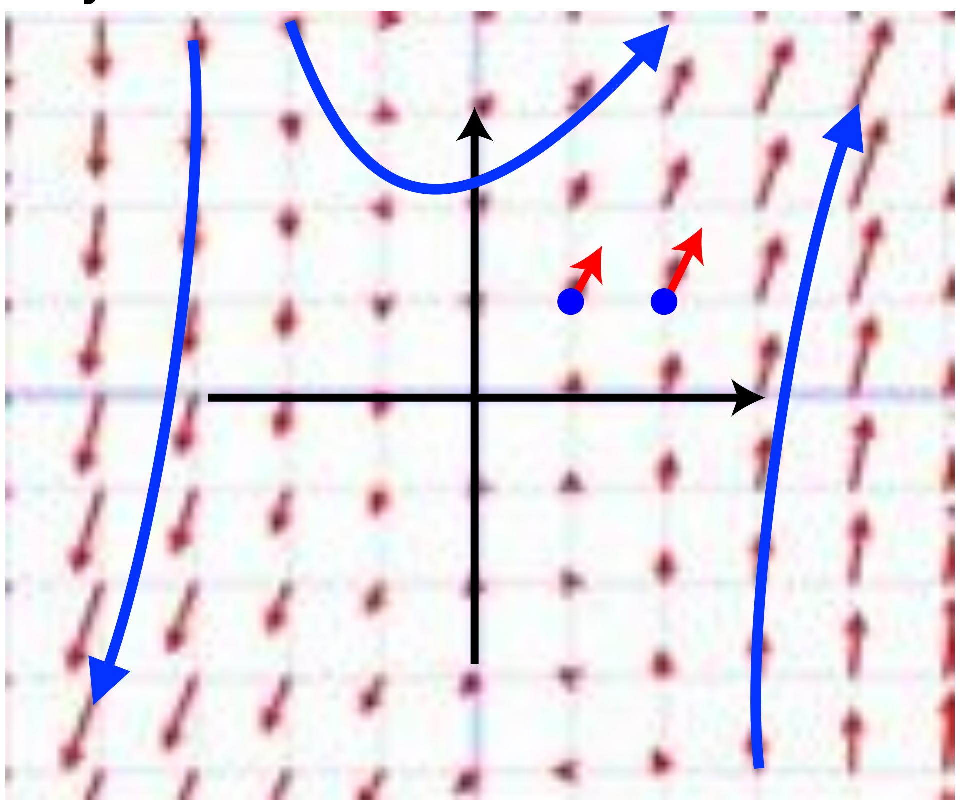
$$A\mathbf{x} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$



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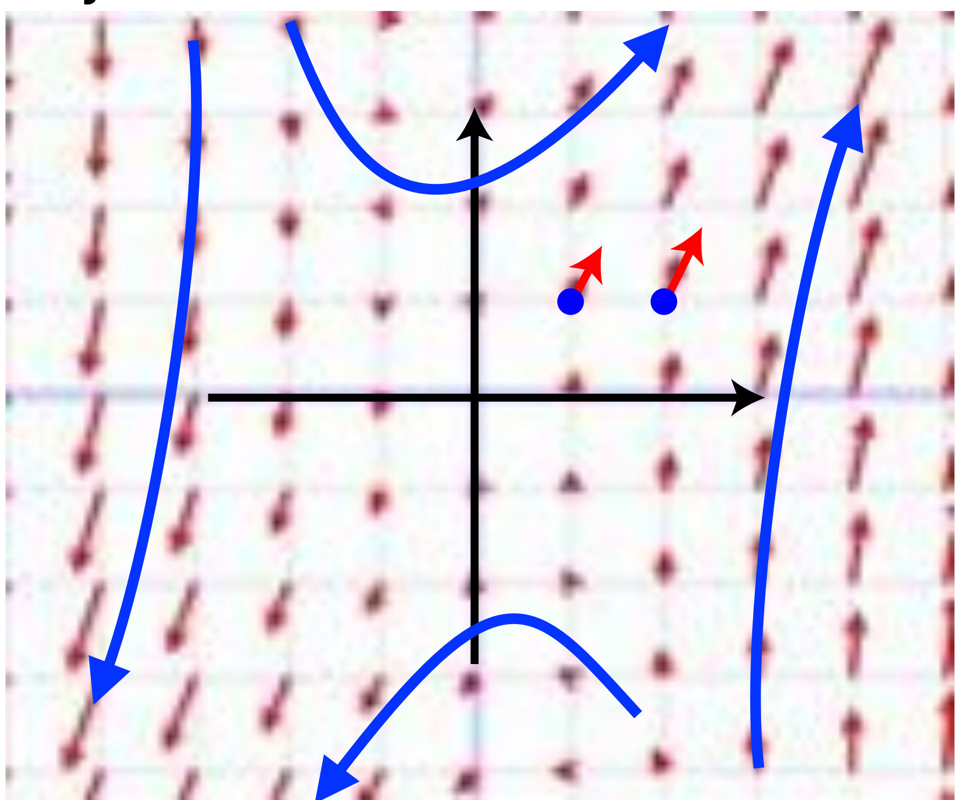

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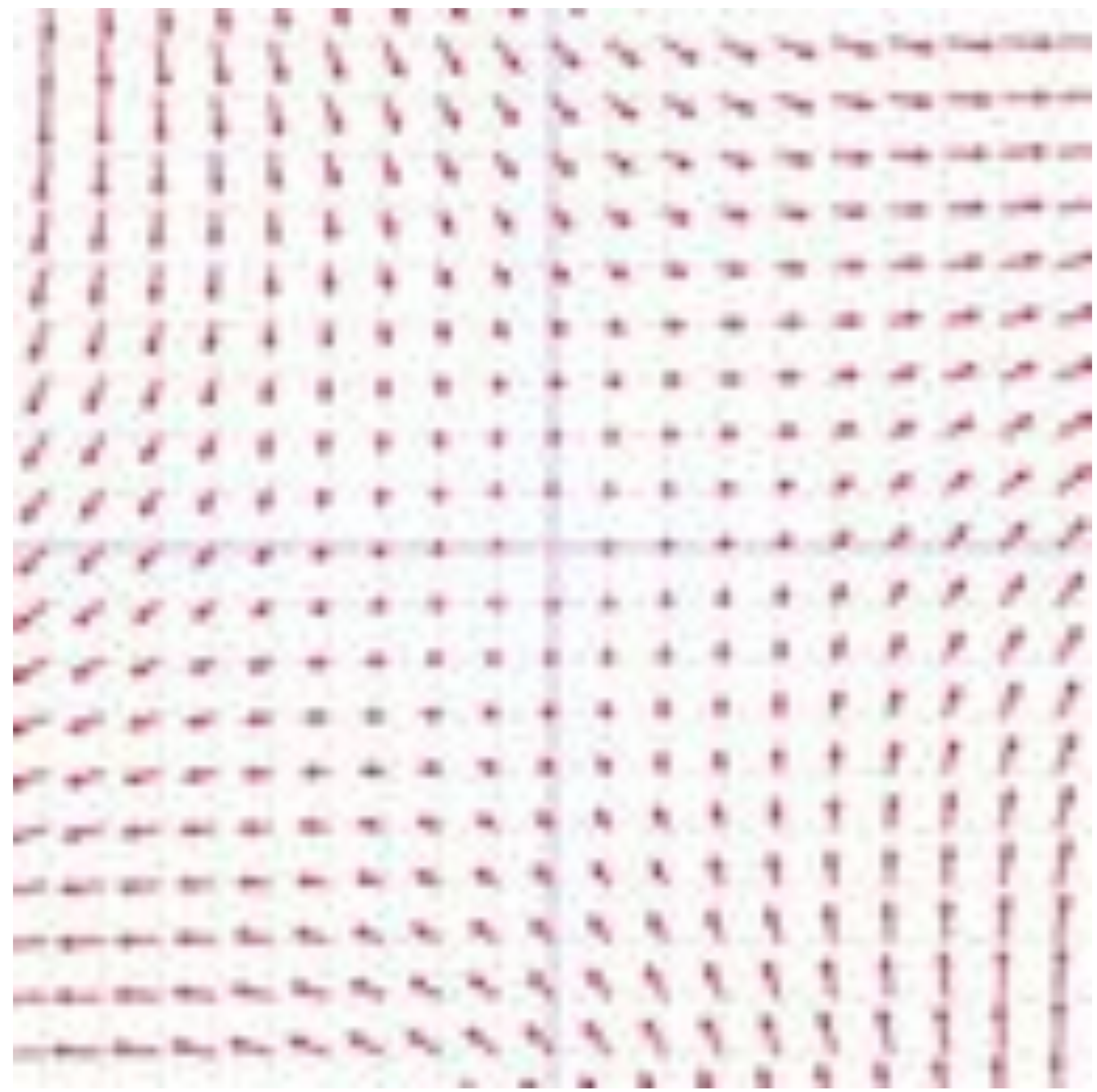
(A) $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(B) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(C) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(D) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(E) Explain, please.



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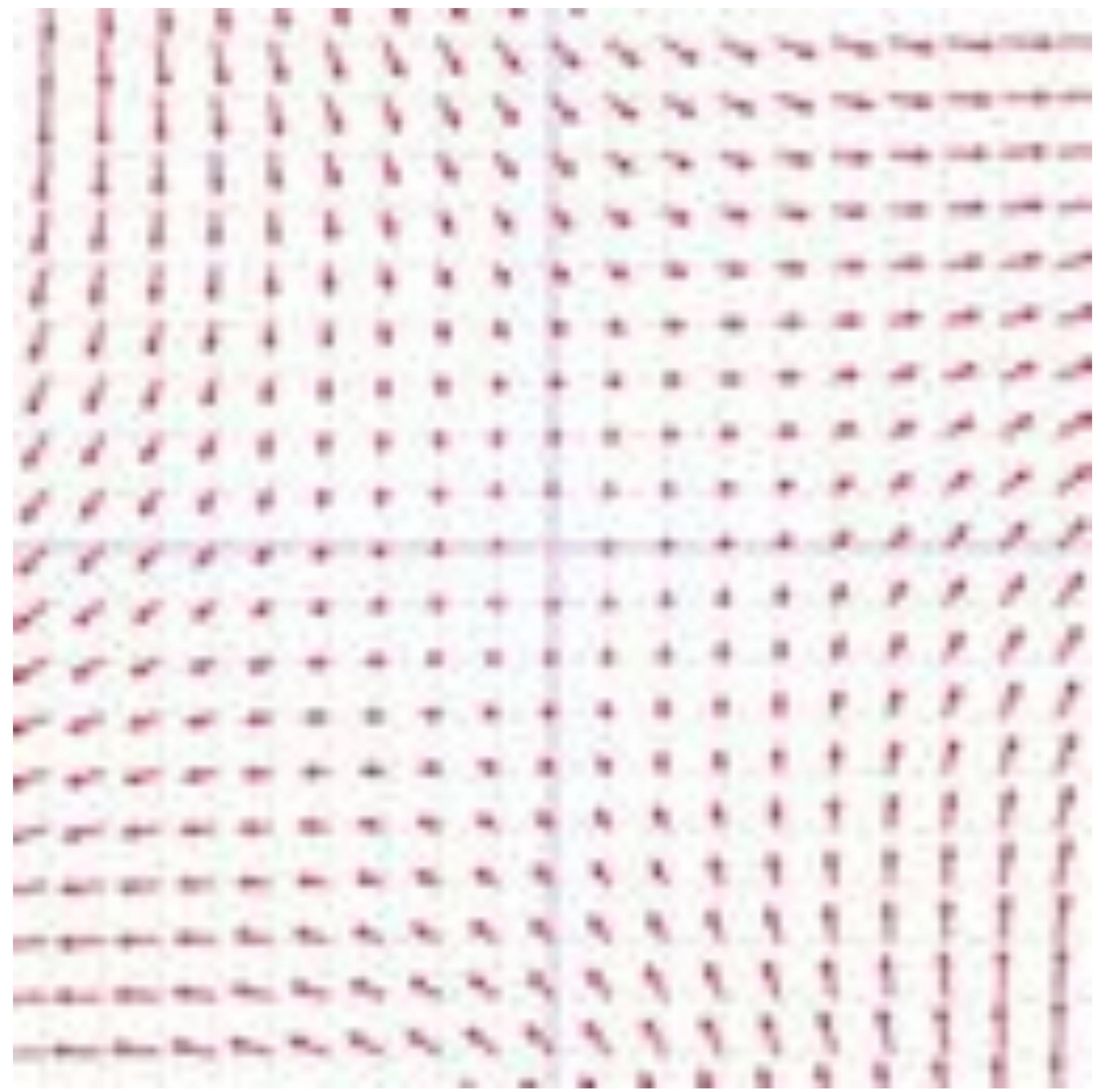

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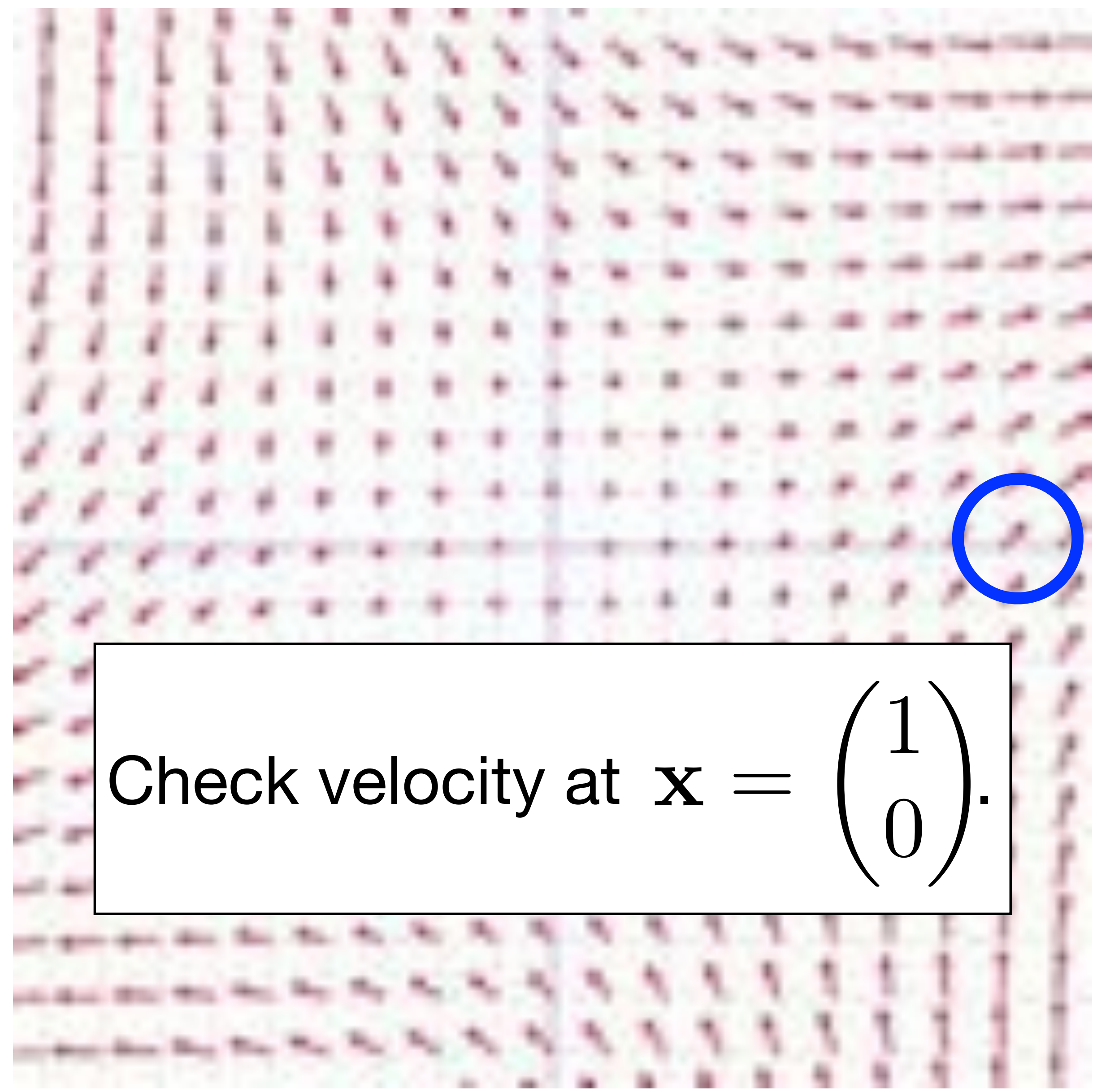


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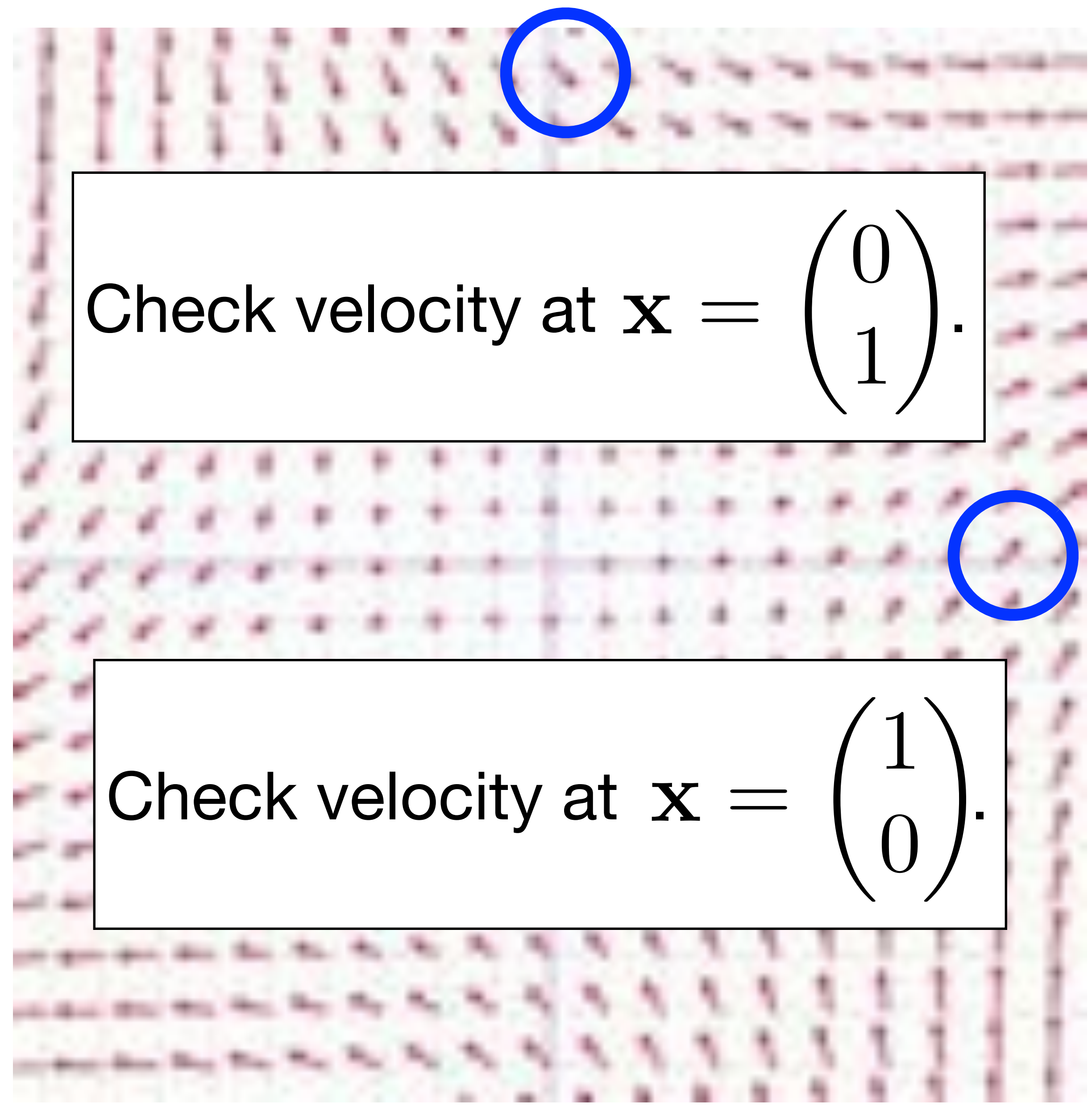


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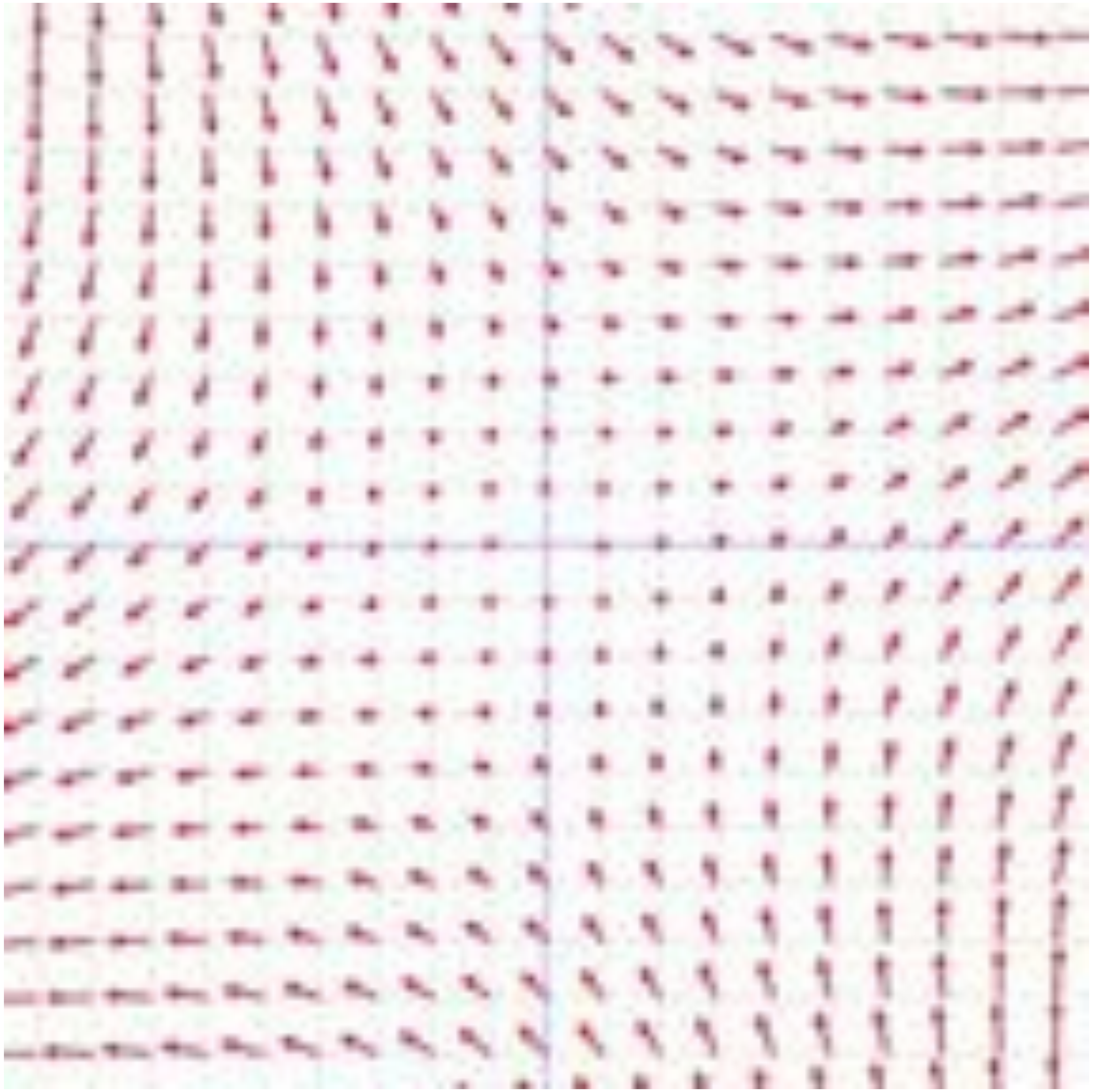
- What are they?
- Directions along which the velocity vector is parallel to the position vector.
- That is, .



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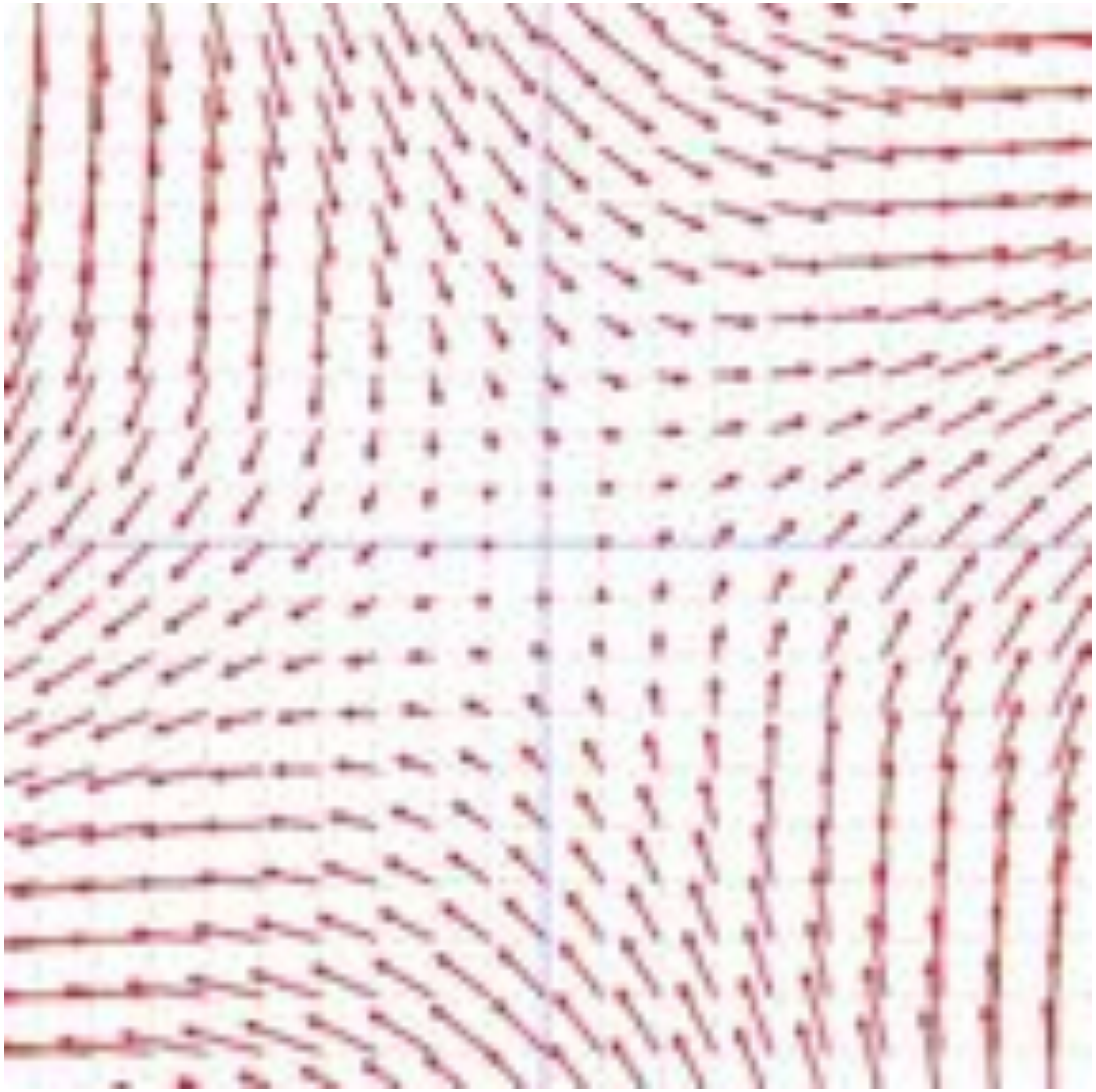

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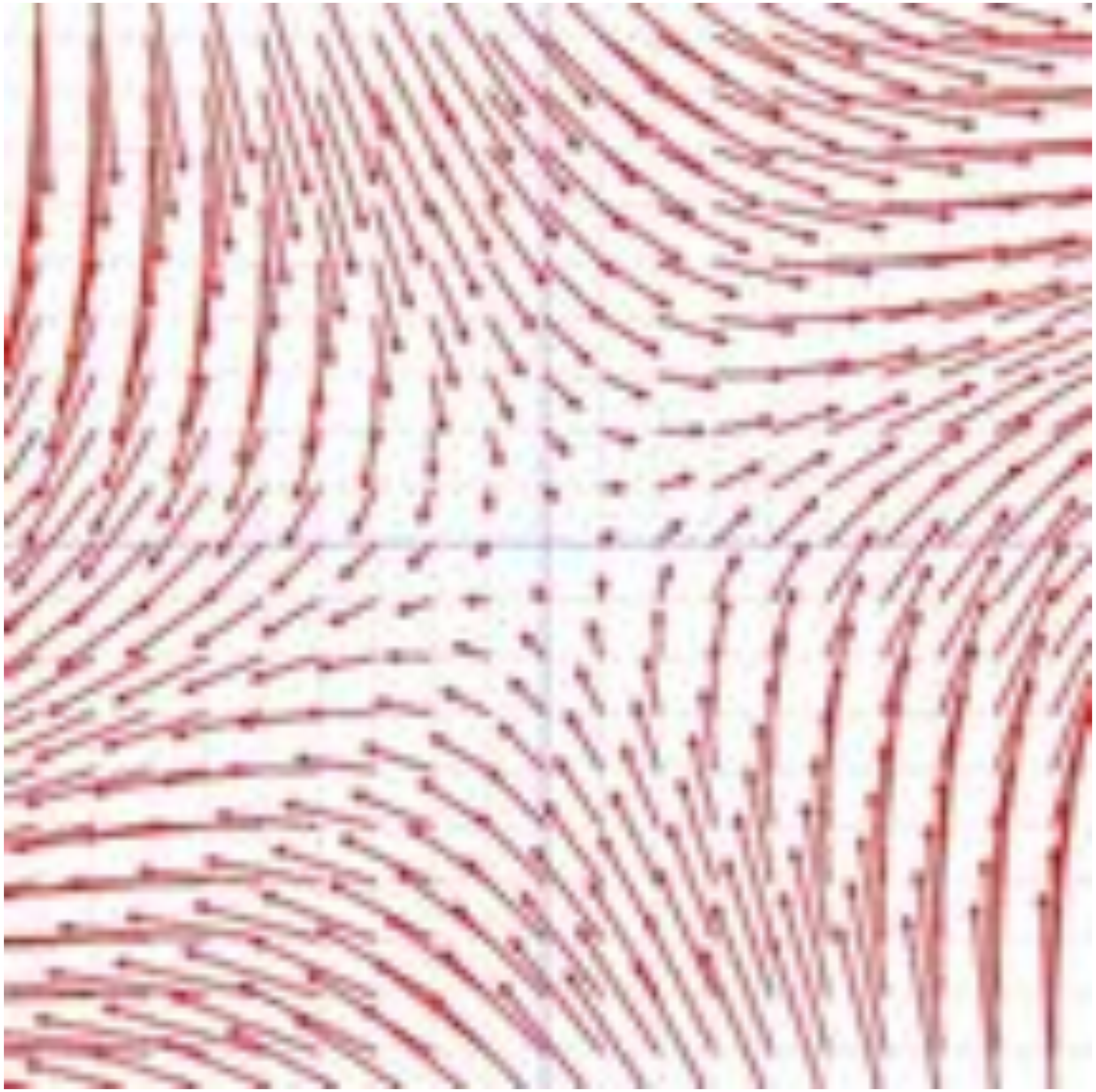

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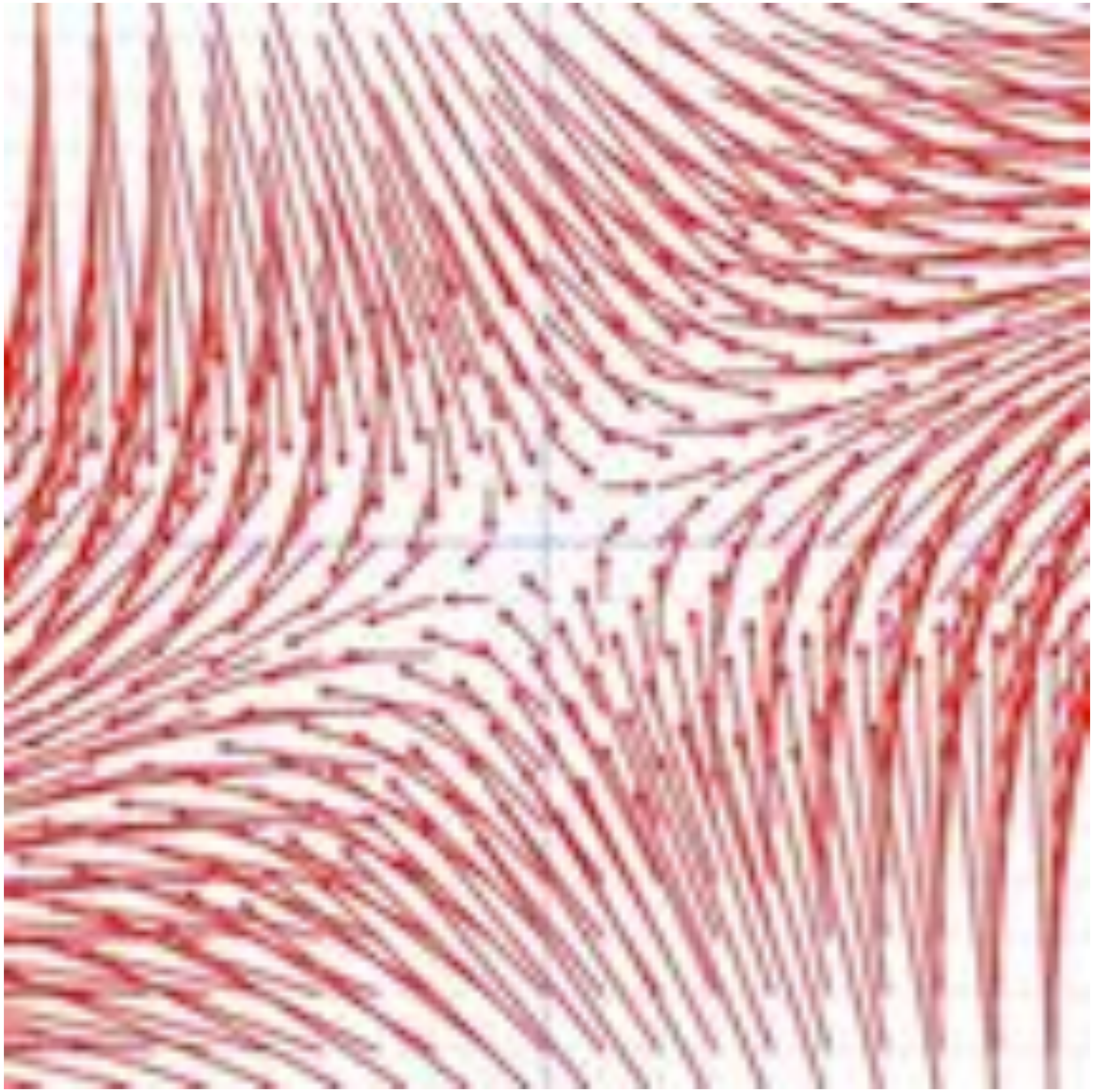

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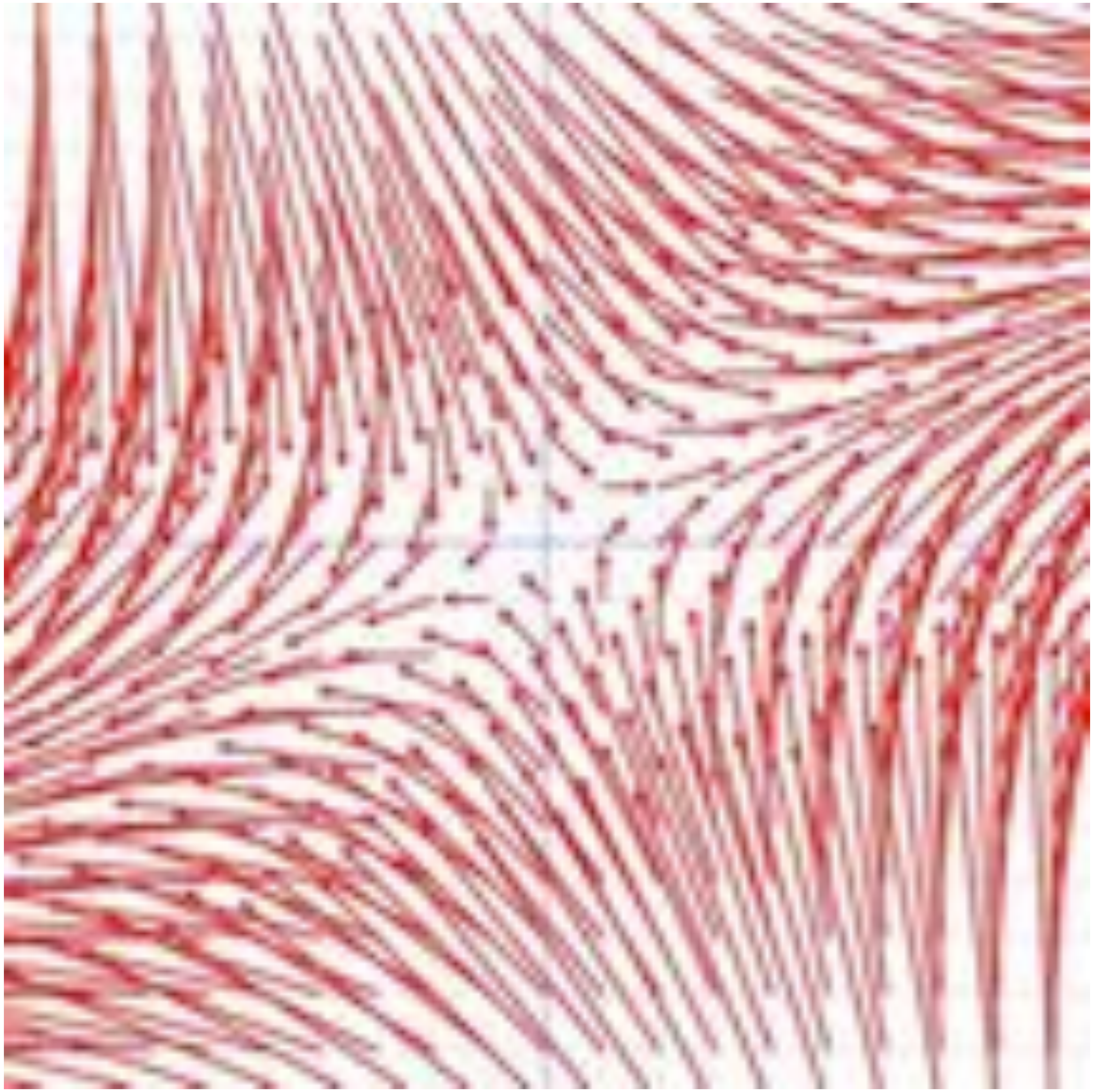

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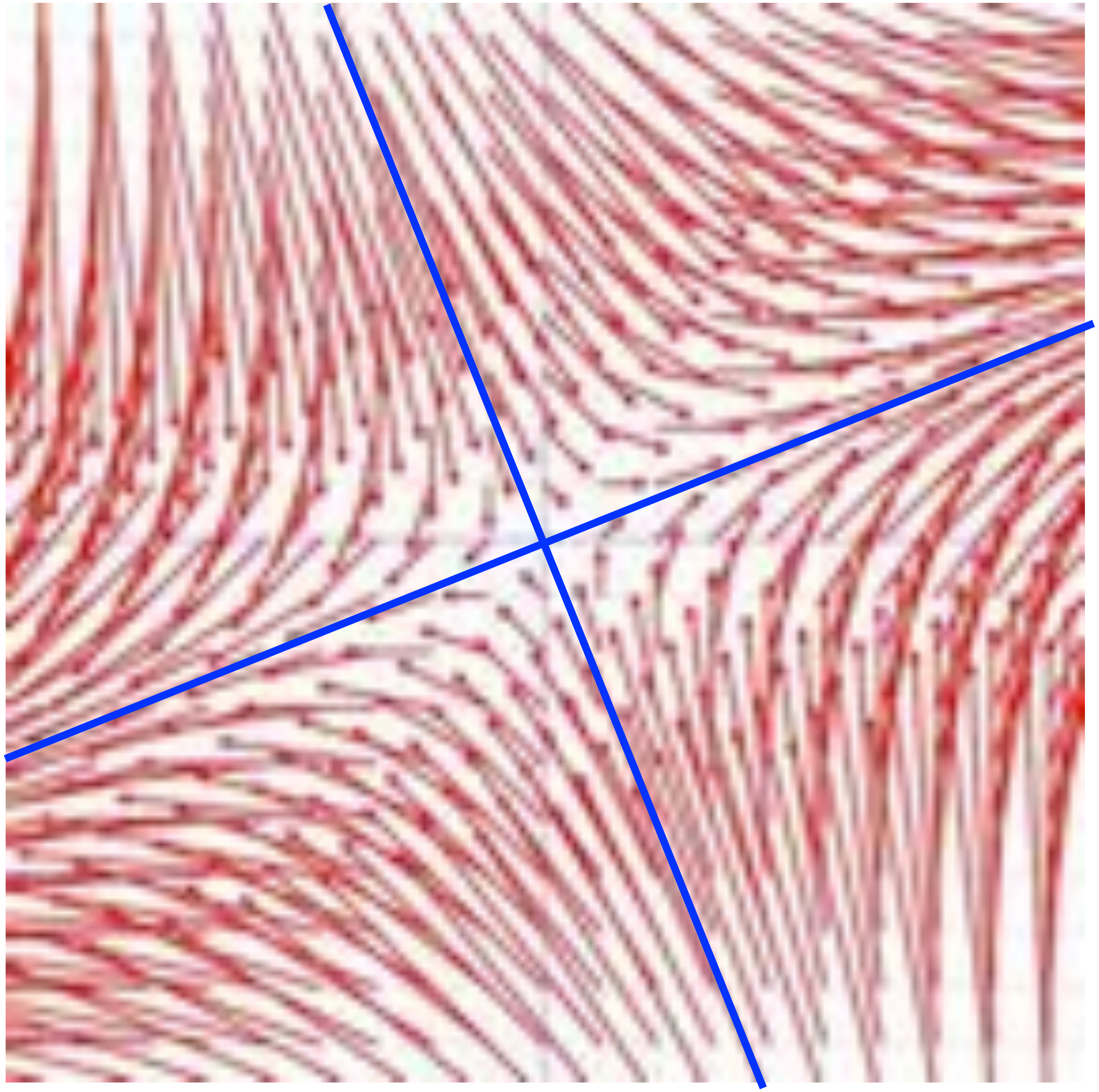

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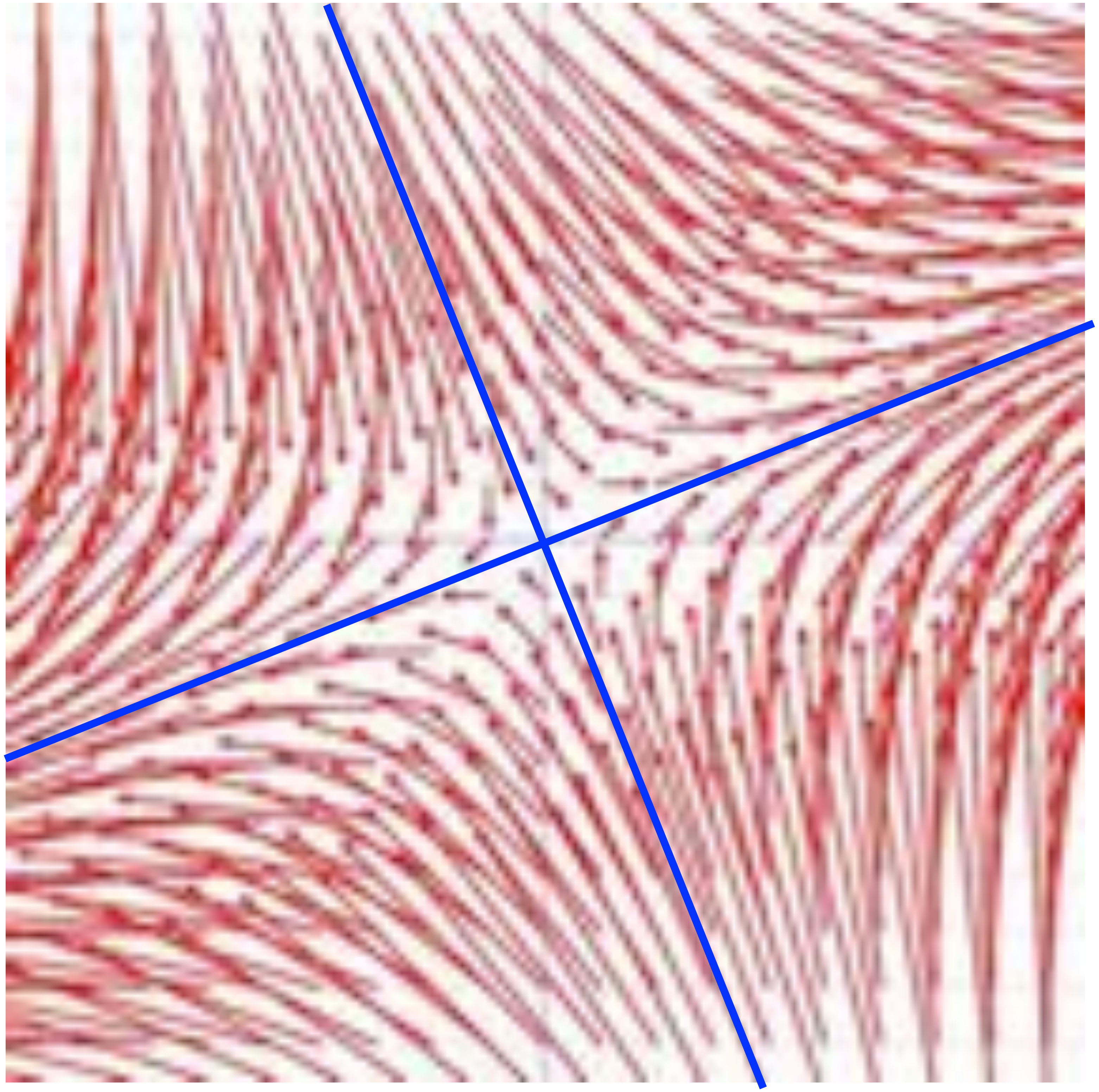

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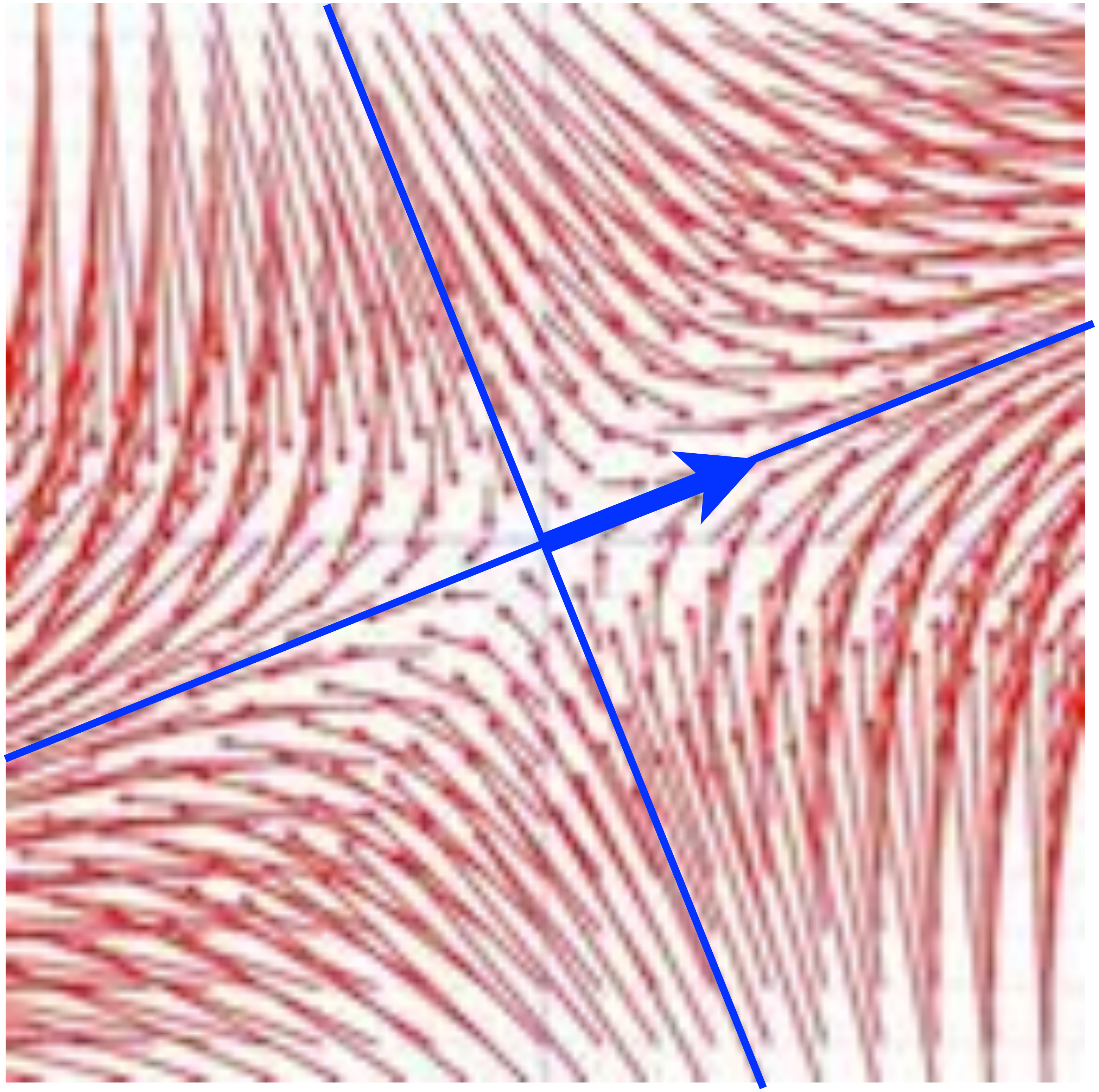

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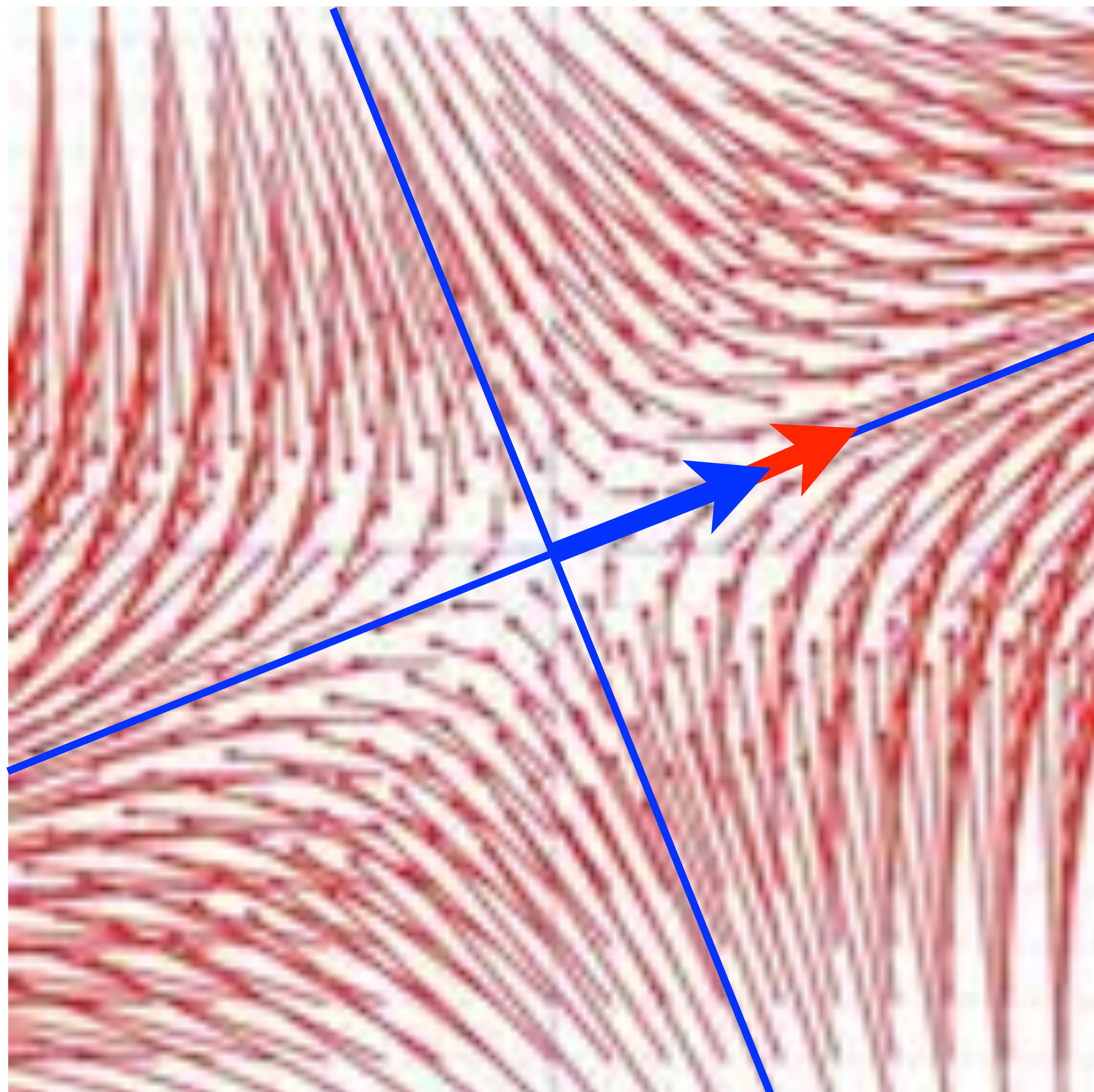

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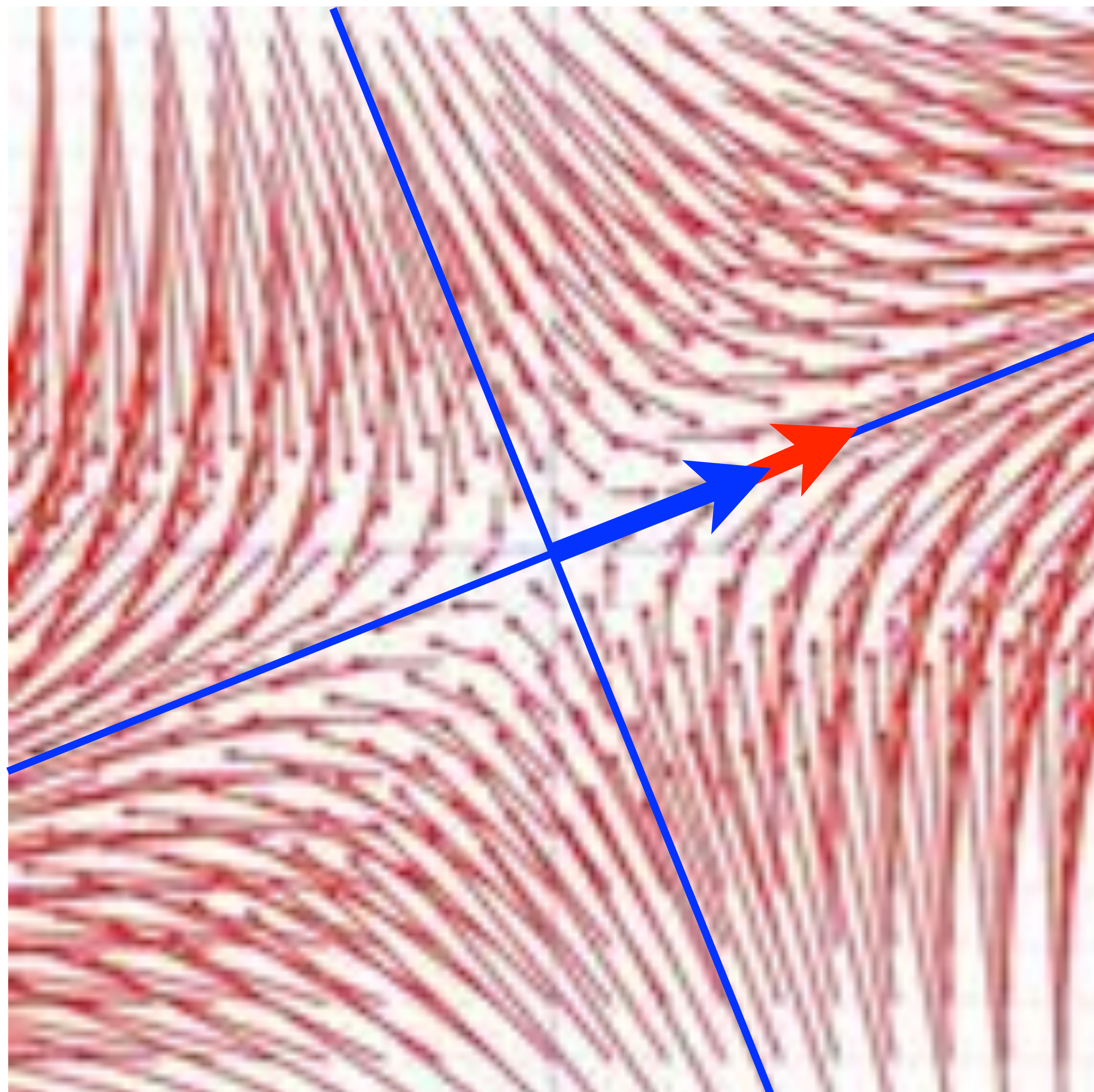
$$\lambda_1 = \sqrt{2}$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ \sqrt{2} - 1 \end{pmatrix}$$



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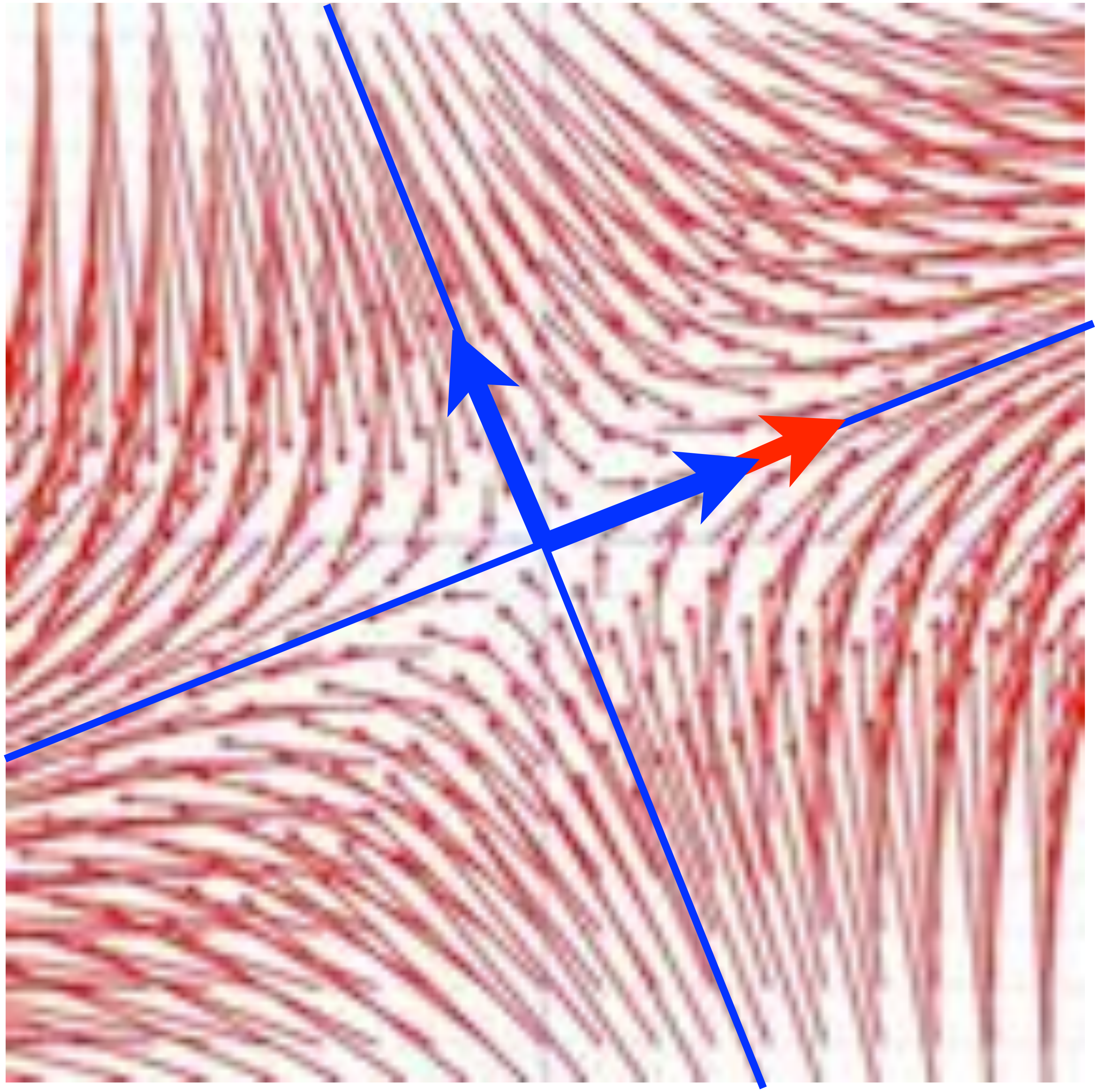

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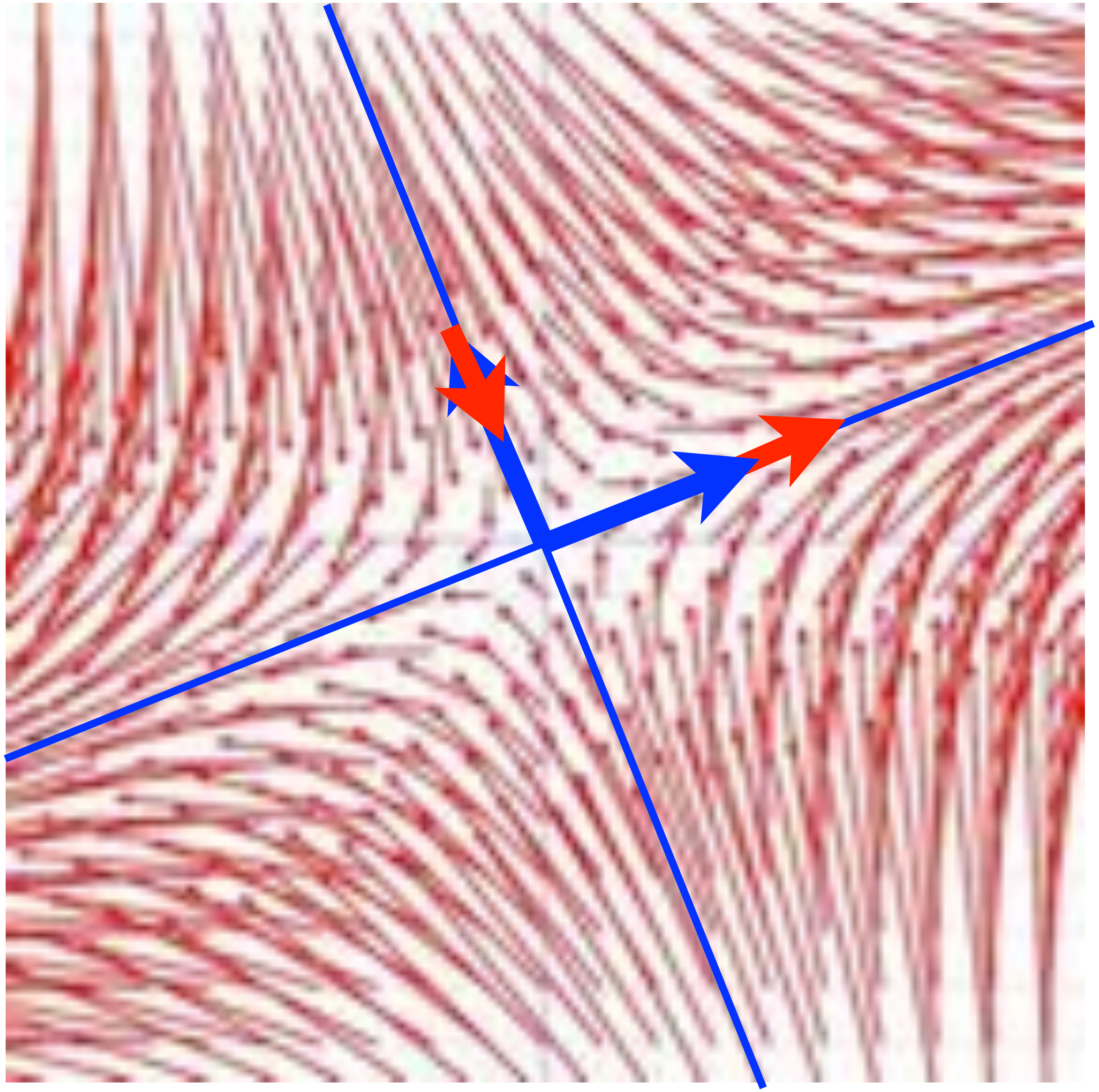

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$$\lambda_2 = -\sqrt{2}$$

$$\mathbf{v_2} = \begin{pmatrix} 1-\sqrt{2} \\ 1 \end{pmatrix}$$