Today

- Solving ODEs with forcing terms using Laplace transforms - examples
- Laplace transforms of step functions
- Applications

Solving IVPs using Laplace transforms (6.2)

- With a forcing term, the transformed equation is

$$ay'' + by' + cy = g(t)$$

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transform of homogeneous solution with two degrees of freedom

transform of particular solution

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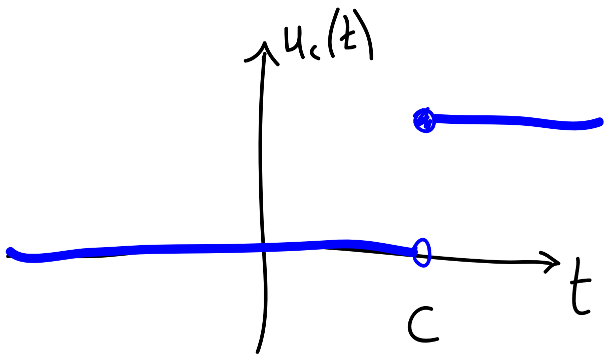
Laplace transforms (so far)

$f(t)$	$F(s)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}f(t)$	$F(s-a)$
$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$

Step function forcing (6.3, 6.4)

- We define the Heaviside function $u_c(t) = \begin{cases} 0 & t < c, \\ 1 & t \geq c. \end{cases}$



- In WW, $u_c(t) = u(t-c) = h(t-a)$

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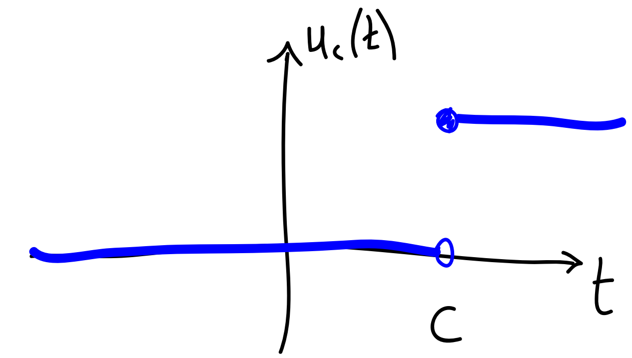

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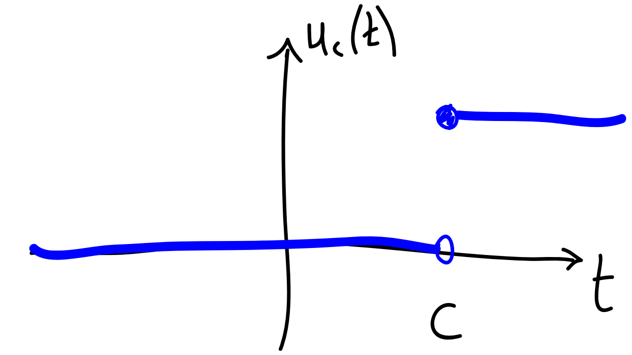

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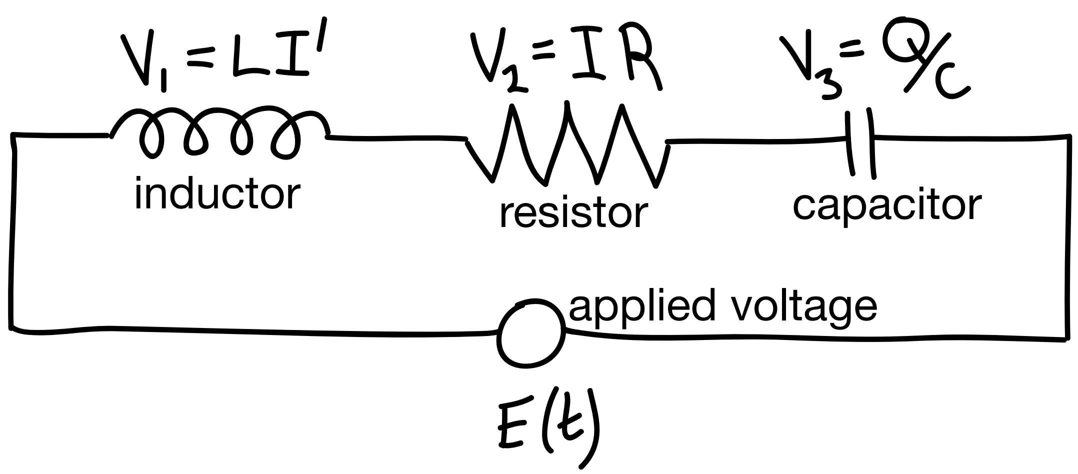
- For example, in LRC circuits, Kirchoff's second law tells us that:



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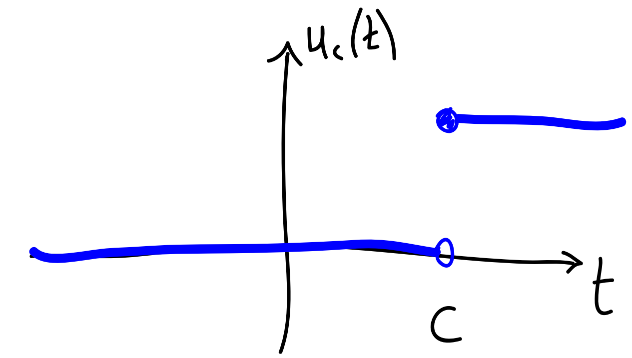

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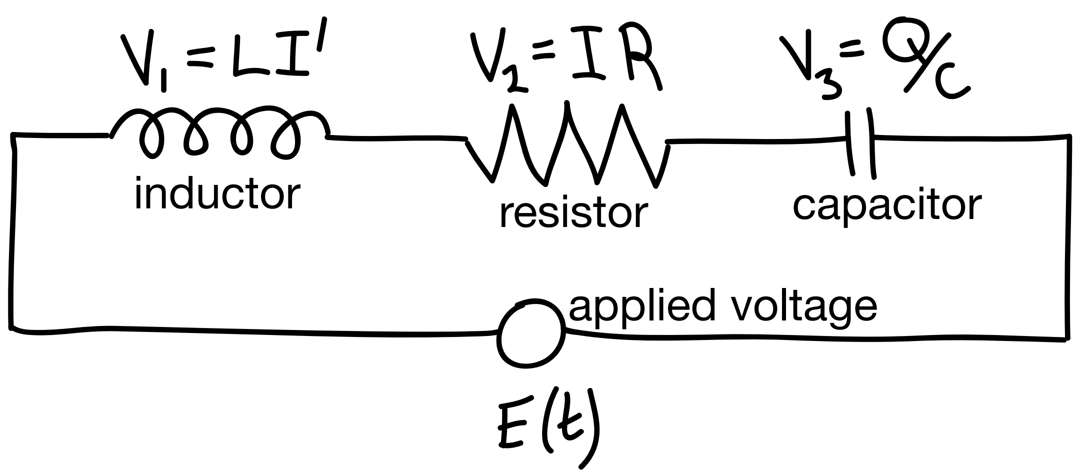

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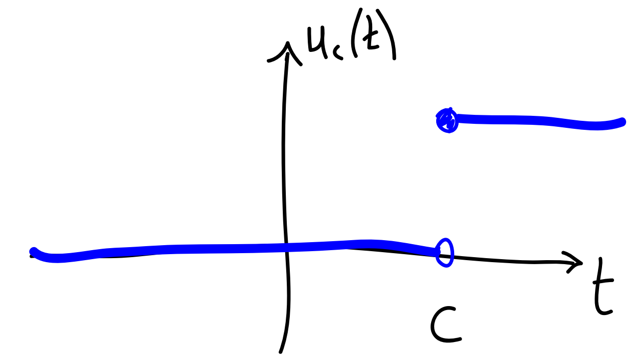

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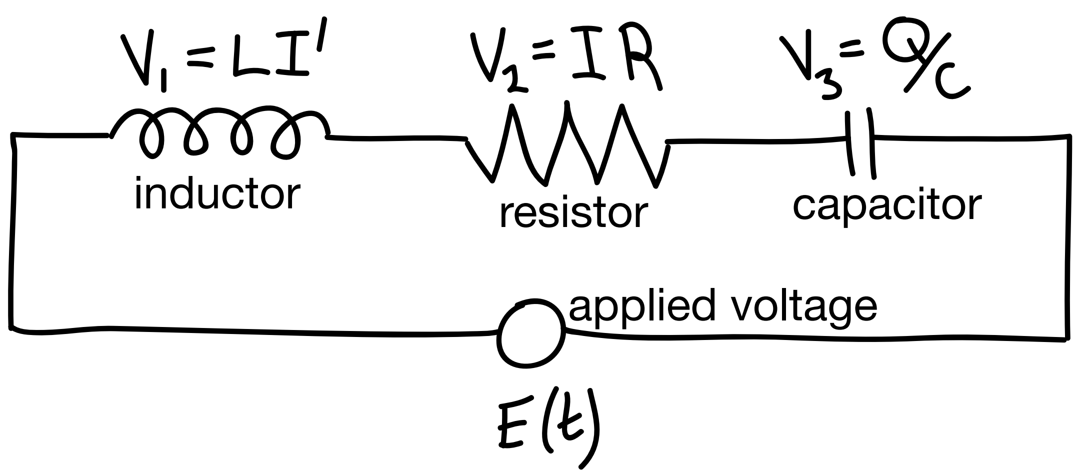

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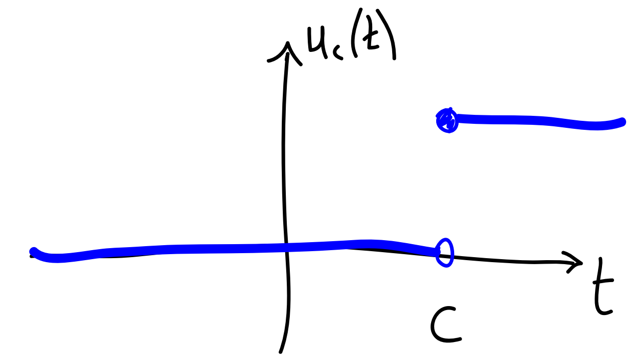

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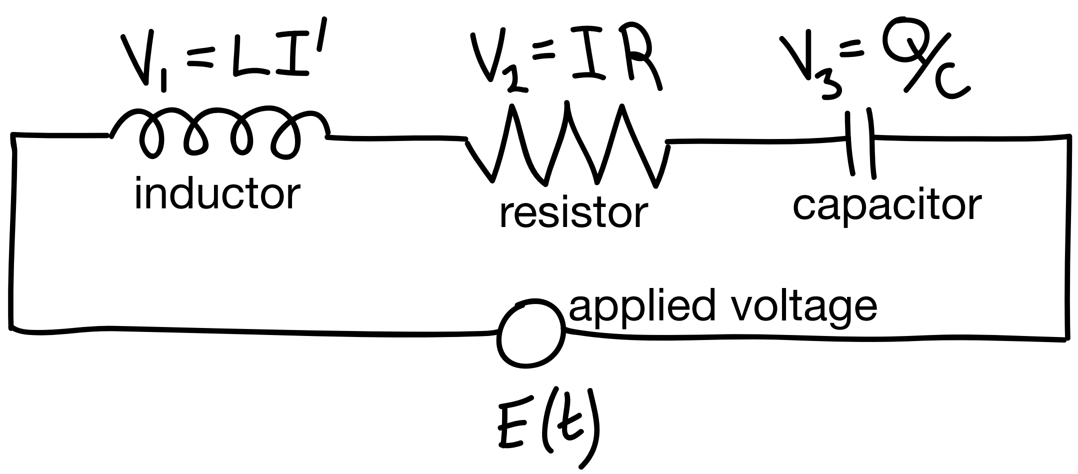

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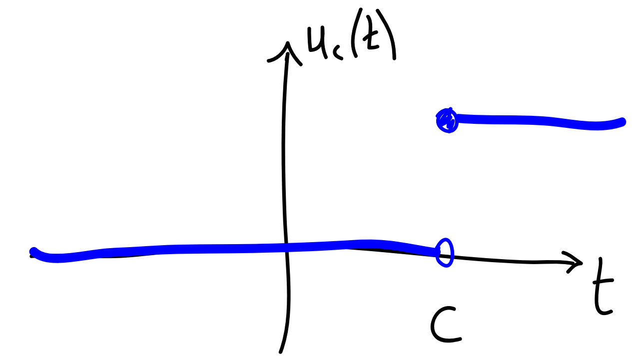

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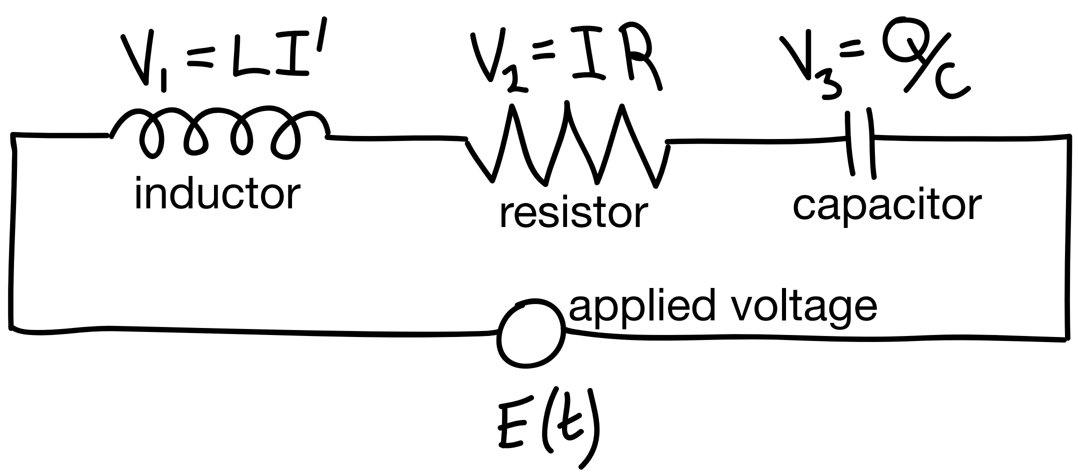

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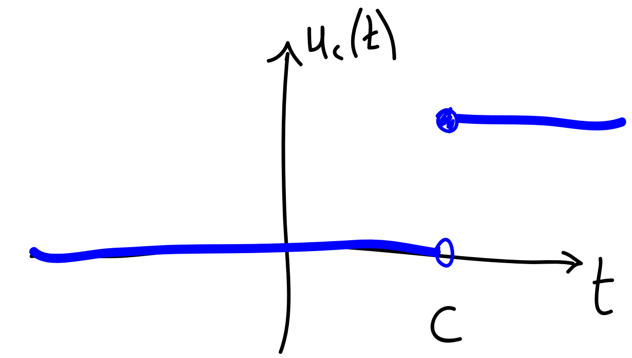

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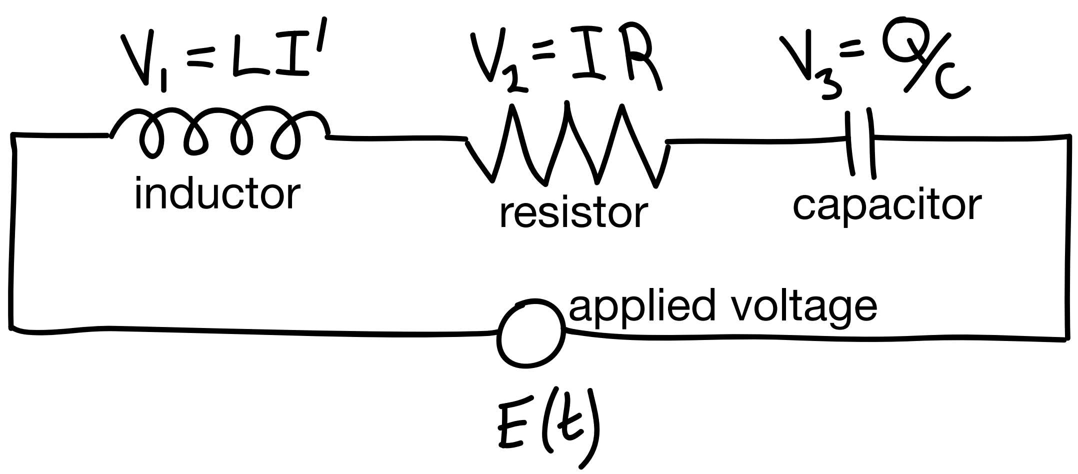

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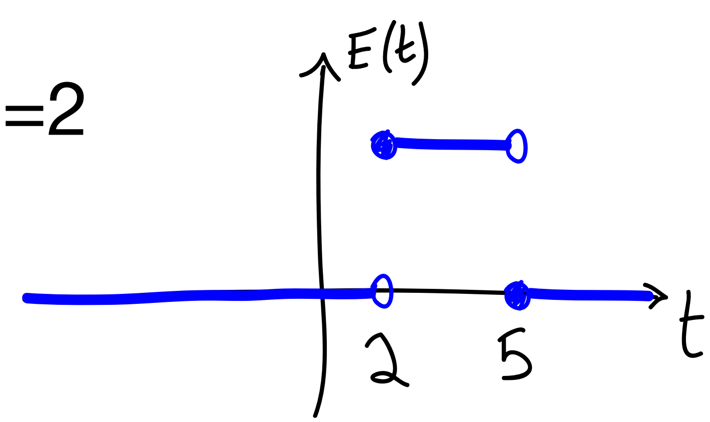
- For example, turn E on at t=2 and off again at t=5:



Step function forcing (6.3, 6.4)

- Use the Heaviside function to rewrite $g(t) = \begin{cases} 0 & \text{for } t < 2 \text{ and } t \geq 5, \\ 1 & \text{for } 2 \leq t < 5. \end{cases}$

(A) $g(t) = u_2(t) + u_5(t)$

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(E) Explain, please.

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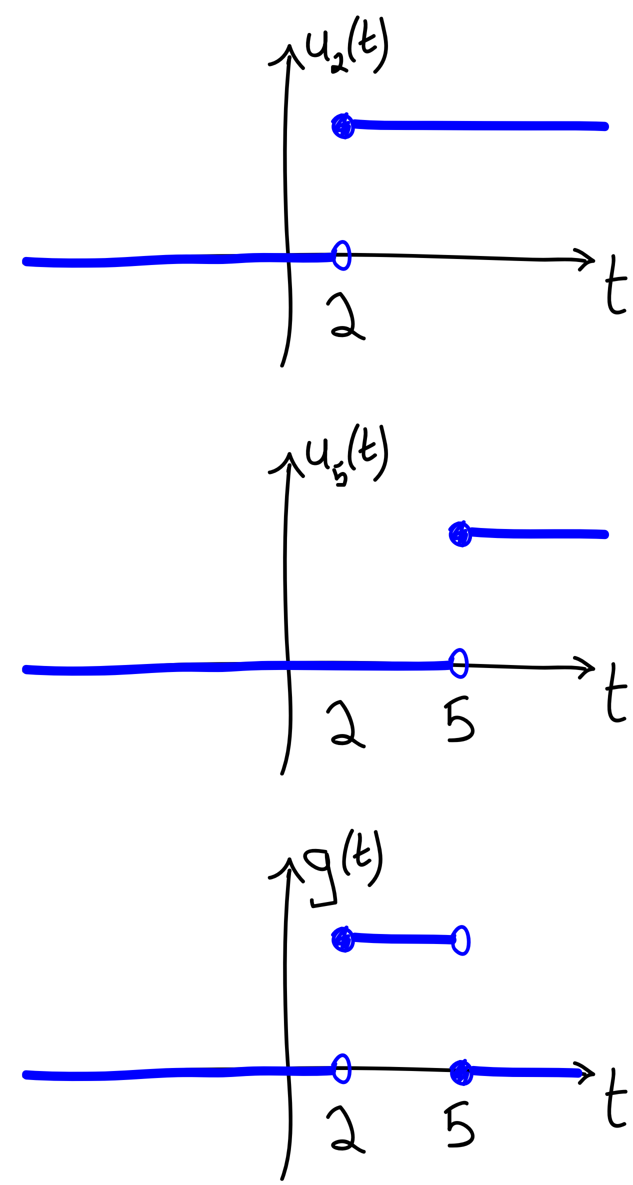

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messier with transforms

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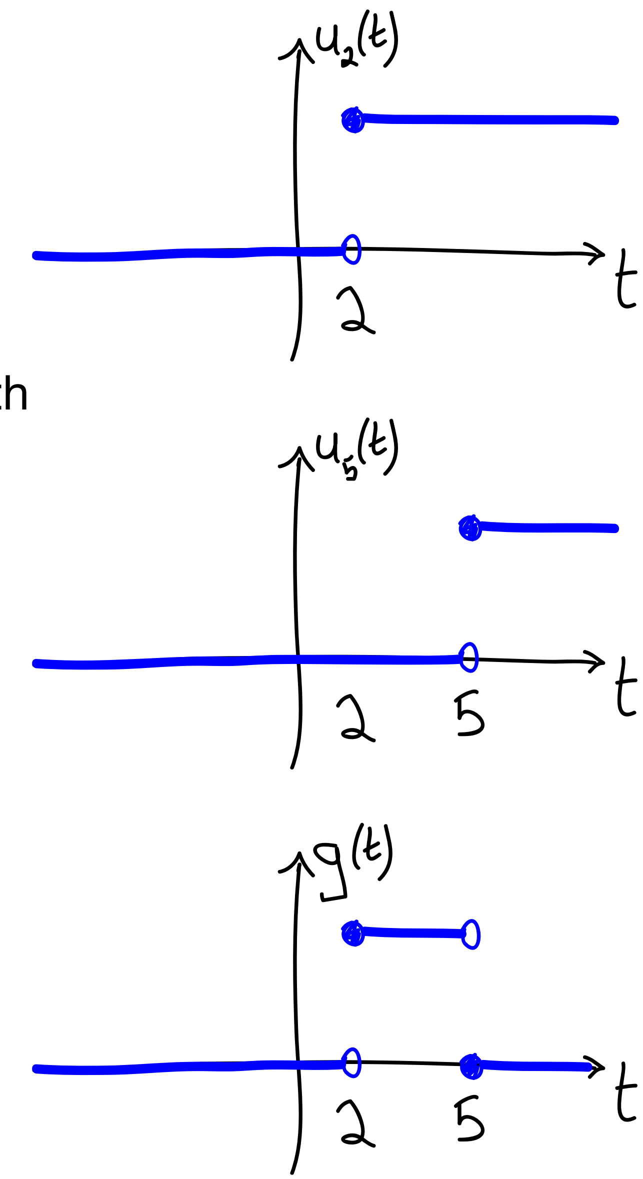

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Recall: $\mathcal{L}\{f(t) + g(t)\} = \int_0^\infty e^{-st}(f(t) + g(t))\ dt$

$$= \int_0^\infty e^{-st} f(t)\ dt + \int_0^\infty e^{-st} g(t)\ dt$$

$$= \mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\}$$

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- Suppose we know the transform of f(t).
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$$k(t) = \begin{cases} 0 & \text{for } t < c, \\ f(t-c) & \text{for } t \geq c. \end{cases}$$

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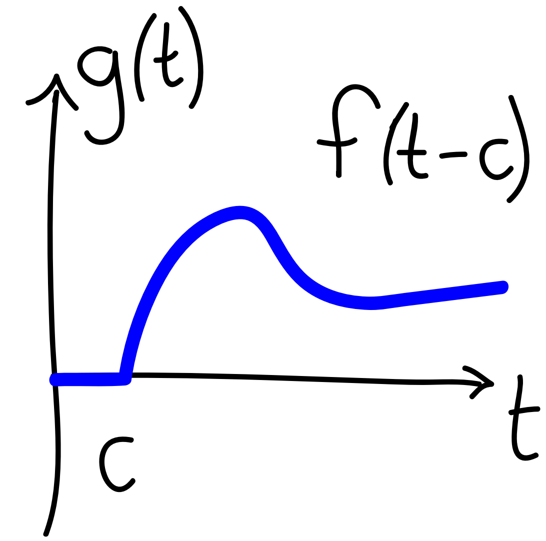

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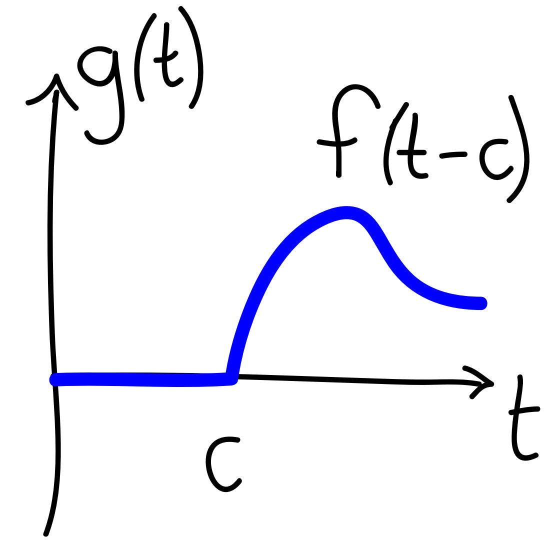

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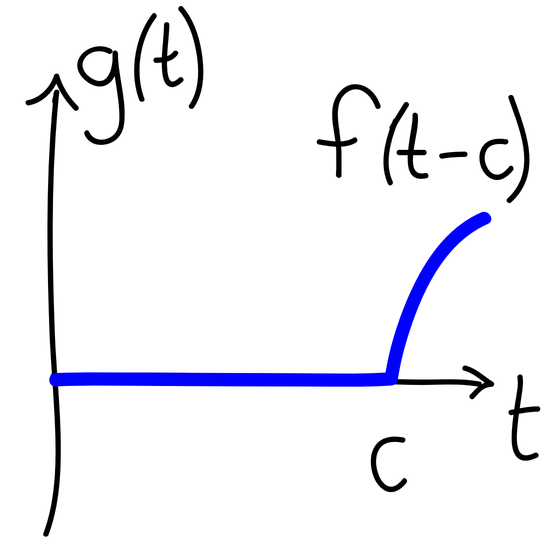

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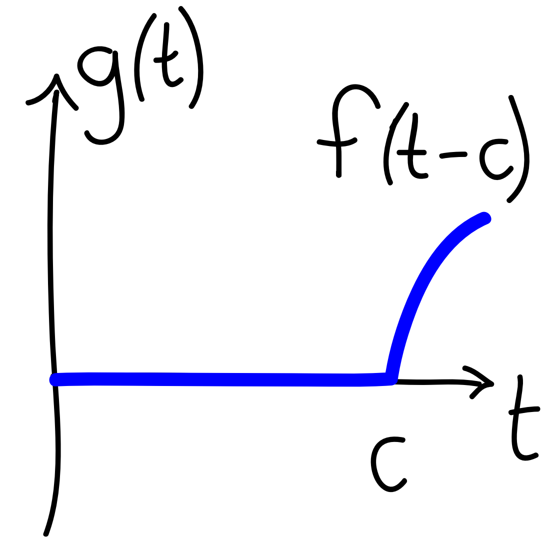

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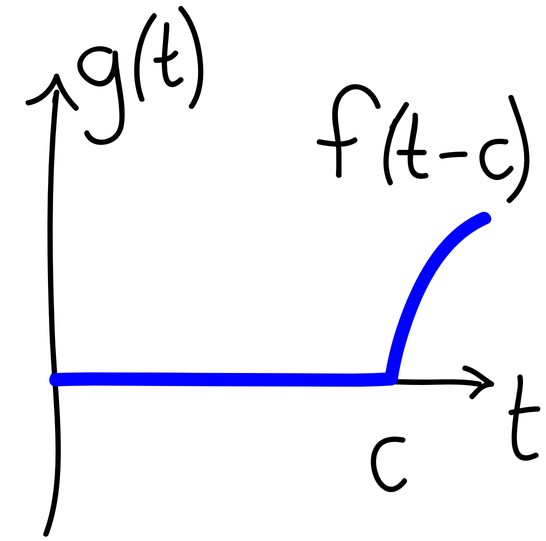

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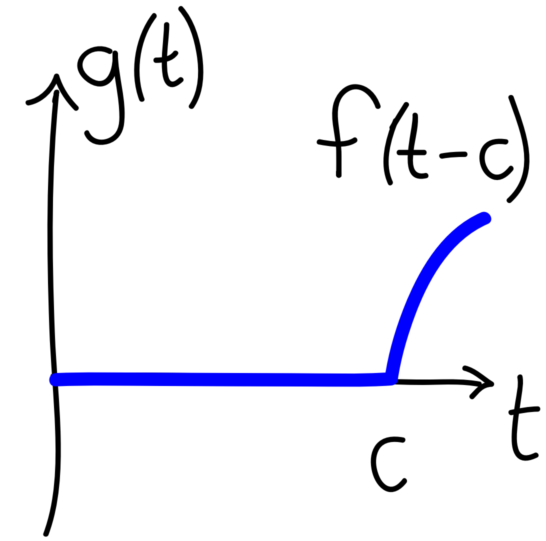

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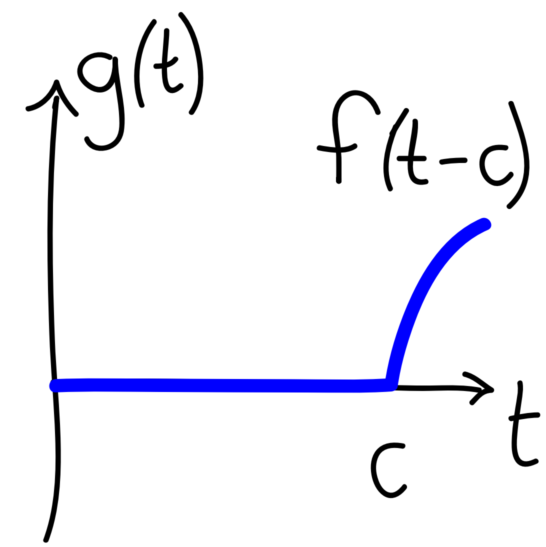

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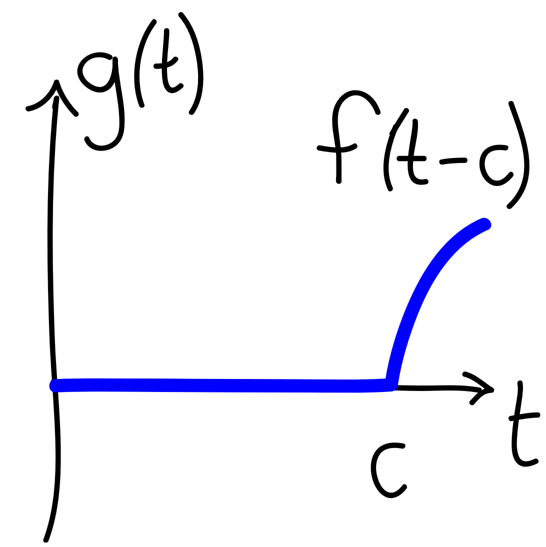

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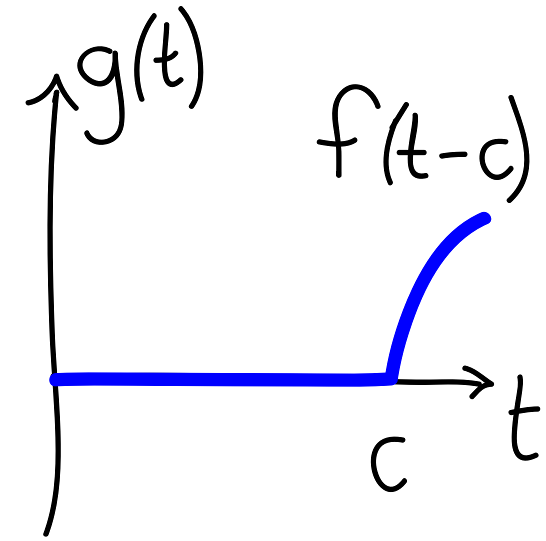

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- Solve using Laplace transforms:

$$y'' + 2y' + 10y = g(t) = \begin{cases} 0 & \text{for } t < 2 \text{ and } t \geq 5, \\ 1 & \text{for } 2 \leq t < 5. \end{cases}$$

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- So we just need h(t) and we're done.

Step function forcing (6.3, 6.4)

- Inverting H(s) to get h(t): $H(s) = \dfrac{1}{s(s^2 + 2s + 10)}$

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Partial fraction decomposition!

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(on the blackboard...)

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Step function forcing (6.3, 6.4)

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Partial fraction decomposition!

- Does $s^2 + 2s + 10$ factor? No real factors.

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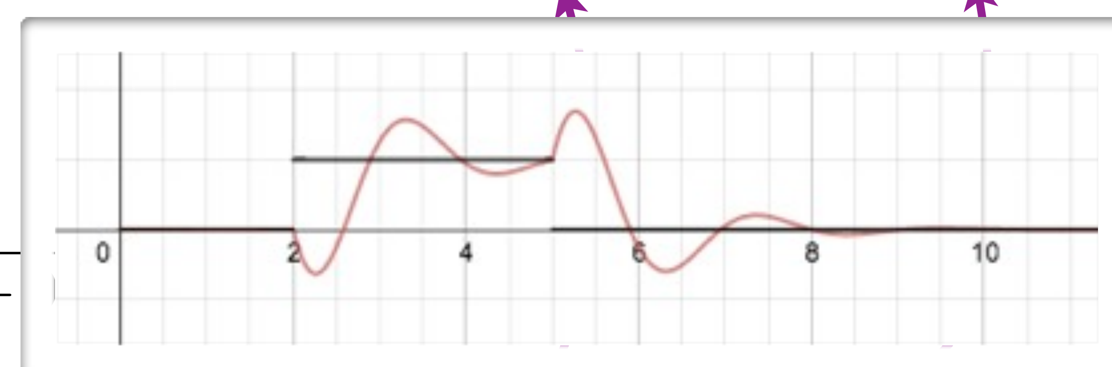

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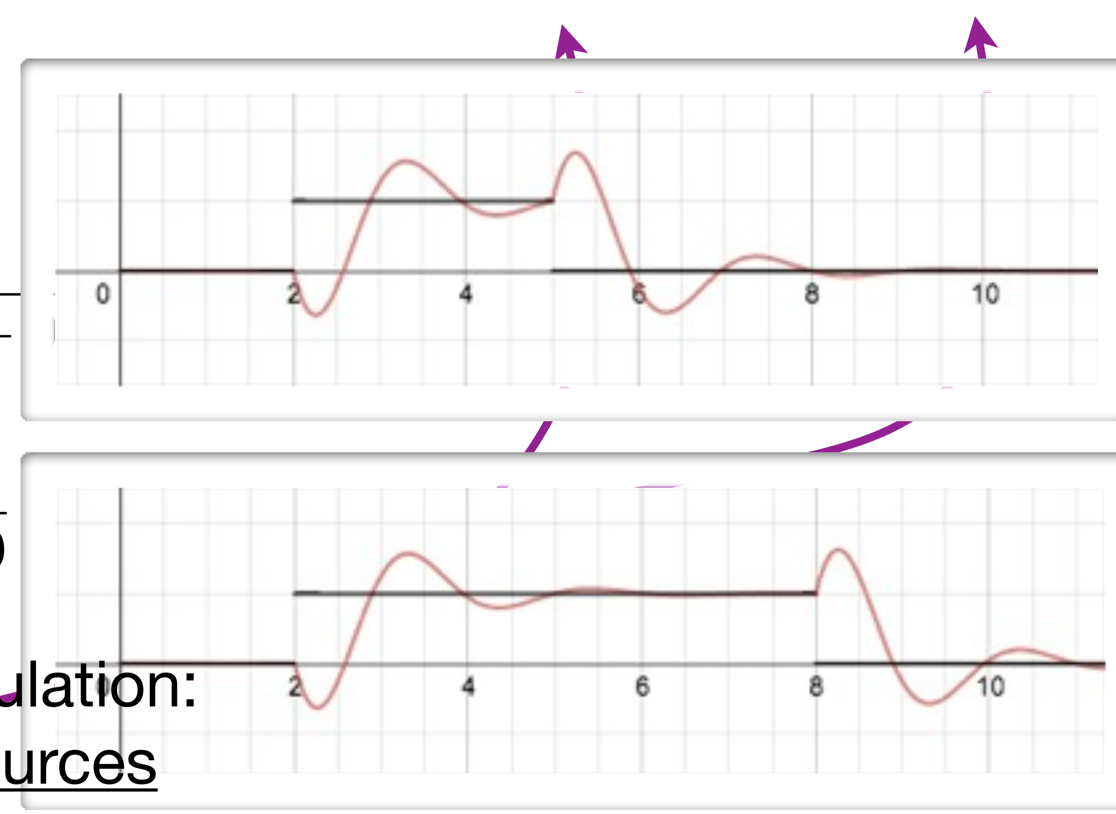

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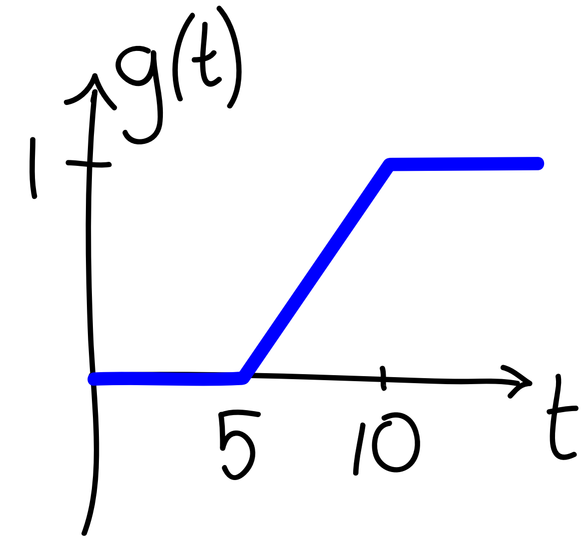
Step function forcing (6.3, 6.4)

- An example with a ramped forcing function:

$$y'' + 4y = \begin{cases} 0 & \text{for } t < 5, \\ \frac{t-5}{5} & \text{for } 5 \le t < 10, \\ 1 & \text{for } t \ge 10. \end{cases}$$

$$y(0) = 0, \;\; y'(0) = 0.$$



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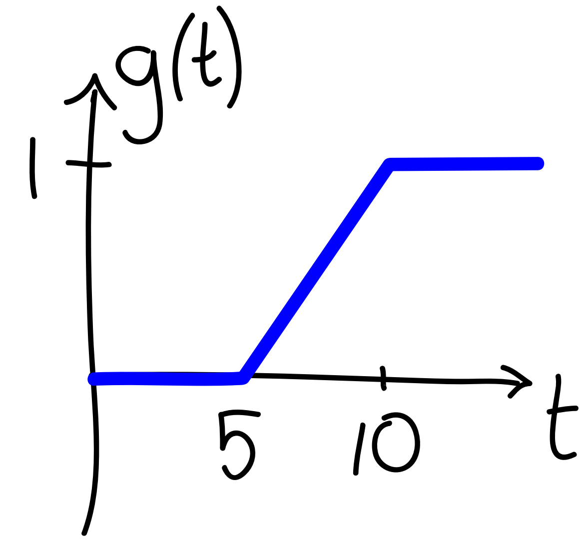

- Write g(t) is terms of $u_c(t)$:

(A) $g(t) = u_5(t) - u_{10}(t)$

(B) $g(t) = u_5(t)(t-5) - u_{10}(t)(t-5)$

(C) $g(t) = (u_5(t)(t-5) - u_{10}(t)(t-10))/5$

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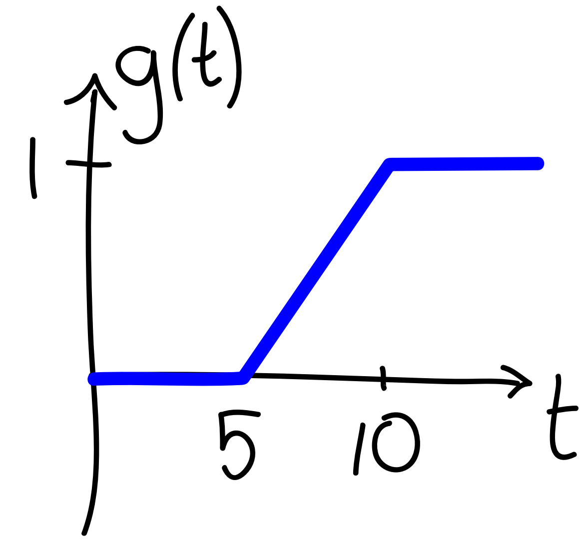

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