Today

- Intro to the Laplace Transform
- Solving ODEs with forcing terms using Laplace transforms - examples
- Laplace transforms of step functions
- Applications

Laplace transforms - intro (6.1)

- Using the Laplace Transform to solve (linear) ODEs.

- Idea:

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- Idea:

Unknown y(t) that satisfies some ODE $\xrightarrow{\text{solve ODE}}$ Found y(t)

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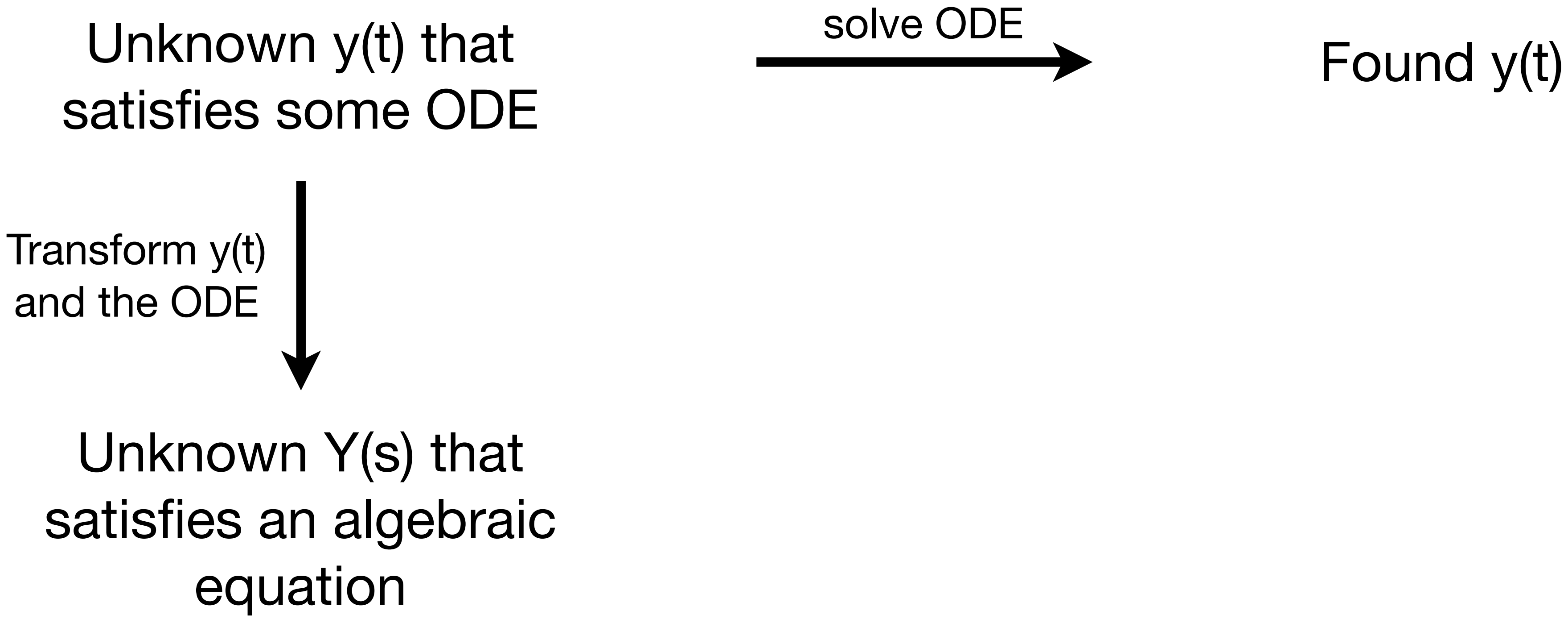

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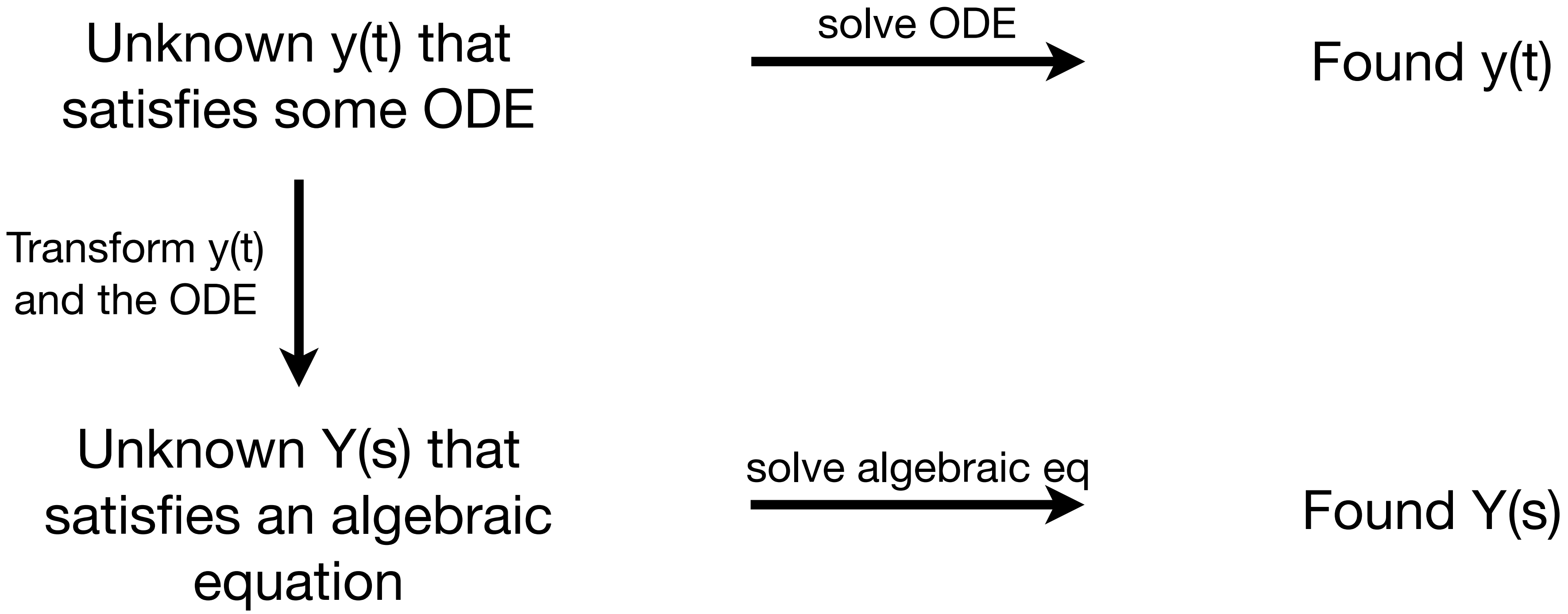

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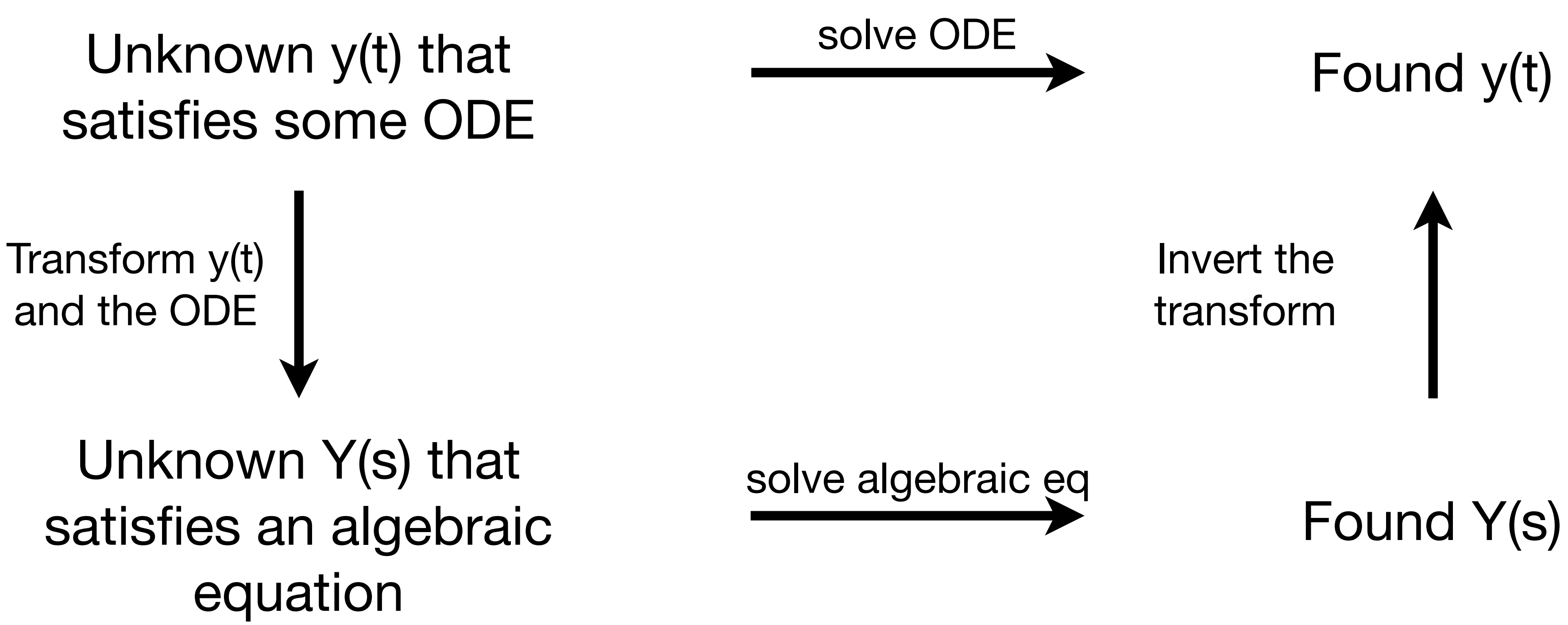

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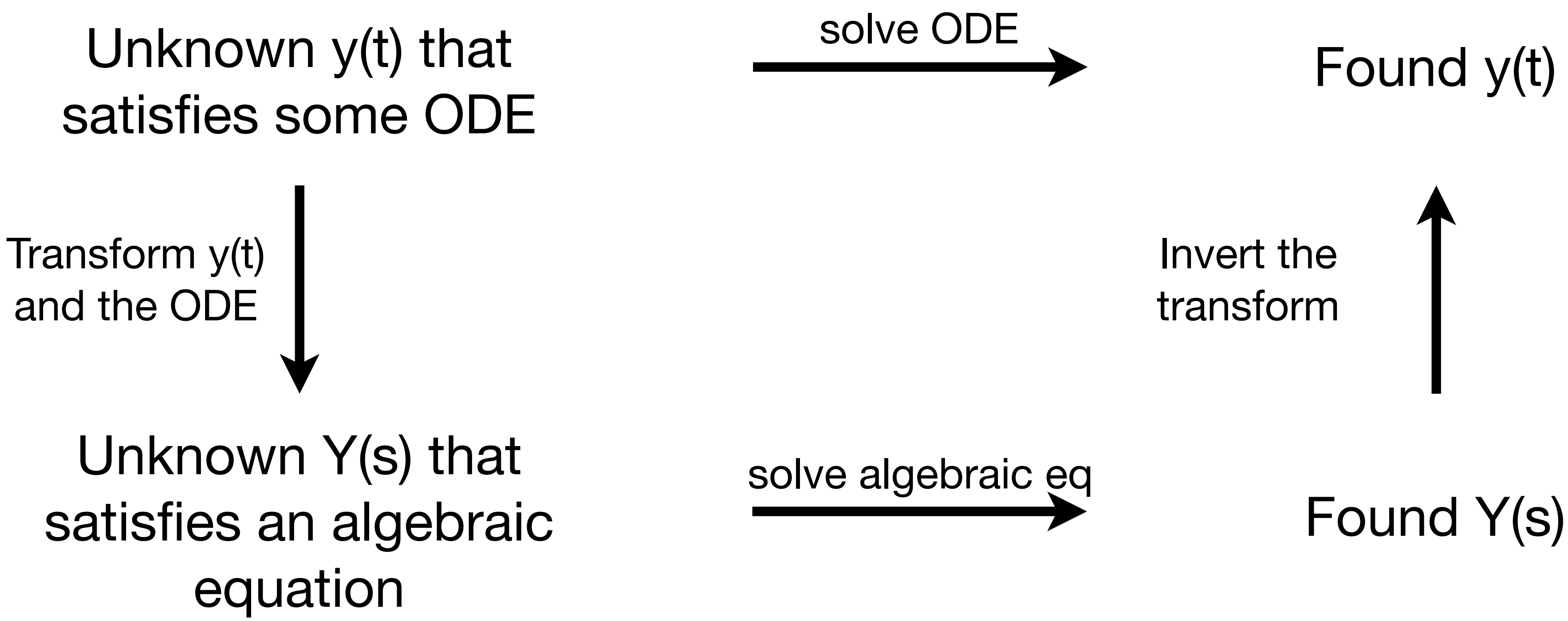

- Laplace transform of y(t): $\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} y(t)\ dt$

Laplace transforms - examples (6.1)

- What is the Laplace transform of $y(t) = 3$?

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$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} 3 \; dt$$

$$= -\frac{3}{s} e^{-st} \Big|_0^\infty$$

$$= \lim_{A \to \infty} -\frac{3}{s} e^{-st} \Big|_0^A$$

$$= -\frac{3}{s} \left(\lim_{A \to \infty} e^{-sA} - 1 \right)$$

$$= \frac{3}{s}$$ provided $s > 0$ and does not exist otherwise.

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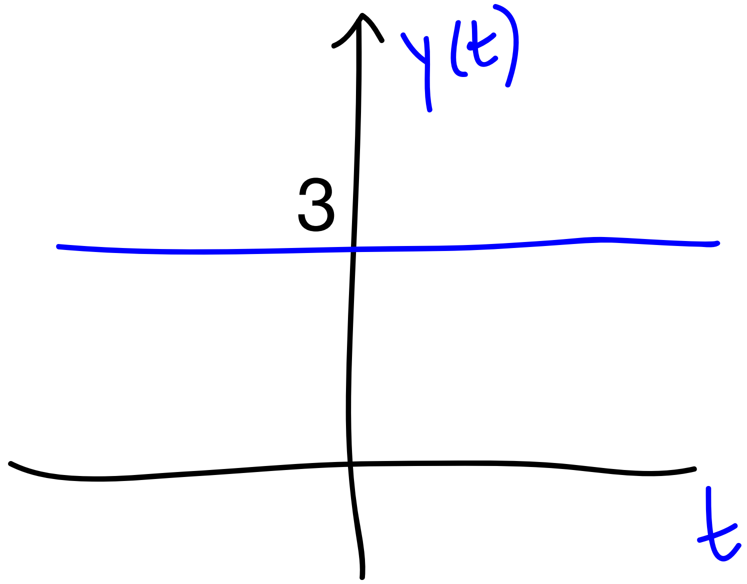

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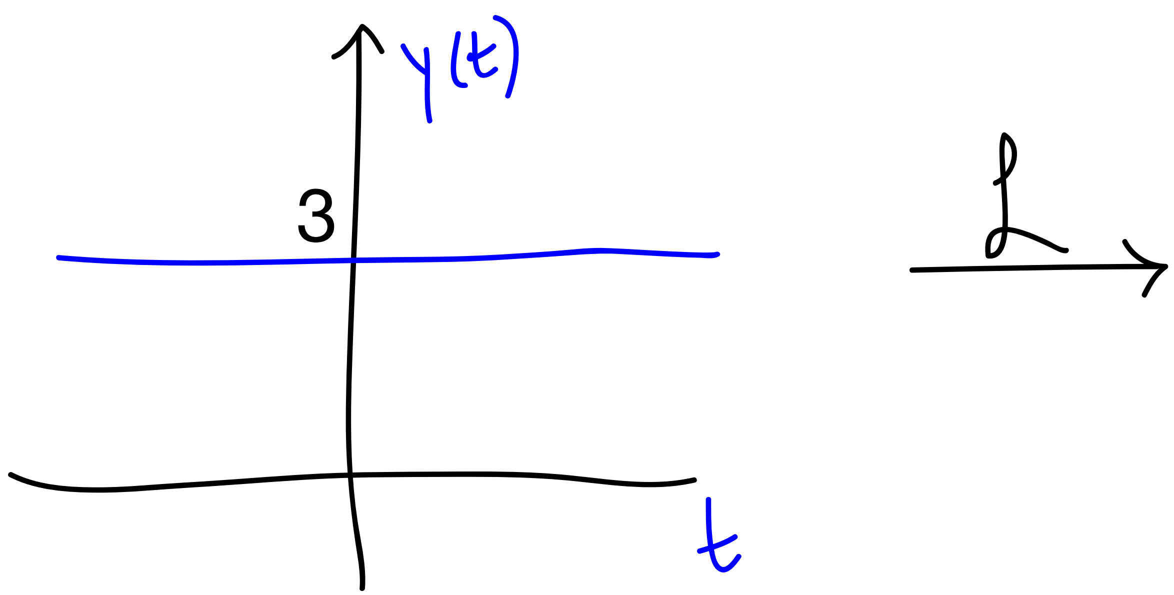

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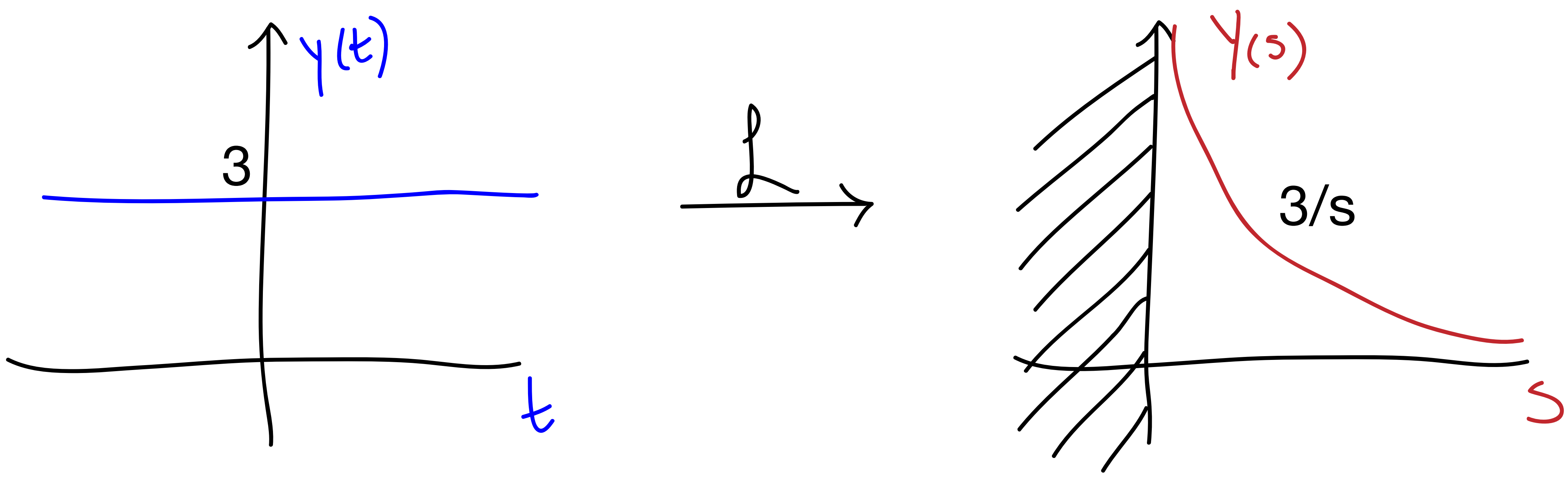

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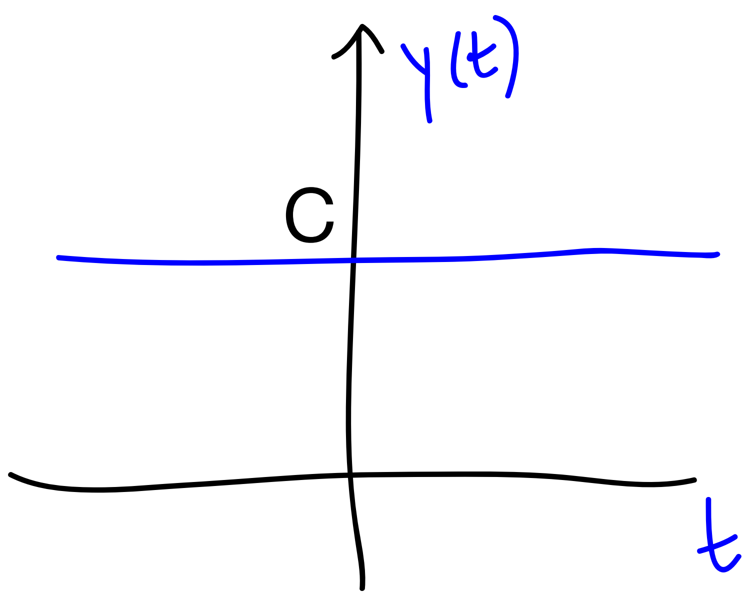

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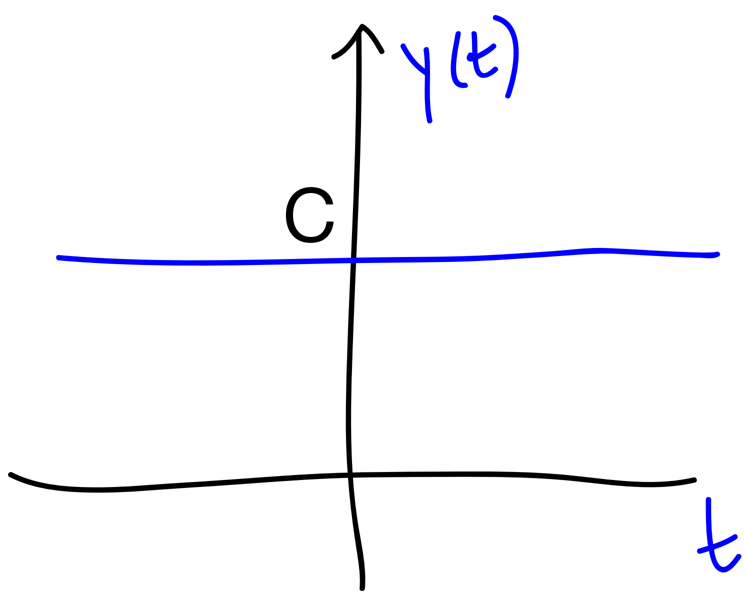

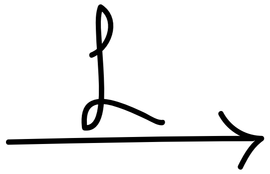

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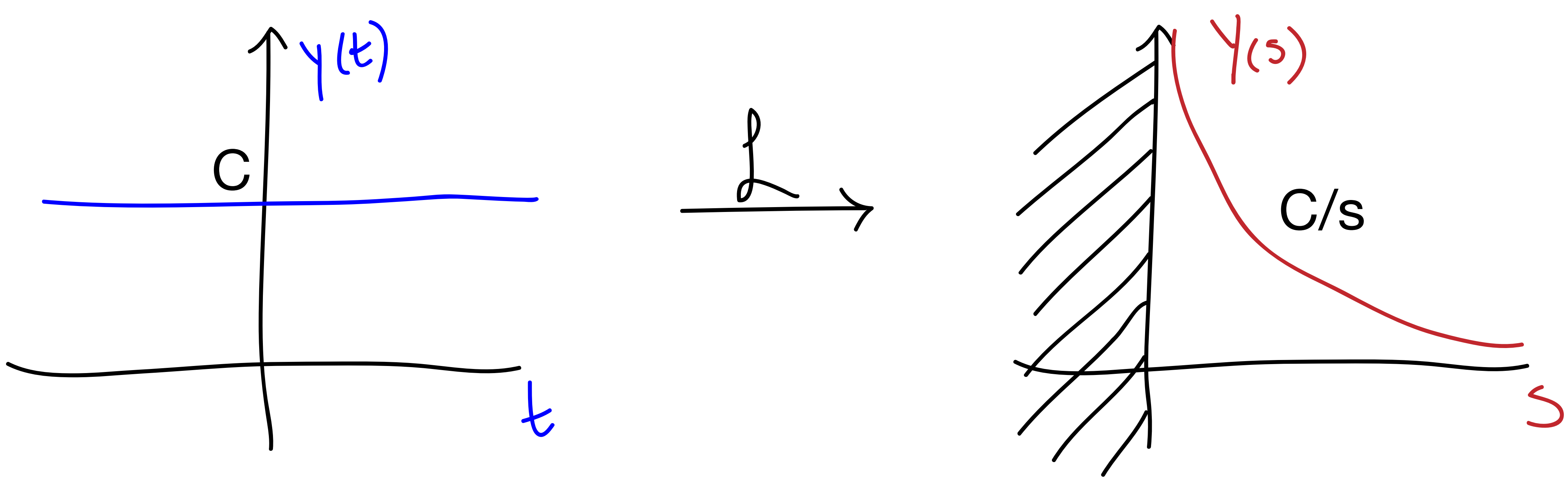

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$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} e^{6t} \, dt$$

(A) $Y(s) = \dfrac{1}{s-6} \qquad s > 0$

(B) $Y(s) = \dfrac{1}{6-s} \qquad s > 6$

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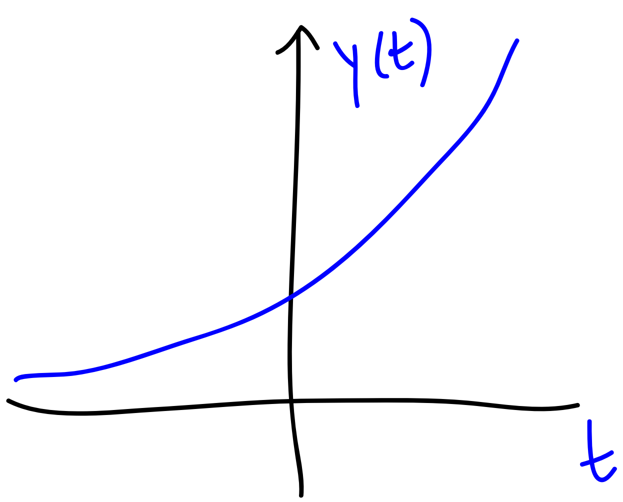

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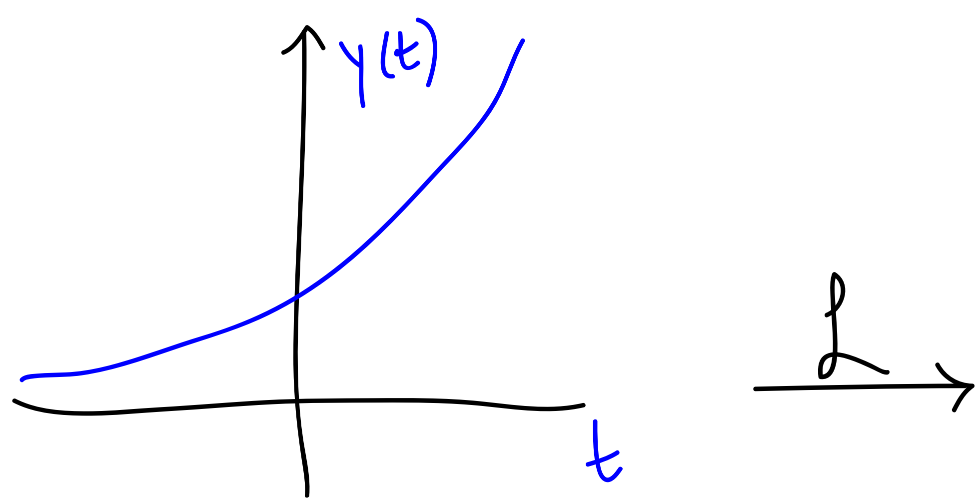

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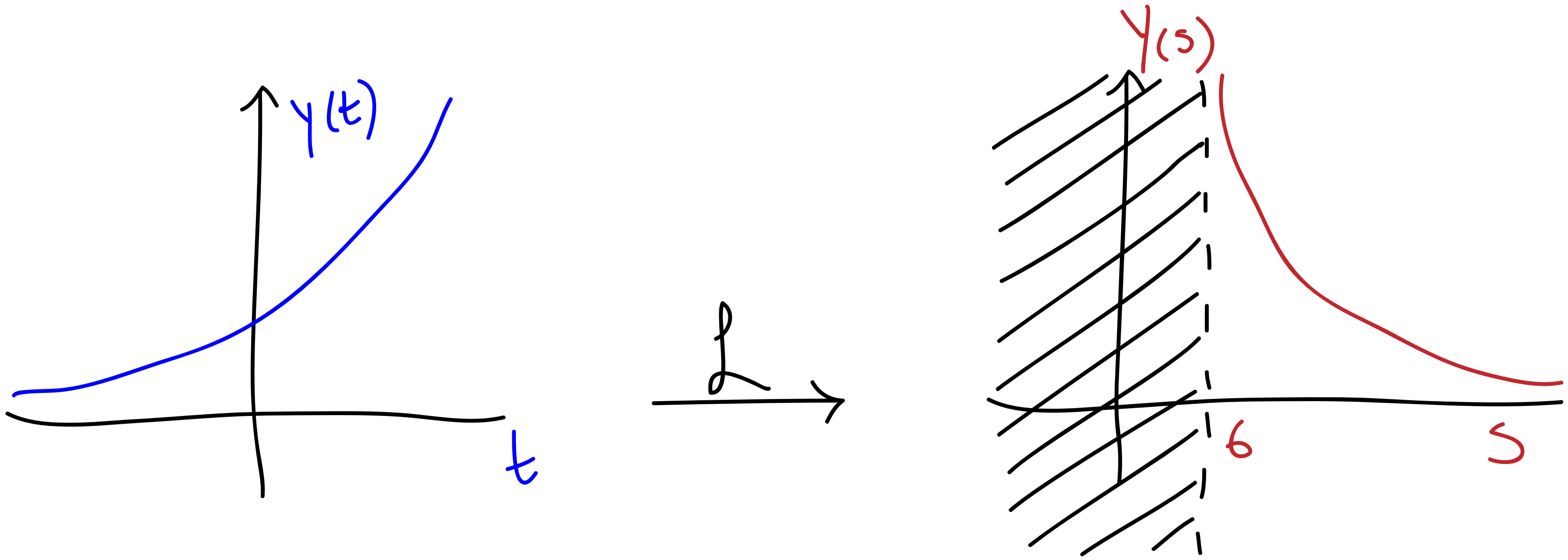

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$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty \underbrace{e^{-st}}_{u} \underbrace{\sin t \; dt}_{dv}$$

$$= e^{-st}(-\cos t)\Big|_0^\infty - \int_0^\infty (-s)e^{-st}(-\cos t)\; dt$$

$$= \lim_{A\to\infty} e^{-sA}(-\cos A) - (-1) - \int_0^\infty (-s)e^{-st}(-\cos t)\; dt$$

$$= 1 - s\int_0^\infty e^{-st}\cos t\; dt \qquad s > 0$$

$$= 1 - s\left(e^{-st}\sin t\Big|_0^\infty + s\int_0^\infty e^{-st}\sin t\; dt \right)$$

0

$F(s)$

$$= 1 - s^2 F(s) \qquad s > 0$$

$$F(s) = \frac{1}{1+s^2} \qquad s > 0$$

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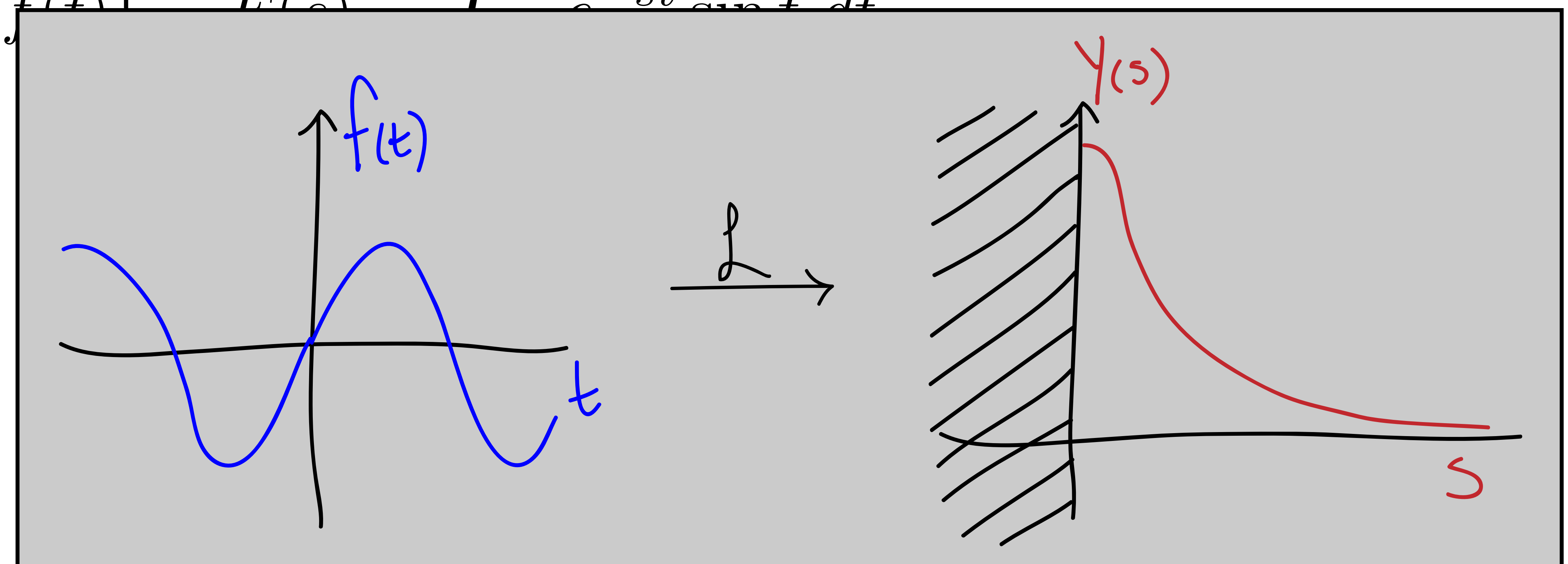

dt

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$$u = \omega t$$
$$du = \omega dt$$

(A) $H(s) = \dfrac{\omega}{\omega^2 + s^2}$

(B) $H(s) = \dfrac{1}{1 + \left(\frac{s}{w}\right)^2}$

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$$= e^{-st} f(t) \Big|_0^\infty + s \int_0^\infty e^{-st} f(t)\ dt$$

$$= -f(0) + sF(s) \qquad s > 0$$

$$= -0 + s\frac{1}{1+s^2} = \frac{s}{1+s^2}$$

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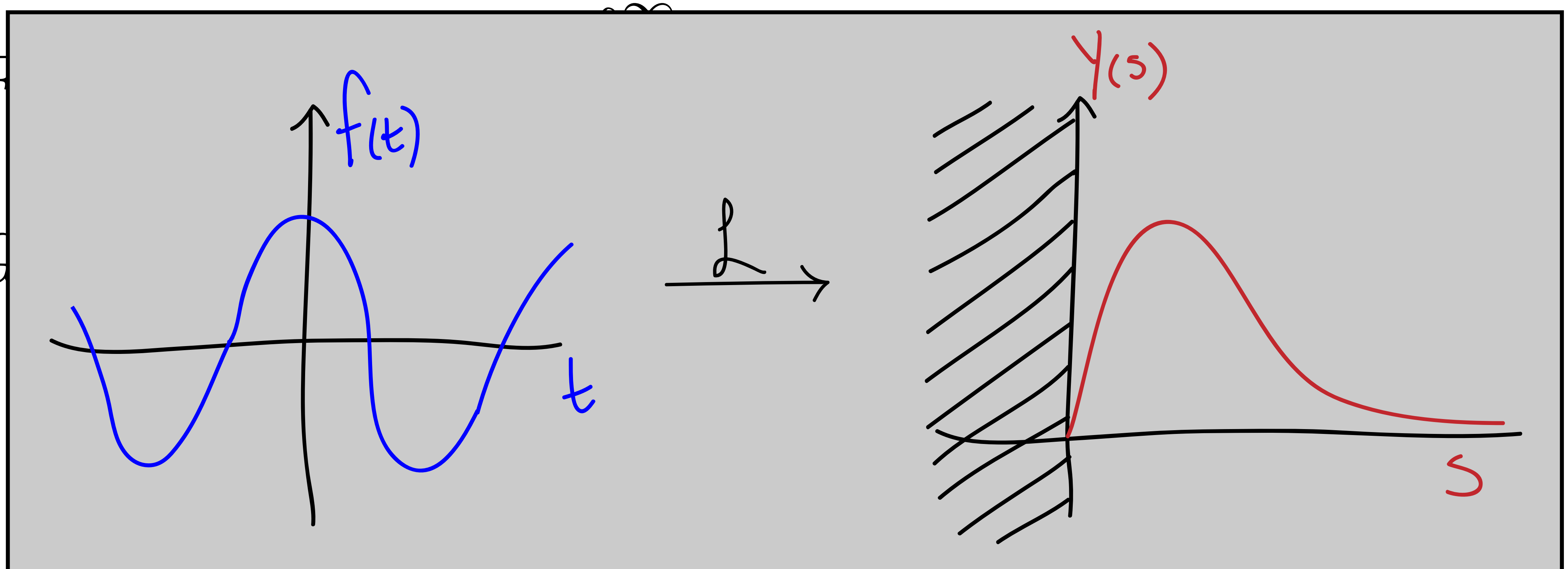

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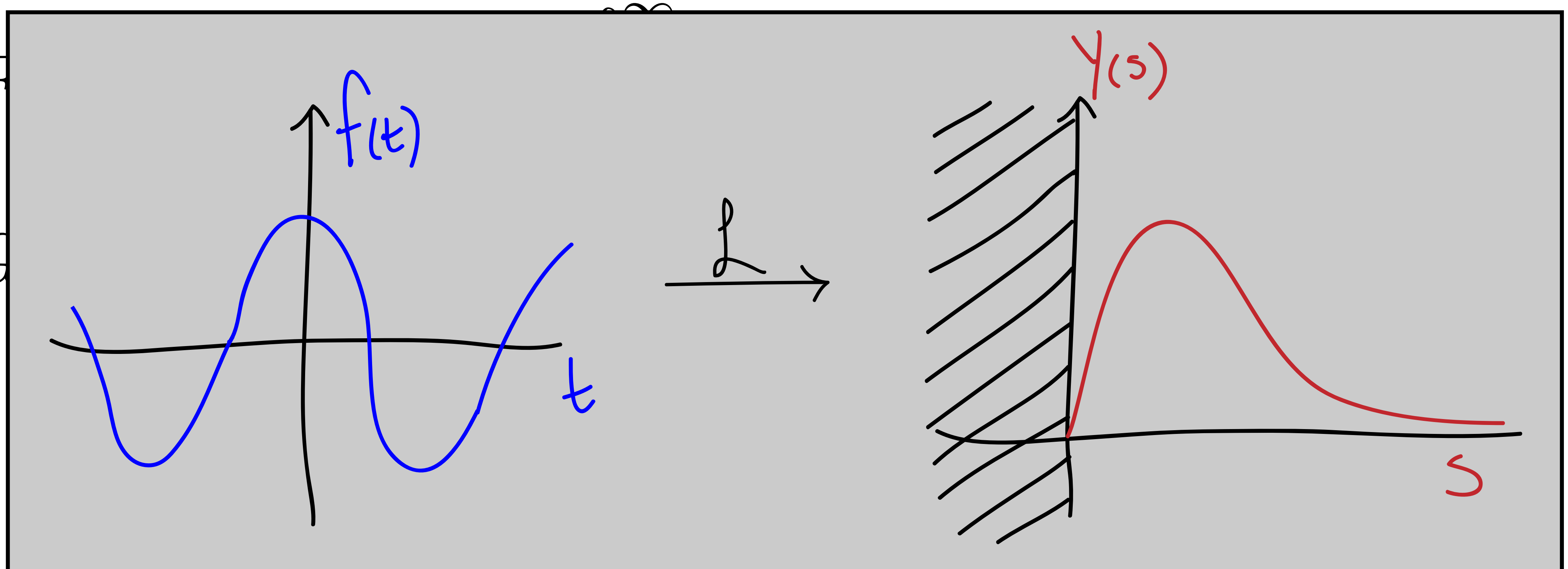
G



$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$s > 0$

$1 + s^2$ $\quad$ $1 + s^2$

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$$a\underline{\mathcal{L}\{y''\}} + b\underline{\mathcal{L}\{y'\}} + c\underline{\mathcal{L}\{y\}} = 0$$

$$a(\underline{s^2Y(s) - sy(0) - y'(0)}) + b(\underline{sY(s) - y(0)}) + c\underline{Y(s)} = 0$$

$$(as^2 + bs + c)Y(s) = asy(0) + ay'(0) + by(0)$$

$$Y(s) = \frac{asy(0) + ay'(0) + by(0)}{as^2 + bs + c}$$

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$$\begin{aligned} Y(s) &= \frac{s+3+3}{s^2+6s+9+4} \\ &= \frac{s+3}{(s+3)^2+4} + \frac{3}{(s+3)^2+4} \\ &= \frac{s+3}{(s+3)^2+4} + \frac{3}{2}\frac{2}{(s+3)^2+4} \end{aligned}$$

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$$\lambda = \frac{-6 \pm i\sqrt{52-36}}{2} = -3 \pm 2i$$

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4. Fix up coefficient of the term with no s in the numerator. 5. Invert.