Today

- The geometry of homogeneous and nonhomogeneous matrix equations
- Solving nonhomogeneous equations
    - Method of undetermined coefficients

Tutorial poll

(A) Hand out worksheet on Friday, print and hand in during tutorial.

(B) Hand out worksheet during tutorial, hand in during Tuesday class.

Second order, linear, constant coeff, **non**homogeneous (3.5)

- Our next goal is to figure out how to find solutions to nonhomogeneous equations like this one:

$$y'' - 6y' + 8y = \sin(2t)$$

- But first, a bit more on the connections between matrix algebra and differential equations . . .

Some connections to linear (matrix) algebra

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$$z = L[y] = \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y$$

- This one is linear because

$$L[cy] = cL[y]$$

$$L[y + z] = L[y] + L[z]$$

Note: y, z are functions of t and c is a constant.

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- A non-homogeneous differential equation has the form

$$L[y] = g(t)$$

Solutions to homogeneous matrix equations

- The matrix equation $A\overline{x} = \overline{0}$ could have (depending on A)

    (A) no solutions.

    (B) exactly one solution.

    (C) a one-parameter family of solutions.

    (D) an n-parameter family of solutions.

Choose the answer that is incorrect.

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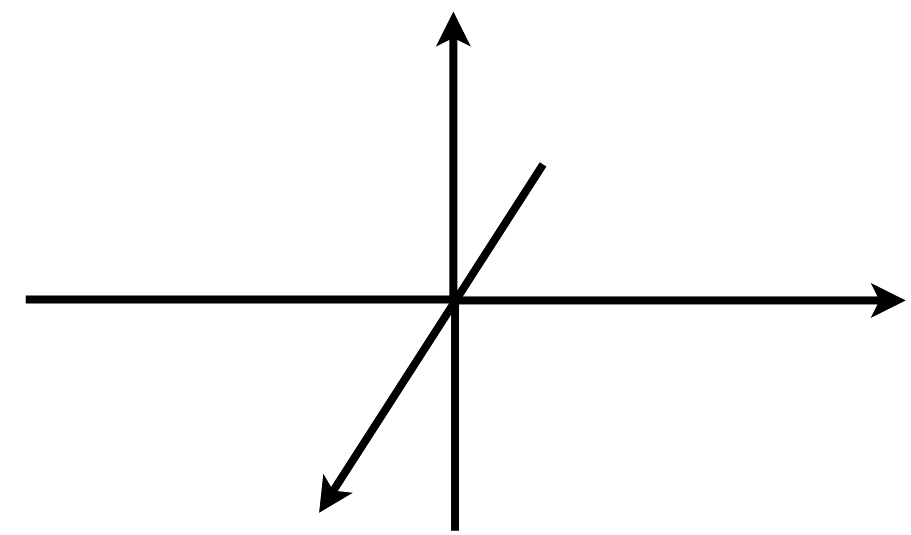

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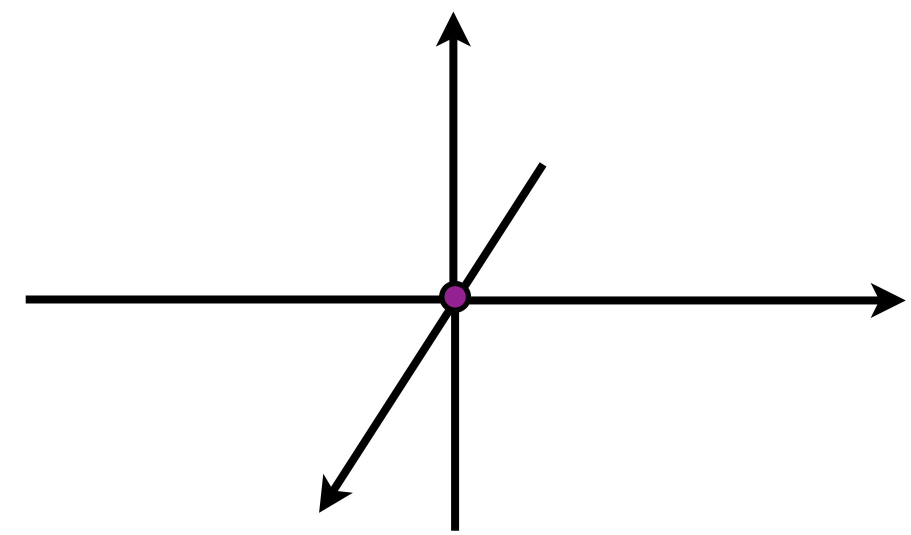

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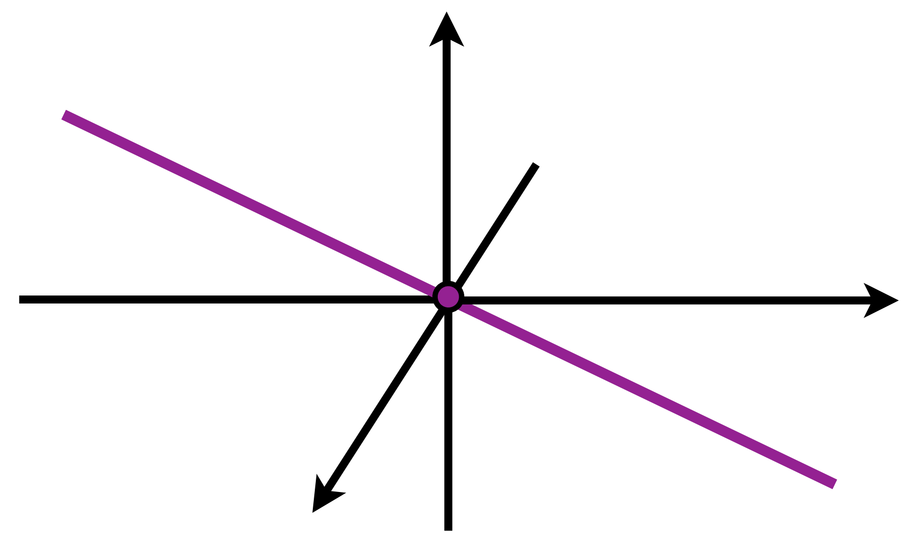

Possibilities:

$$\overline{x} = C \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

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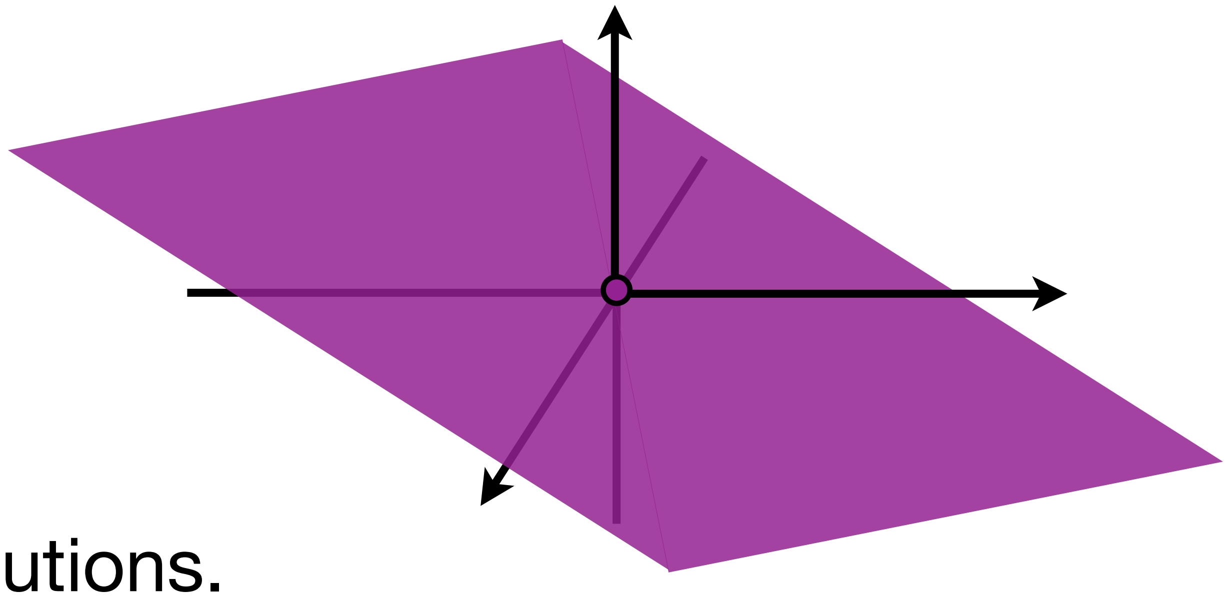

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Possibilities:

$$\overline{x} = C_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

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Solutions to homogeneous matrix equations

- Example 1. Solve the equation $A\overline{x} = \overline{0}$ where

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- so $x_1 - \frac{1}{3}x_3 = 0$ and $x_2 + \frac{5}{3}x_3 = 0$ and x_3 can be whatever (because it doesn't have a leading one).

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- Thus, the solution can be written as $\overline{x} = \frac{C}{3}\begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix}$.

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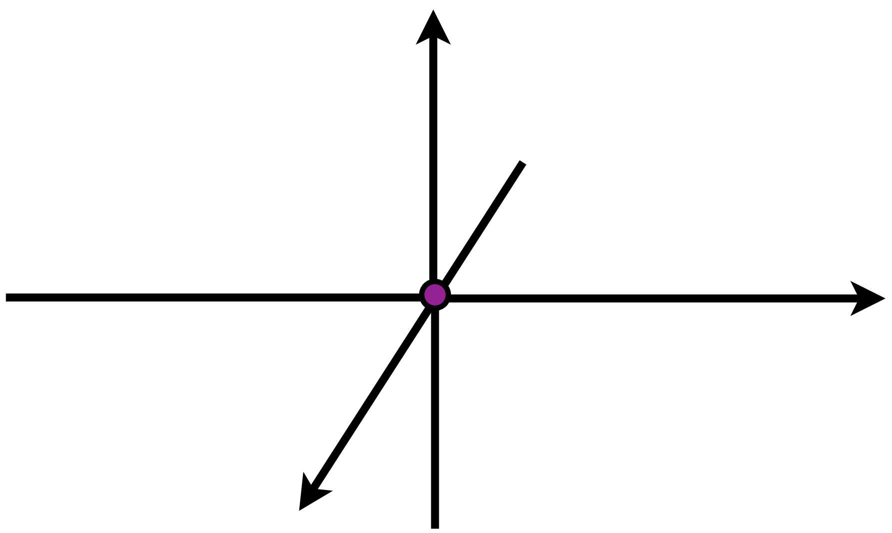

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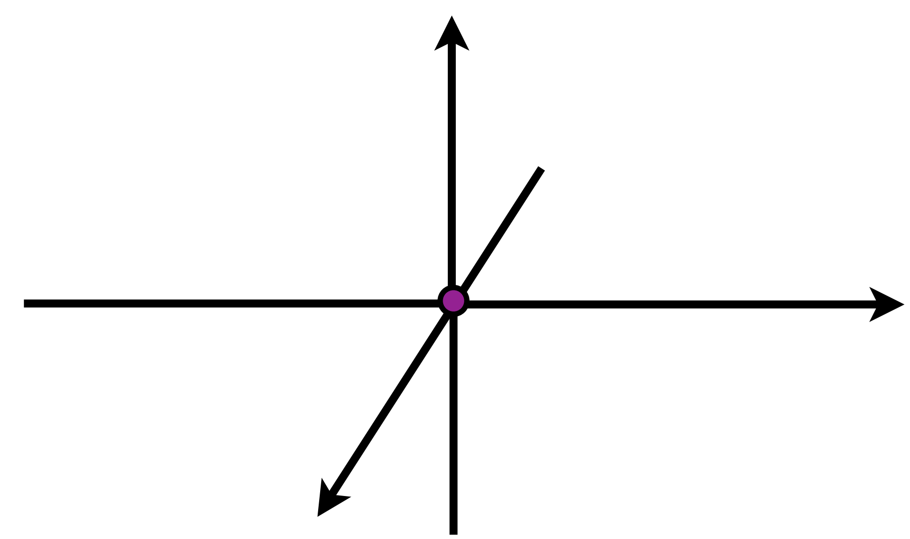

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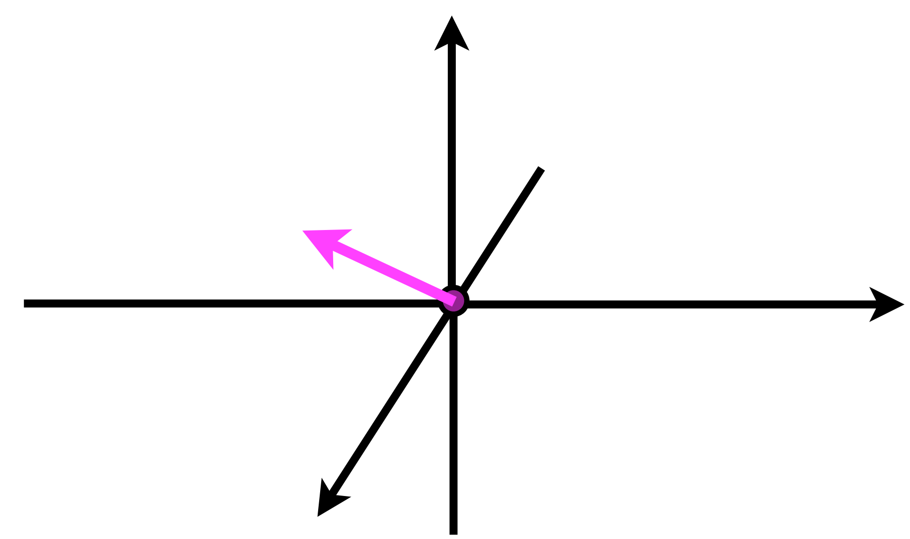

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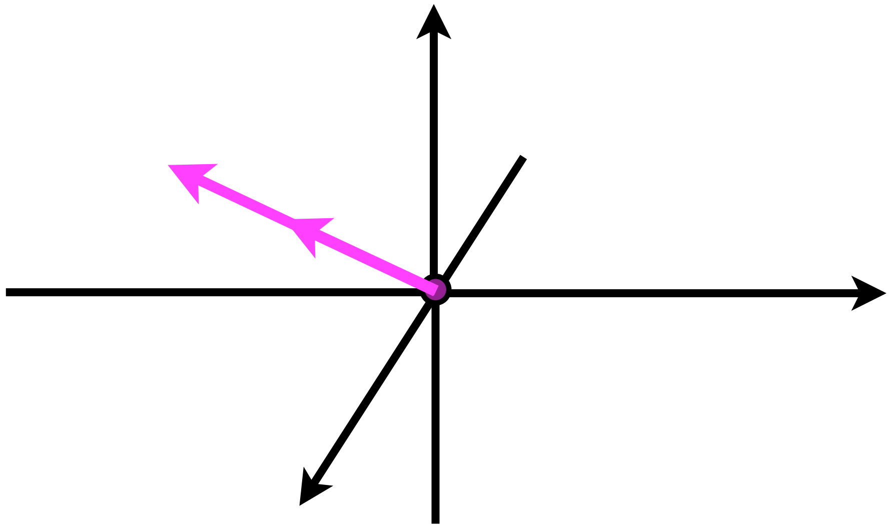

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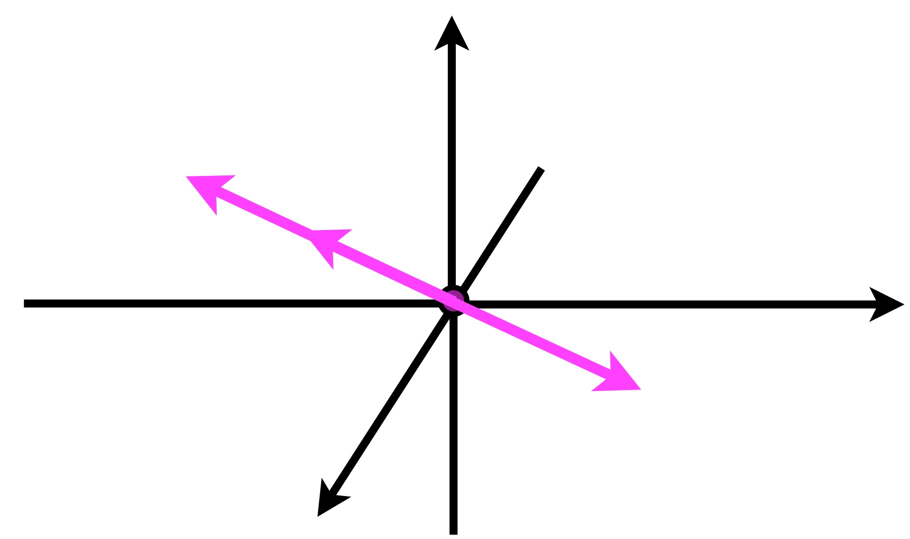

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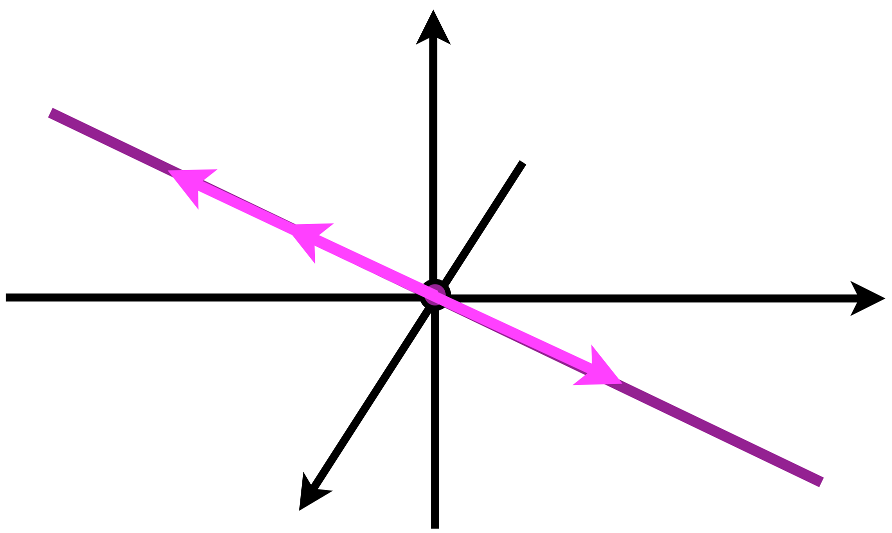

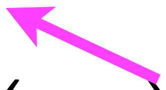

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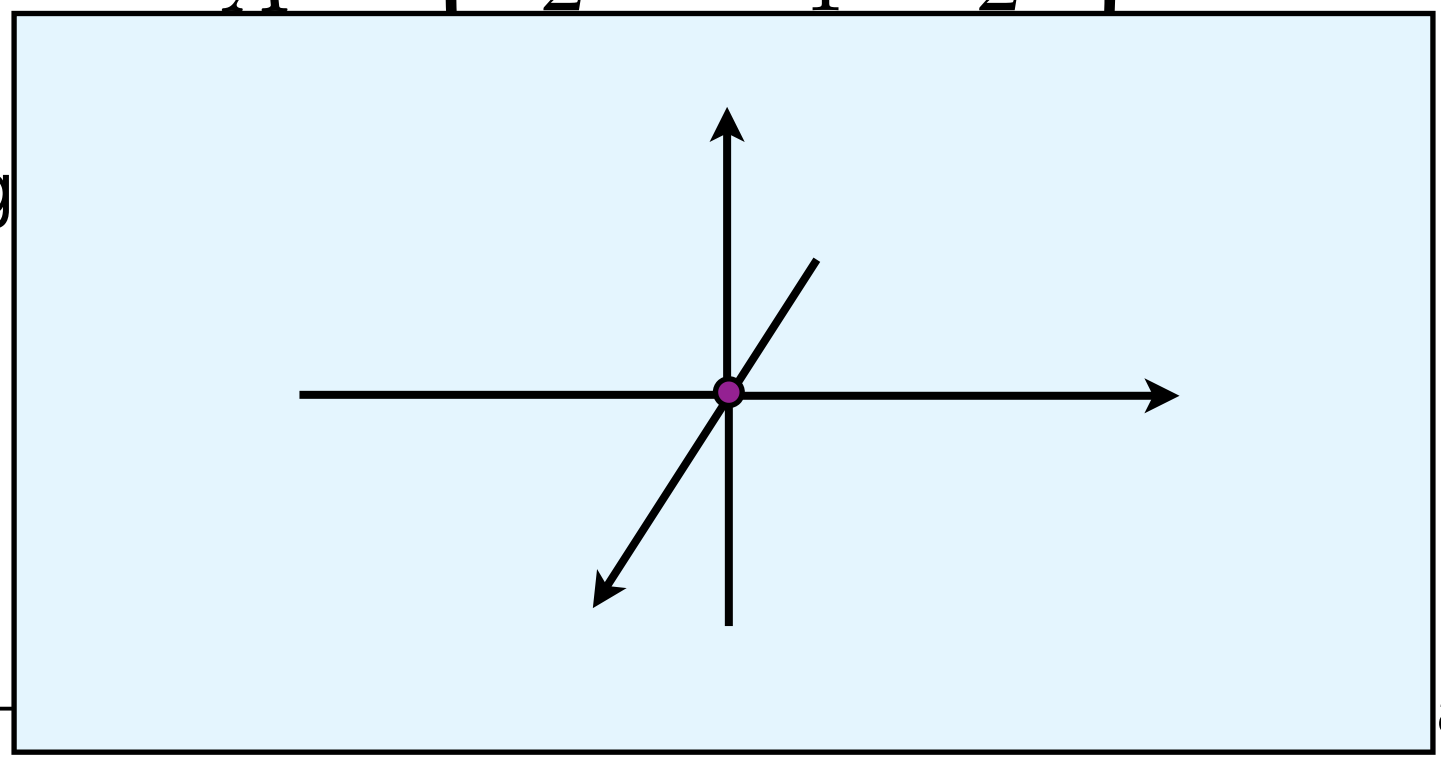

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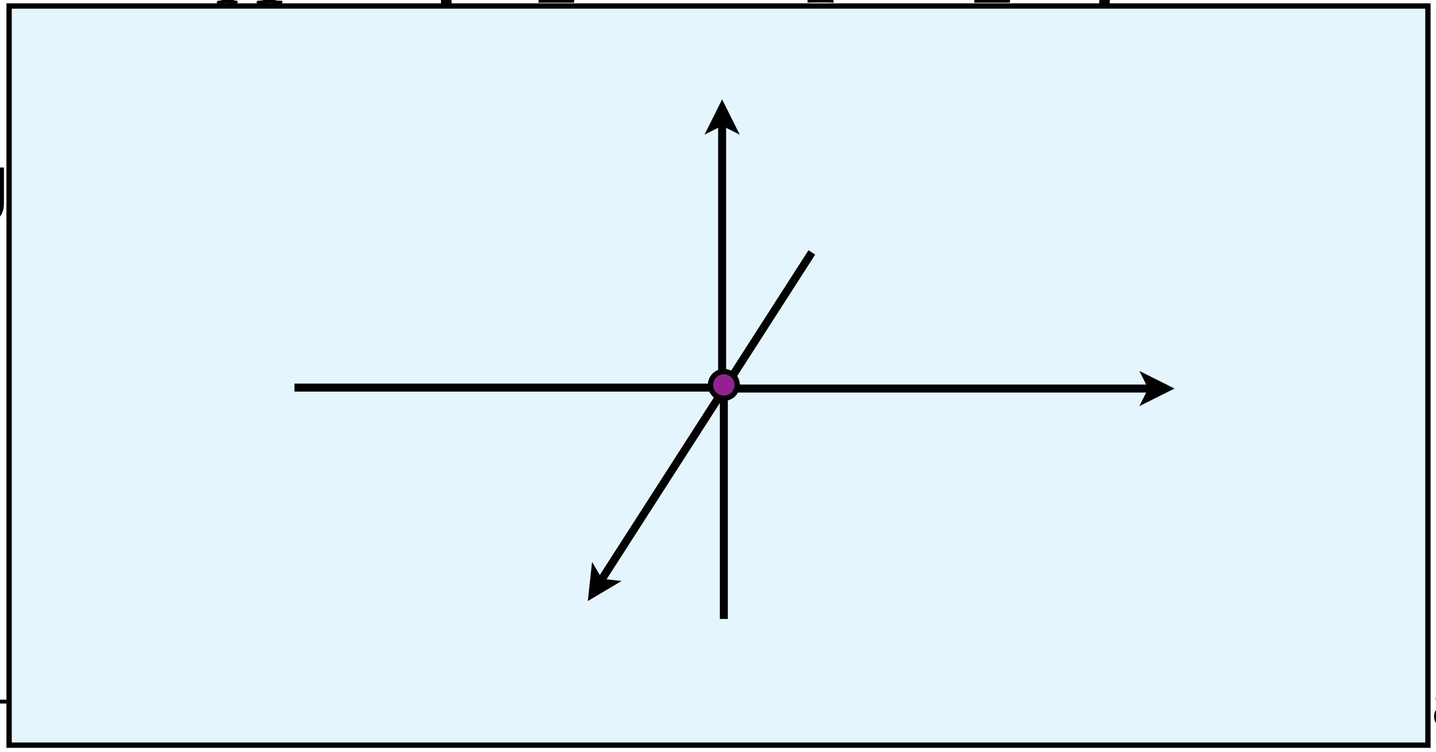

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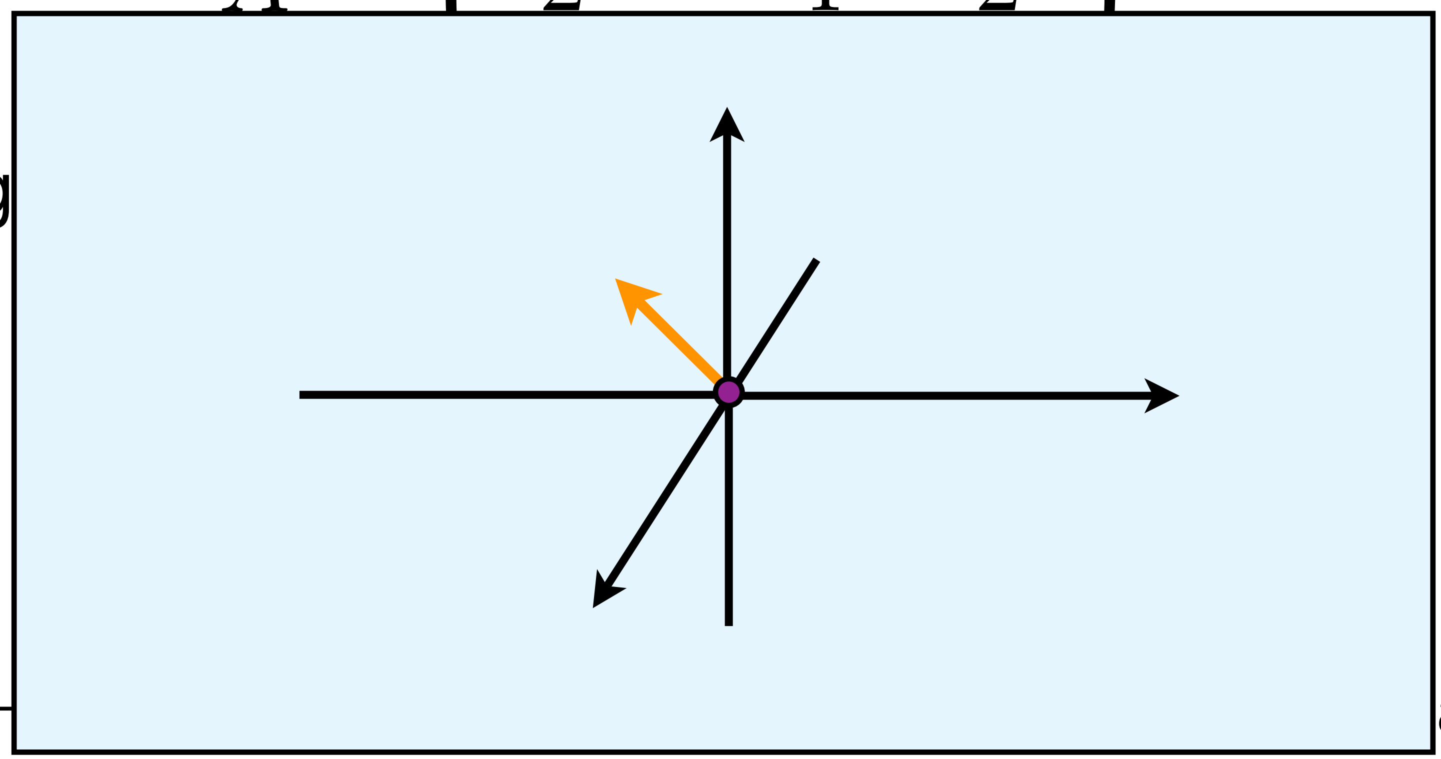

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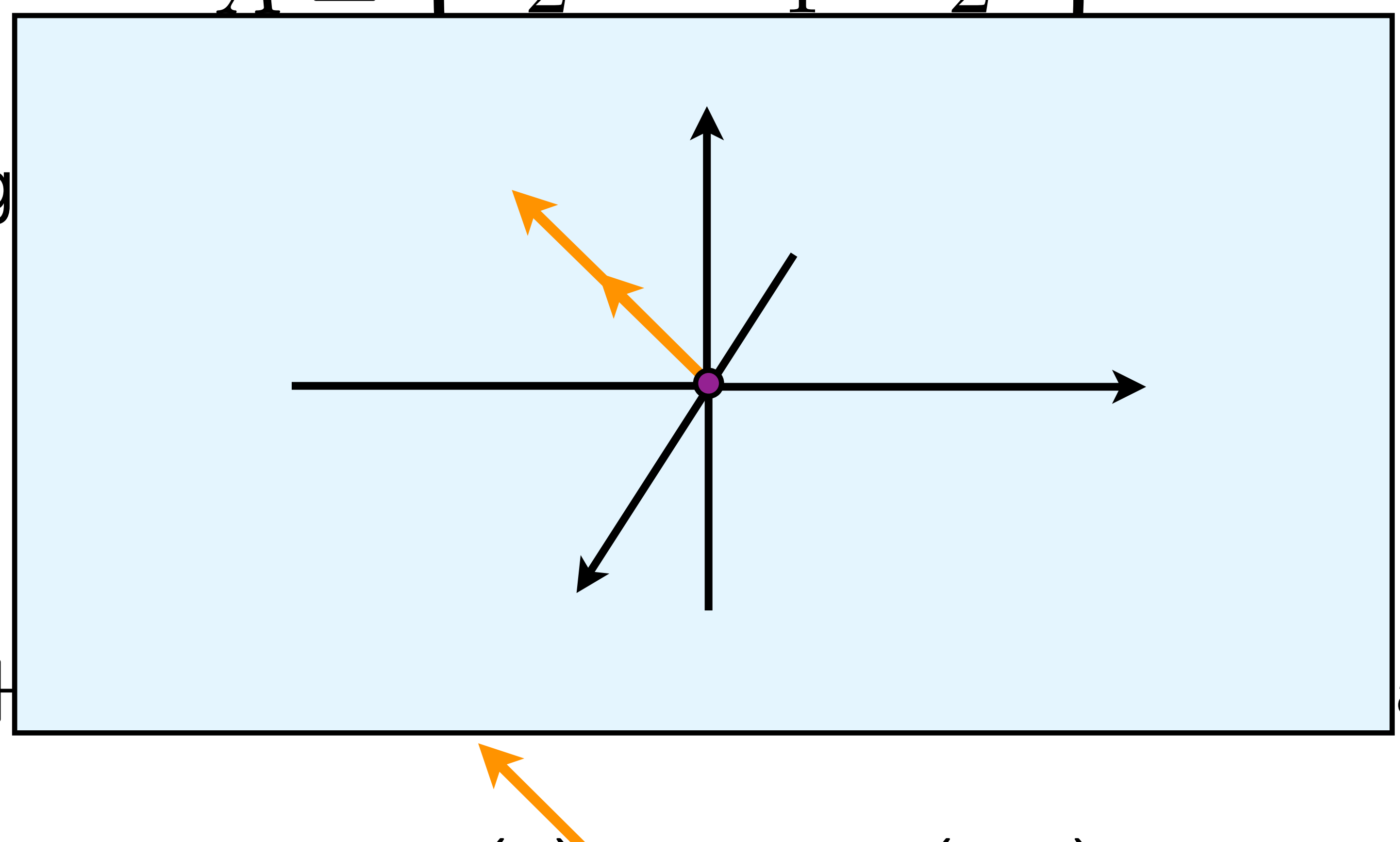

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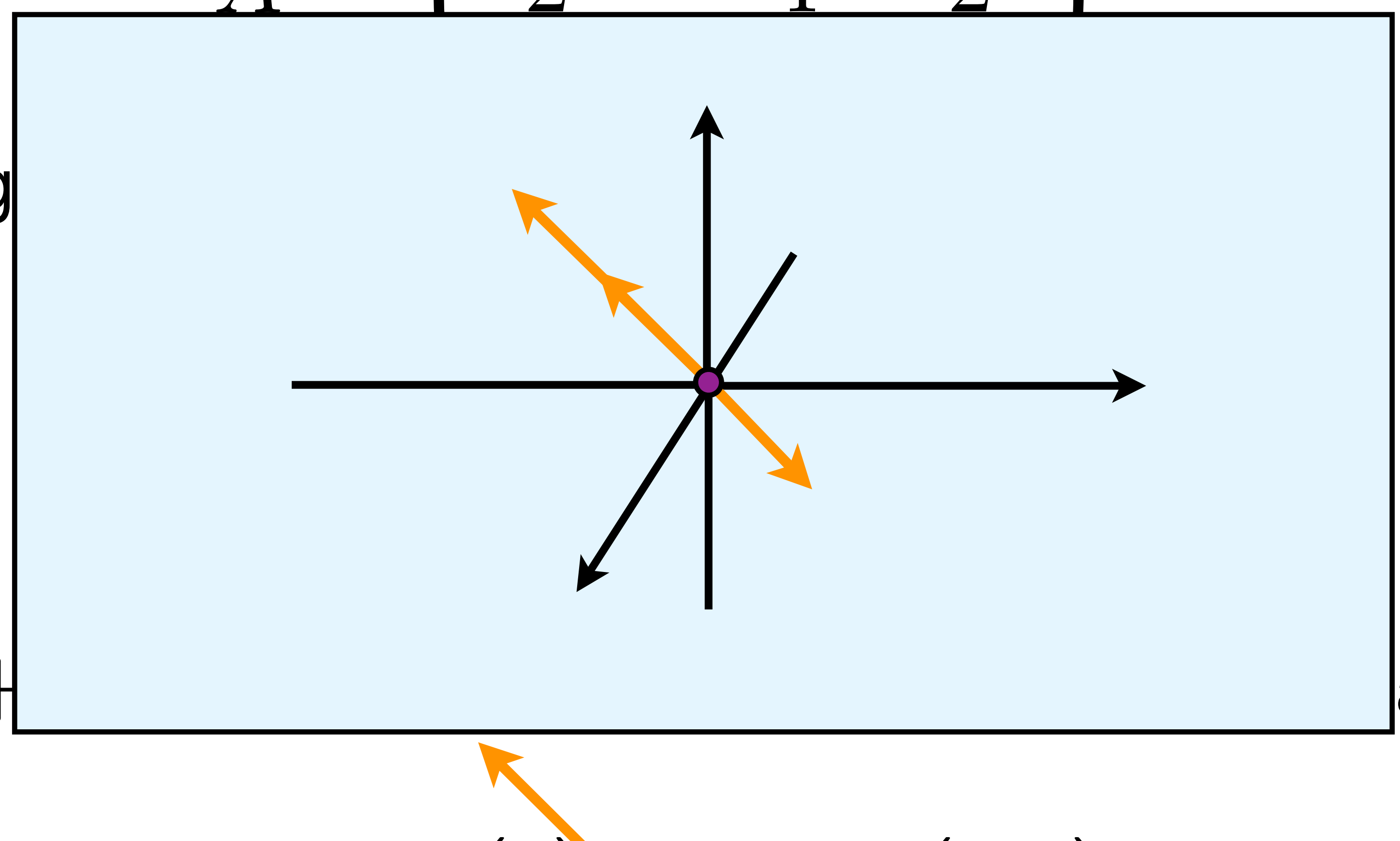

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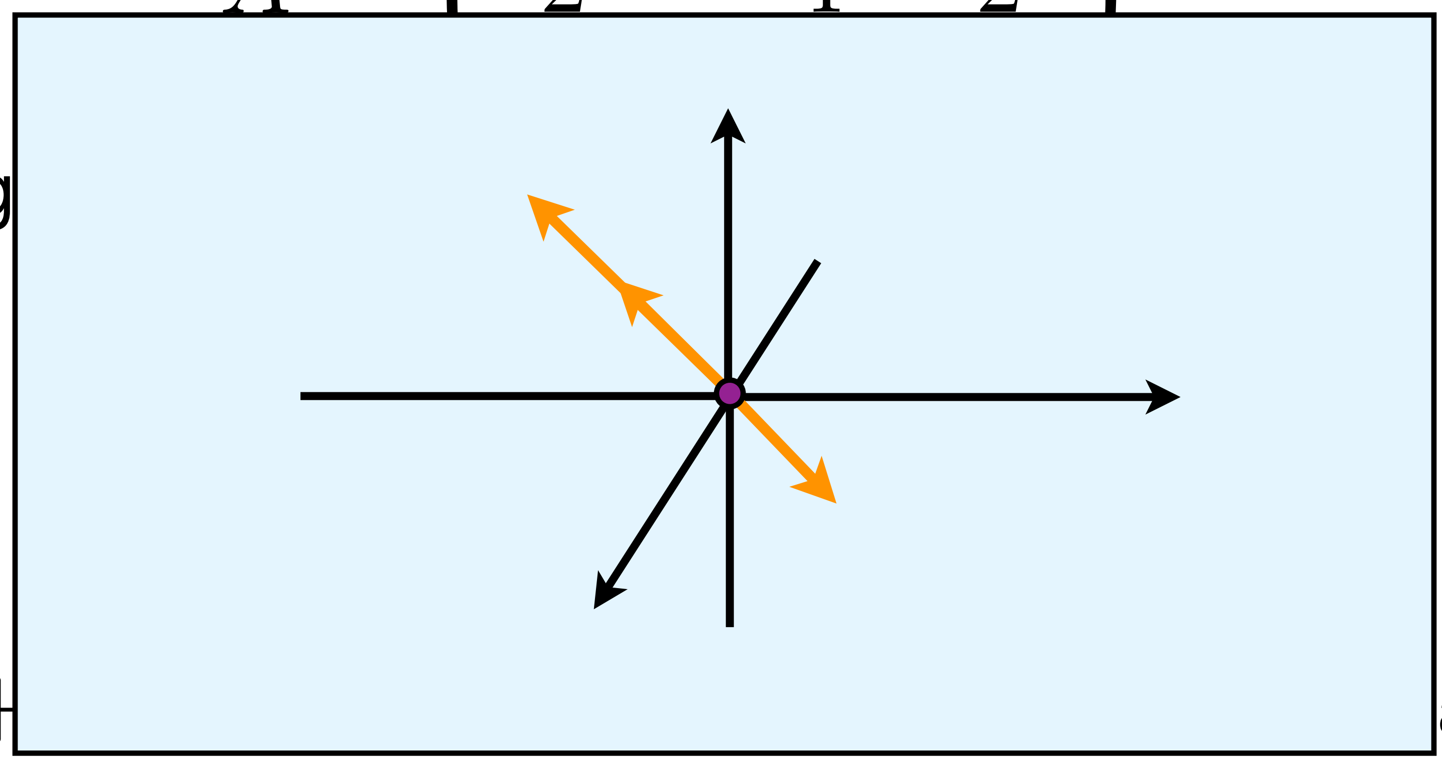

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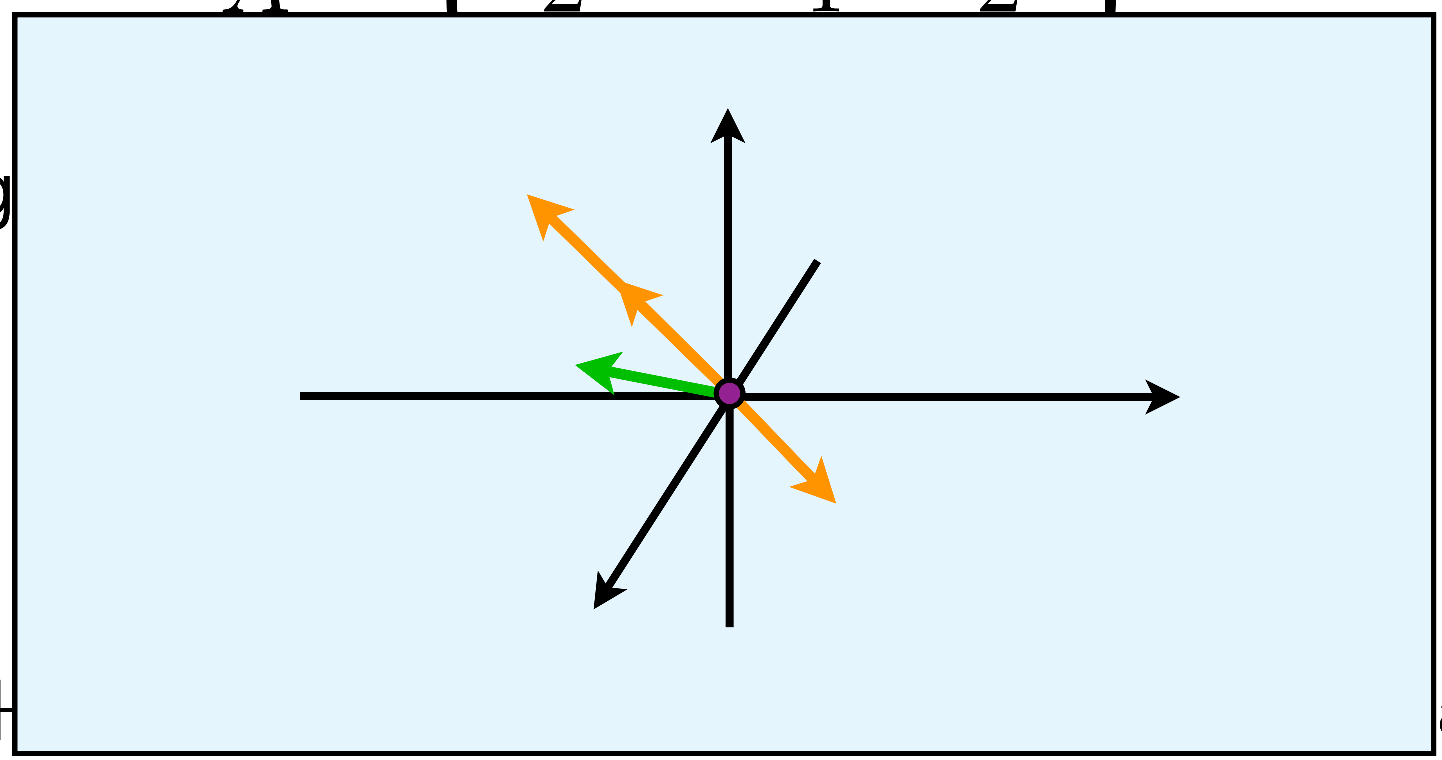

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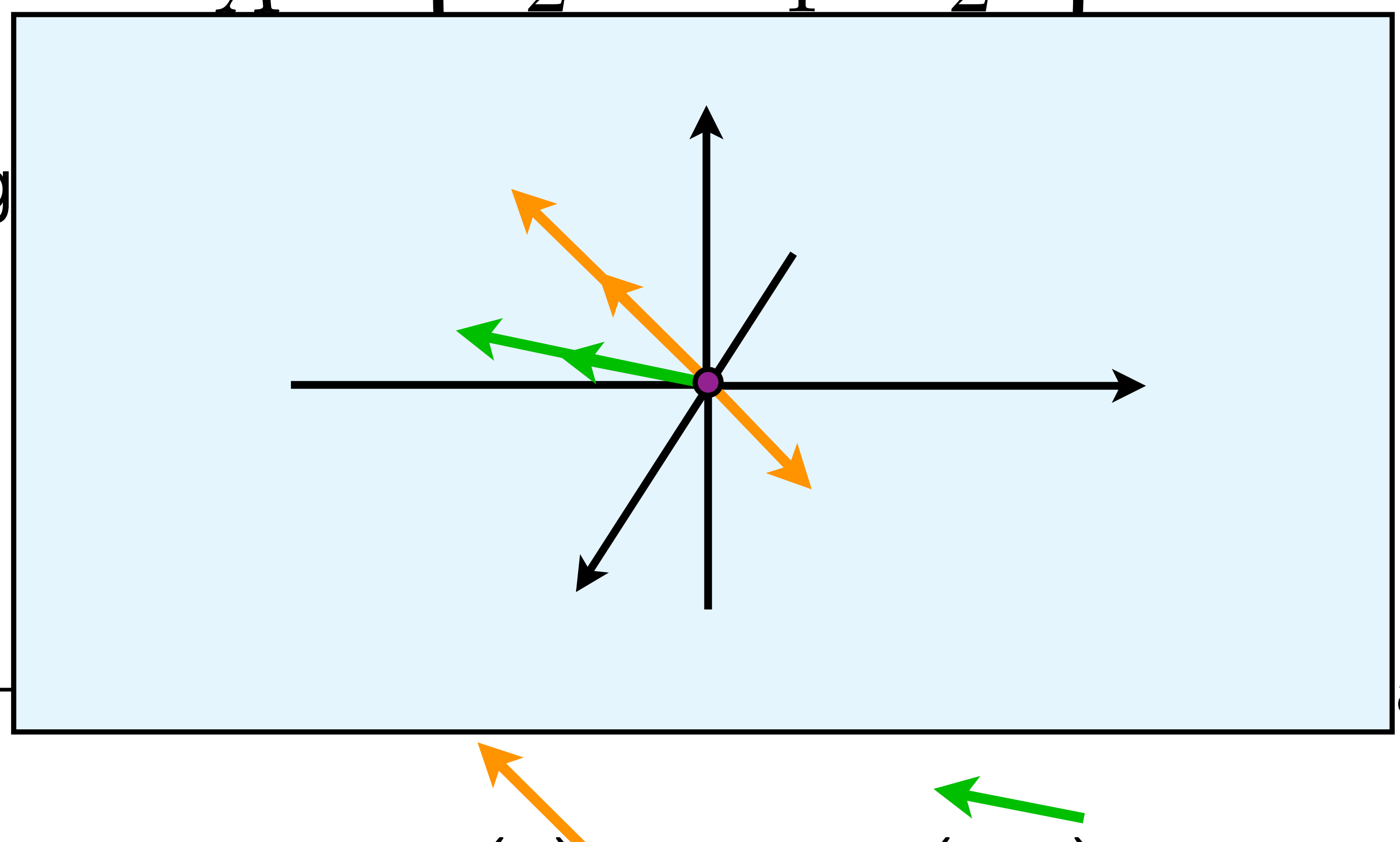

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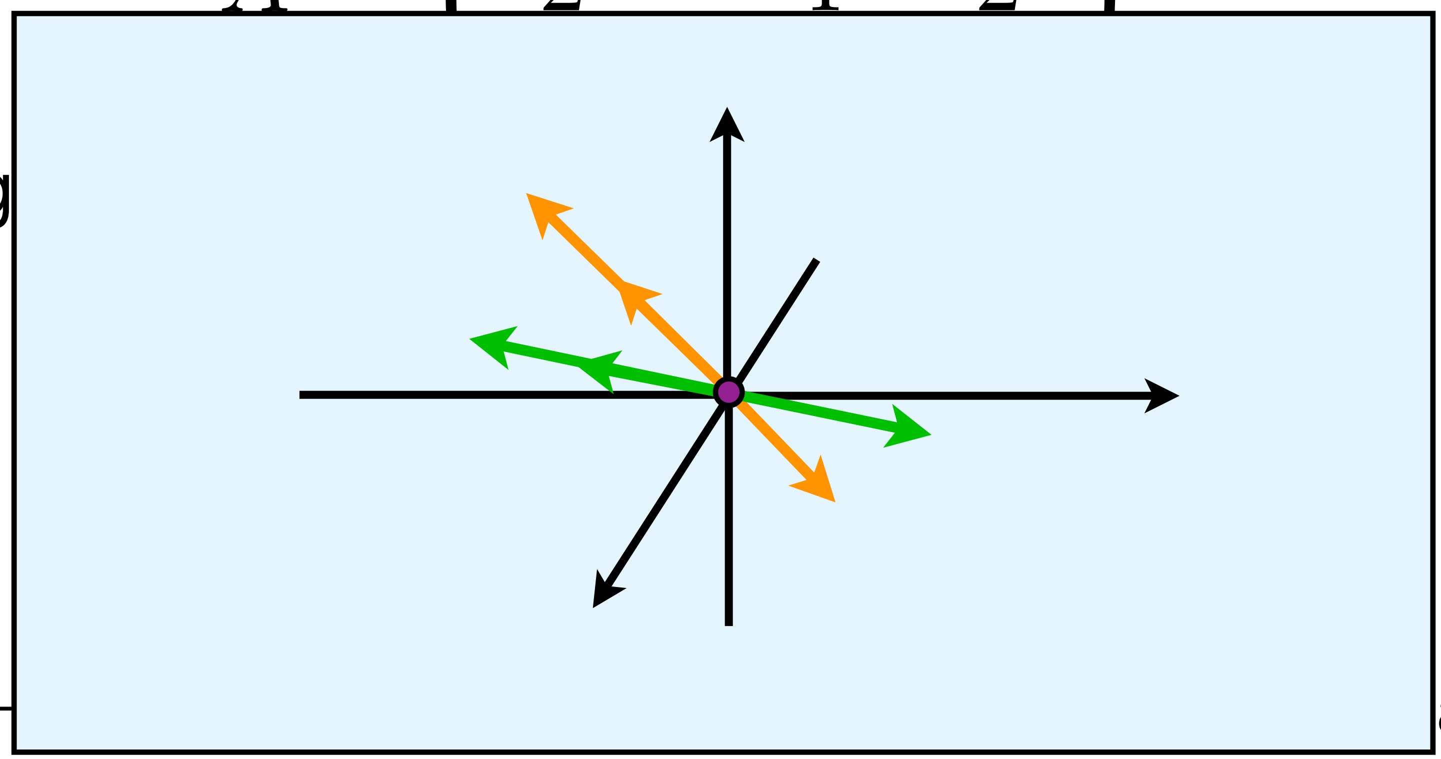
- Example 2. Solve the equation $A\overline{x} = \overline{0}$ where

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & -4 & 2 \end{pmatrix}$$



- Row reduction g

- so $x_1 - 2x_2 +$ atever.

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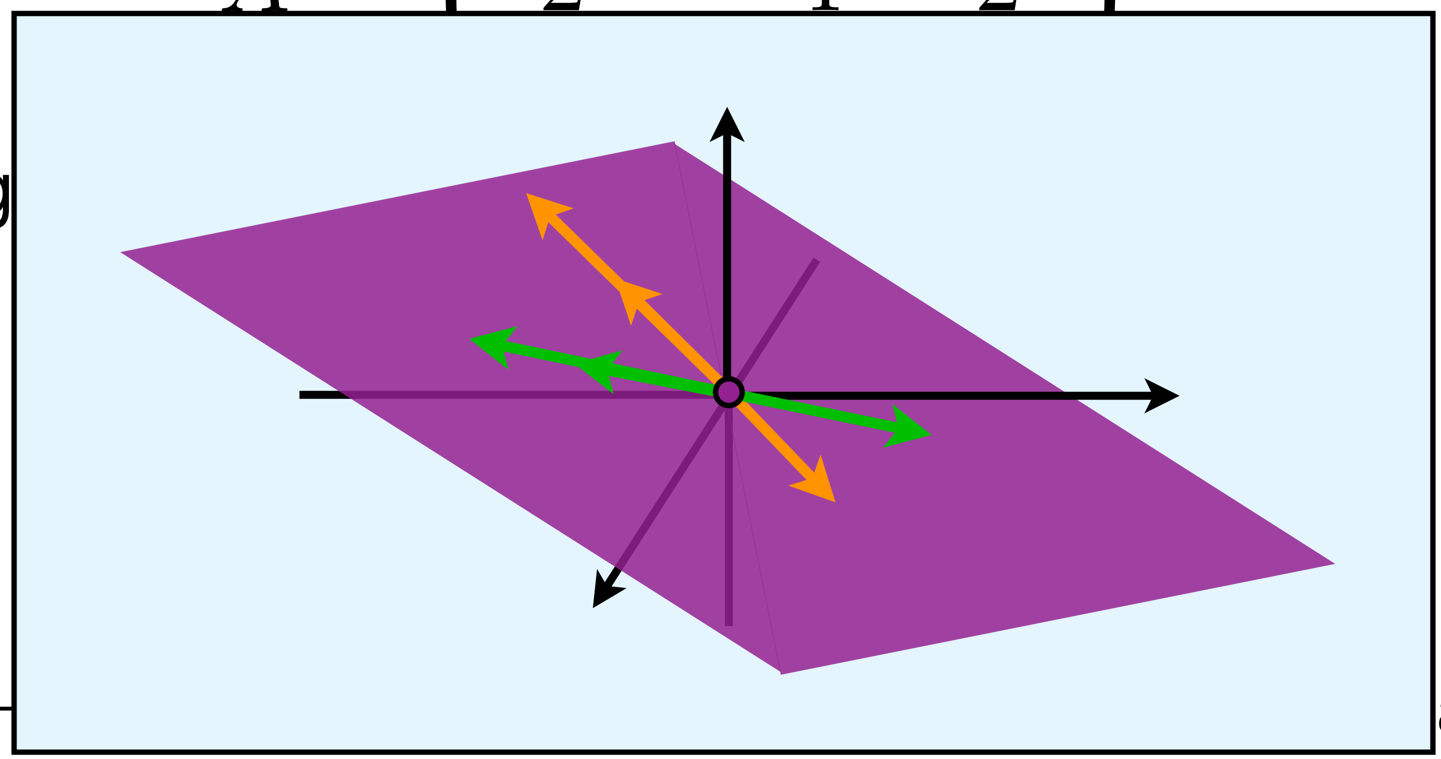

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Solutions to non-homogeneous matrix equations

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$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & -2 \\ 2 & 1 & 1 \end{pmatrix} \text{ and } \overline{b} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}.$$

Solutions to non-homogeneous matrix equations

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the general solution to the homogeneous problem

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the general solution to the homogeneous problem (first term)

one particular solution to nonhomogeneous problem (second term)

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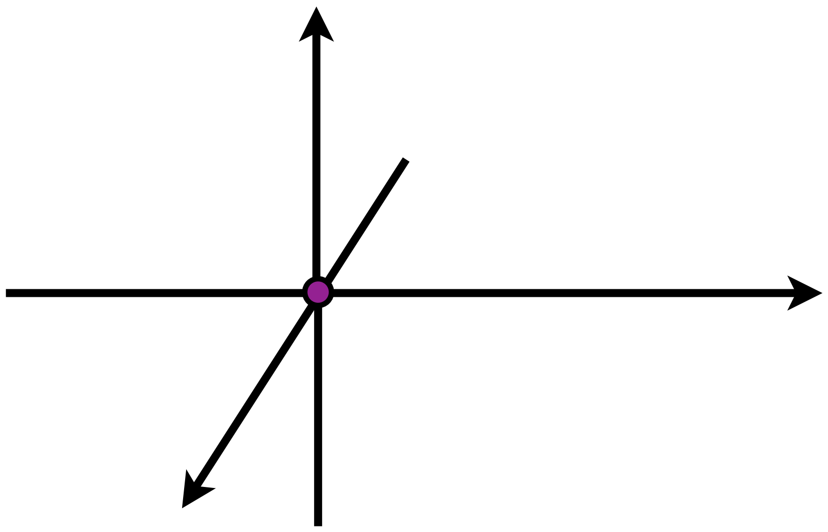

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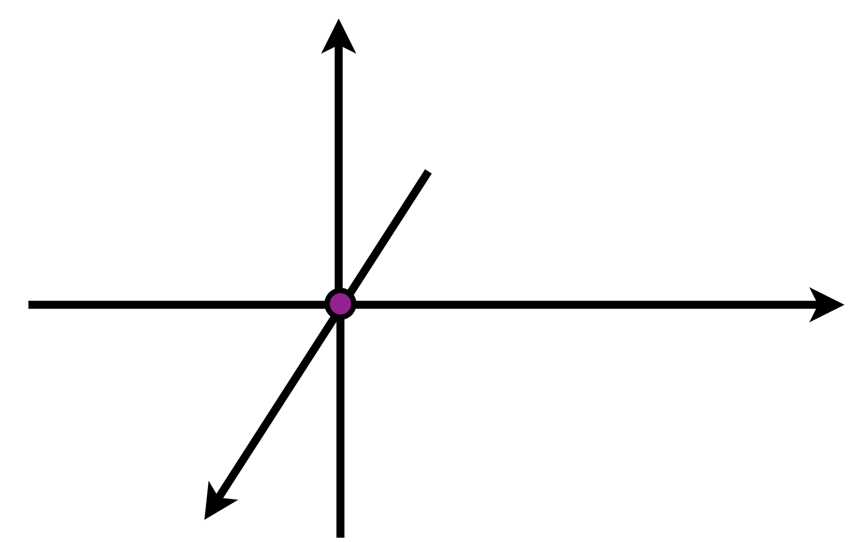

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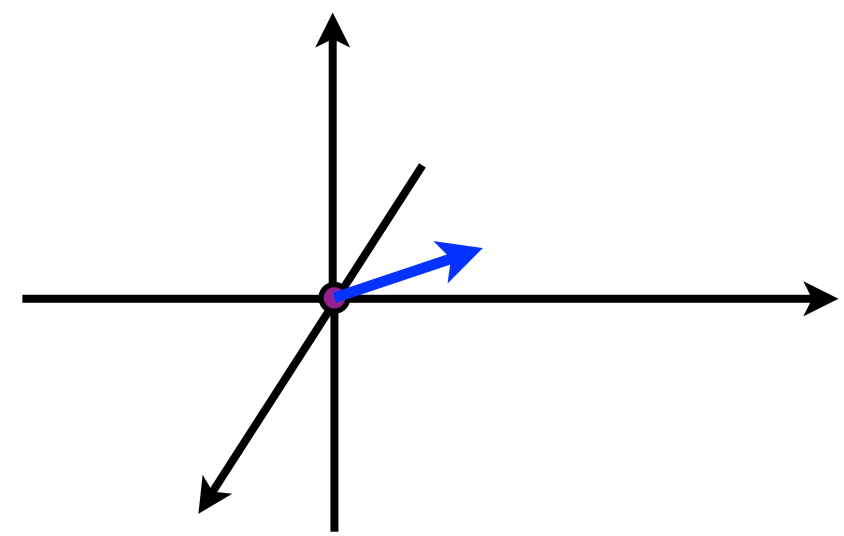

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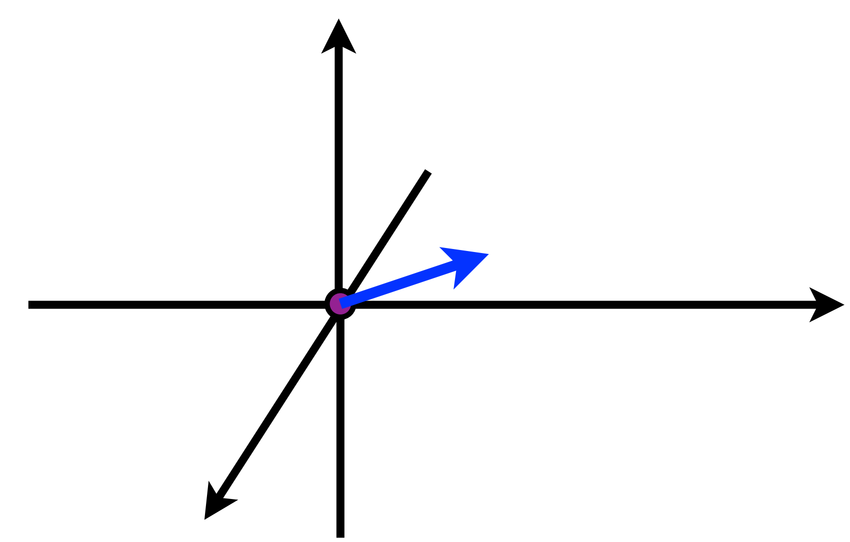

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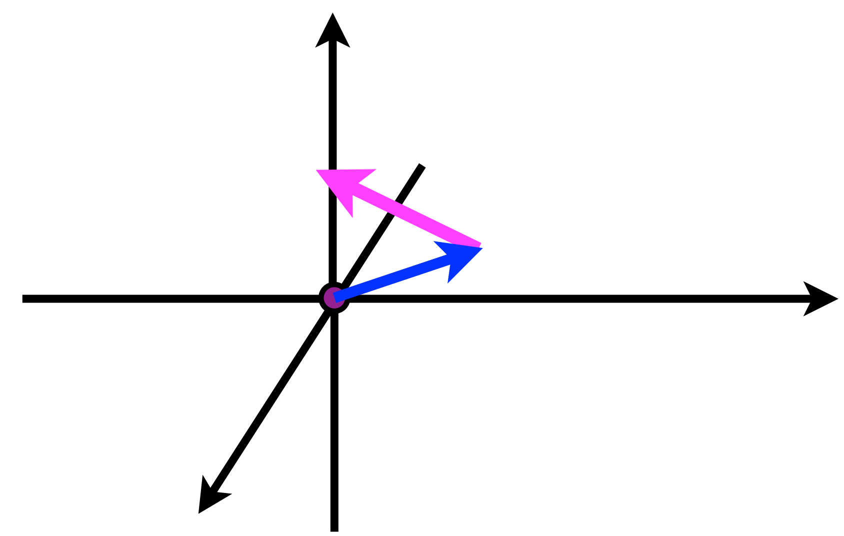

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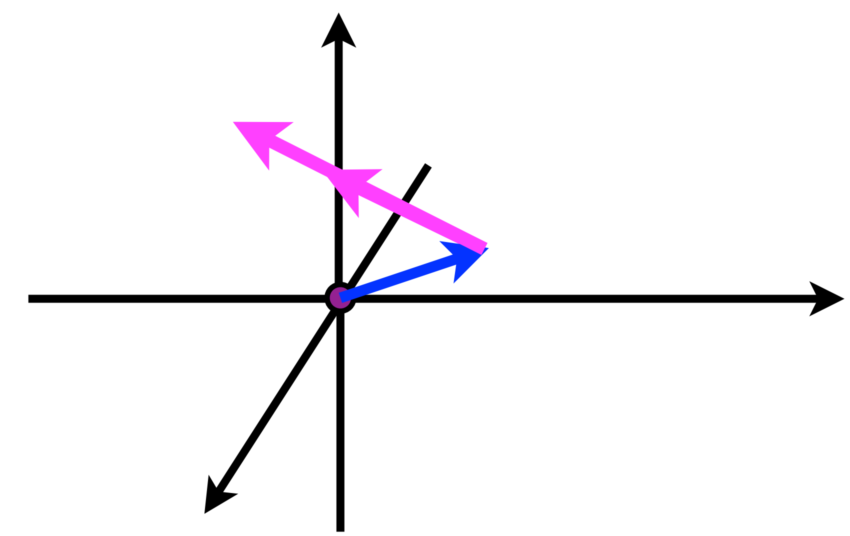

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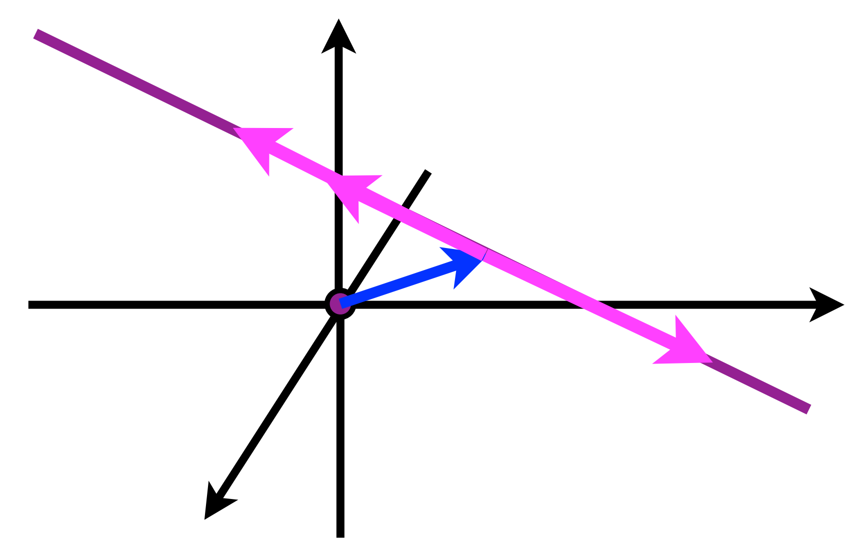

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Solutions to nonhomogeneous differential equations

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- To solve a nonhomogeneous differential equation:

Solutions to nonhomogeneous differential equations

- To solve a nonhomogeneous differential equation:
    1. Find the general solution to the associated homogeneous problem, $y_h(t)$.

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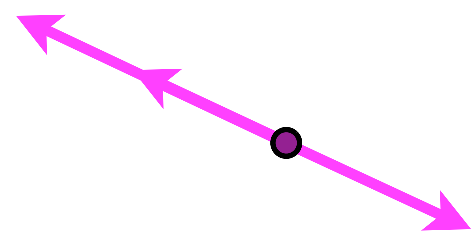

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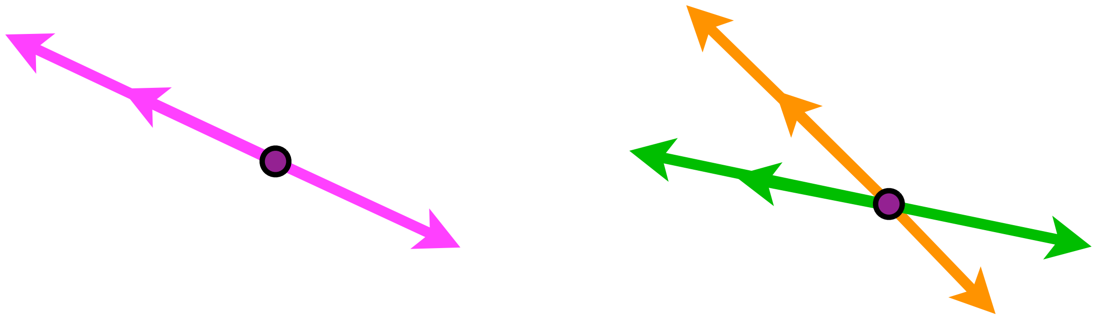

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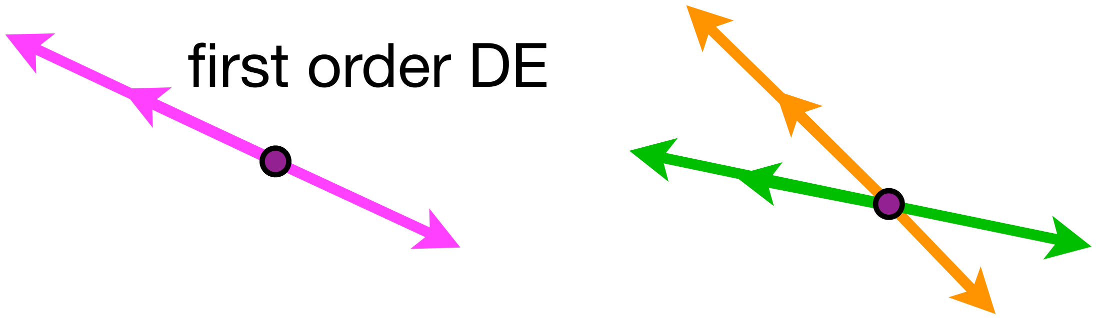

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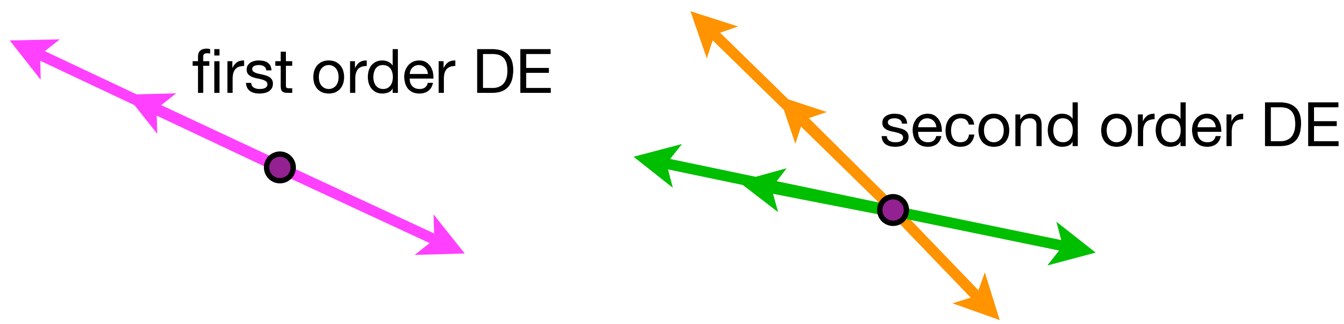

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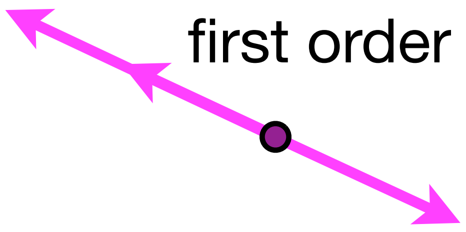

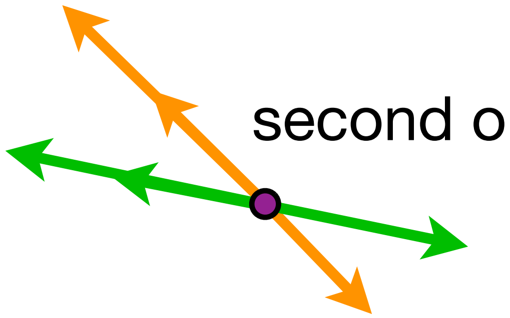

    2. Find a particular solution to the nonhomogeneous problem, $y_p(t)$.

Solutions to nonhomogeneous differential equations

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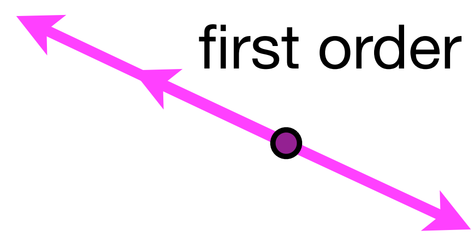
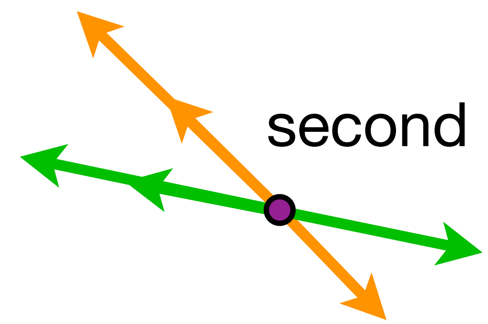
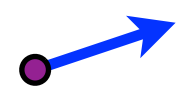


first order DE



second order DE

2. Find a particular solution to the nonhomogeneous problem, $y_p(t)$.



Solutions to nonhomogeneous differential equations

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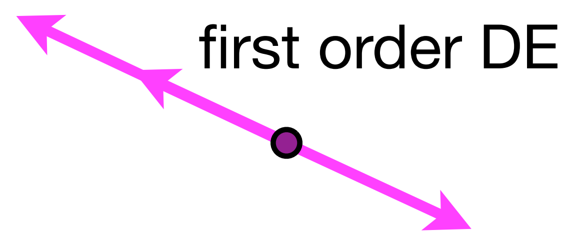

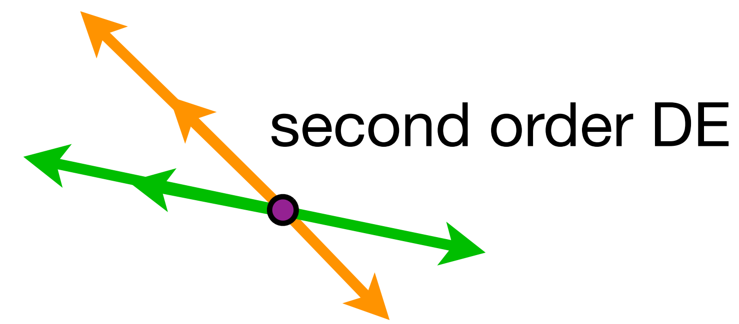

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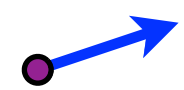

3. The general solution to the nonhomogeneous problem is their sum:

$$y = y_h + y_p = C_1 y_1 + C_2 y_2 + y_p$$

Solutions to nonhomogeneous differential equations

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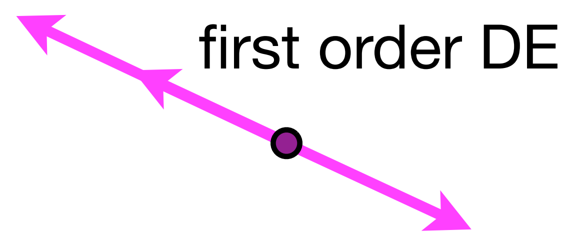

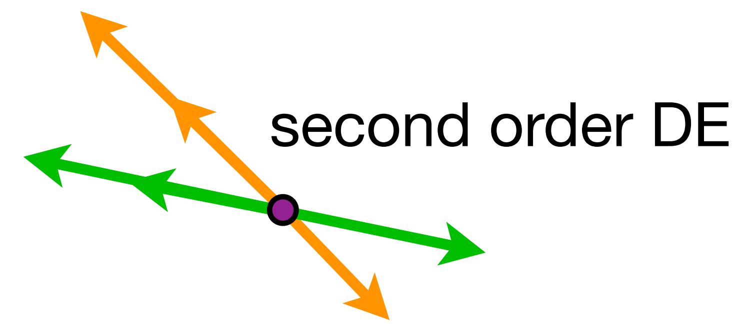

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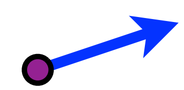

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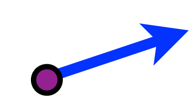

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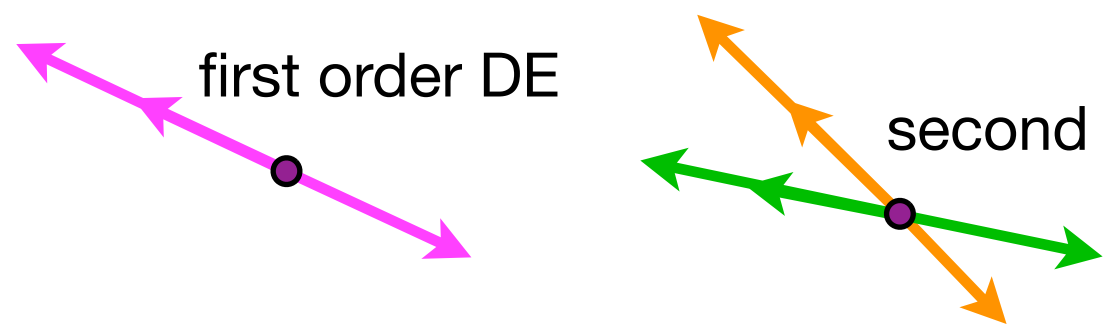

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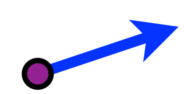

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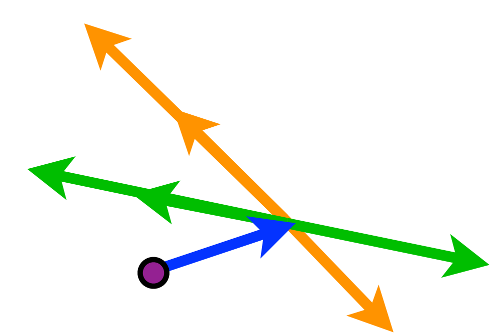

Solutions to nonhomogeneous differential equations

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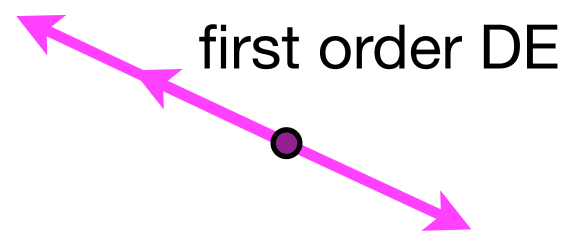

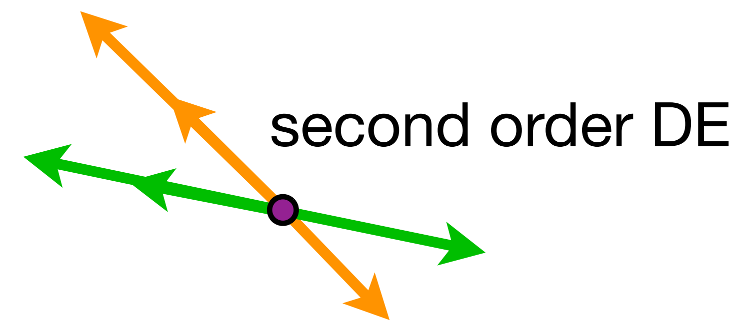

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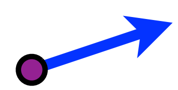

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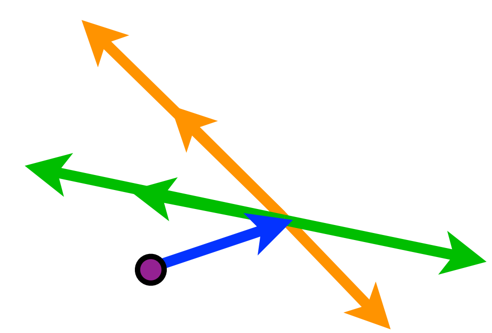

- For step 2, try "Method of undetermined coefficients"...

Method of undetermined coefficients

Method of undetermined coefficients

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.

Method of undetermined coefficients

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
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$$y_h(t) = C_1 e^t + C_2 e^{-3t}$$

Method of undetermined coefficients

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
  - Step 1: Solve the associated homogeneous equation
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    - Try $y_p(t) = Ae^{2t}$.
    - $L[y_p(t)] = L[Ae^{2t}] =$
      - (A) $5e^{2t}$
      - (B) $5Ae^{2t}$
      - (C) $4e^{2t}$
      - (D) $4Ae^{2t}$

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      - (A) $5e^{2t}$
      - (B) $5Ae^{2t}$ ★
      - (C) $4e^{2t}$
      - (D) $4Ae^{2t}$
    - A is an undetermined coefficient (until you determine it).

Method of undetermined coefficients

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    - Summarizing:

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    - We know that, for any C_1 and C_2,

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    - We know that, for any C_1 and C_2,

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    - We also know that

$$L[Ae^{2t}] = 5Ae^{2t}$$

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    - Finally, by linearity, we know that

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients

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    - So what's left to do to find our general solution? Pick A =?

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    - Finally, by linearity, we know that

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

    - So what's left to do to find our general solution? Pick A = 1/5.

Method of undetermined coefficients

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the solution to the associated homogeneous equation?

(A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

(B) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t)$

(C) $y_h(t) = C_1 e^{2t} + C_2 t e^{2t}$

(D) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t) + e^t$

(E) Don’t know.

Method of undetermined coefficients

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(E) Don't know.

Method of undetermined coefficients

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
    - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

(B) $y_p(t) = Ae^{-2t}$

(C) $y_p(t) = Ae^{t}$

(D) $y_p(t) = Ate^{t}$

(E) Don't know

Method of undetermined coefficients

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    (B) $y_p(t) = Ae^{-2t}$

    ☆ (C) $y_p(t) = Ae^t$

    (D) $y_p(t) = Ate^t$

    (E) Don't know

Method of undetermined coefficients

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the value of A that gives the particular solution (Ae^t) ?

(A) A = 1

(B) A = 3

(C) A = 1/3

(D) A = -1/3

(E) Don't know.

Method of undetermined coefficients

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the value of A that gives the particular solution (Ae^t) ?

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(B) A = 3

(C) A = 1/3

(D) A = -1/3

(E) Don’t know.

Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the solution to the associated homogeneous equation?

(A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

(B) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t)$

(C) $y_h(t) = C_1 e^{2t} + C_2 t e^{2t}$

(D) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t} + e^t$

(E) Don't know.

Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
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    ☆ (A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

    (B) $y_h(t) = C_1 \cos(2t) + C$

    (C) $y_h(t) = C$

    $C_1 e^{2t} + C_2 e^{-2t} + e^t$

    (E) Don't know.

Same as the last example

Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

    (A) $y_p(t) = Ae^{2t}$

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    (C) $y_p(t) = Ate^{2t}$

    (D) $y_p(t) = Ae^{t}$

    (E) $y_p(t) = Ate^{t}$

Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \; !$$

(B) $y_p(t) = Ae^{-2t}$

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(D) $y_p(t) = Ae^{t}$

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- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^{t}$$

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$$y' - y = e^t$$

$$e^{-t}y' - e^{-t}y = 1$$

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$$(Ae^{2t})'' - 4Ae^{2t} = 0 \ !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^t$$

$$e^{-t}y' - e^{-t}y = 1$$

$$y = te^t + Ce^t$$

Method of undetermined coefficients

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- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^{t}$$

$$e^{-t}y' - e^{-t}y = 1$$

$$y = te^{t} + Ce^{t}$$

- General rule: when your guess at y_p makes LHS=0, try multiplying it by t.

Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the value of A that gives the particular solution (Ate^{2t})?

(A) A = 1

(B) A = 4

(C) A = -4

(D) A = 1/4

(E) A = -1/4

Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
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(A) A = 1

$$(Ate^{2t})' = Ae^{2t} + 2Ate^{2t}$$

(B) A = 4

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(B) A = 4

$$(Ate^{2t})'' = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t}$$

(C) A = -4

$$= 4Ae^{2t} + 4Ate^{2t}$$

(D) A = 1/4

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Method of undetermined coefficients

- Example 6. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the value of A that gives the particular solution $\left(Ate^{2t}\right)$?

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$$(Ate^{2t})' = Ae^{2t} + 2Ate^{2t}$$

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$$= 4Ae^{2t} + 4Ate^{2t}$$

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Need: $= e^{2t}$

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$$= 4Ae^{2t} + 4Ate^{2t}$$

$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) = 4Ae^{2t}$$

Need: $= e^{2t}$

Method of undetermined coefficients

- Example 7. Find the general solution to $y'' - 4y = \cos(2t)$.
  - What is the form of the particular solution?

(A) $y_p(t) = A\cos(2t)$

(B) $y_p(t) = A\sin(2t)$

(C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Method of undetermined coefficients

- Example 7. Find the general solution to $y'' - 4y = \cos(2t)$.
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(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Challenge: What small change to the DE makes (D) correct?

Method of undetermined coefficients

- Example 7. Find the general solution to $y'' + y' - 4y = \cos(2t)$.
    - What is the form of the particular solution?

(A) $y_p(t) = A\cos(2t)$

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Method of undetermined coefficients

- Example 7. Find the general solution to $y'' + y' - 4y = \cos(2t)$.
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(C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Method of undetermined coefficients

- Example 8. Find the general solution to $y'' - 4y = t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = At^3$

(B) $y_p(t) = At^3 + Bt^2 + Ct$

(C) $y_p(t) = At^3 + Bt^2 + Ct + D$

(D) $y_p(t) = At^3 + Be^{2t} + Ce^{-2t}$

(E) Don’t know.

Method of undetermined coefficients

- Example 8. Find the general solution to $y'' - 4y = t^3$.
  - What is the form of the particular solution?

    (A) $y_p(t) = At^3$

    (B) $y_p(t) = At^3 + Bt^2 + Ct$

    ☆ (C) $y_p(t) = At^3 + Bt^2 + Ct + D$

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    (E) Don't know.

Method of undetermined coefficients

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  - What is the form of the particular solution?

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(B) $y_p(t) = At^3 + Bt^2 + Ct$

☆ (C) $y_p(t) = At^3 + Bt^2 + Ct + D$

(D) $y_p(t) = At^3 + Be^{2t} + Ce^{-2t}$ ← waste of time including solution to homogeneous eq.

(E) Don't know.

Method of undetermined coefficients

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or

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$$y_{p_2}(t) = Ct^3 + Dt^2 + Et + F$$

$$y_p(t) = y_{p_1}(t) + y_{p_2}(t)$$