Today

- Summary of 2x2 systems with constant coefficients.

- Nonhomogeneous example

Summary - homogeneous 2x2 systems

$$= 0$$

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- For the system of equations $\mathbf{x}' = A\mathbf{x}$, we always have $\mathbf{x}(t) = \mathbf{0}$ as a steady state solution.

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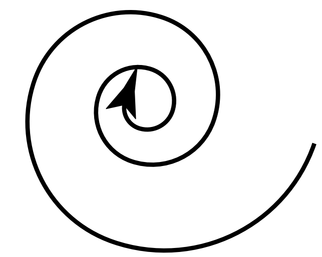
- When do the solutions spiral in to the origin?

$$\lambda^2 - (a+d)\lambda + ad - bc = 0$$



(A) $\begin{cases} \mathrm{tr}A < 0 \\ (\mathrm{tr}A)^2 < 4\det A \end{cases}$

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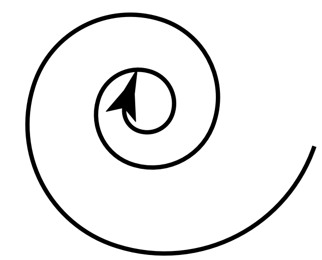

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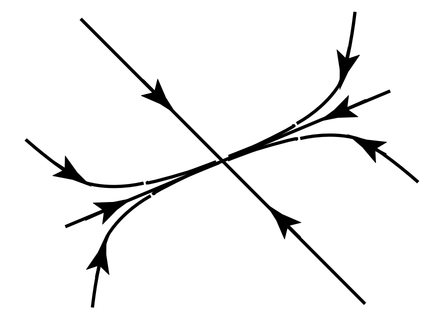

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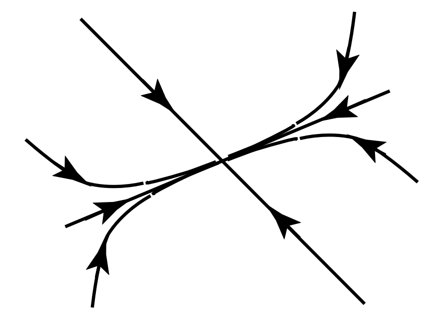

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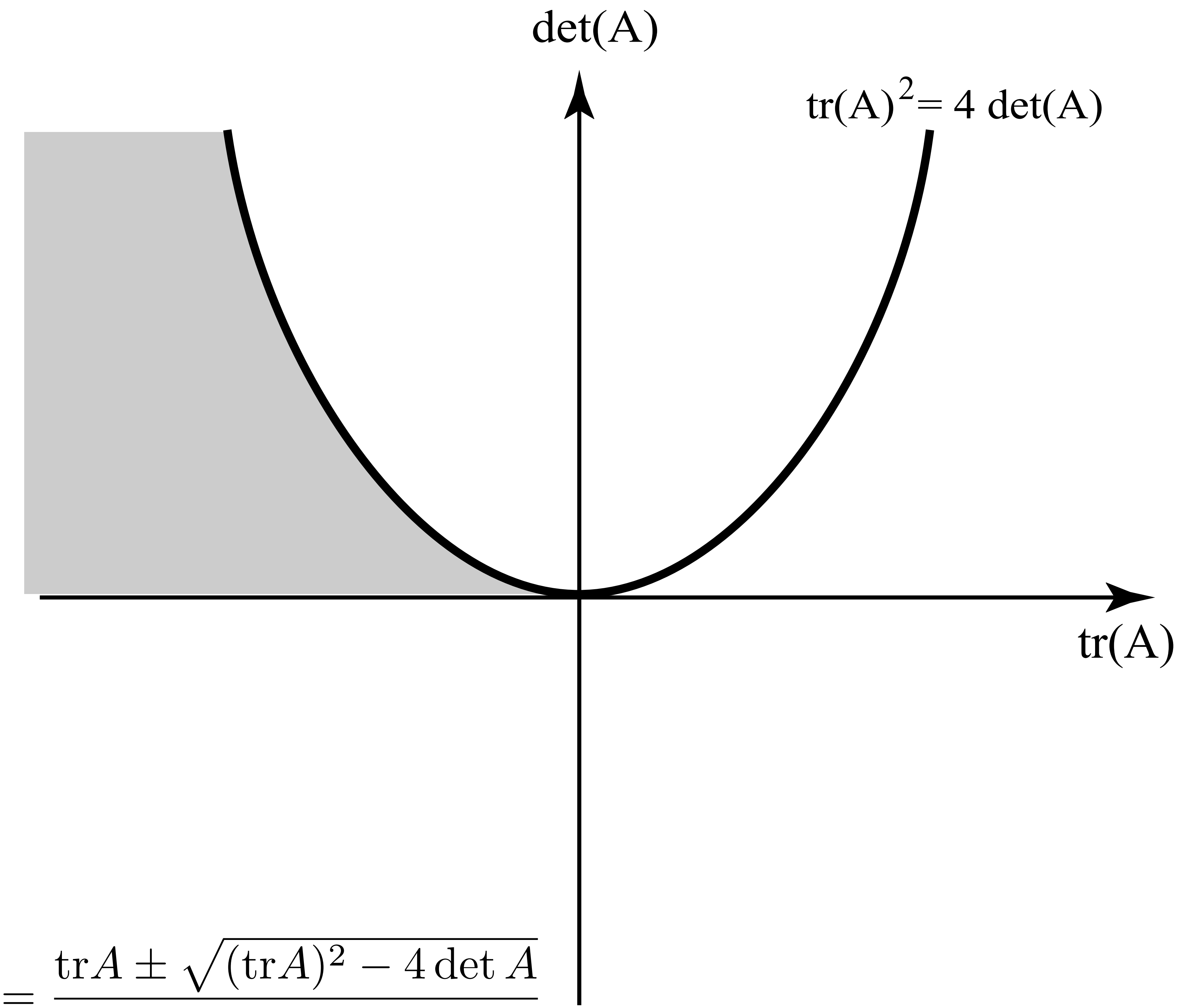

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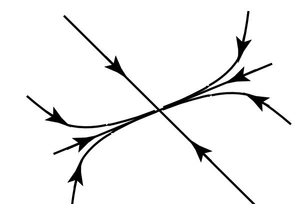

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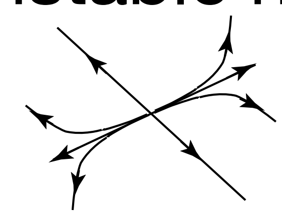

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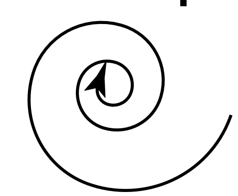

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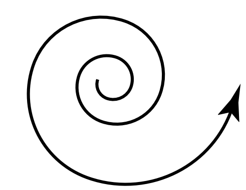

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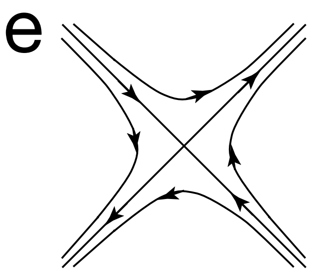

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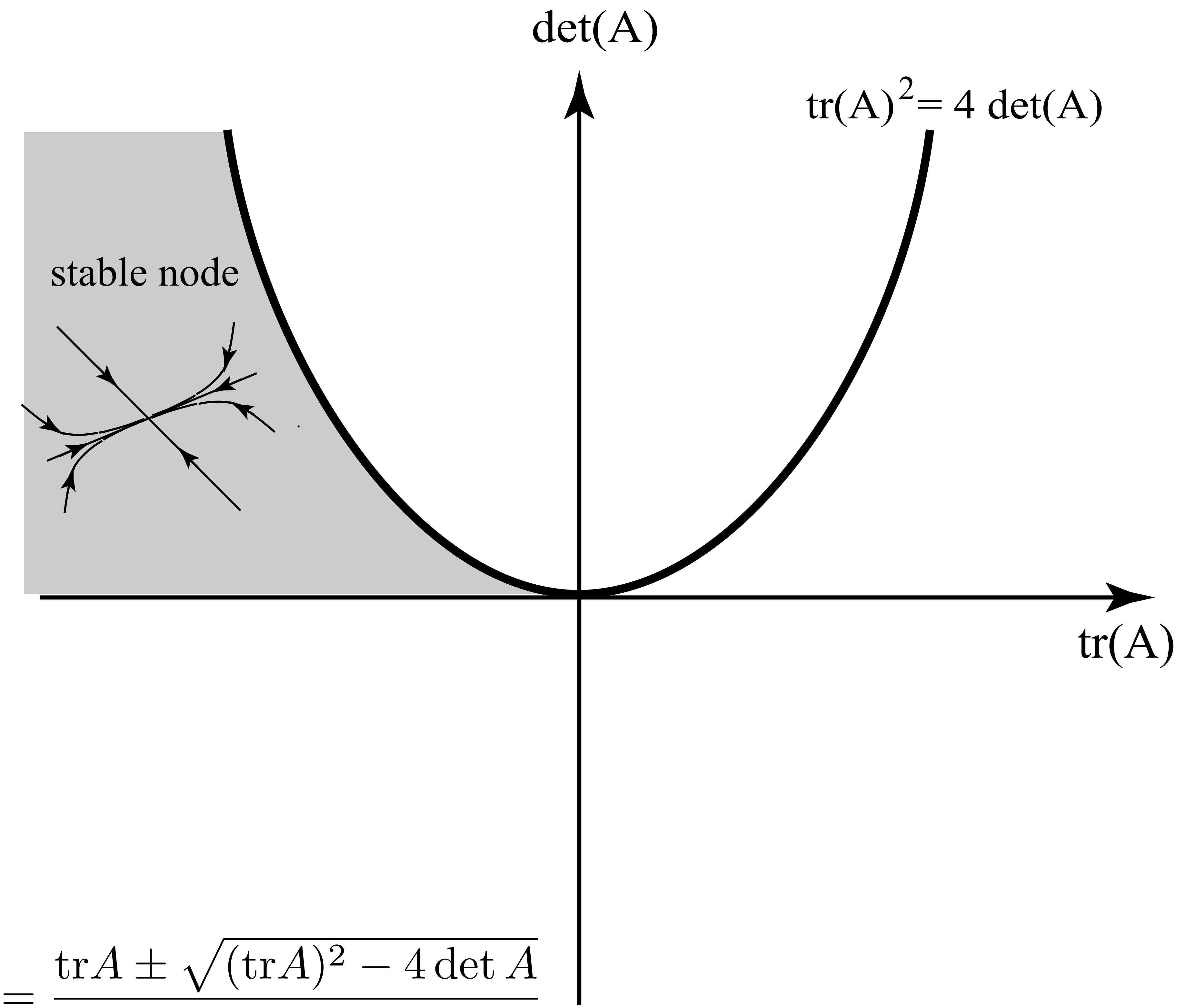

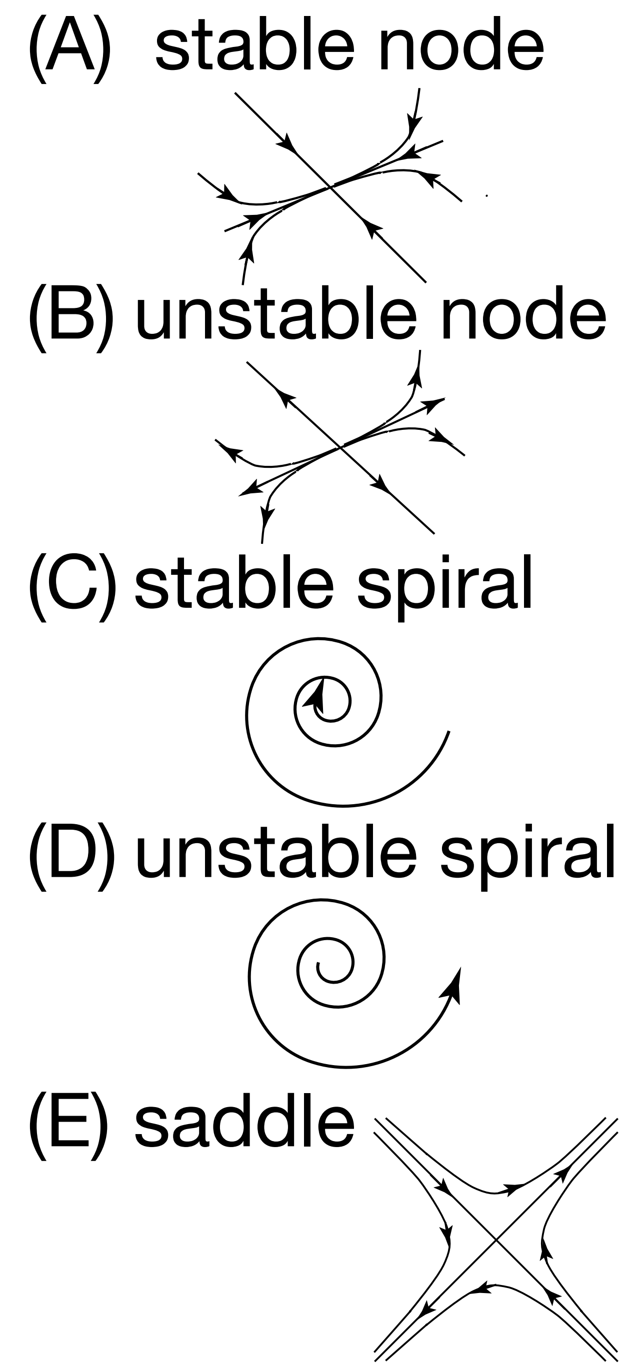

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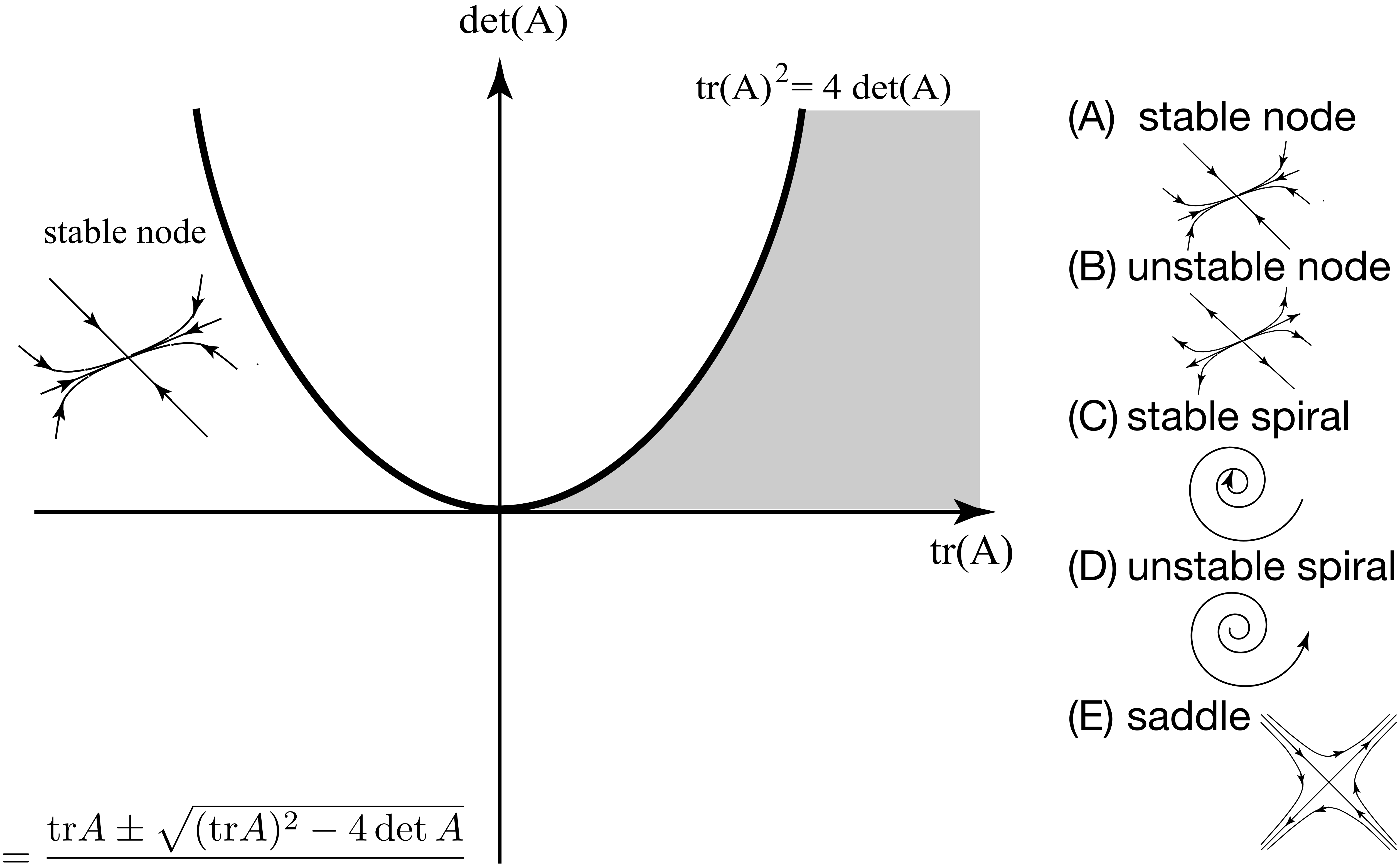

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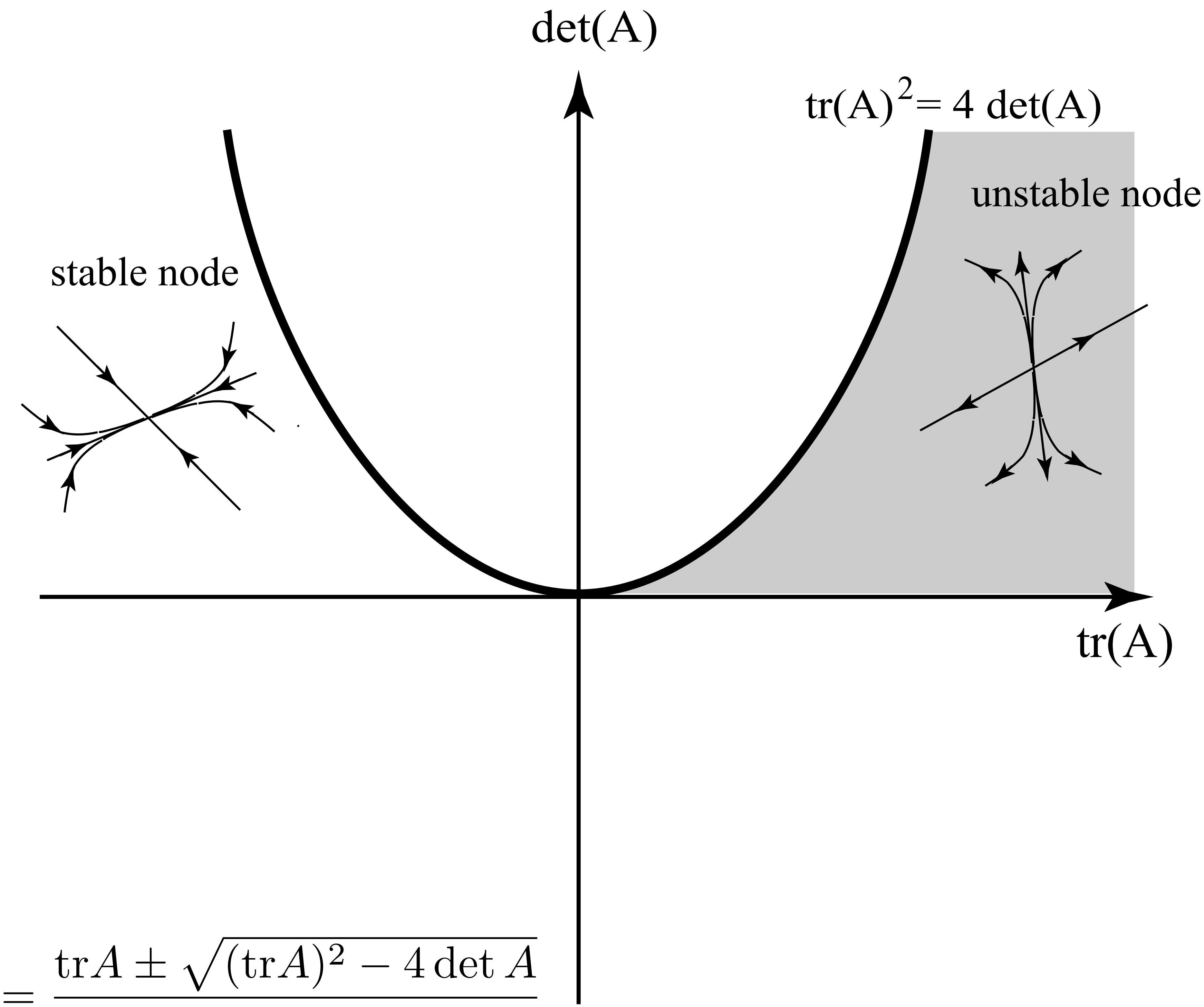

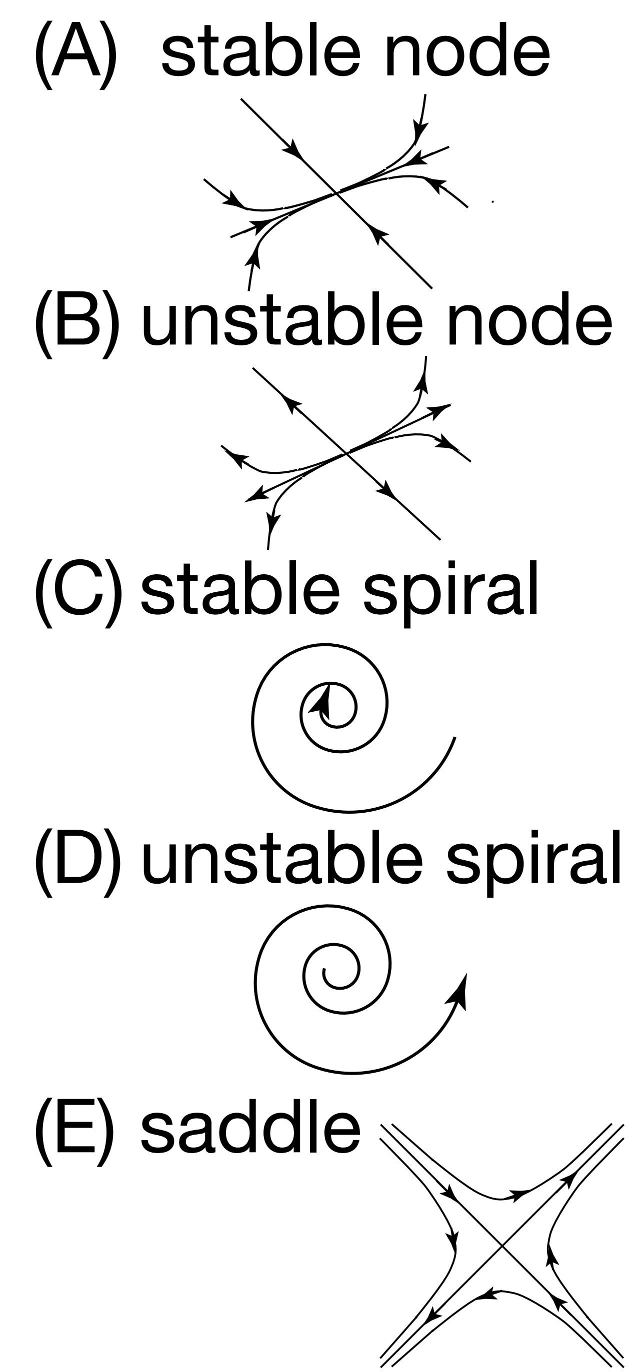

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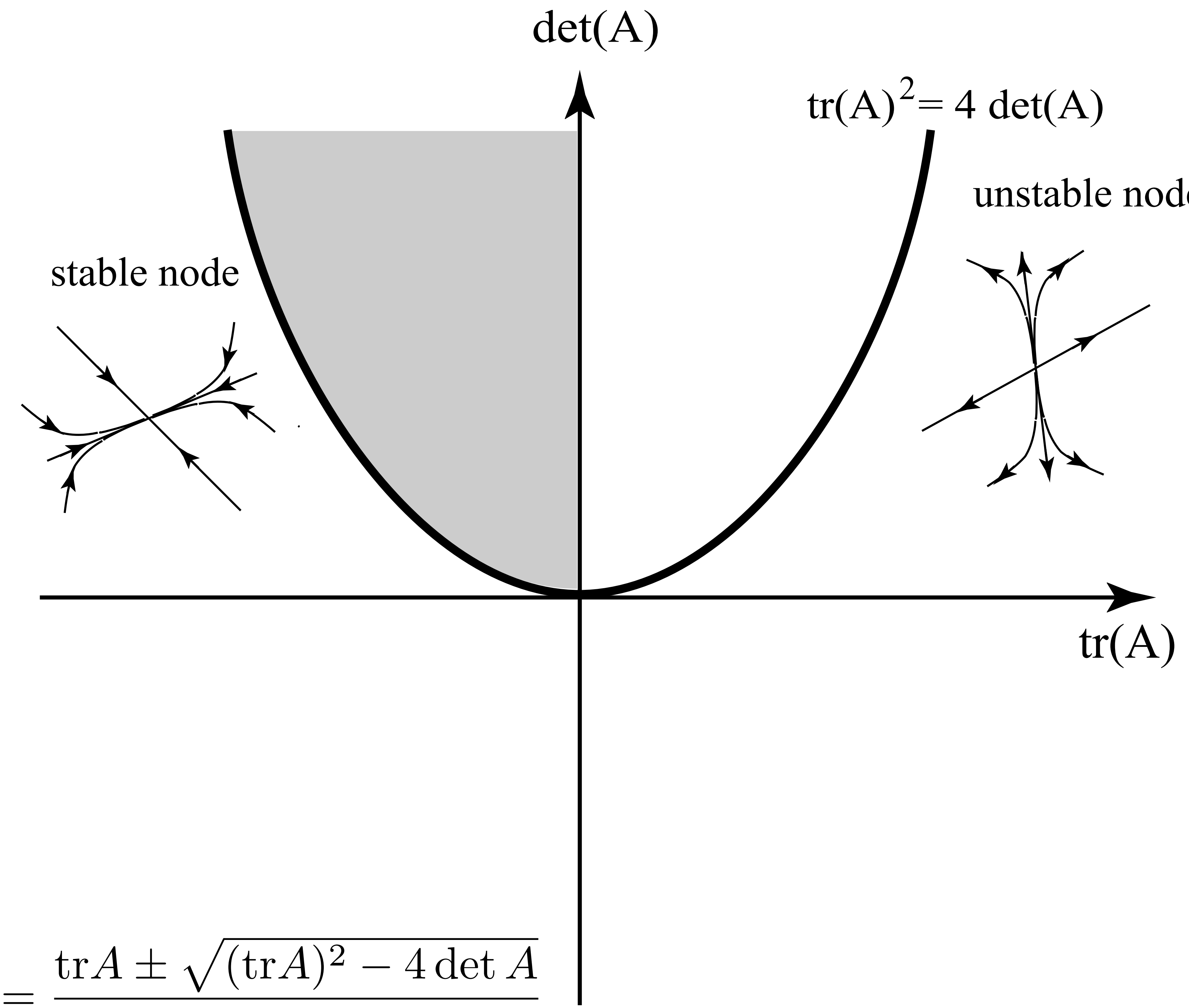

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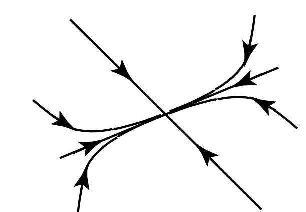

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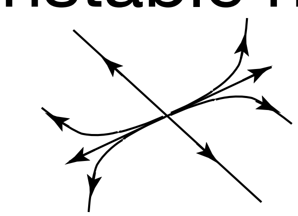

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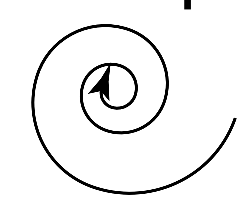

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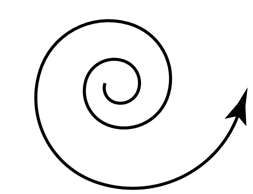

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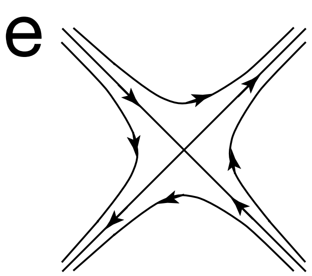

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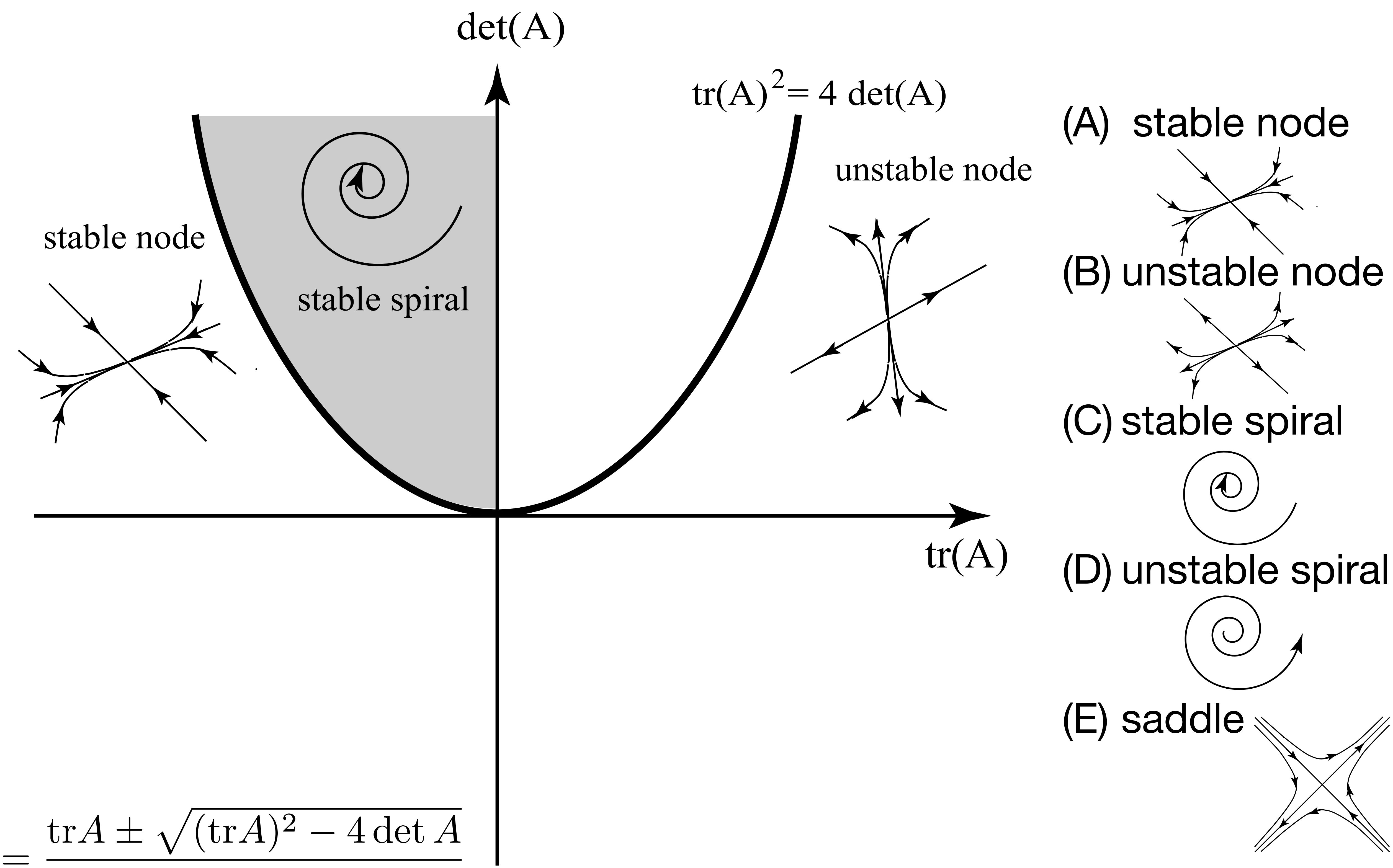

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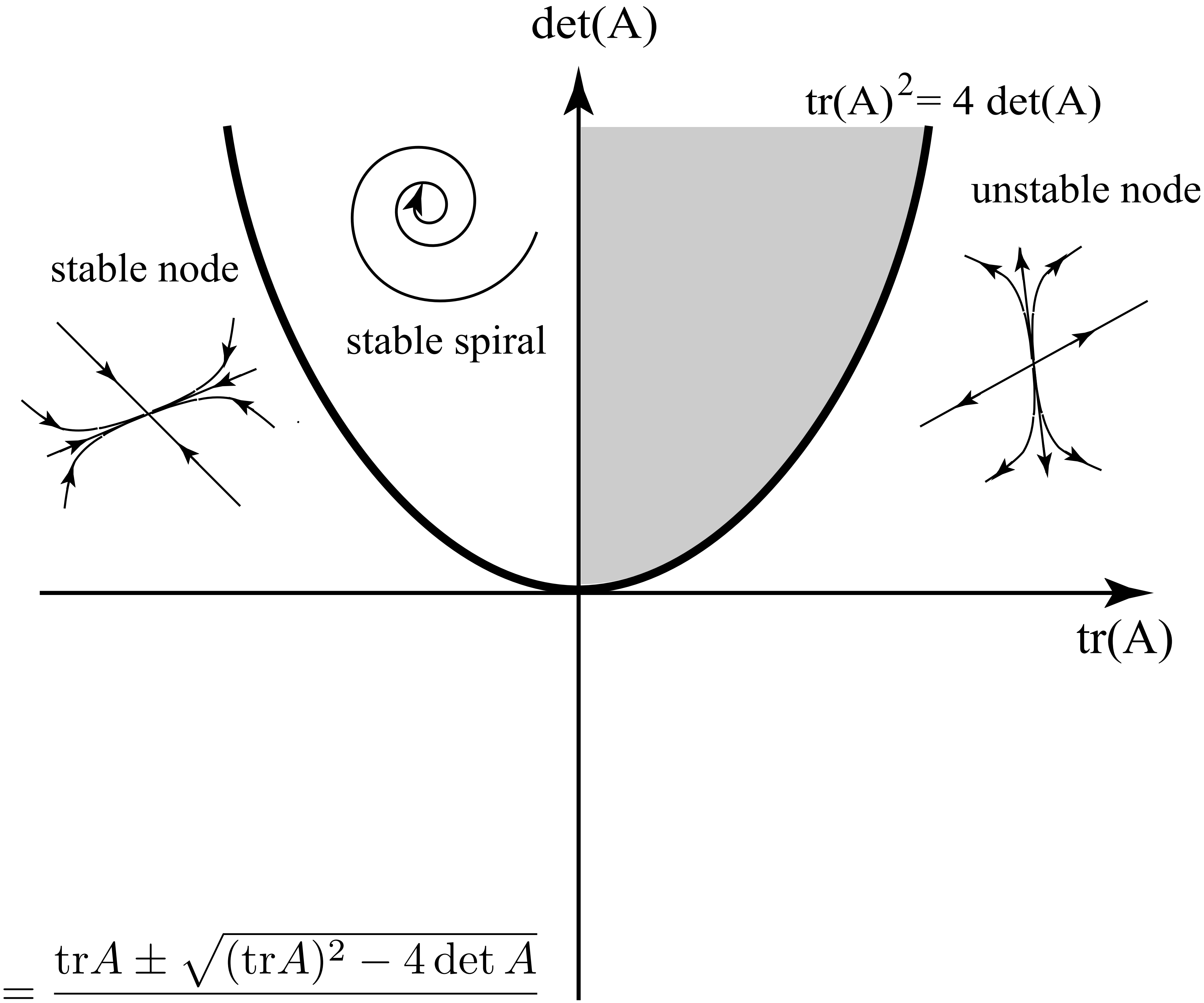

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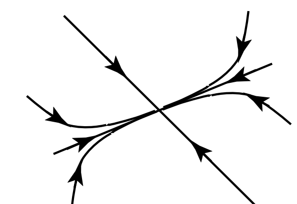

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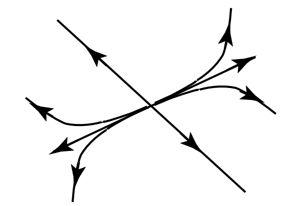

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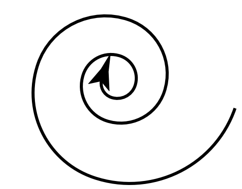

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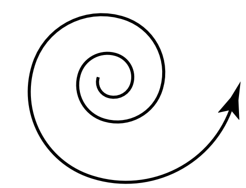

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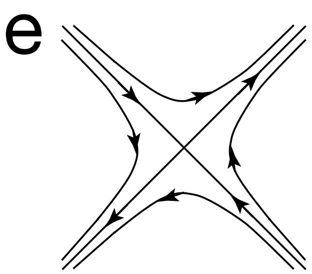

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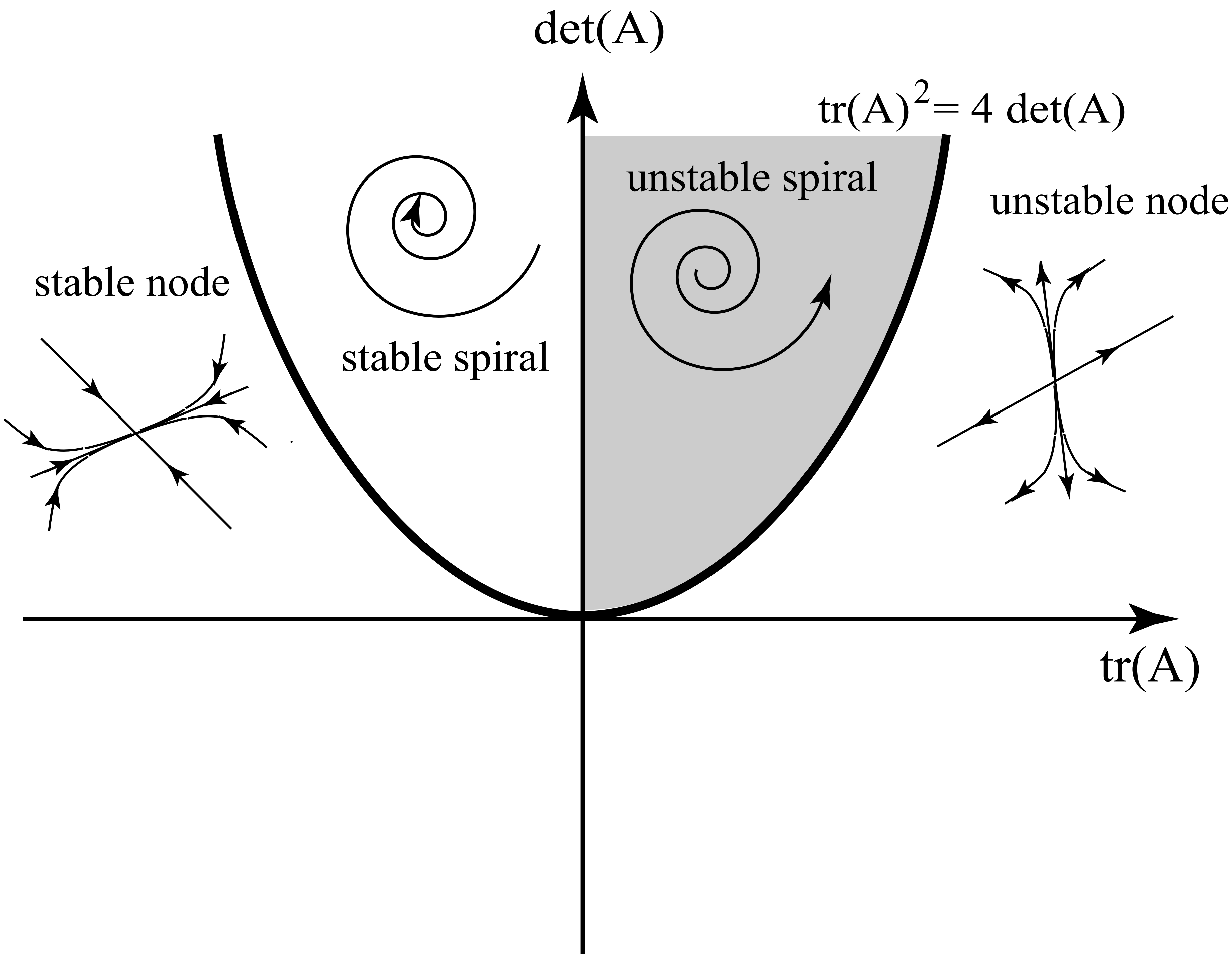

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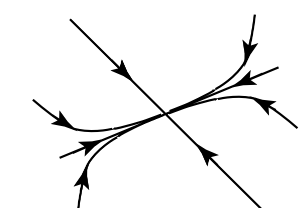

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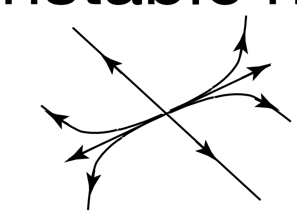

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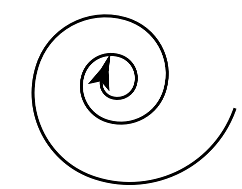

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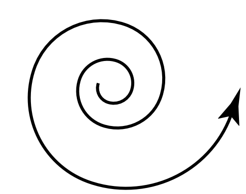

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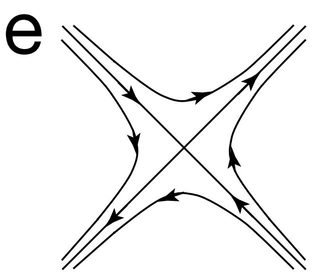

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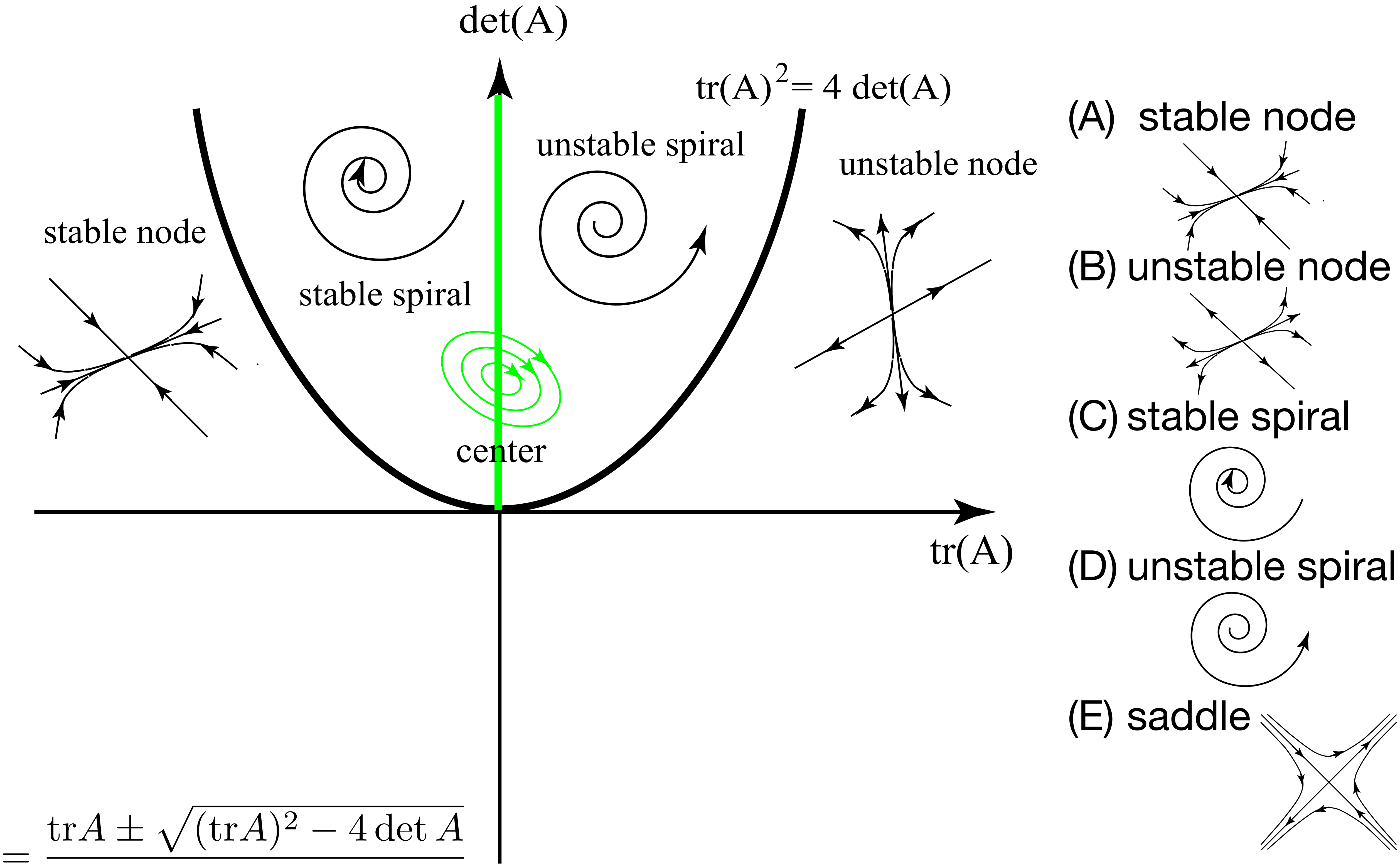

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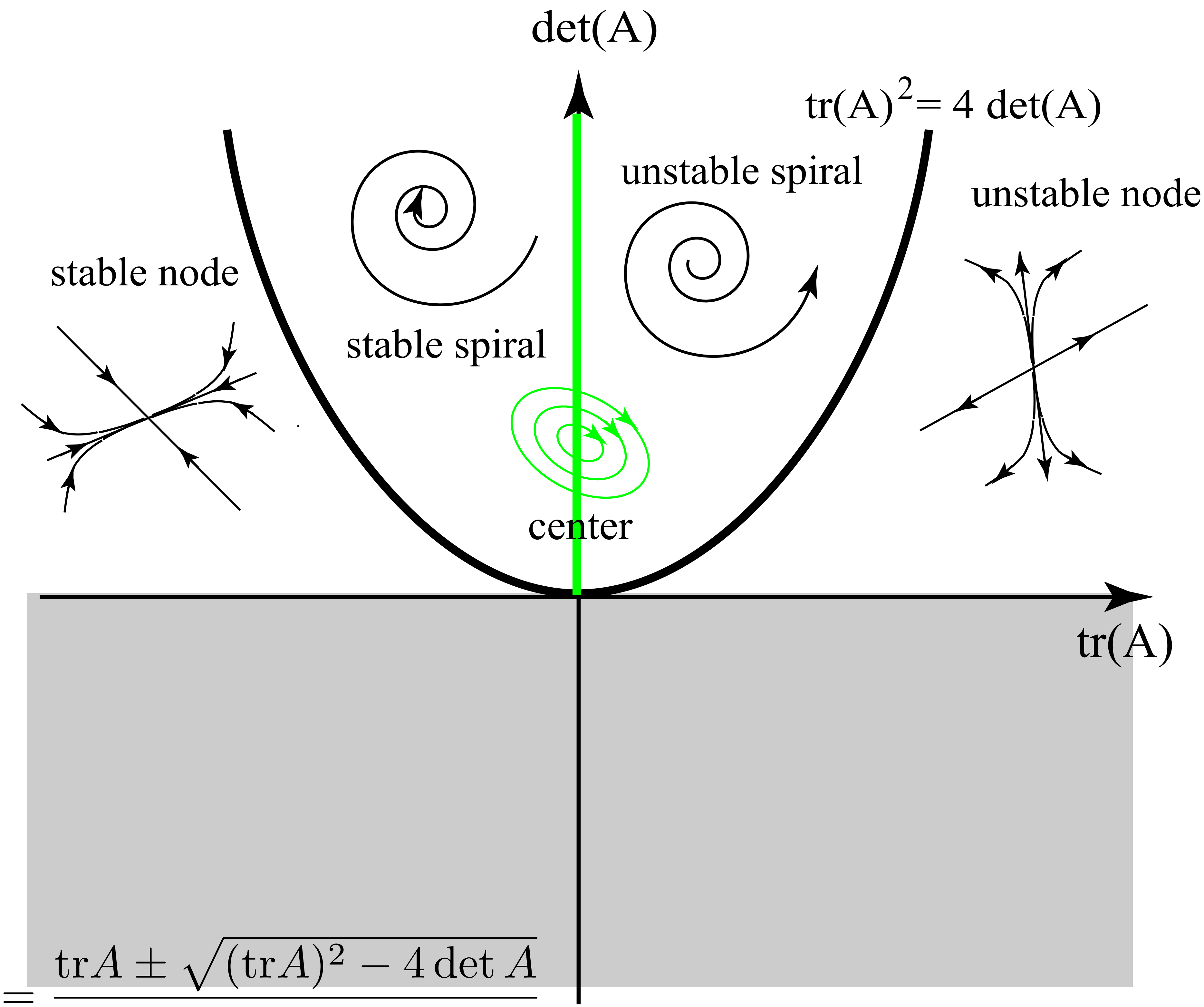

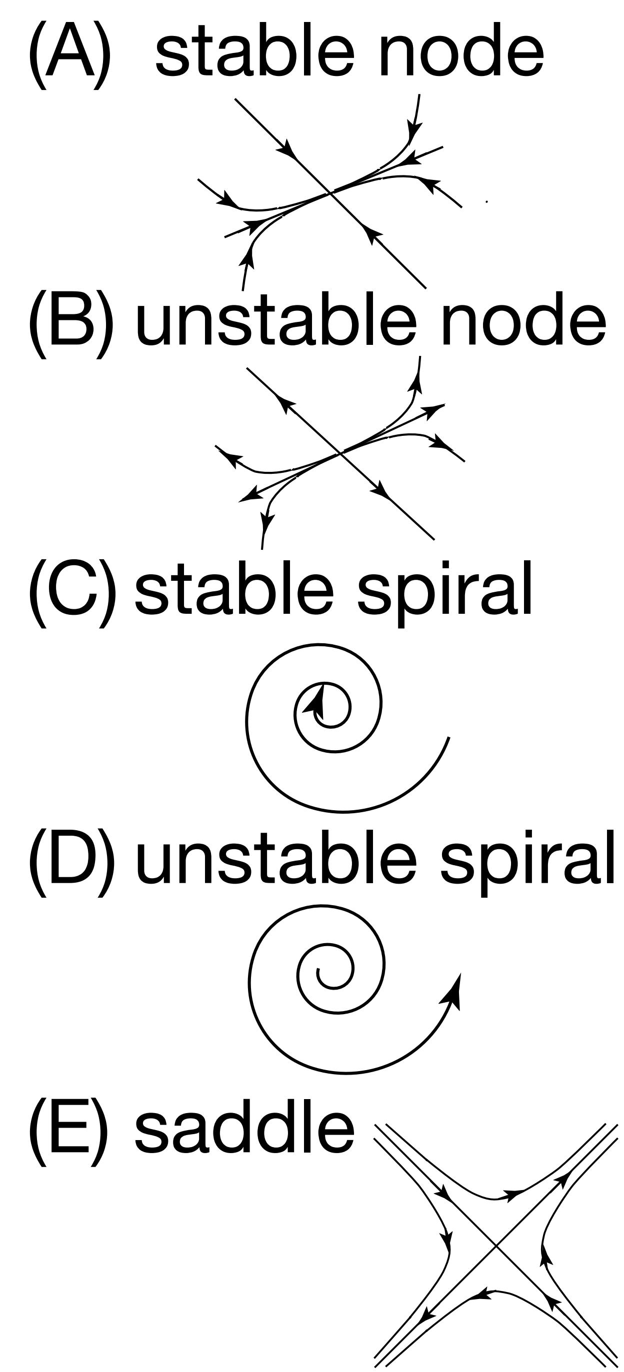

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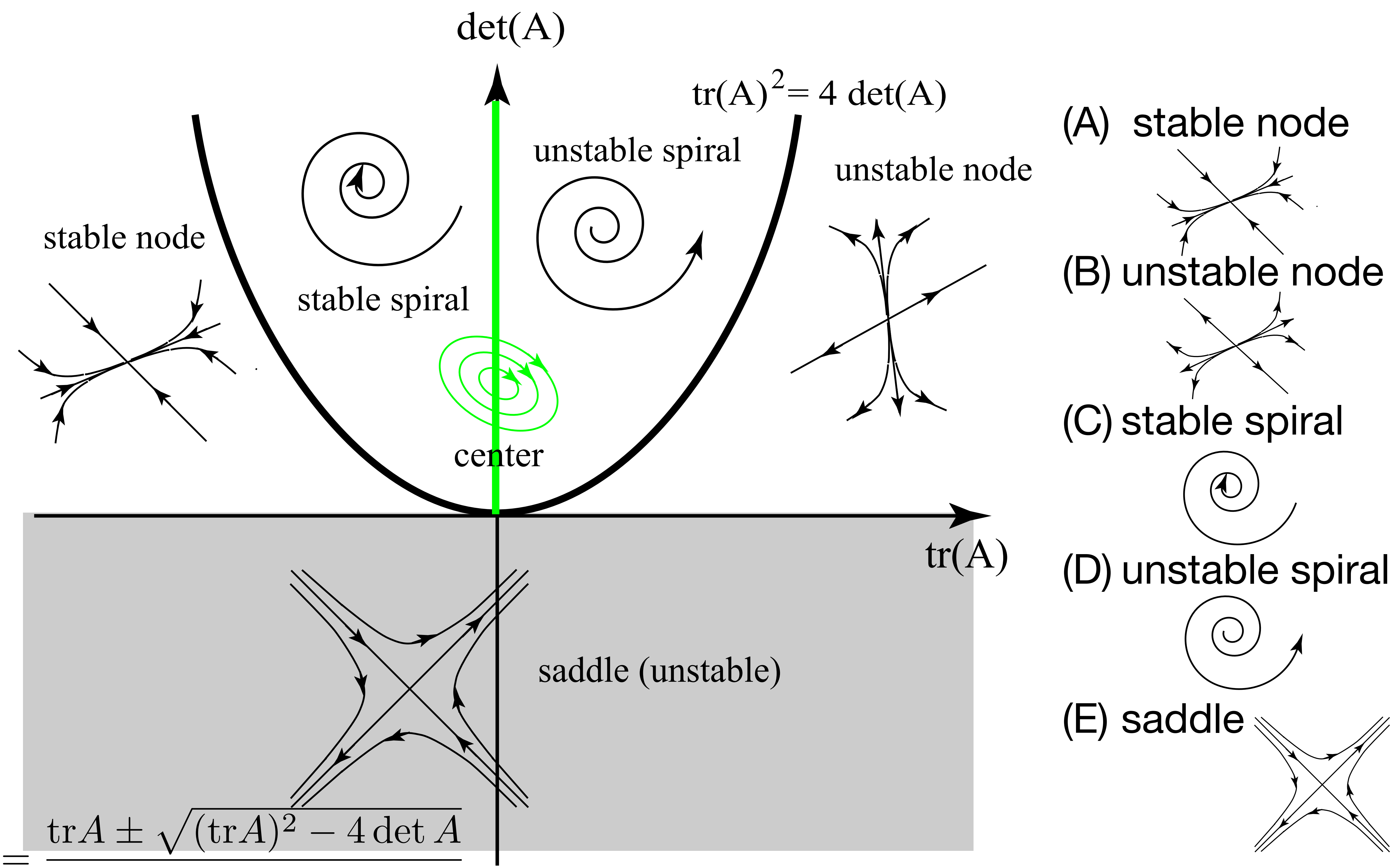

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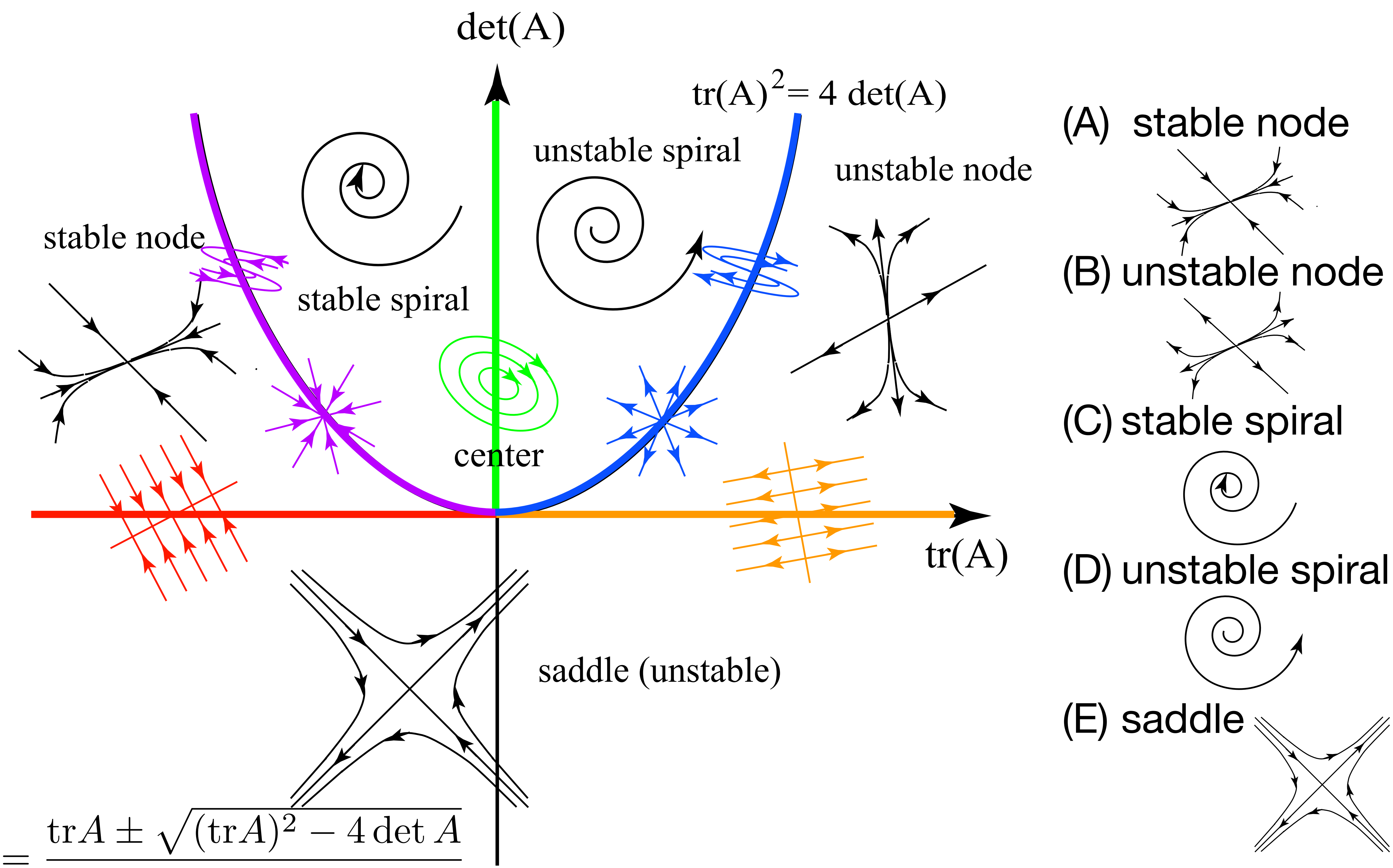

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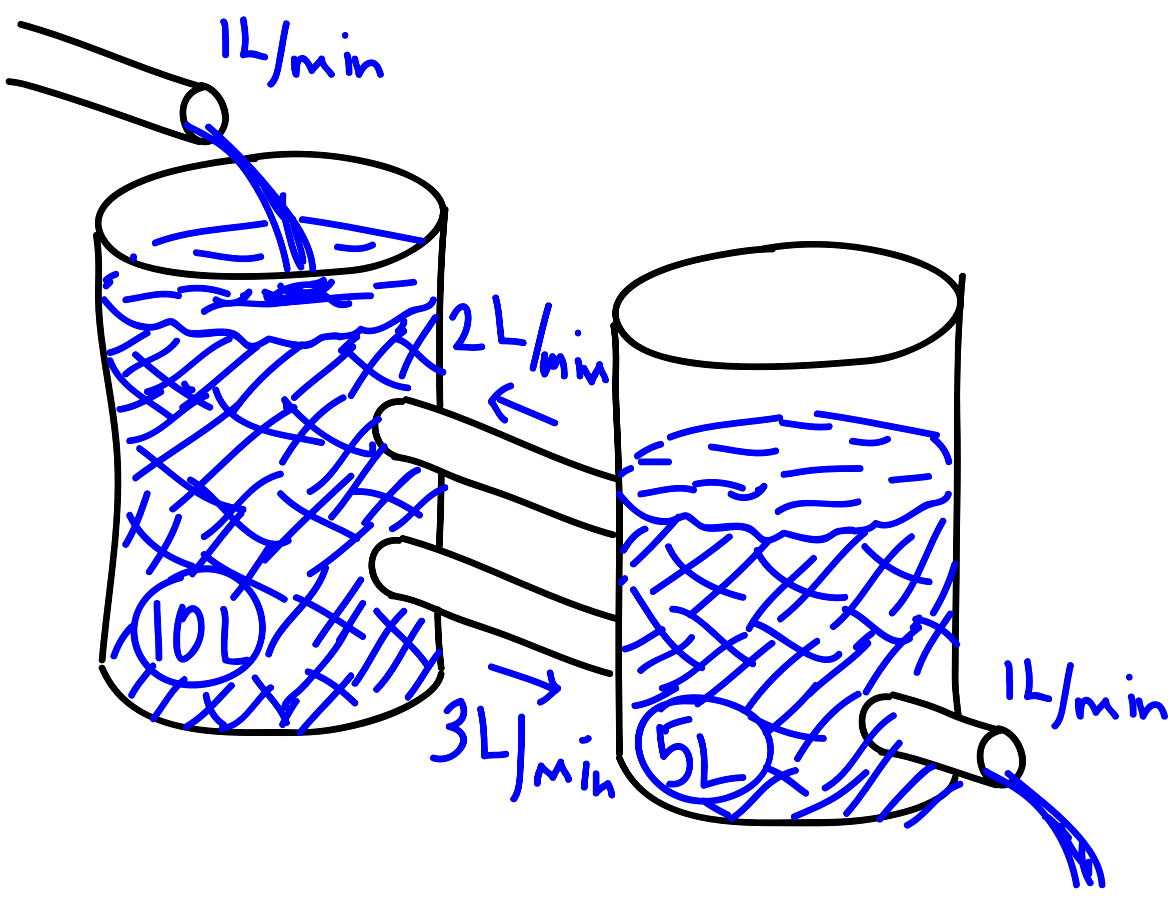
Nonhomogeneous case - example

- Salt water flows into a tank holding 10 L of water at a rate of 1 L/min with a concentration of 200 g/L. The well-mixed solution flows from that tank into a tank holding 5 L through a pipe at 3 L/min. Another pipe takes the solution in the second tank back into the first at a rate of 2 L/min. Finally, solution drains out of the second tank at a rate of 1 L/min.
- Write down a system of equations in matrix form for the mass of salt in each tank.



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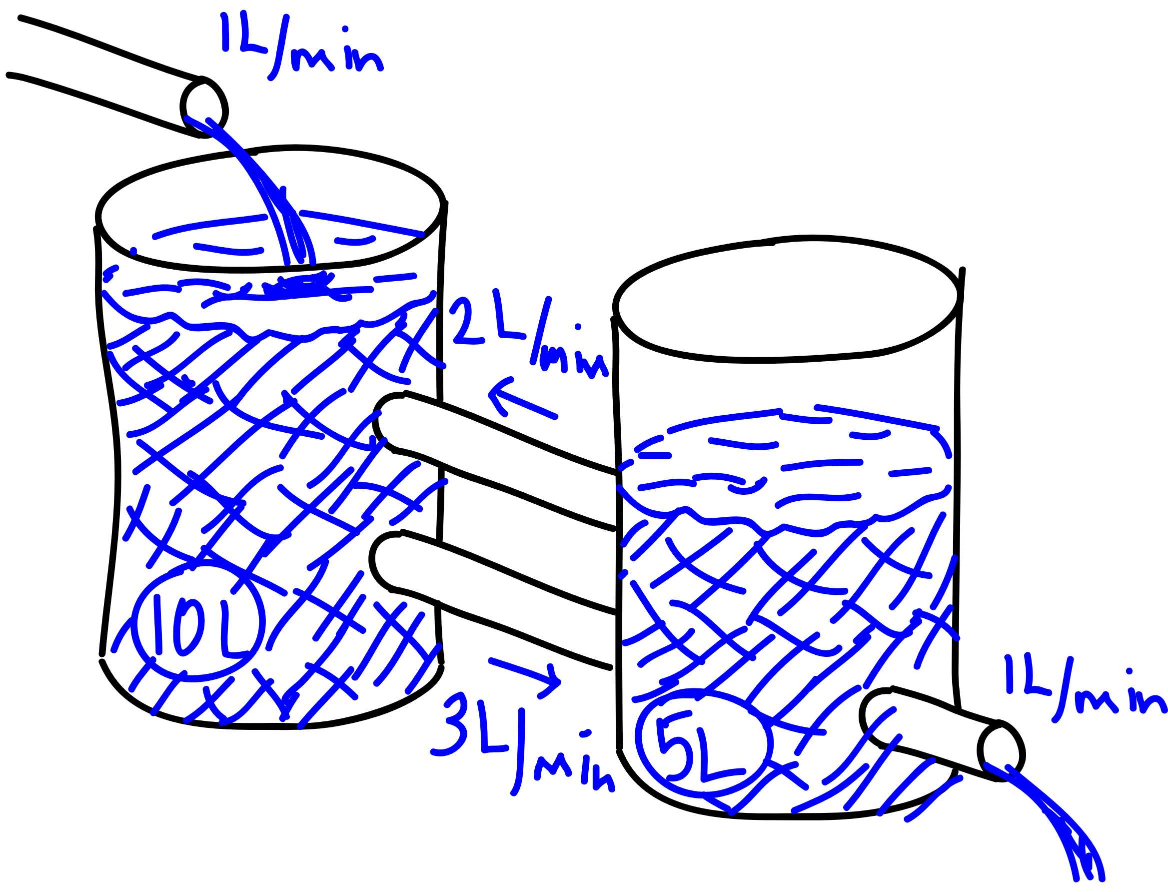
- Write down a system of equations in matrix form for the mass of salt in each tank.

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



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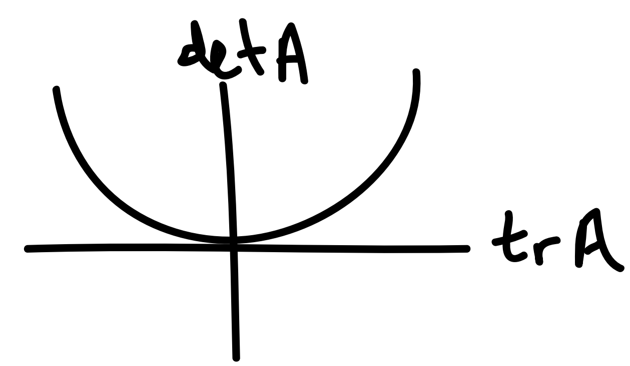

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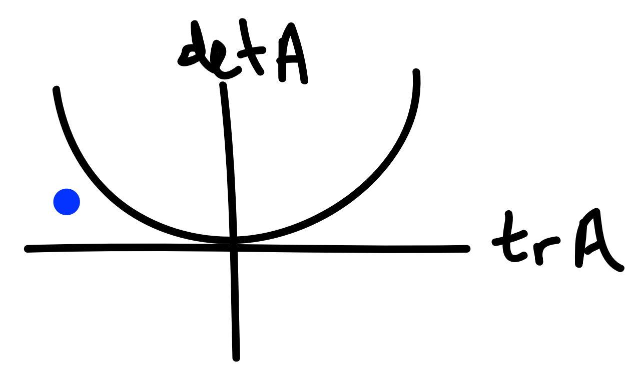

Both evalues negative!

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Nonhomogeneous case - example

- A “Method of undetermined coefficients” similar to what we saw for second order equations can be used for systems.

- For this course, I’ll only test you on constant nonhomogeneous terms (like the previous example) in the context of mixing problems.

Laplace transforms

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  - These can be handled by previous techniques (modified) but it isn't pretty.