Laplace transforms - intro

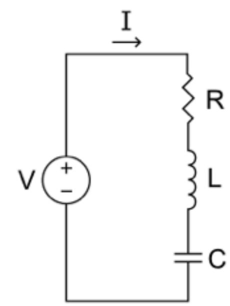
- Motivation for Laplace transforms:
  - We know how to solve $ay'' + by' + cy = g(t)$ when $g(t)$ is polynomial, exponential, trig.
  - In applications, $g(t)$ is often "piece-wise continuous" meaning that it consists of a finite number of pieces with jump discontinuities in between, or include "impulses". For example,

$$g(t) = \begin{cases} \sin(\omega t) & 0 < t < 10, \\ 0 & t \geq 10. \end{cases}$$

  - These can be handled by previous techniques (modified) but it isn't pretty (solve from t=0 to t=10, use y(10) as the IC for a new problem starting at t=10).

Laplace transforms - intro

- Motivation for Laplace transforms - example RLC circuit

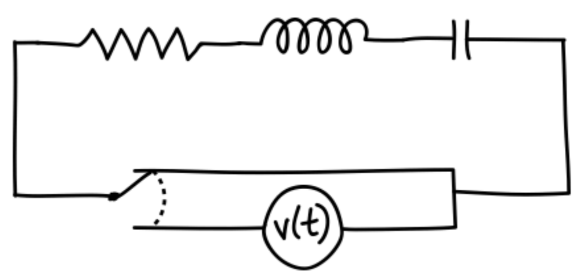


  - Resistor, inductor and capacitor in series

$$I''(t) + \frac{R}{L}I'(t) + \frac{1}{LC}I(t) = v(t)$$

  - If v(t) comes from radio waves then $v(t) = A\cos(\omega t)$ and the circuit is called a radio receiver.

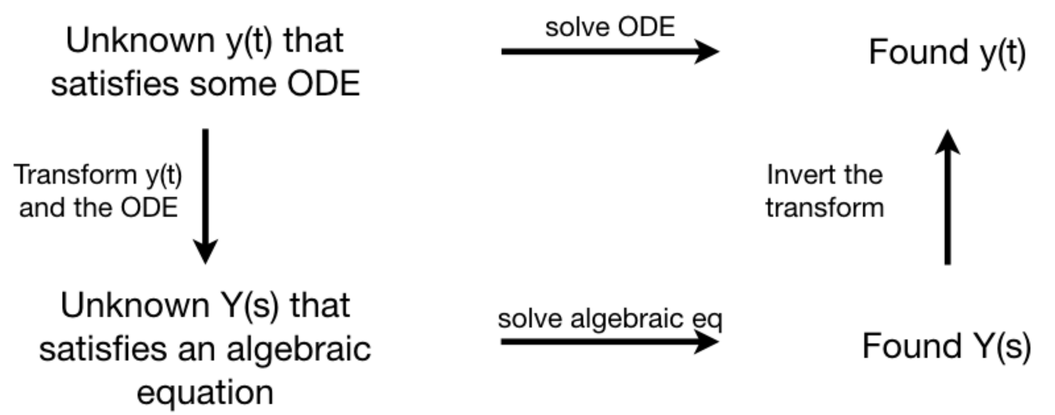
  - For $v(t) = \begin{cases} 1 & 0 < t < 10 \\ 0 & t \geq 10 \end{cases}$, the circuit has a switch that gets flipped at t=10.



Laplace transforms - intro

- Instead of not-so-pretty techniques, we use Laplace transforms.

- Idea:



- Laplace transform of y(t): $\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} y(t)\ dt$

Laplace transforms - examples

- What is the Laplace transform of $y(t) = 3$?

$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} 3 \, dt$$

$$= -\frac{3}{s} e^{-st} \Big|_0^\infty$$

$$= \lim_{A \to \infty} -\frac{3}{s} e^{-st} \Big|_0^A$$

$$= -\frac{3}{s} \left(\lim_{A \to \infty} e^{-sA} - 1 \right)$$

$$= \frac{3}{s}$$ provided $s > 0$ and does not exist otherwise.

Laplace transforms - examples

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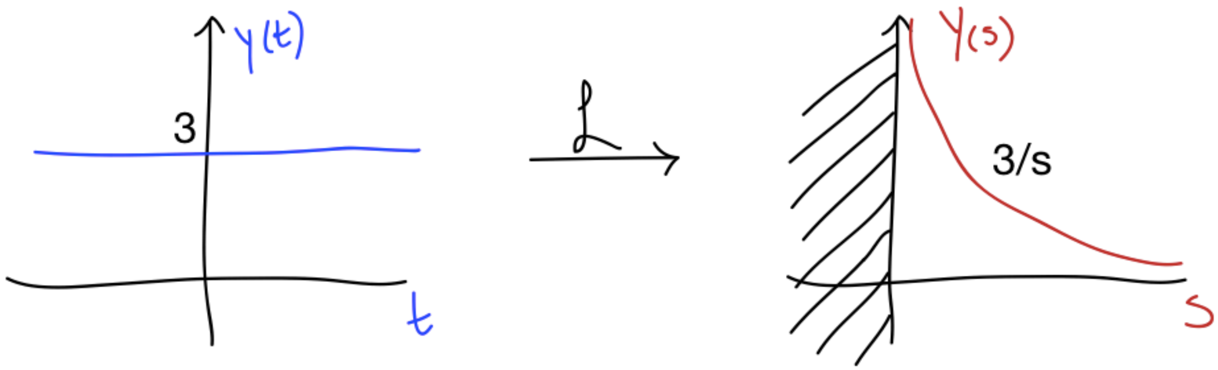
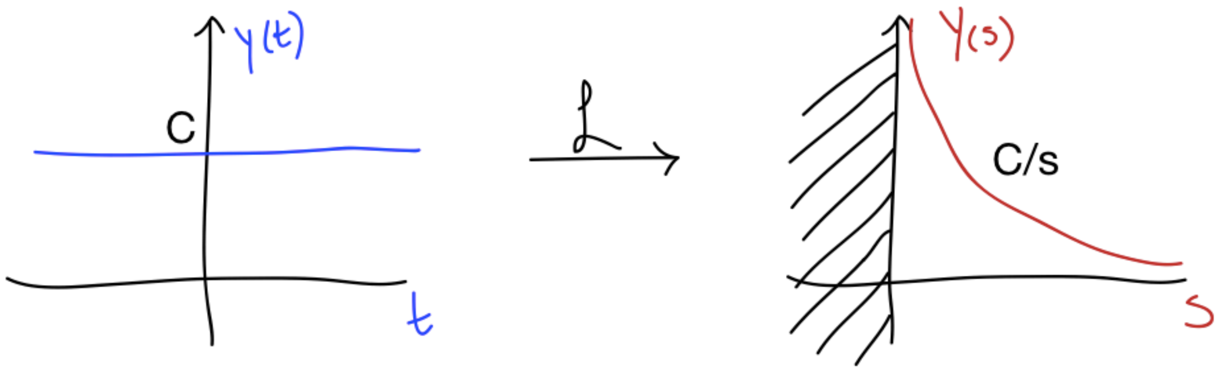


Laplace transforms - examples

- What is the Laplace transform of $y(t) = C$?

$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} C \; dt$$

$$= \frac{C}{s}$$ provided $s > 0$ and does not exist otherwise.



Laplace transforms - examples

- What is the Laplace transform of $y(t) = e^{6t}$?

$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} e^{6t} \, dt$$

(A) $Y(s) = \frac{1}{s-6} \quad s > 0$

(C) $Y(s) = \frac{1}{s-6} \quad s > 6$

(B) $Y(s) = \frac{1}{6-s} \quad s > 6$

(D) $Y(s) = \frac{1}{6-s} \quad s > 0$

Laplace transforms - examples

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