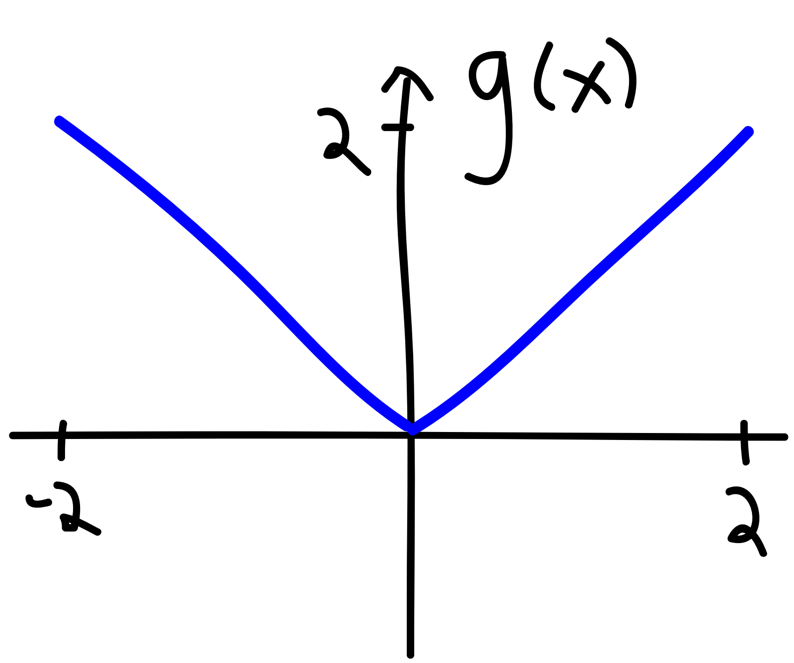
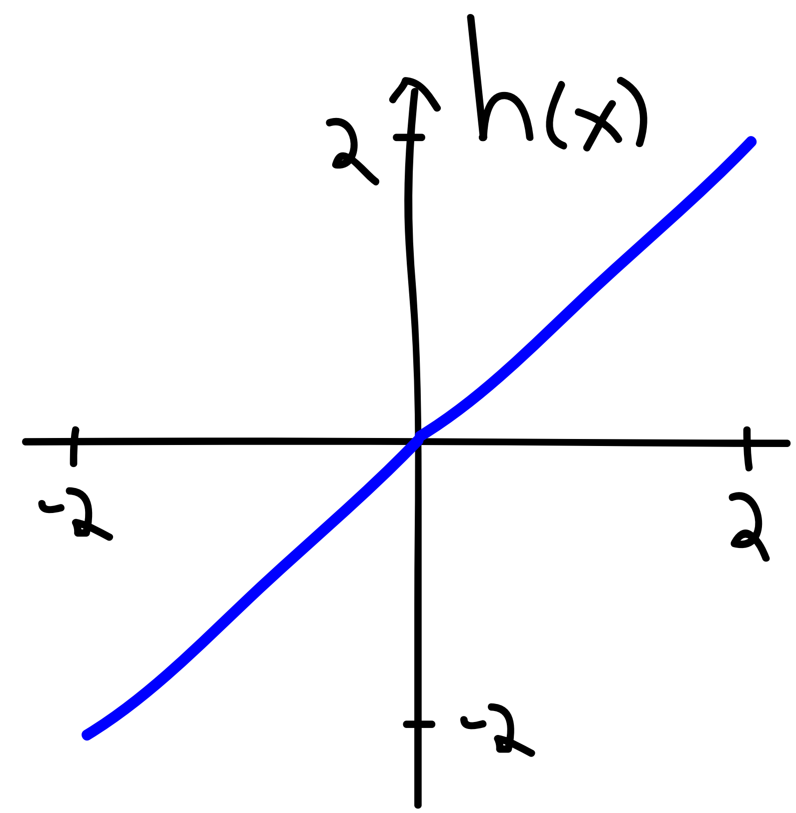
Today

- Fourier Series examples - even and odd extensions, other symmetries
- Using Fourier Series to solve the Diffusion Equation

Examples - calculate the Fourier Series





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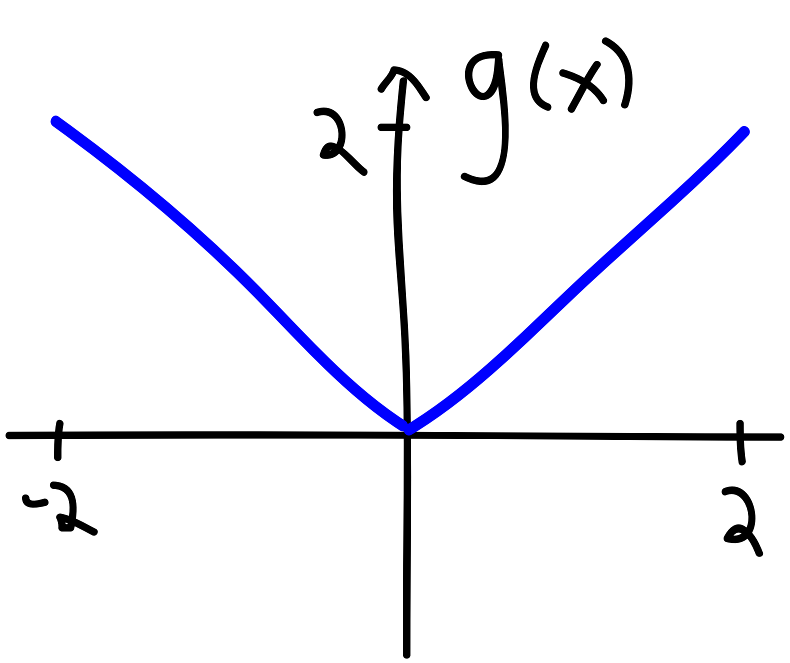

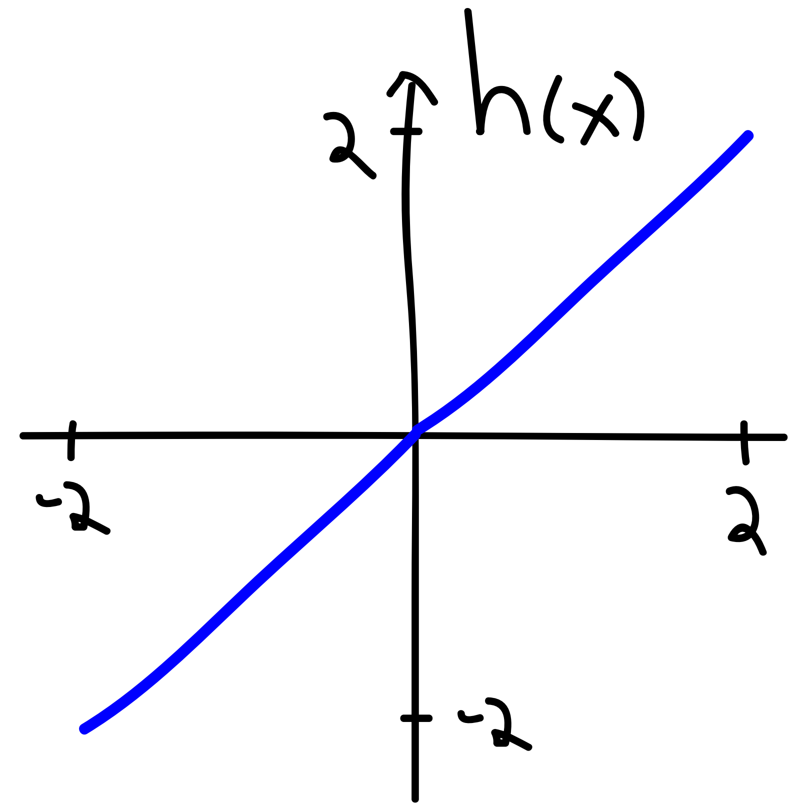

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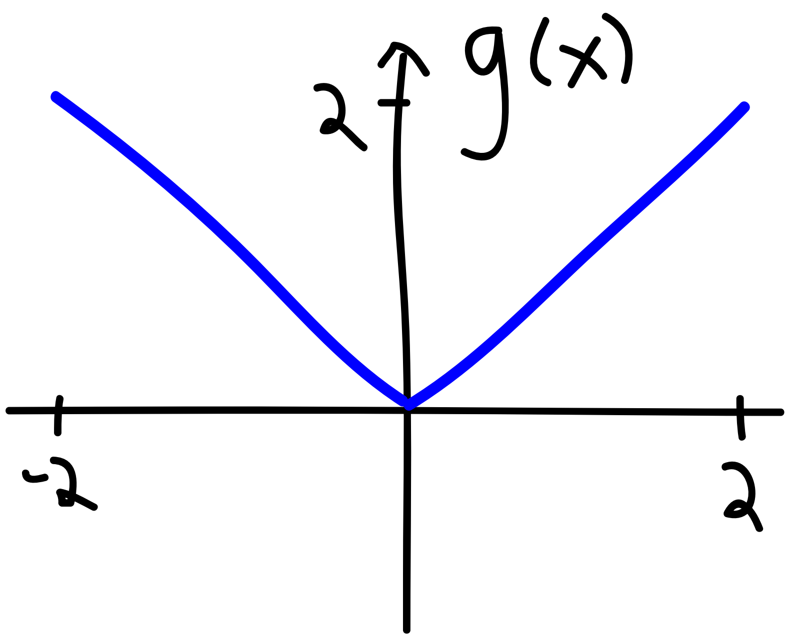

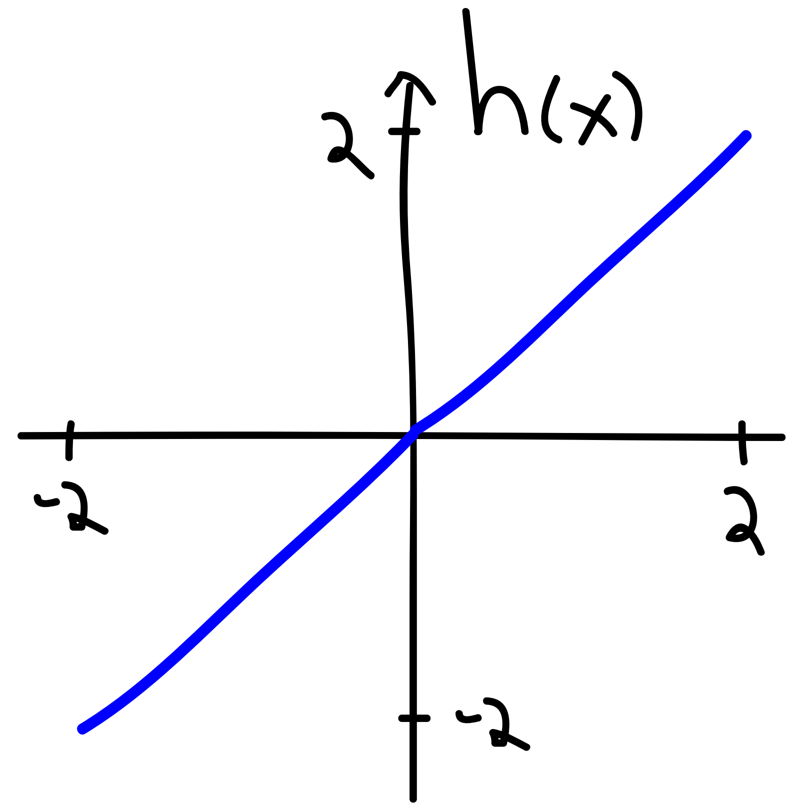

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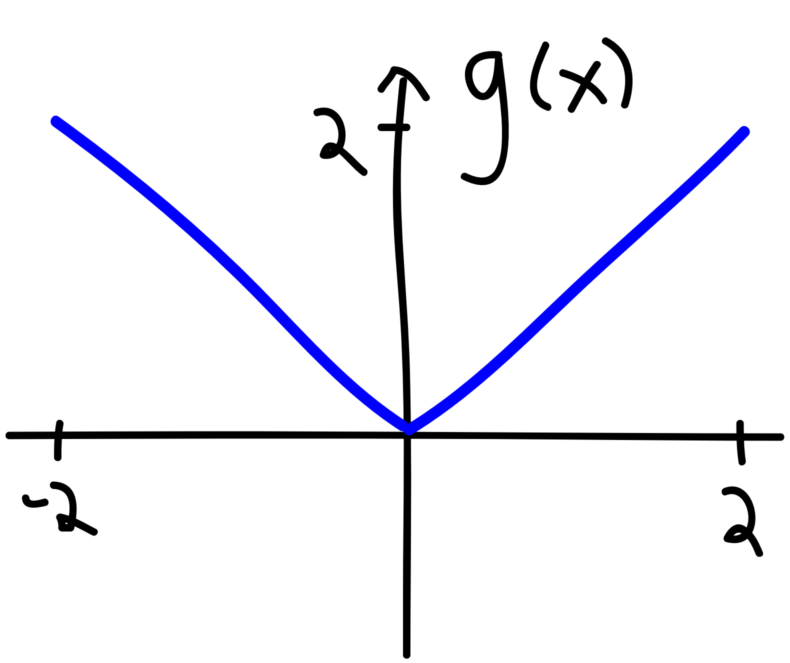

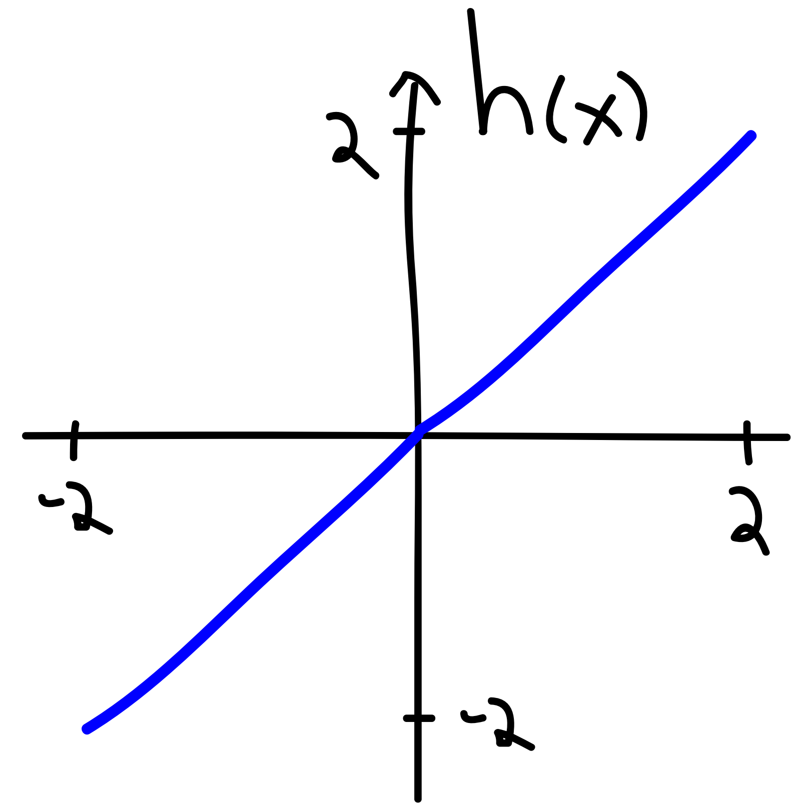

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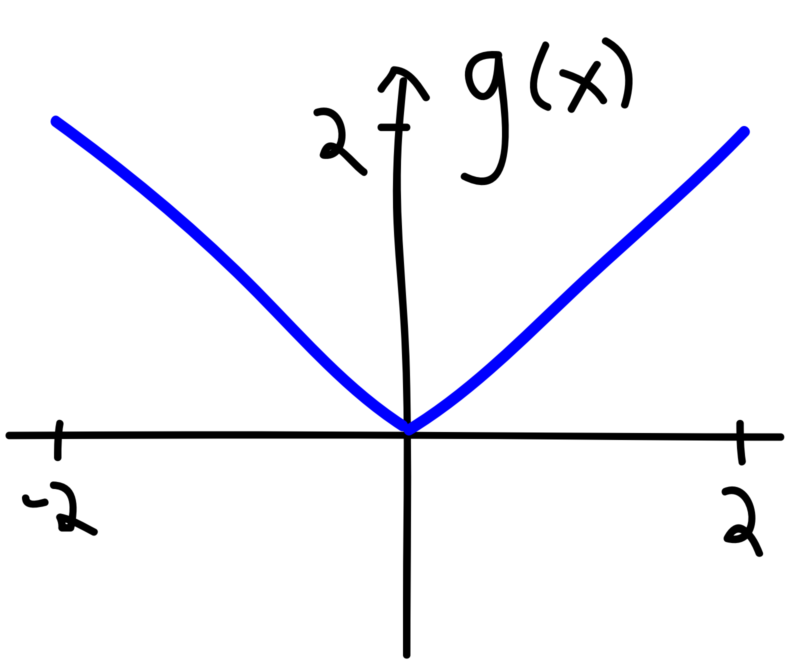

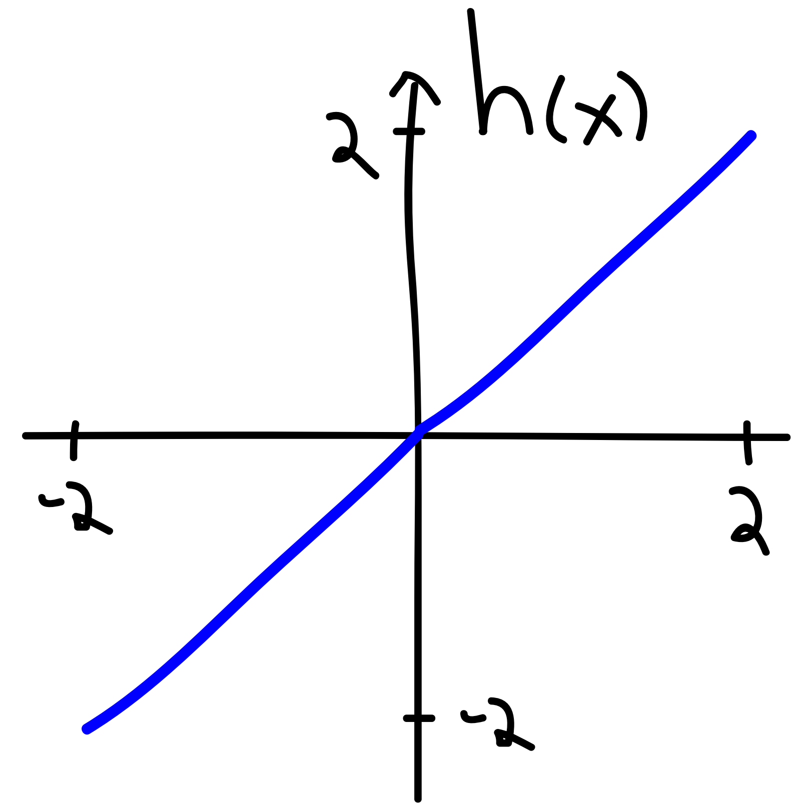

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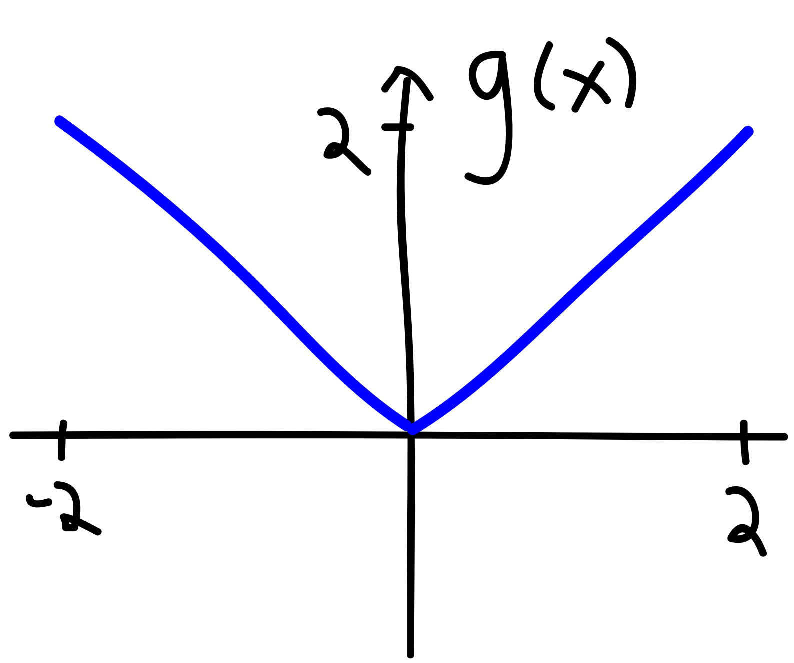

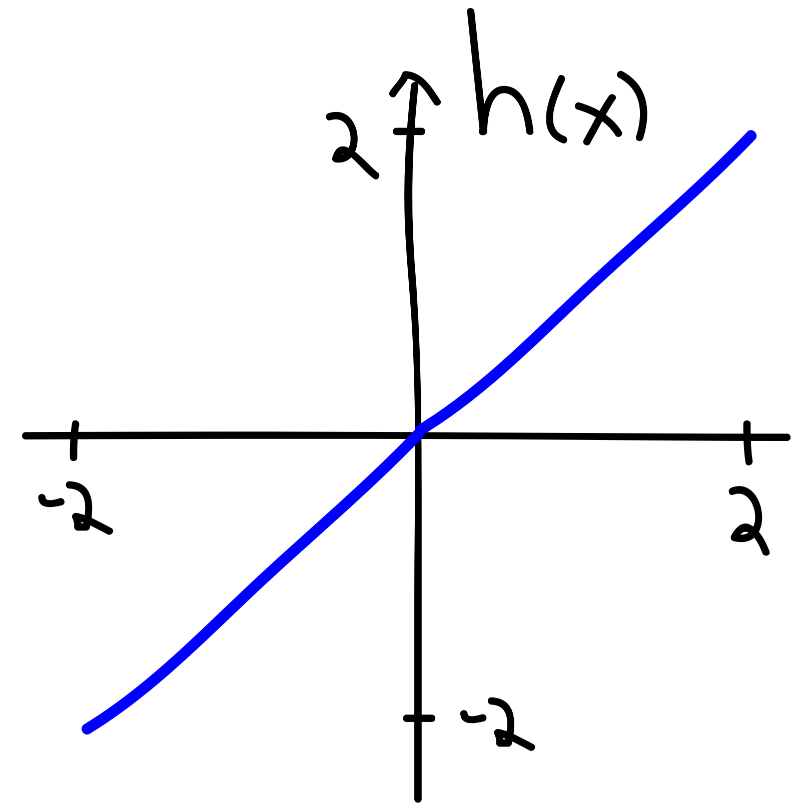

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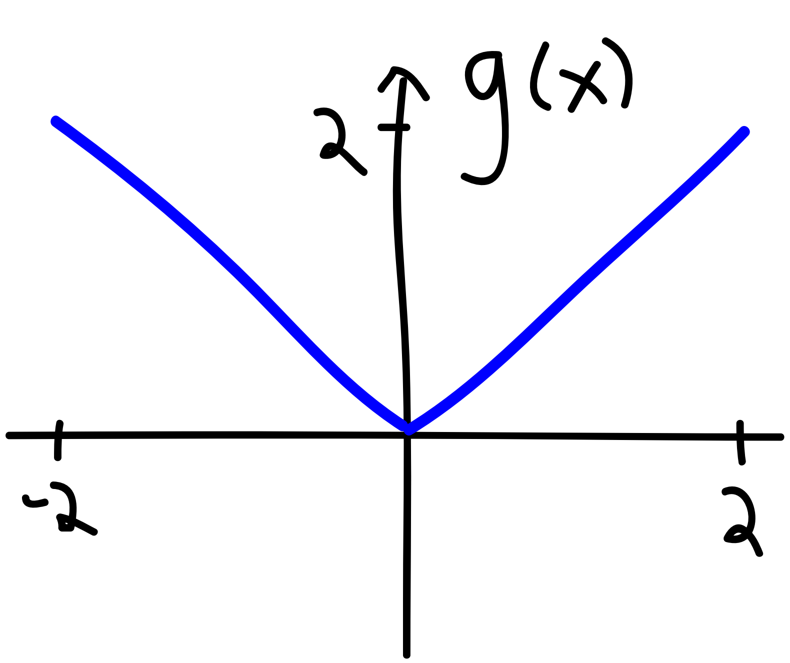

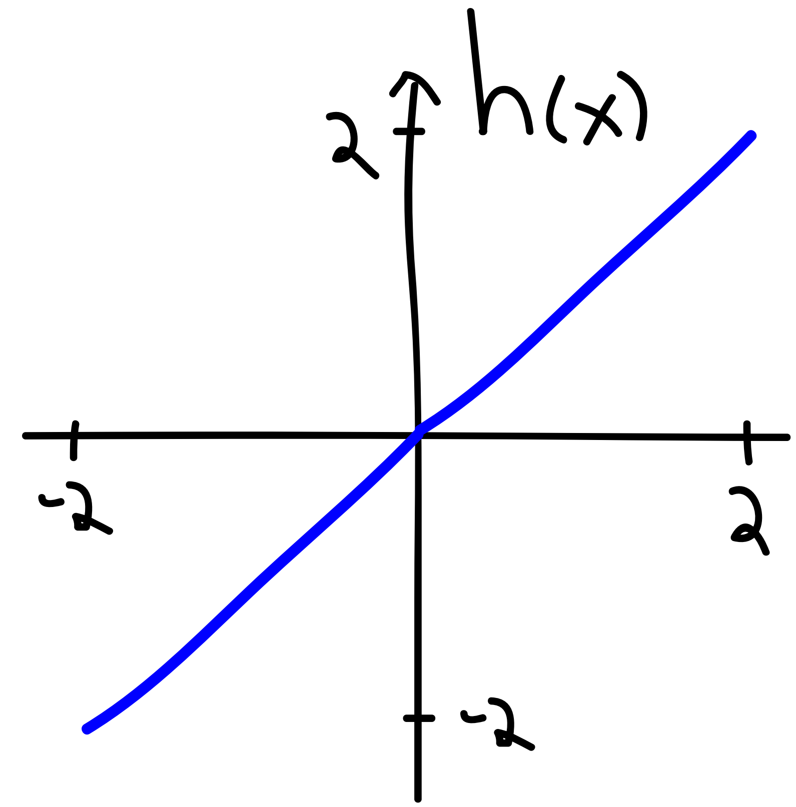

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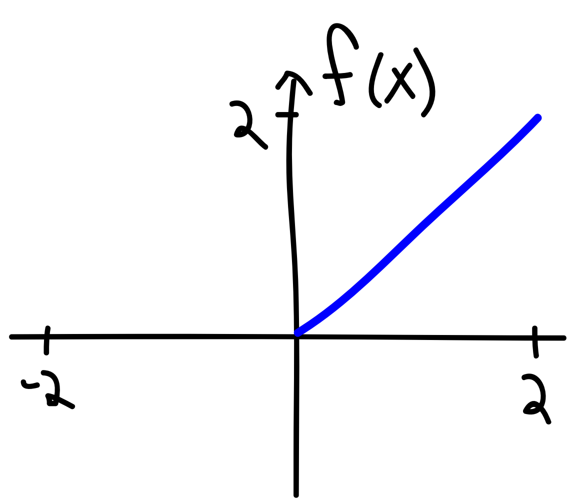
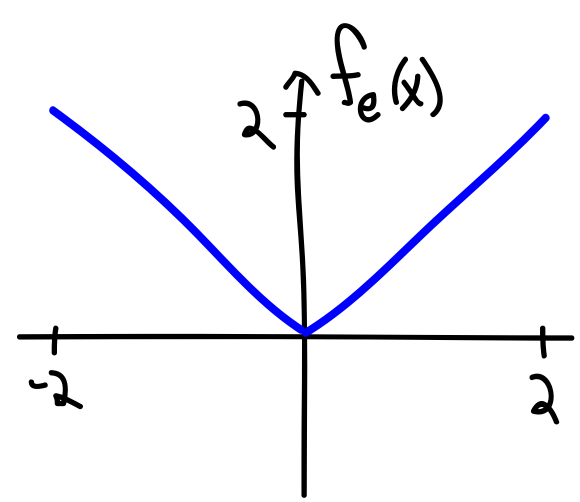
$$h(x) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin\frac{n\pi x}{2}$$

Even and odd extensions

- For a function *f(x)* defined on [0,L], the even extension of *f(x)* is the function

$$f_e(x) = \begin{cases} f(x) & \text{for } 0 \le x \le L, \\ f(-x) & \text{for } -L \le x < 0. \end{cases}$$





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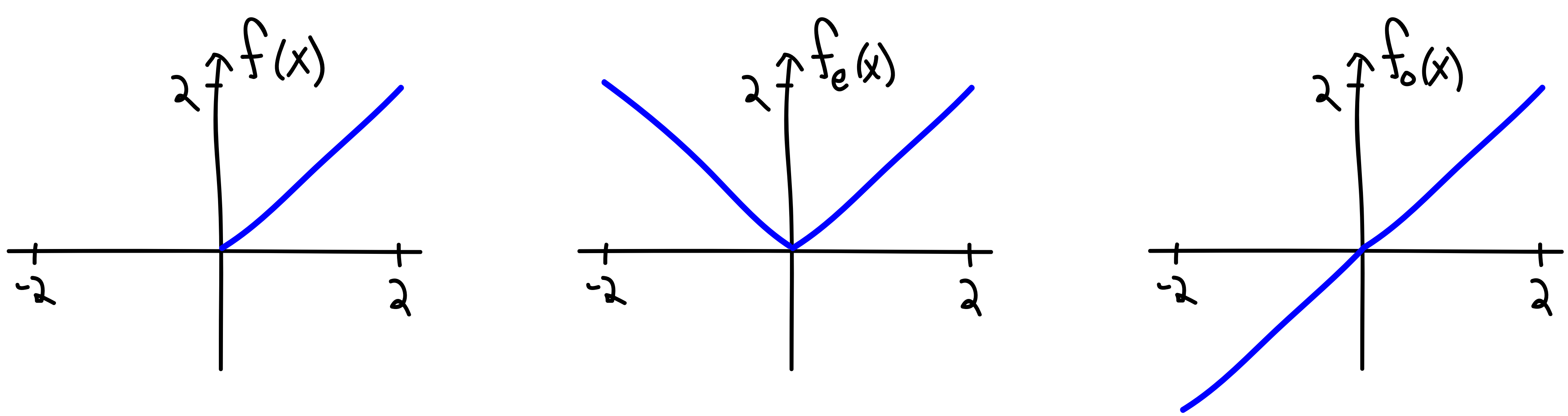

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Fourier Series for functions with other symmetries

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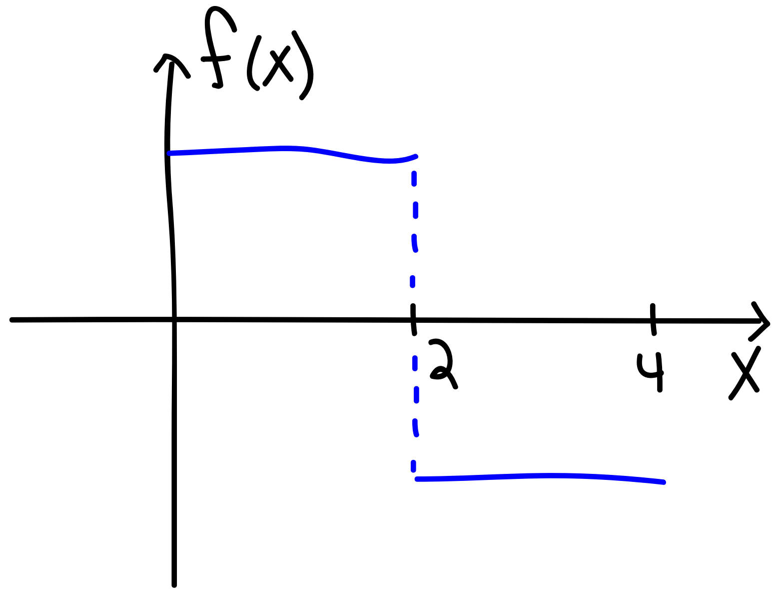

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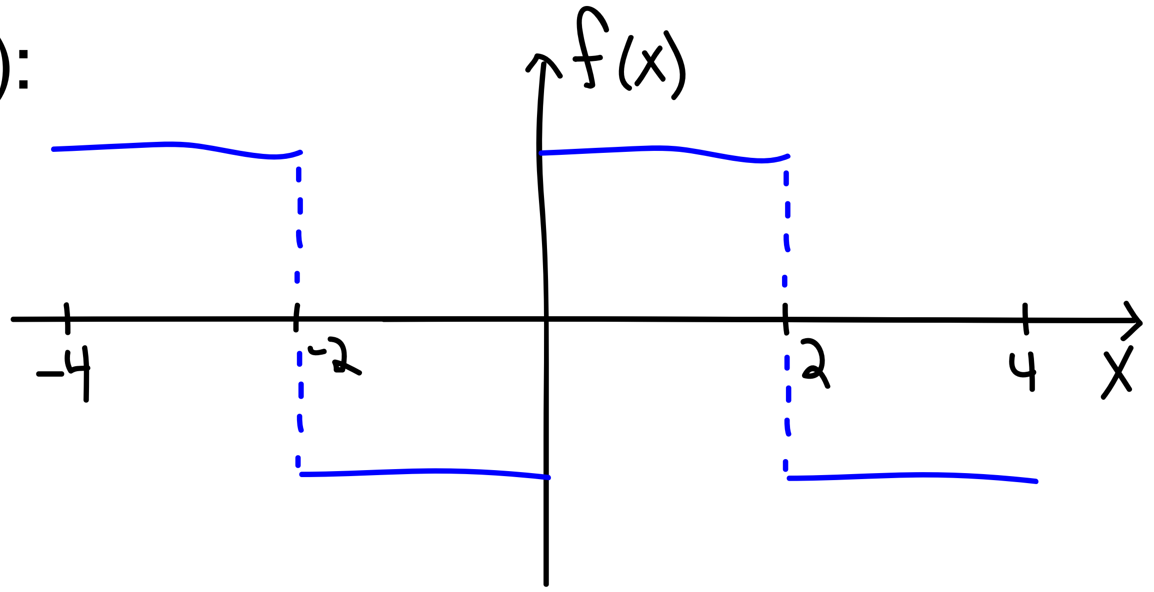

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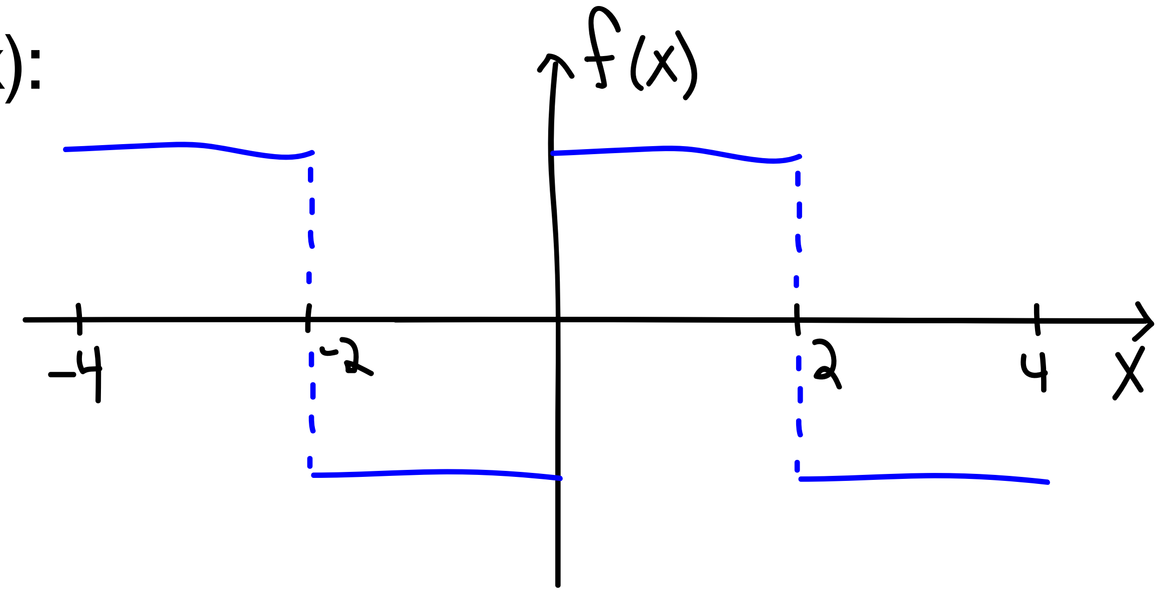

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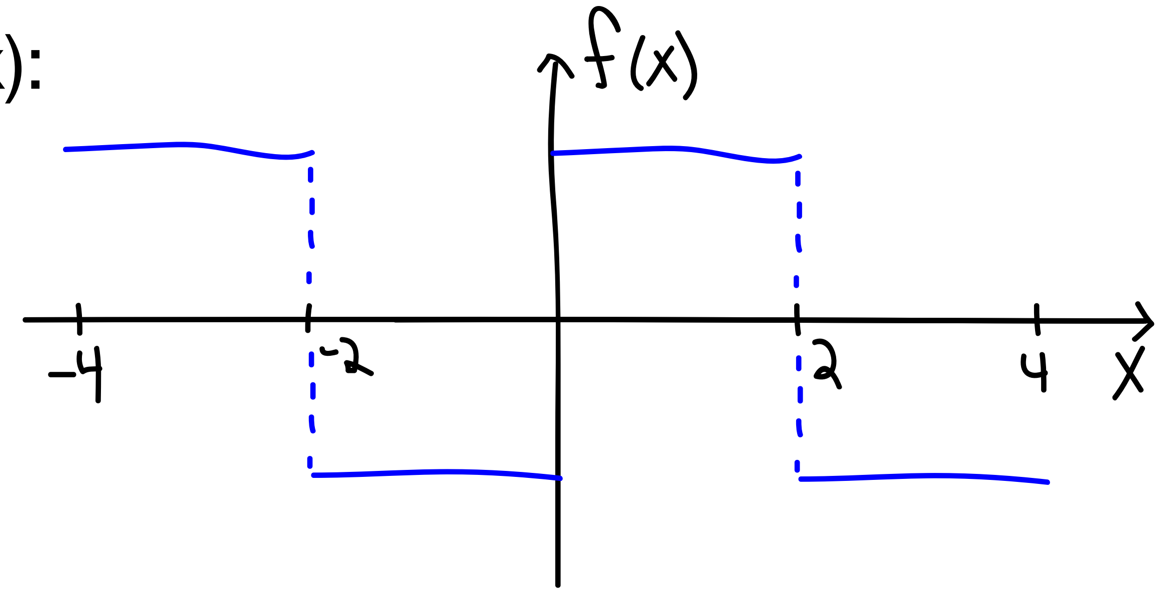

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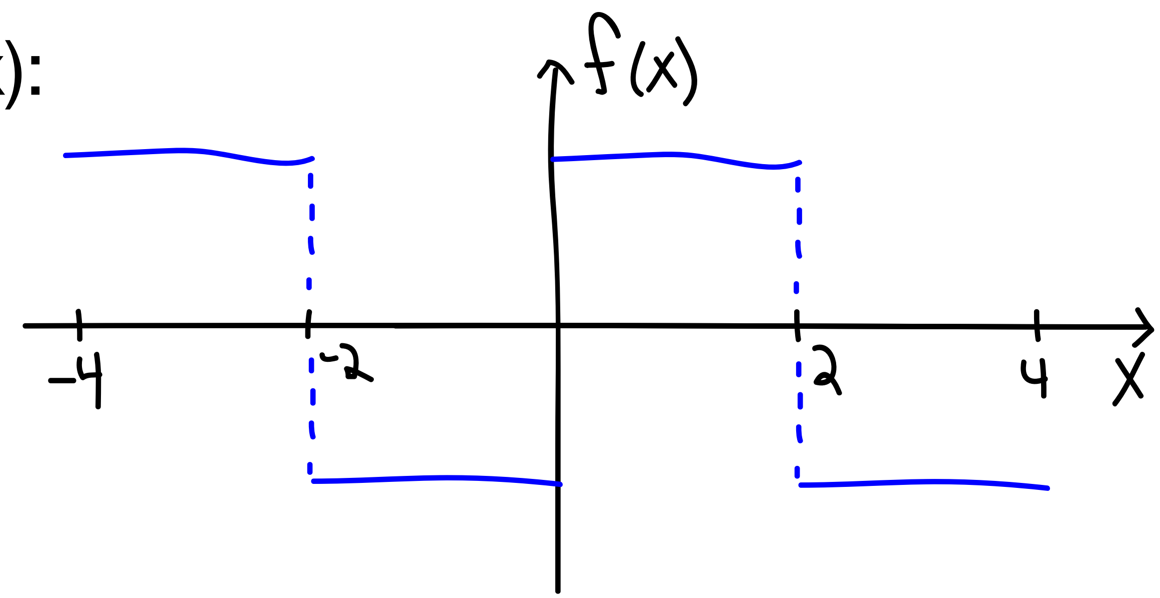

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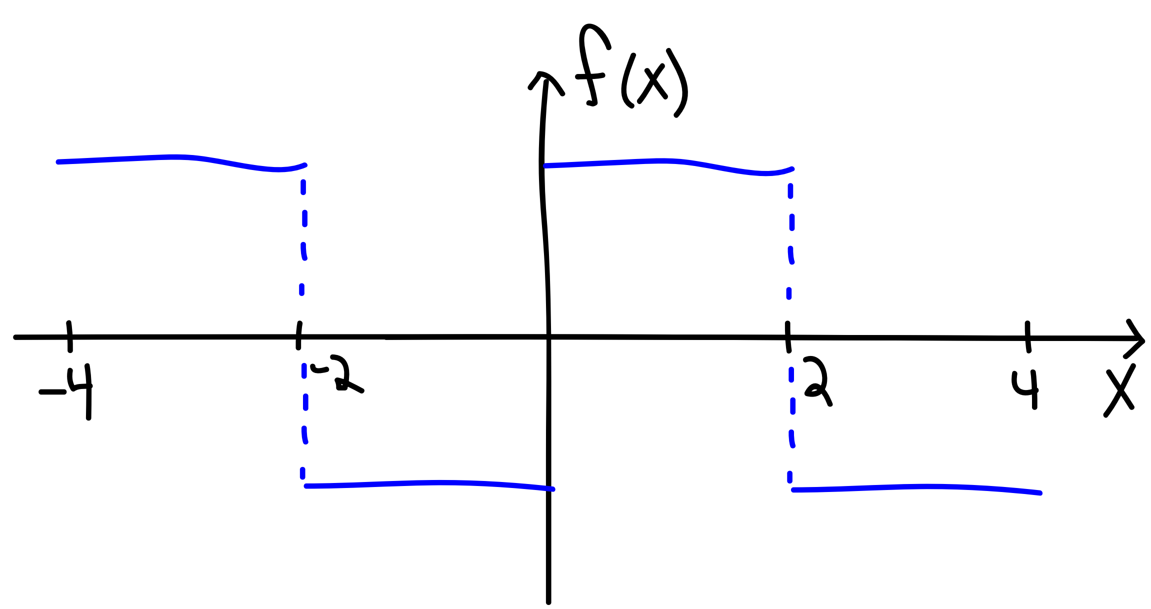

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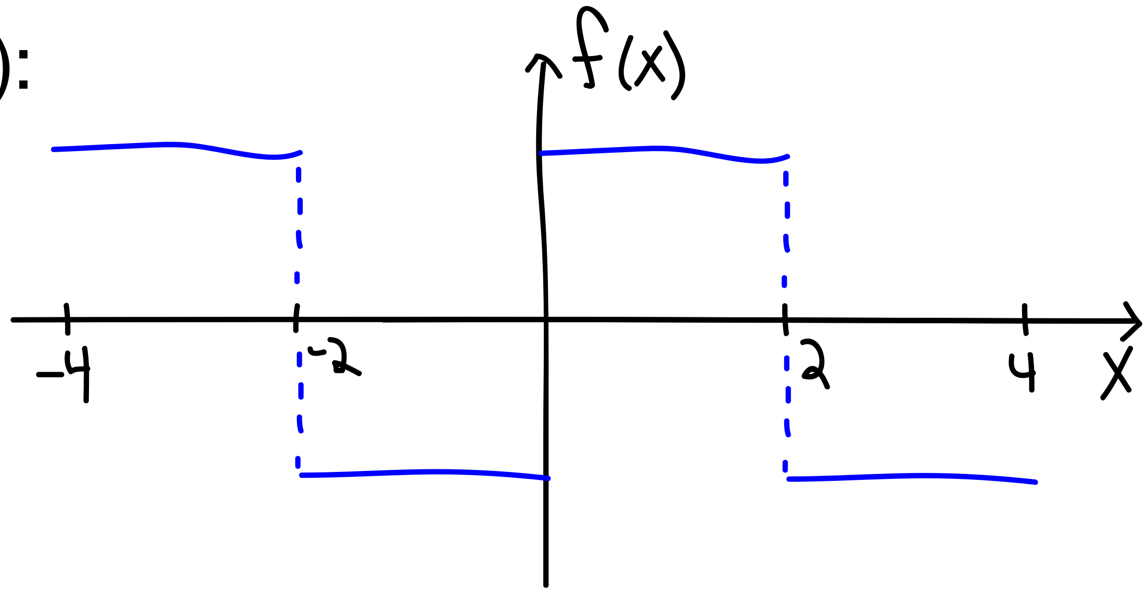

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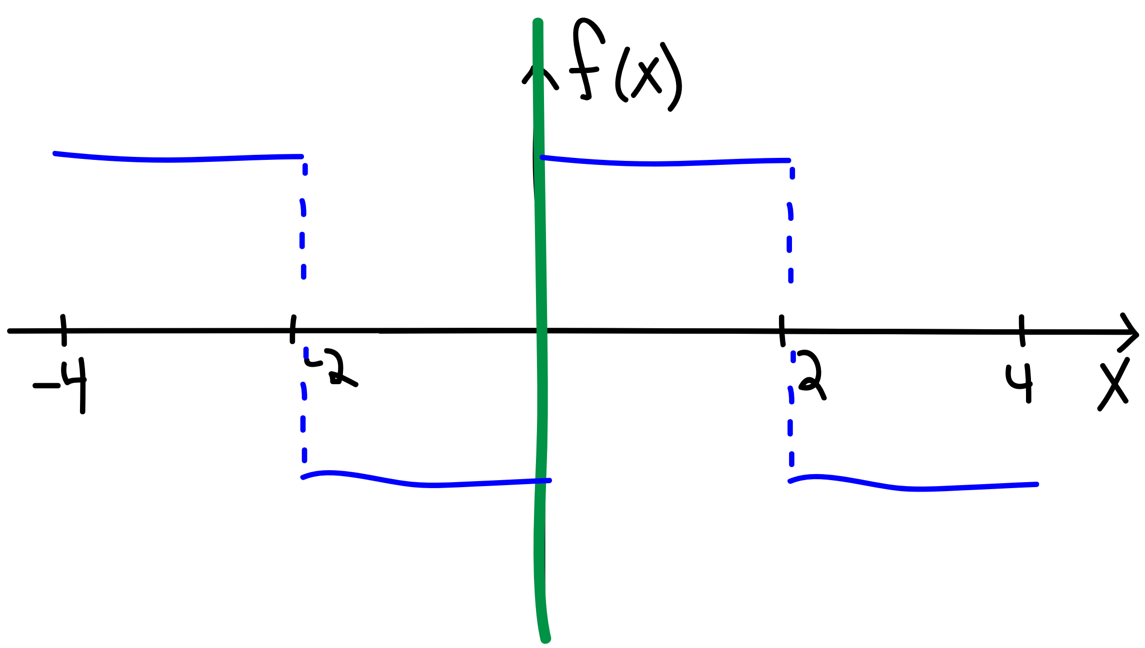

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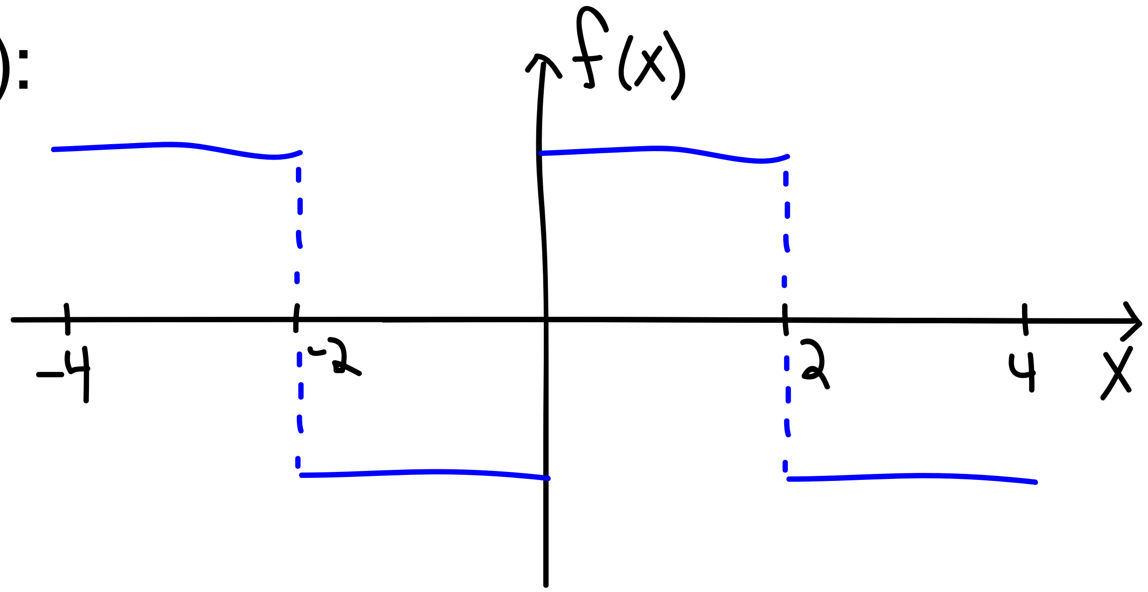

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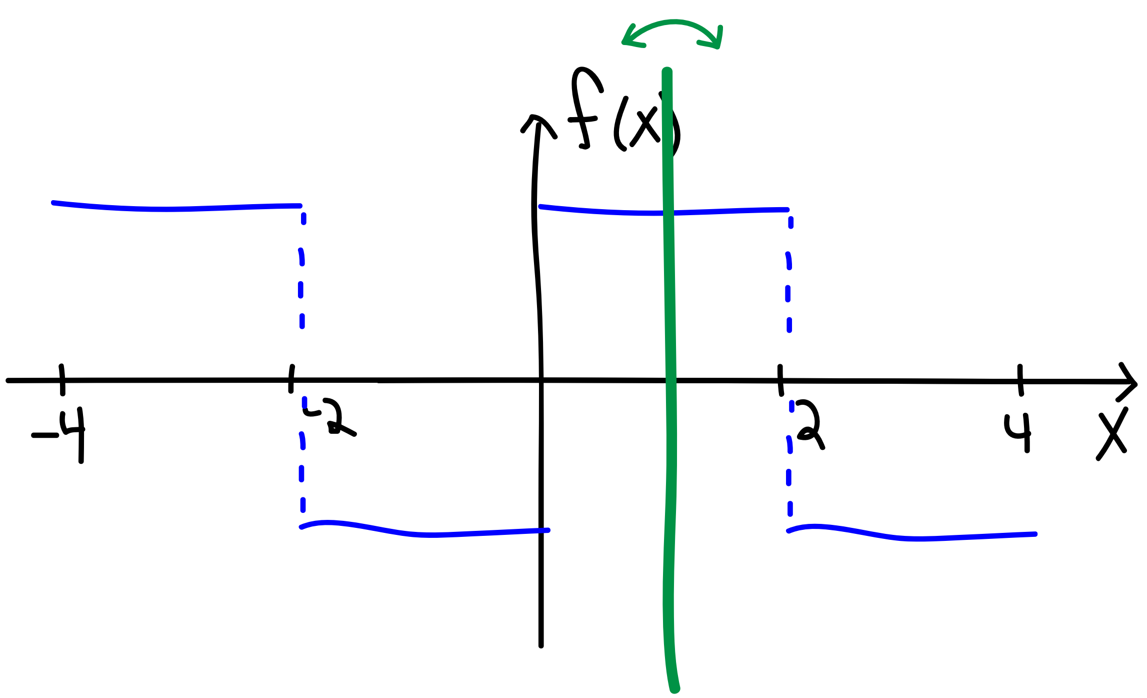

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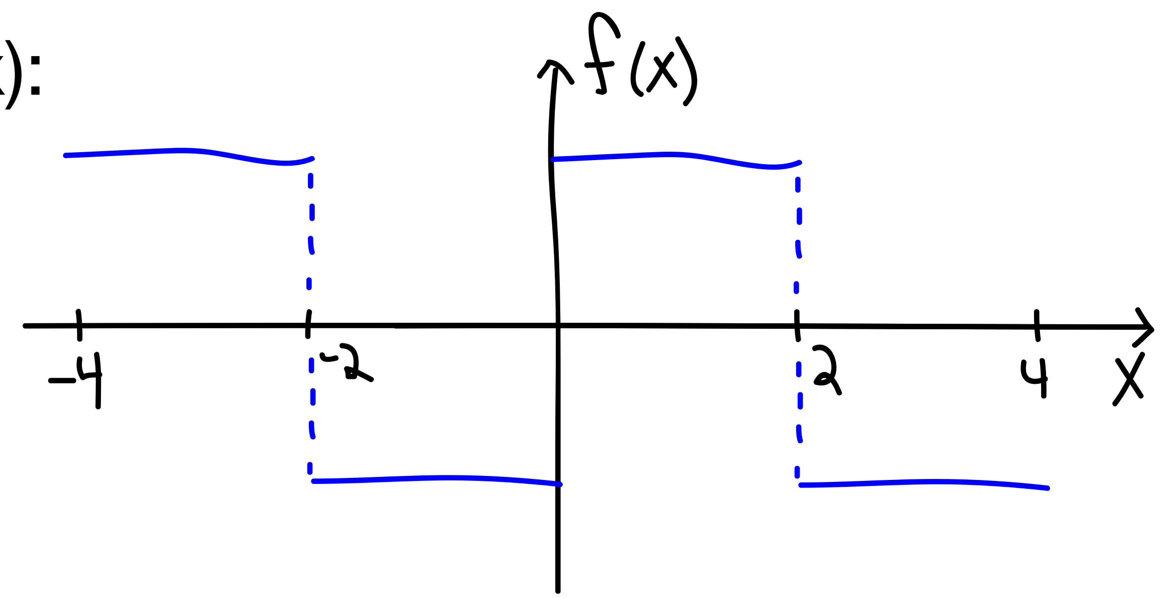

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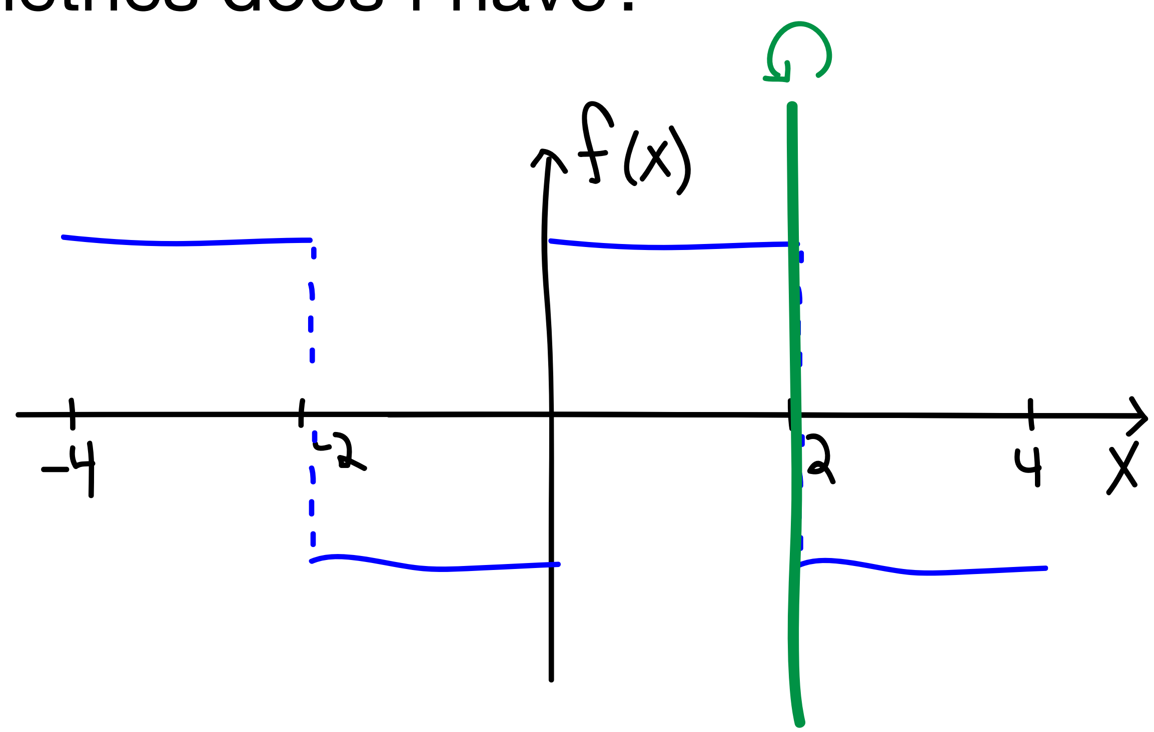

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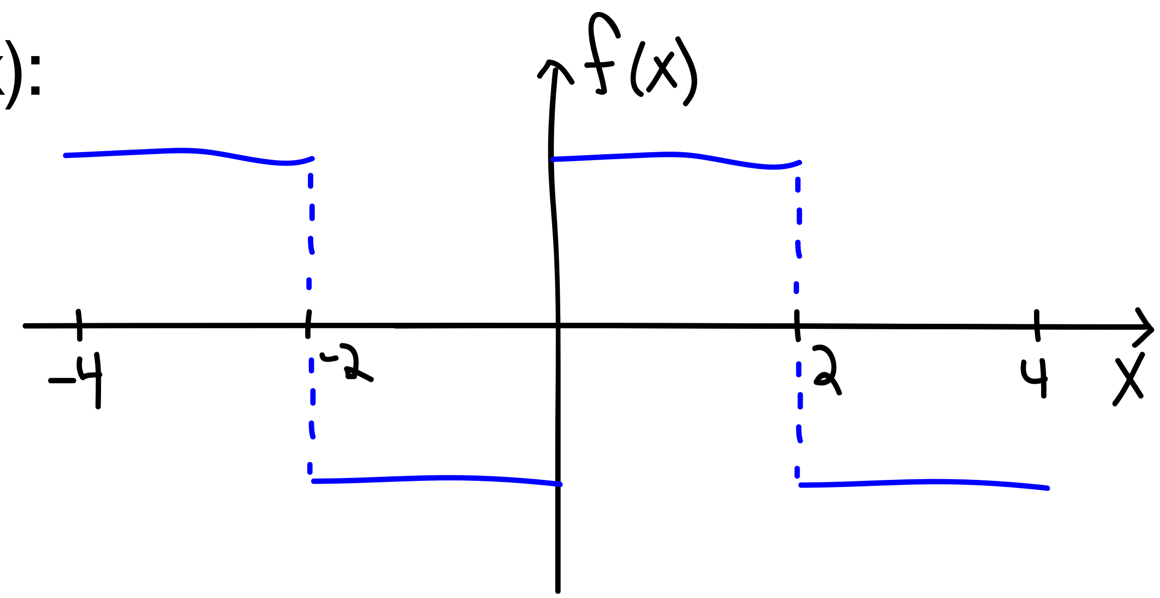

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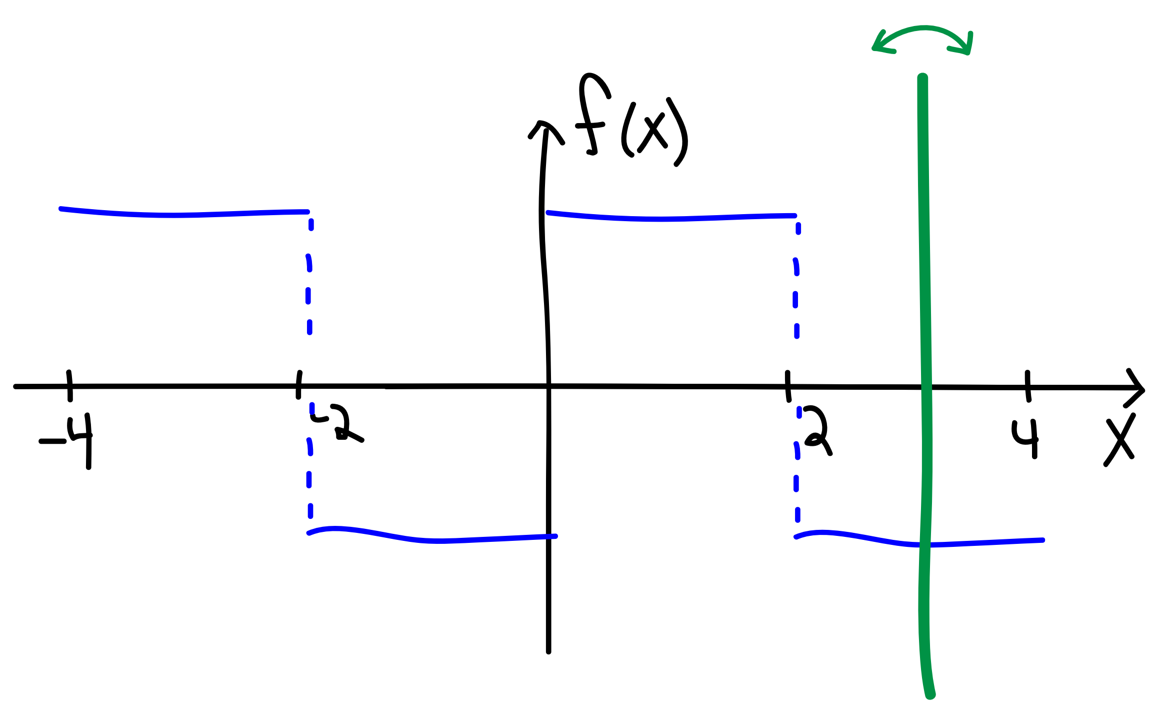

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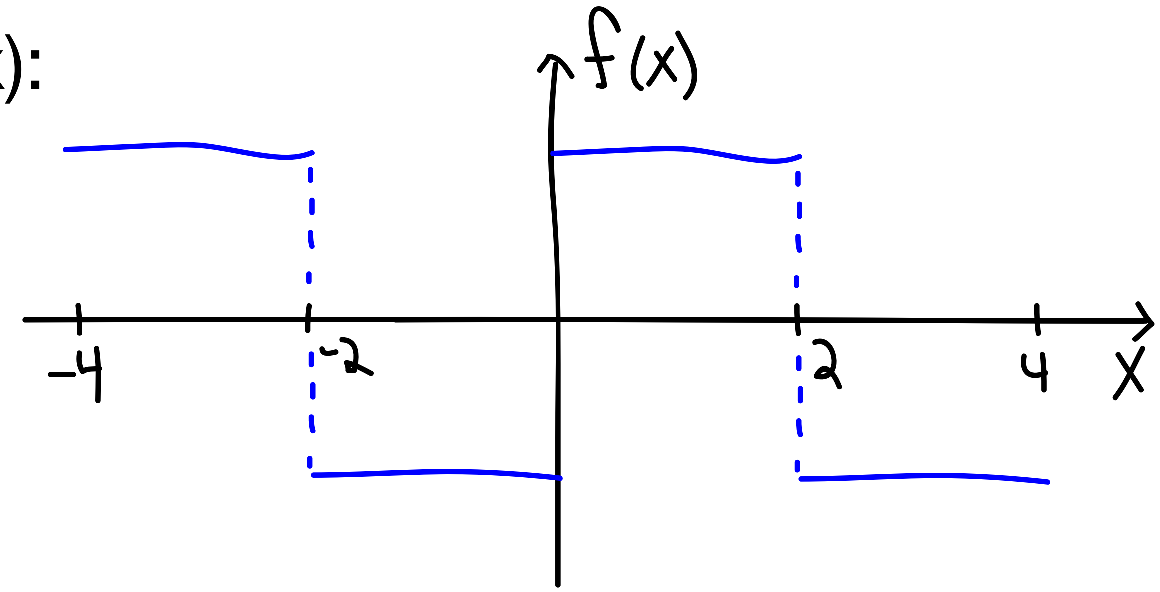

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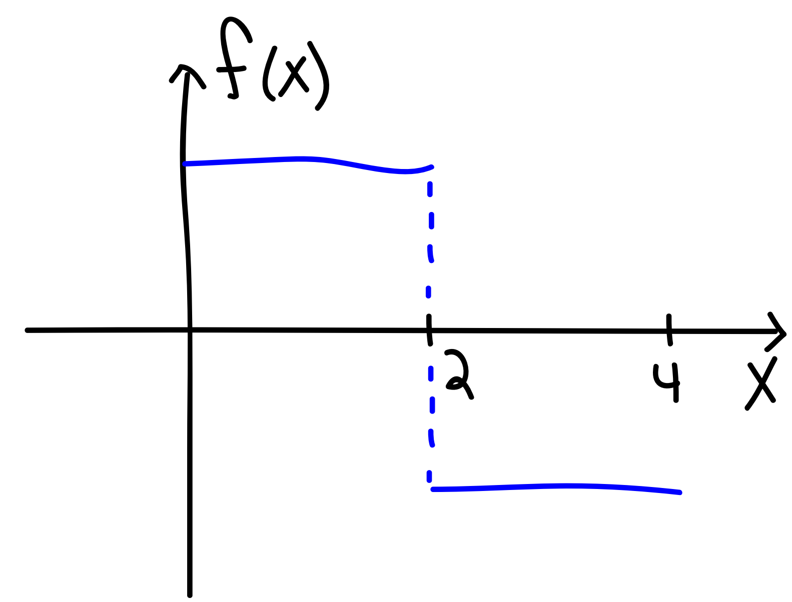

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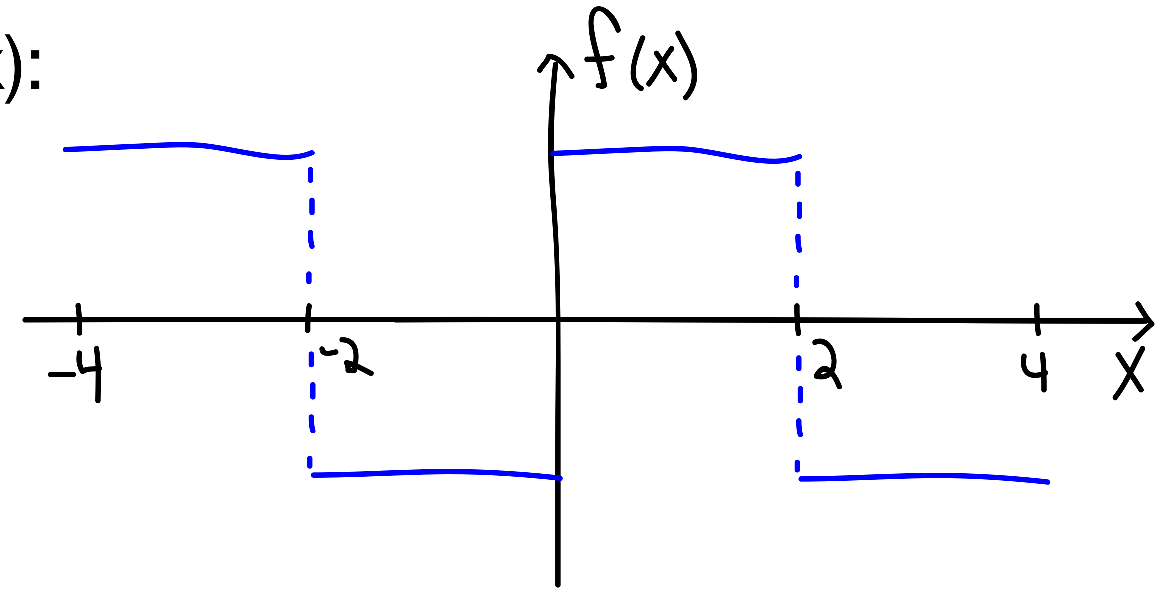

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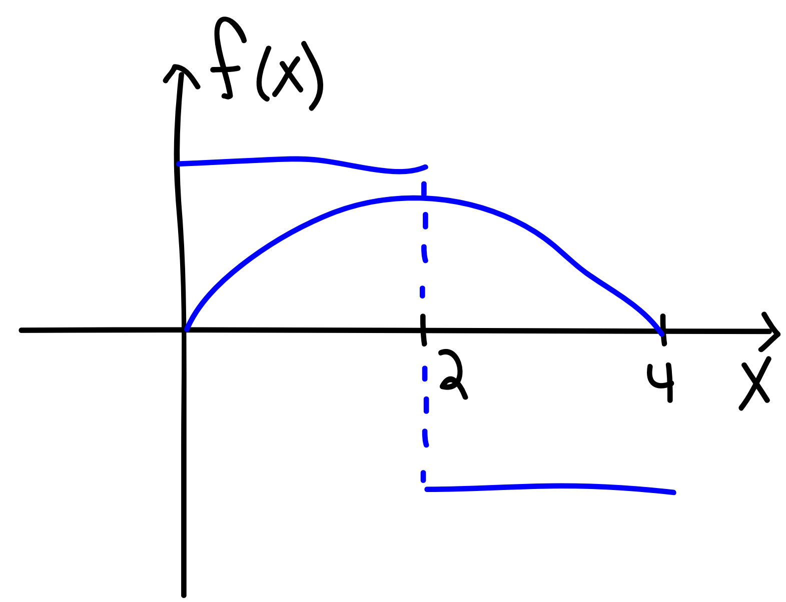

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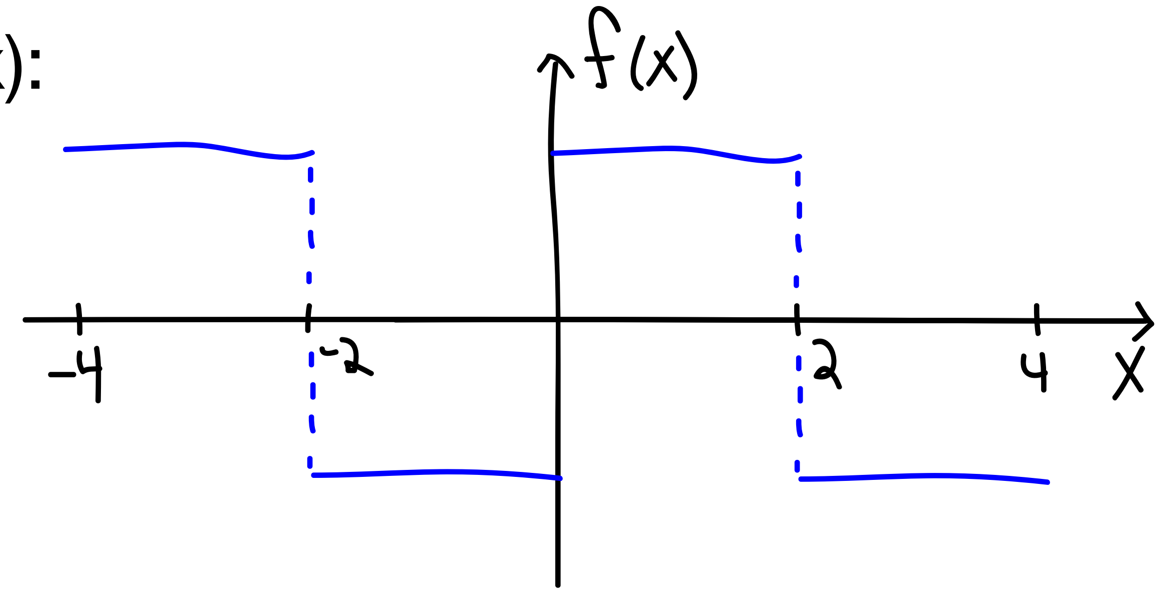

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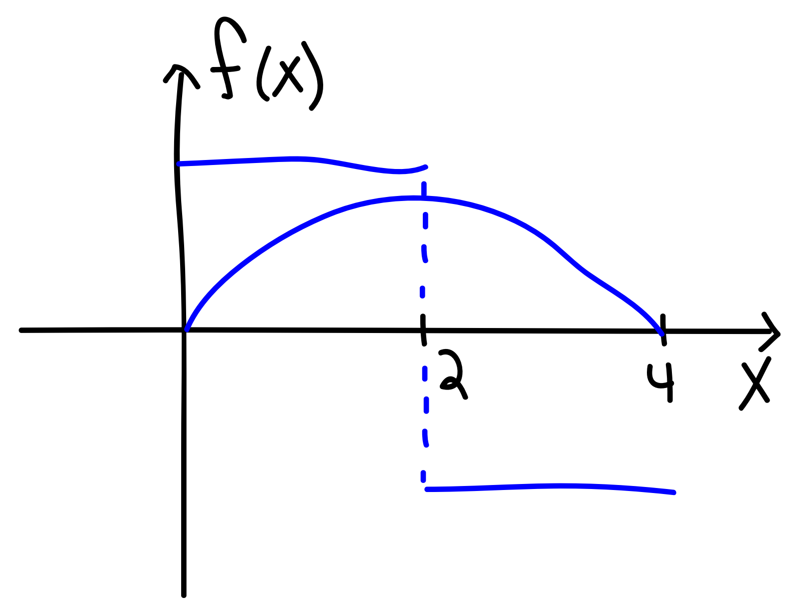

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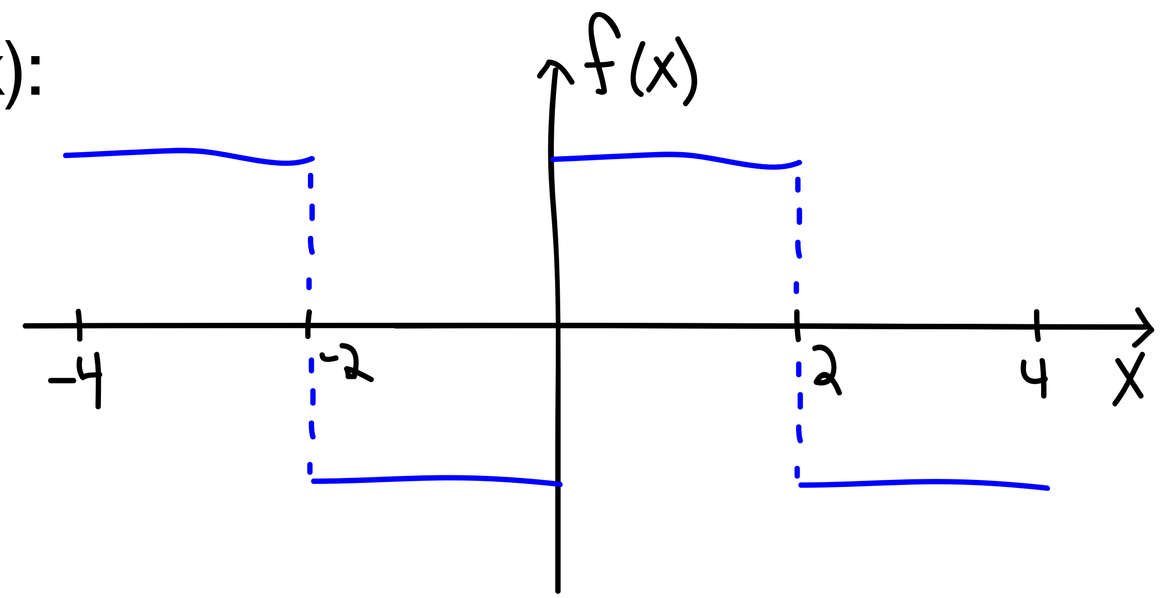

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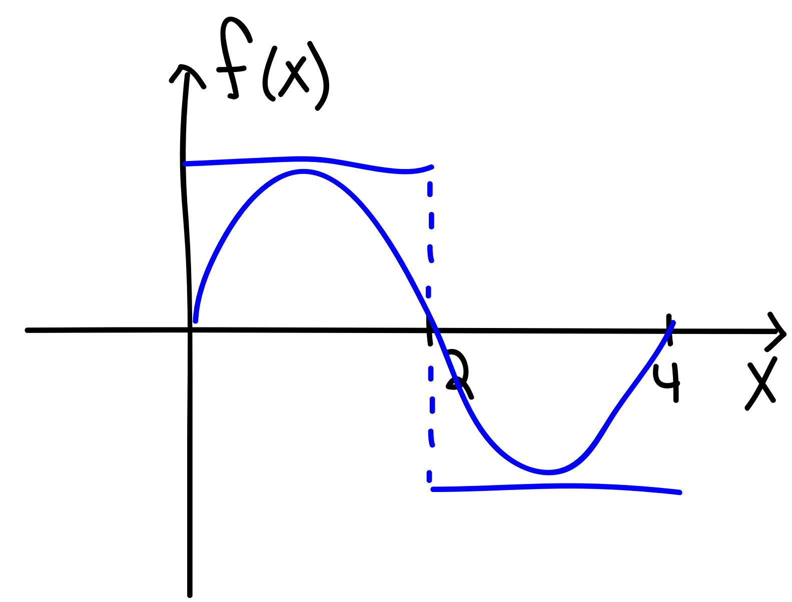

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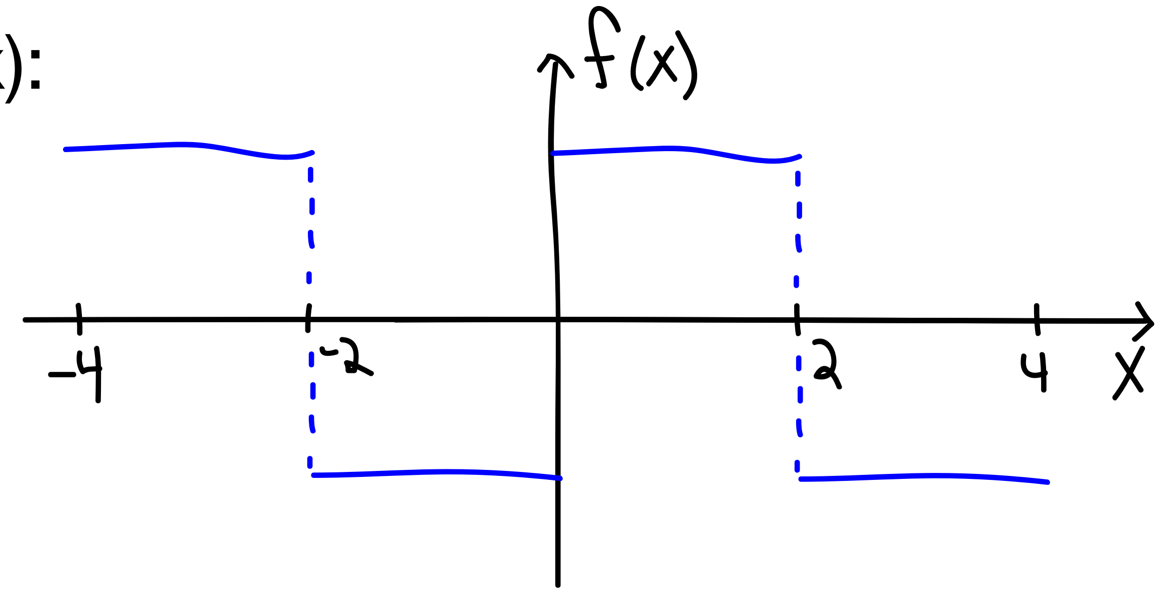

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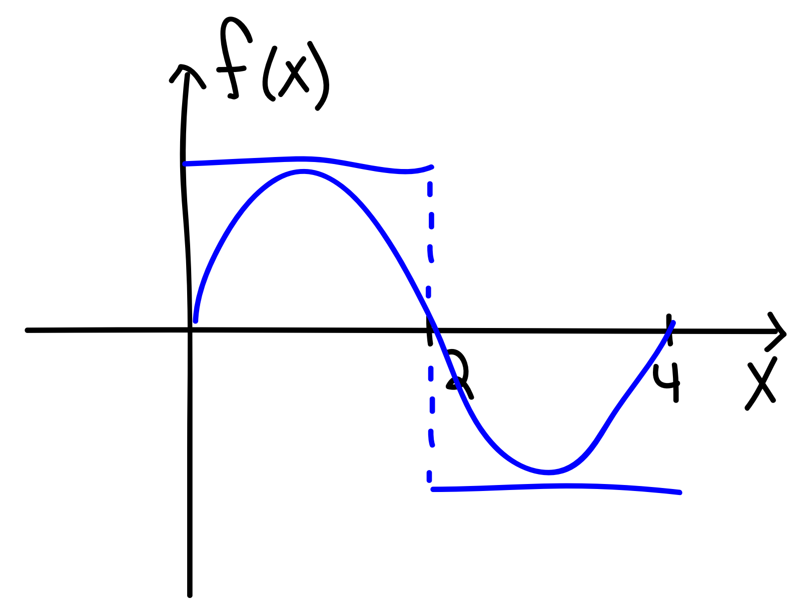
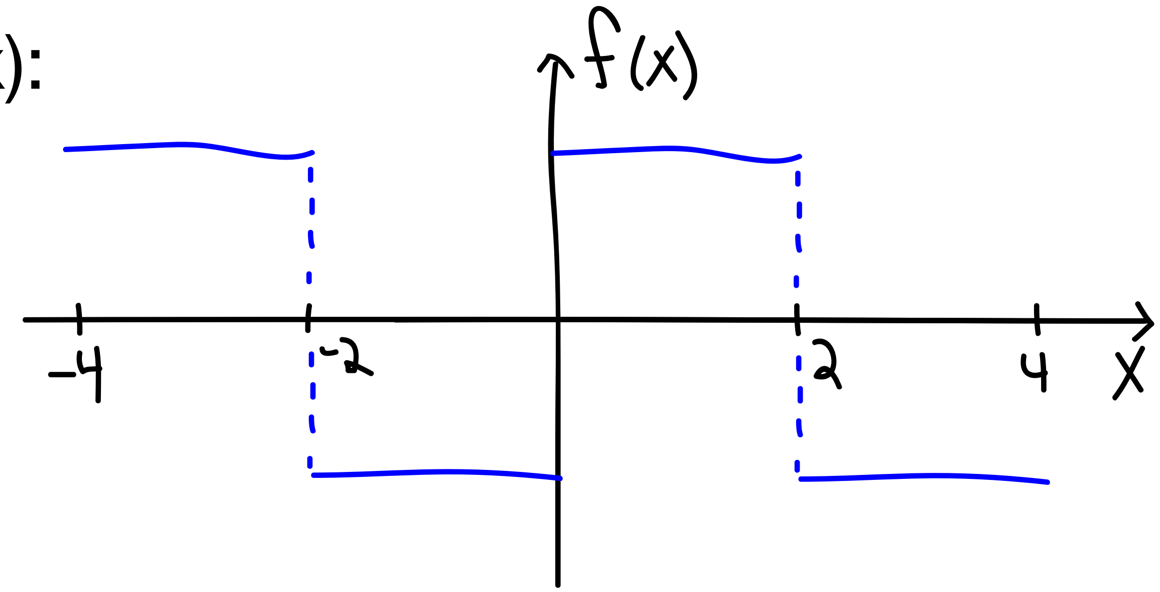


Fourier Series for functions with other symmetries

- Find the Fourier Sine Series for f(x):



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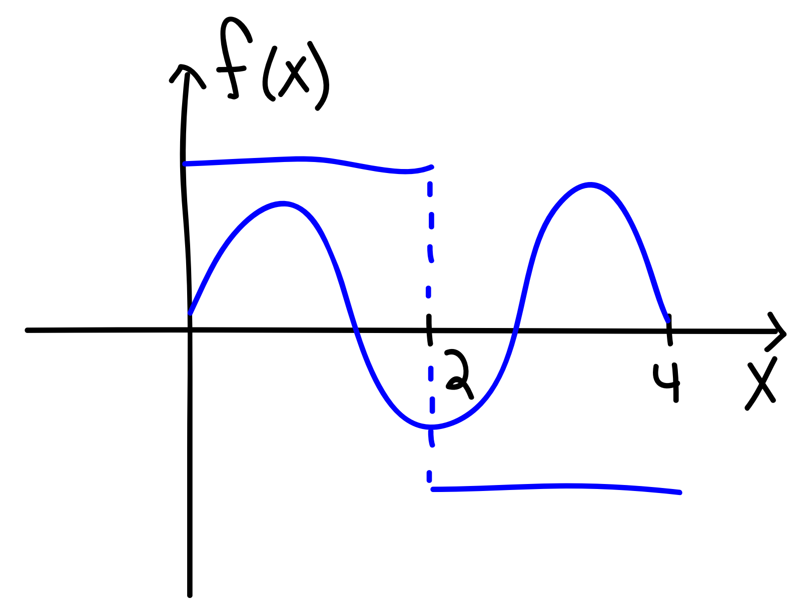

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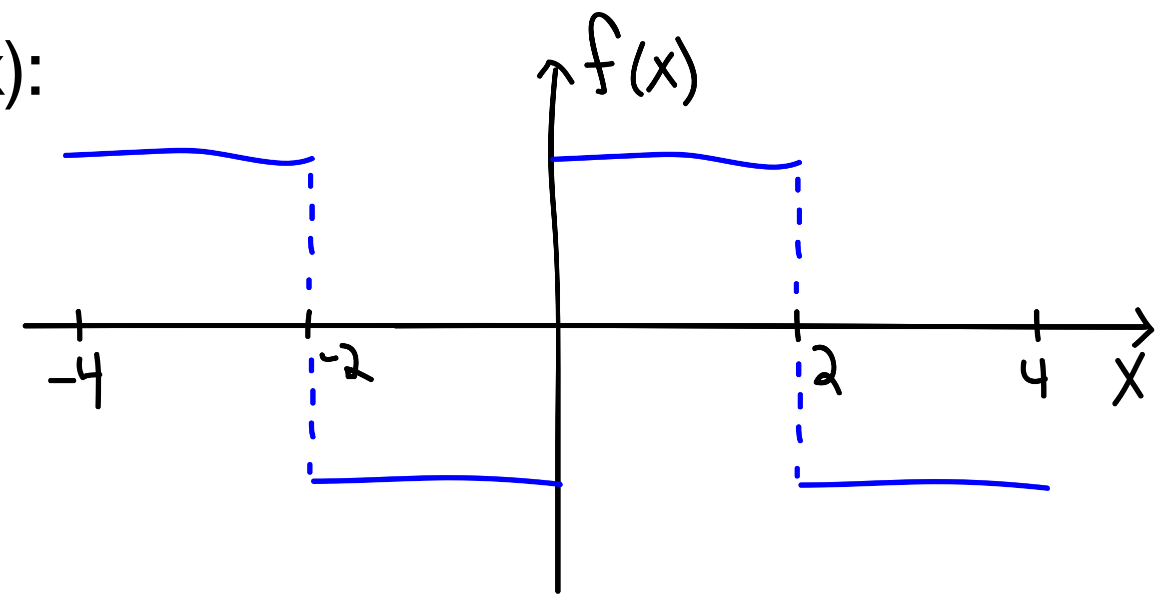

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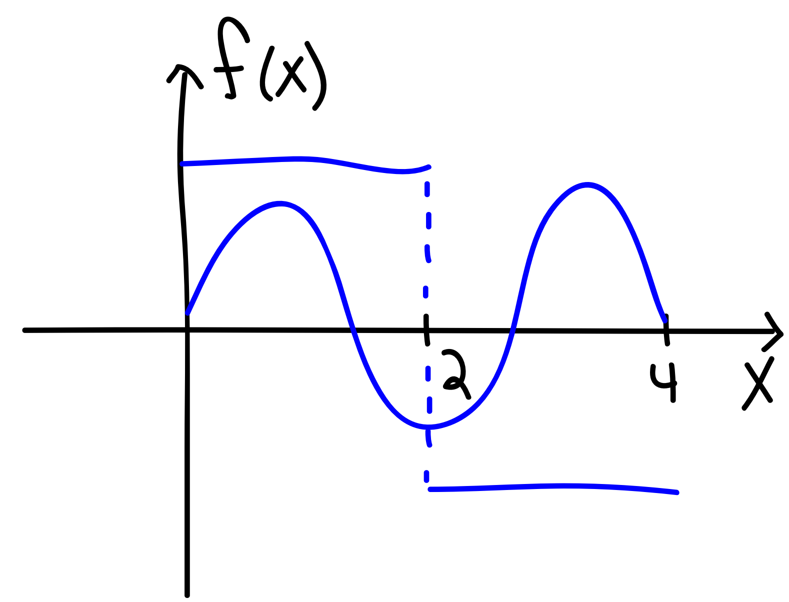

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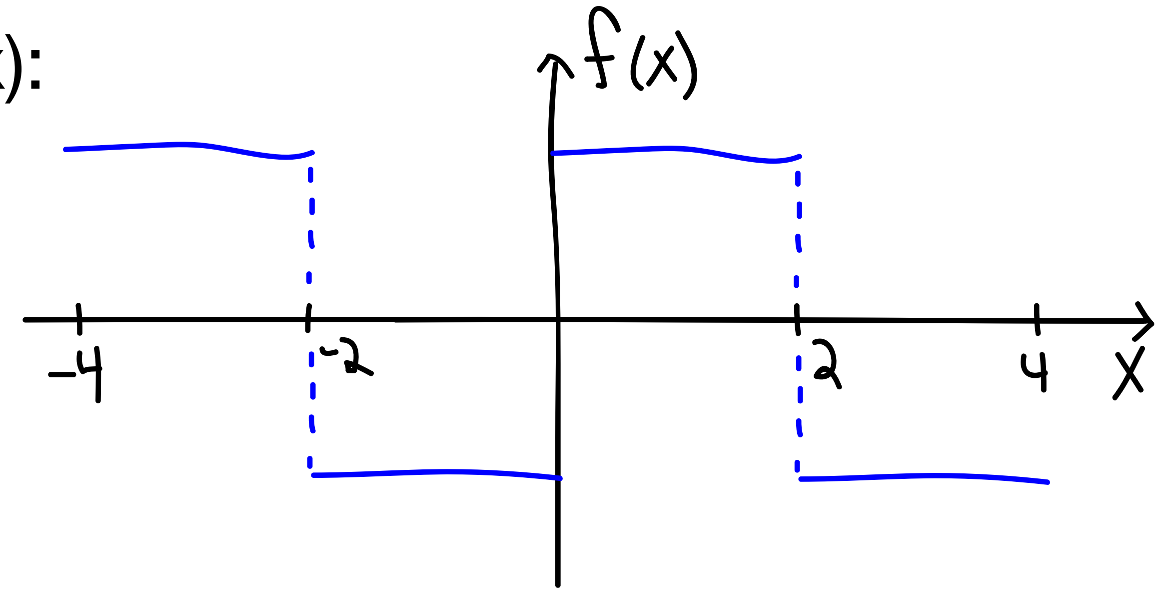

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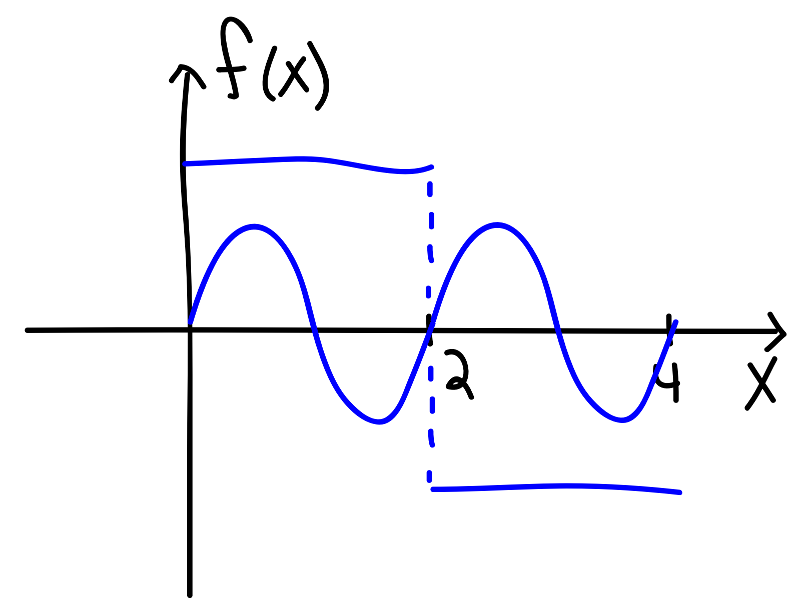

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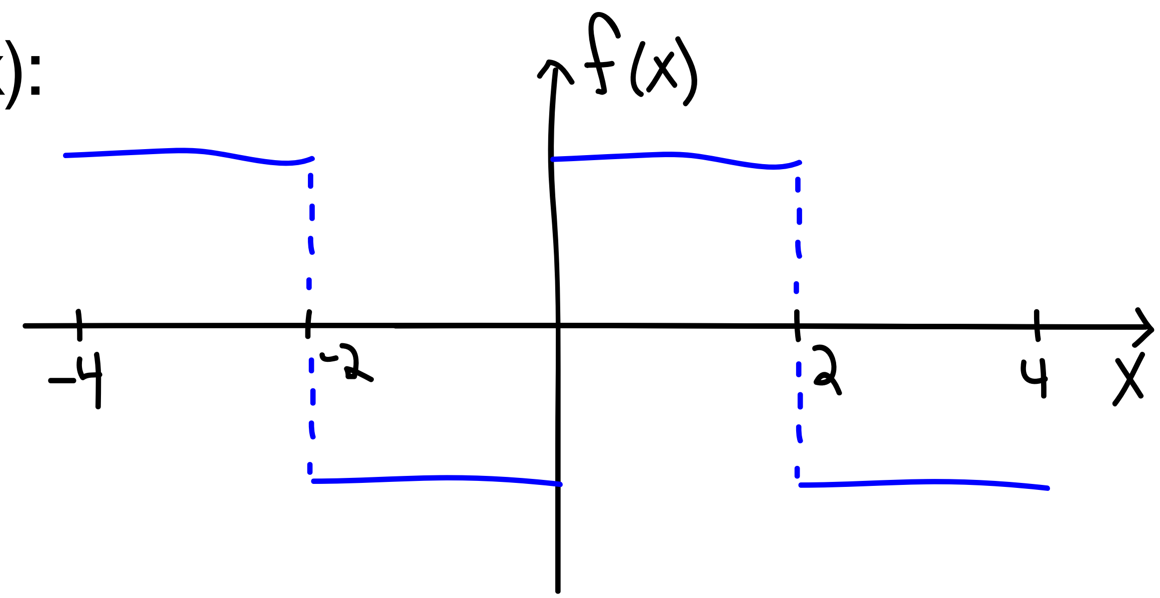

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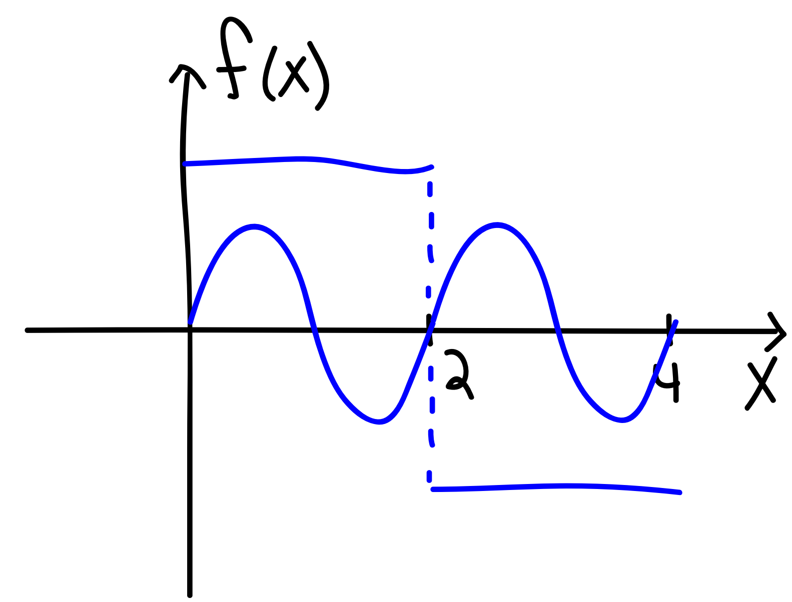

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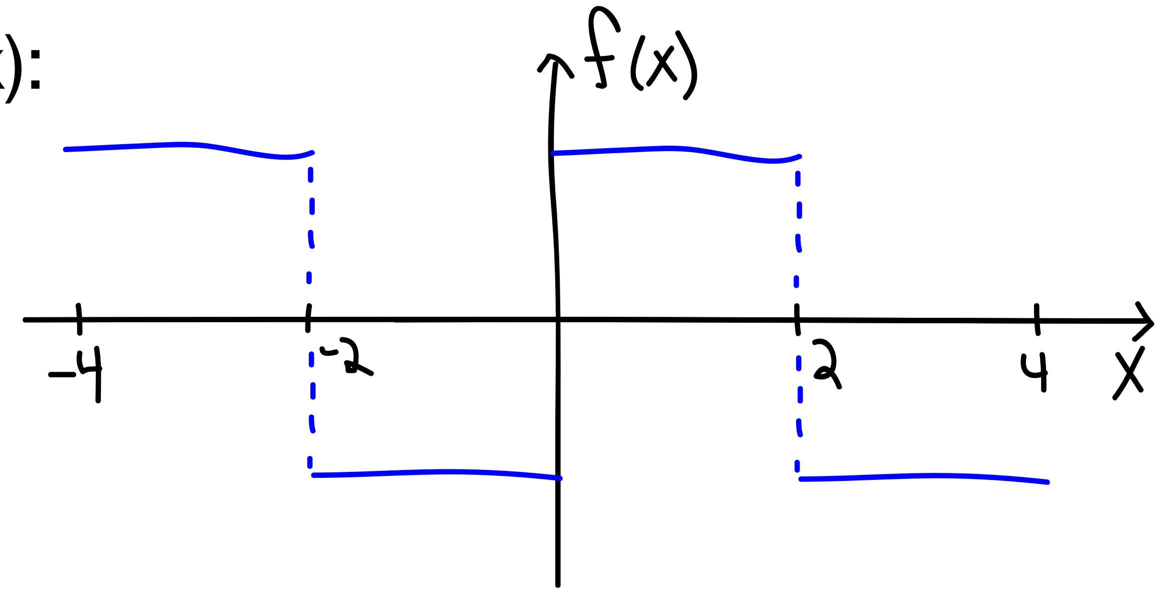

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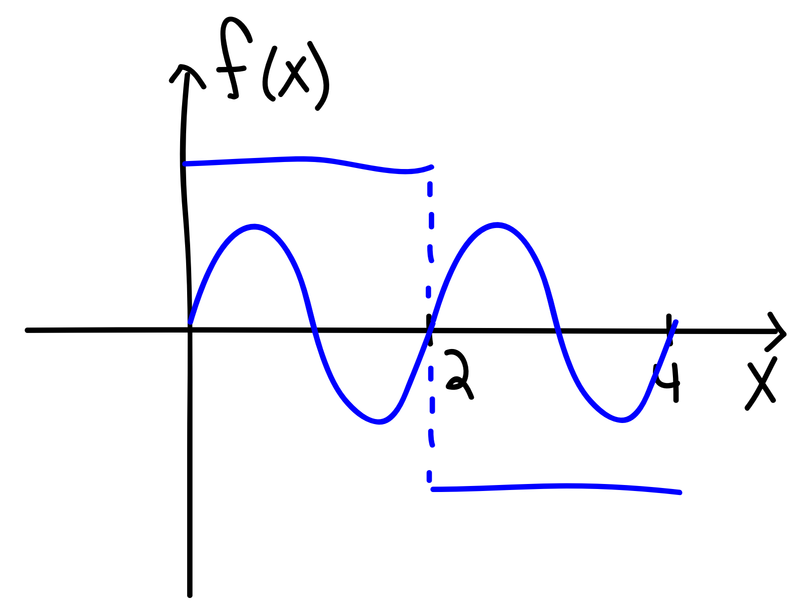

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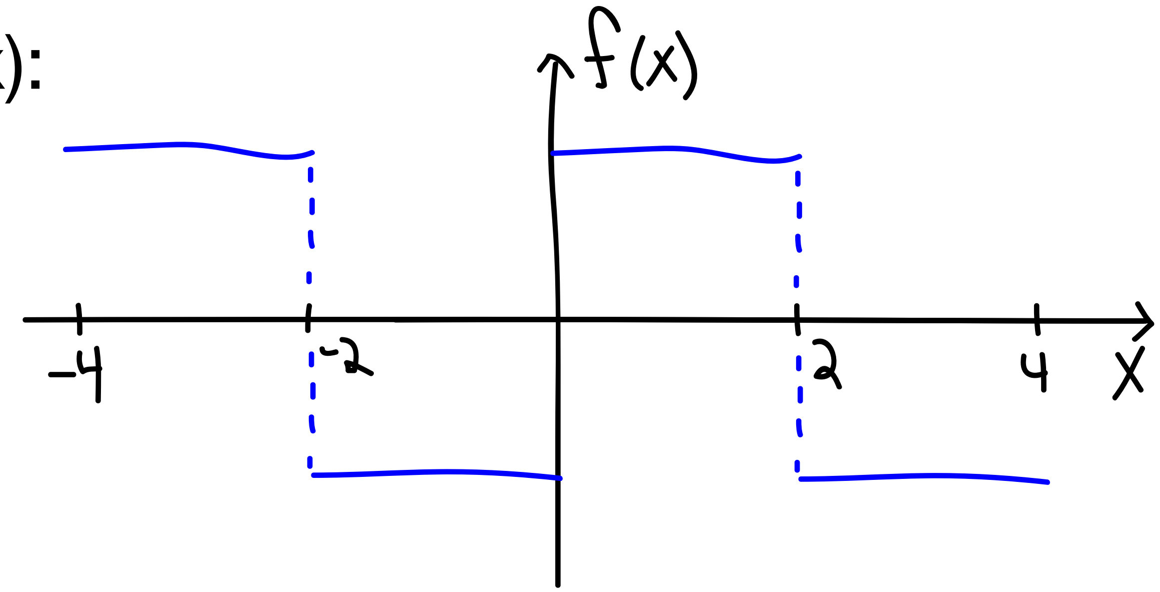

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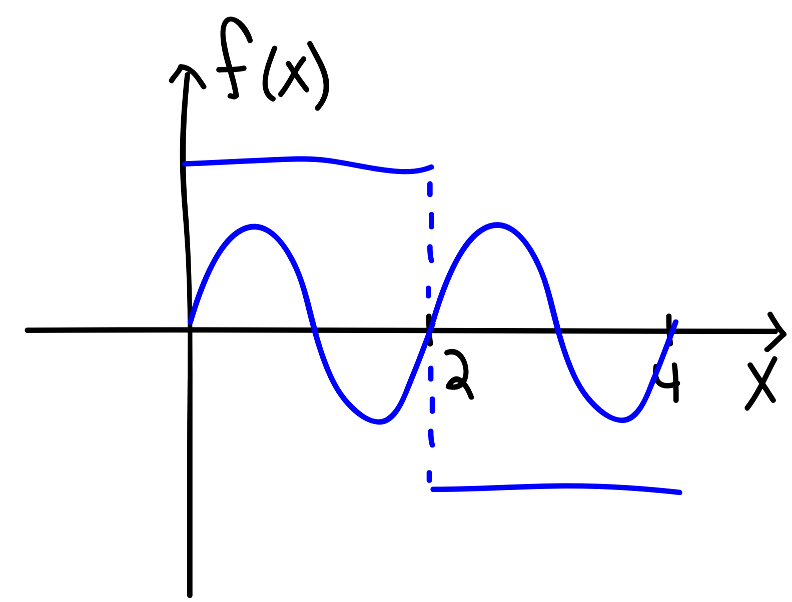

- b_n=0 for n = odd or 4k
- Calculate b_n

Using Fourier Series to solve the Diffusion Equation

$$u_t = 4u_{xx}$$

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So the solution is

$$u(x,t) = e^{-9\pi^2 t}\cos\frac{3\pi x}{2}$$

Using Fourier Series to solve the Diffusion Equation

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$$u(x,0) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$

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The IC is the sum of eigenvectors!

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(B) $u(x,t) = \displaystyle\sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 t} \sin\frac{n\pi x}{2}$ $\qquad b_n = \dfrac{(-1)^{n+1}4}{n\pi}$

Using Fourier Series to solve the Diffusion Equation

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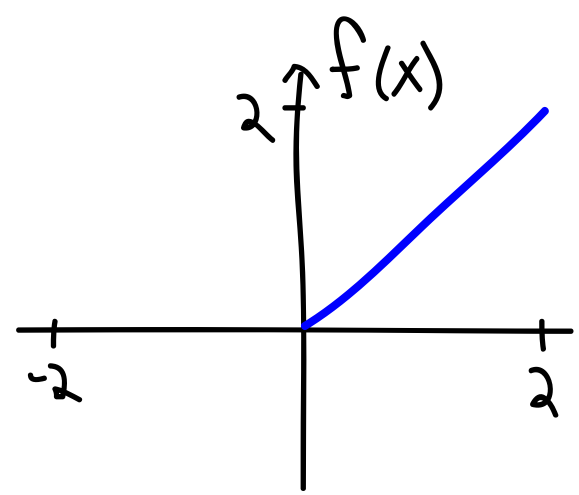

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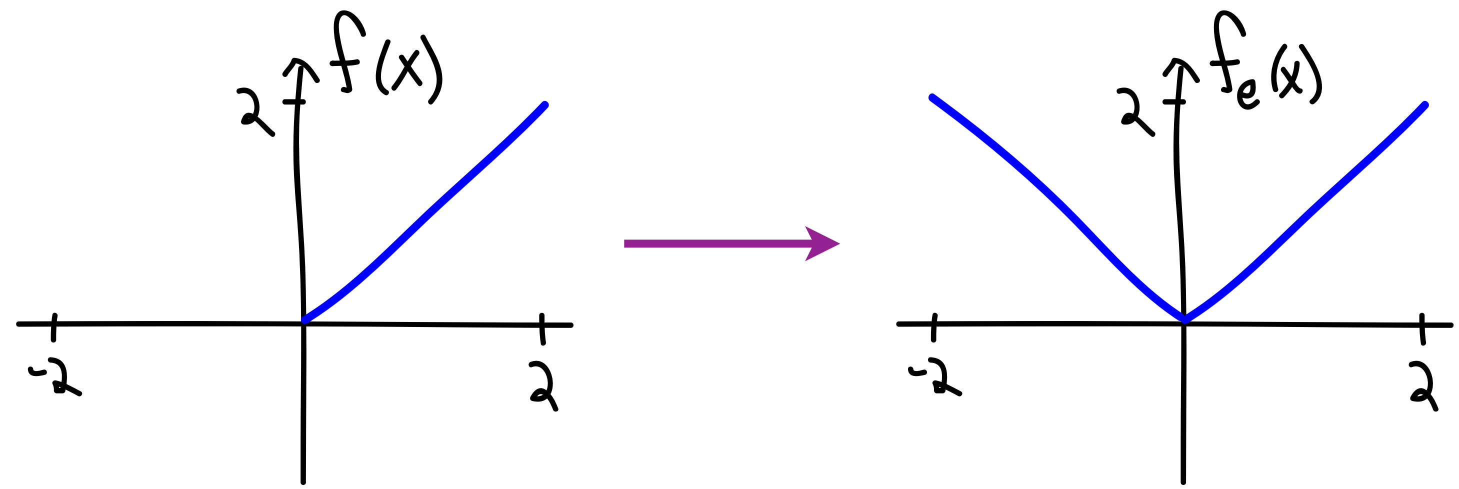

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$$u_t = 4u_{xx}$$

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https://www.desmos.com/calculator/yt7kztckeu

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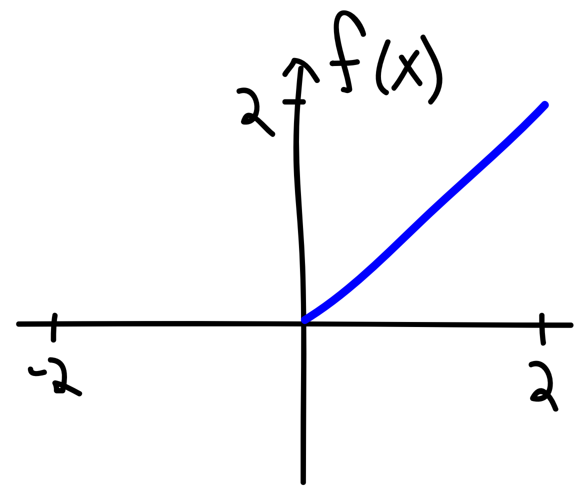

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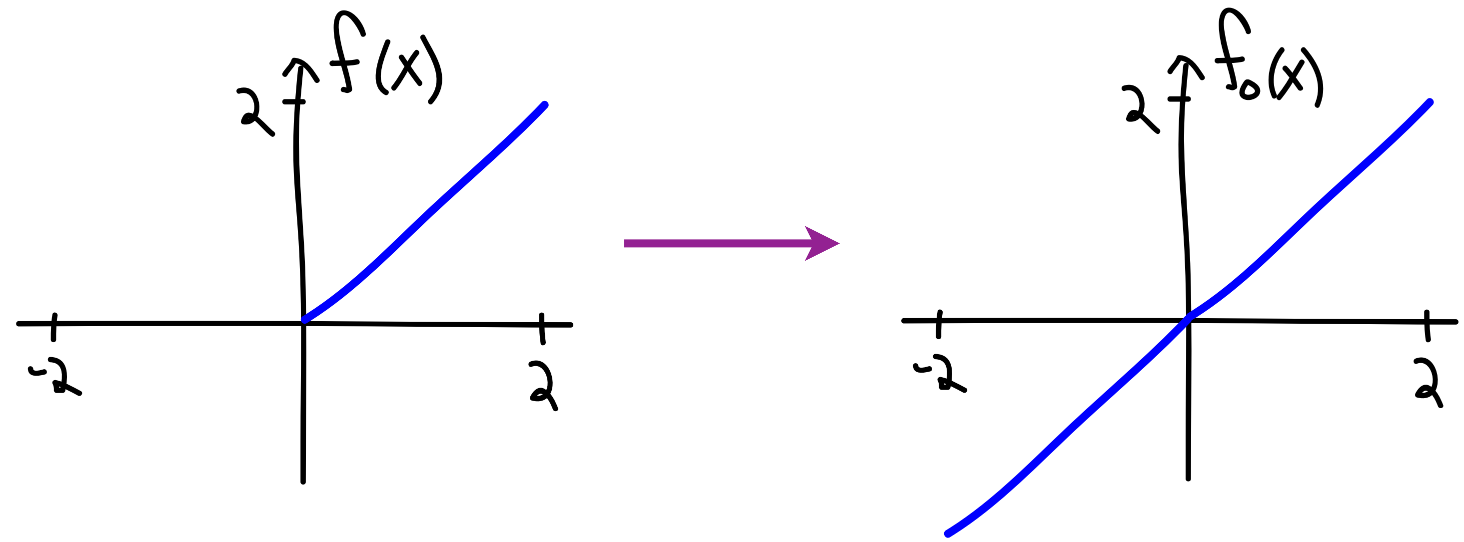

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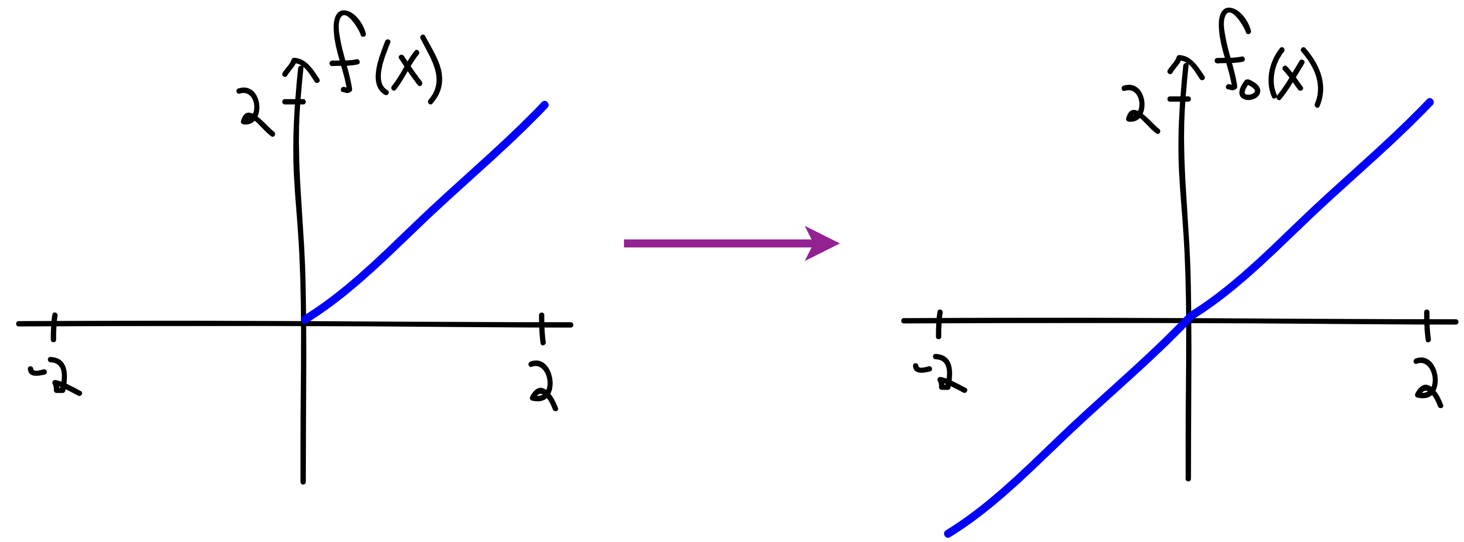
$$u(x,0) = x$$



(A) $$u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 t} \cos\frac{n\pi x}{2}$$

$$a_0 = 1,\ a_n = -\frac{8}{n^2\pi^2} \text{ for } n \text{ even}$$
$$(0 \text{ for } n \text{ odd})$$

★ (B) $$u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 t} \sin\frac{n\pi x}{2}$$

$$b_n = \frac{(-1)^{n+1}4}{n\pi}$$

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