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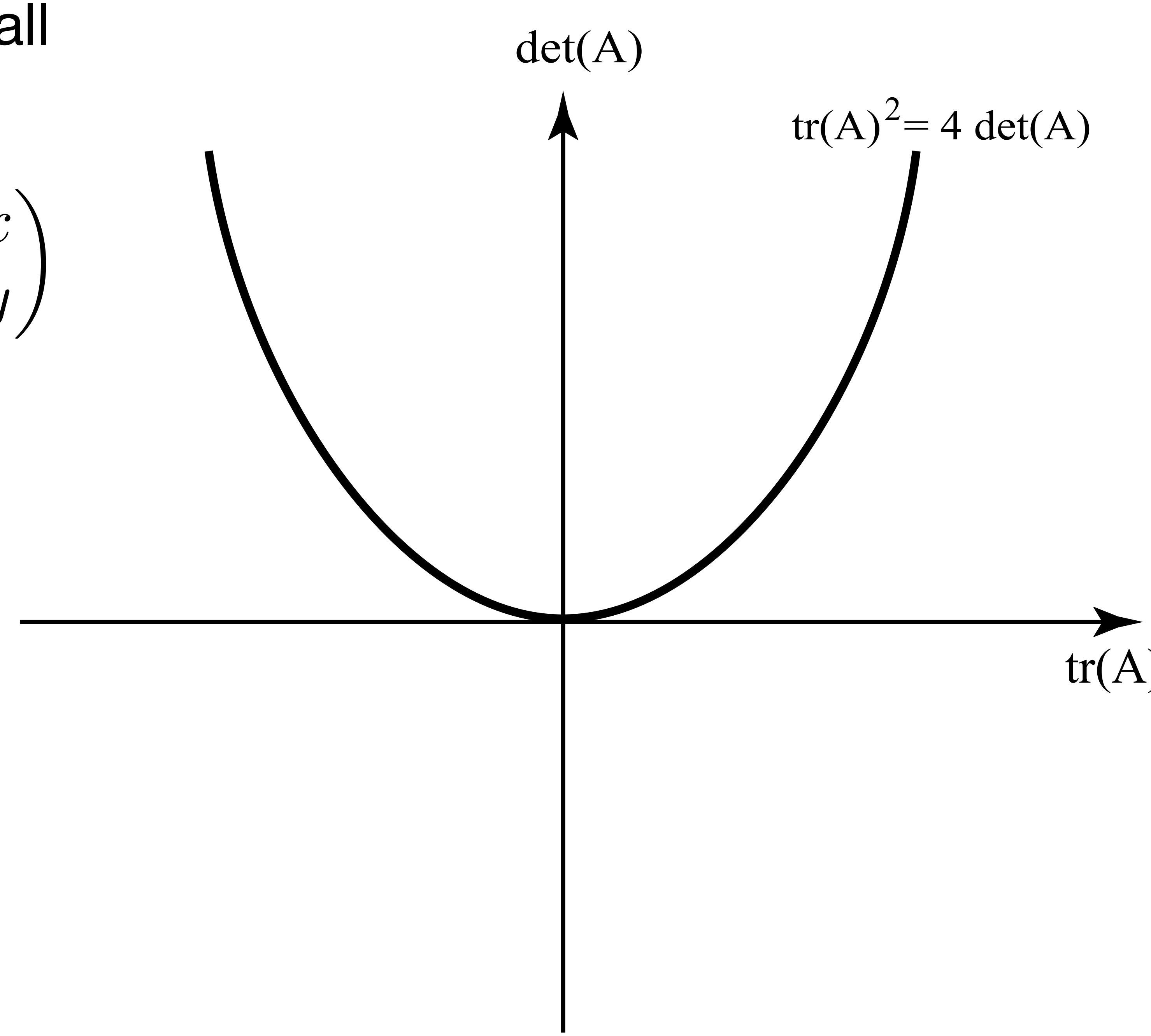

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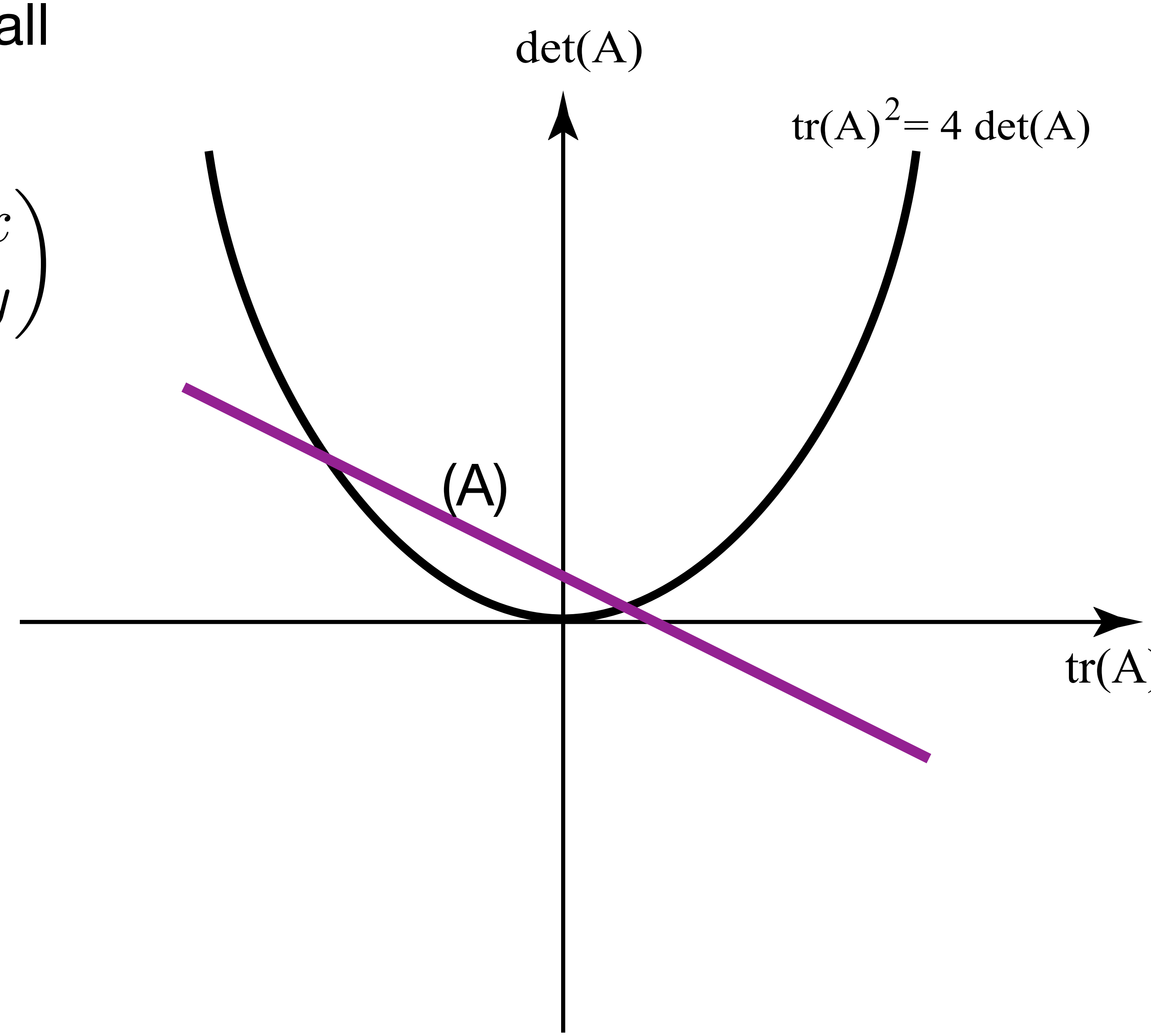

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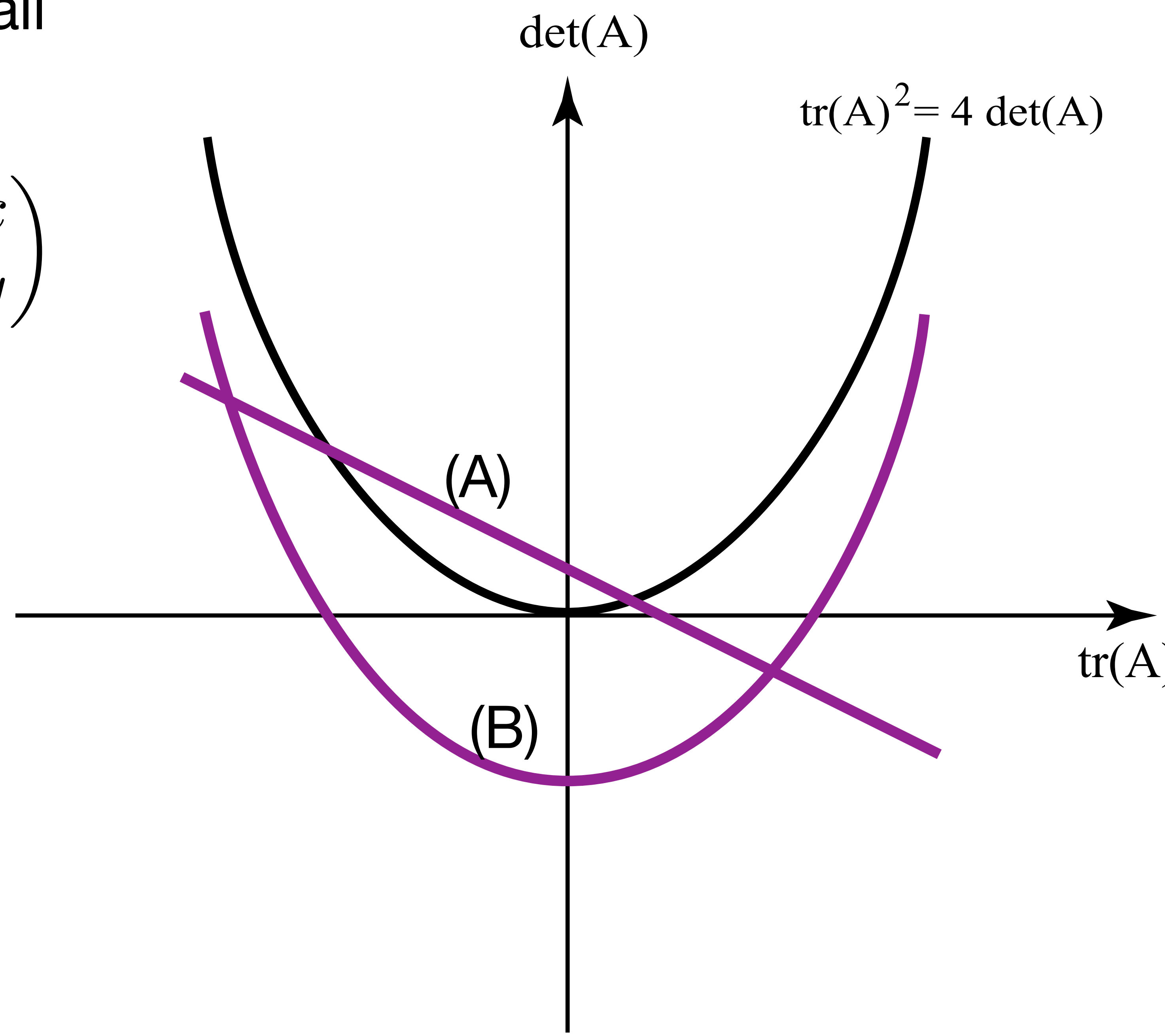

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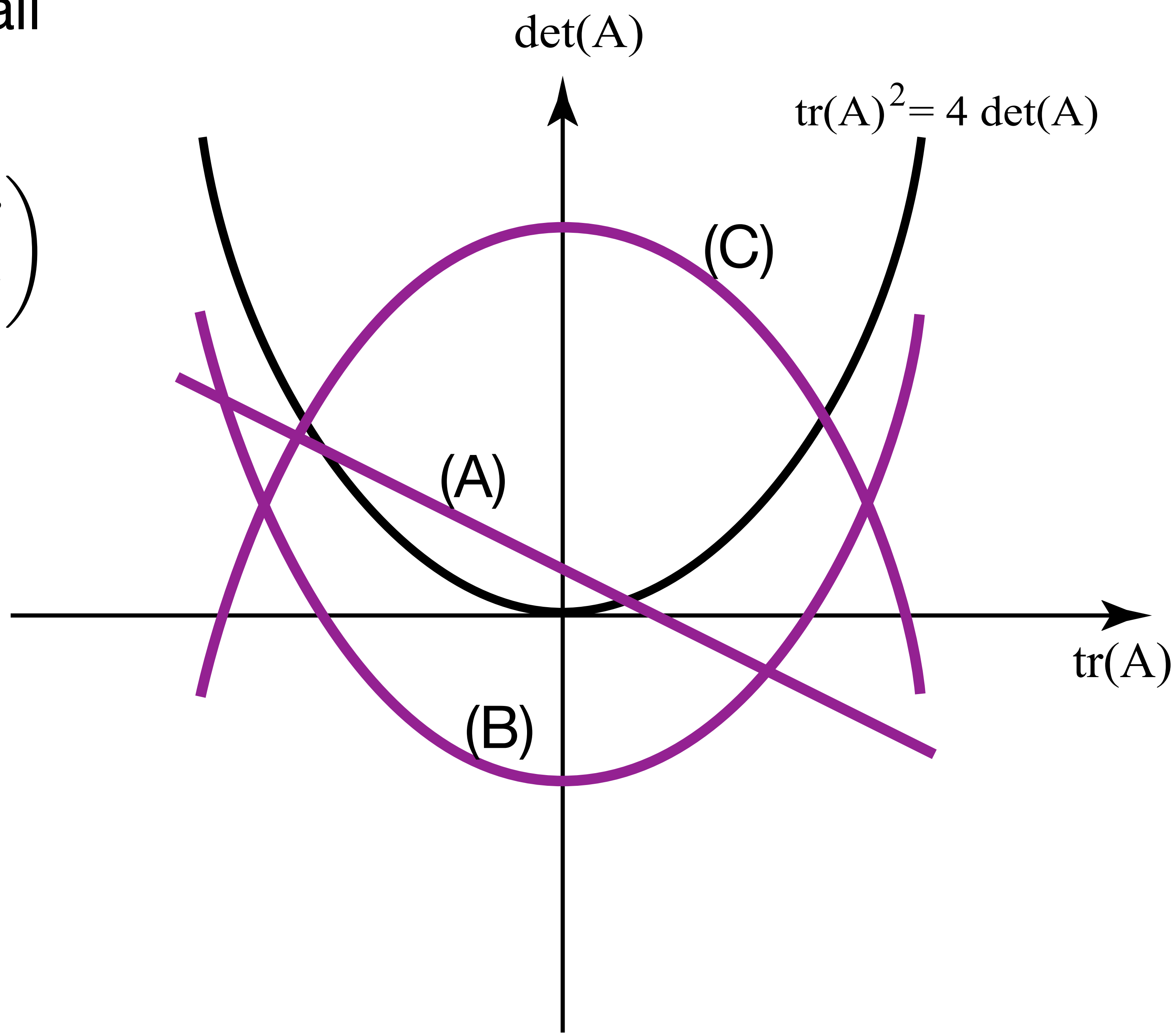

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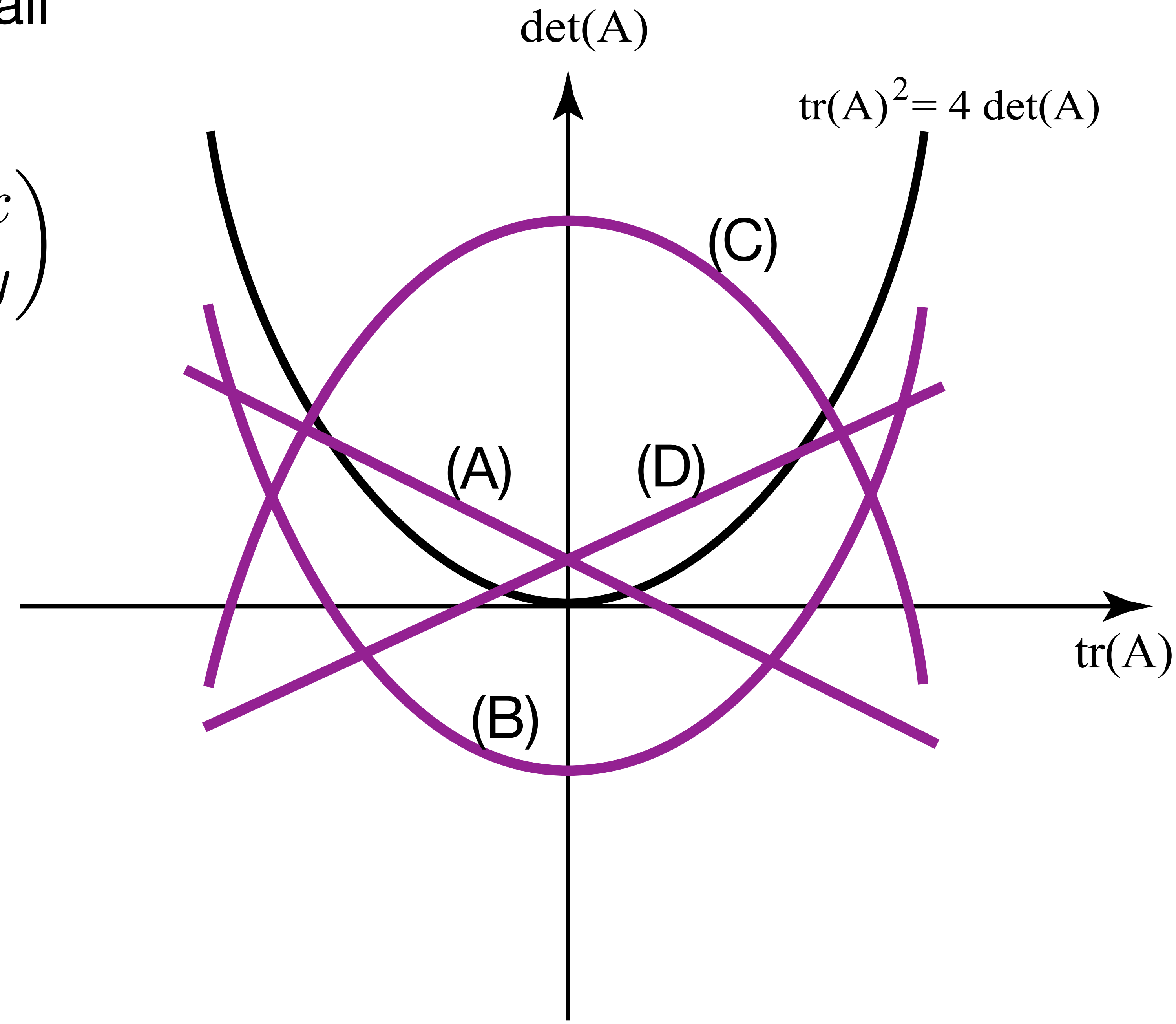

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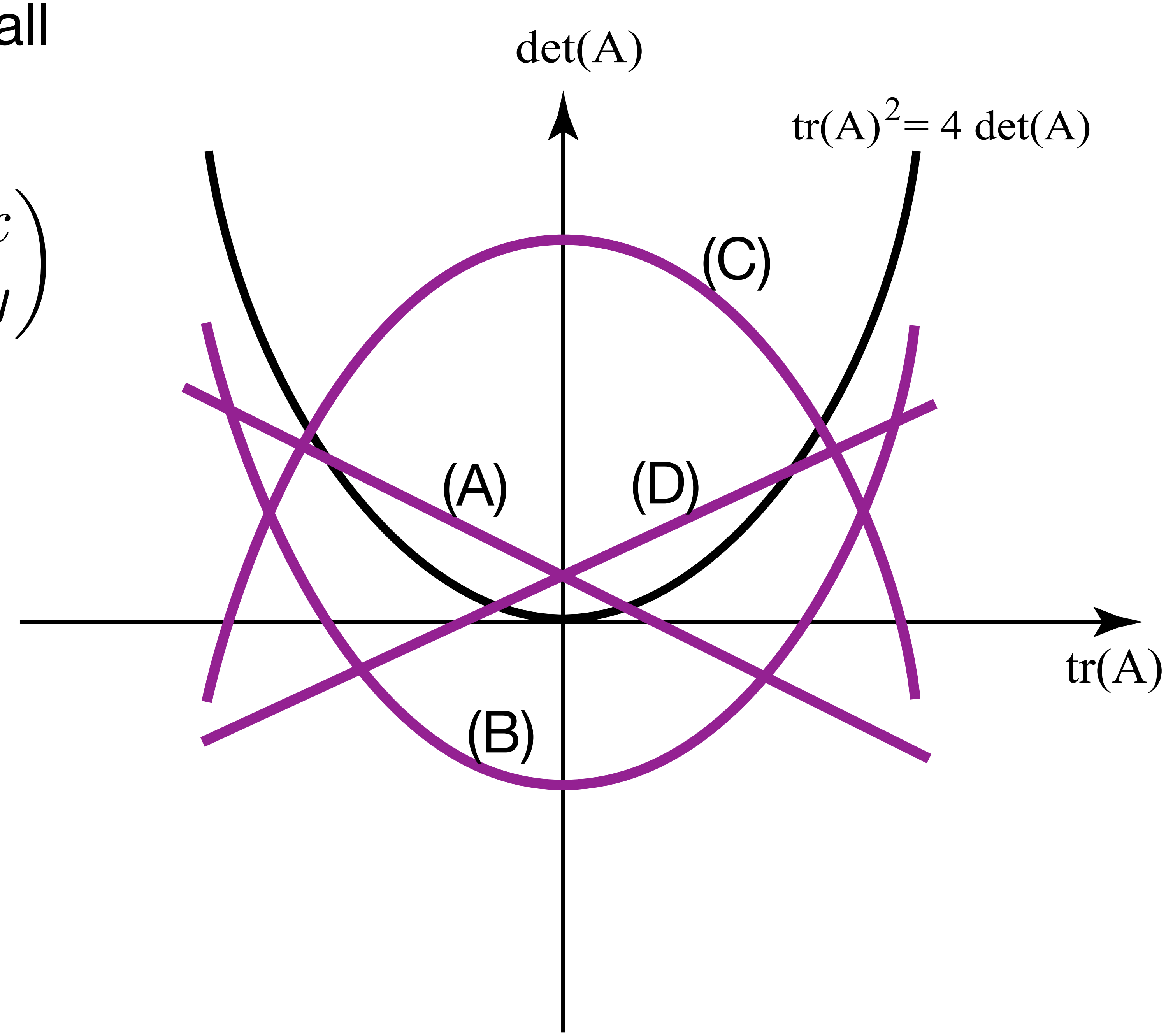

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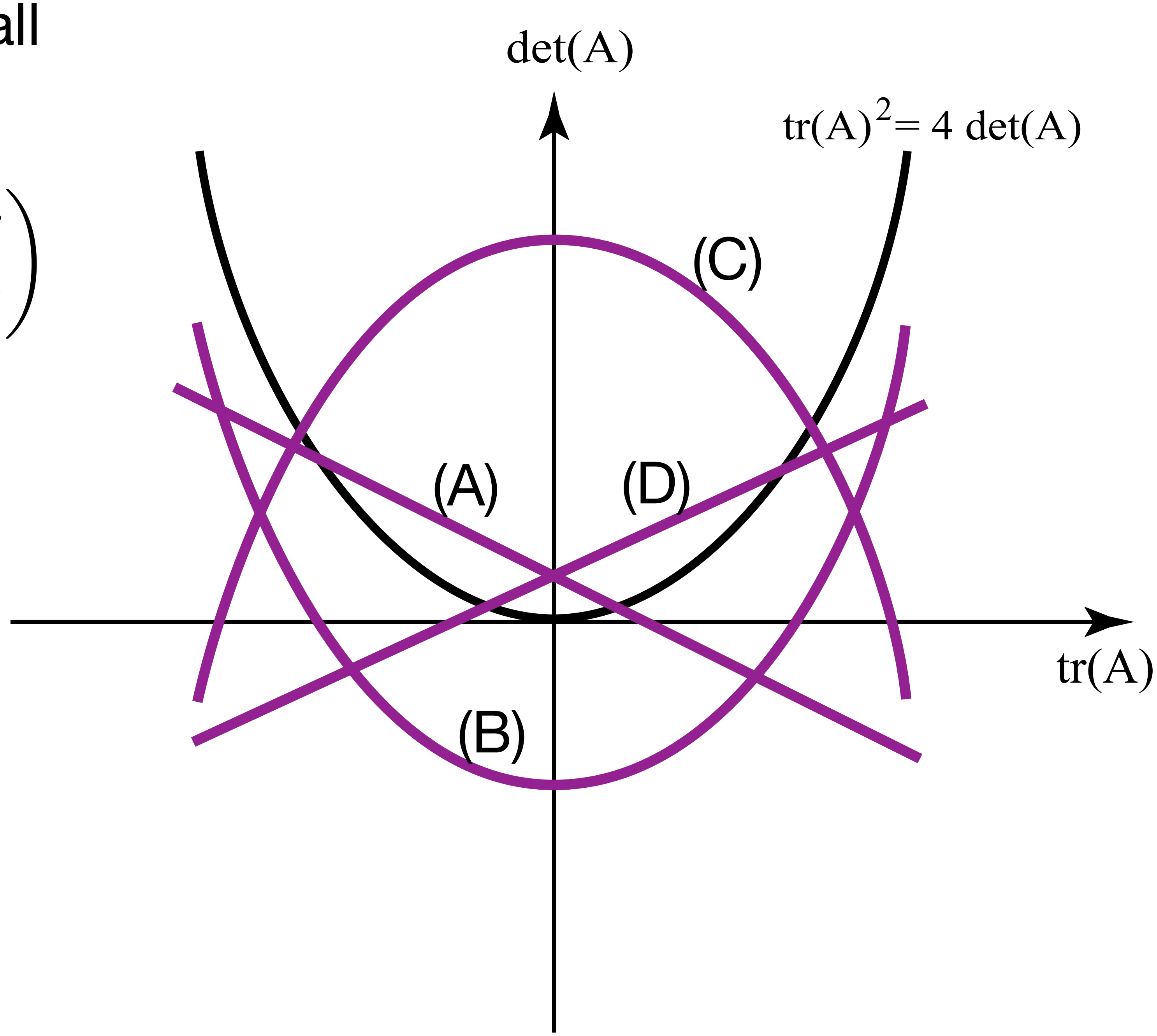

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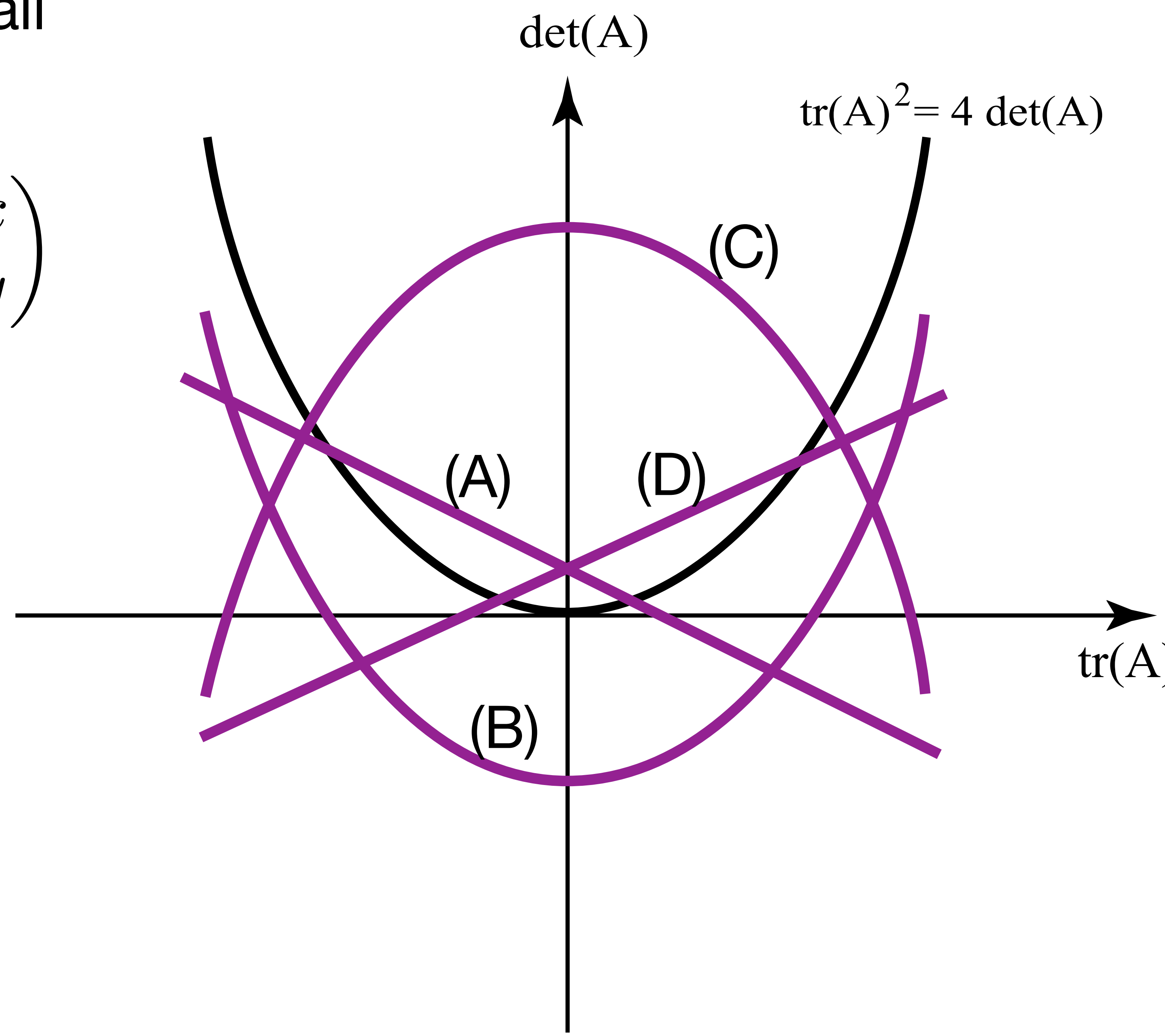

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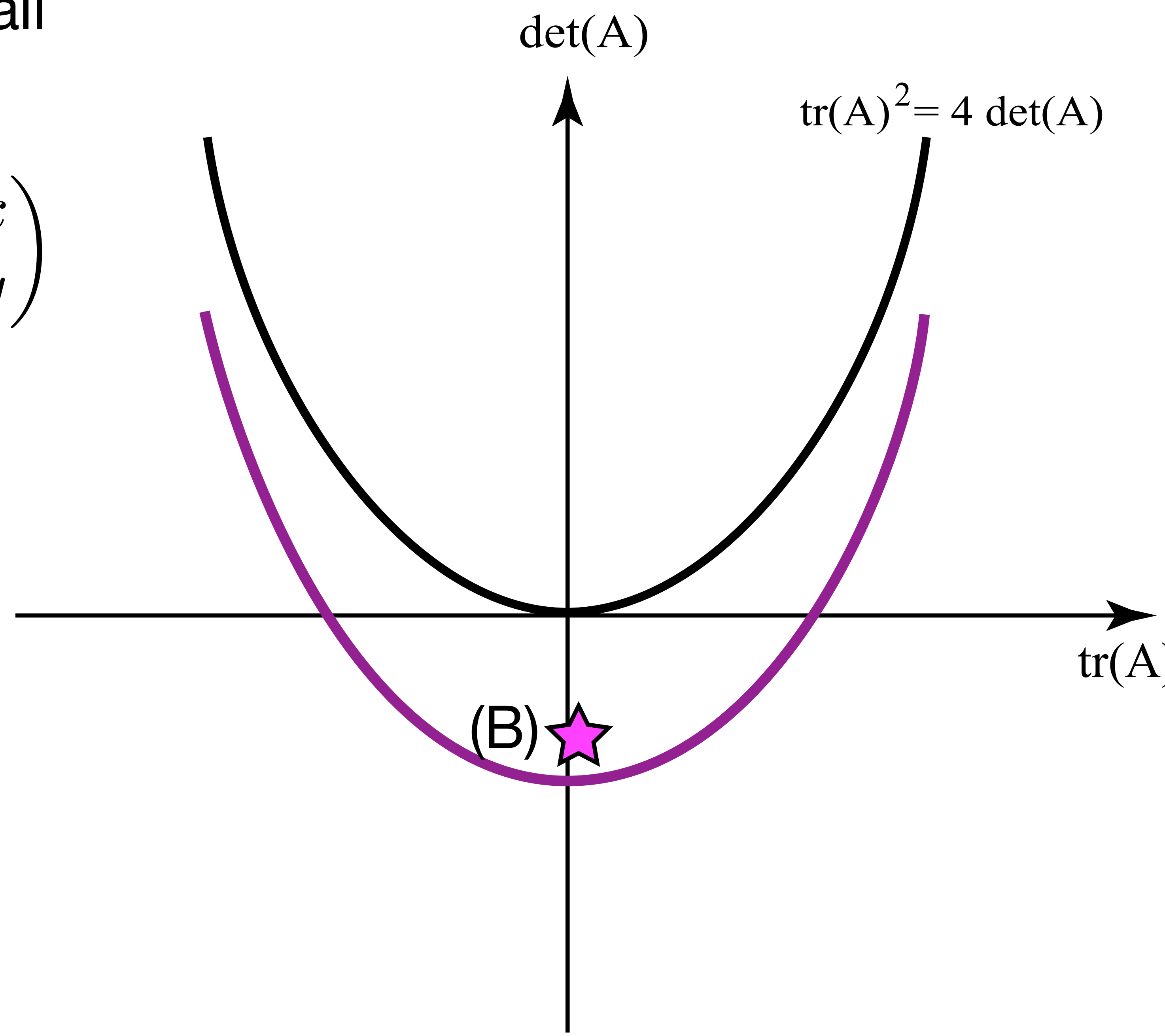

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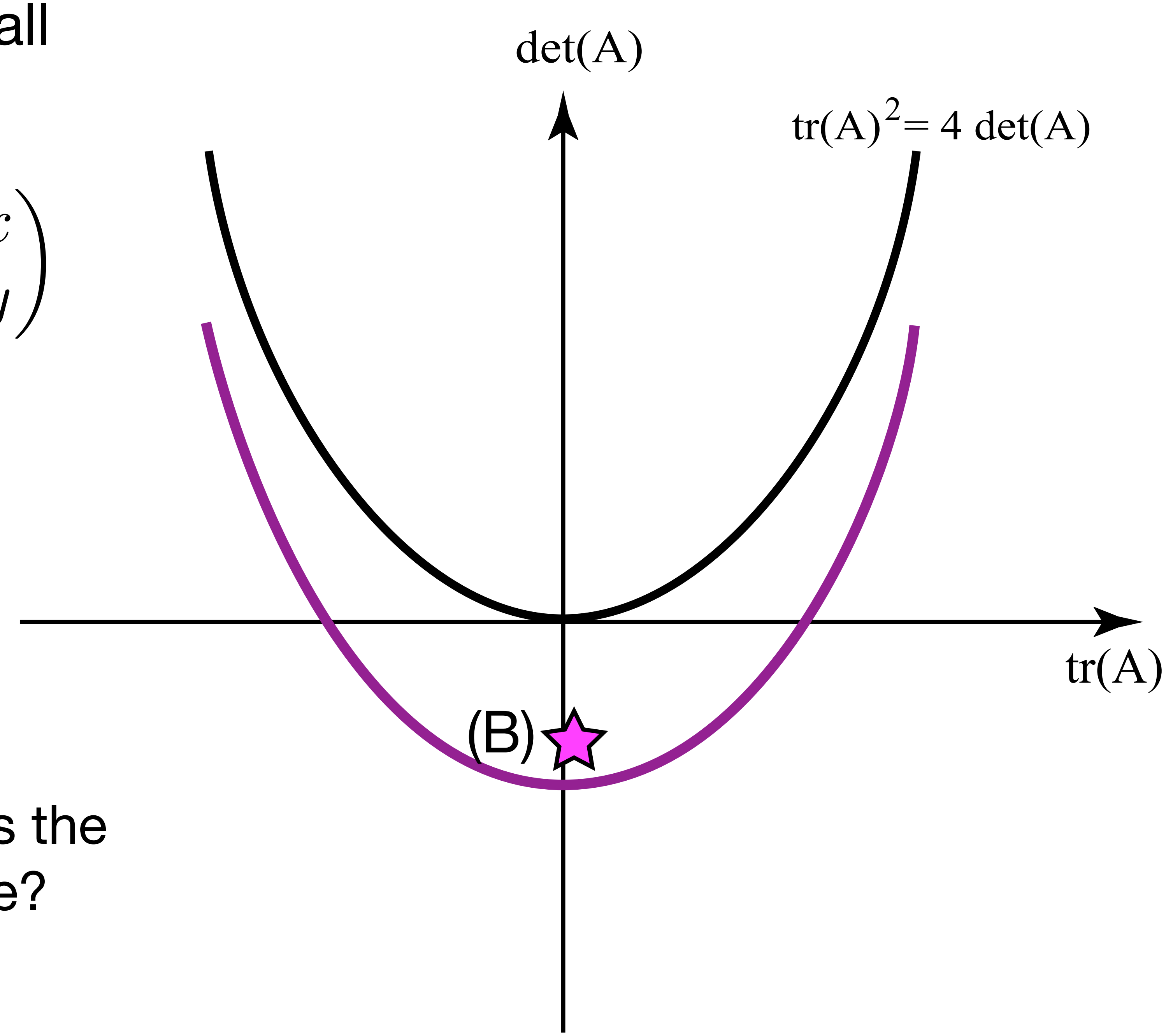

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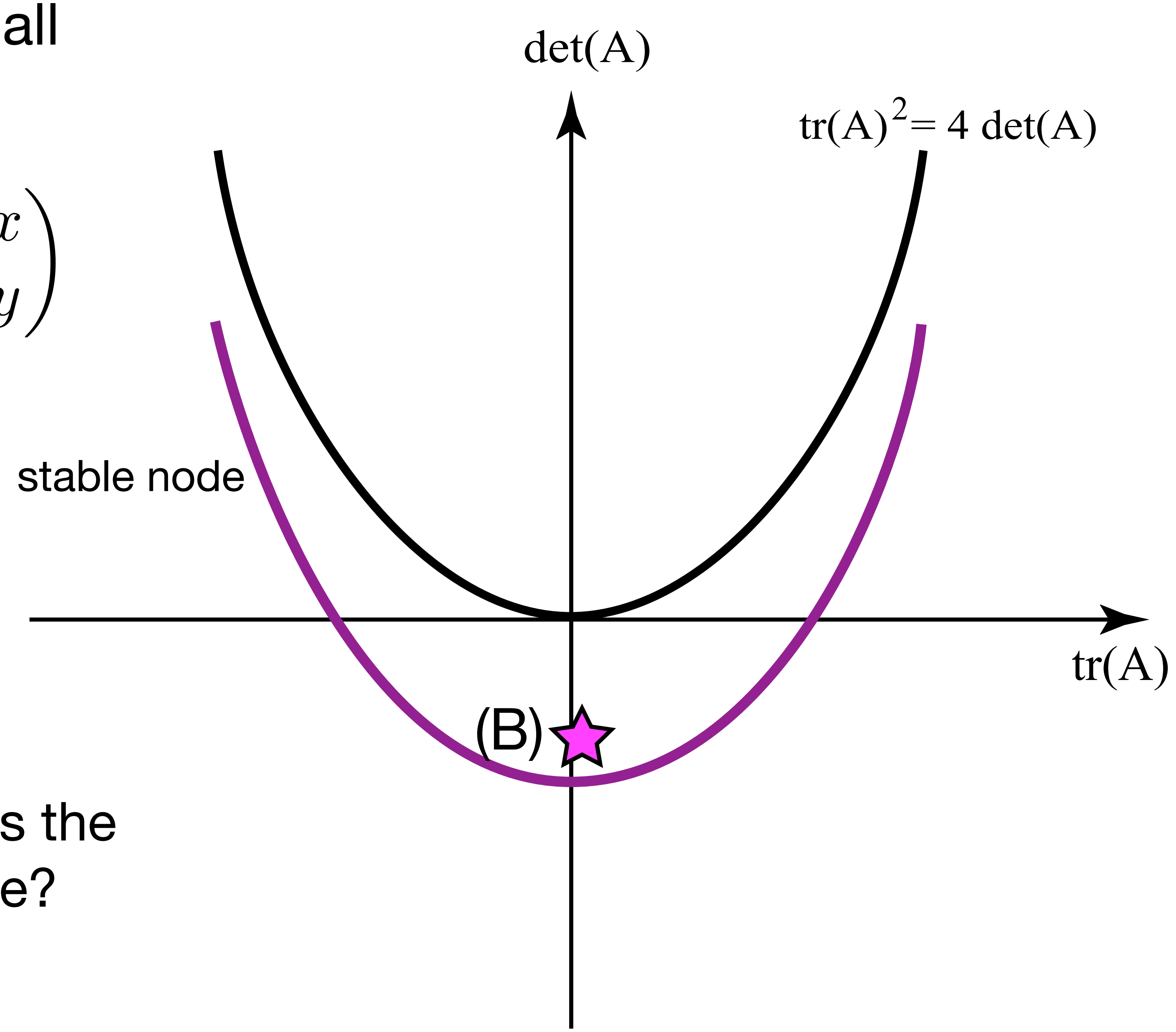

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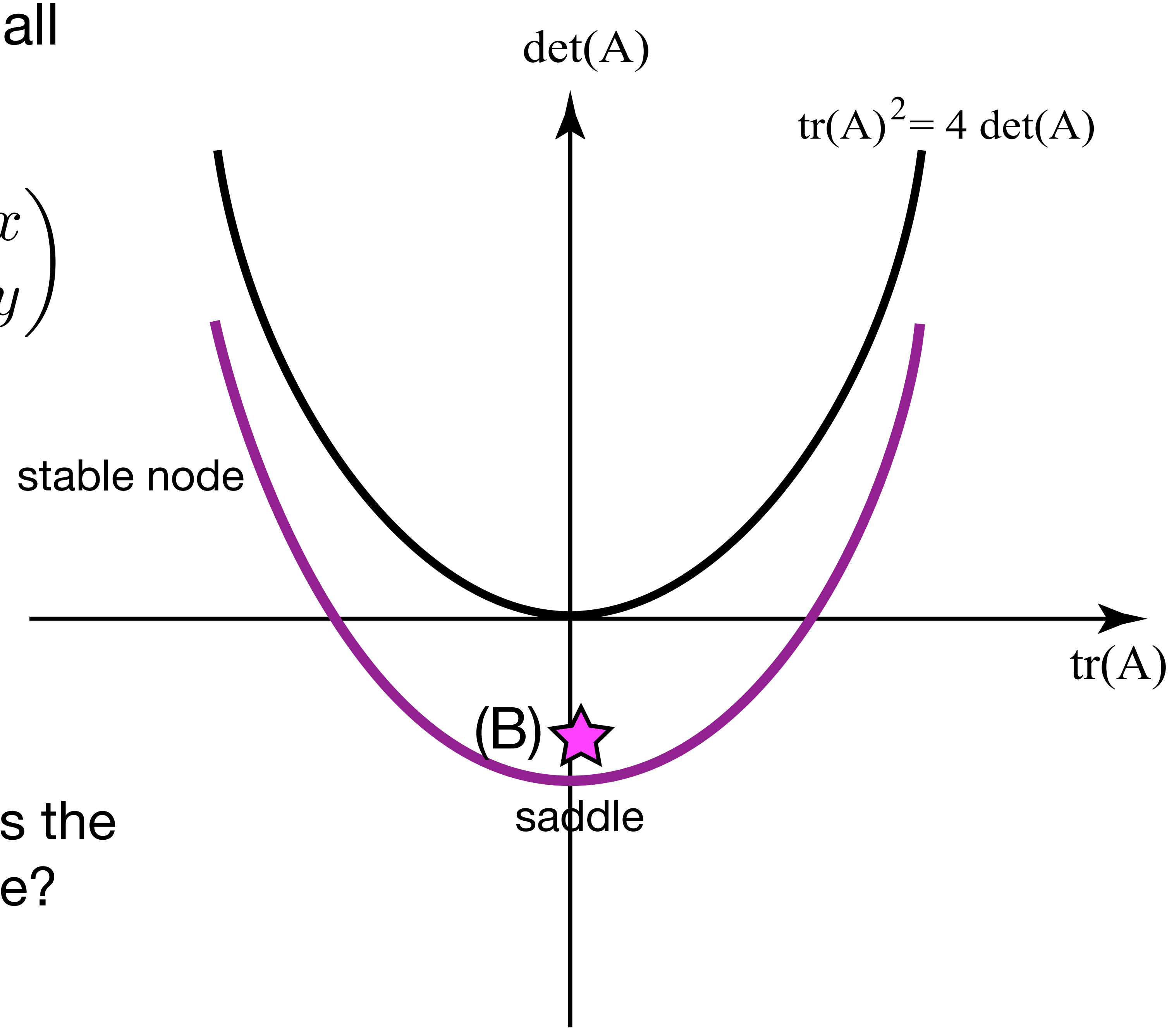

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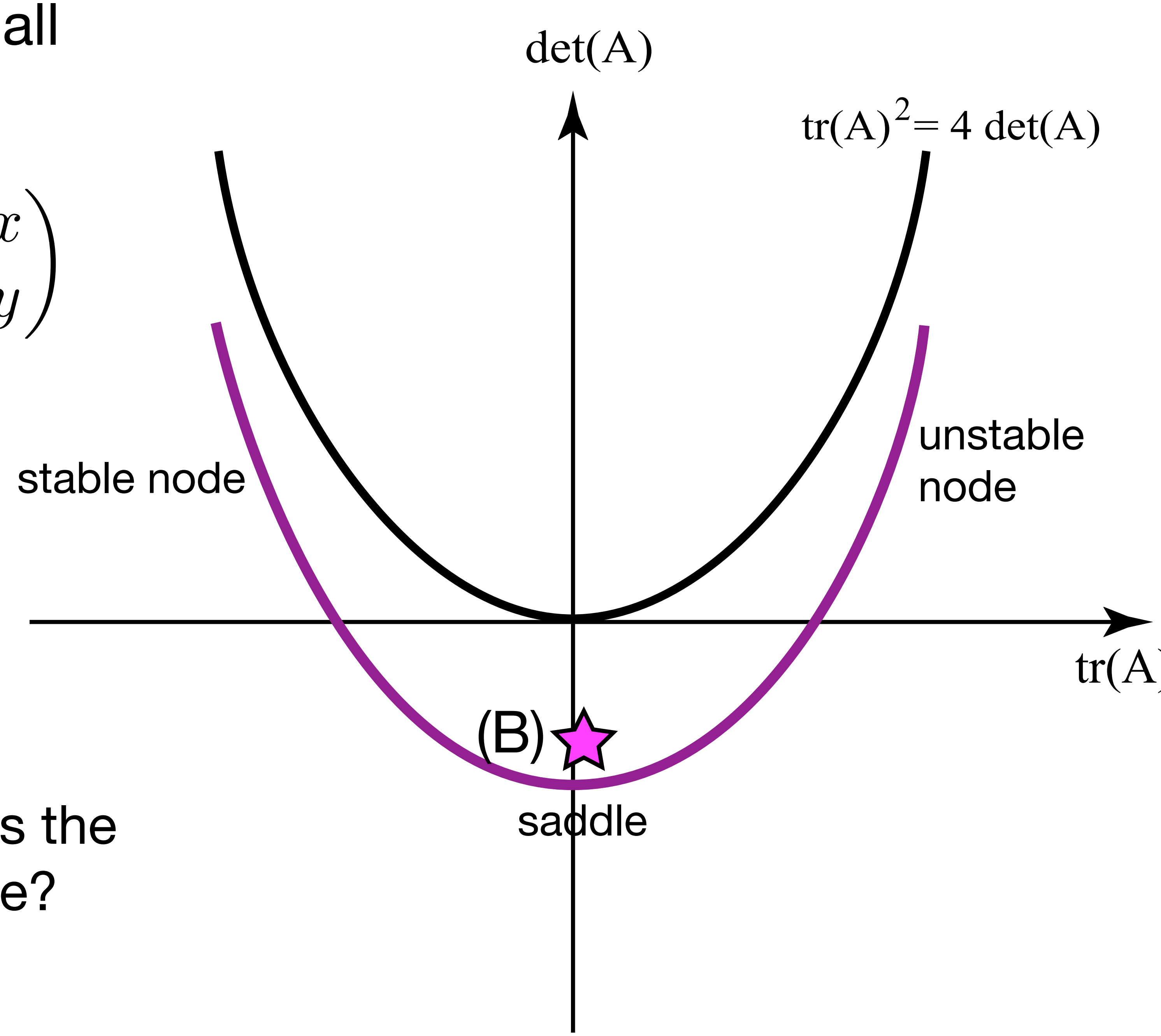

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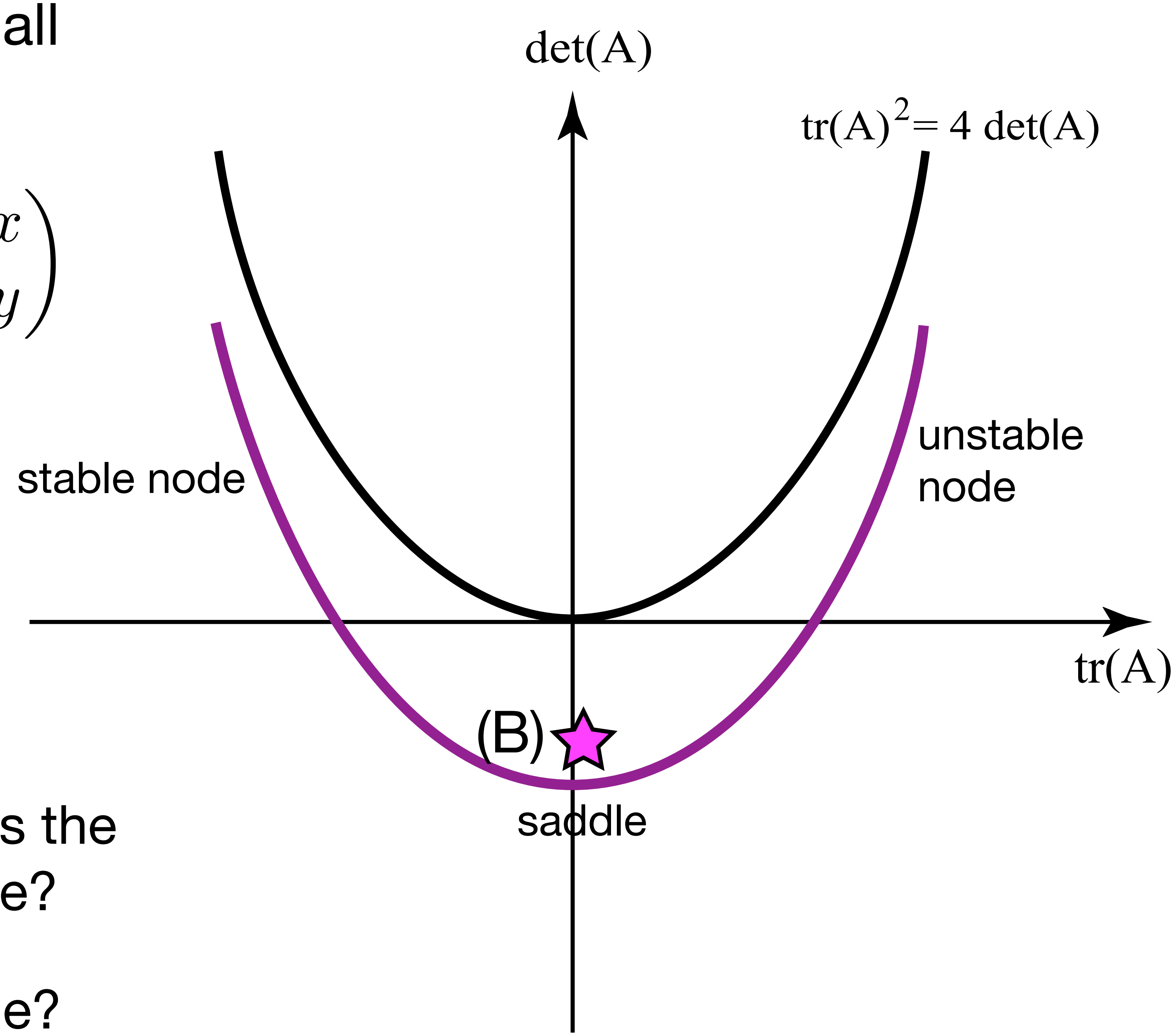

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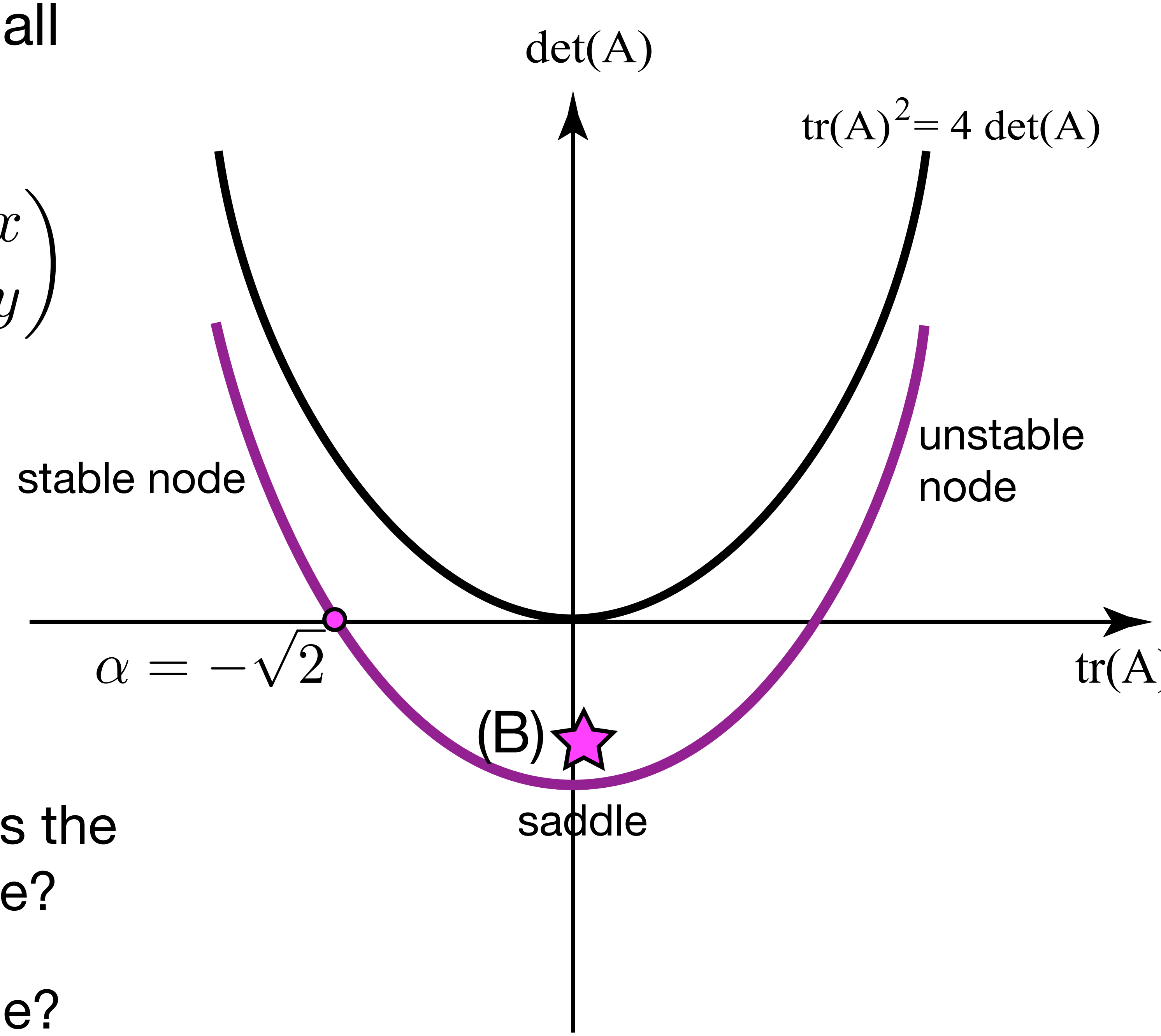

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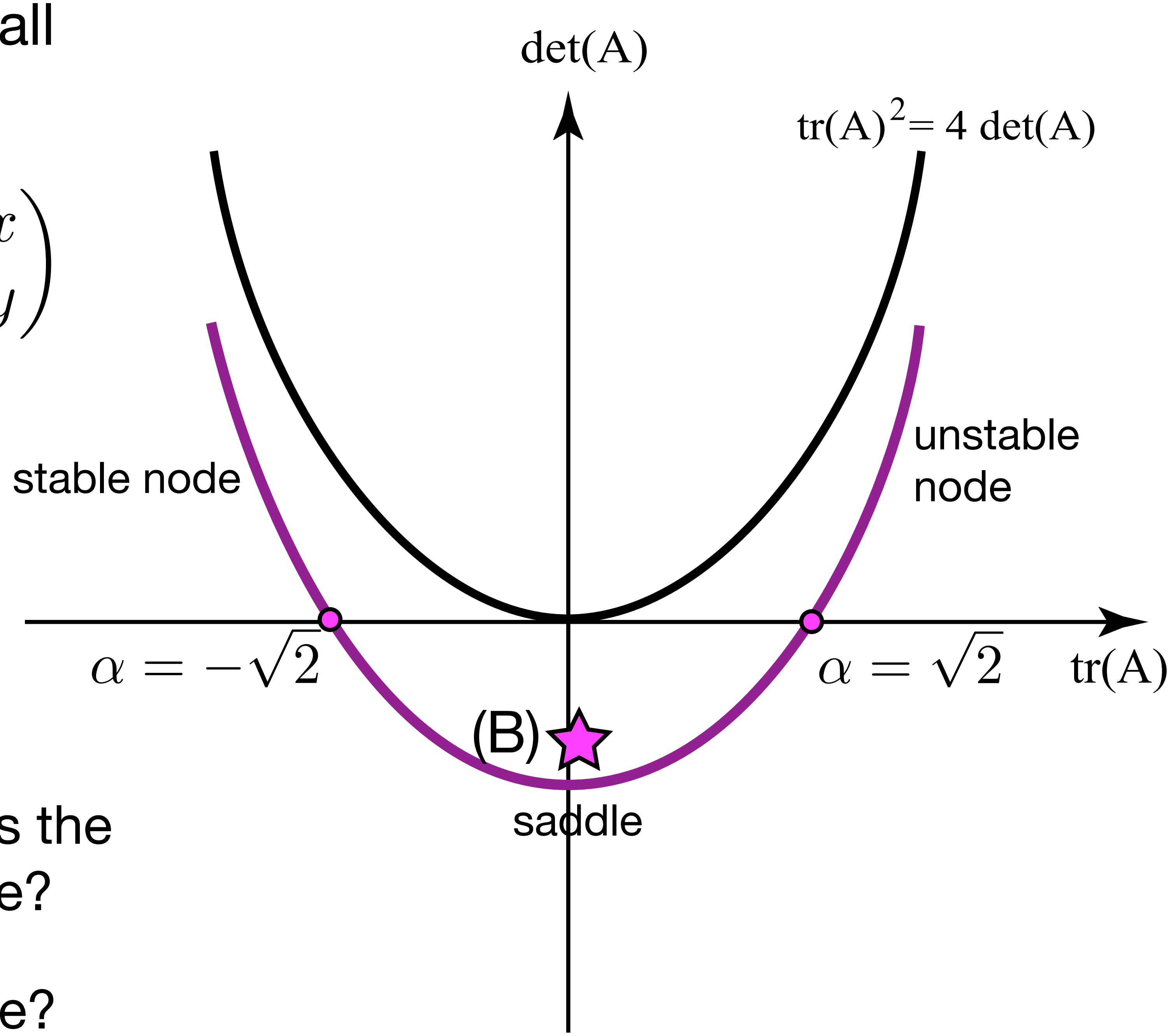

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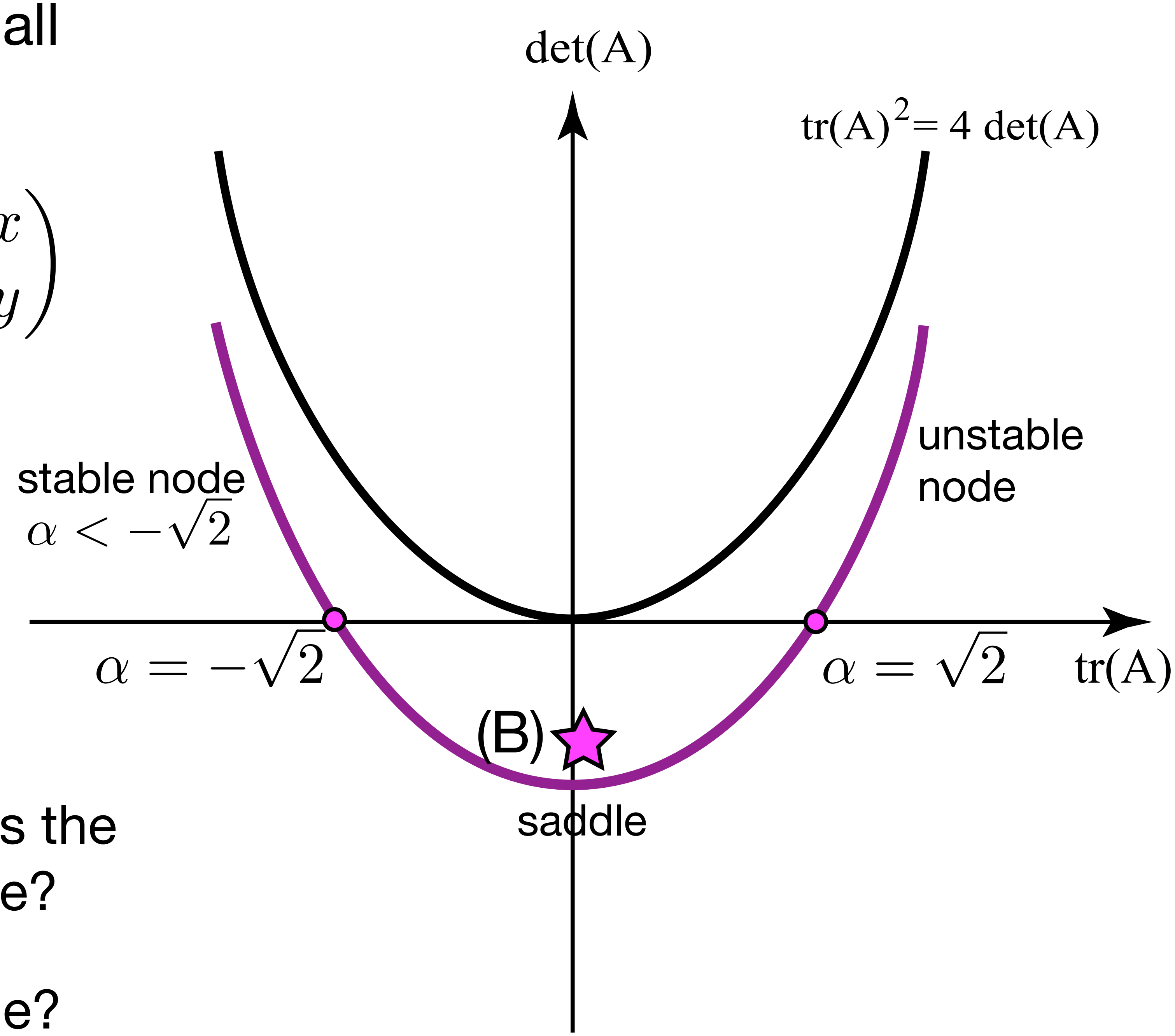

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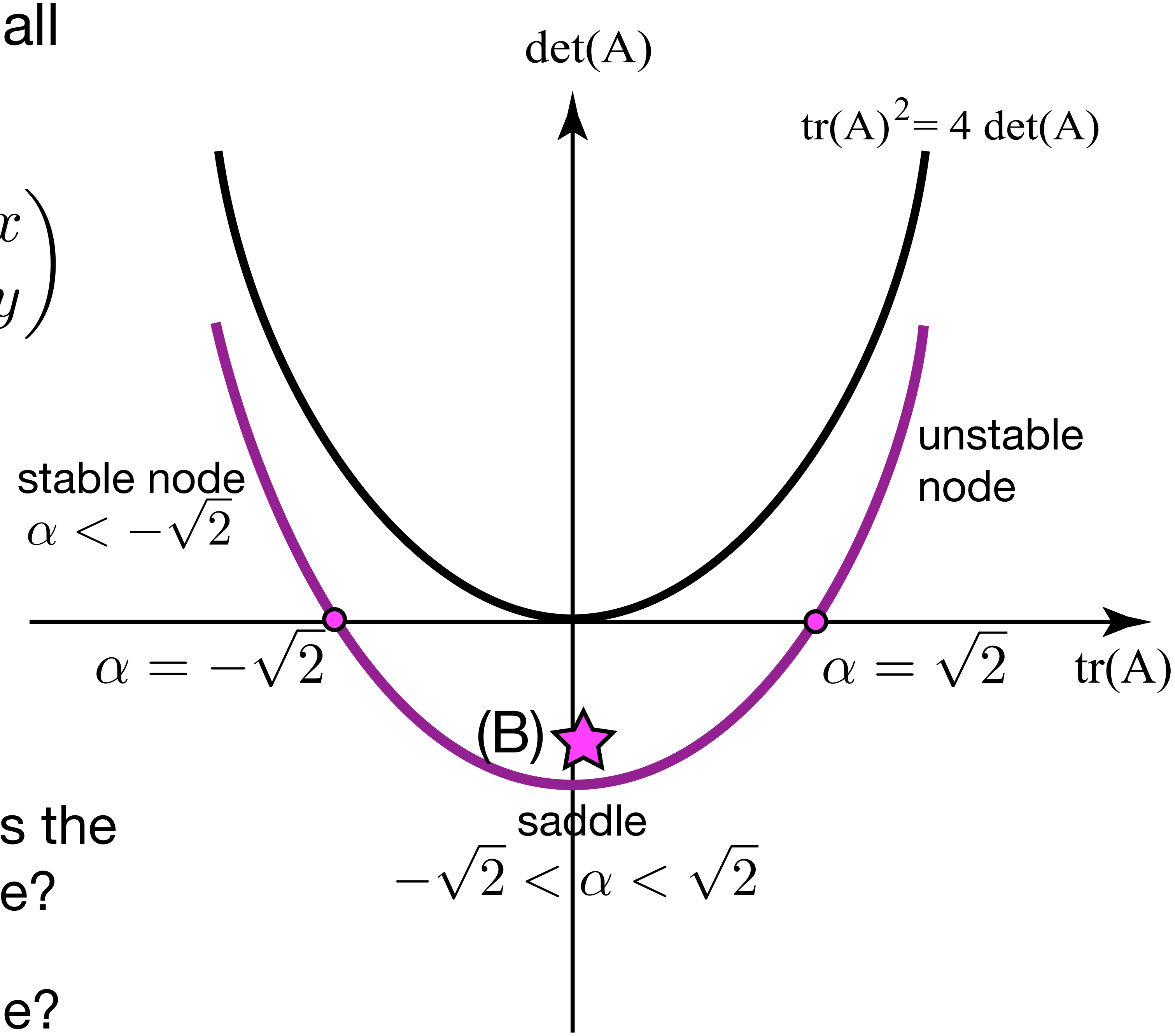

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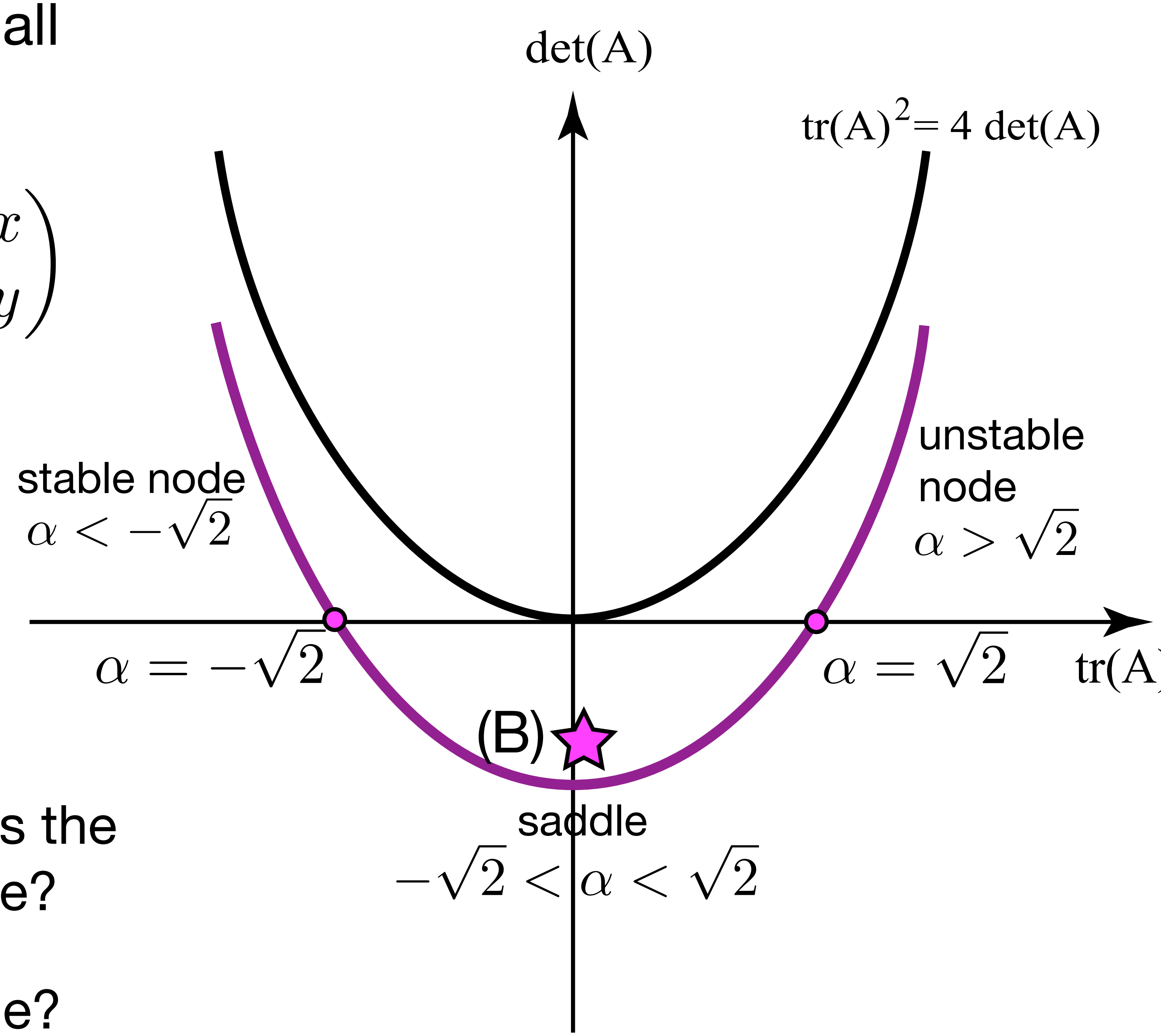

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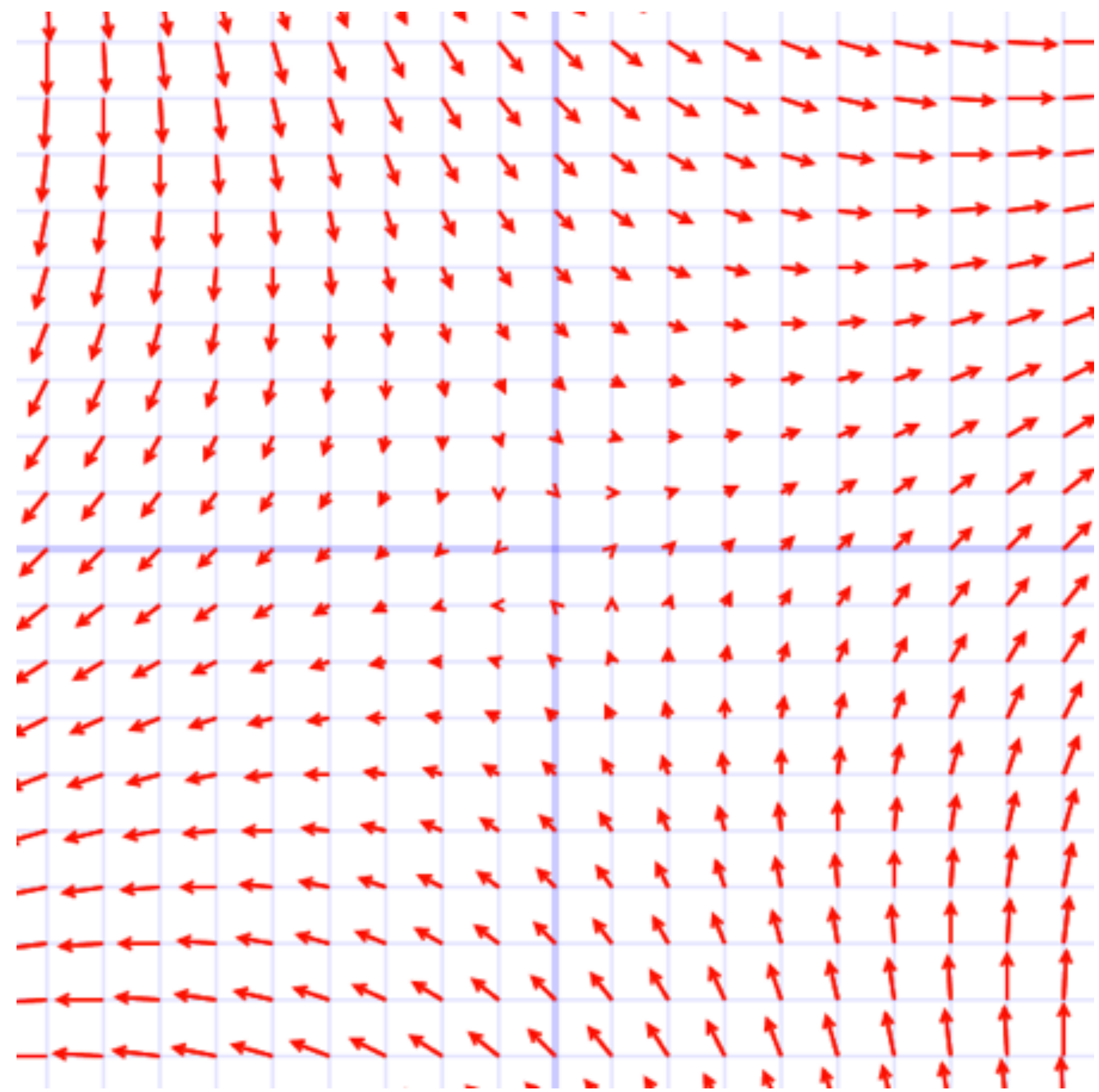
(E) Explain, please.



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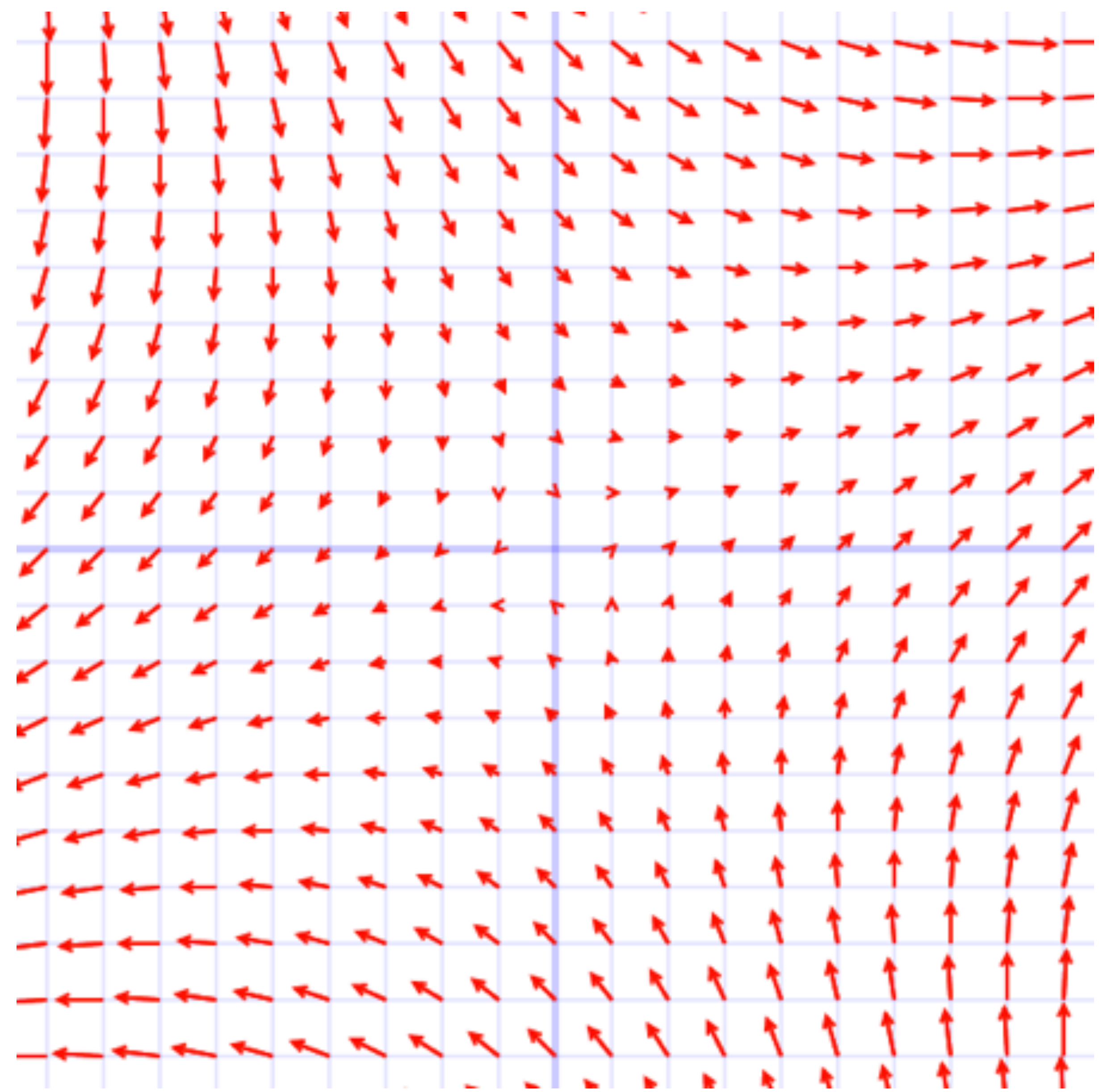

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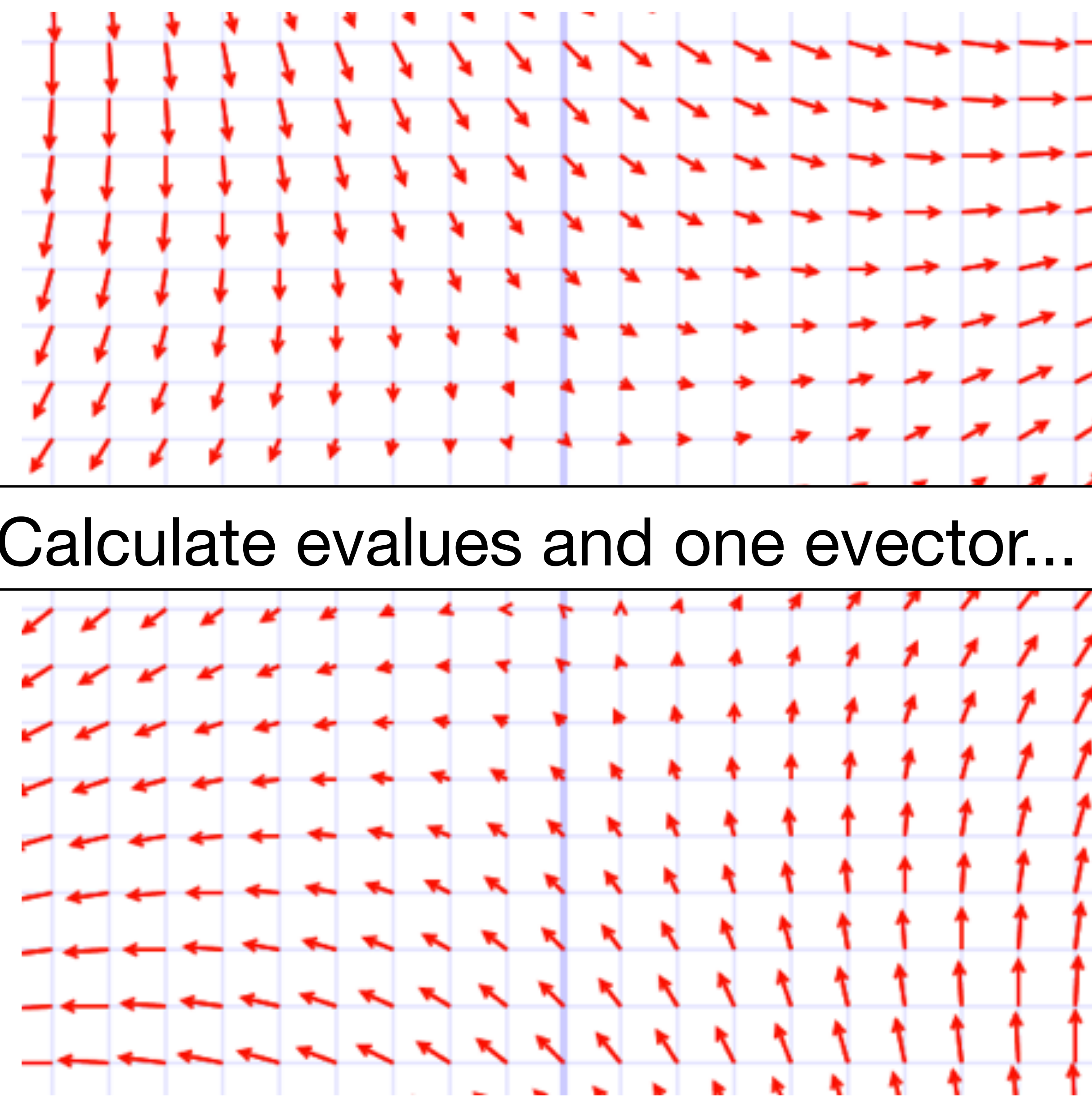

- Calculate evalues and one evector... OR

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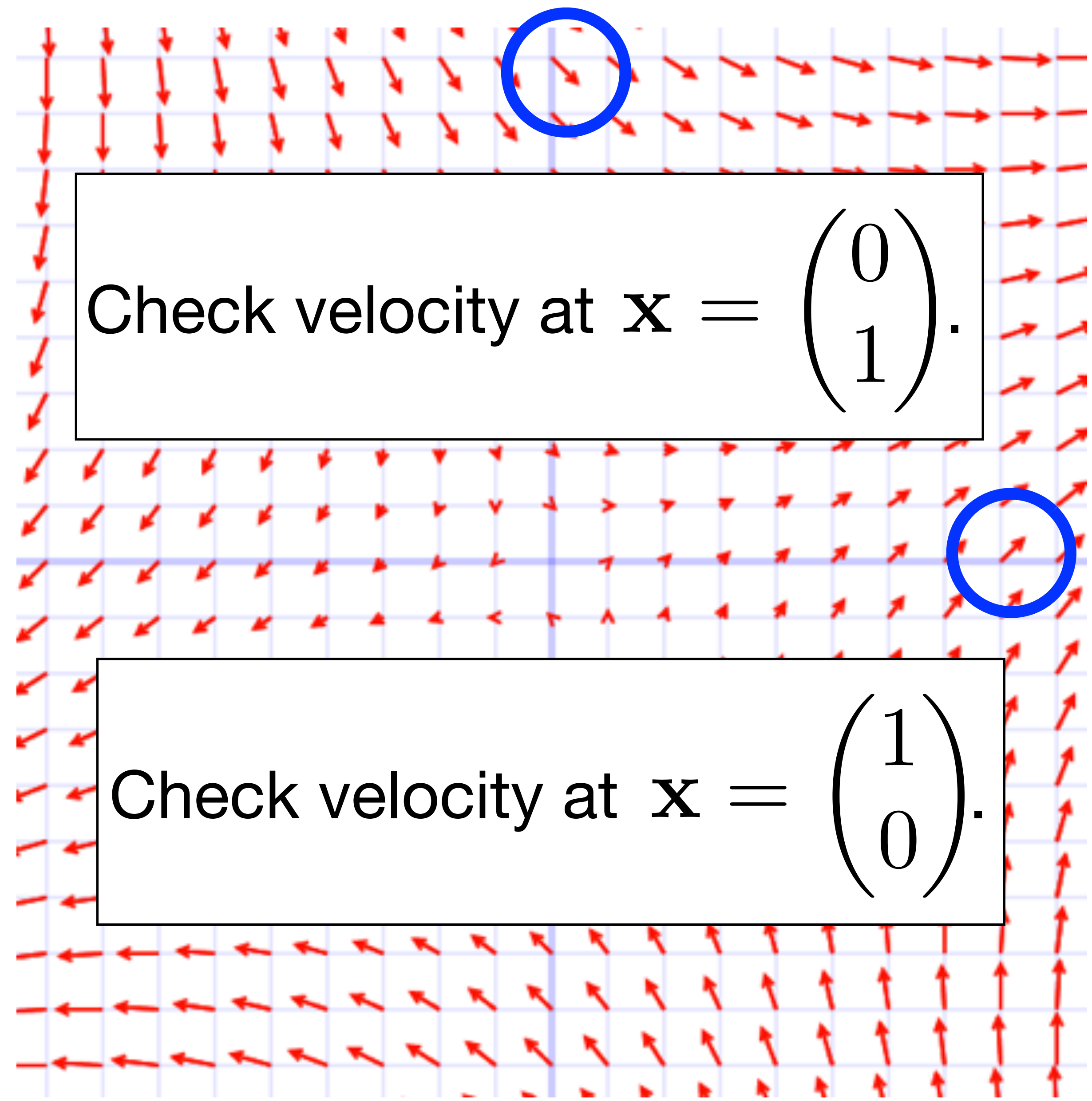

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Review problems

- A mass-spring system is at rest. At t=3, a linearly increasing force is applied until the force reaches $F_0 = 10$ N at t=8. After that moment, the force remains constant at that level (F_0). Write down the forcing function for this scenario.

  (A) $2t(u_3(t) - u_8(t))$

  (B) $2u_3(t)(t-3) - 2u_8(t)(t-8)$

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- Solve this equation using Laplace transform techniques.

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- Two tanks are connected by pipes. They initially contain large quantities of salt. Freshwater is added to the tanks so that the volumes of water are constant. The mass of salt in each tank is given by the system of equations

$$\frac{d}{dt}\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$$

where time is measured in minutes. How long does it take for the concentration in both tanks to decrease to less than one tenth of their original values?

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Require $e^{\lambda t} < 1/10$ for both evalues $\lambda_1=-2$ & $\lambda_2=-3$.

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$$\begin{pmatrix} -2 & 1/2 \\ 2 & -2 \end{pmatrix}$$

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$$\mathbf{v_{-1}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \mathbf{v_{-3}} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Review problems

- Consider the solution to the IVP

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 1/2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

For $t>0$, do we ever have $y(t)<0$?

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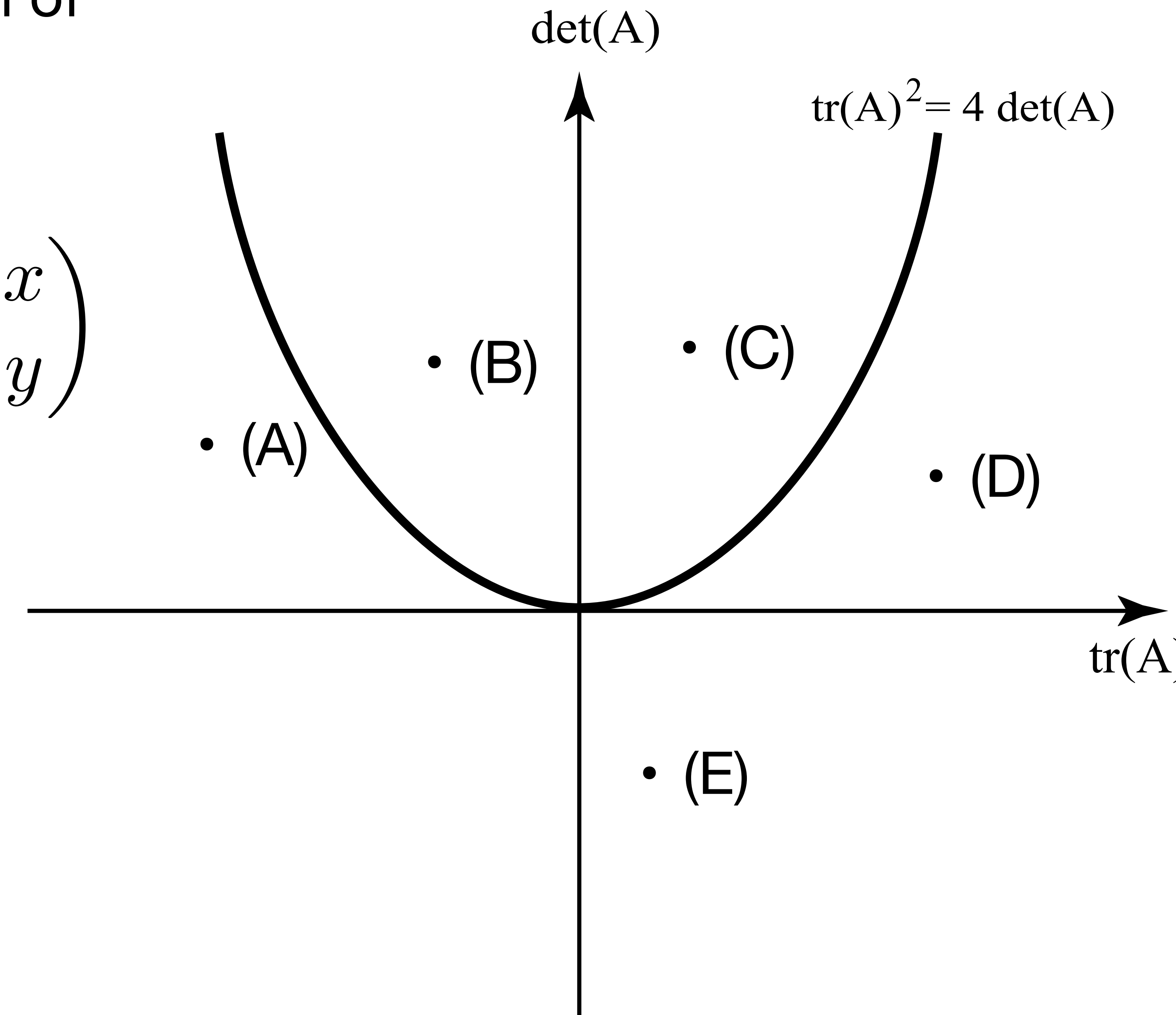

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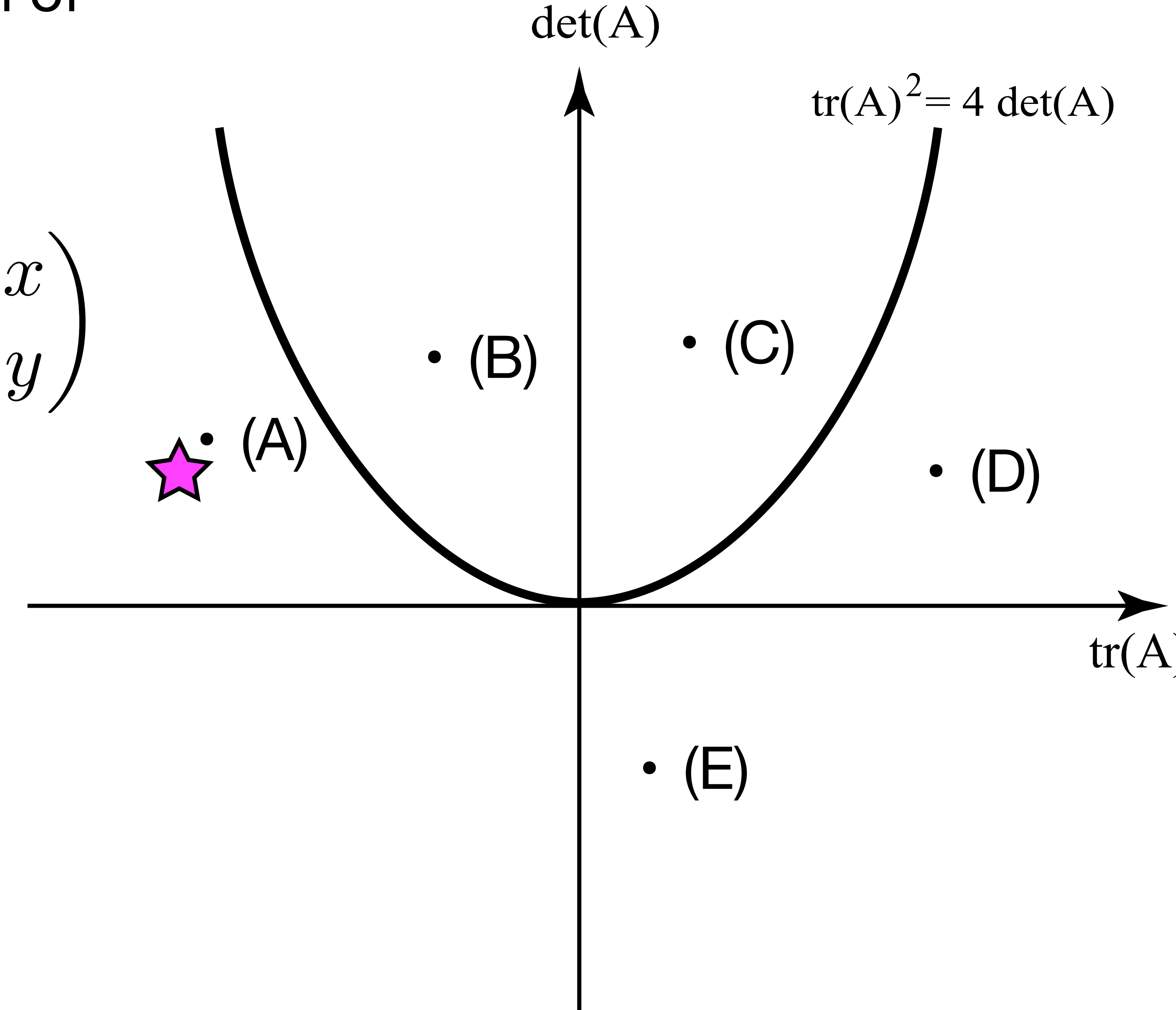

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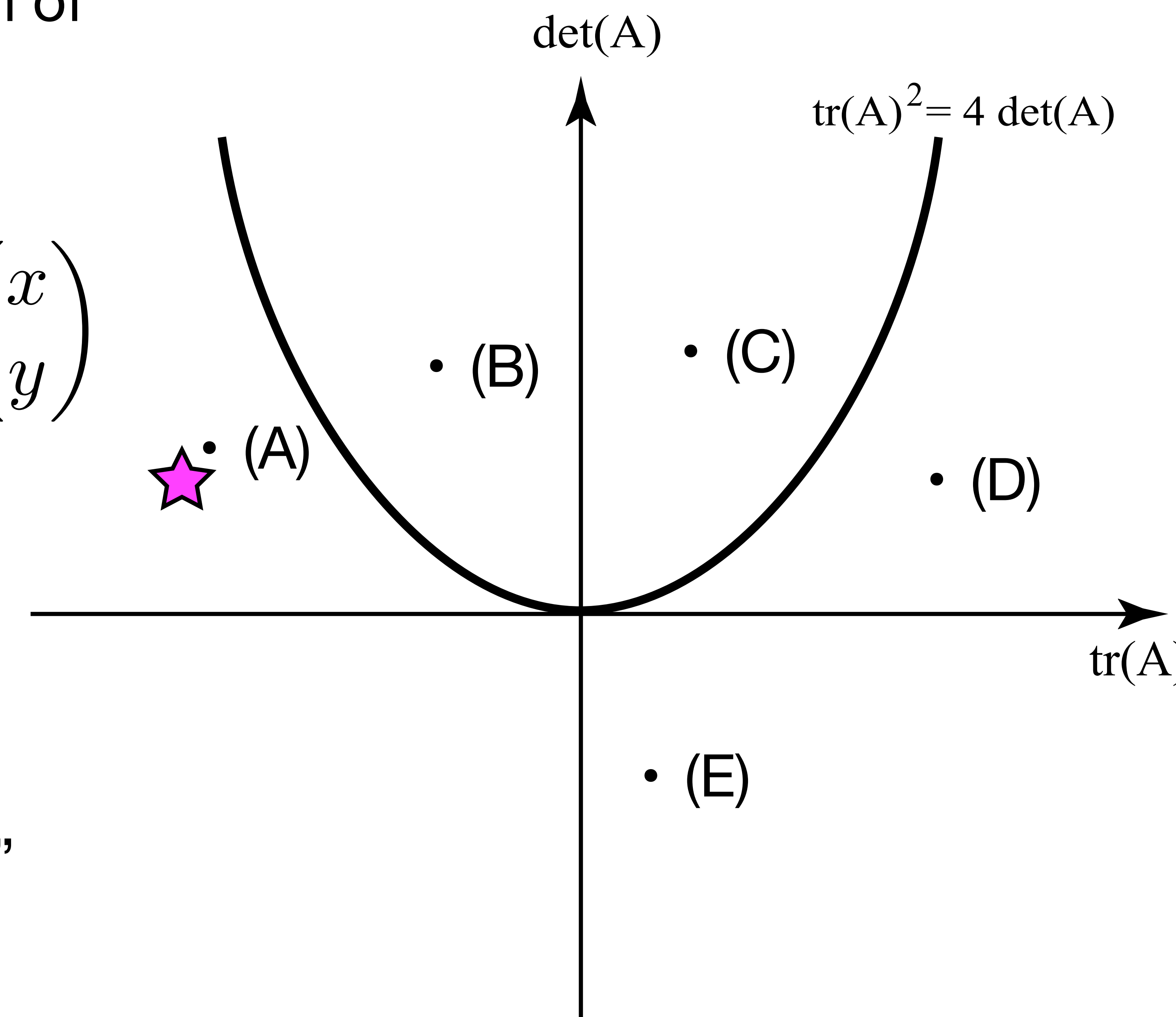
  - tr(A) = -4
  - det(A) = 3.
  - $(\mathrm{tr}(A))^2 > 4\det(A)$ so it lies below the "repeated root" parabola.



Review problems

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$$y'' + 4y' + 8y = 0,$$
$$y(0) = 1,\ y'(0) = 0$$

Review problems

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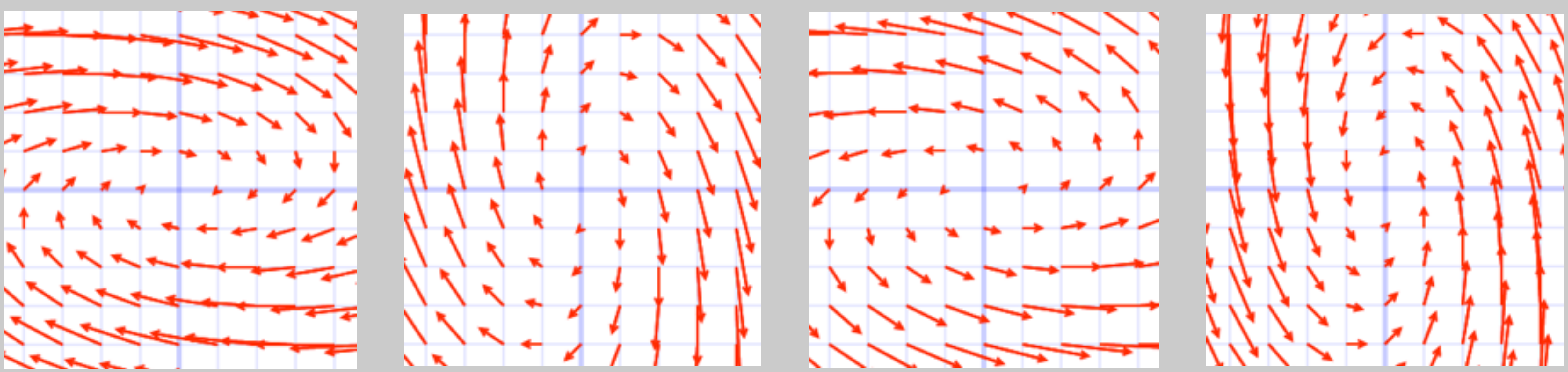
$$y'' + 4y' + 8y = \delta(t-3)$$

Review problems

$$\mathbf{x}(\mathbf{t}) = e^t \left(C_1 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \sin(2t) \right) + C_2 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \cos(2t) \right) \right)$$

(A) (B) (C) (D)



(E) Explain, please.

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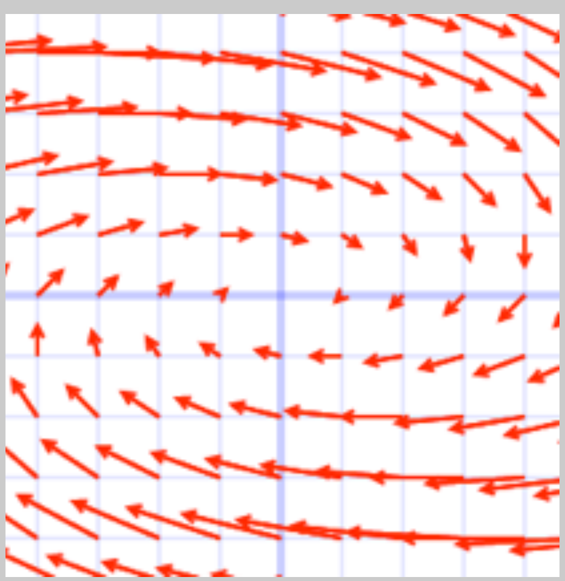
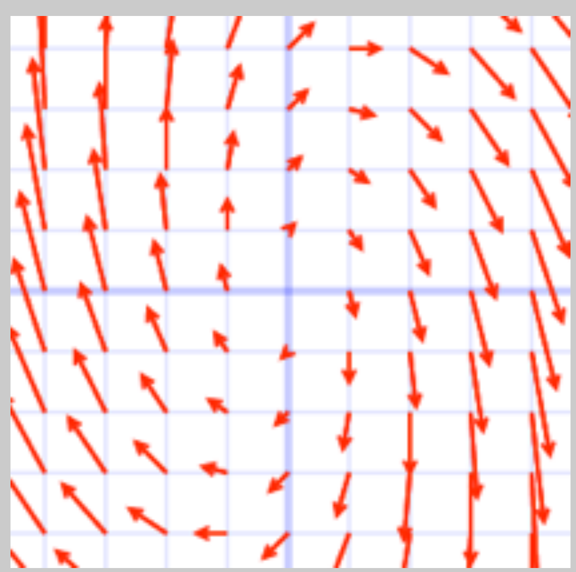


(B)



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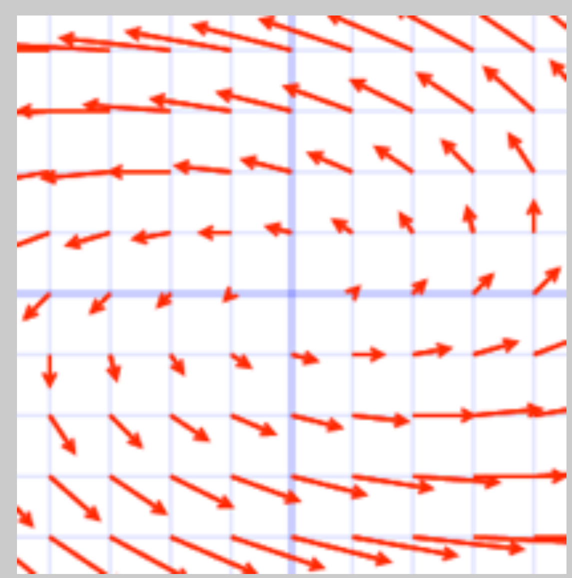

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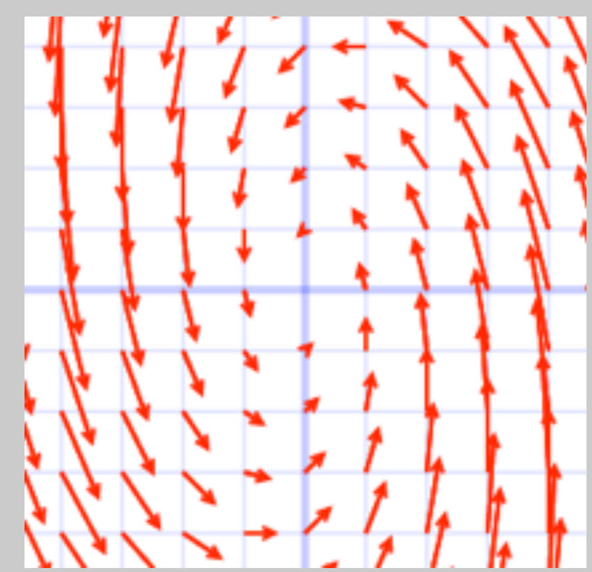

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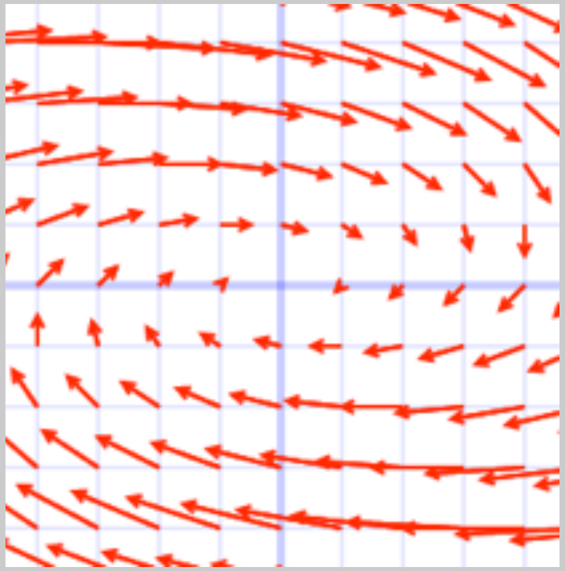

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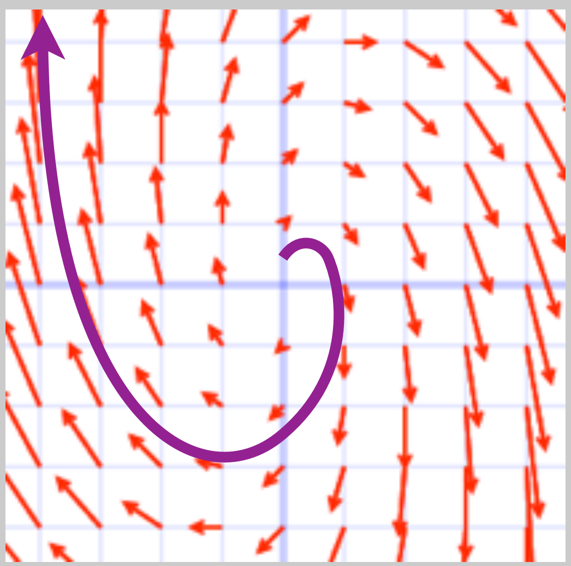

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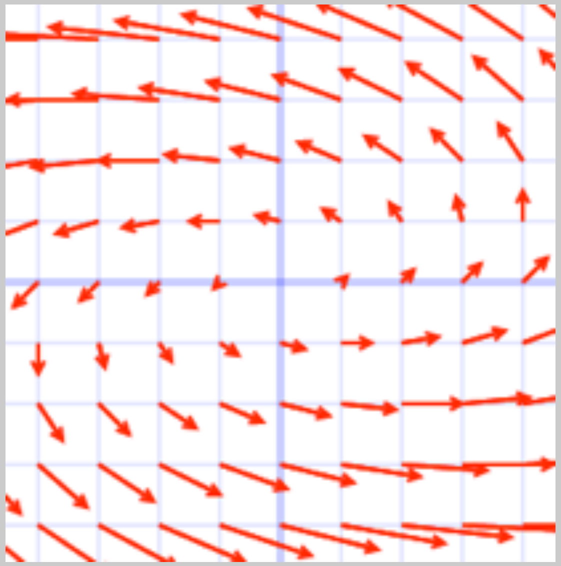

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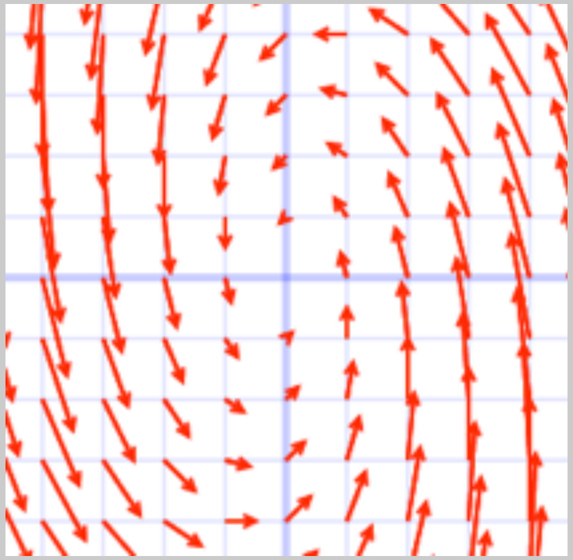

(E) Explain, please.

Review problems

- A patient is given a 100 mg injection of a medication every 4 hours for weeks. The mean life of the drug in the bloodstream is 10 hours (so it is cleared at a rate 1/10 hour $^{-1}$). Sketch the amount of the drug in the patient's system as a function of time.

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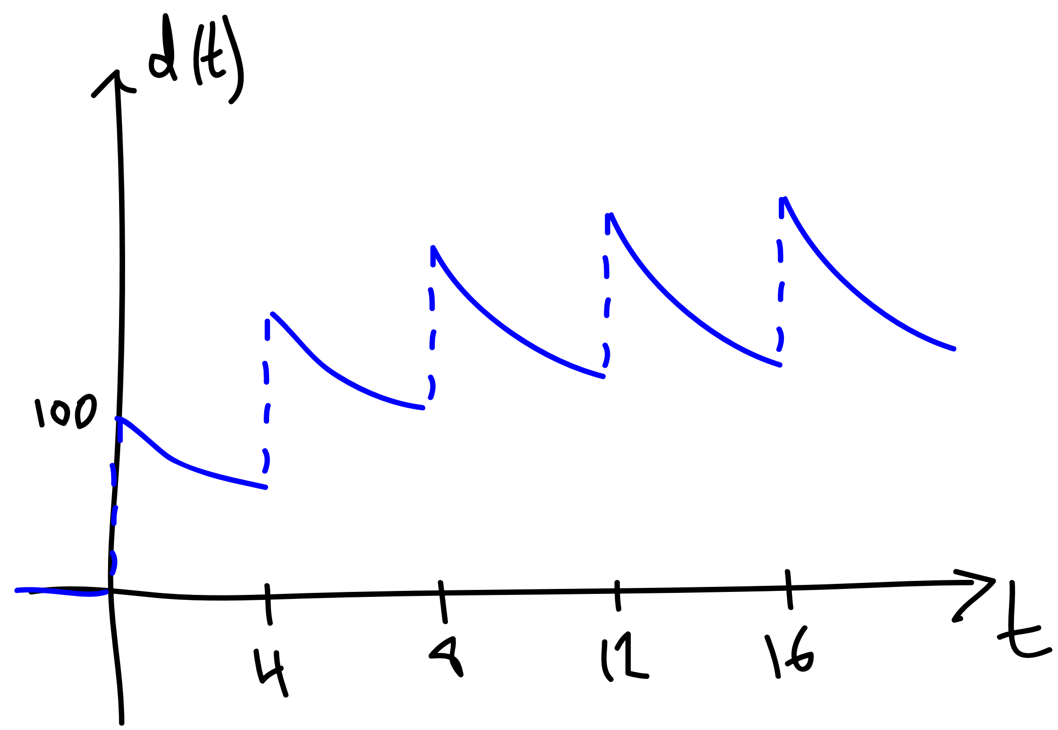

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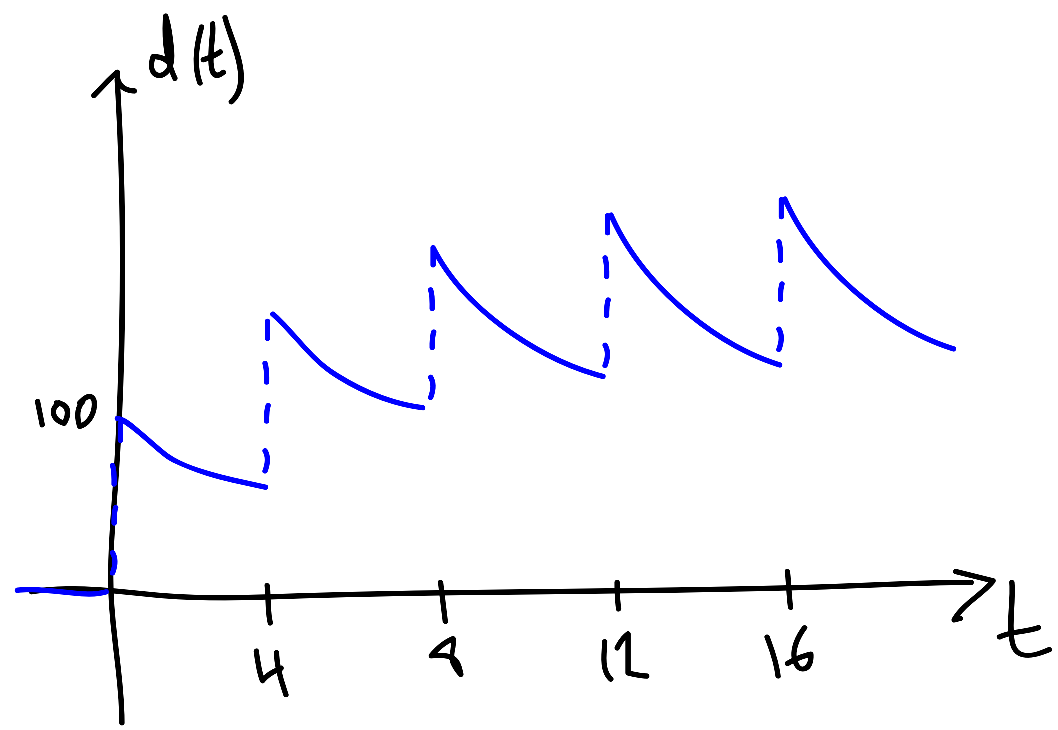
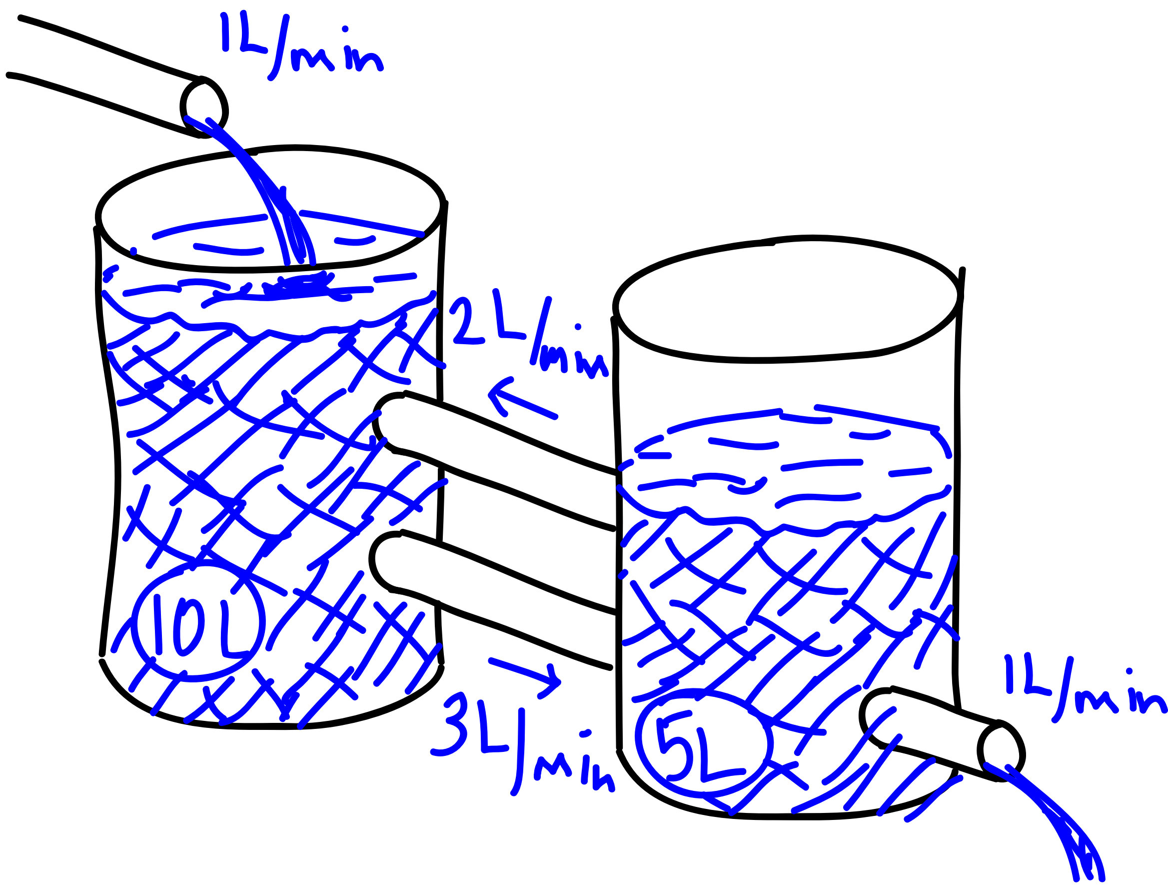


- Exercise (tricky): calculate the longterm minimum and maximum concentration.

Review problems

- Salt water flows into a tank holding 10 L of water at a rate of 1 L/min with a concentration of 200 g/L. The well-mixed solution flows from that tank into a tank holding 5 L through a pipe at 3 L/min. Another pipe takes the solution in the second tank back into the first at a rate of 2 L/min. Finally, solution drains out of the second tank at a rate of 1 L/min.
- Write down a system of equations in matrix form for the mass of salt in each tank.



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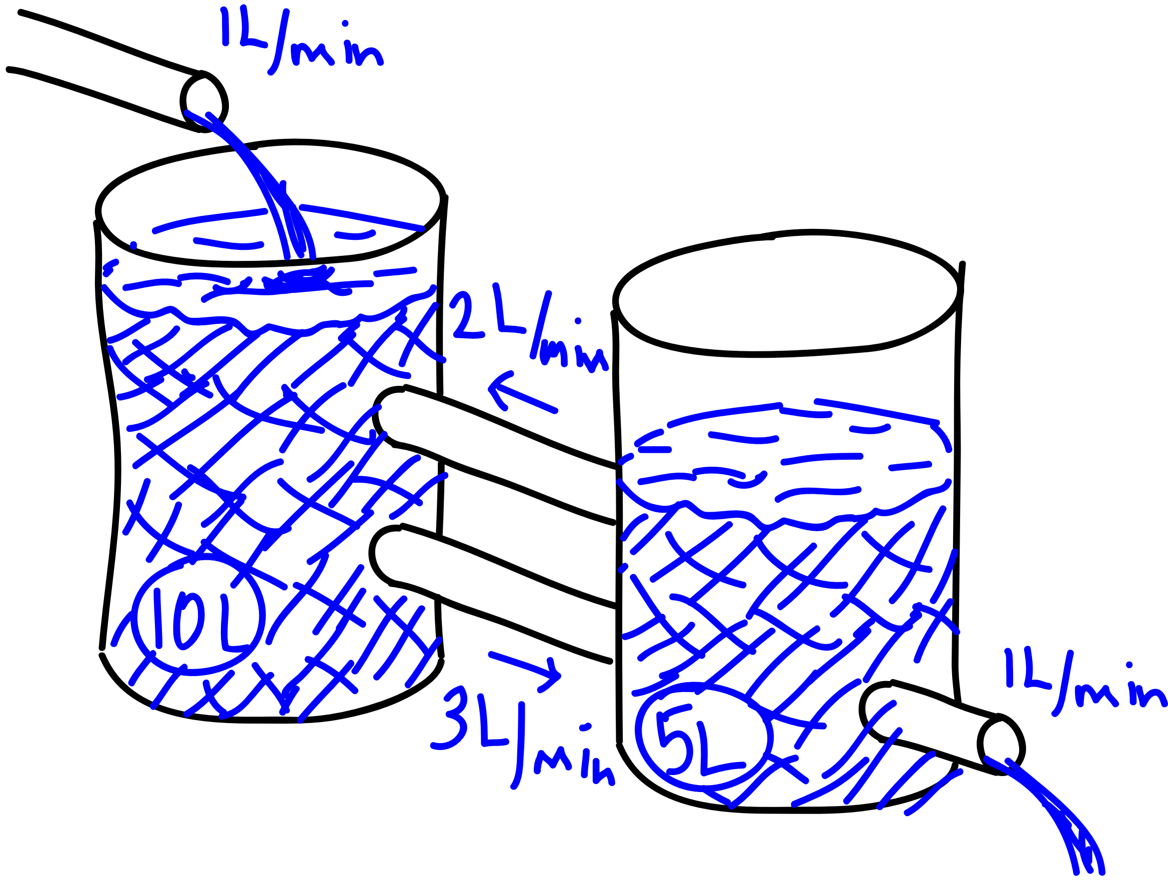

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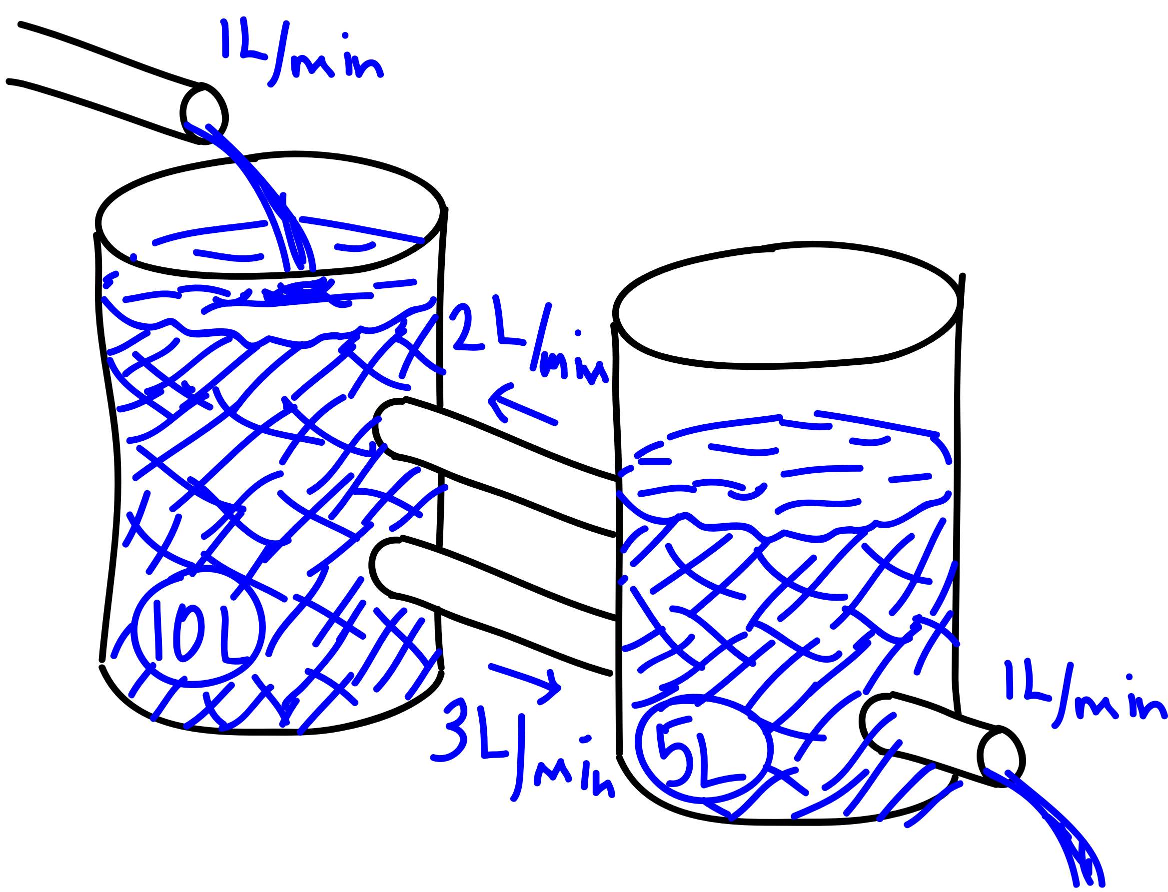
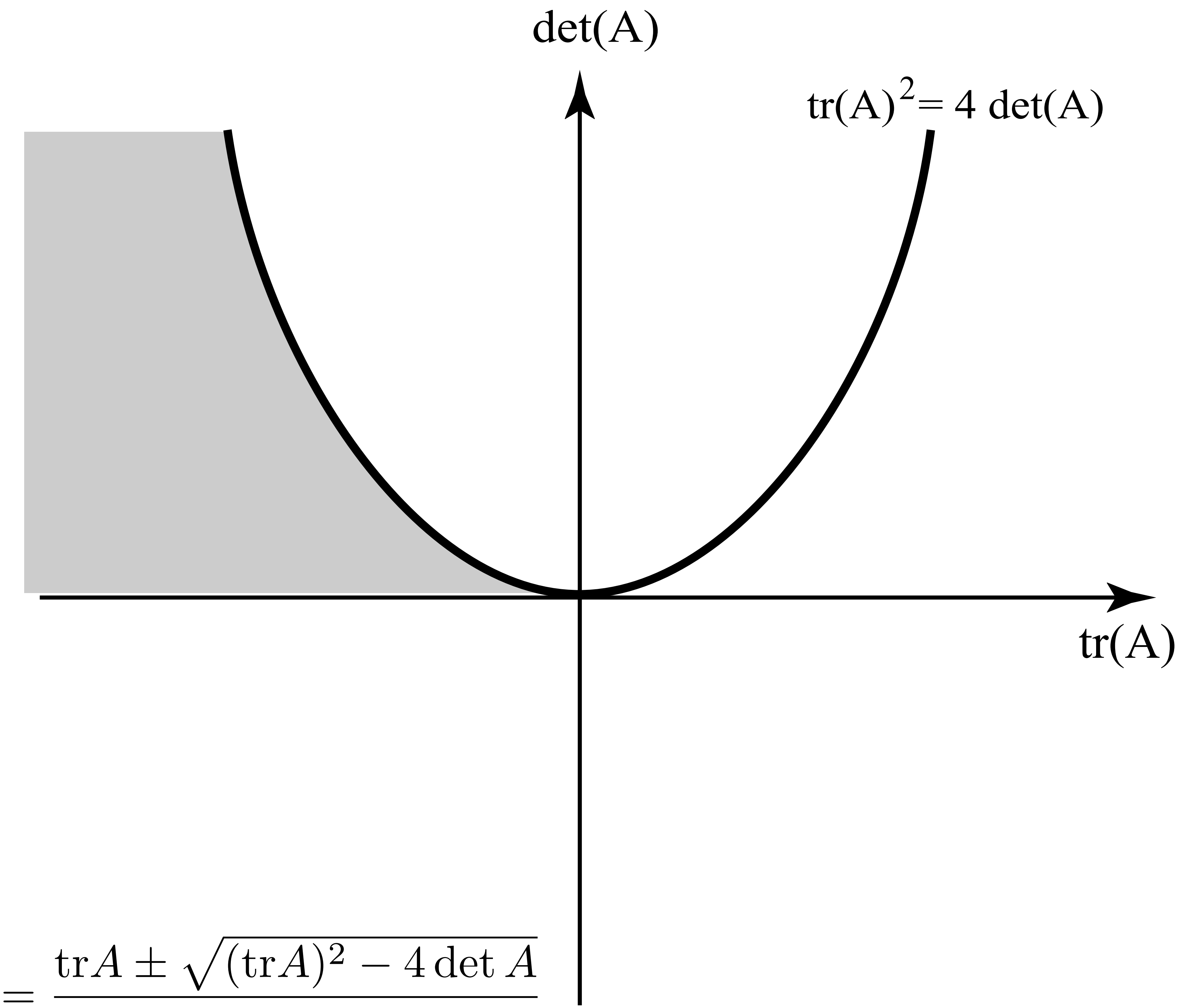
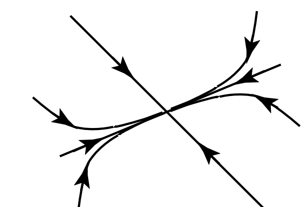


- What are $m_1(t)$ and $m_2(t)$ as $t \to \infty$?

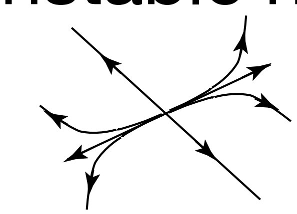
Review problems



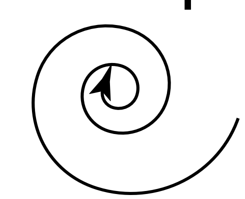
(A) stable node



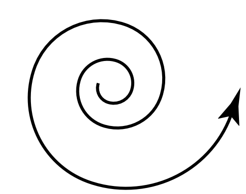
(B) unstable node



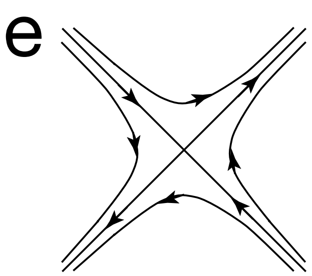
(C) stable spiral



(D) unstable spiral



(E) saddle



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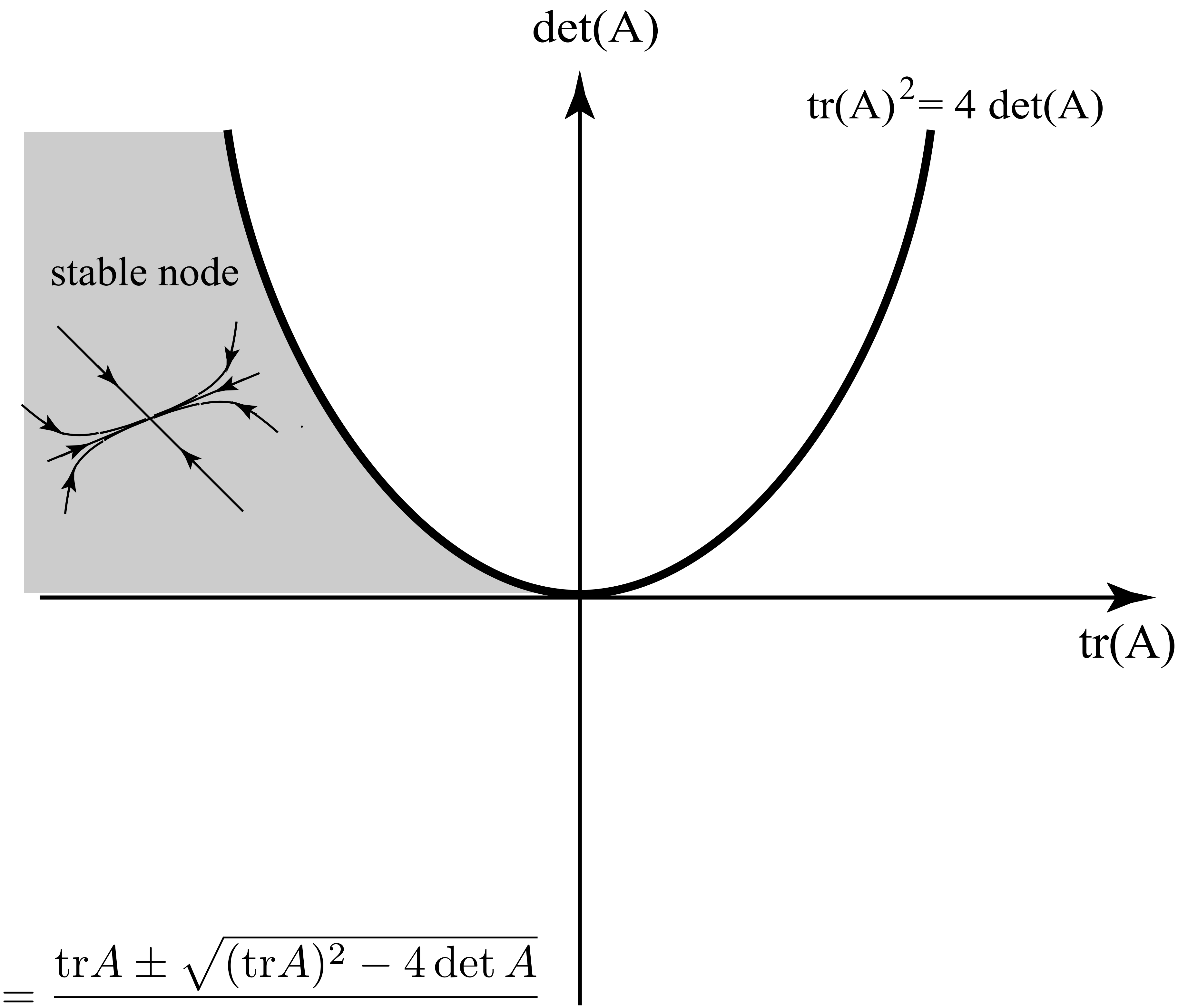

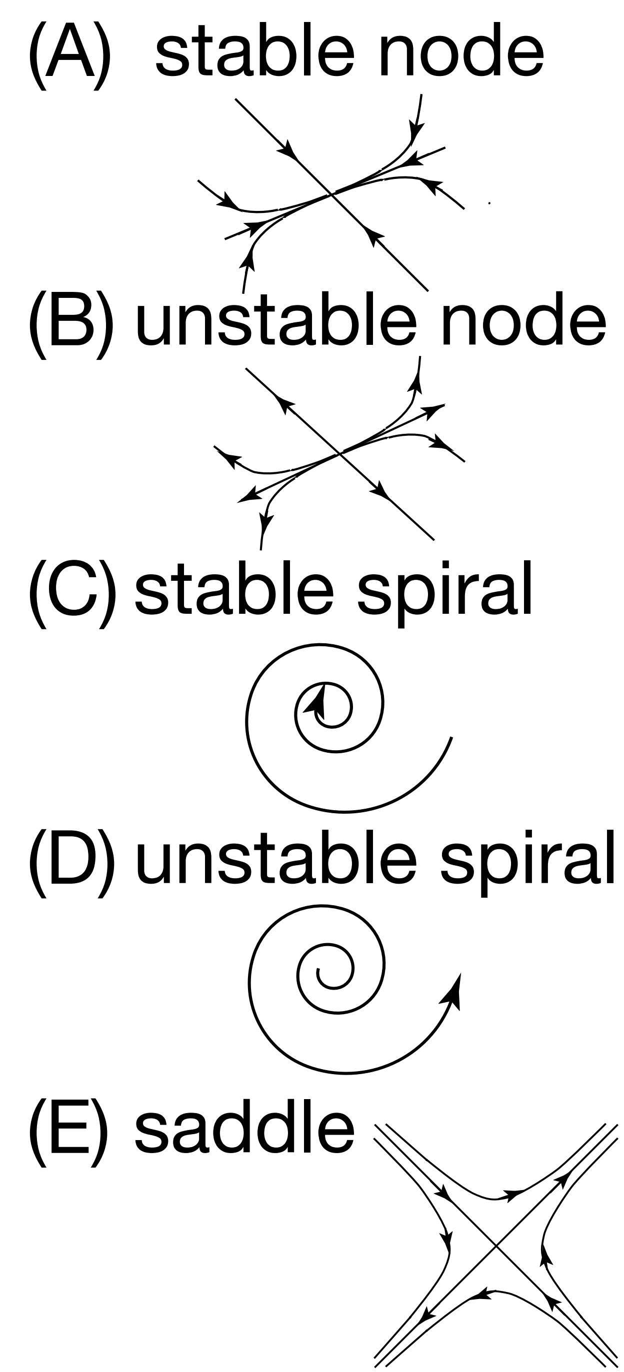

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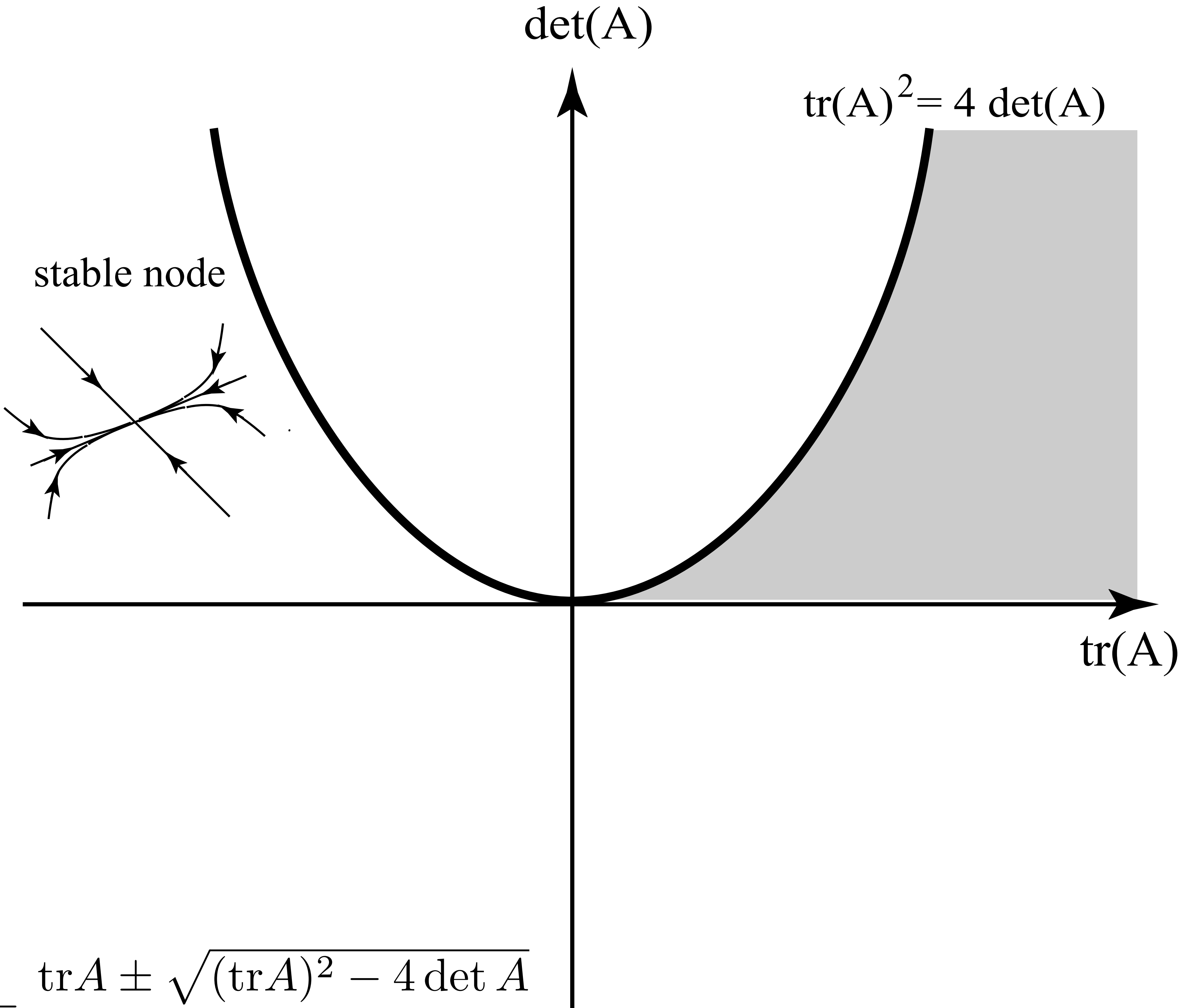

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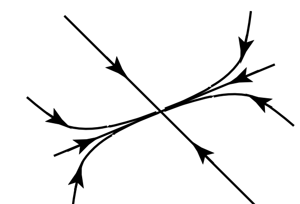

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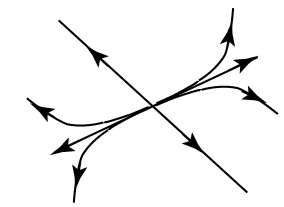

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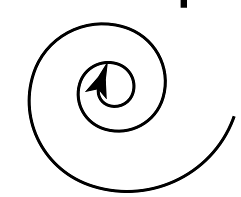

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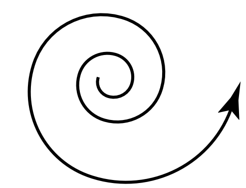

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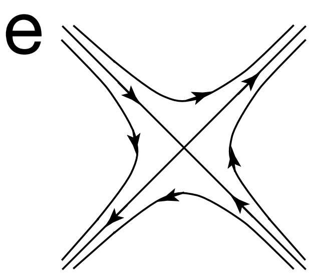

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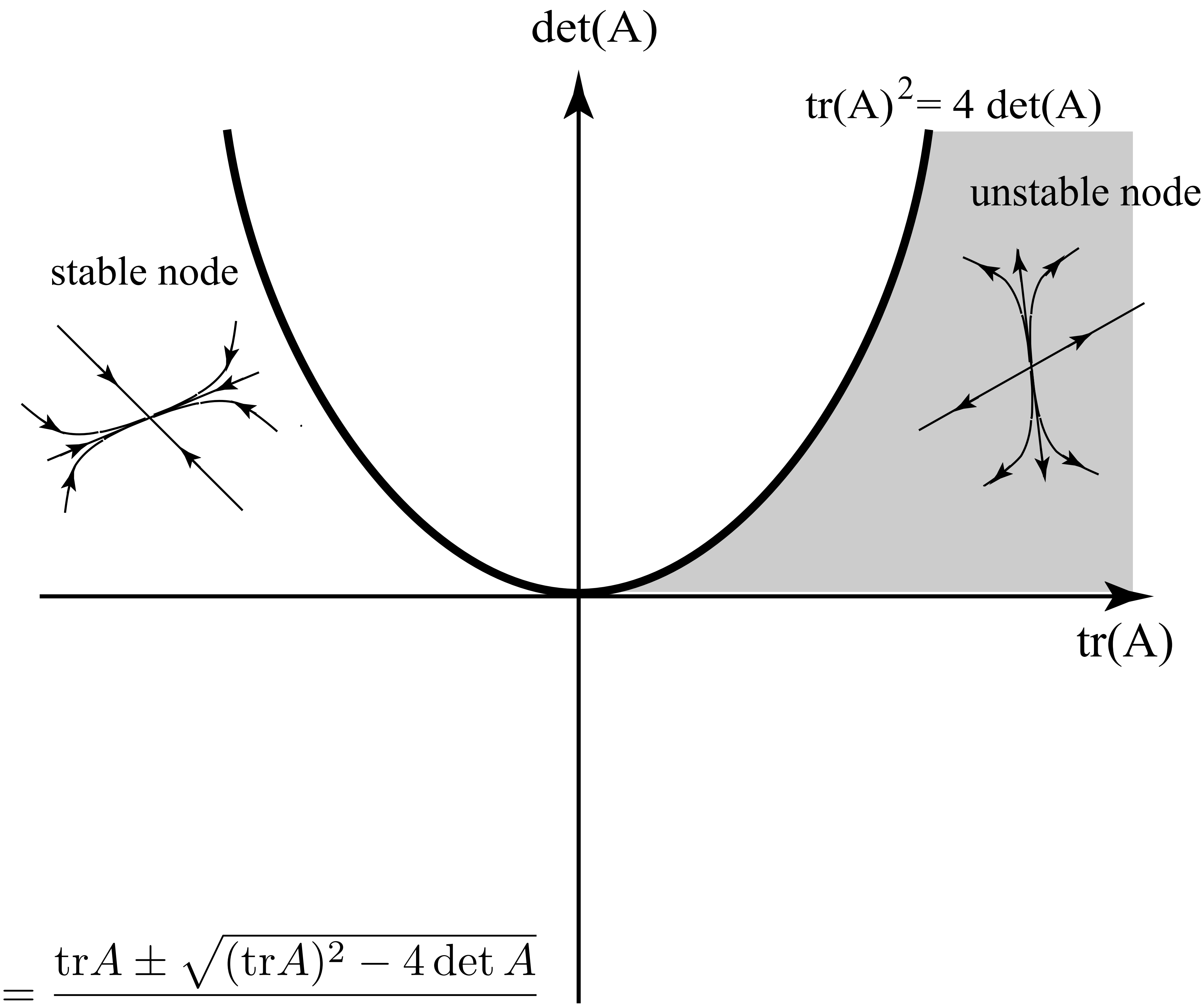

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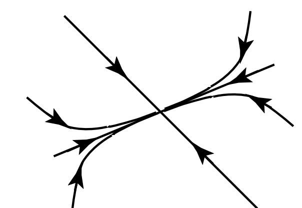

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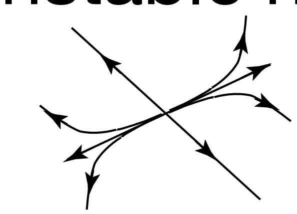

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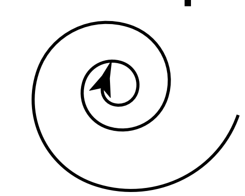

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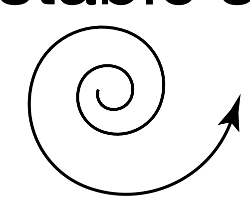

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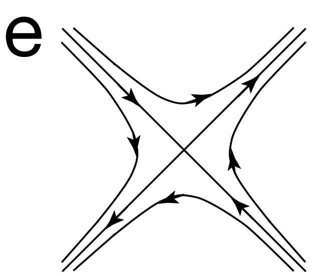

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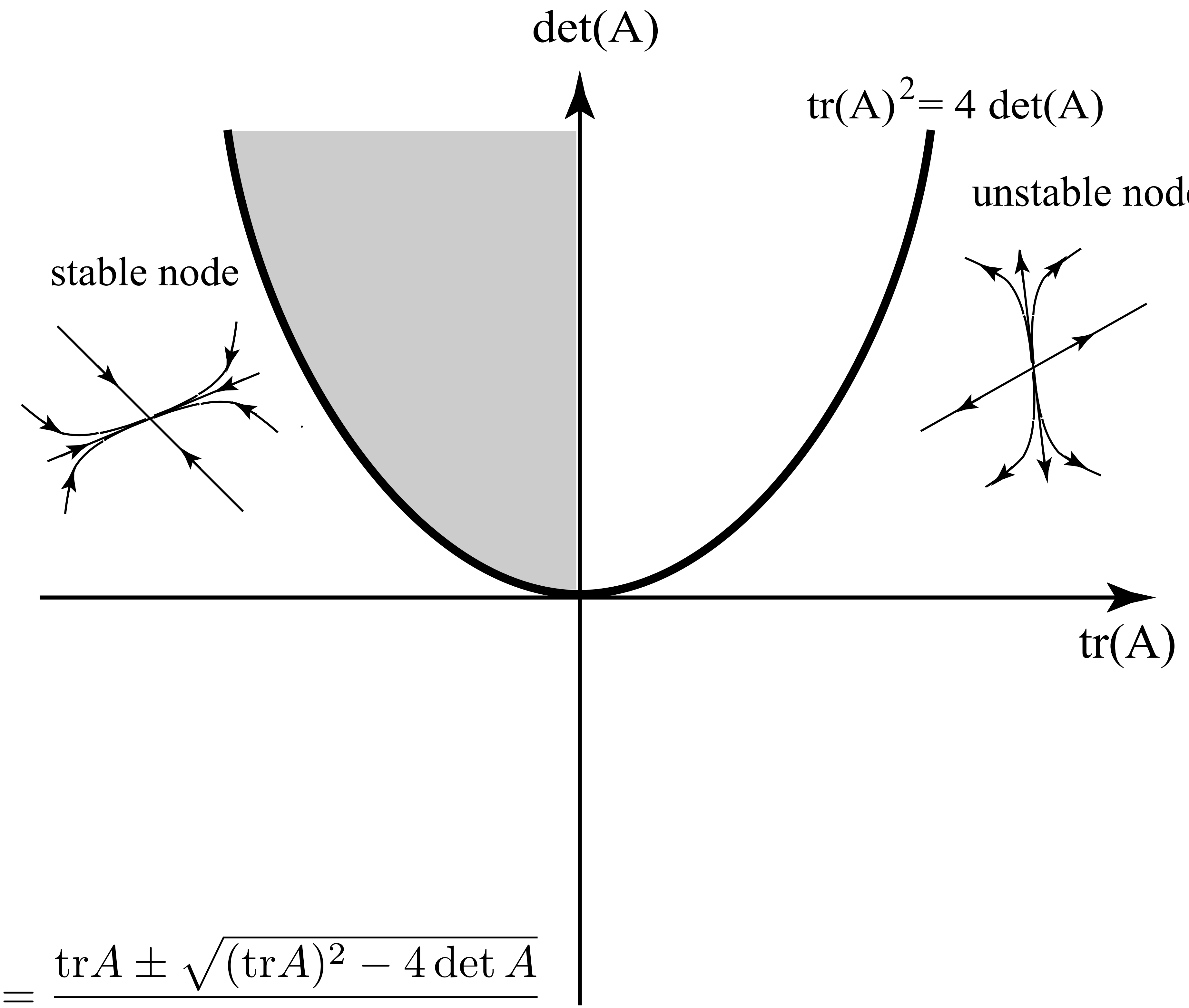

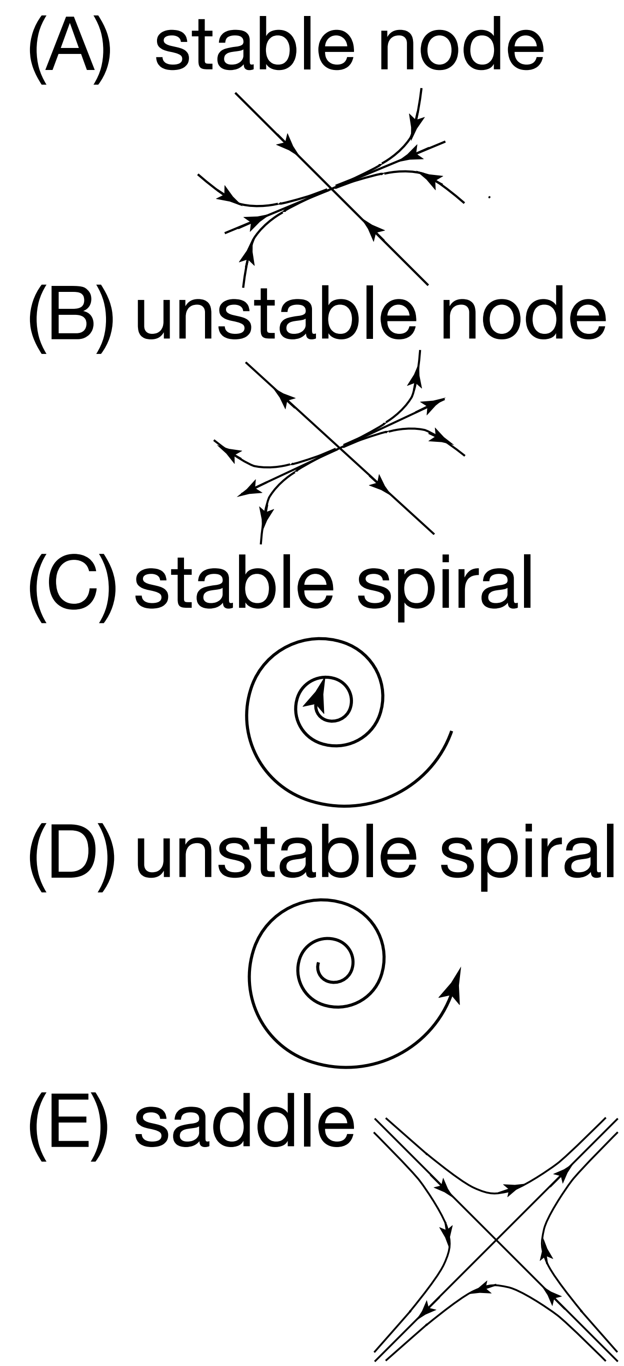

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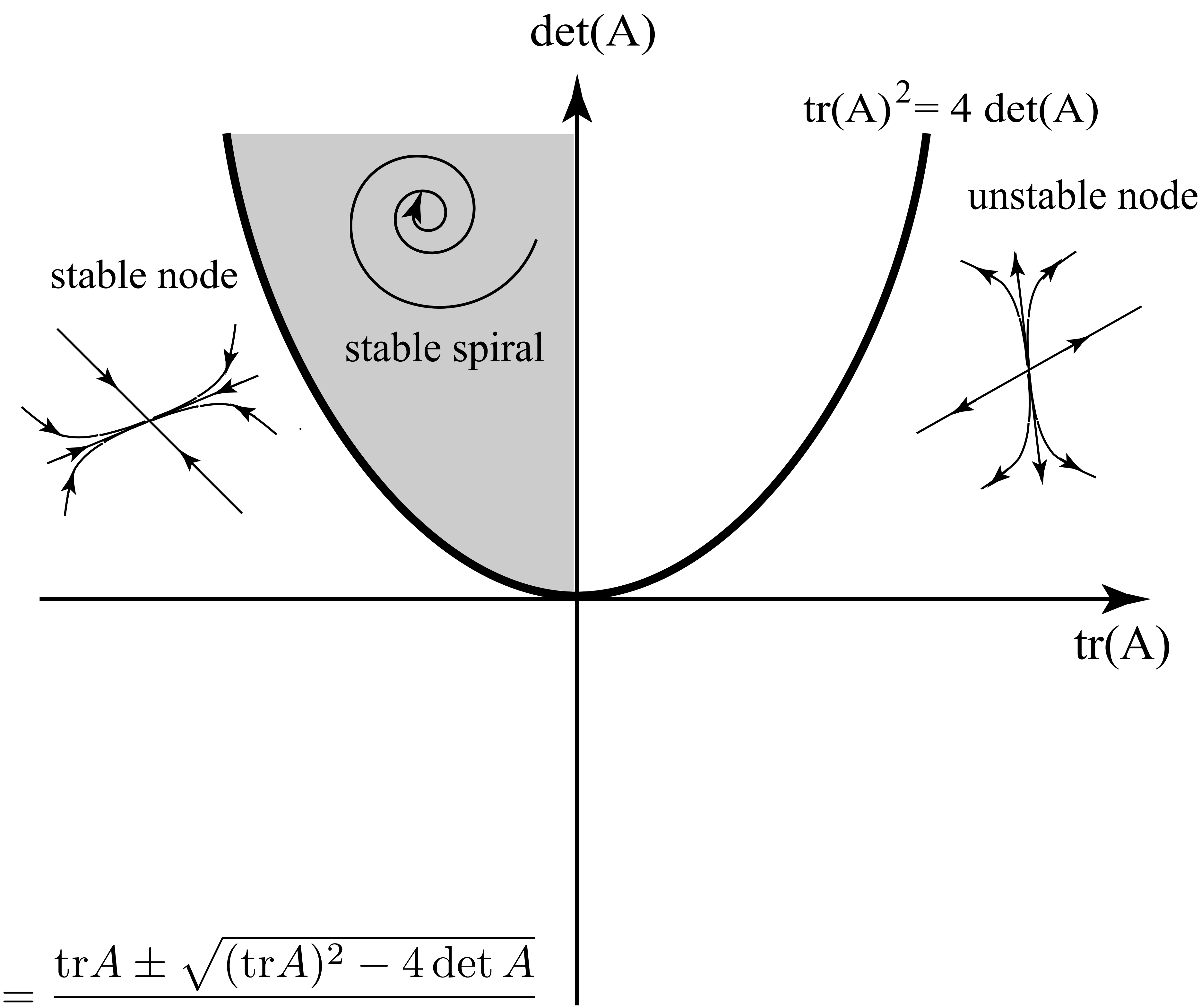

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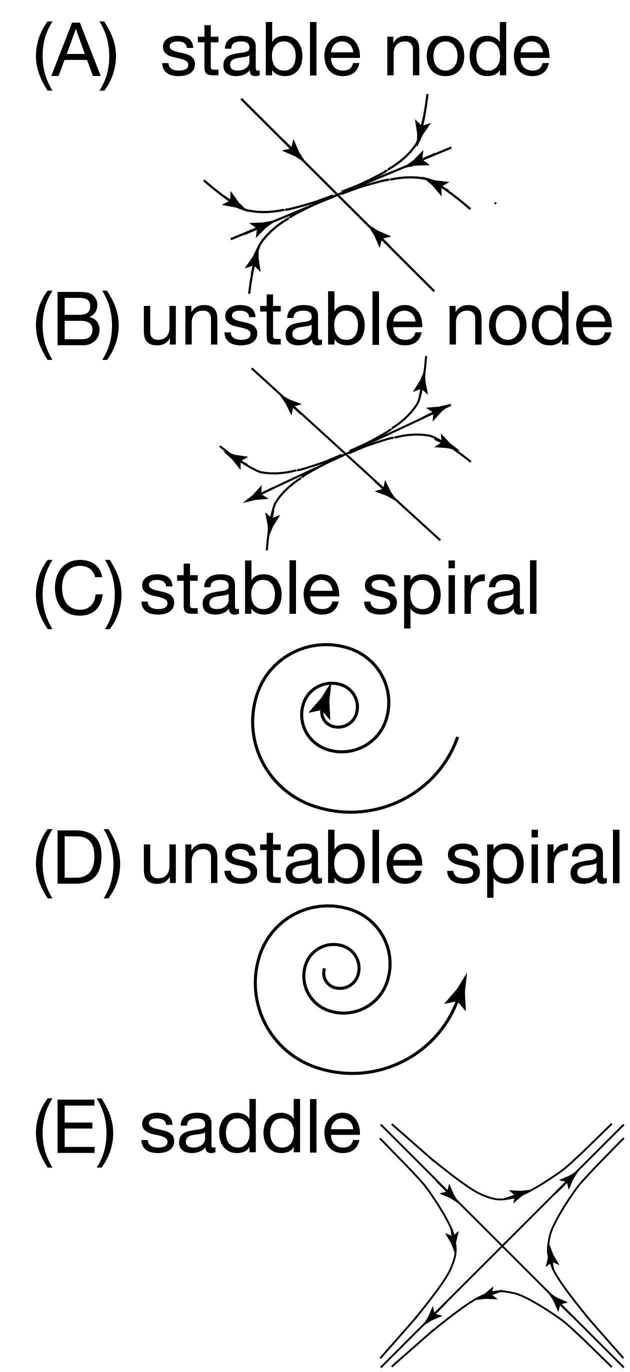

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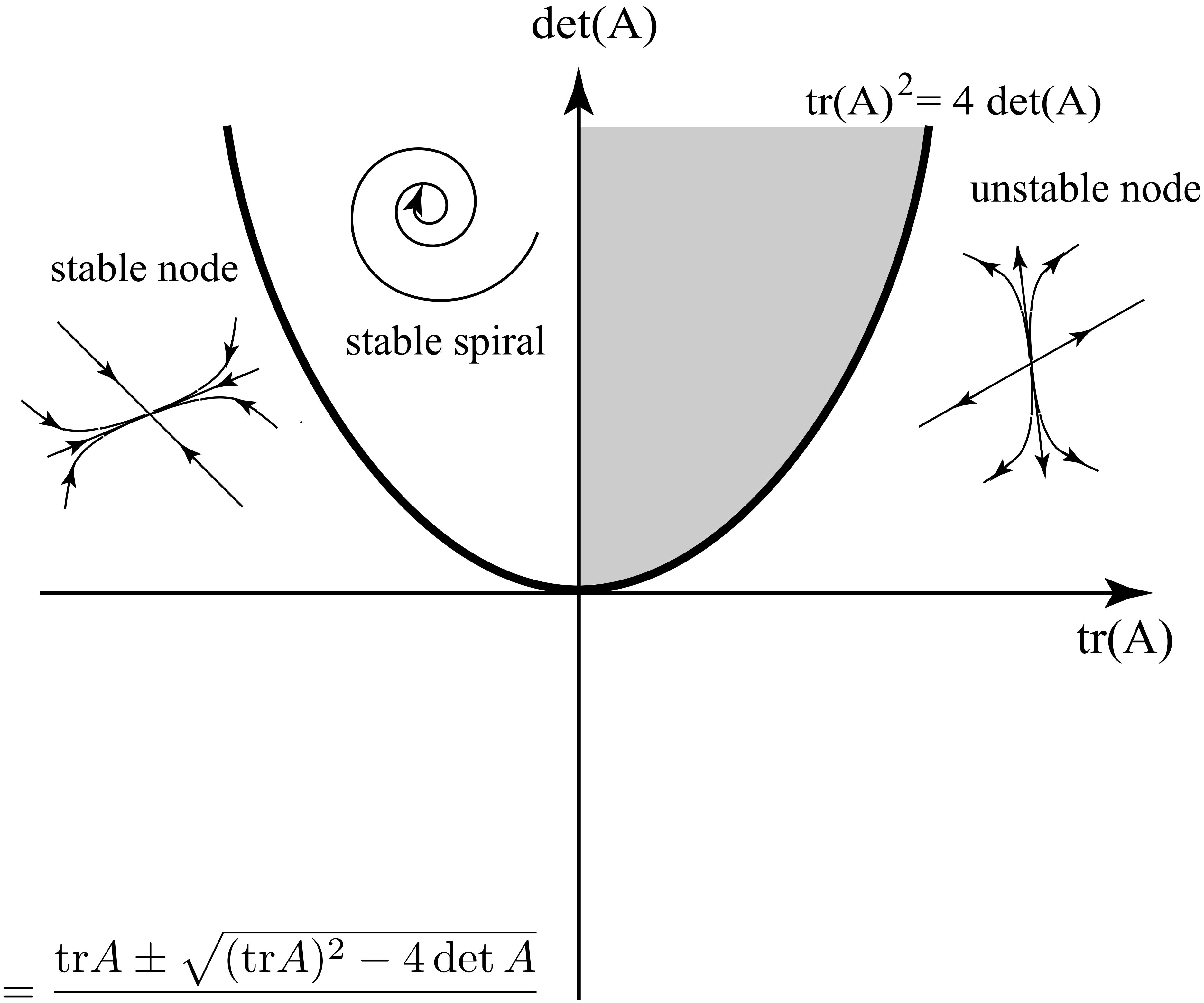

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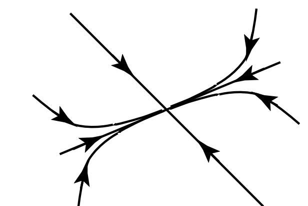

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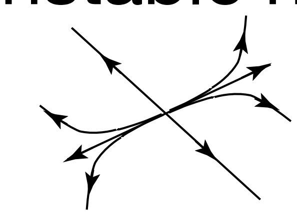

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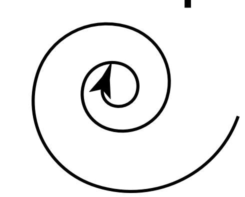

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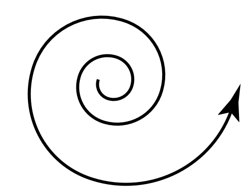

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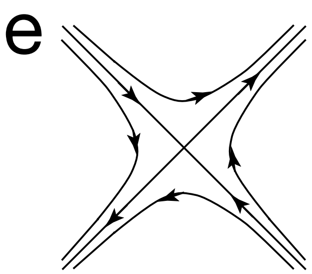

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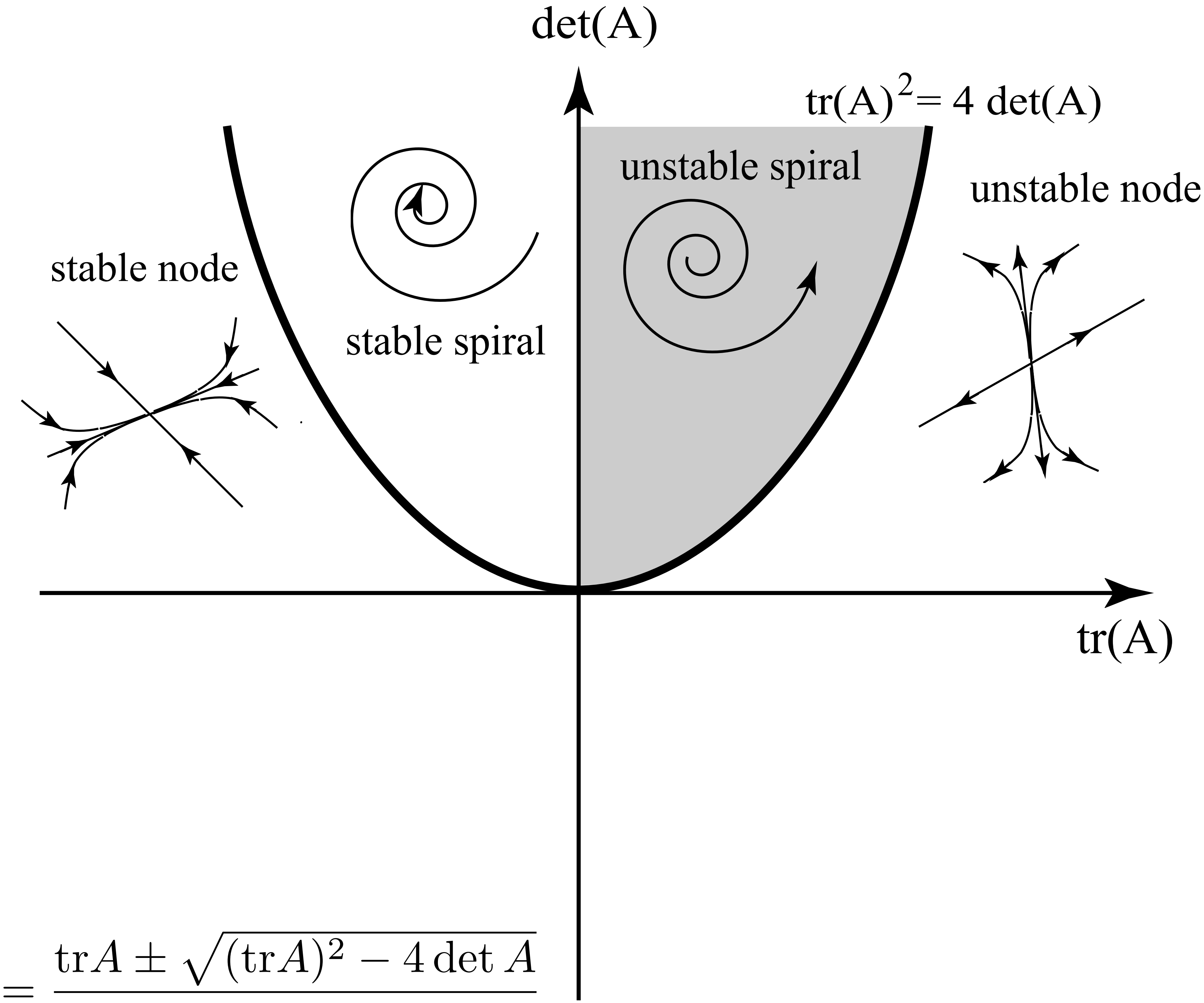

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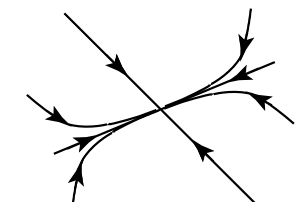

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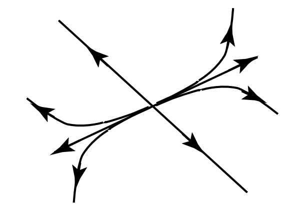

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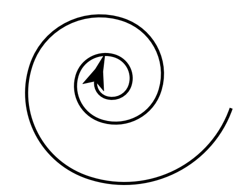

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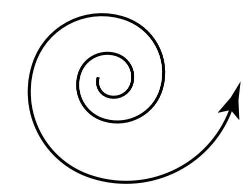

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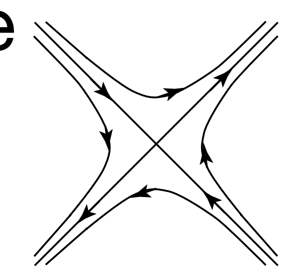

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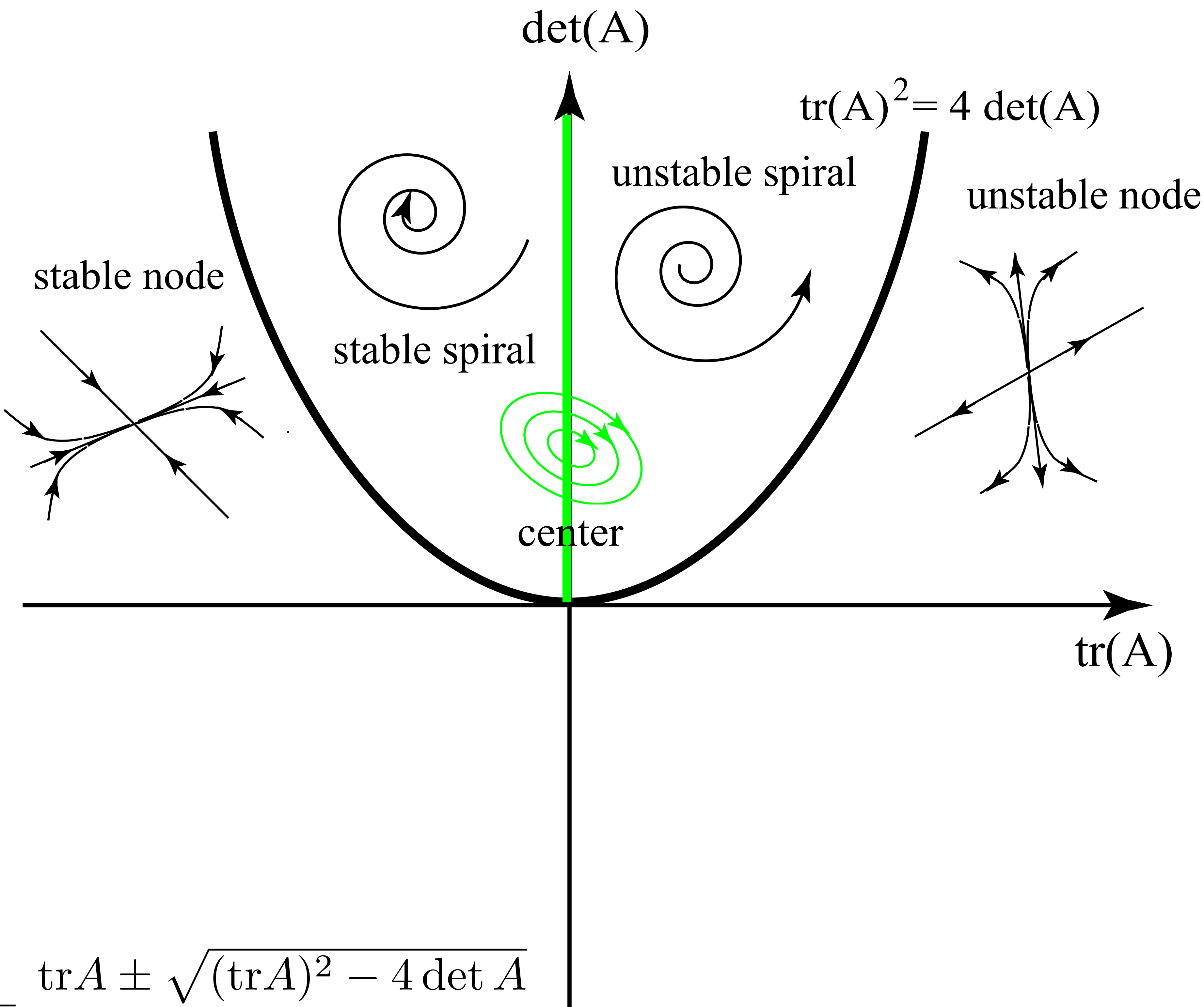

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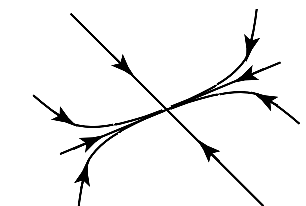
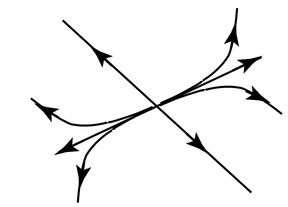
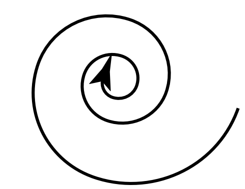


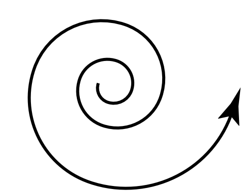
(B) unstable node



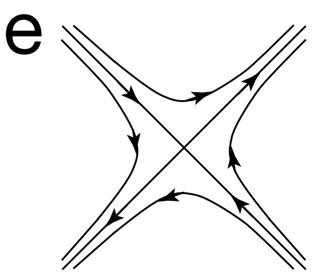
(C) stable spiral



(D) unstable spiral

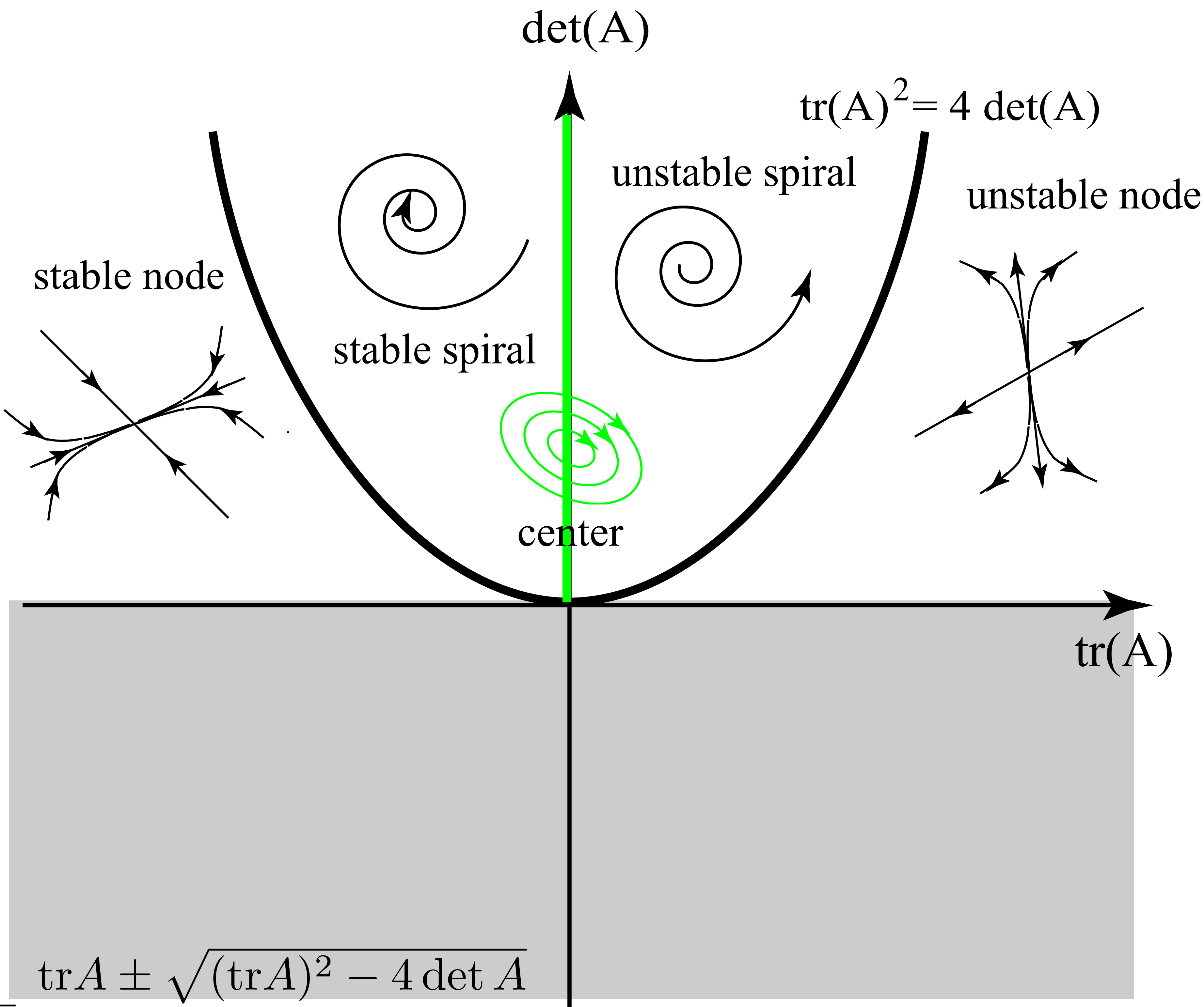


(E) saddle

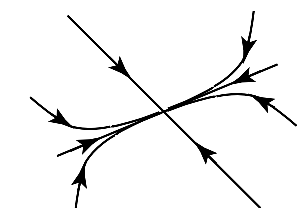


$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

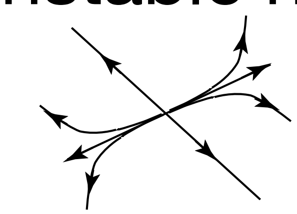
Review problems



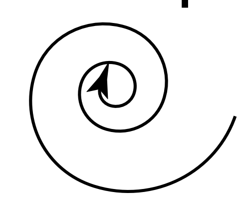
(A) stable node



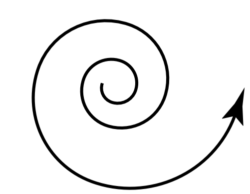
(B) unstable node



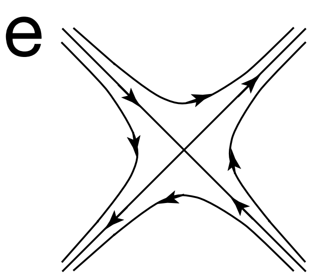
(C) stable spiral



(D) unstable spiral

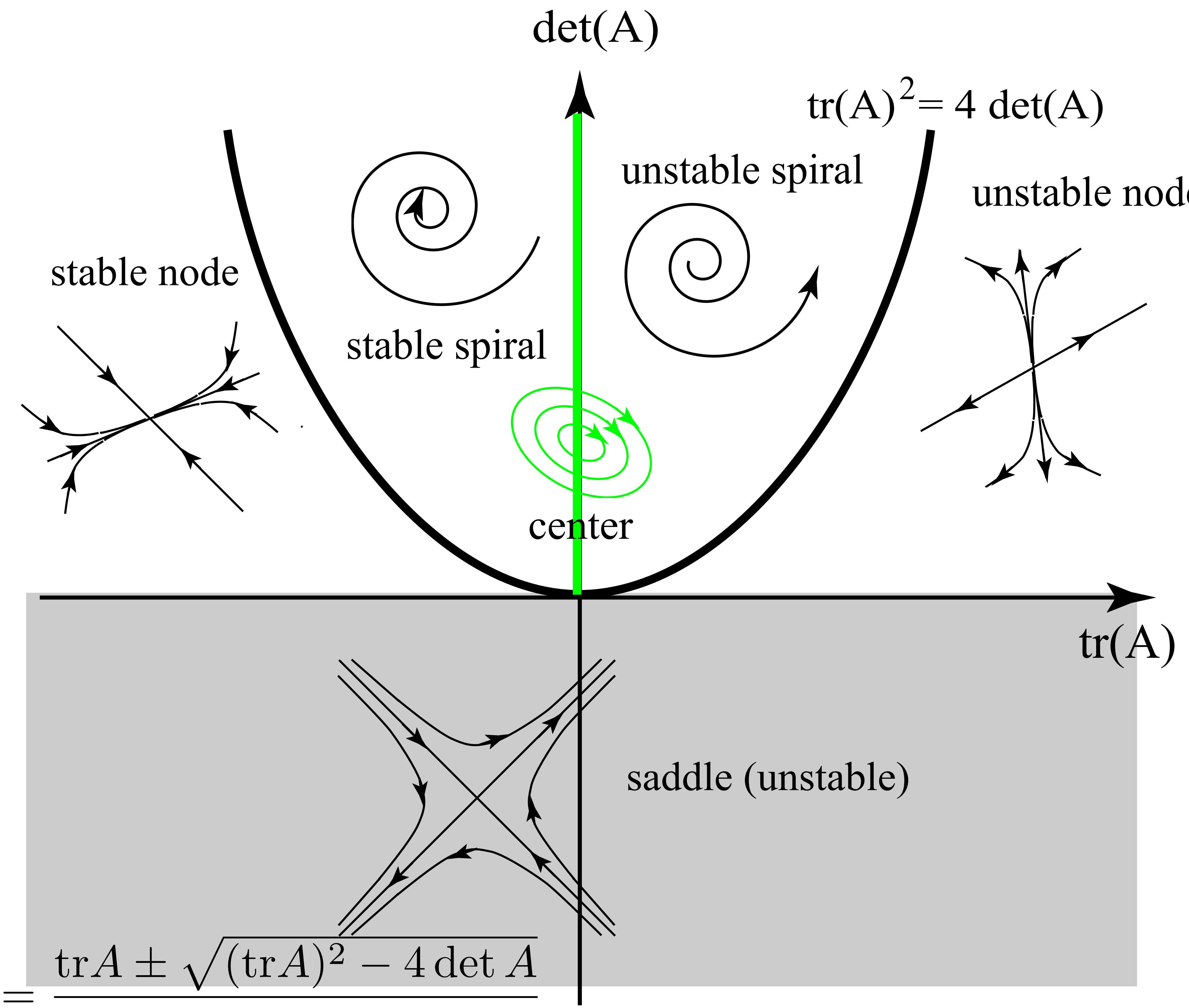


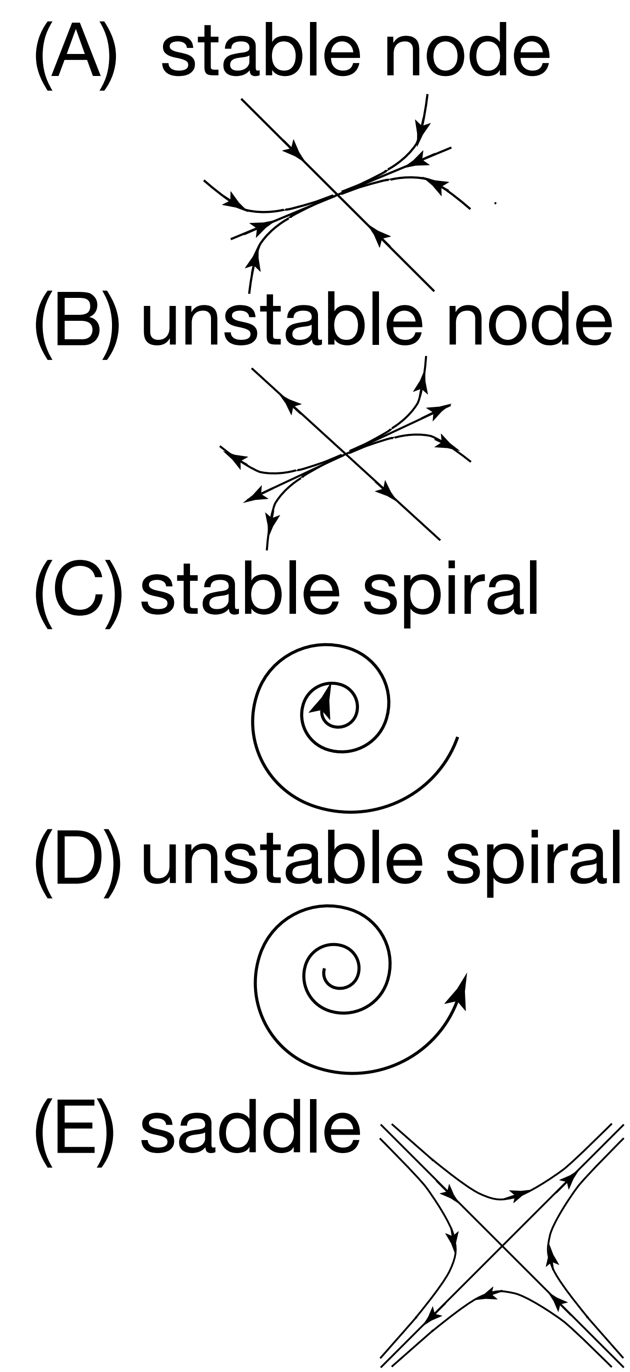
(E) saddle



$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

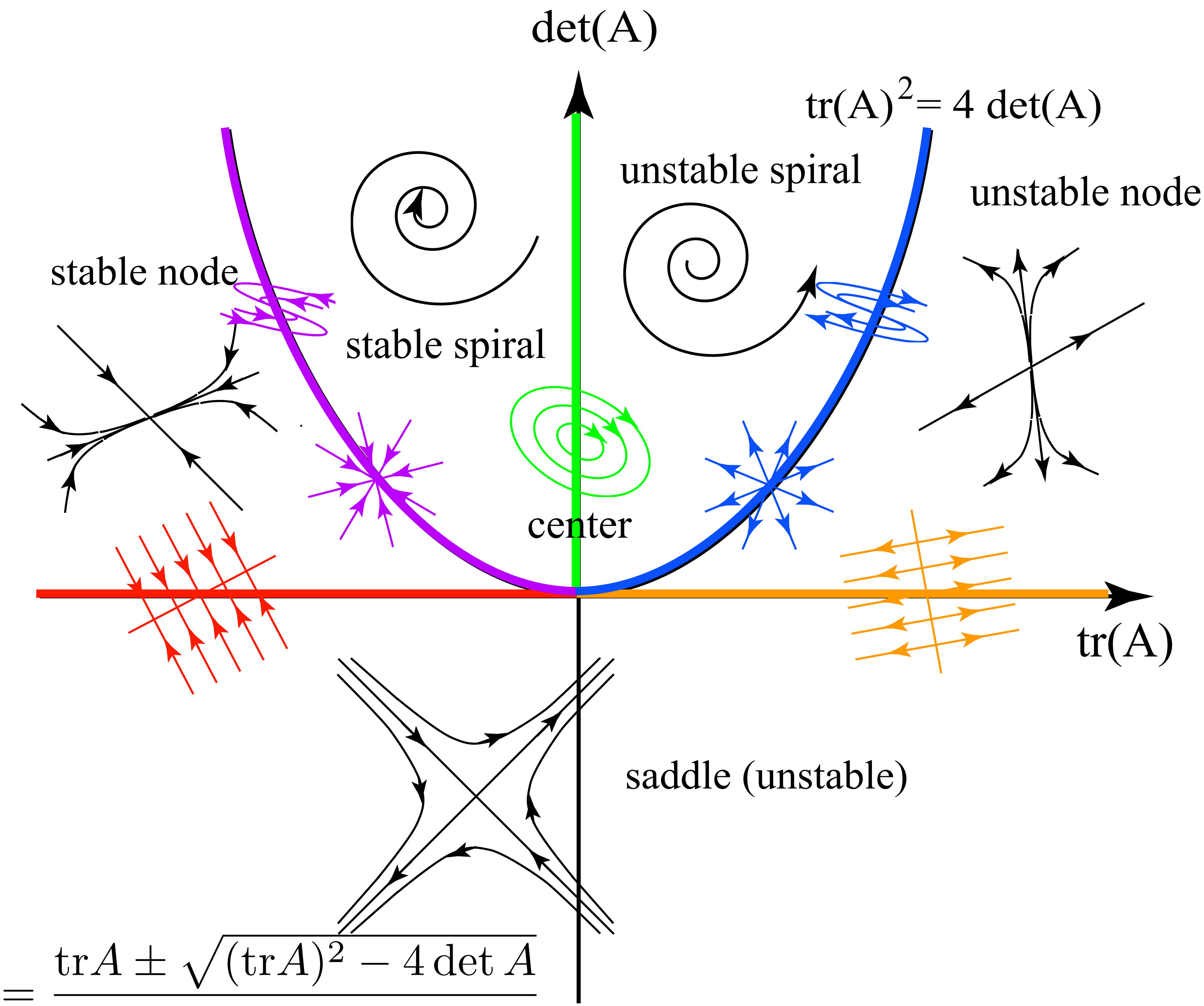
Review problems





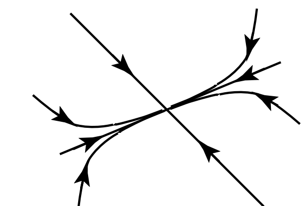
$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

Review problems

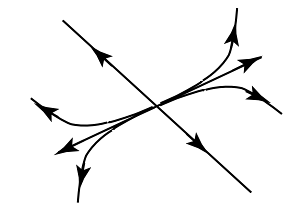


$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

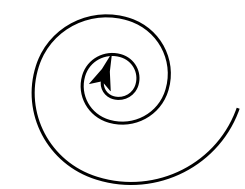
(A) stable node



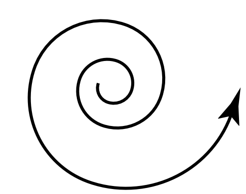
(B) unstable node



(C) stable spiral



(D) unstable spiral



(E) saddle