Today

- Summary of 2x2 systems all in one picture
- Non-homogeneous systems of ODEs
- Non-homogeneous two-tank example
- Intro to Laplace transforms

Summary - homogeneous 2x2 systems

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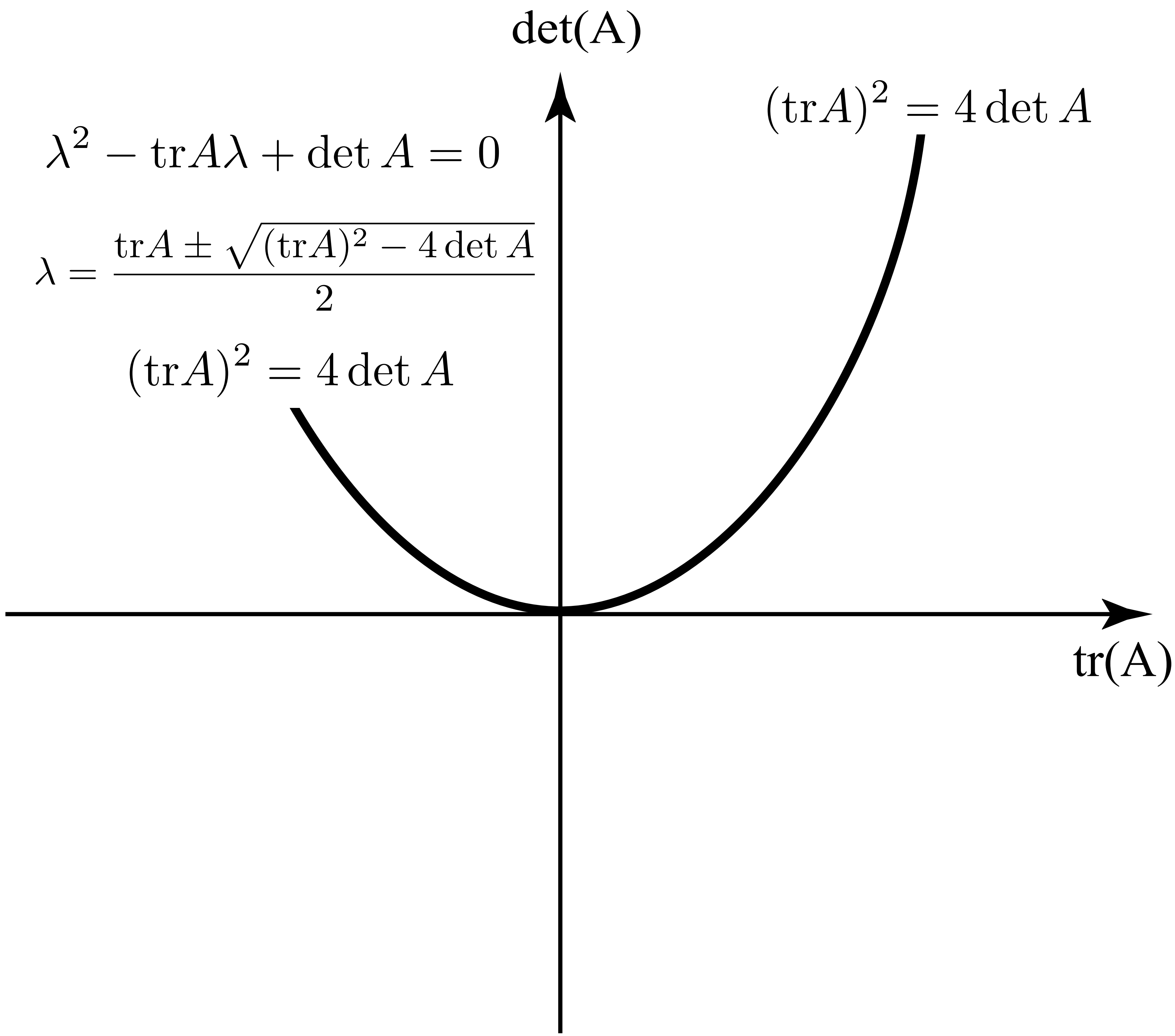

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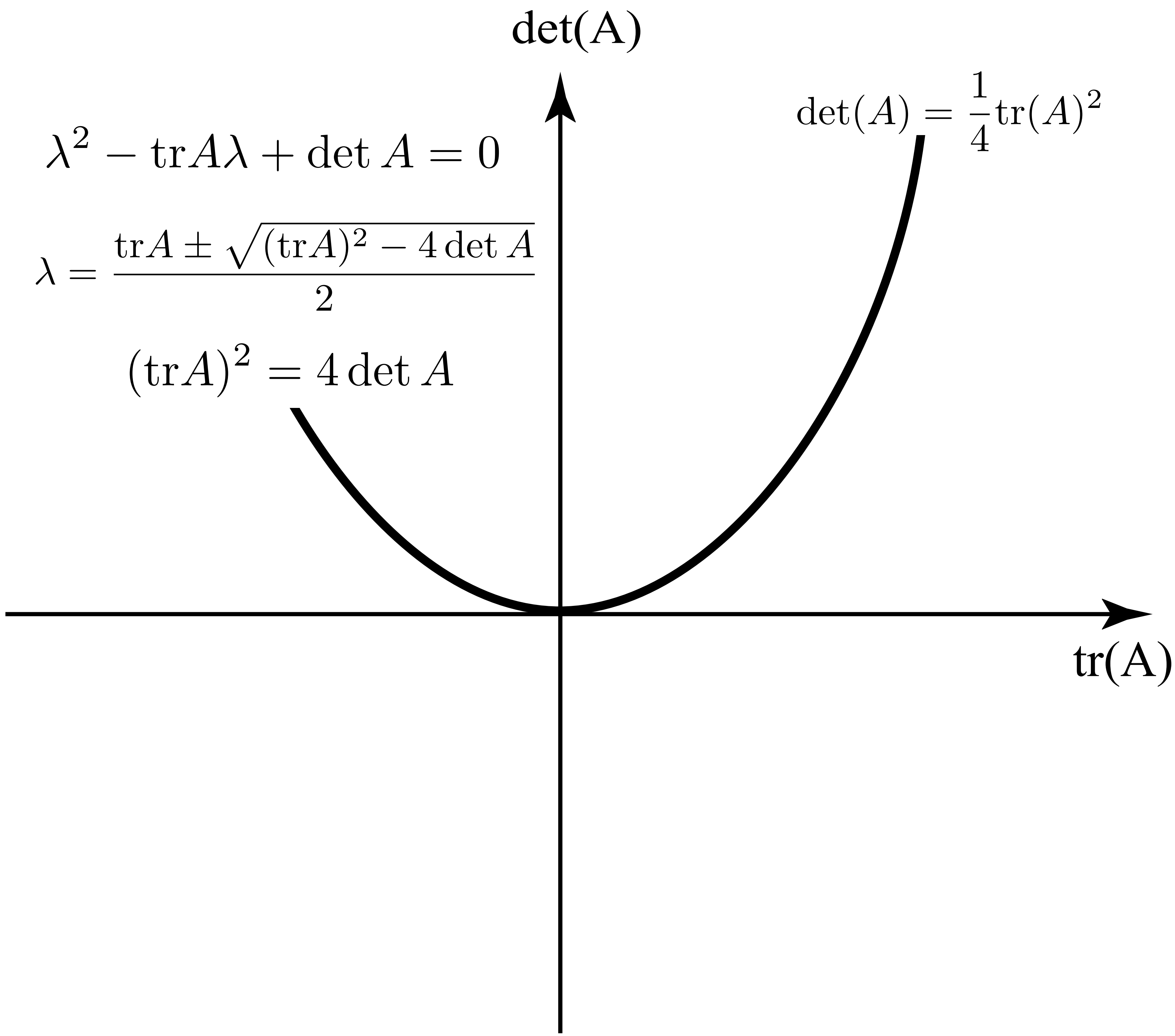

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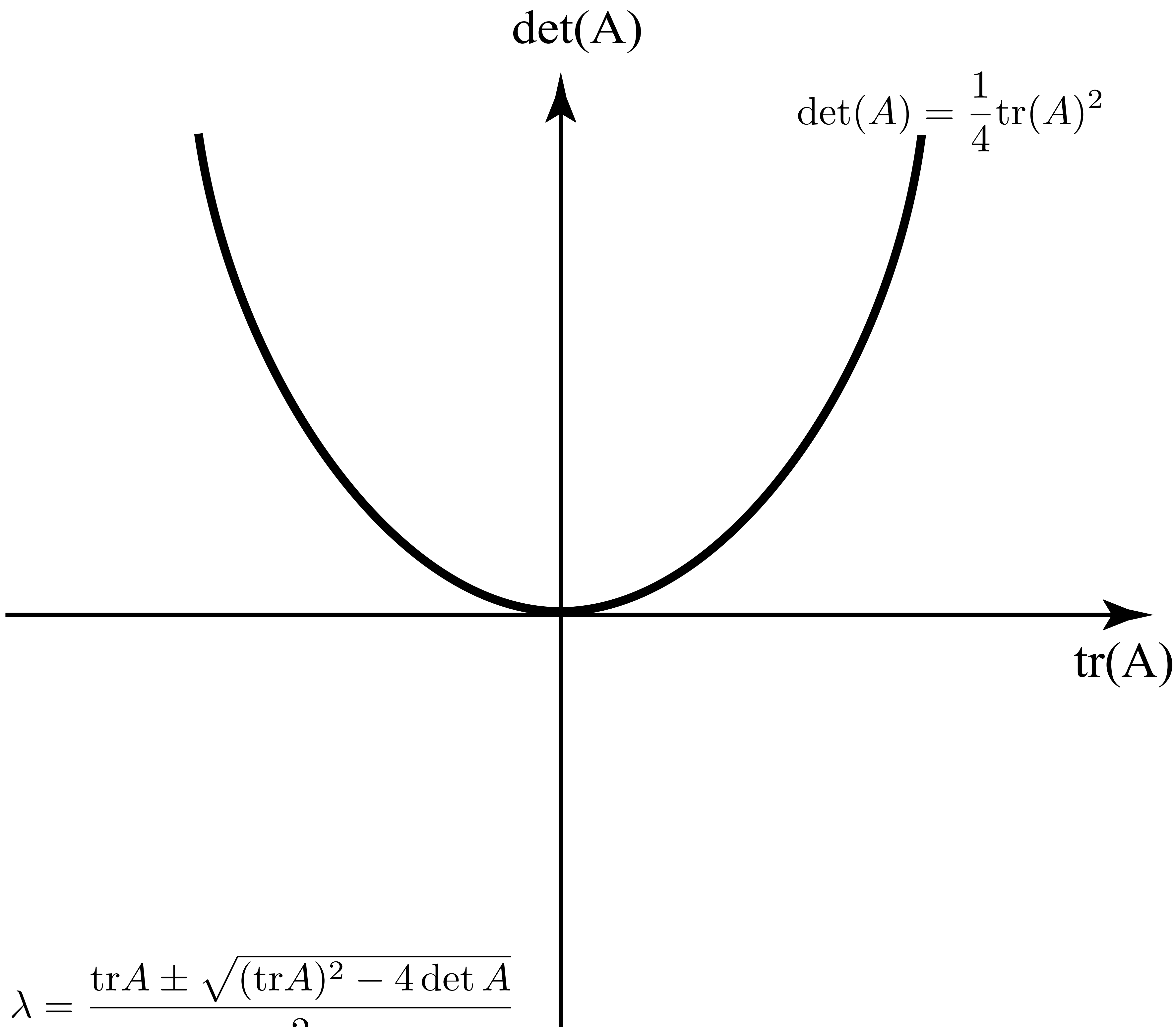

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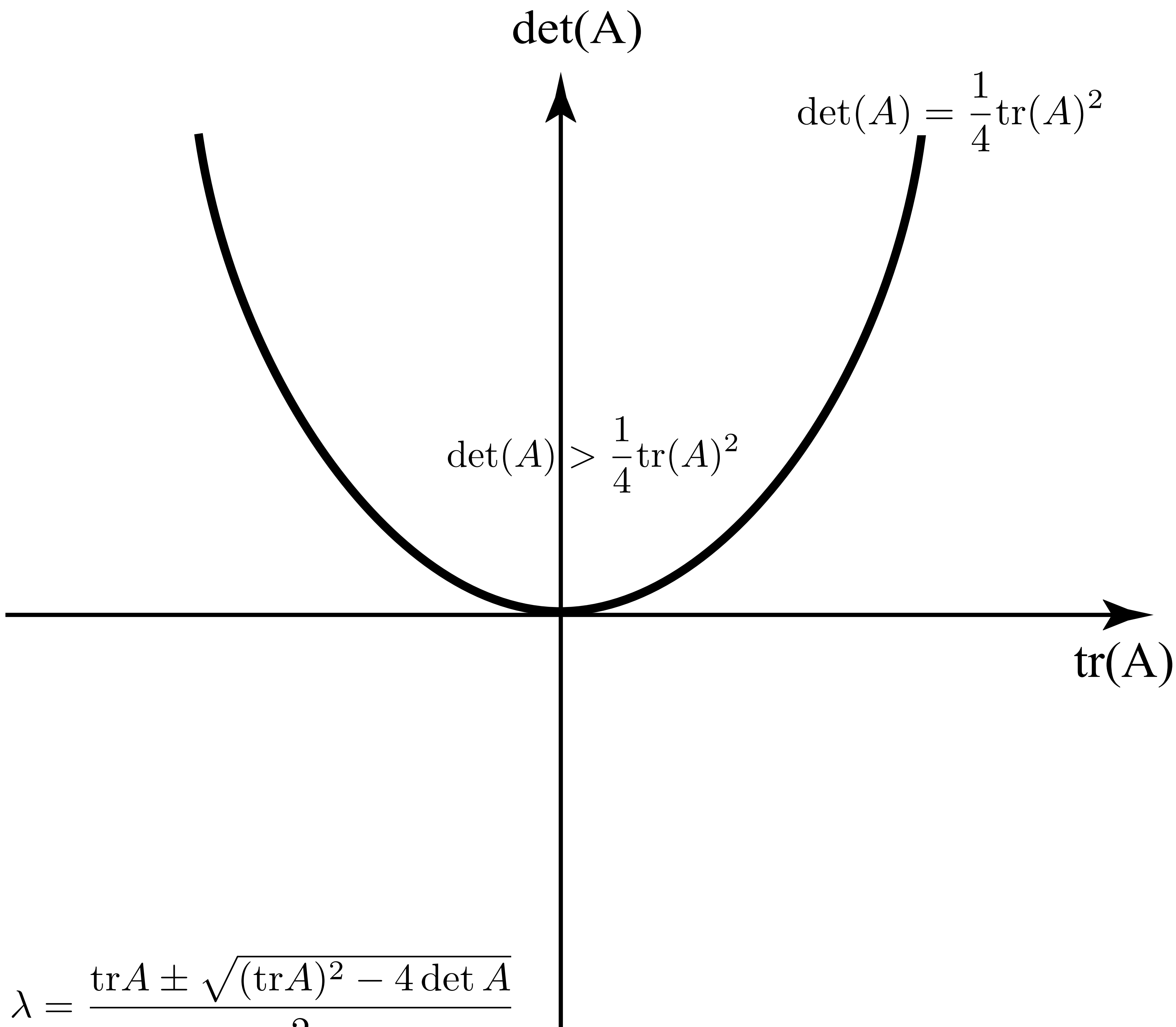

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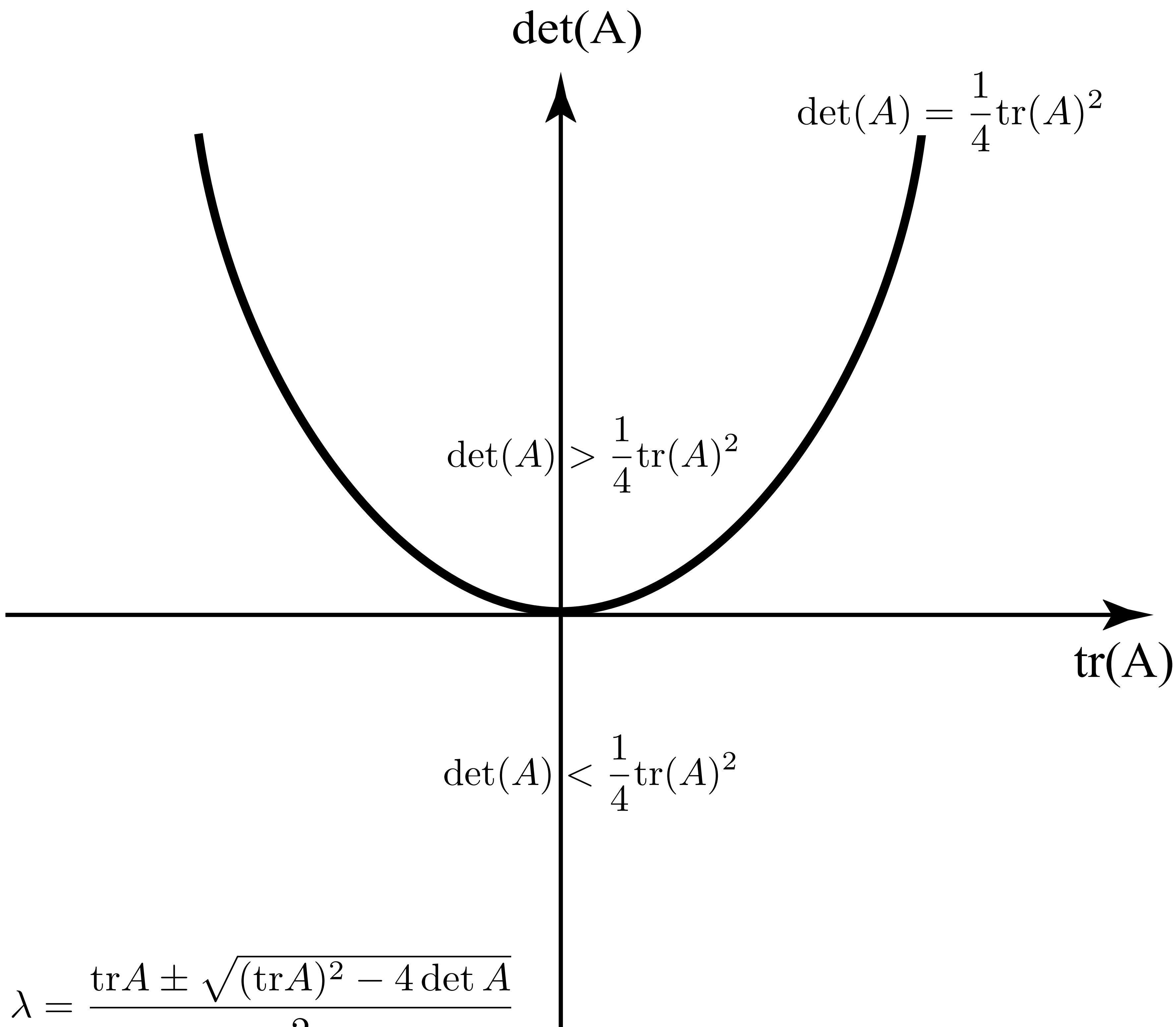

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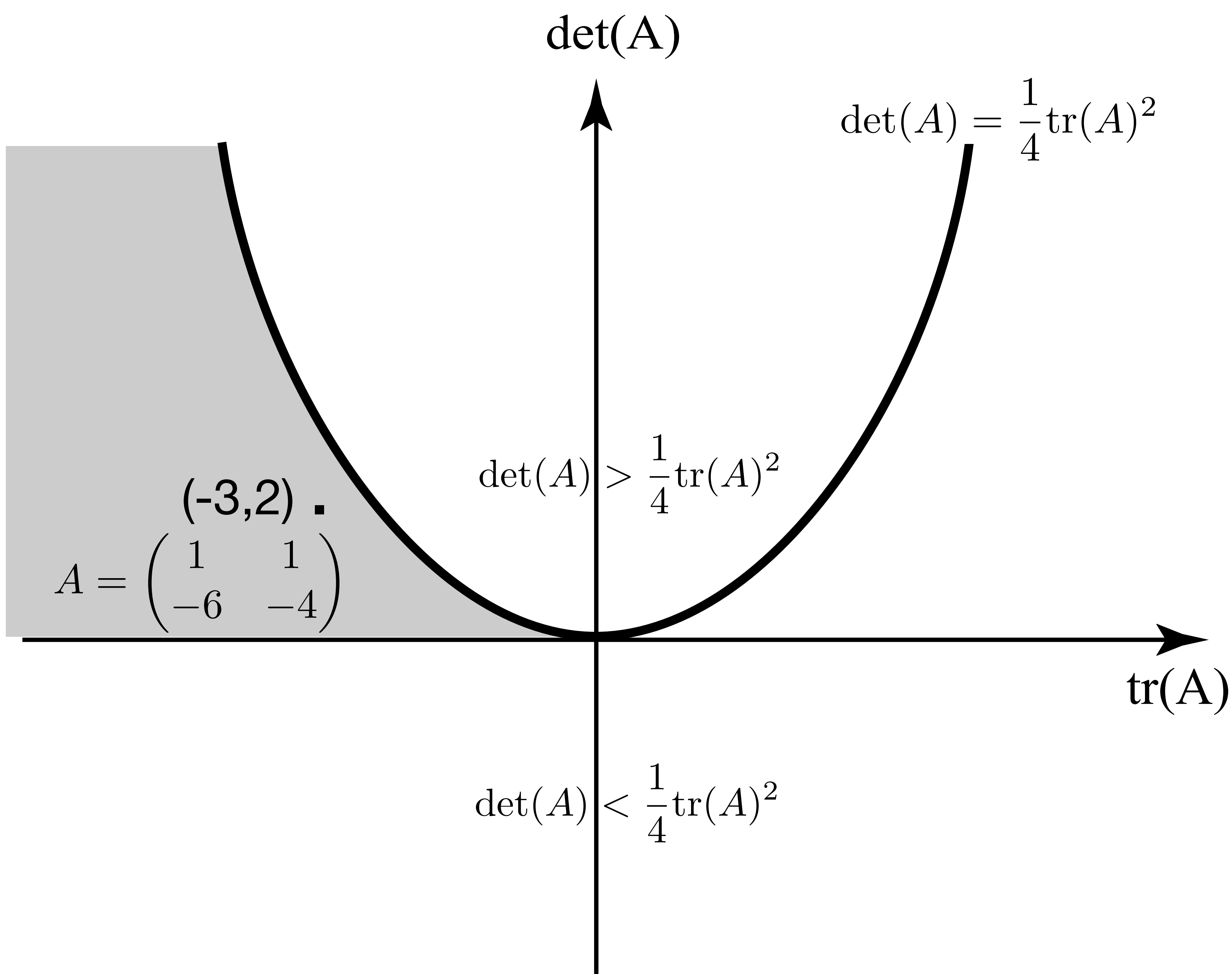

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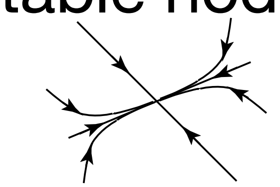

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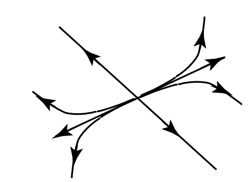

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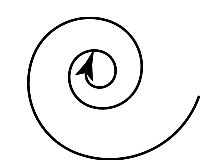

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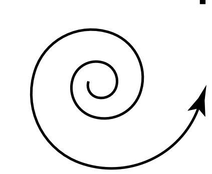

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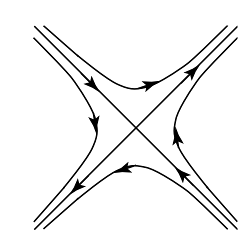

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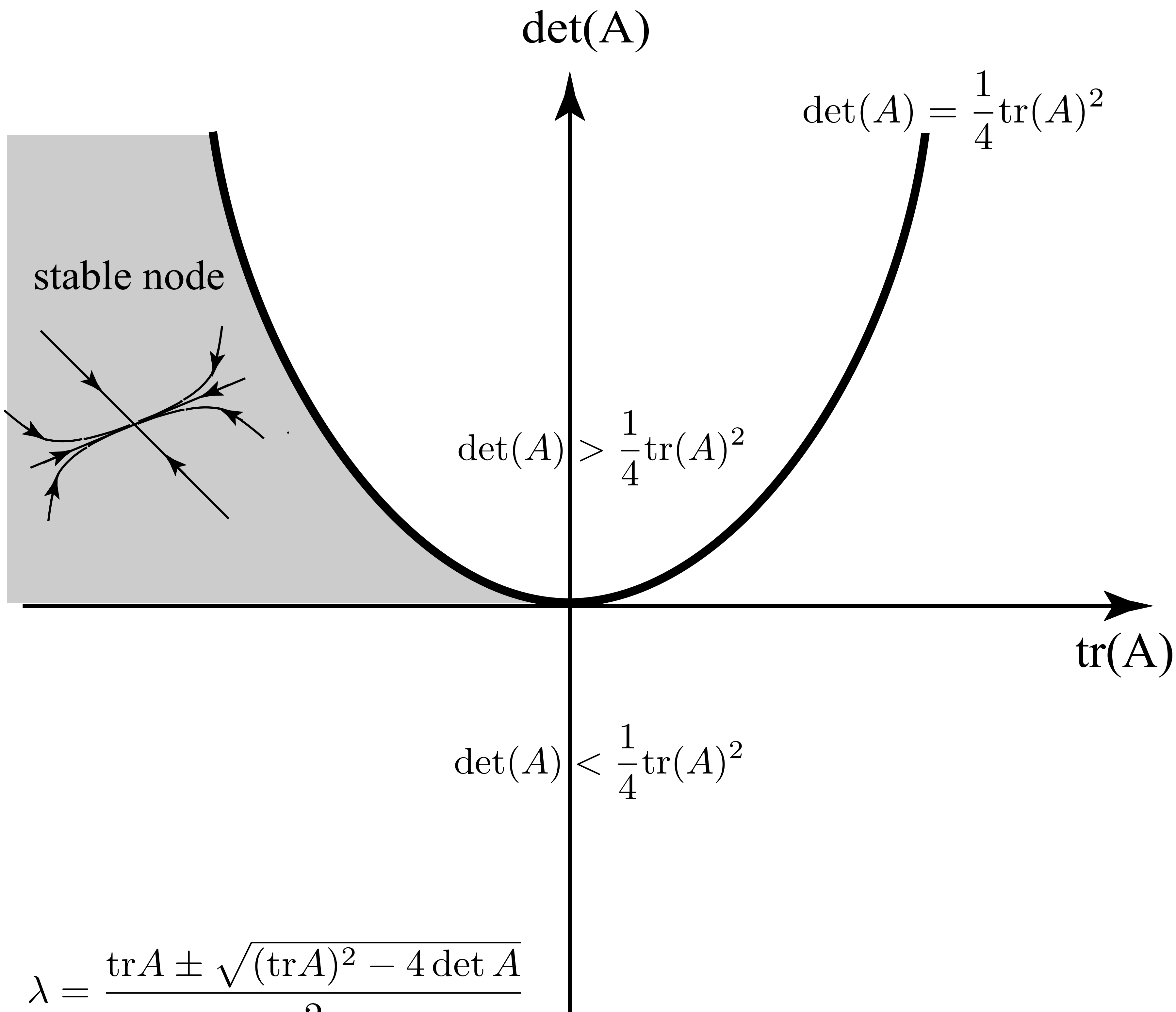

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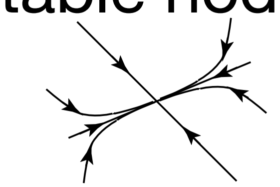

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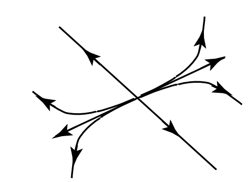

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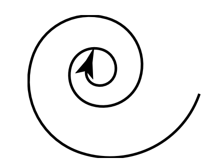

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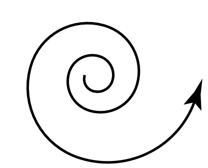

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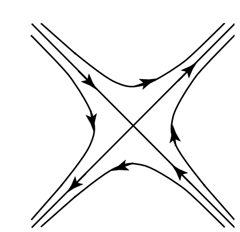

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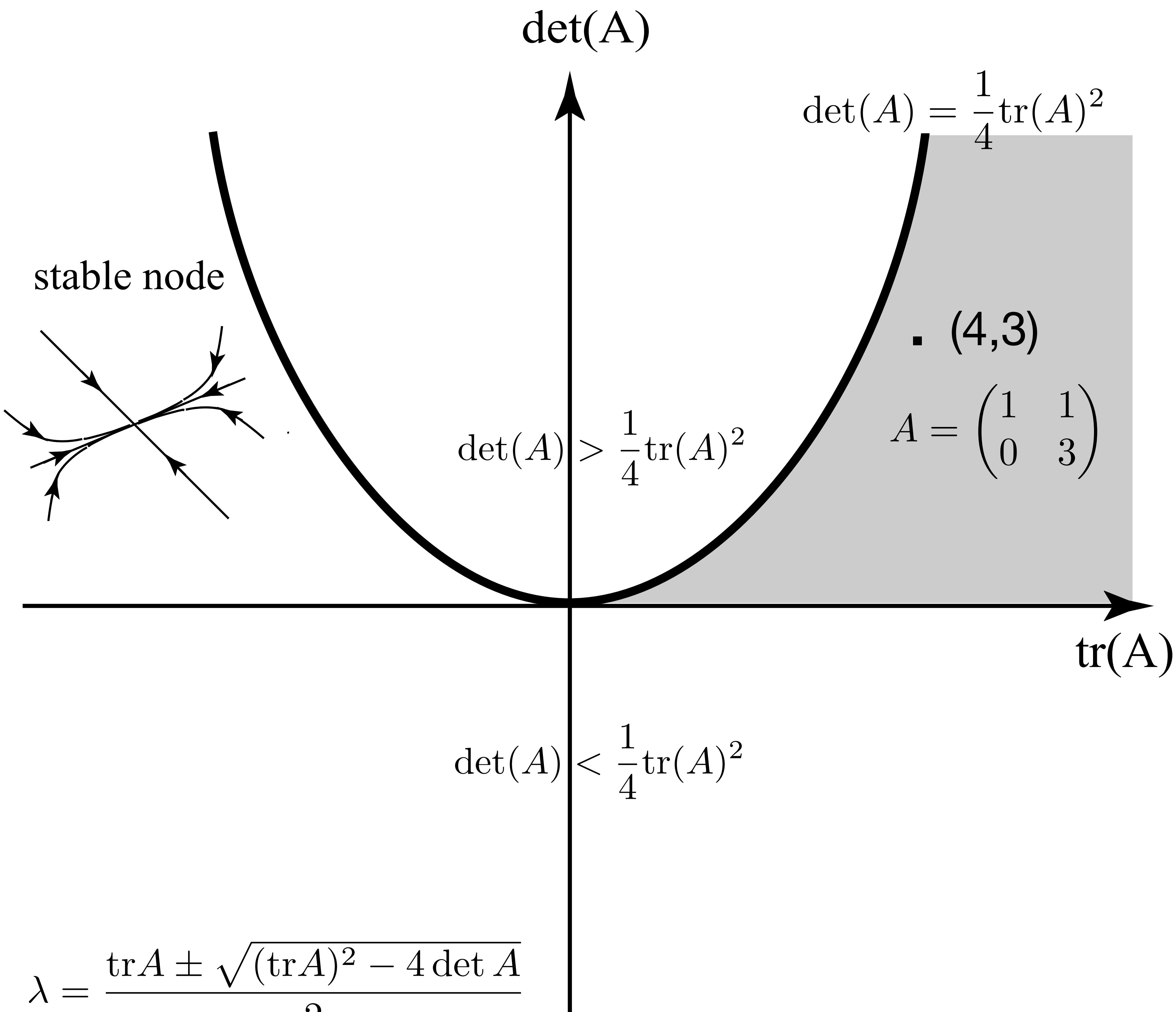

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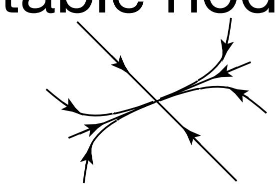

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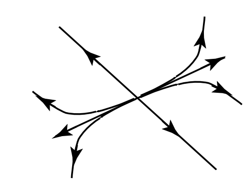

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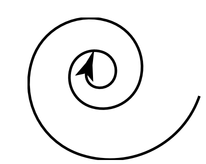

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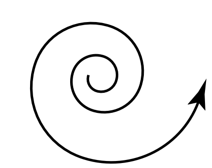

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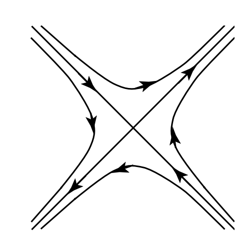

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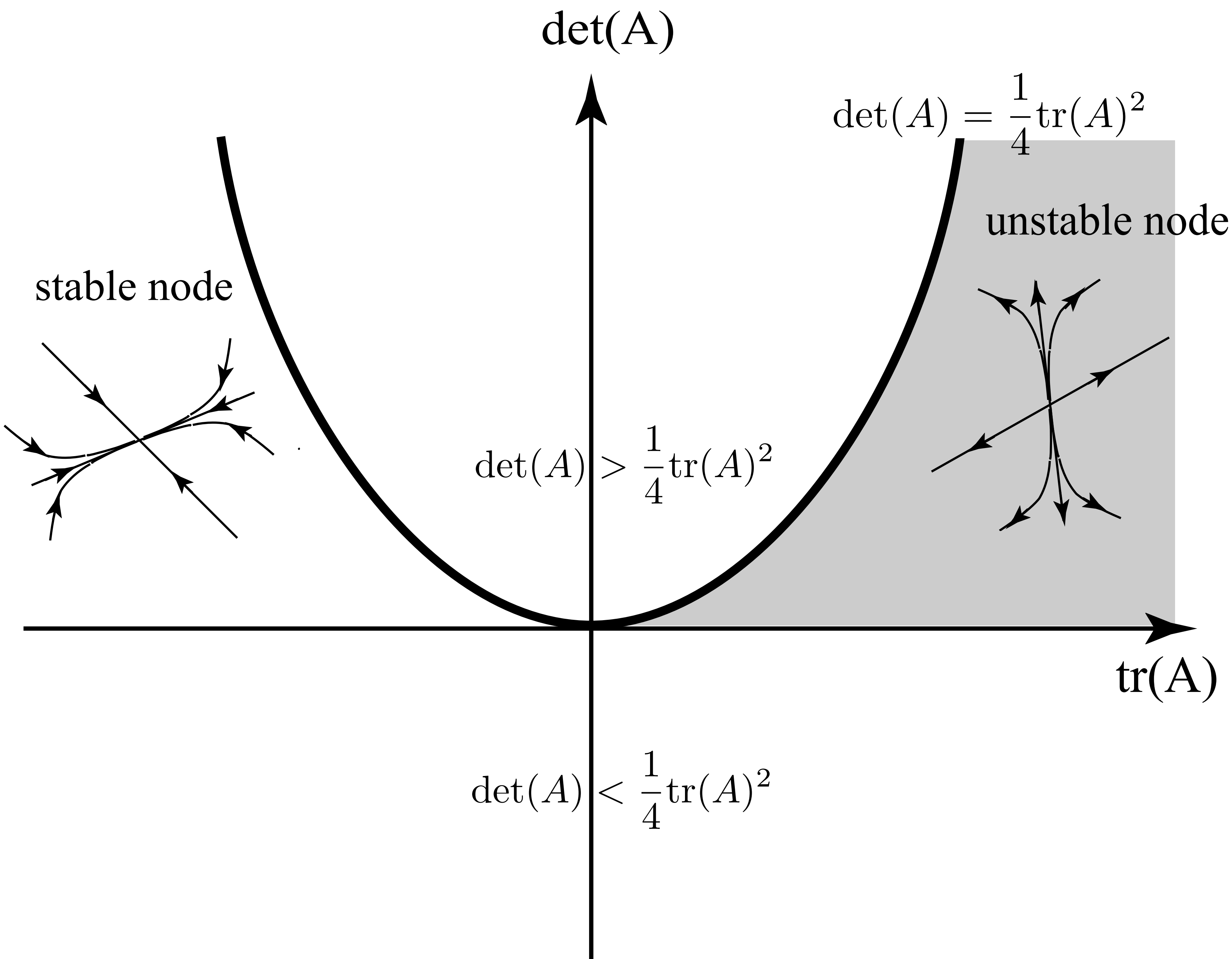

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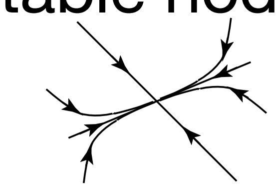

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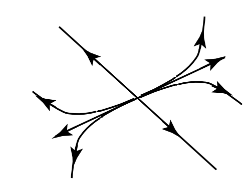

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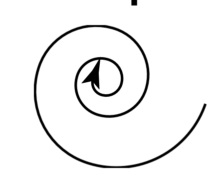

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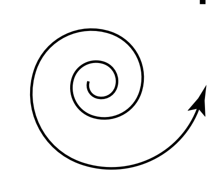

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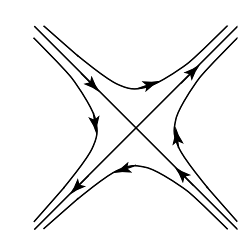

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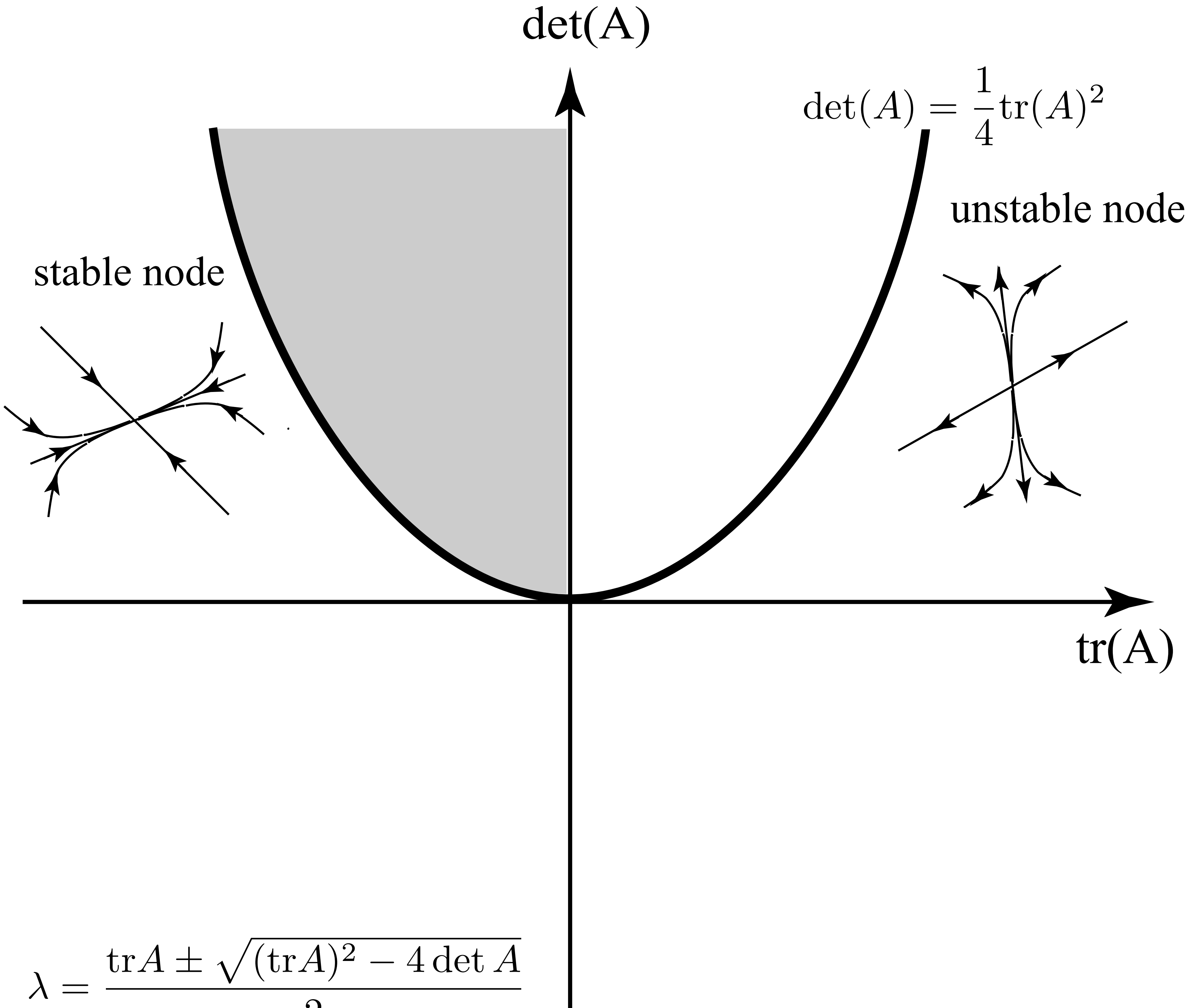

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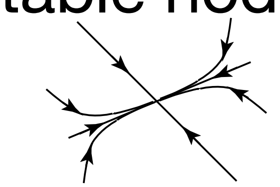

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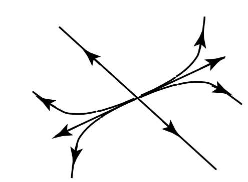

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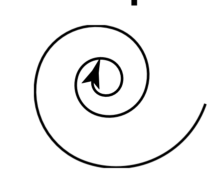

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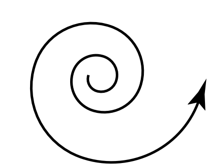

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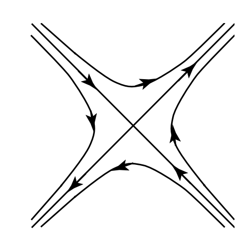

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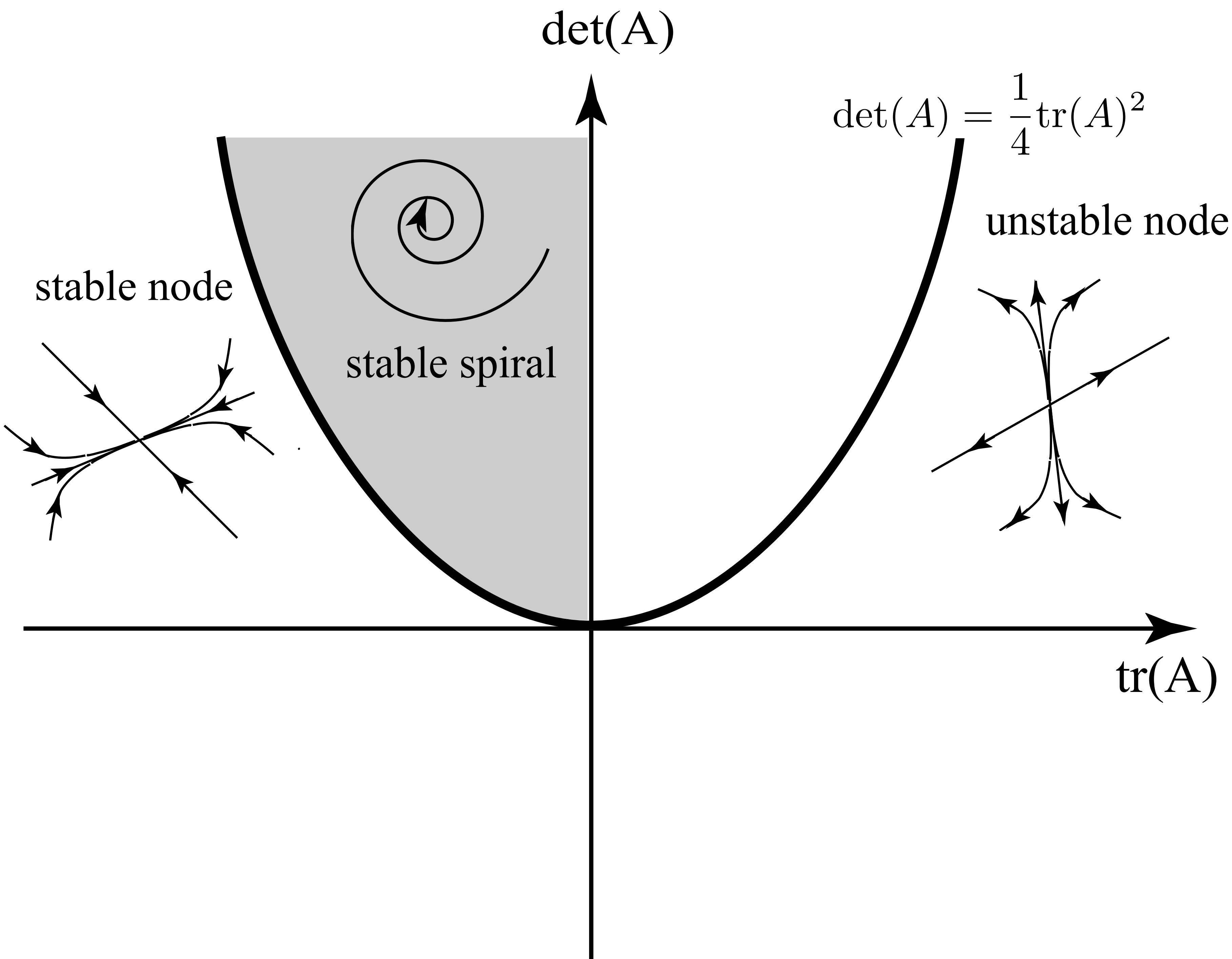

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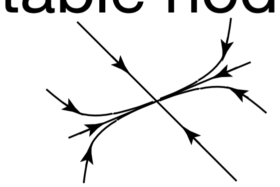

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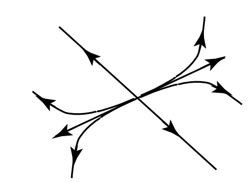

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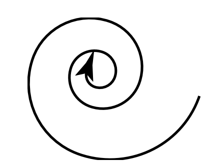

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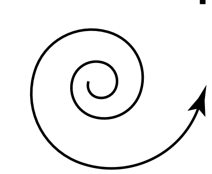

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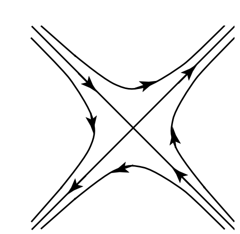

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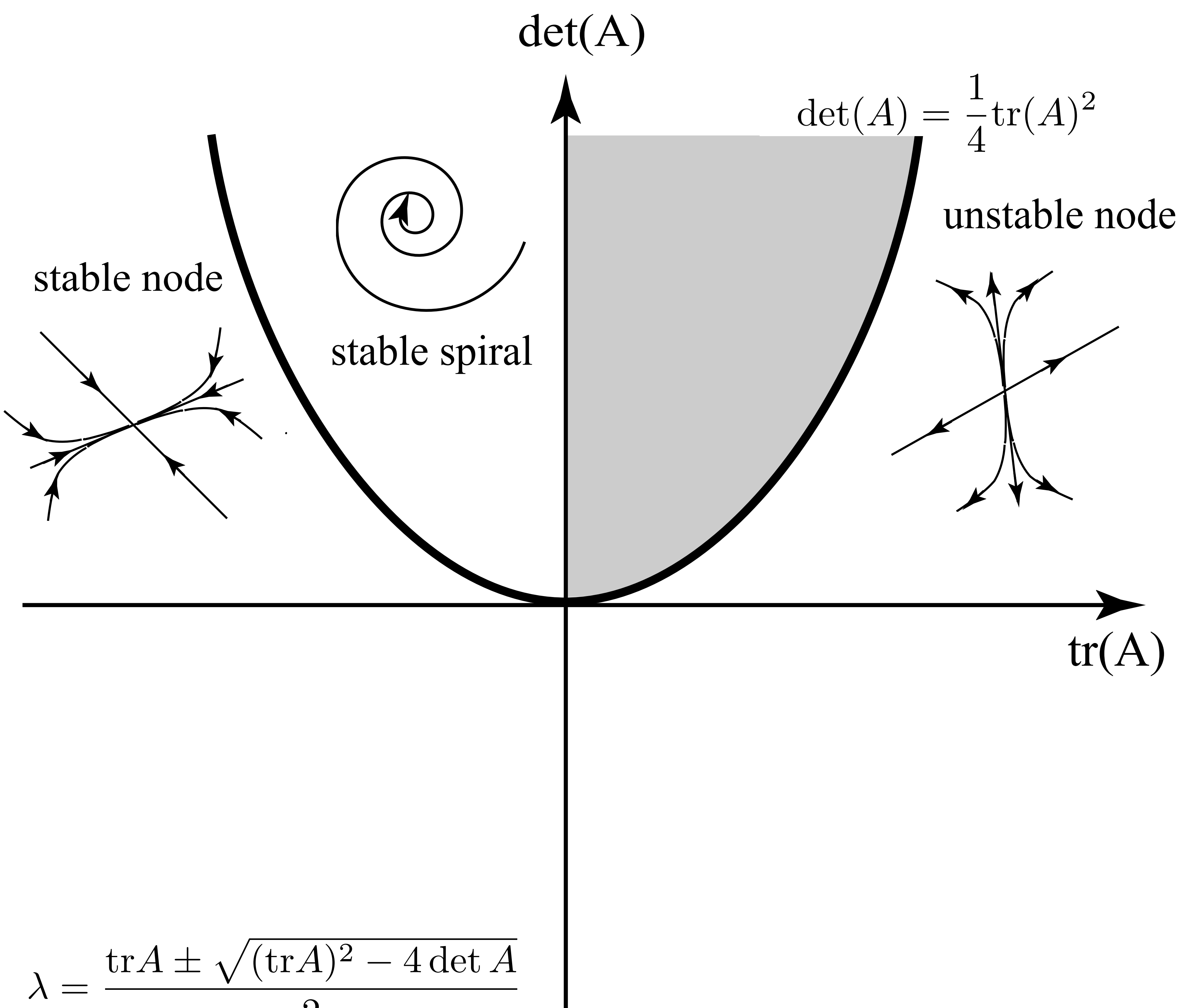

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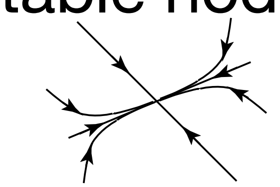

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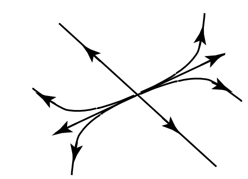

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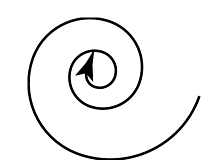

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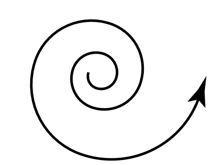

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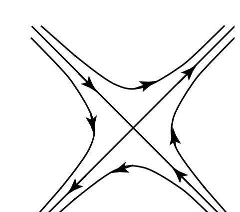

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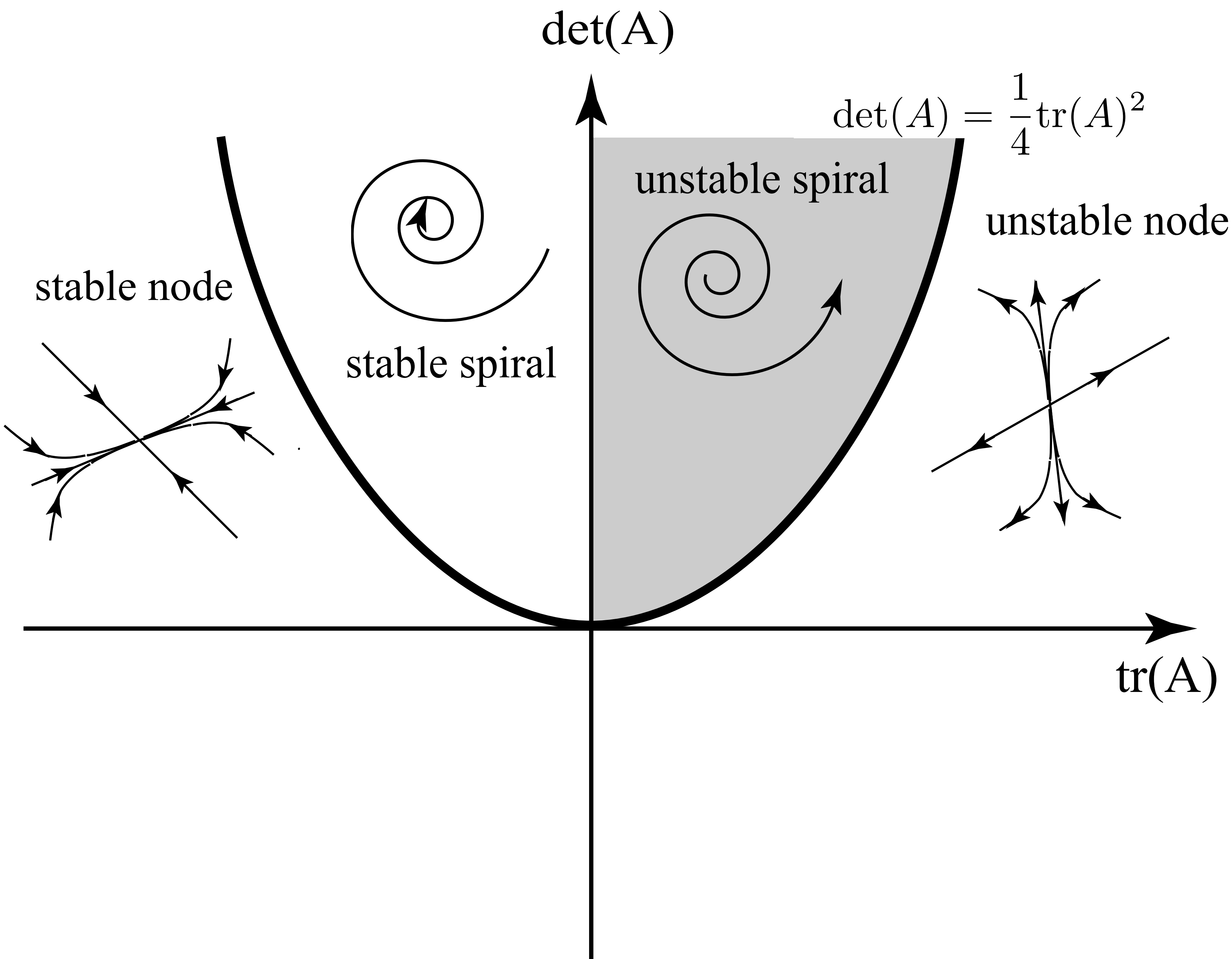

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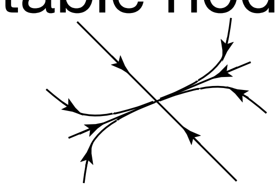

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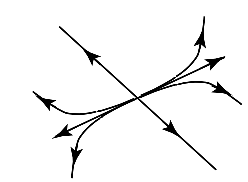

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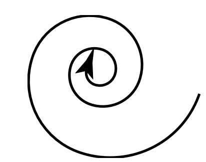

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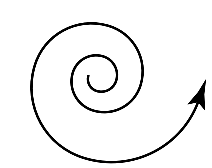

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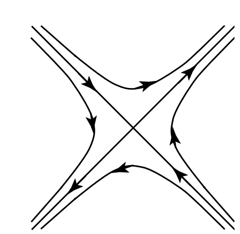

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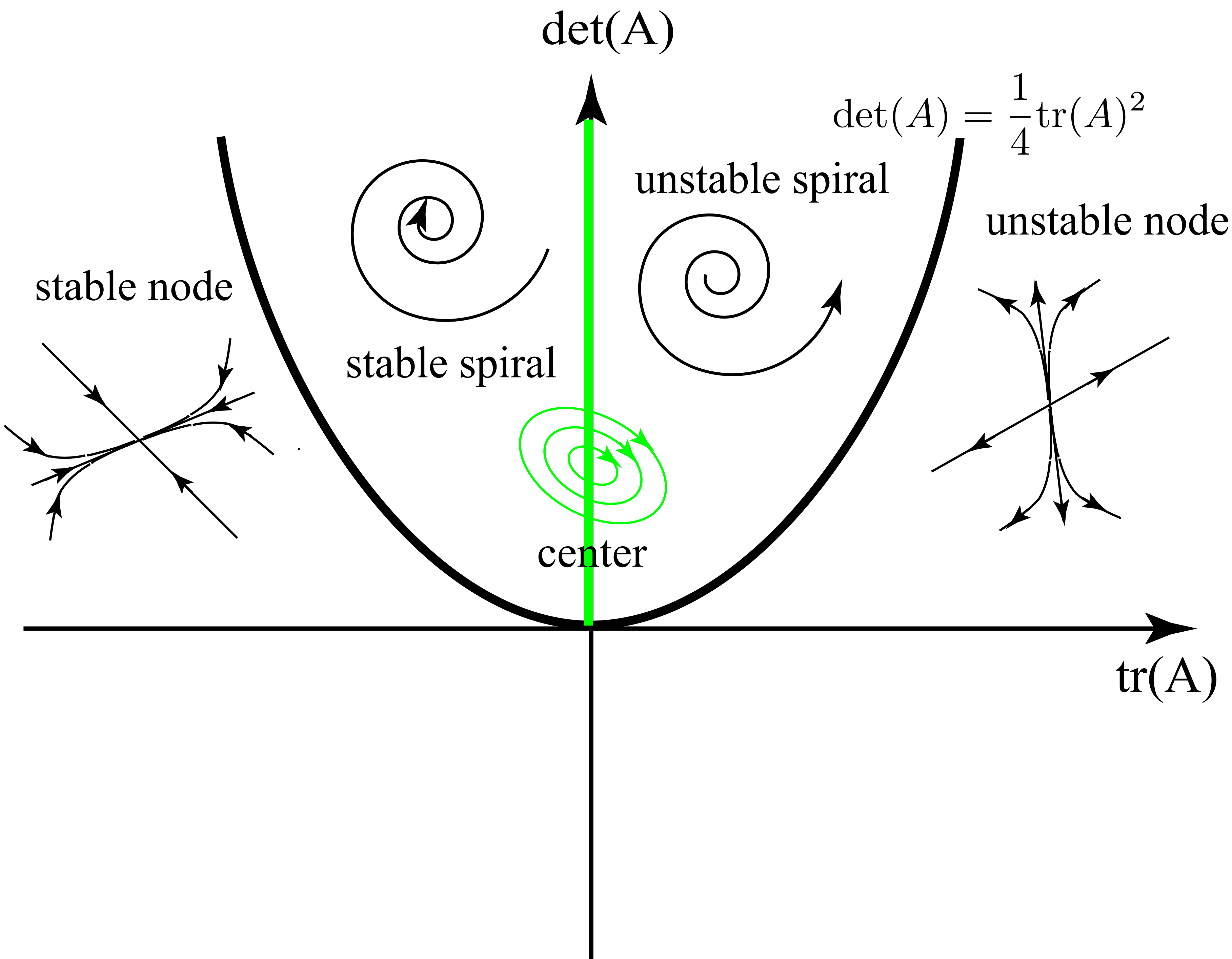

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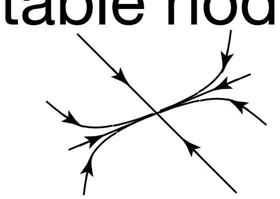

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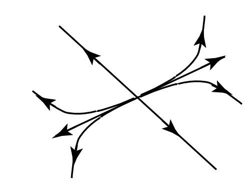

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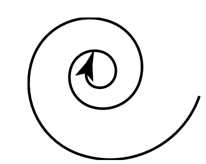

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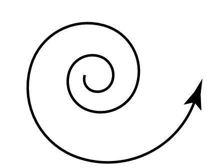

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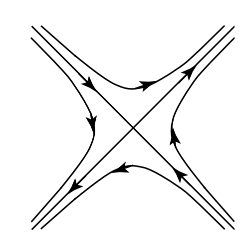

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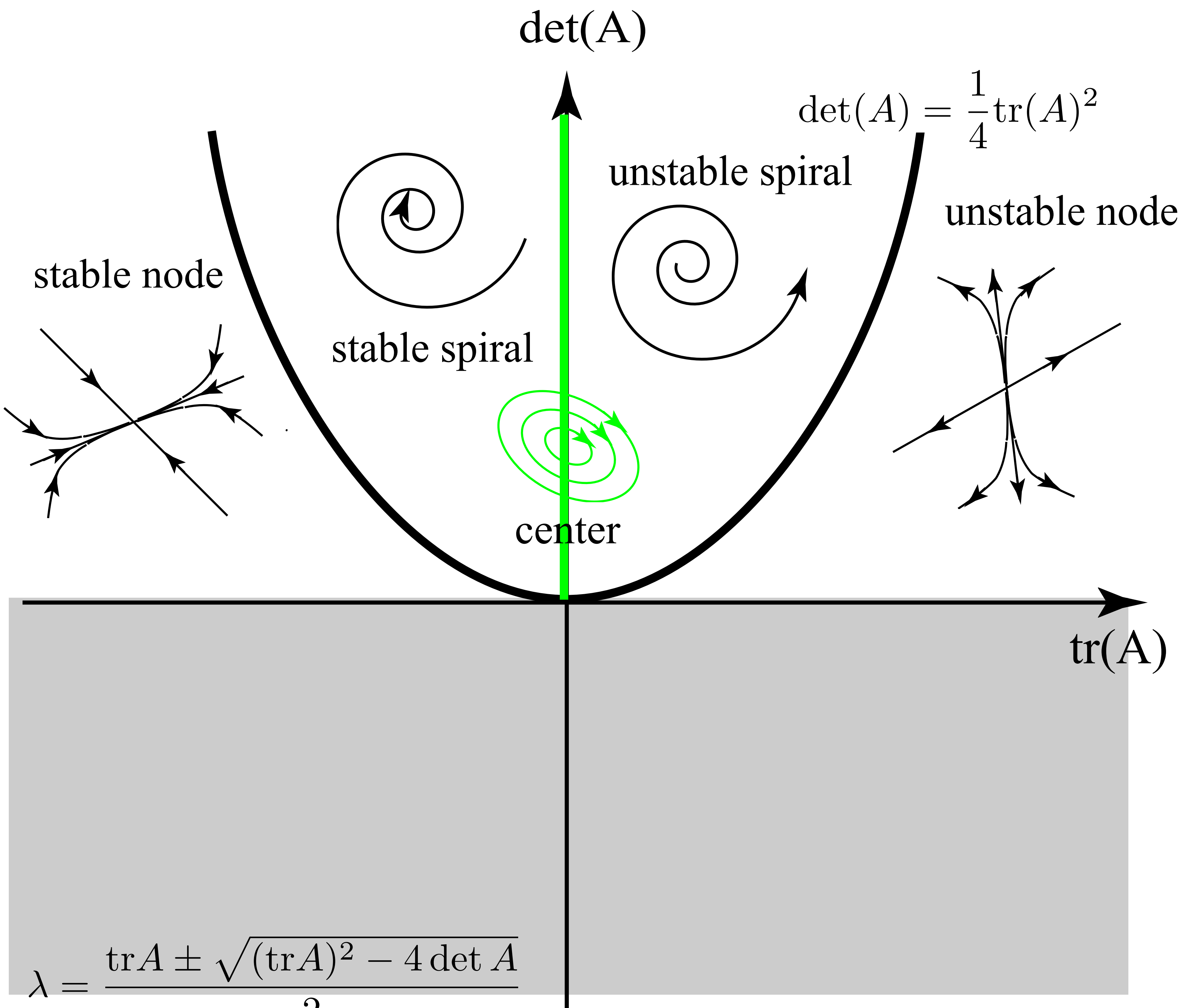

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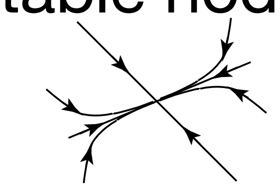

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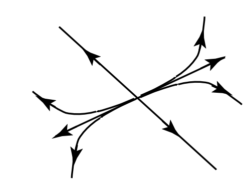

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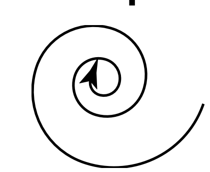

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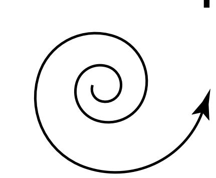

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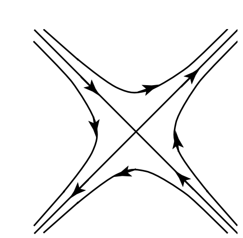
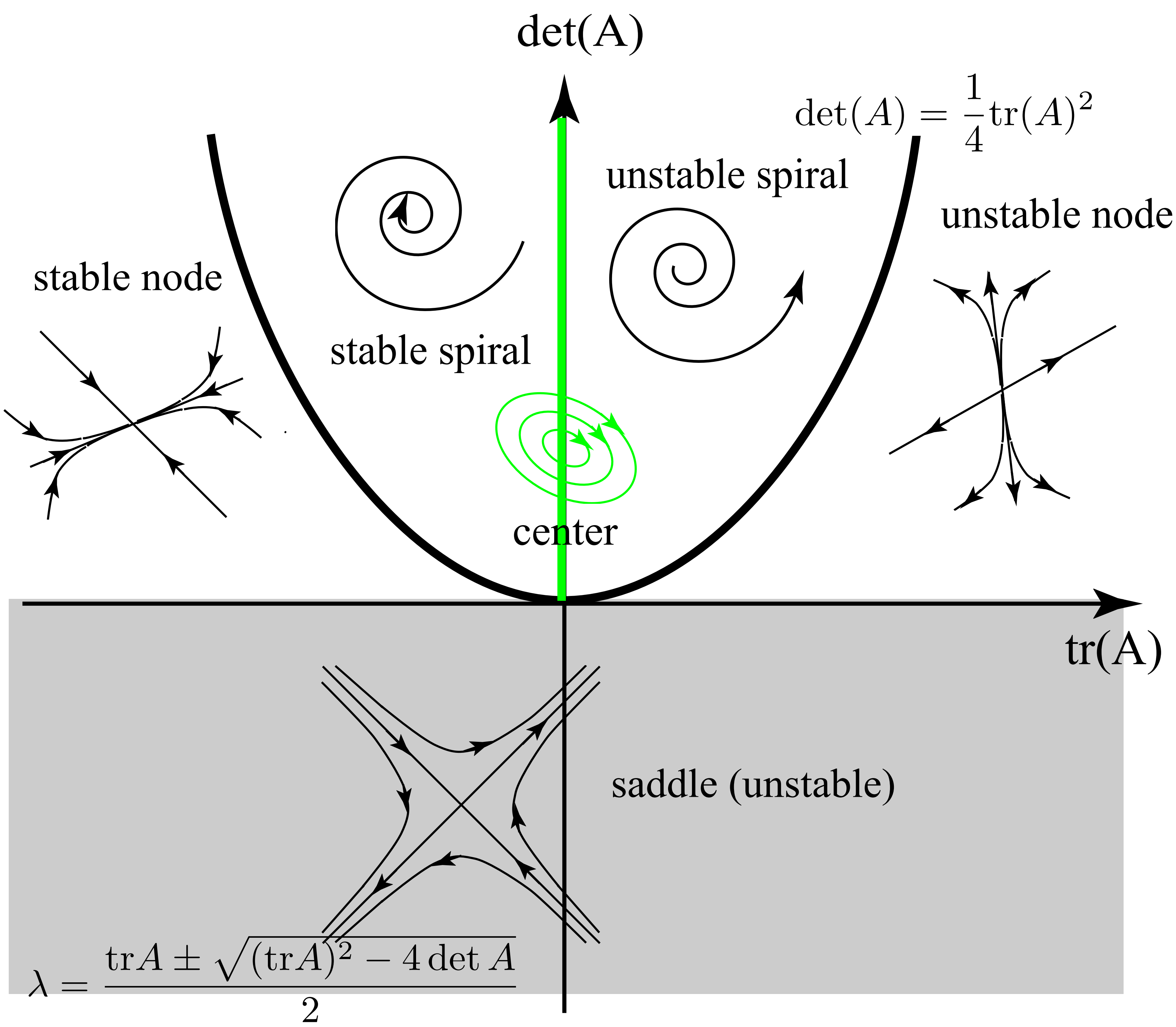
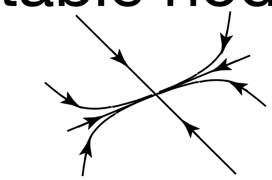


Summary - homogeneous 2x2 systems

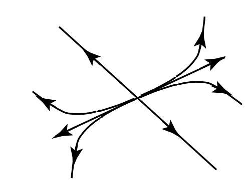


When (tr(A), det(A)) is in the shaded region, we have a…

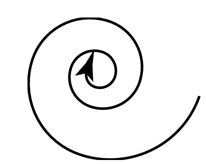
(A) stable node



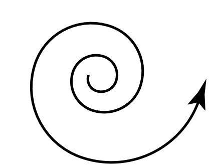
(B) unstable node



(C) stable spiral



(D) unstable spiral



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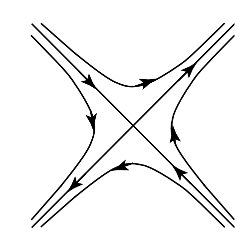

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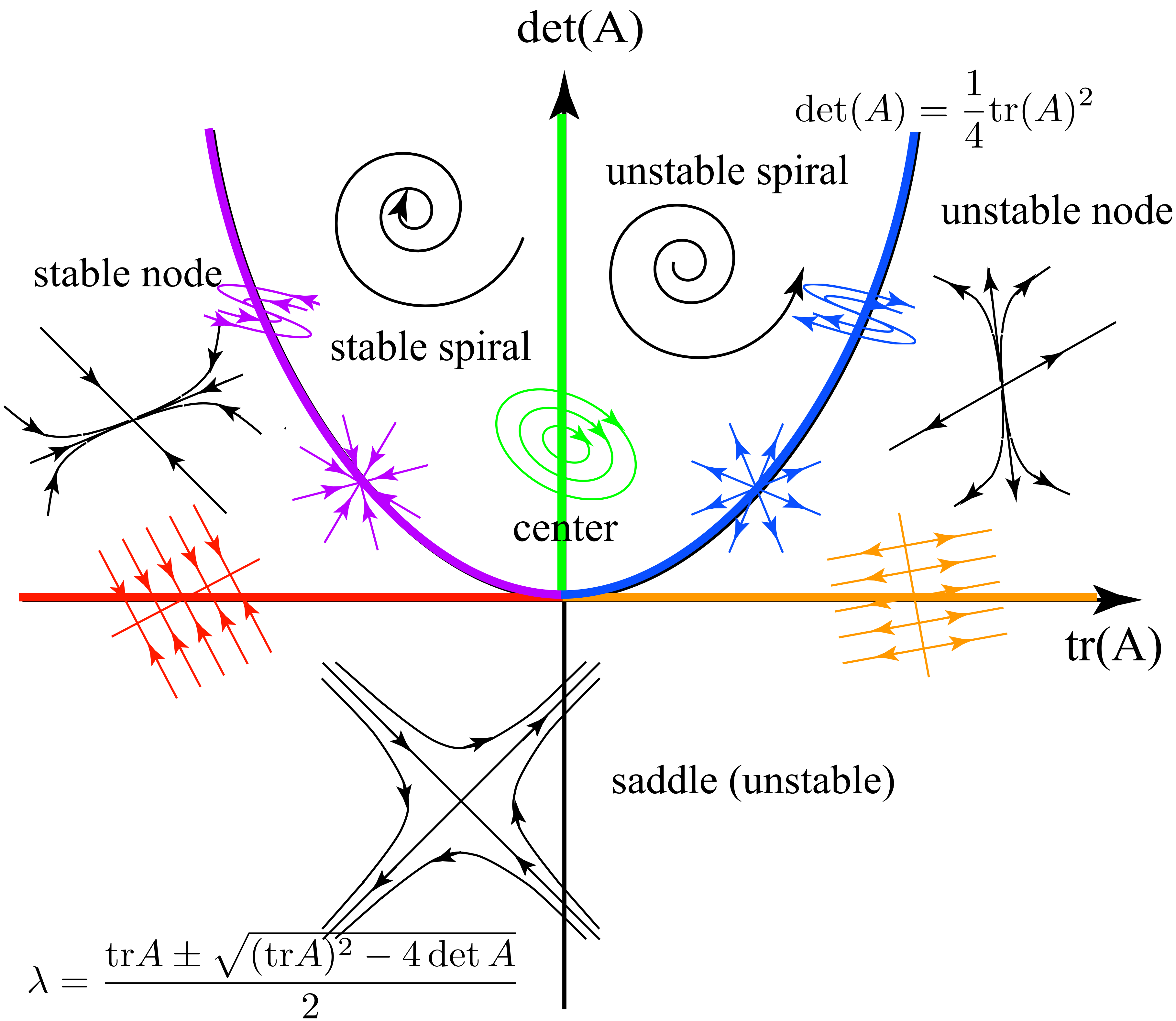

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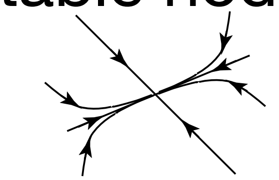
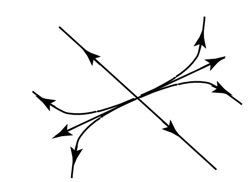


(B) unstable node



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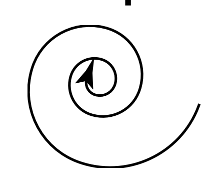

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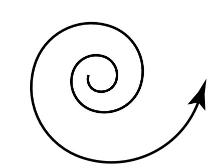

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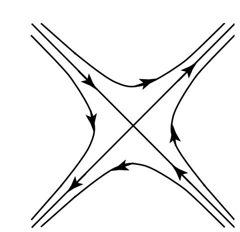
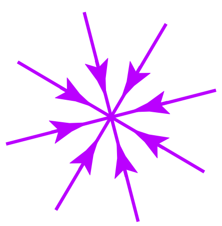
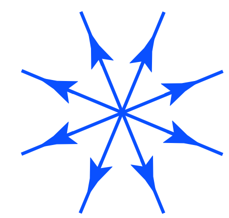


Summary - homogeneous 2x2 systems

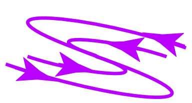
Repeated evalue cases:



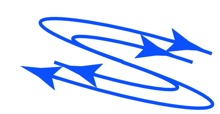
$\lambda<0$, two indep. evectors.



$\lambda>0$, two indep. evectors.

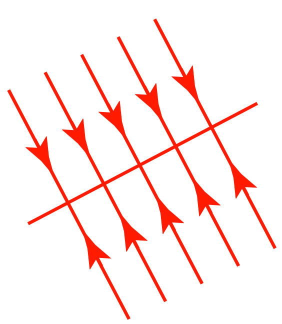


$\lambda<0$, only one evector.

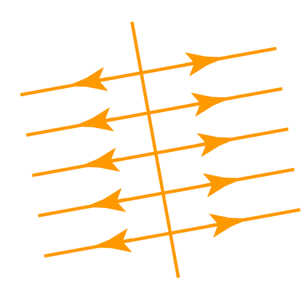


$\lambda>0$, only one evector.

One zero evalue (singular matrix):



$\lambda_1=0$, $\lambda_2<0$,



$\lambda_1=0$, $\lambda_2>0$,

Nonhomogeneous system of DEs

- How do you solve the equation

$$\mathbf{x}'(\mathbf{t}) = A\mathbf{x}(\mathbf{t}) + \mathbf{b} \ \ ?$$

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$$L[\mathbf{x}] = \mathbf{x}'(\mathbf{t}) - A\mathbf{x}(\mathbf{t})$$

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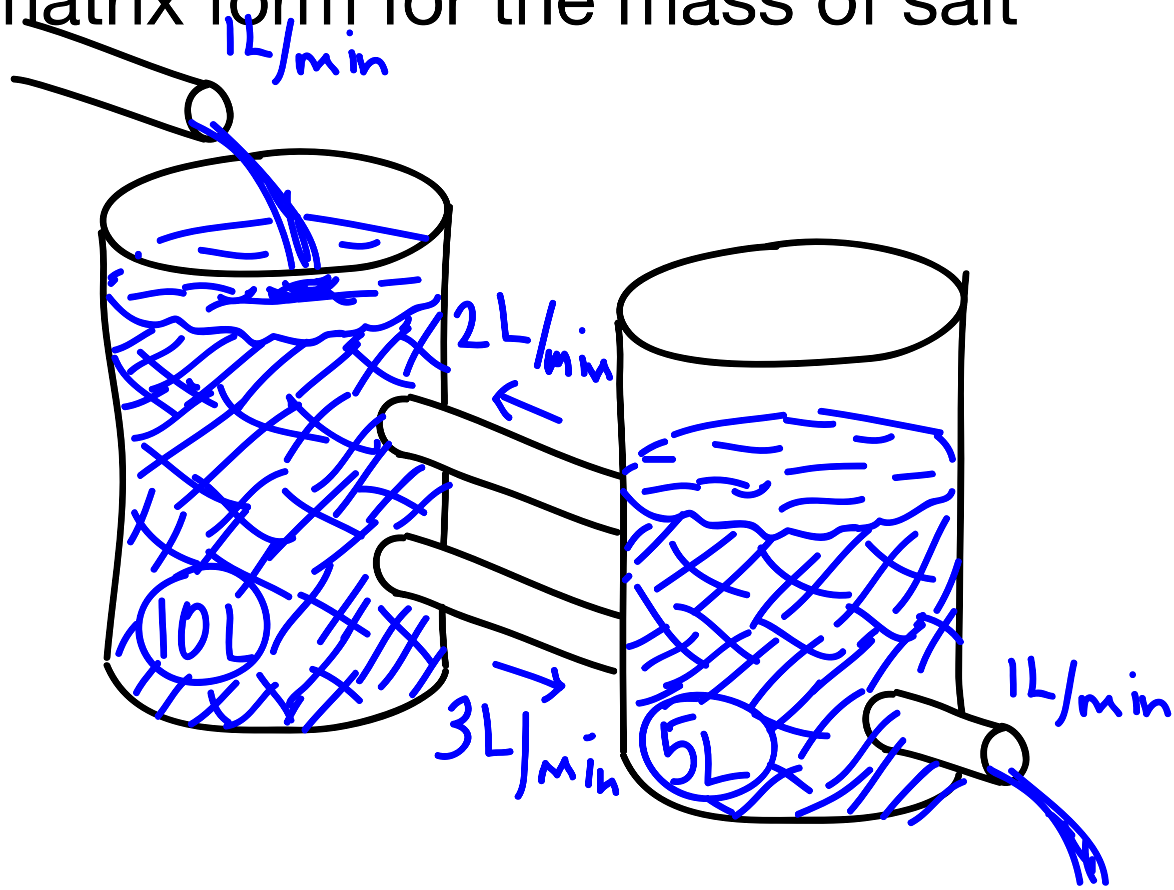
Nonhomogeneous system of DEs - example

- Salt water flows into a tank holding 10 L of water at a rate of 1 L/min with a concentration of 200 g/L. The well-mixed solution flows from that tank into a tank holding 5 L through a pipe at 3 L/min. Another pipe takes the solution in the second tank back into the first at a rate of 2 L/min. Finally, solution drains out of the second tank at a rate of 1 L/min.
- Write down a system of equations in matrix form for the mass of salt in each tank.



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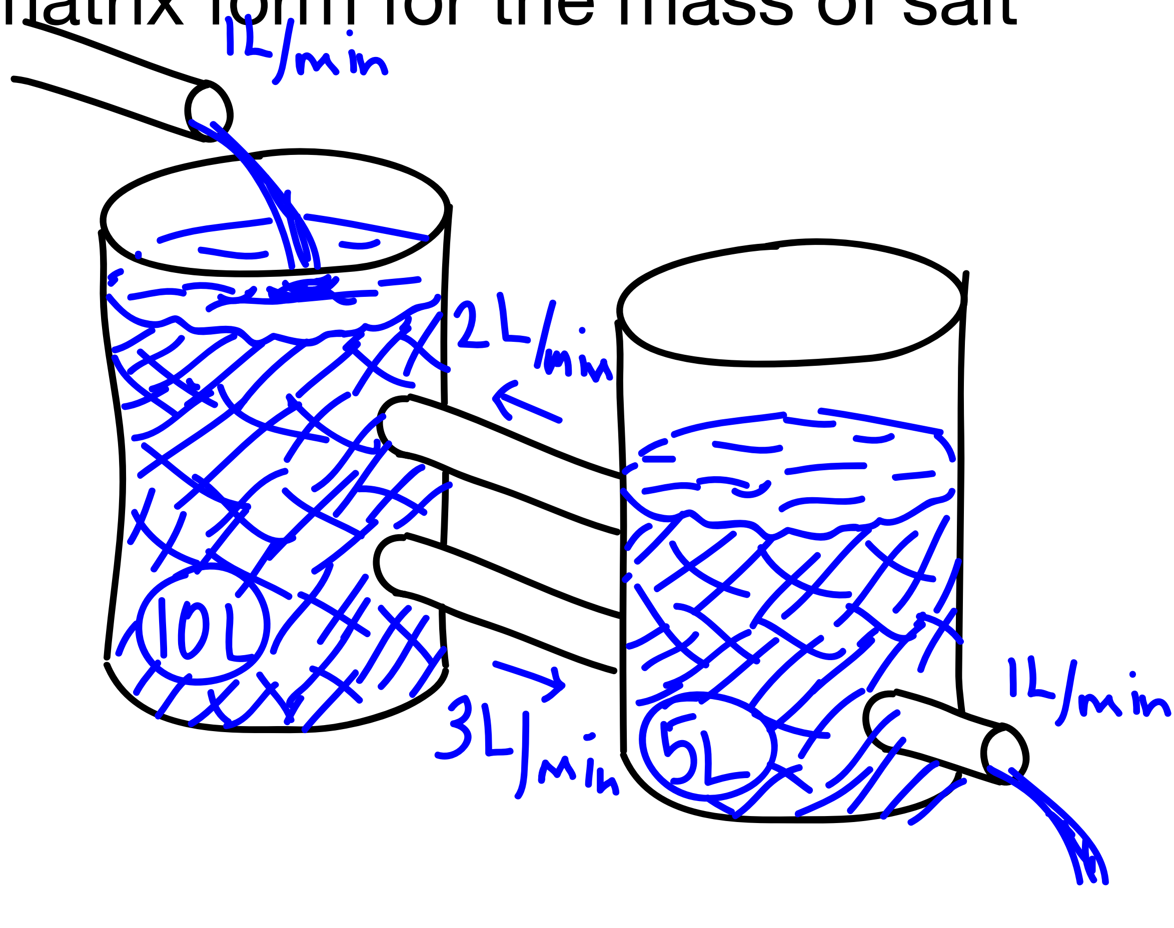

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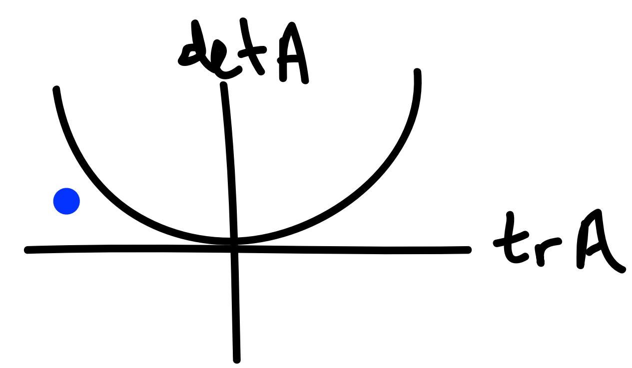
$$\mathrm{tr}A = -\frac{9}{10} \qquad (\mathrm{tr}A)^2 = \frac{81}{100}$$

$$\det A = \frac{9}{50} - \frac{6}{50} = \frac{3}{50} \qquad 4\det A = \frac{12}{50}$$



Both evalues negative!

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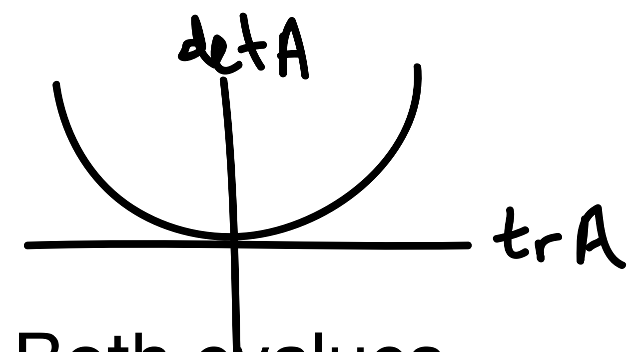

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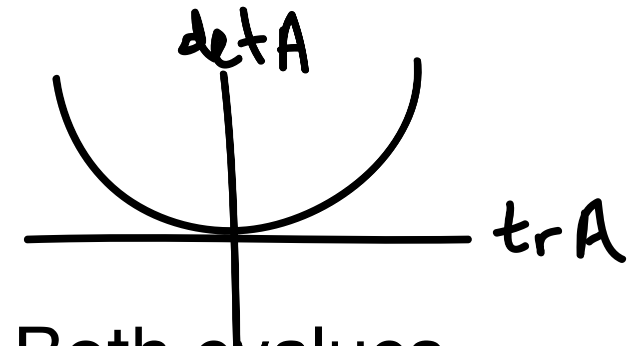

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$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2} \qquad \left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20}\right)$$

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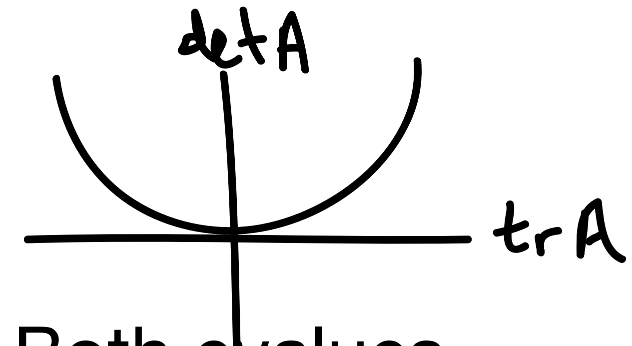


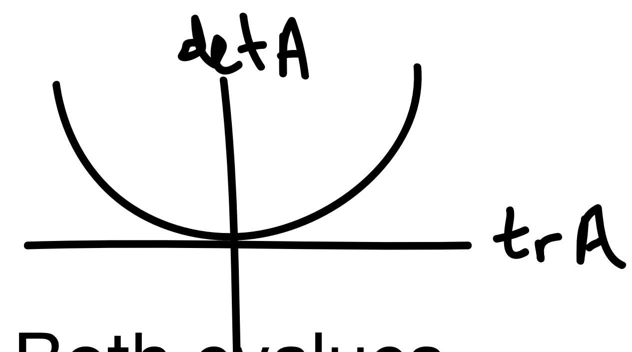
Both evalues negative!

$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2} \qquad \left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20}\right)$$

$$\mathbf{m_p}(t) =$$

Nonhomogeneous case - example

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



Both evalues negative!

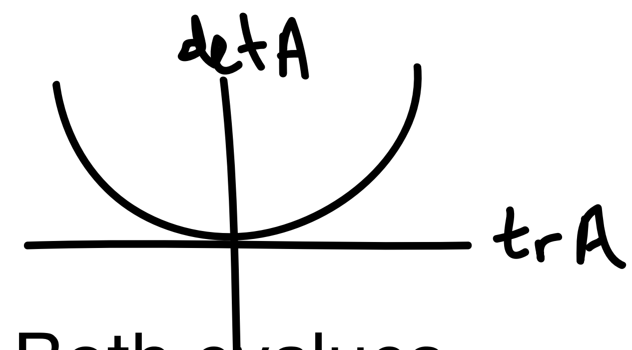
$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

$$\left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20}\right)$$

$$\mathbf{m_p}(t) = \ \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

Nonhomogeneous case - example

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



Both evalues negative!

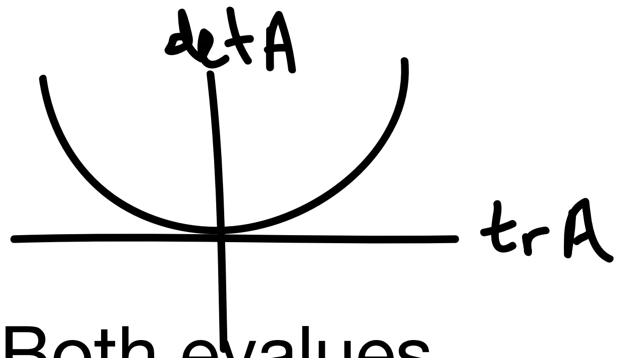
$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2} \qquad \left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20}\right)$$

$$\mathbf{m_p}(t) = \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\mathbf{0} = A\mathbf{w} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$

Nonhomogeneous case - example

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



Both evalues negative!

$$\left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20} \right)$$

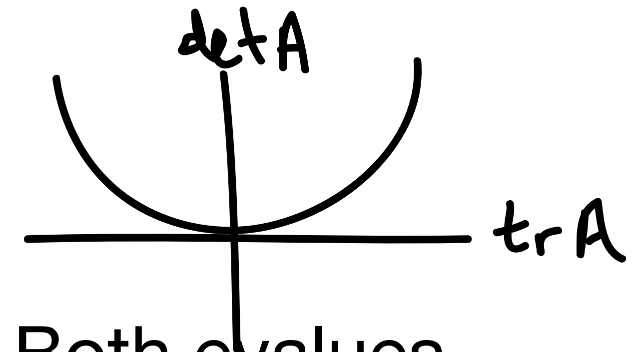
$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

$$\mathbf{m_p}(t) = \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\mathbf{0} = A\mathbf{w} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$

Nonhomogeneous case - example

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



Both evalues negative!

$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

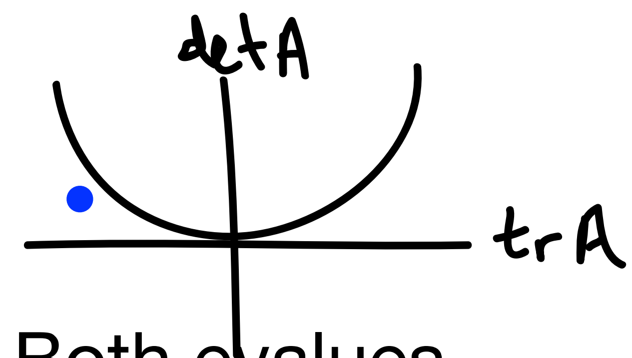
$$\left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20} \right)$$

$$\mathbf{m_p}(t) = \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\mathbf{0} = A\mathbf{w} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \rightarrow A\mathbf{w} = -\begin{pmatrix} 200 \\ 0 \end{pmatrix}$$

Nonhomogeneous case - example

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



Both evalues negative!

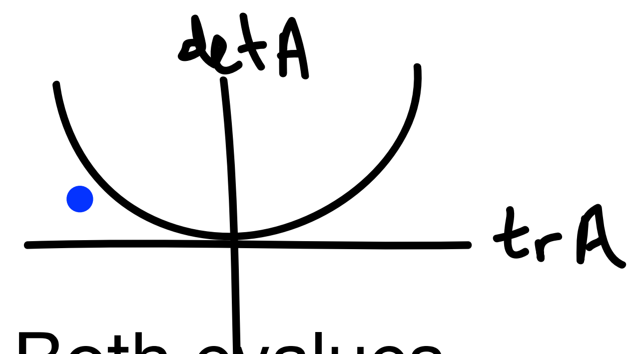
$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2} \qquad \left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20}\right)$$

$$\mathbf{m_p}(t) = \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\mathbf{0} = A\mathbf{w} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \longrightarrow A\mathbf{w} = -\begin{pmatrix} 200 \\ 0 \end{pmatrix} \longrightarrow \mathbf{w} = \begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$$

Nonhomogeneous case - example

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}' = \begin{pmatrix} -\frac{3}{10} & \frac{2}{5} \\ \frac{3}{10} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix}$$



Both evalues negative!

$$\mathbf{m_h}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2} \qquad \left(\lambda_{1,2} = -\frac{9}{20} \pm \frac{\sqrt{57}}{20} \right)$$

$$\mathbf{m_p}(t) = \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\mathbf{0} = A\mathbf{w} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \rightarrow A\mathbf{w} = -\begin{pmatrix} 200 \\ 0 \end{pmatrix} \rightarrow \mathbf{w} = \begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$$

$$\mathbf{m}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2} + \begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$$

Nonhomogeneous case - example

- A “Method of undetermined coefficients” similar to what we saw for second order equations can be used for systems.

- For this course, I’ll only test you on constant nonhomogeneous terms (like the previous example).