MATH 256-201

Tutorial 5 Worksheet

Feb 6, 2017

Surname: _____ Given name: _____

Student number: _____

1. Salt water with concentration of K in g/L flows into Tank A at a rate a in L/min. The mixed solution in Tank A flows into Tank B at a rate b L/min. Another pipe takes the solution in Tank B back into Tank A at a rate c in L/min. Finally, solution drains out of Tank B at a rate a in L/min. *Note*: the initial volume V in each tank is the same and $a + c = b$ (to ensure that the volumes in the tanks are constant).

   (a) Write down the system in matrix form that gives the amount of salt in each tank (call these $A(t)$ and $B(t)$) at any given time.

   (b) Show that the eigenvalues of the matrix for this system are real, and therefore it is impossible for this system to have oscillations (damped or otherwise). *Hint*: express the matrix in terms of b and c only. *Optional*: show that both eigenvalues are negative.

2. Consider the matrix

$$A = \begin{pmatrix} 3\alpha - 8 & 7 \\ -4 & 3\alpha + 8 \end{pmatrix}.$$

(a) Find the eigenvalues and eigenvectors of A.

(b) For what values of α (i) are both eigenvalues postive, (ii) are both eigenvalues negative, (iii) do the eigenvalues have opposite sign, (iv) is one eigenvalue zero?

(c) Sketch the eigenvectors and some solution curves of the system $\mathbf{x}' = A\mathbf{x}$ in the case when $\alpha = 0$.