

Welcome to MATH 256

Differential equations (for Chemical and Biological Engineering students)

Instructor:

Prof. Eric Cytrynbaum

Course goals

- **Primary:** Learn to solve ordinary and partial differential equations (mostly linear first and second order DEs).
- **Secondary:** Learn to use DEs to model physical, chemical, biological systems (really just an intro to this skill).

Prerequisites

- First year calculus (MATH 100/101).
- Linear algebra (MATH 152).
- Multivariable calculus (MATH 200 or 253).
- Talk to me if you aren't sure that you're prepared for this course.

Tools we'll be using this term

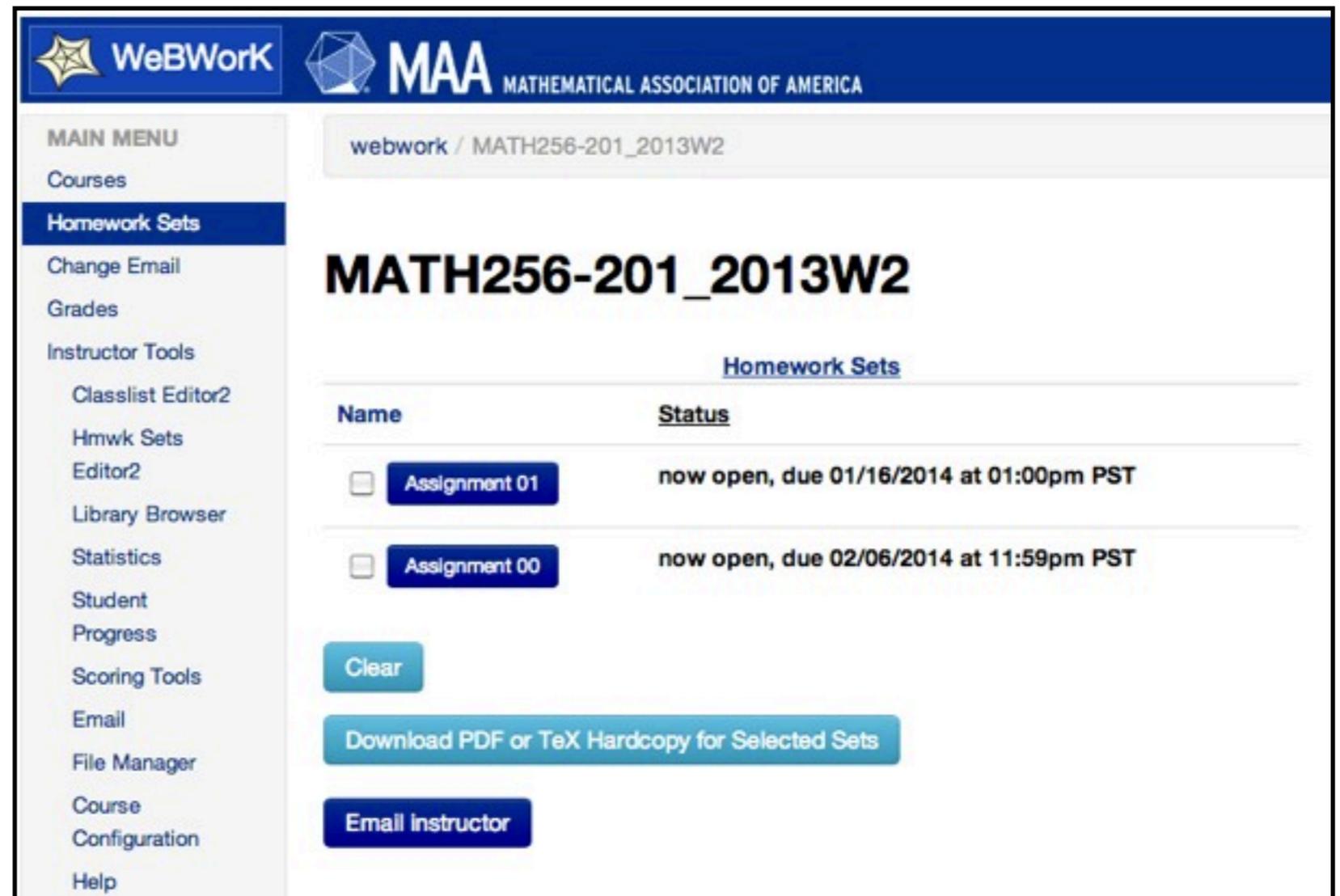
- WeBWork for homework assignments.
- Piazza for online discussion.
- Clickers for in-class responses.
- Cell phones and facebook for getting distracted during lectures and while studying.

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WeBWork

- Online homework system.
- https://webwork.elearning.ubc.ca/webwork2/MATH256-201_2013W2
- Log in using your CWL.



The screenshot displays the WeBWork interface for the course MATH256-201_2013W2. The top navigation bar includes the WeBWork logo and the MAA (Mathematical Association of America) logo. The main menu on the left lists various options such as Courses, Homework Sets, Change Email, Grades, Instructor Tools, Classlist Editor2, Hmwk Sets Editor2, Library Browser, Statistics, Student Progress, Scoring Tools, Email, File Manager, Course Configuration, and Help. The main content area shows the course breadcrumb "webwork / MATH256-201_2013W2" and the course title "MATH256-201_2013W2". Below the title, there is a section for "Homework Sets" with a table listing the sets and their statuses.

Name	Status
<input type="checkbox"/> Assignment 01	now open, due 01/16/2014 at 01:00pm PST
<input type="checkbox"/> Assignment 00	now open, due 02/06/2014 at 11:59pm PST

Below the table, there are three buttons: "Clear", "Download PDF or TeX Hardcopy for Selected Sets", and "Email Instructor".

Why WeBWork?

- Automated marking (instant feedback).
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- Have you used WeBWork previously? (A) Yes. (B) No.

Piazza

- Online discussion forum.
- Sign up at <https://piazza.com>

The screenshot shows the Piazza interface for a course named MATH 256. The top navigation bar includes the Piazza logo, the course name, and tabs for Q & A, Course Page, and Manage Class. A user profile for Eric Cytrynbaum is visible in the top right. Below the navigation, there are filters for Unread, Updated, Unresolved, and Following. A search bar and a 'New Post' button are also present. The main content area displays a pinned post titled 'Welcome to Piazza!' by the instructor, Eric Cytrynbaum, dated 12:37PM. The post text reads: 'MATH 256 students, Welcome to Piazza! We'll be using Piazza as our online discussion forum this term. The quicker you welcome to Piazza! We'll be using Piazza as our online discussion forum this term. The quicker you (rather than via emails), the quicker you'll benefit from the collective knowledge of your classmates and instructors. We encourage you to ask questions when you're struggling to understand a concept—you can even do so anonymously.' Below the post, there is an 'edit' button, a 'good note' button, and a '0' count. A 'followup discussions' section is also visible, with a text input field for starting a new discussion.

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Clickers

- Personal response system.
- Register your clicker at <https://connect.ubc.ca>

Why clickers?

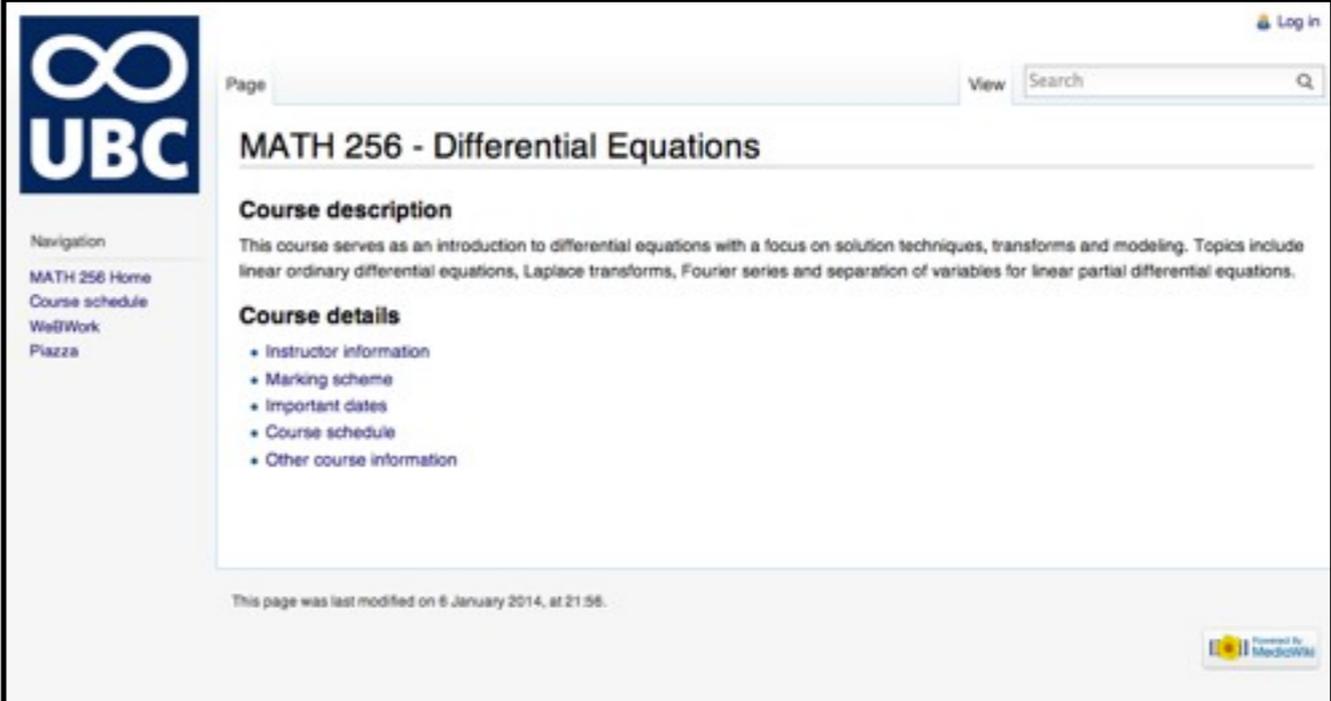
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- My goal is to make clicker Qs that many of you get wrong - they help us to target what you don't understand yet.
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More info online...

- Check the course website for
 - office hour info,
 - info on additional help,
 - textbook,
 - course policies (e.g. marking scheme)
 - week-by-week schedule.
- [https://wiki.math.ubc.ca/mathbook/M256/MATH_256 -
_Differential_Equations](https://wiki.math.ubc.ca/mathbook/M256/MATH_256_-_Differential_Equations)



The screenshot shows a web page for the course "MATH 256 - Differential Equations" on the UBC Wiki. The page features the UBC logo in the top left corner. A navigation menu on the left includes links for "MATH 256 Home", "Course schedule", "We@Work", and "Piazza". The main content area is titled "MATH 256 - Differential Equations" and contains a "Course description" section stating that the course is an introduction to differential equations with a focus on solution techniques, transforms, and modeling. Below this is a "Course details" section with a bulleted list of links: "Instructor information", "Marking scheme", "Important dates", "Course schedule", and "Other course information". At the bottom of the page, a footer indicates the page was last modified on 6 January 2014 at 21:56. A "Powered by MediaWiki" logo is visible in the bottom right corner.

Felix Baumgartner's freefall from 40 km up



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$$ma = -mg + kv^2$$



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- Simple model to predict how fast he'll go, how long it will take etc.



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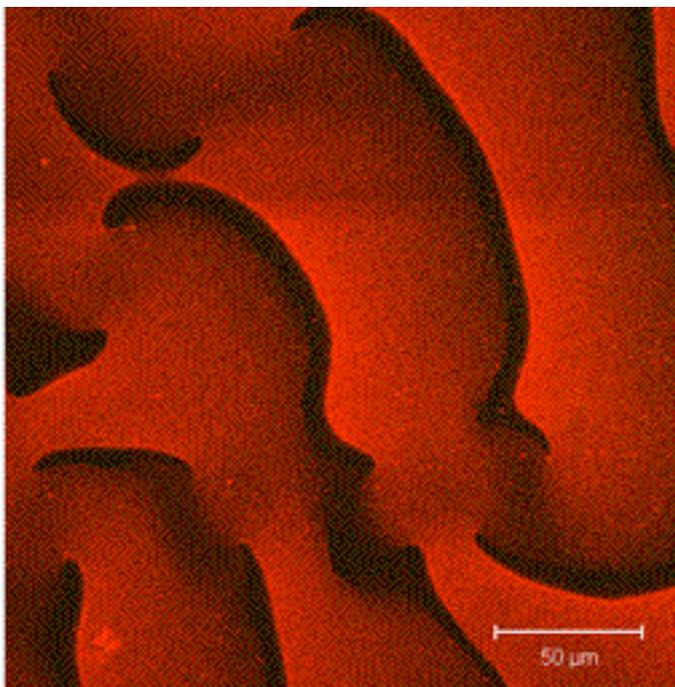
- Flaws with this model?
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- k is not constant either (depends on air density) - this is significant!



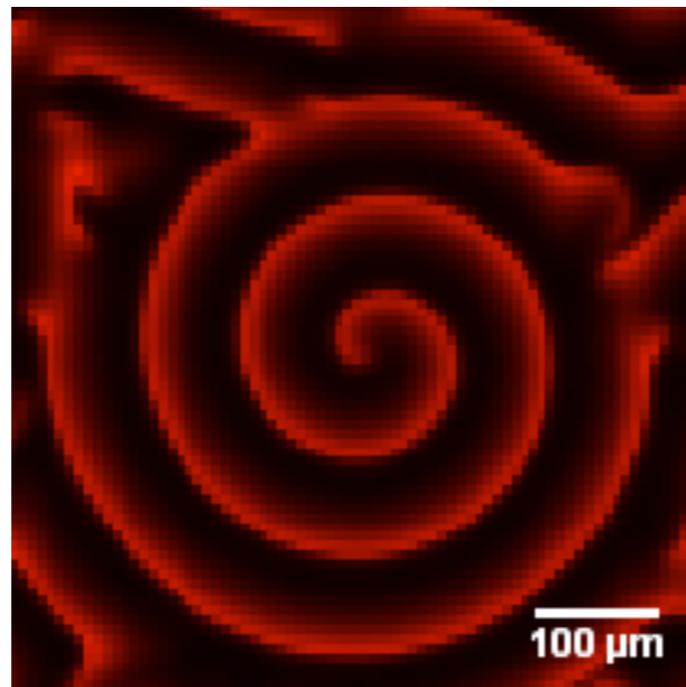
A bacterial cell division regulator

- Two interacting bacterial proteins that undergo complicated dynamics.
- Differential equation model help understand how they work.

Experiment



Model



$$\frac{\partial u}{\partial t} = u - uv + D \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = uv - v + D \frac{\partial^2 v}{\partial x^2}$$

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- **A particular solution** - a solution with no arbitrary constants in it.
- **The general solution** - a solution with one or more arbitrary constants that encompass ALL possible solutions to the DE.

Verifying that a function is a solution

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A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket ($h(0) = 4$), what is the solution to this differential equation?

- A.** $h(t) = (2 - 3t)^2$
- B.** $h(t) = \sqrt{16 - 2t}$
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Method of integrating factors (Section 2.1)

$$\frac{d}{dt} (t^2 y(t)) =$$

(A) $2t \frac{dy}{dt}$

(B) $t^2 \frac{dy}{dt}$

(C) $2ty$

(D) $t^2 \frac{dy}{dt} + 2ty$

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Method of integrating factors (Section 2.1)

- Given that $\frac{d}{dt} (t^2 y(t)) = t^2 \frac{dy}{dt} + 2ty$

- if you're given the equation $t^2 \frac{dy}{dt} + 2ty = 0$

- you can rewrite it as $\frac{d}{dt} (t^2 y(t)) = 0$

- so the solution is $t^2 y(t) = C$ or equivalently $y(t) = \frac{C}{t^2}$.

arbitrary constant
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Method of integrating factors (Section 2.1)

- Solve the equation $t^2 \frac{dy}{dt} + 2ty(t) = \sin(t)$ (not brute force checking).

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general solution
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- An Initial Value Problem (IVP) is a ODE together with an IC.

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$$\frac{dy}{dt} + g'(t)y(t) = 0$$

Method of integrating factors (Section 2.1)

$$t \frac{dy}{dt} + 2y(t) = 1 \quad \rightarrow f(t) = t$$

$$t^2 \frac{dy}{dt} + 4ty(t) = \frac{1}{t} \quad \rightarrow f(t) = t^2$$

$$\frac{dy}{dt} + y(t) = 0 \quad \rightarrow f(t) = e^t$$

$$\frac{dy}{dt} + \cos(t)y(t) = 0 \quad \rightarrow f(t) = e^{\sin(t)}$$

$$\frac{dy}{dt} + g'(t)y(t) = 0 \quad \rightarrow f(t) = e^{g(t)}$$

Method of integrating factors (Section 2.1)

- General case - all first order linear ODEs can be written in the form

$$\frac{dy}{dt} + p(t)y = q(t)$$

- The appropriate integrating factor is $e^{\int p(t)dt}$.
- The equation can be rewritten $\frac{d}{dt} \left(e^{\int p(t)dt} y \right) = e^{\int p(t)dt} q(t)$ which is solvable provided you can find the antiderivative of the right hand side.