Today

- Reminders:
    - WeBWorK assignment 1 due Thursday 1 pm.
    - Quiz 2 on Monday in tutorial sections.
    - If you have questions, post them on Piazza (don’t email me) and/or come to office hours.
- Modeling (Section 2.3)
- Existence and uniqueness (Section 2.4 - not going test on the theory)
- Second order linear equations - constant coefficients, Wronskian (3.1, 3.2)

Office hours

- Eric - MATX 1219 (might change to MATX 1215 during the term)
  - Tues 11 am -12:30 pm
  - Wed 1 pm - 2 pm
- Ye - location TBA in tutorial
  - Fri 1 pm - 2 pm
- Mengdi - location TBA in tutorial
  - Mon 1 pm - 2 pm

Modeling (Section 2.3)

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  - Choose a small interval of time, Δt, and add up all the changes.
  - Note that $q(t + \Delta t) = q(t) +$ change during intervening Δt.
  - Take limit as $\Delta t \to 0$ to get an equation for q(t).

Modeling (Section 2.3) - Example

- Freshwater flows into a tank at a rate 2 L/min. The tank starts with a concentration of 100 g / L of salt in it and holds 10 L. The tank is well mixed and the mixed water drains out at the same rate as the inflow.

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(B) $\Delta m \approx -2\ \mathrm{L/min}\ \times 100\ \mathrm{g/L} \times \Delta t$

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- $m(t+\Delta t) \approx m(t) - \Delta t \ \times\ 2\ \mathrm{L/min}\ \times\ m(t)\ /\ 10\ \mathrm{L}$
- Rearranging: $\dfrac{m(t+\Delta t) - m(t)}{\Delta t} \approx -\dfrac{1}{5}m(t)$
- Finally, taking a limit:

$$\boxed{\frac{dm}{dt} = -\frac{1}{5}m(t)}$$

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- m(0) = 100 g / L.

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- What method could you use to solve the ODE $\dfrac{dm}{dt} = -\dfrac{1}{5}m(t)$?

  (A) Integrating factors.

  (B) Separating variables.

  (C) Just knowing some derivatives.

  (D) All of these.

  (E) None of these.

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To think about: what is the most general equation that can be solved using (A) and (B)?

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    (A) $m(t) = Ce^{-t/5}$

    (B) $m(t) = 100e^{-t/5}$

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Answer to (b)?

$$\lim_{t\to\infty} m(t) = 0$$

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(A) $m' = 200 - 2m, \quad m(0) = 0$

(B) $m' = 400 - 2m, \quad m(0) = 200$

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- $m=2000$. Called steady state - a constant solution.
- What happens when $m < 2000$? ---> $m' > 0$.
- What happens when $m > 2000$? ---> $m' < 0$.
- Limiting mass: 2000 g (Long way: solve the eq. and let $t \rightarrow \infty$.)

Existence and uniqueness (Section 2.4)

Theorem 2.4.2 Let the functions f and $\dfrac{\partial f}{\partial y}$ be continuous in some rectangle $\alpha < t < \beta, \quad \gamma < y < \delta$ containing the point (t_0, y_0). Then, in some interval $t_0 - h < t_0 < t_0 + h$ contained in $\alpha < t < \beta$, there is a unique solution $y = \phi(t)$ of the IVP

$$y' = f(t, y), \quad y(t_0) = y_0.$$

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- A couple questions/examples to explore on your own:

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- A couple questions/examples to explore on your own:
  - Why don't we get a solution all the way to the ends of the t interval?
    - Example: $$\frac{dy}{dt} = y^2, \quad y(0) = 1$$
  - How does a non-continuous RHS lead to more than one solution?
    - Example: $$\frac{dy}{dt} = \sqrt{y}, \quad y(0) = 0$$

Second order linear equations (Chapter 3)

- The general form for a second order linear equation:

$$y'' + p(t)y' + q(t)y = g(t)$$

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- As with first order linear equations, we have homogeneous (g=0) and non-homogeneous second order linear equations.

- We'll start by considering the homogeneous case with constant coefficients:

$$ay'' + by' + cy = 0$$

Homog. eq. with constant coeff. (Section 3.1)

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- Notice that $y(t) = C_1 y_1(t)$ is also a solution. Plug it in and check:

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$$a(C_1y_1)'' + b(C_1y_1)' + c(C_1y_1)$$

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$$\begin{aligned} &a(C_1y_1)'' + b(C_1y_1)' + c(C_1y_1) \\ &\quad = aC_1(y_1)'' + bC_1(y_1)' + cC_1(y_1) \end{aligned}$$

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Homog. eq. with constant coeff. (Section 3.1)

$$ay'' + by' + cy = 0$$

- Suppose you already found a couple solutions, $y_1(t)$ and $y_2(t)$. This means that

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Homog. eq. with constant coeff. (Section 3.1)

- Which of the following functions are also solutions?

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  (B) $y(t) = y_1(t)+y_2(t)$

  (C) $y(t) = y_1(t)\ y_2(t)$

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- The general solution will be $y(t) = C_1y_1(t) + C_2y_2(t)$.

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$$y'' + y' = 0$$

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