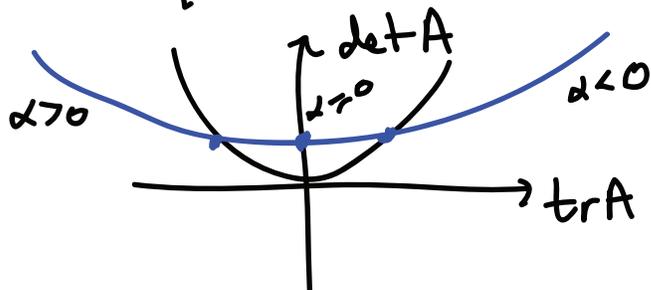


$$2. A = \begin{pmatrix} -\alpha & 1 \\ -1 & -3\alpha \end{pmatrix}$$

$$\lambda^2 + 4\alpha\lambda + 3\alpha^2 + 1 = 0$$



$$\begin{cases} \text{tr}A = -4\alpha \\ \det A = 3\alpha^2 + 1 \\ \det A = \frac{1}{4}(\text{tr}A)^2 \leftarrow \text{nodes or spirals} \end{cases}$$

$$\alpha = \frac{\text{tr}A}{-4}$$

$$\det A = 3 \left(\frac{\text{tr}A}{-4} \right)^2 + 1 = \frac{3}{16} (\text{tr}A)^2 + 1$$

$$\frac{3}{16} < \frac{1}{4}$$

$$\frac{3}{16} t^2 + 1 = \frac{1}{4} t^2$$

$$1 = \frac{1}{16} t^2$$

$$t^2 = 16$$

$$t = \pm 4$$

$$\text{tr}A = -4\alpha = \pm 4$$

$$\alpha = \pm 1$$

$\alpha \gg 1$	⊖	sn
$0 < \alpha < 1$	⊖	ss
$-1 < \alpha < 0$	⊖	us
$\alpha < -1$	⊖	un
ND	⊖	saddle

$\alpha > 0$ stability
 $\alpha < 0$ instab
 $\alpha^2 > 1$ nodes
 $\alpha^2 < 1$ spiral

$$3. (a) \quad Y(s) = \frac{20}{s(s^2+4s+20)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+20}$$

$$20 = A(s^2+4s+20) + Bs^2 + Cs$$

$$A+B=0 \Rightarrow A=-B$$

$$4A+C=0 \Rightarrow C=-4A$$

$$20A=20 \Rightarrow A=1, B=-1, C=-4$$

$$\begin{aligned} Y(s) &= \frac{1}{s} - \frac{s+4}{s^2+4s+20} = \frac{1}{s} - \frac{s+4}{s^2+4s+4+16} = \frac{1}{s} - \frac{s+4}{(s+2)^2+4^2} \\ &= \frac{1}{s} - \frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2+4^2} = \frac{1}{s} - \frac{s+2}{(s+2)^2+4^2} - \frac{1}{2} \frac{4}{(s+2)^2+4^2} \end{aligned}$$

$$y(t) = 1 - e^{-2t} \cos 4t - \frac{1}{2} e^{-2t} \sin 4t$$

$$(b) \quad y'' + 4y' + 20y = 20 \quad y(0) = 0, \quad y'(0) = 0$$

$$\text{or } y' = 5 e^{-2t} \sin 4t \quad y(0) = 0, \quad y'(0) = 0$$

$$4. (a) A' = -5 \cdot \frac{A}{5} + 5 \cdot \frac{B}{5} = -A + B$$

$$B' = A - B$$

$$\begin{pmatrix} A \\ B \end{pmatrix}' = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \quad \textcircled{1}$$

$$A(0) = 400 \quad \textcircled{1}$$

$$B(0) = 0$$

$$(b) \lambda = 0, -2 \quad \textcircled{1}$$

$$\lambda_1 = 0 \quad \bar{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \textcircled{1}$$

$$\lambda_2 = -2 \quad \bar{v}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \textcircled{1}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = c_1 e^{0t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$A(0) = c_1 + c_2 = 400$$

$$c_1 = c_2 = 200$$

$$B(0) = c_1 - c_2 = 0$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = 200 \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) \quad \textcircled{1}$$

$$(c) A(t) = 200 (1 + e^{-2t}) \quad \leadsto \quad t = 1 \quad \textcircled{1}$$

$$A(1) = 200 + 200 \frac{1}{e^2}$$

$$\left[t = 1 + \frac{1}{2} \ln(200) \right]$$

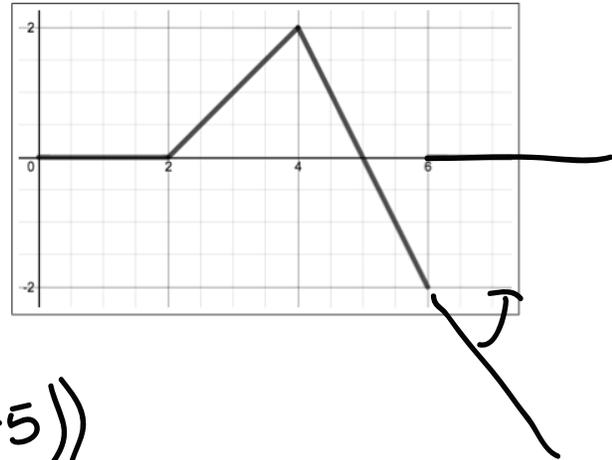
(d) No. Steady state is at 200 so he'll never drop below that. \textcircled{1}

$$5. (a) g(t) = (t-2)u_2(t) - 3(t-4)u_4(t)$$

$$+ 2(t-6)u_6(t)$$

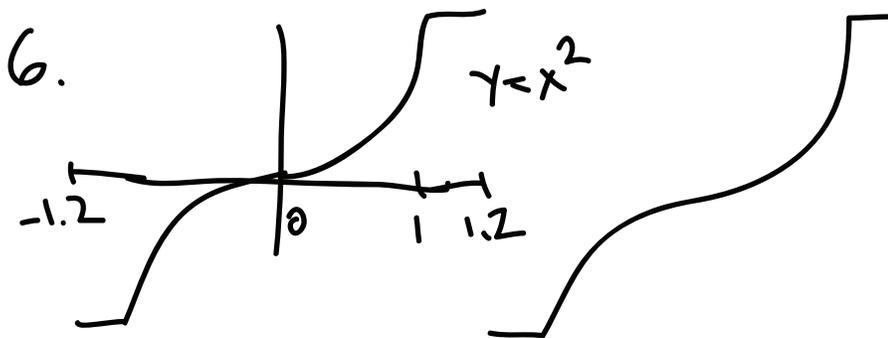
$$+ 2u_6(t)$$

$$= (u_2(t) - u_4(t))(t-2) + (u_4(t) - u_6(t))(-2(t-5))$$



$$(b) f(t) = u_2(t) \cdot (t-2) - 2u_3(t)(t-3) + u_4(t)(t-4) + \delta(t-3)$$

$$F(s) = e^{-2s} \frac{1}{s^2} - 2e^{-3s} \frac{1}{s^2} + e^{-4s} \frac{1}{s^2} + e^{-3s}$$



$$\text{Period} = 2.4$$

$$L = 1.2$$