Today

- Solving a second order linear homogeneous equation with constant coefficients
    - complex roots to the characteristic equation,
    - repeated roots to the characteristic equation (Reduction of Order).
- Connections to matrix algebra.
- Solving a second order linear **non**homogeneous equation.

Reminder: Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Complex roots (Section 3.3)

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- General solution:

$$y(t) = C_1 e^{\alpha t}\cos(\beta t) + C_2 e^{\alpha t}\sin(\beta t)$$

Complex roots (Section 3.3)

- To be sure this is a general solution, we must check the Wronskian:

$$W(e^{\alpha t}\cos(\beta t), e^{\alpha t}\sin(\beta t))(t) =$$

(for you to fill in later - is it non-zero?)

Recall: $W(y_1, y_2)(t) = y_1(t)y_2'(t) - y_1'(t)y_2(t)$

Complex roots (Section 3.3)

- Example: Find the (real valued) general solution to the equation

$$y'' + 2y' + 5y = 0$$

- Step 1: Assume $y(t) = e^{rt}$, plug this into the equation and find values of r that make it work.

(A) $r_1 = 1+2i$, $r_2 = 1-2i$

(B) $r_1 = -1+2i$, $r_2 = -1-2i$

(C) $r_1 = 1-2i$, $r_2 = -1+2i$

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- Step 2: Real part of r goes in the exponent, imaginary part goes in the trig functions.

(A) $y(t) = e^{-t}(C_1 \cos(2t) + C_2 \sin(2t))$

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(C) $y(t) = C_1 \cos(2t) + C_2 \sin(2t) + C_3 e^{-t}$

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$$y'' + 2y' + 5y = 0, \;\; y(0) = 1, \;\; y'(0) = 0$$

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- There are three cases.

  i. Two distinct real roots: $b^2 - 4ac > 0$. ($r_1 \neq r_2$)

  ii. A repeated real root: $b^2 - 4ac = 0$.

  iii. Two complex roots: $b^2 - 4ac < 0$.

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- Reduction of order - a method for guessing another solution.

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- Example - $y'' + 4y' + 4y = 0.$ Only one root to the characteristic equation, r=-2, so we only get one solution that way: $y_1(t) = e^{-2t}$.
- Use Reduction of order to find a second solution.

$$y_2(t) = v(t)e^{-2t}$$

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$$v'' = 0$$

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$$v'' = 0 \quad \Rightarrow \quad v' = C_1$$

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Guess $y_2(t) = v(t)e^{-2t}$. $y_2'(t) = v'(t)e^{-2t} - 2v(t)e^{-2t}$

$$\Downarrow \qquad\qquad \Downarrow$$

$$4y_2(t) = 4v(t)e^{-2t} \qquad 4y_2'(t) = 4v'(t)e^{-2t} - 8v(t)e^{-2t}$$

$$y_2''(t) = v''(t)e^{-2t} - 2v'(t)e^{-2t} - 2v'(t)e^{-2t} + 4v(t)e^{-2t}$$

$$\searrow \quad y_2''(t) = v''(t)e^{-2t} - 4v'(t)e^{-2t} + 4v(t)e^{-2t}$$

$$0 = y_2'' + 4y_2' + 4y_2 = v''e^{-2t}$$

$$v'' = 0 \quad \Rightarrow \quad v' = C_1 \quad \Rightarrow \quad v(t) = C_1 t + C_2$$

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So yes!

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$$y = e^{\alpha t} \left(C_1 \cos(\beta t) + C_2 \sin(\beta t) \right)$$

Second order, linear, constant coeff, homogeneous

- Find the general solution to the equation

$$y'' - 6y' + 8y = 0$$

(A) $y(t) = C_1 e^{-2t} + C_2 e^{-4t}$

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(C) $y(t) = e^{2t}(C_1 \cos(4t) + C_2 \sin(4t))$

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Second order, linear, constant coeff, **non**homogeneous (3.5)

- Our next goal is to figure out how to find solutions to nonhomogeneous equations like this one:

$$y'' - 6y' + 8y = \sin(2t)$$

- But first, a bit more on the connections between matrix algebra and differential equations . . .

Some connections to linear (matrix) algebra

- An $m \times n$ matrix is a gizmo that takes an n-vector and returns an m-vector:

$$\overline{y} = A\overline{x}$$

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$$z = L[y] = \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y$$

- This one is linear because

$$L[cy] = cL[y]$$

$$L[y + z] = L[y] + L[z]$$

Note: y, z are functions of t and c is a constant.

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Systems of equations written in operator notation.

System of equations	Operator definition	Equation in operator notation

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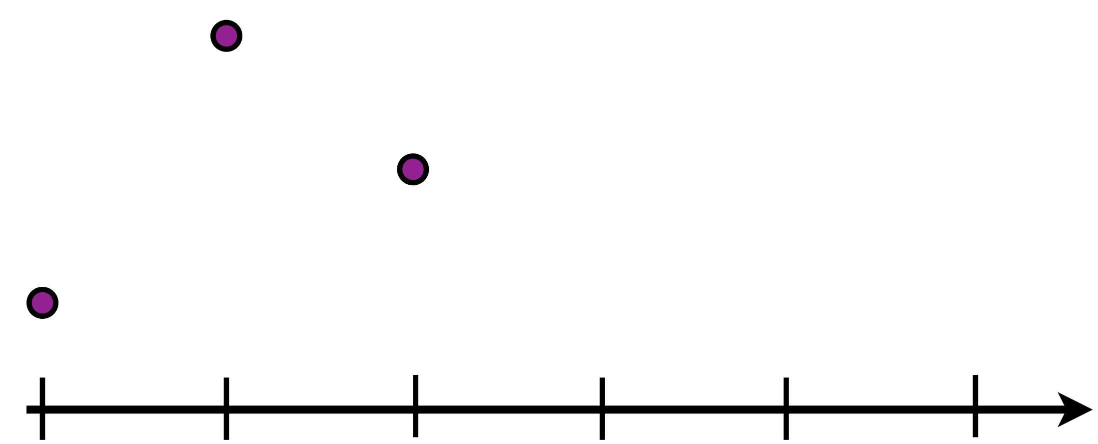

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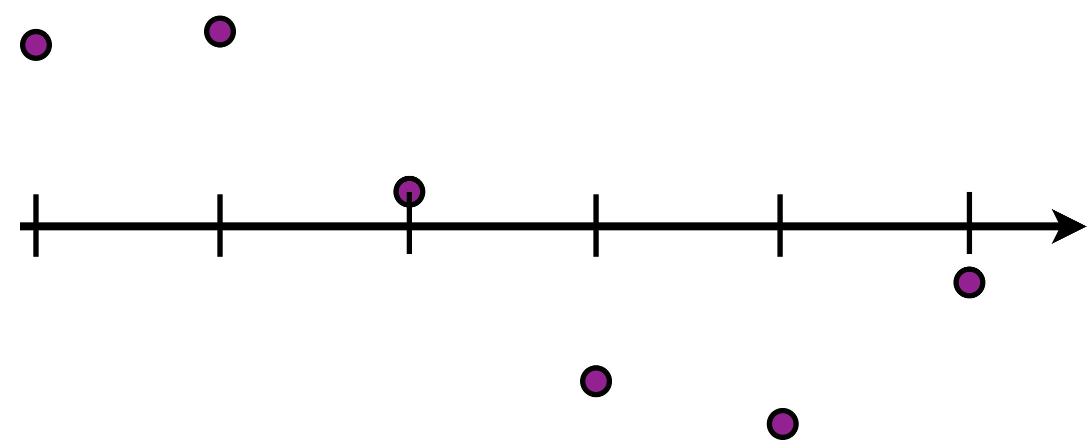

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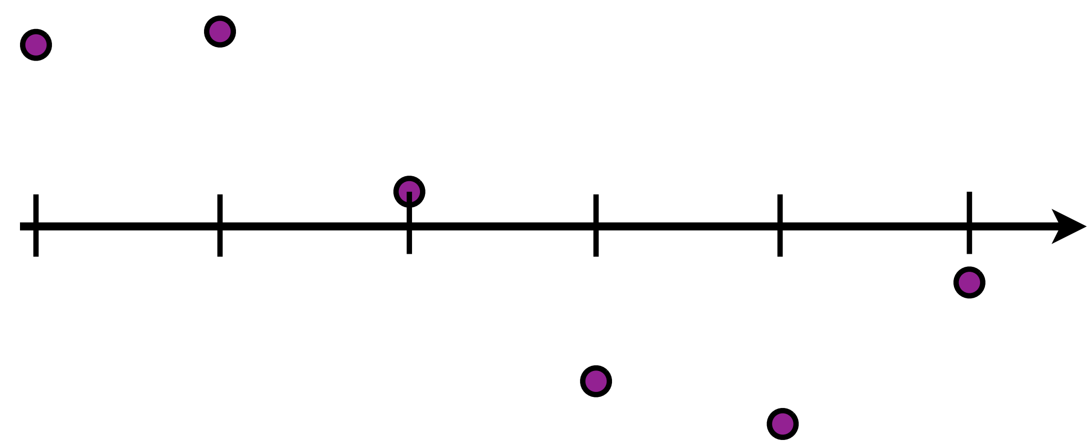

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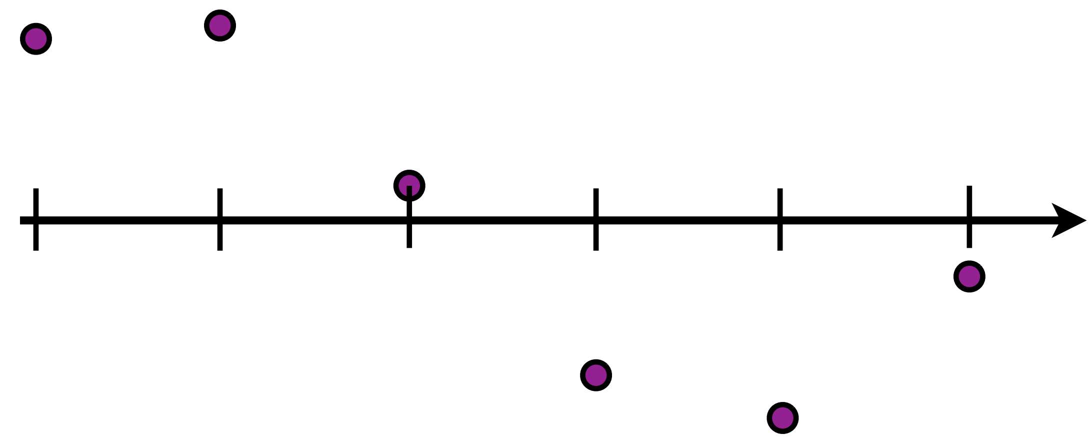

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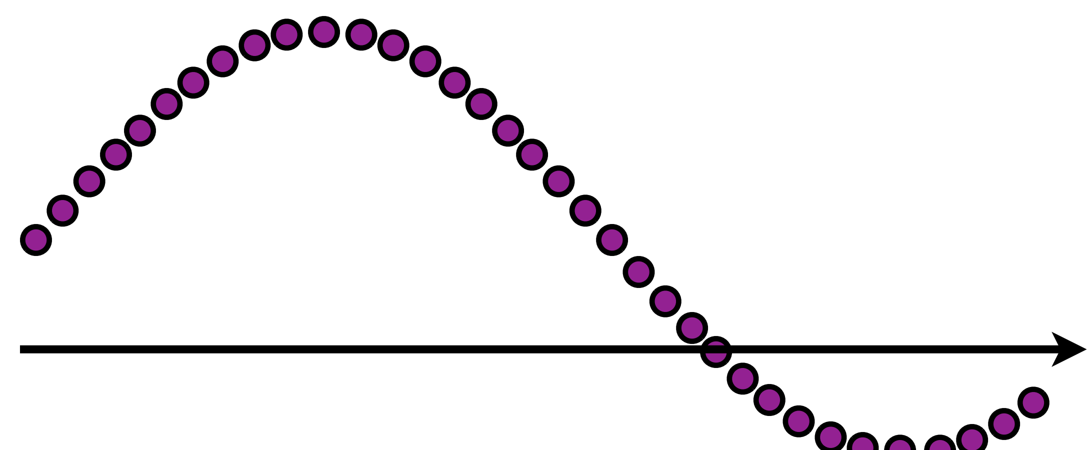
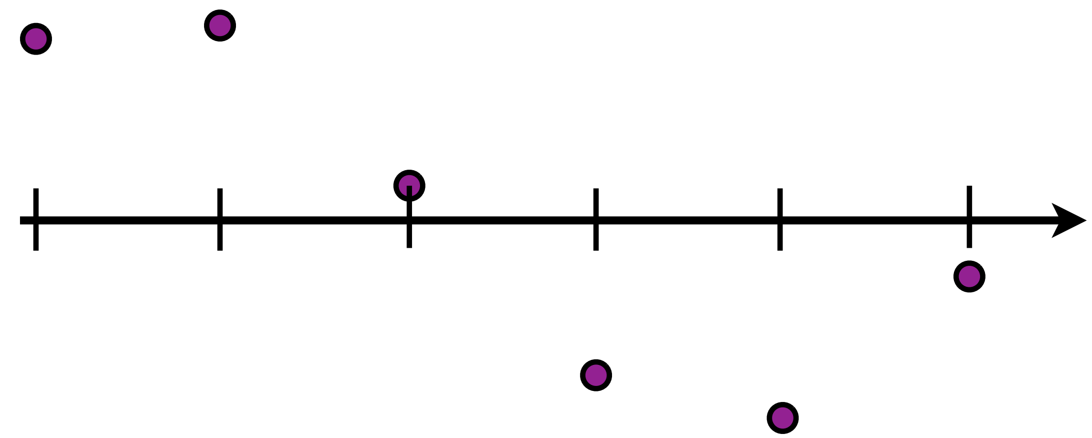


Some connections to linear (matrix) algebra

- A more detailed connection between matrix equations and DEs:
    - A vector as a function

$$\bar{b} = \begin{pmatrix} \sin(1) \\ \sin(2) \\ \sin(3) \\ \sin(4) \\ \sin(5) \\ \sin(6) \end{pmatrix}$$



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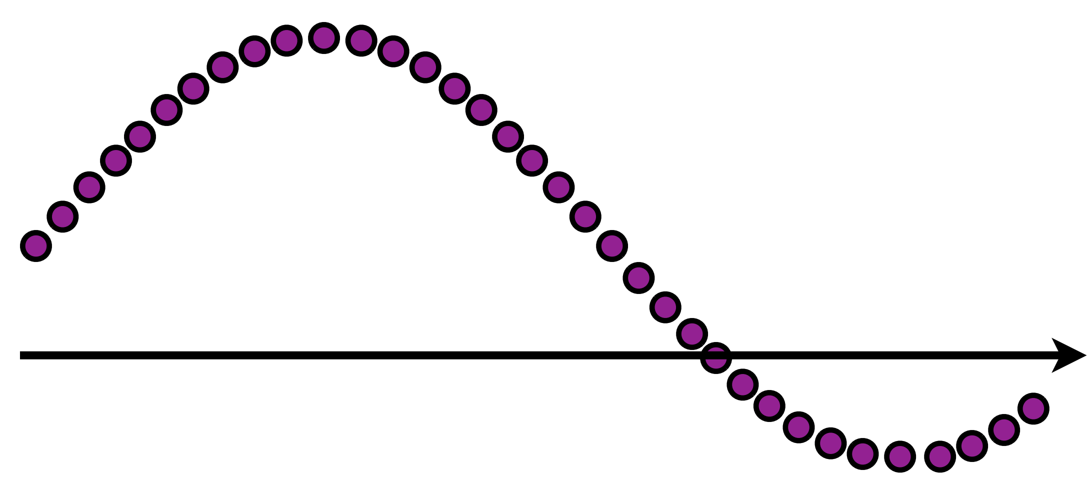

- A differential operator is just a really big matrix.