Today

- Introduction to systems of equations
- Direction fields
- Eigenvalues and eigenvectors
- Finding the general solution (distinct e-value case)
- Return midterm 1

Introduction to systems of equations

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$$\begin{aligned} x' &= v \\ v' &= -\frac{k}{m}x - \frac{\gamma}{m}v \end{aligned} \qquad \begin{pmatrix} x \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\gamma}{m} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}$$

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    - populations of two species (e.g. predator and prey).

Introduction to systems of equations

- As with single equations, we have linear and nonlinear systems:

$$\frac{dx}{dt} = t^2 x - y + \cos(2t) \qquad \frac{dx}{dt} = t^2 x - y^2$$

$$\frac{dy}{dt} = x + 4\sin(t)y + t^3 \qquad \frac{dy}{dt} = \sqrt{x} - y$$

- And we also have nonhomogeneous and homogeneous systems.

$$\frac{dx}{dt} = t^2 x - y + \cos(2t) \qquad \frac{dx}{dt} = t^2 x - y$$

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- Any linear system can be written in matrix form:

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- We'll focus on the case in which the matrix has constant entries. And homogeneous, to start. For example,

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

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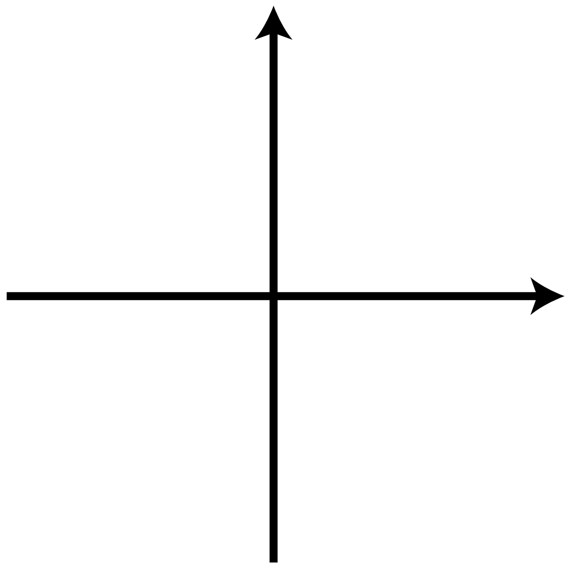

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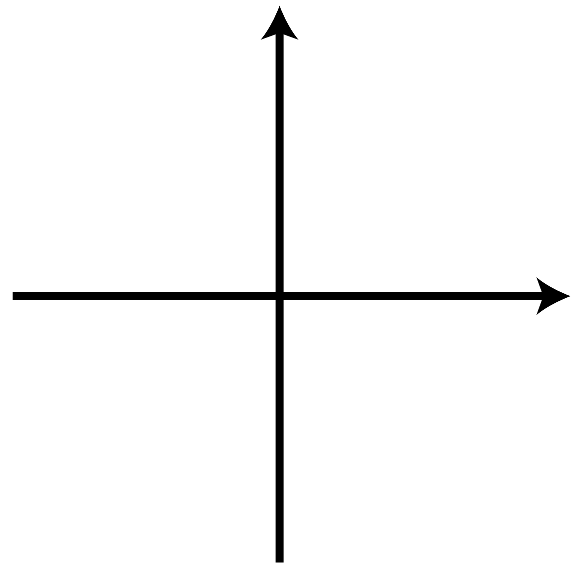

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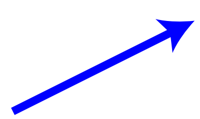

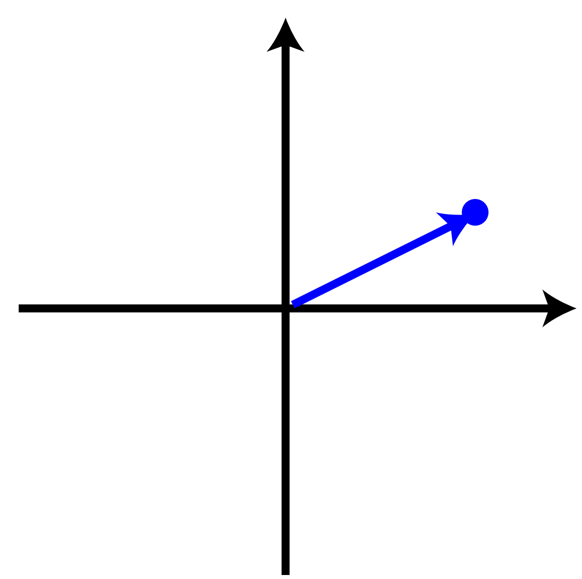

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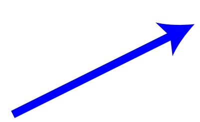

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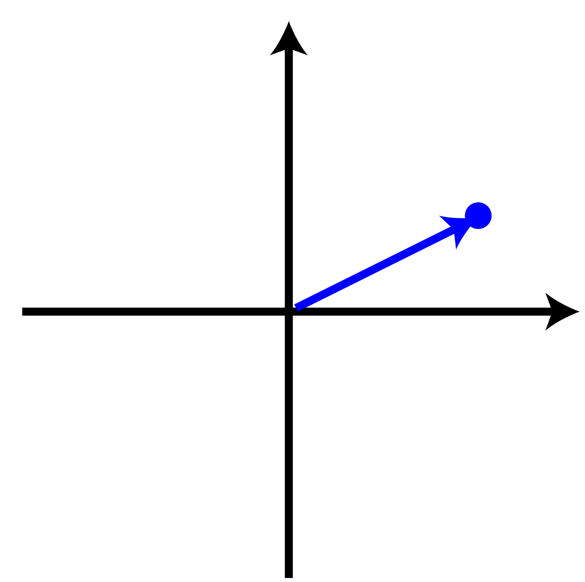

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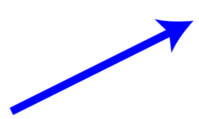

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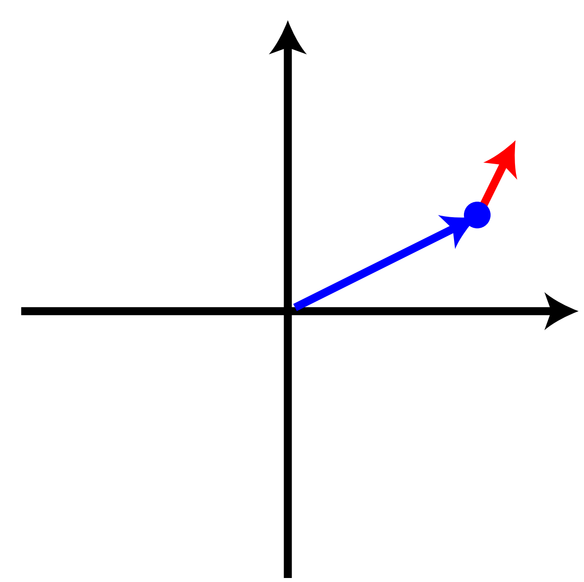

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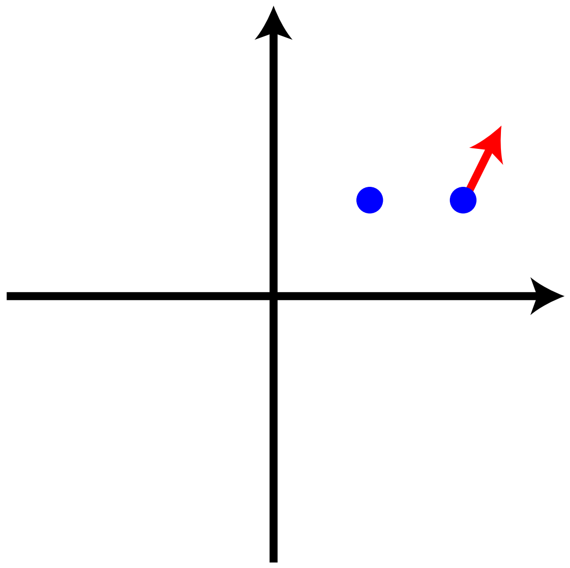

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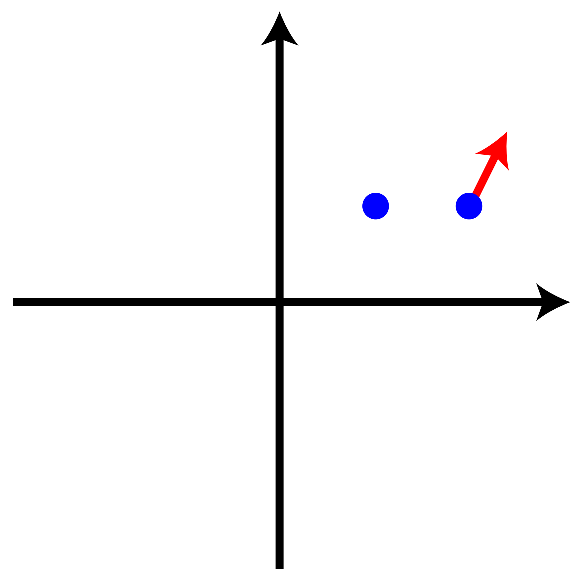

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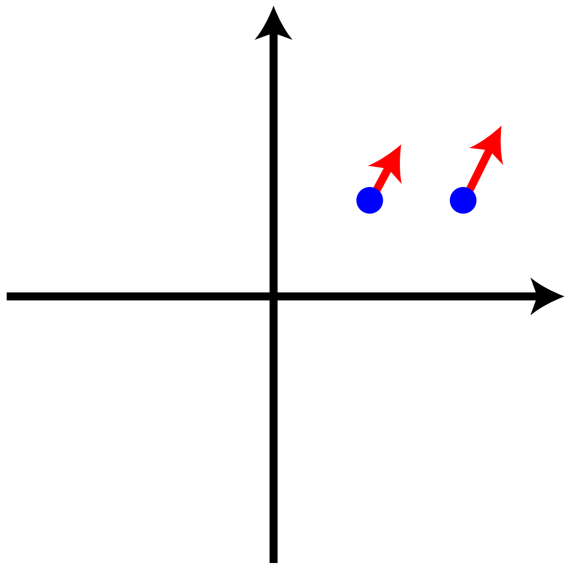

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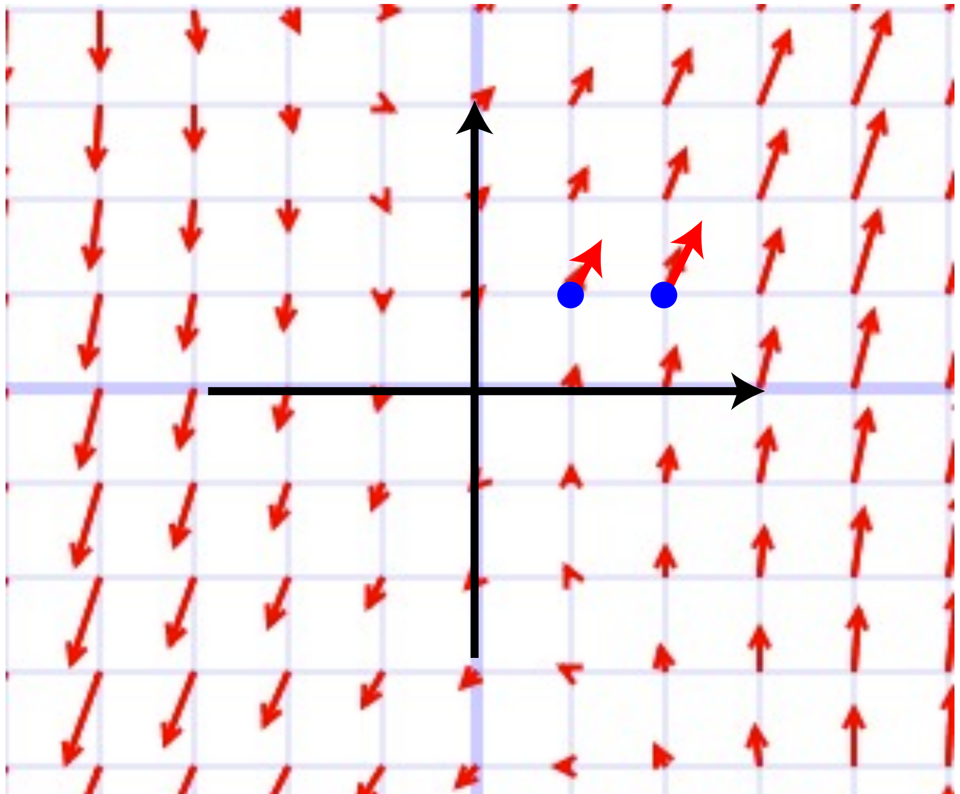

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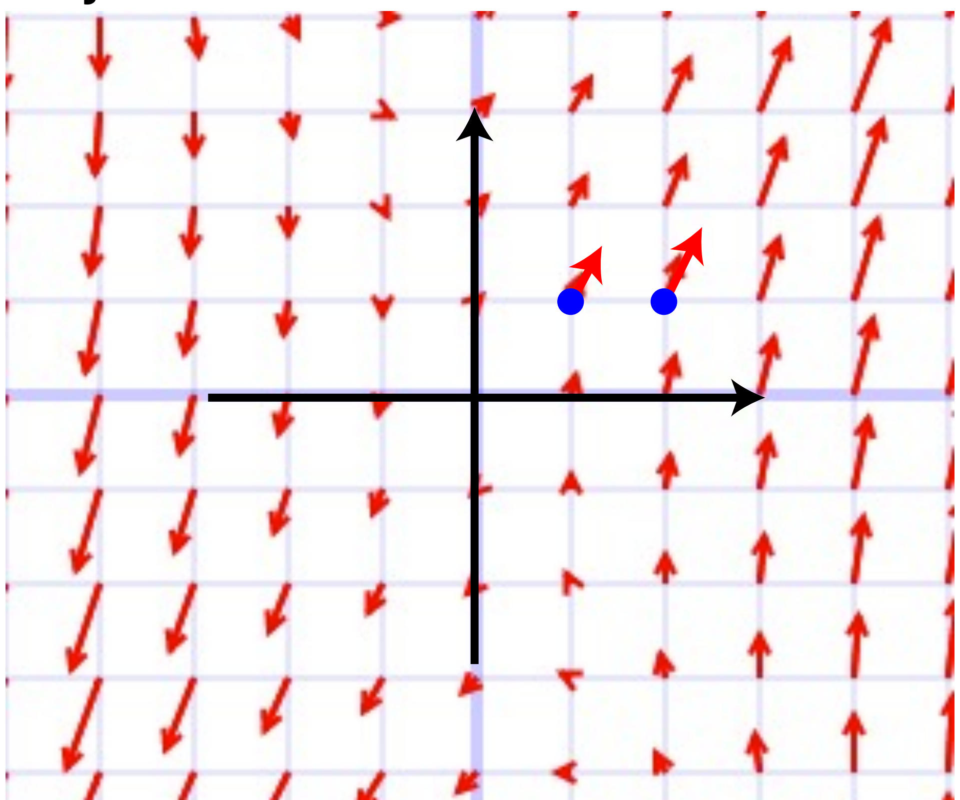

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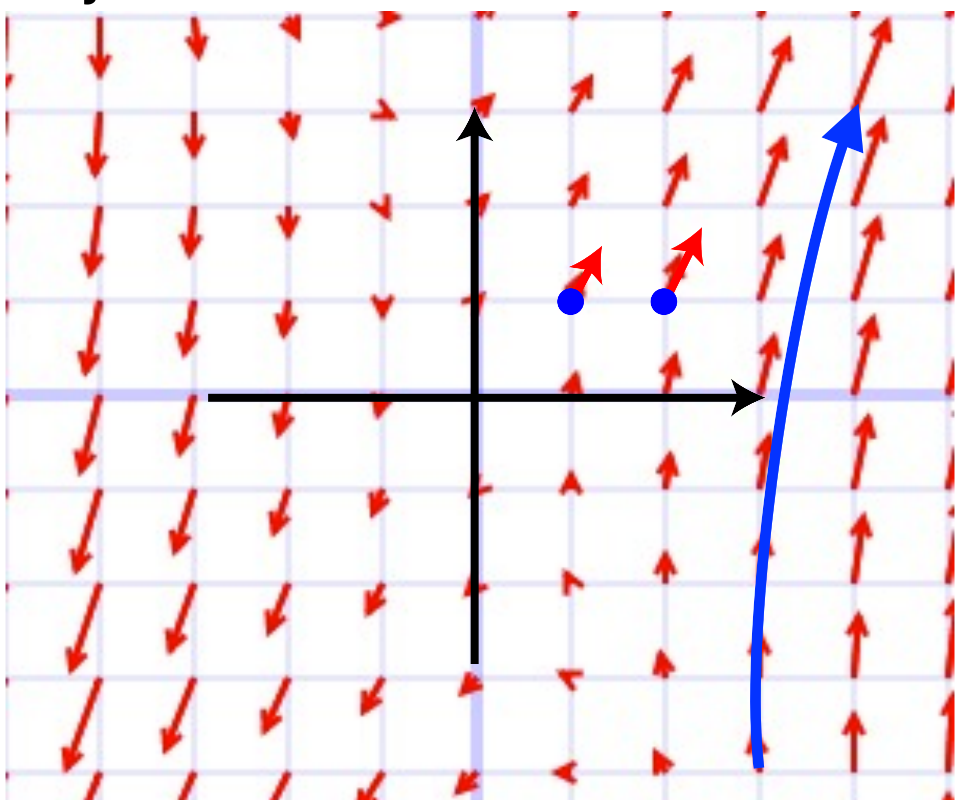

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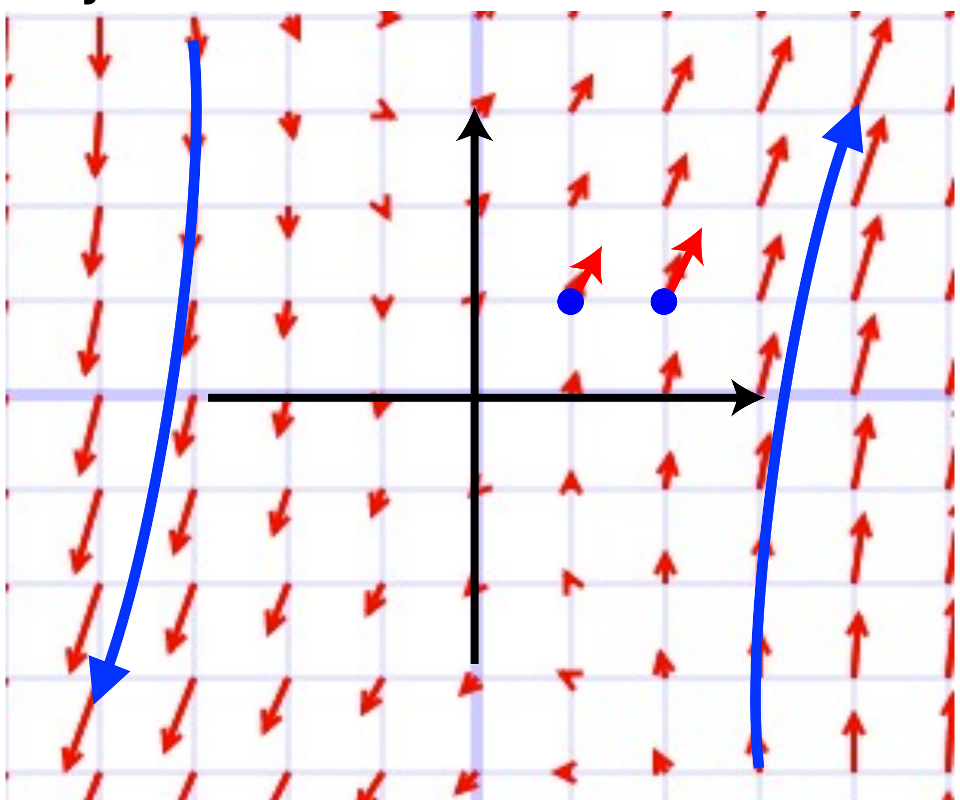

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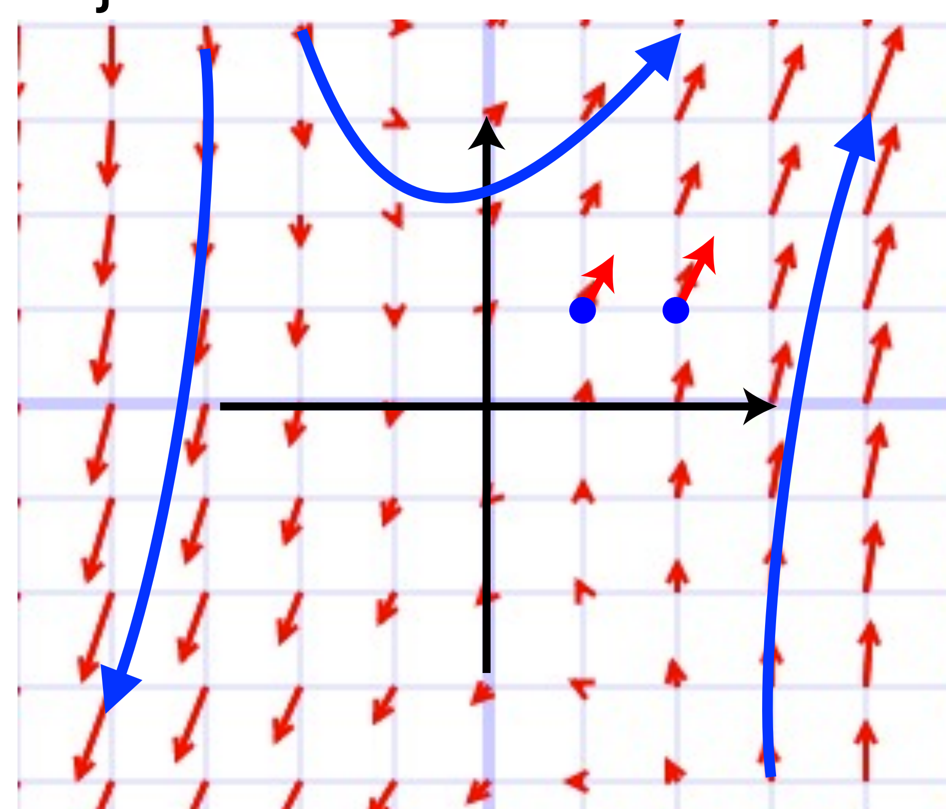

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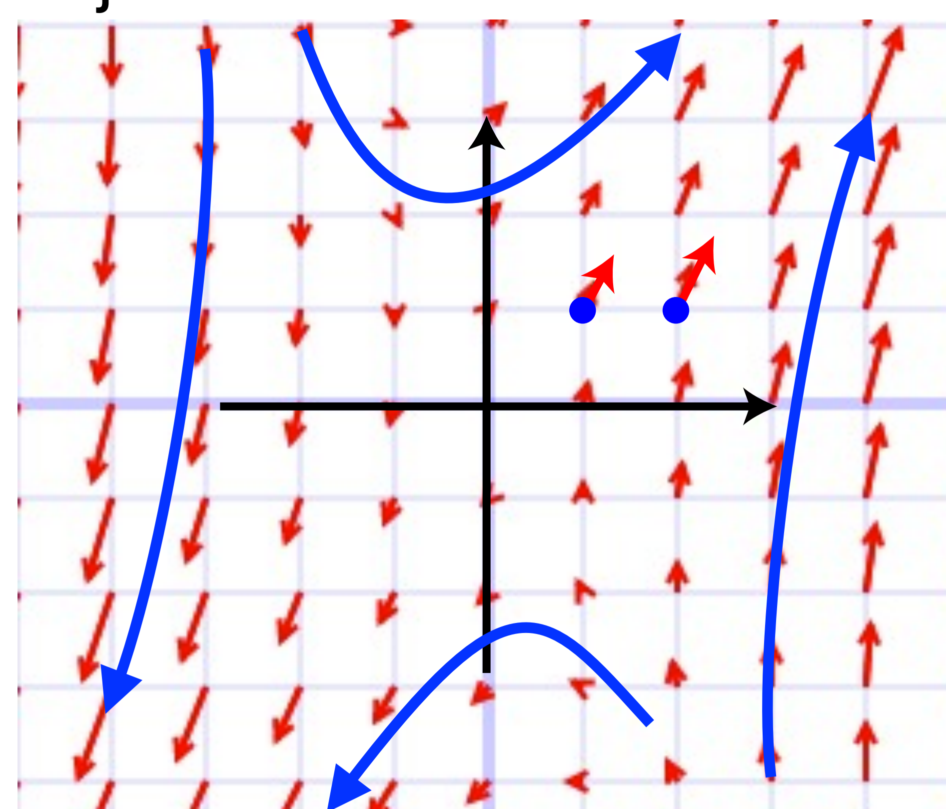

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- Which of the following equations matches the given direction field?

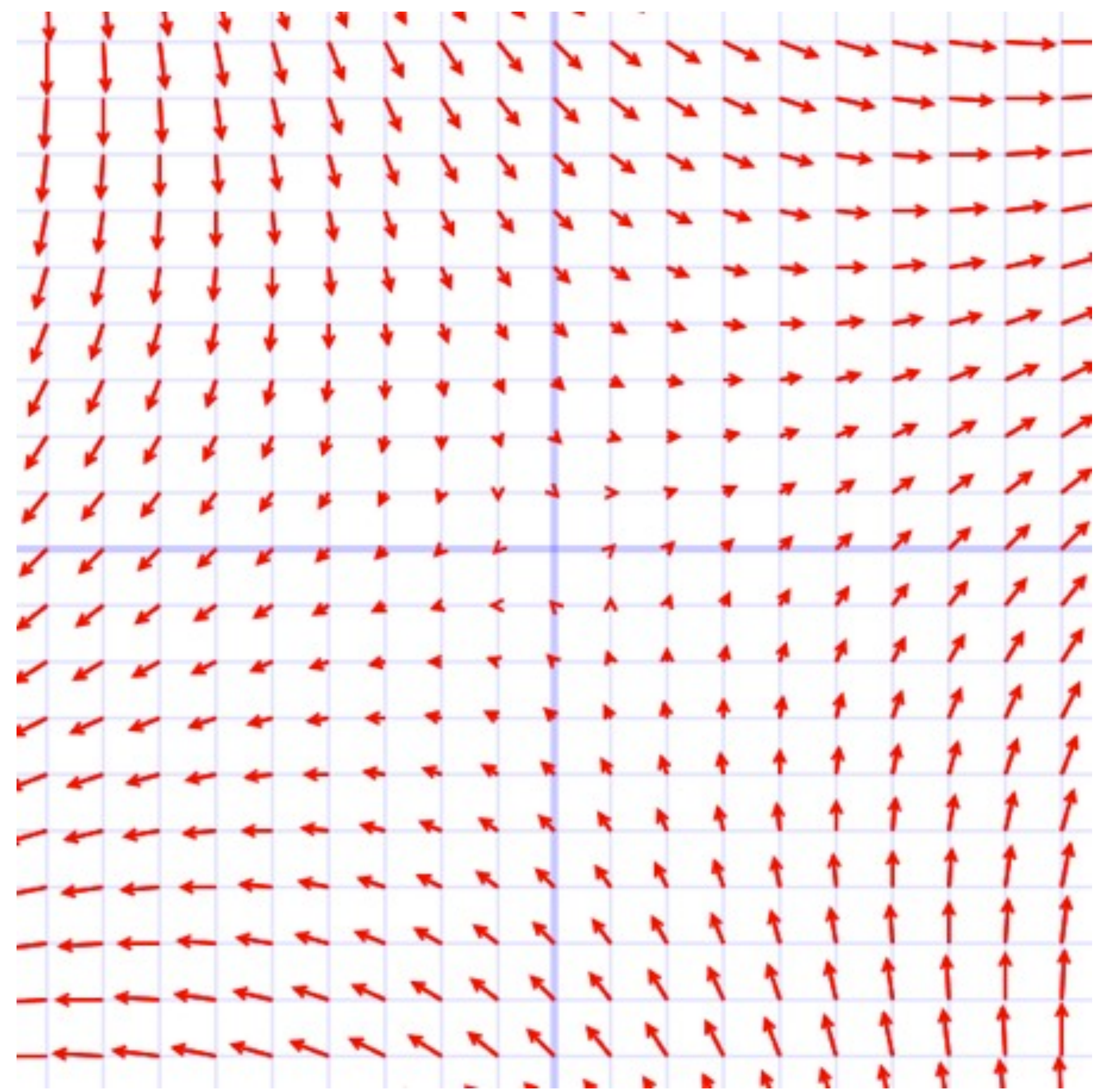
(A) $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(B) $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(C) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(D) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

(E) Explain, please.



http://kevinmehall.net/p/equationexplorer/vectorfield.html#(x+y)i+(x-y)j%7C%5B-10,10,-10,10%5D

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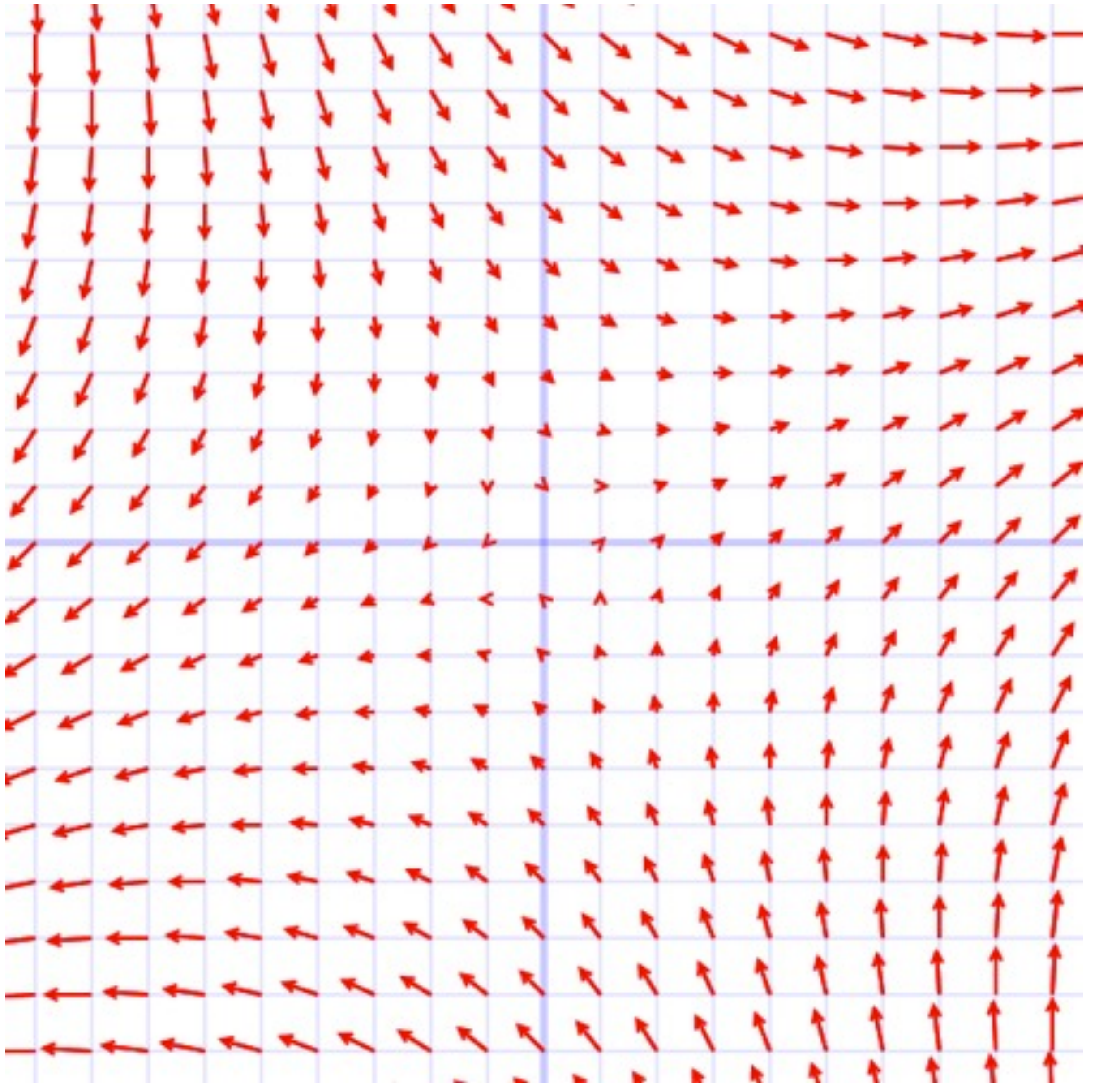

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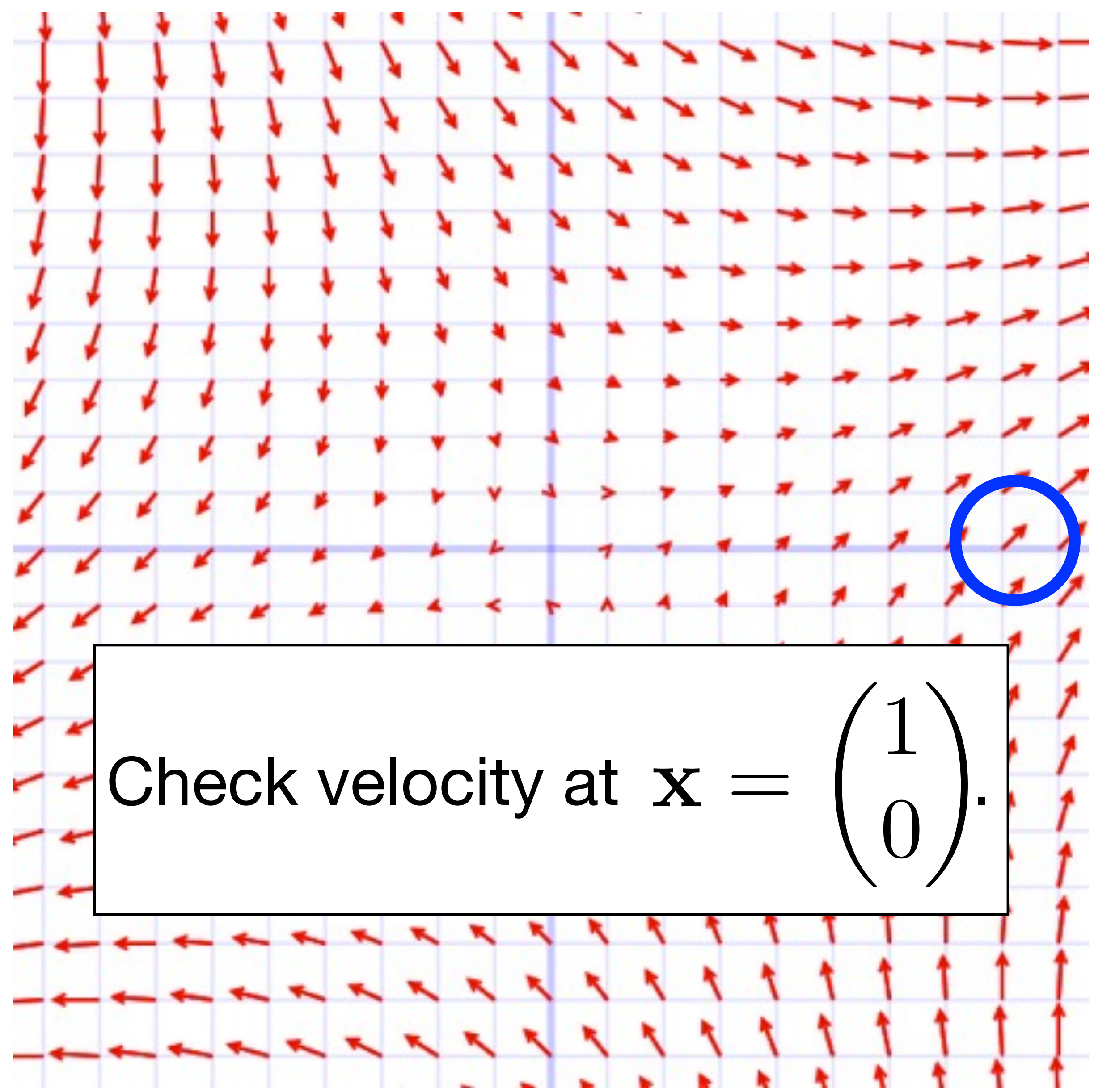

Check velocity at $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

http://kevinmehall.net/p/equationexplorer/vectorfield.html#(x+y)i+(x-y)j%7C%5B-10,10,-10,10%5D

Introduction to systems of equations

- Which of the following equations matches the given direction field?

(A) $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

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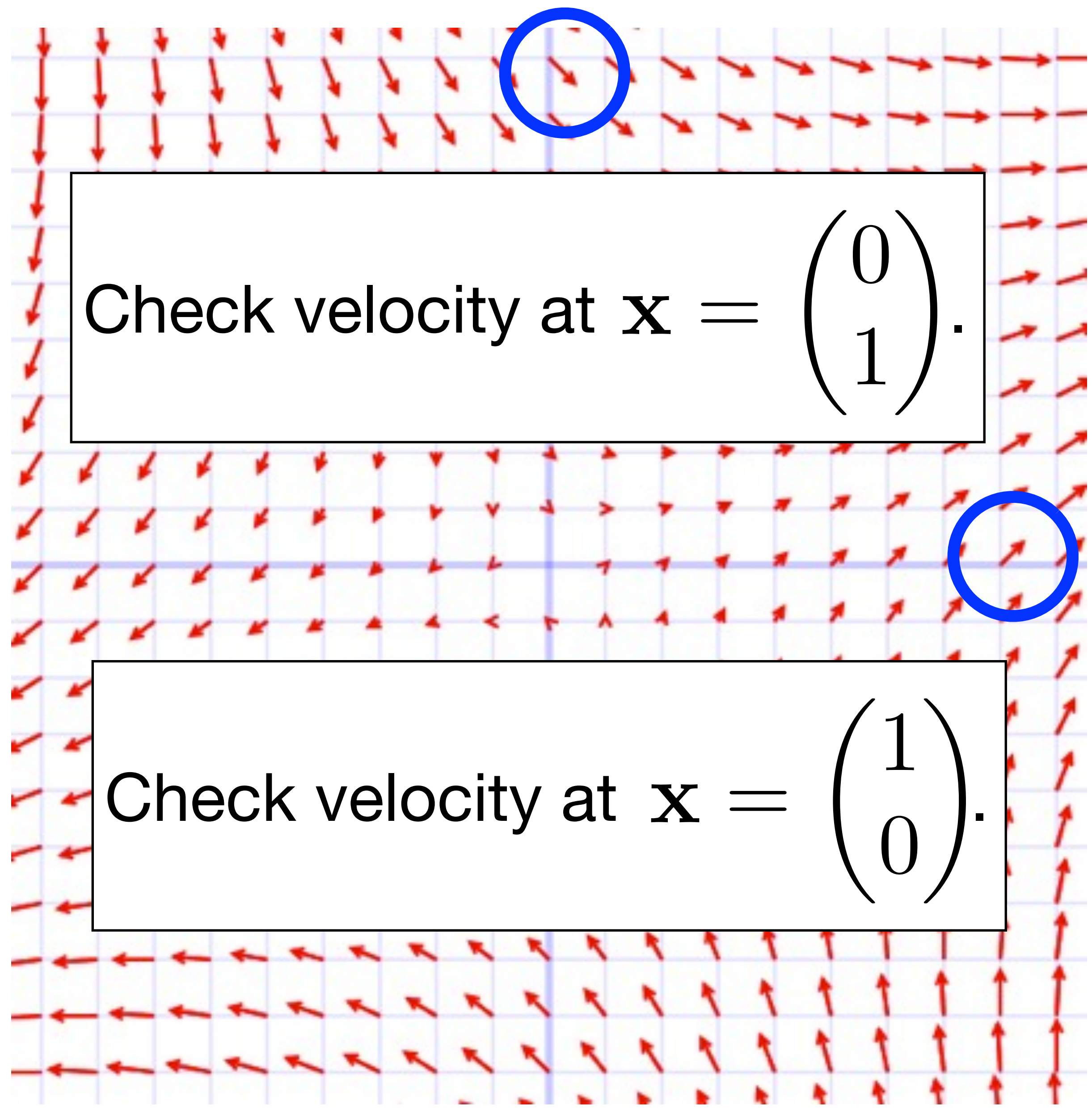

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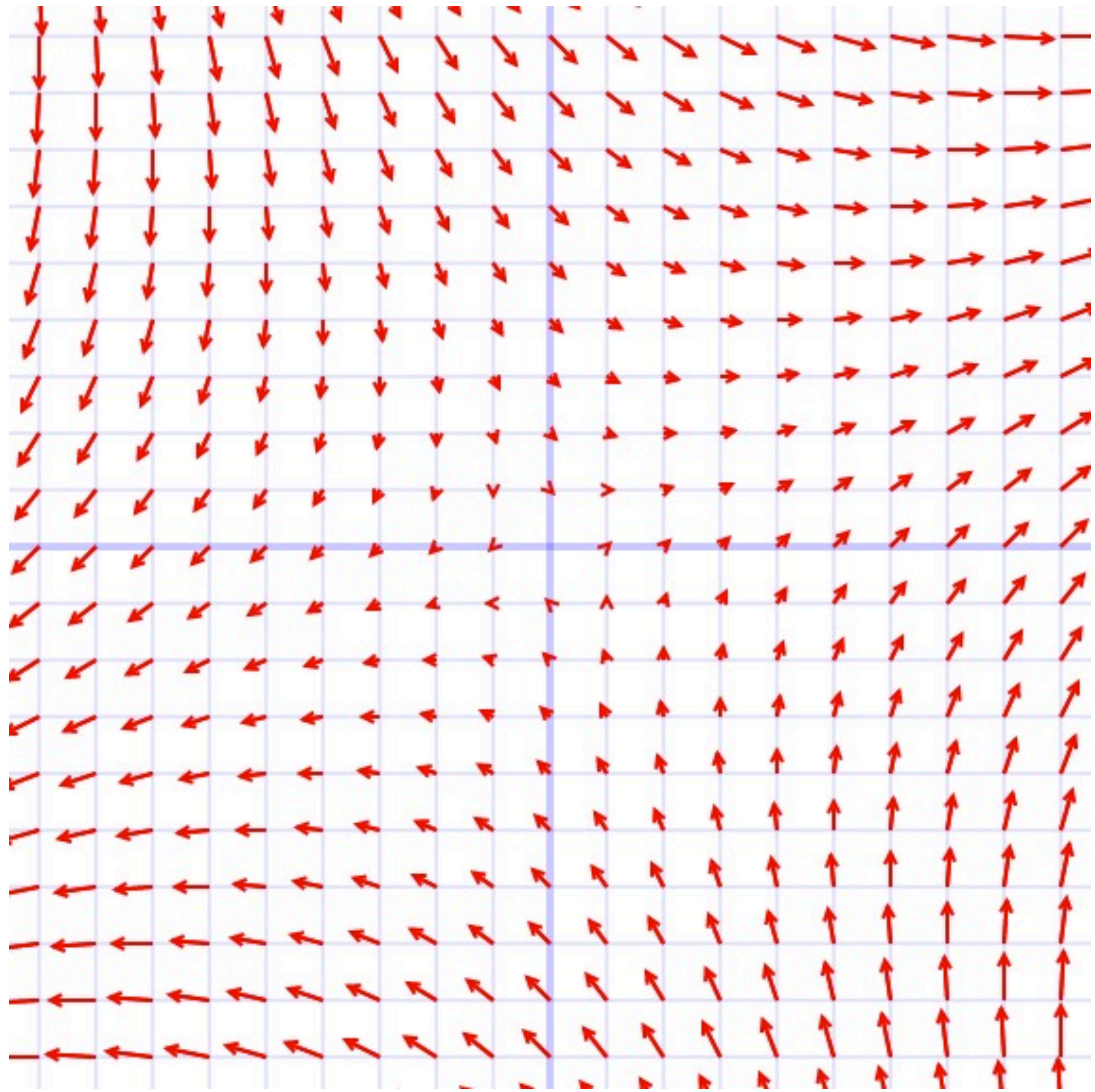
Introduction to systems of equations

- You should see two "special" directions.
- What are they?
- Directions along which the velocity vector is parallel to the position vector.
- That is, .



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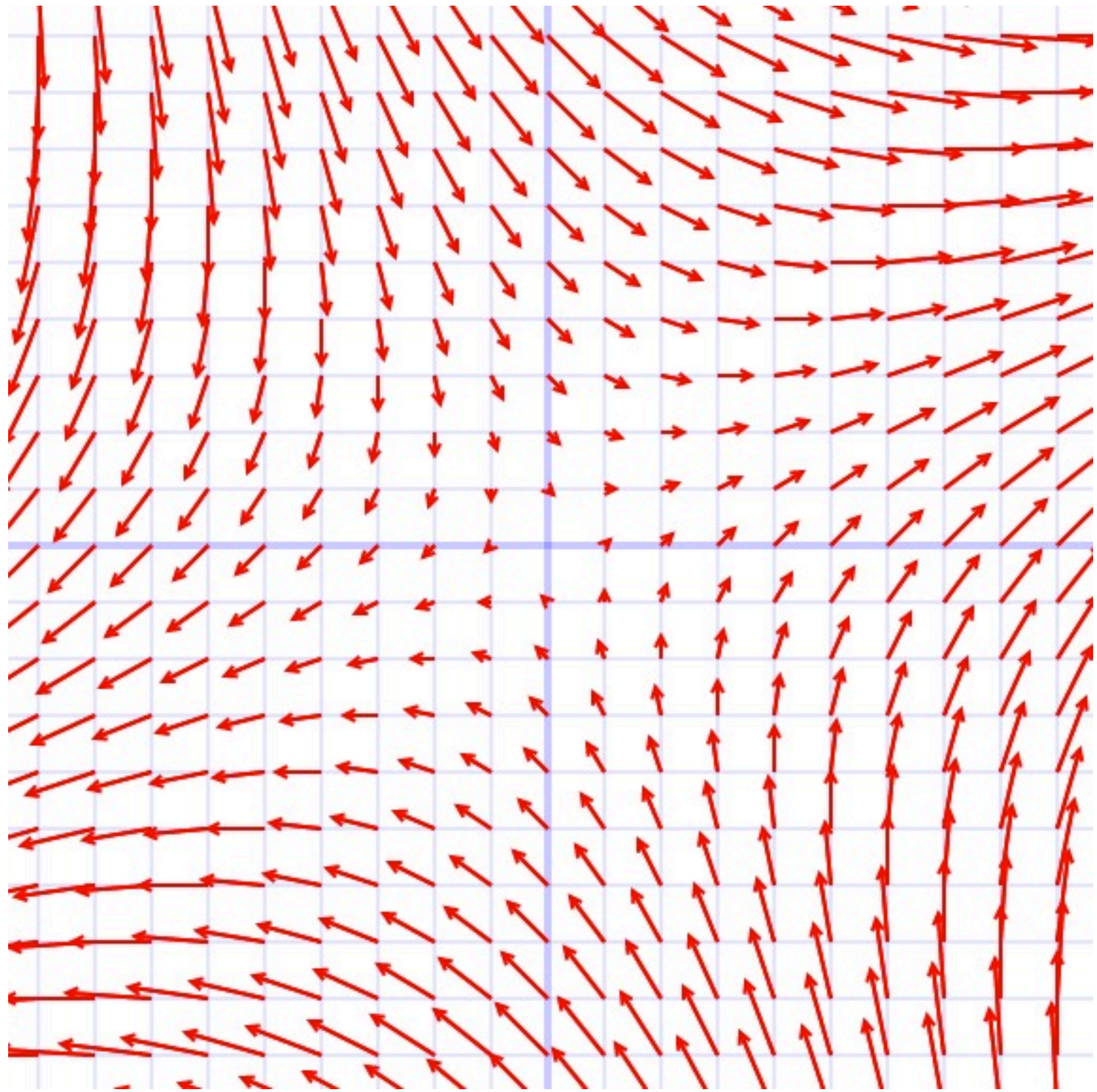

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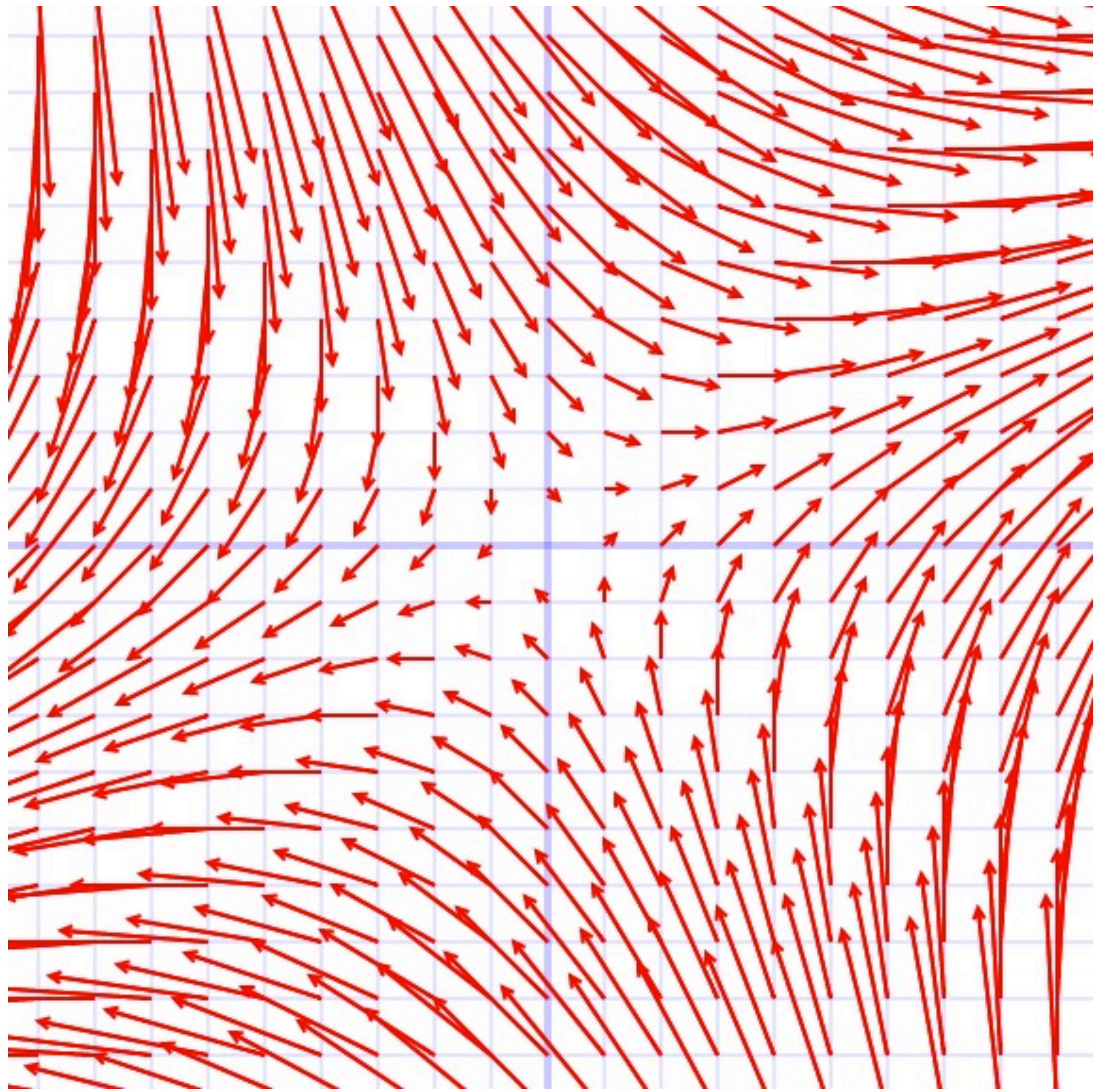

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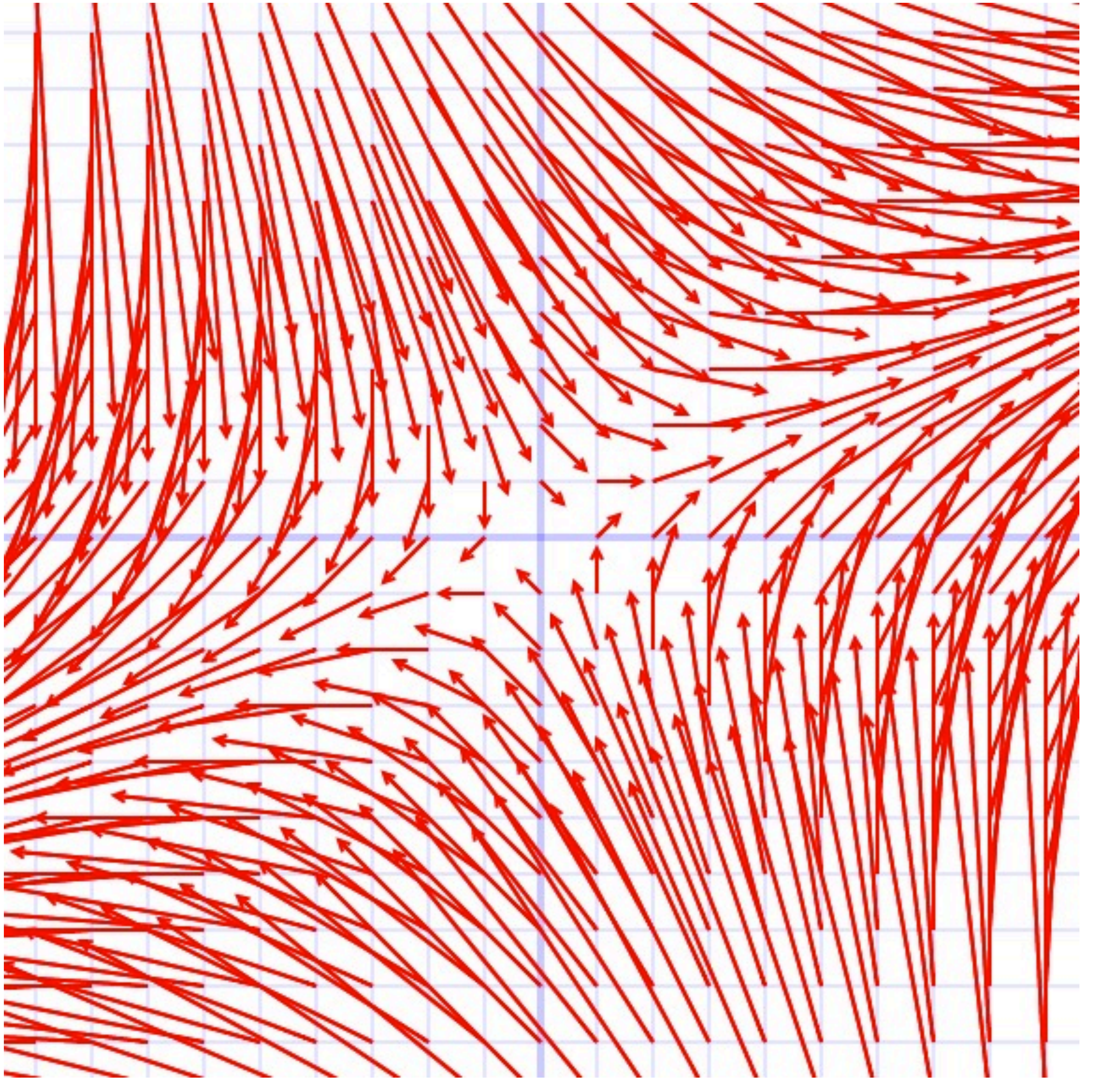

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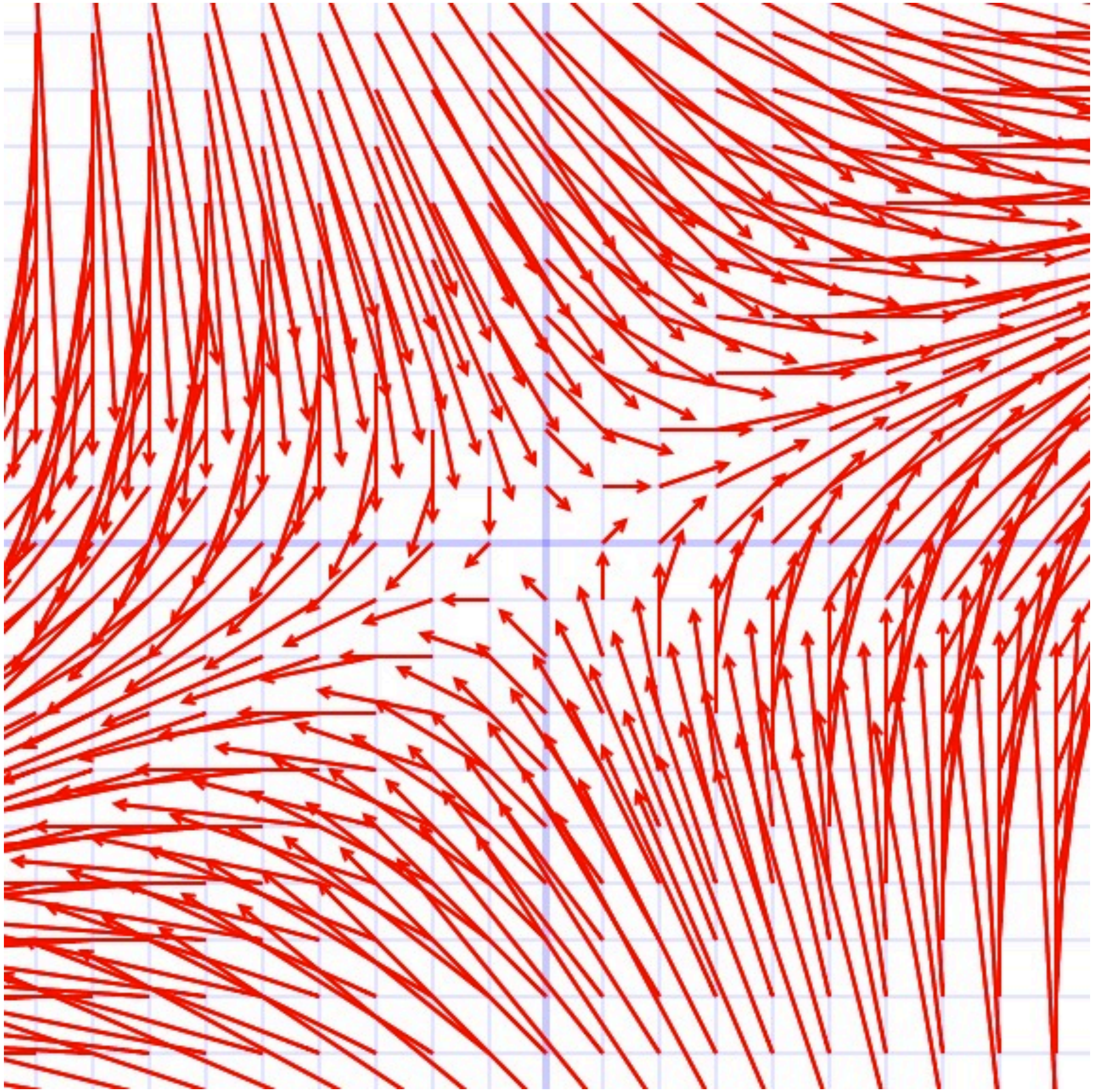

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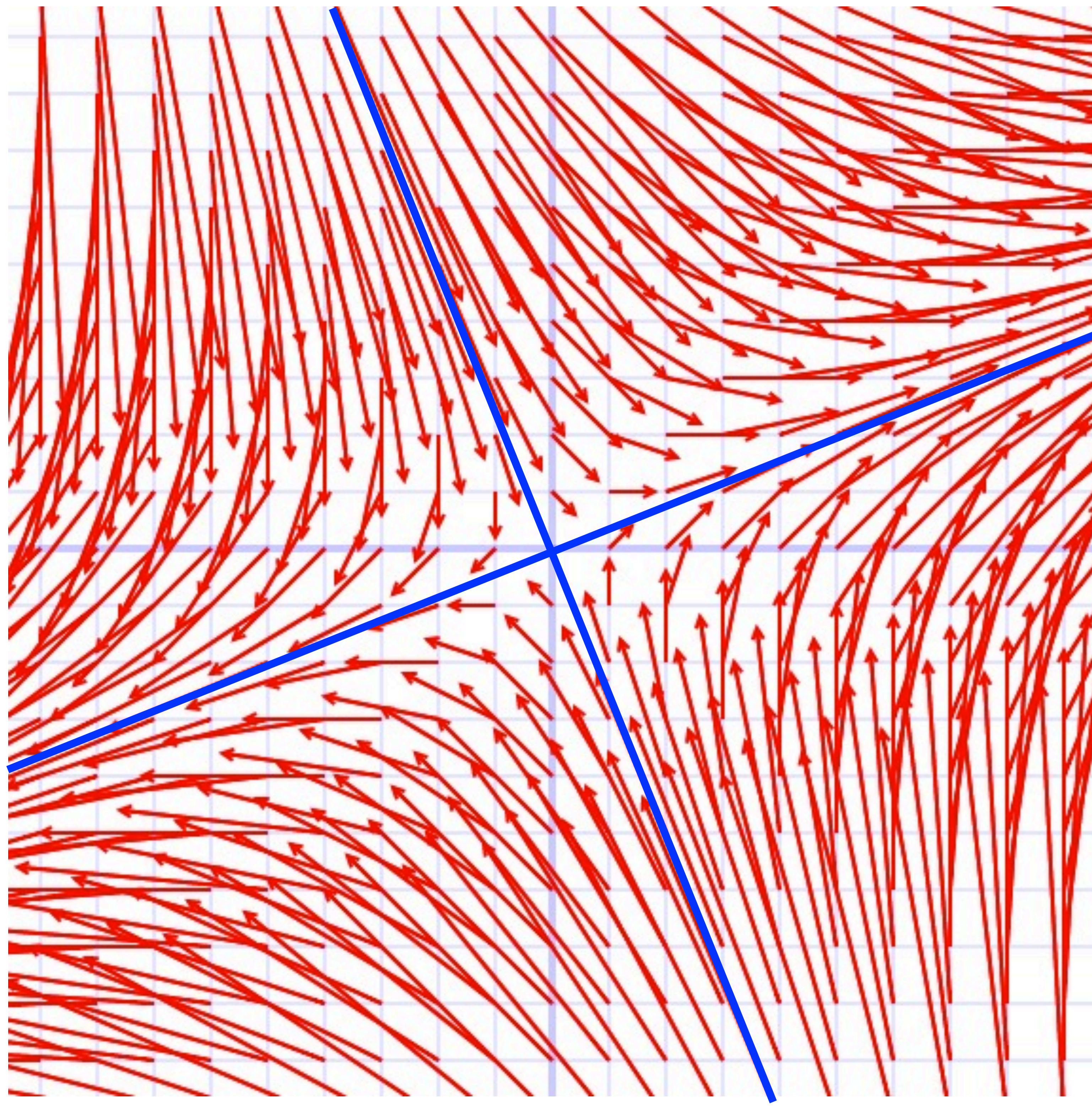

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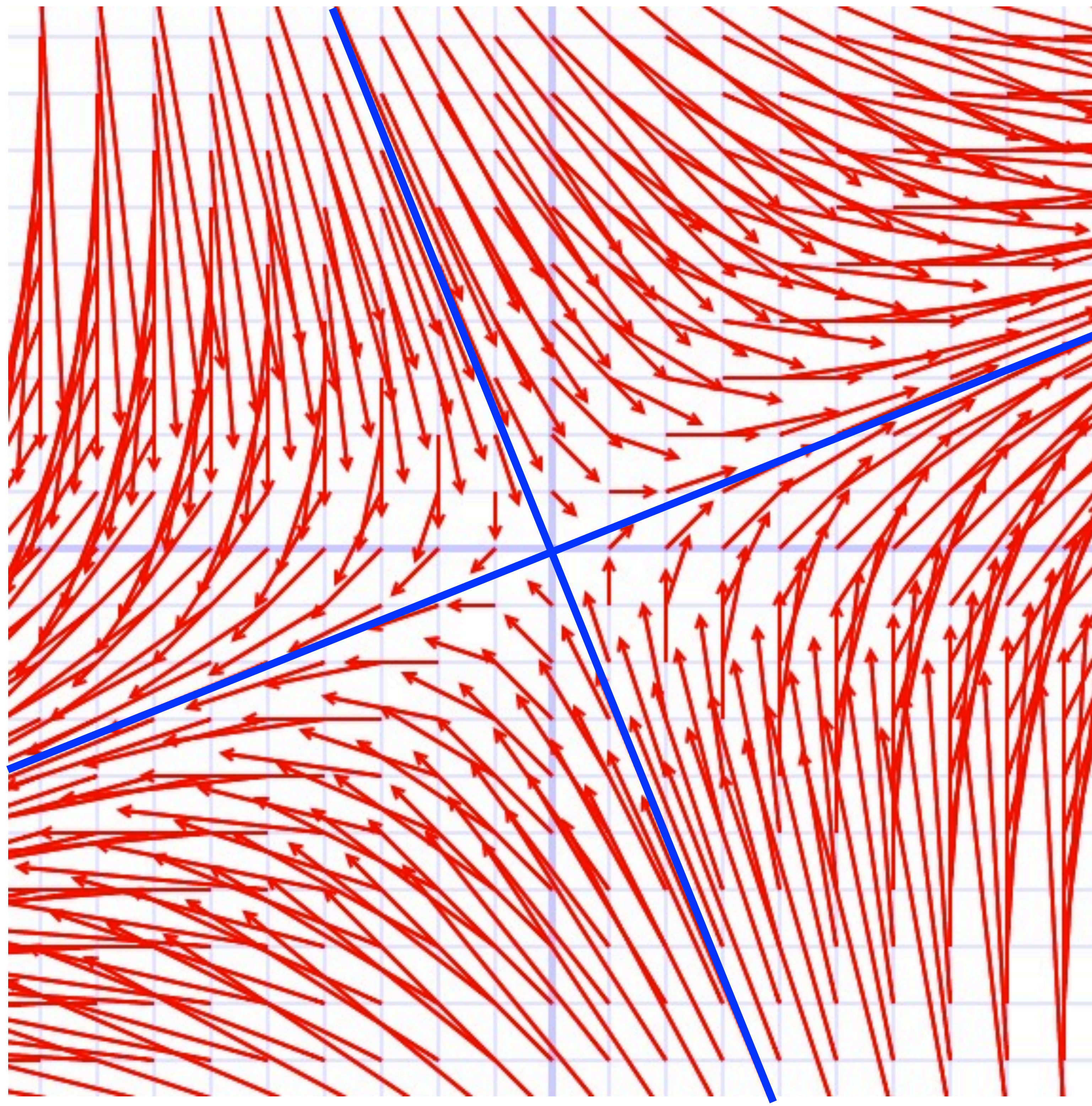

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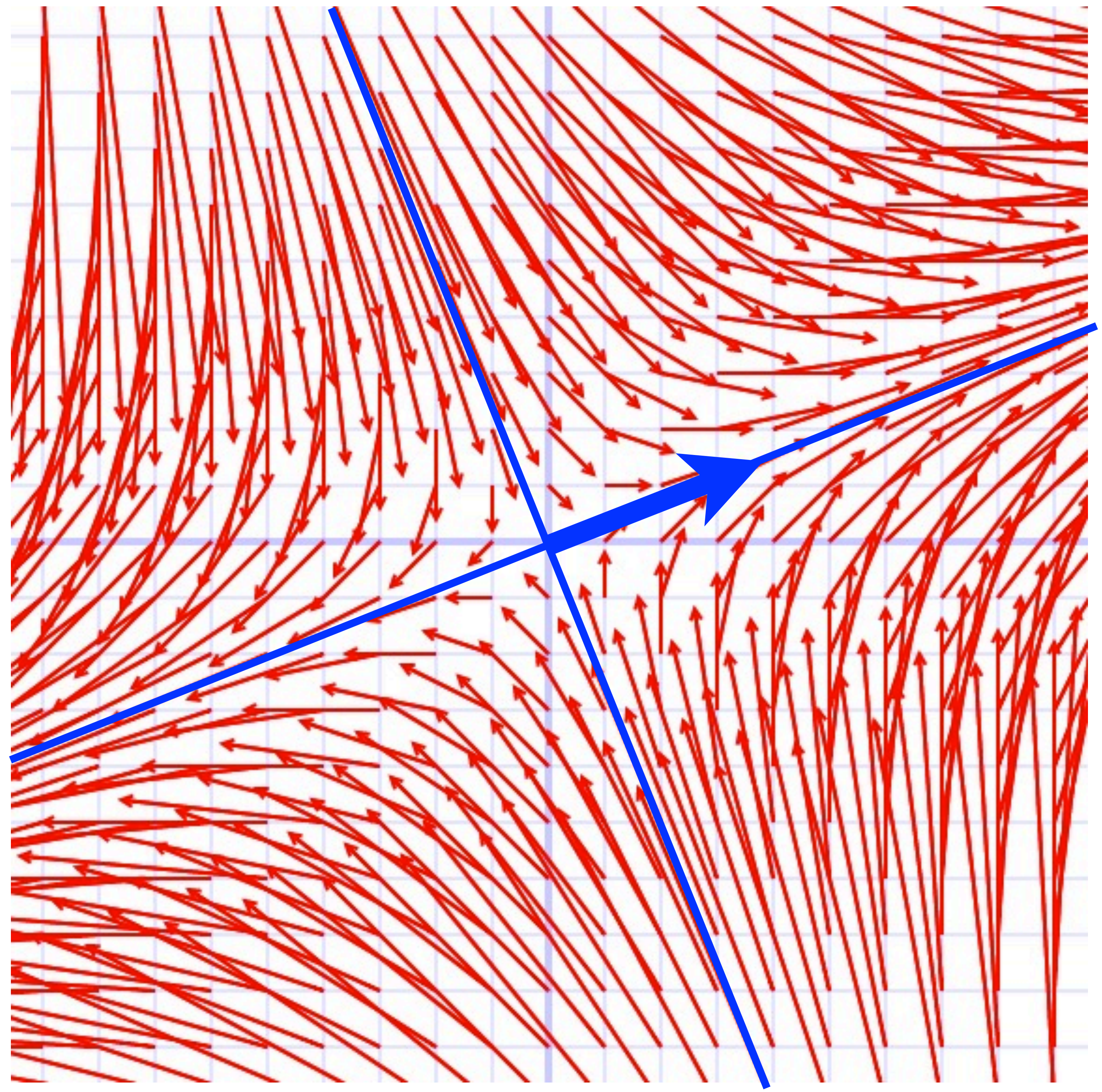

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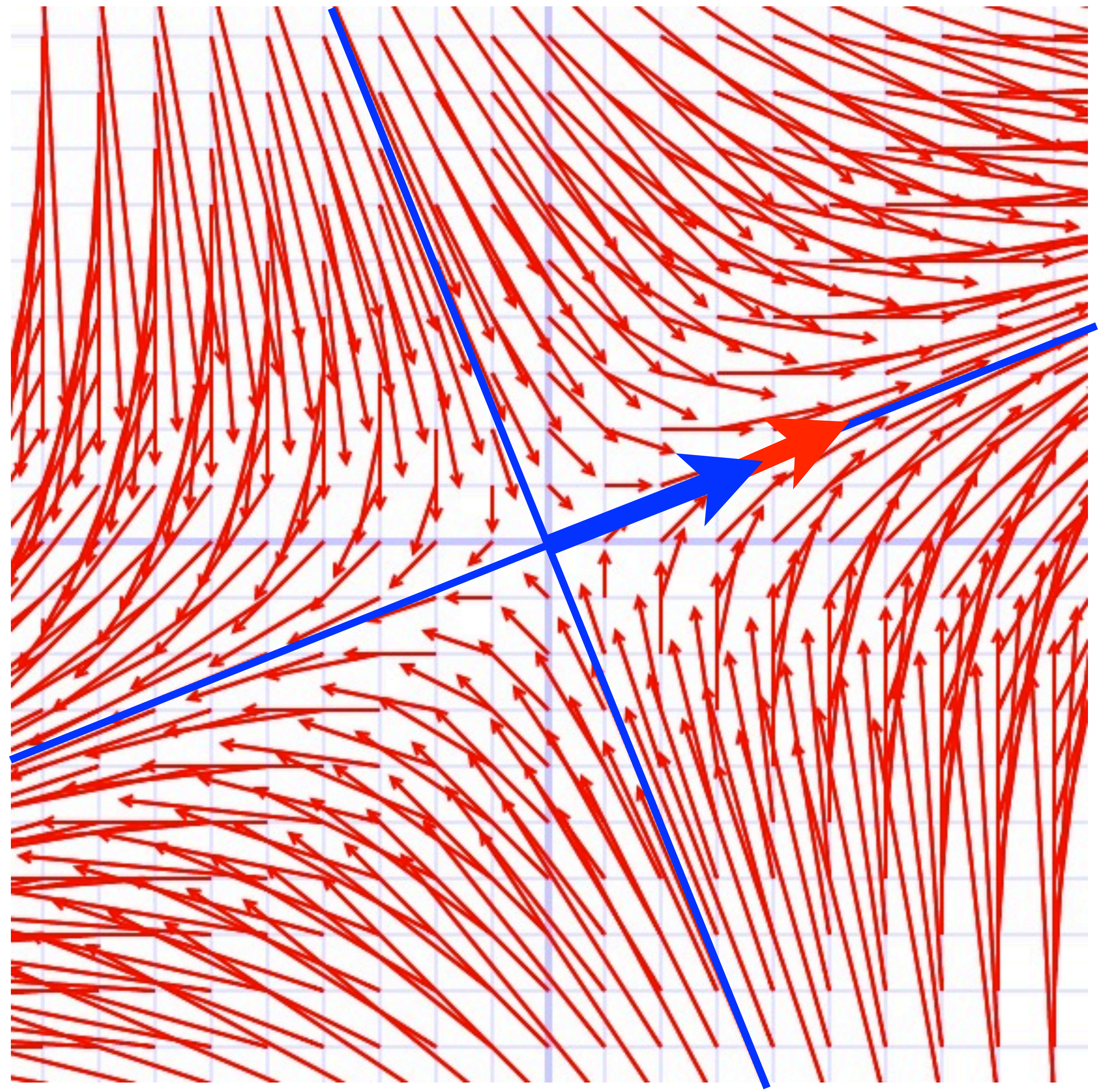

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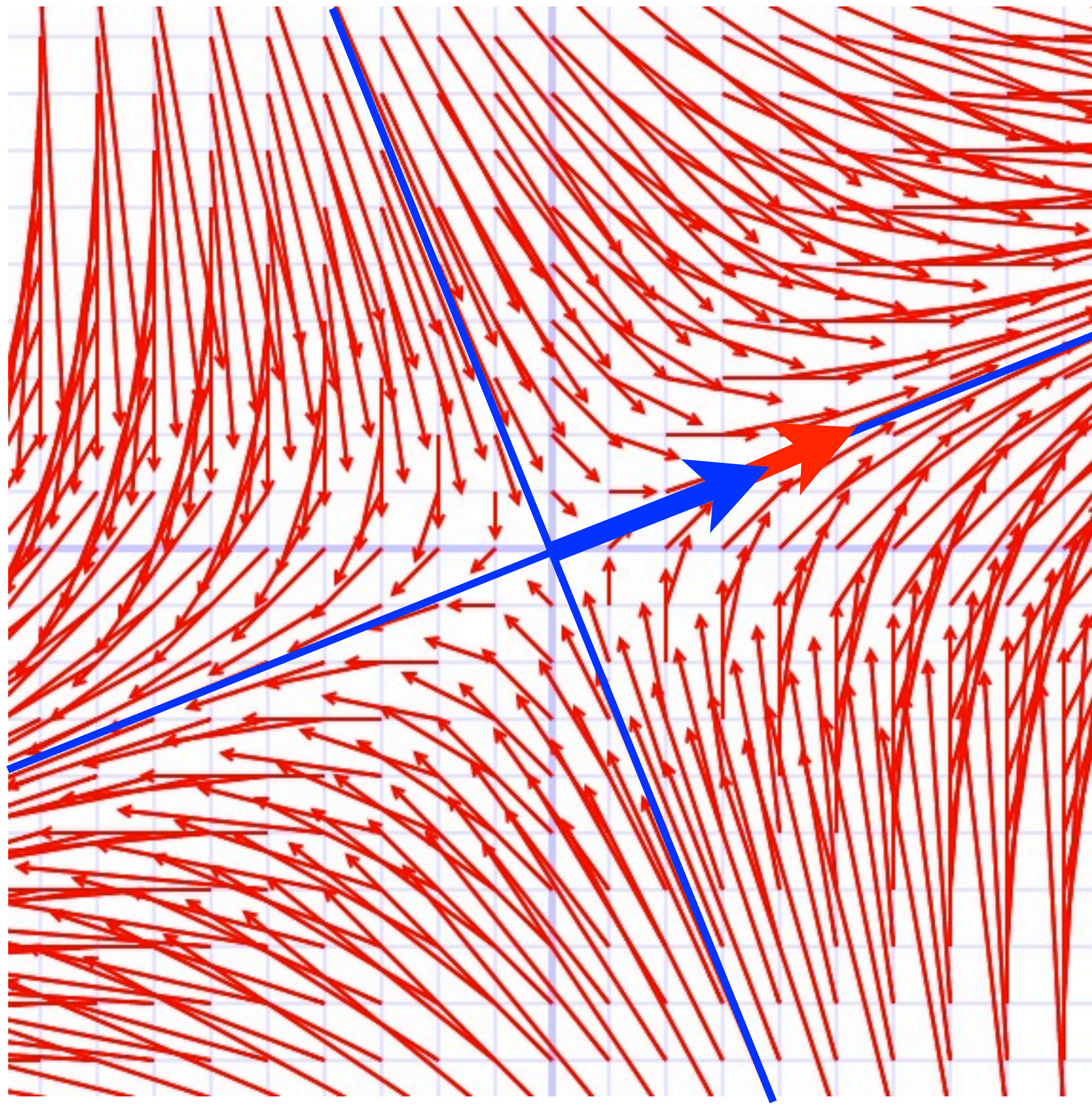
$$\lambda_1 = \sqrt{2}$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ \sqrt{2} - 1 \end{pmatrix}$$



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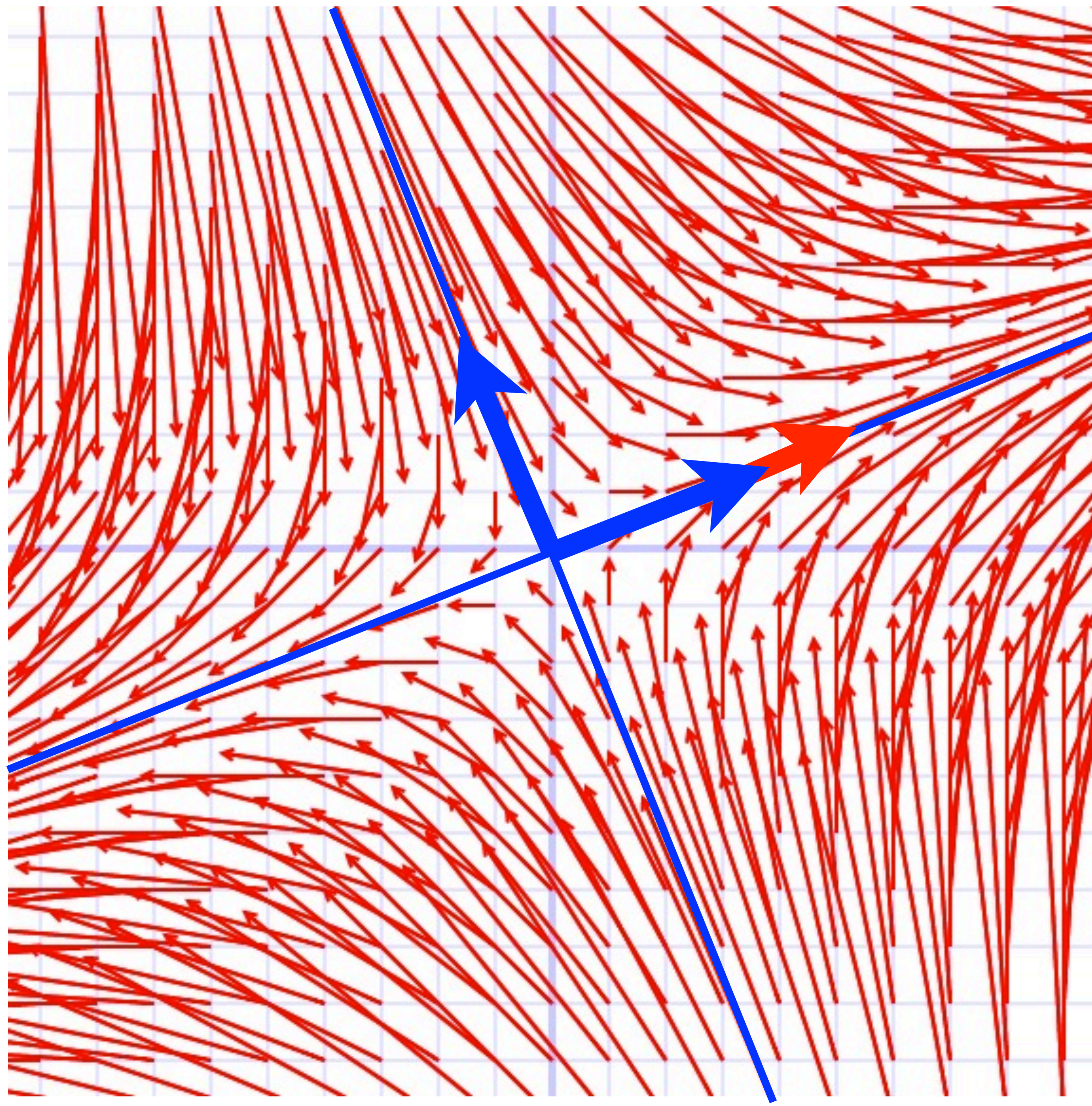

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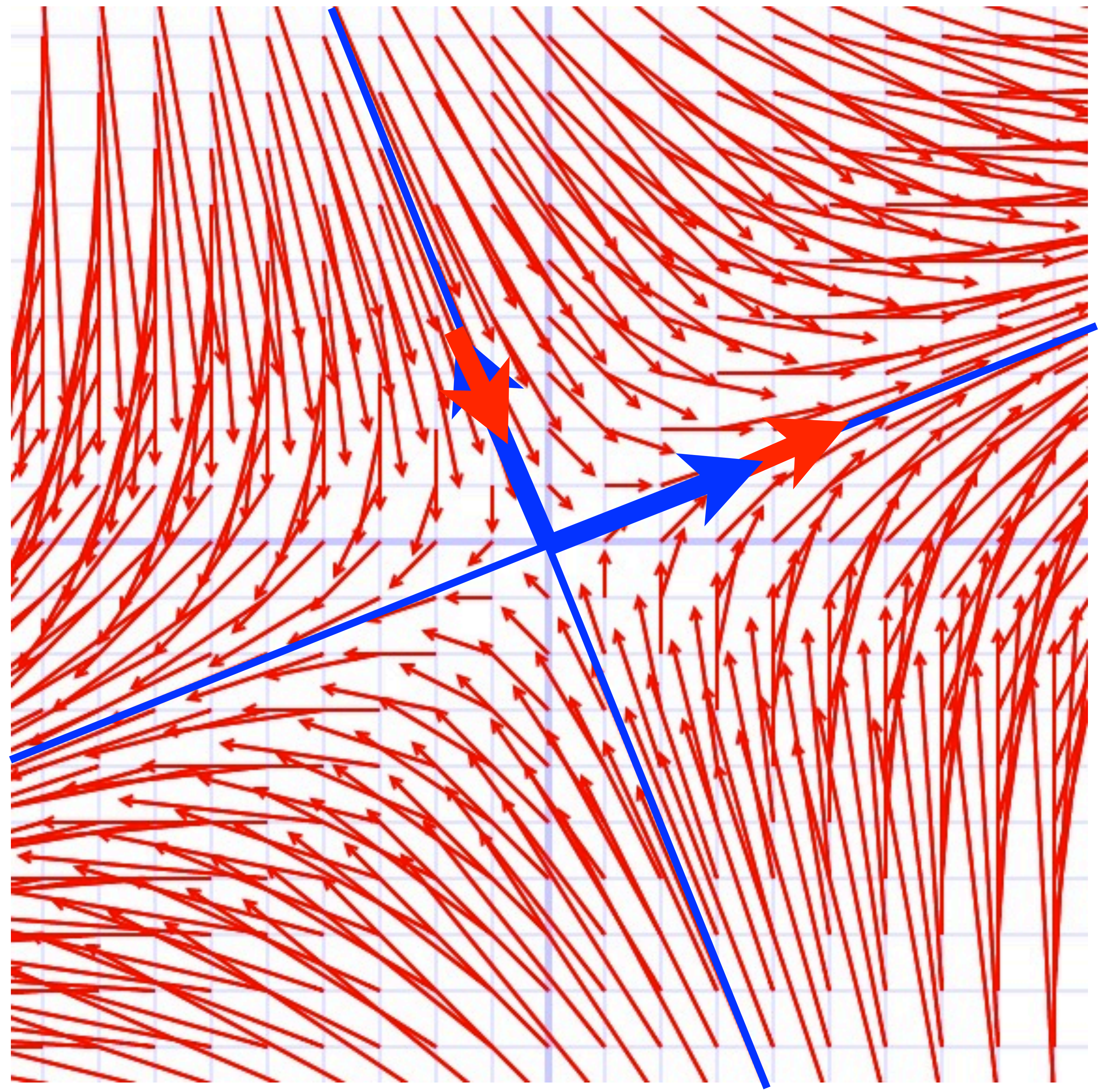

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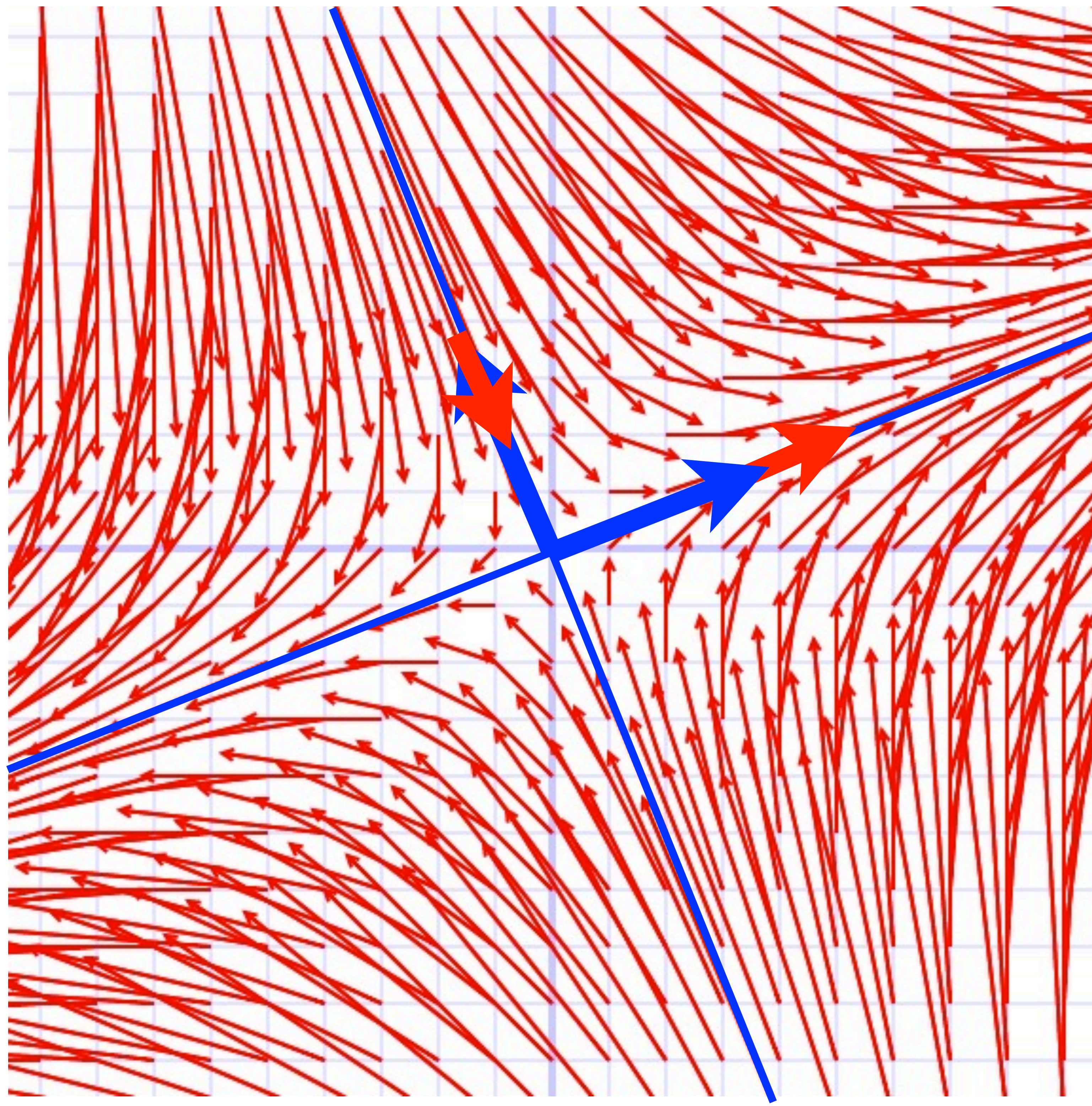
$$\mathbf{v_2} = \begin{pmatrix} 1-\sqrt{2} \\ 1 \end{pmatrix}$$



Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

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- What are the eigenvalues of A?

(A) 1 and -3

(B) -1 and 3

(C) 1 and 3

(D) -1 and -3

(E) Explain, please.

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(C) $\mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

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(and any scalar multiple of it)

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    - Other cases (not enough e-vectors or complex e-values) Thursday.