Today

- Shapes of solutions for distinct eigenvalues case.
- General solution for complex eigenvalues case.
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- Office hours: Friday 1-2 pm, Monday 1-3 pm (to be confirmed)

Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to $\mathbf{x}'=A\mathbf{x}$ is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

- What do solutions look like in the x_1-x_2 plane (called the phase plane)?

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- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$

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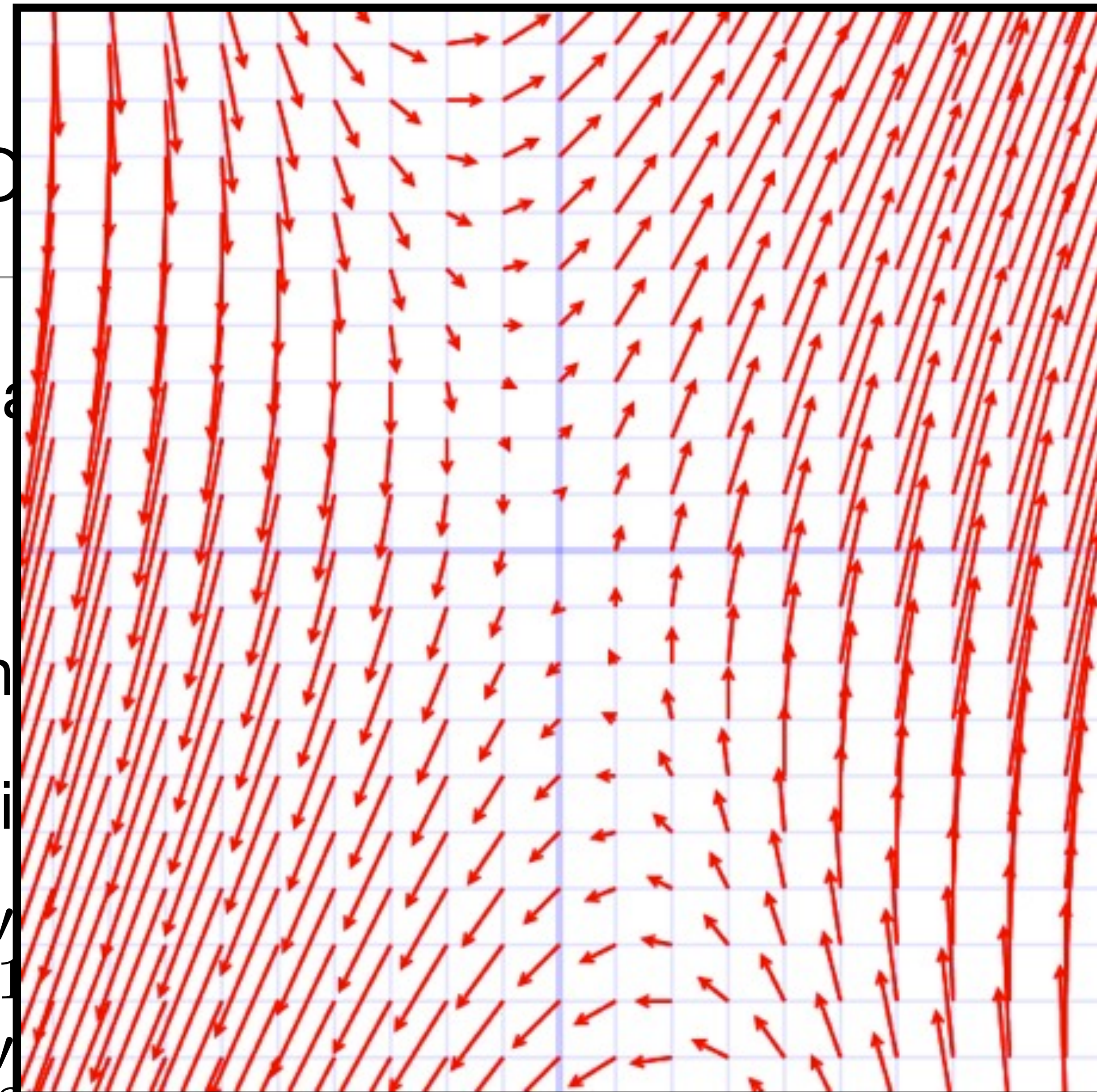

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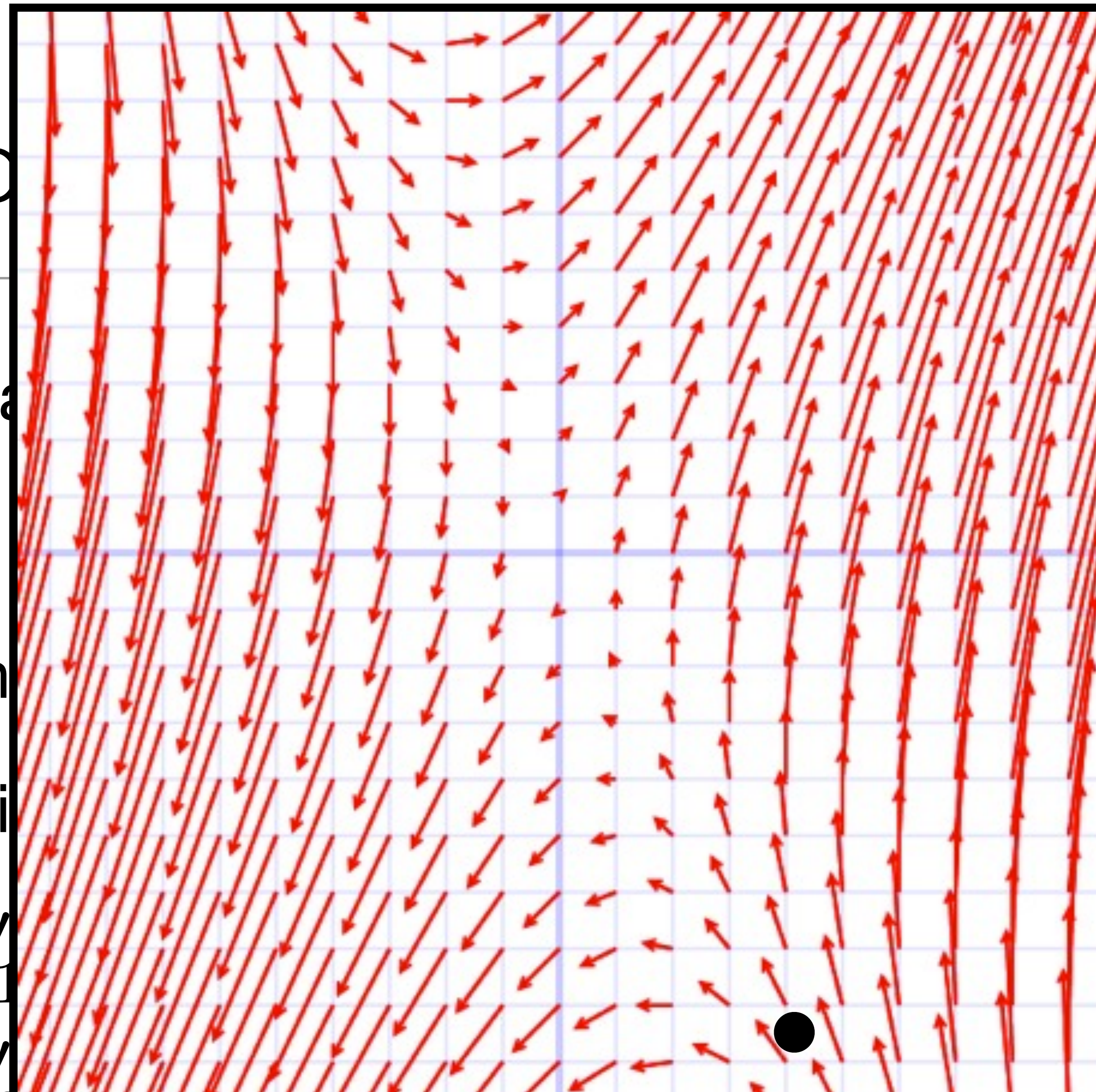

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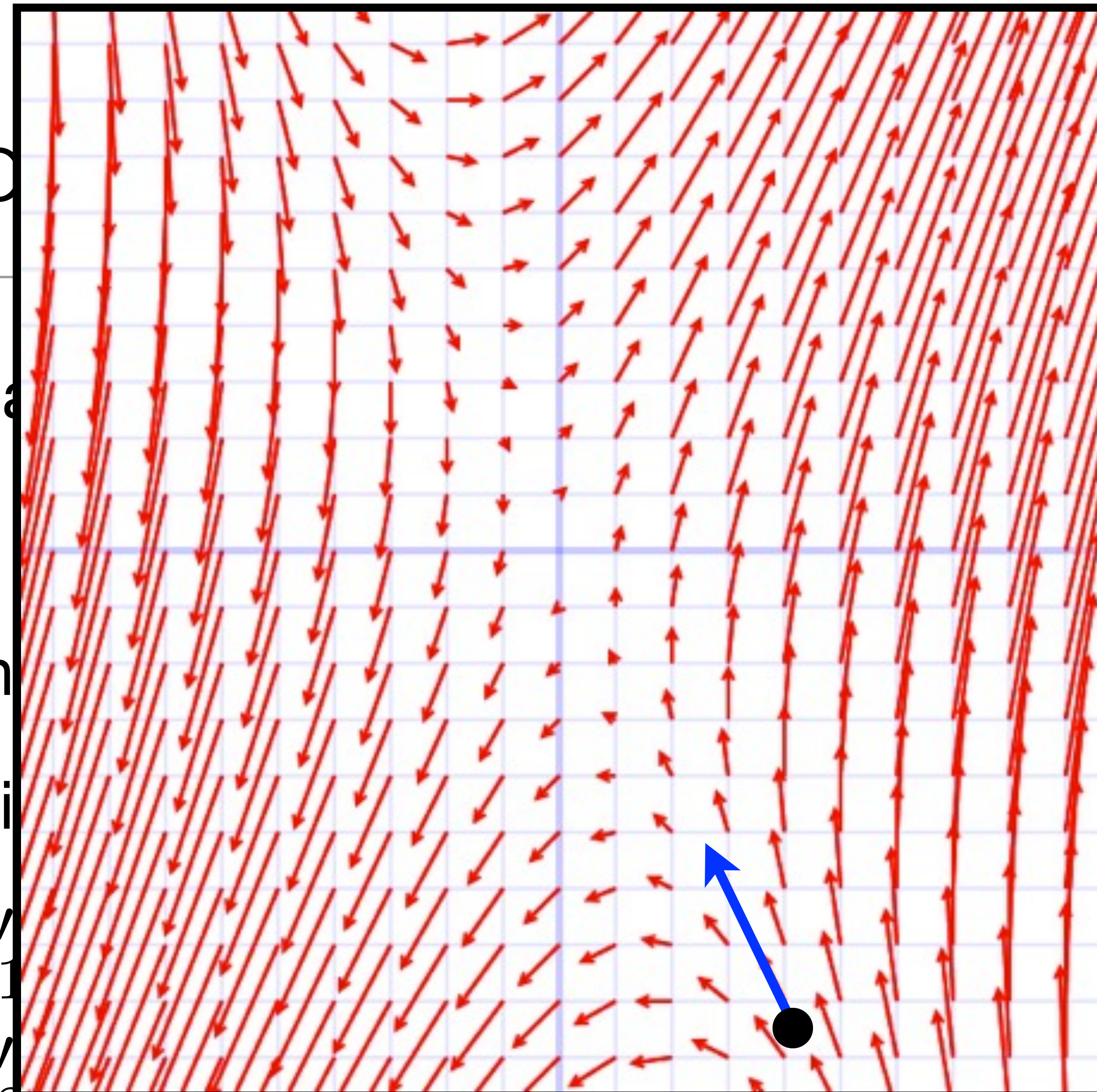

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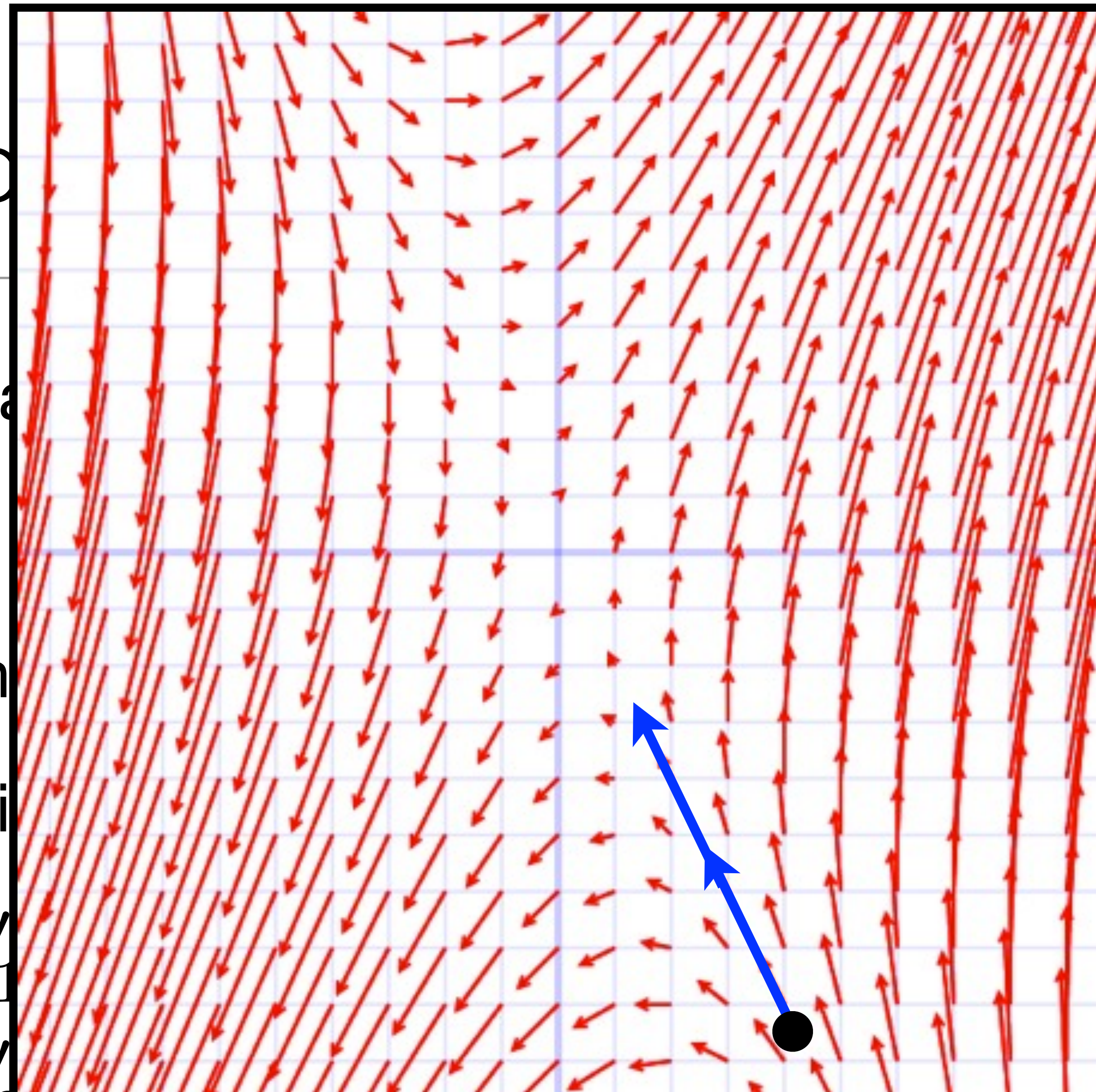

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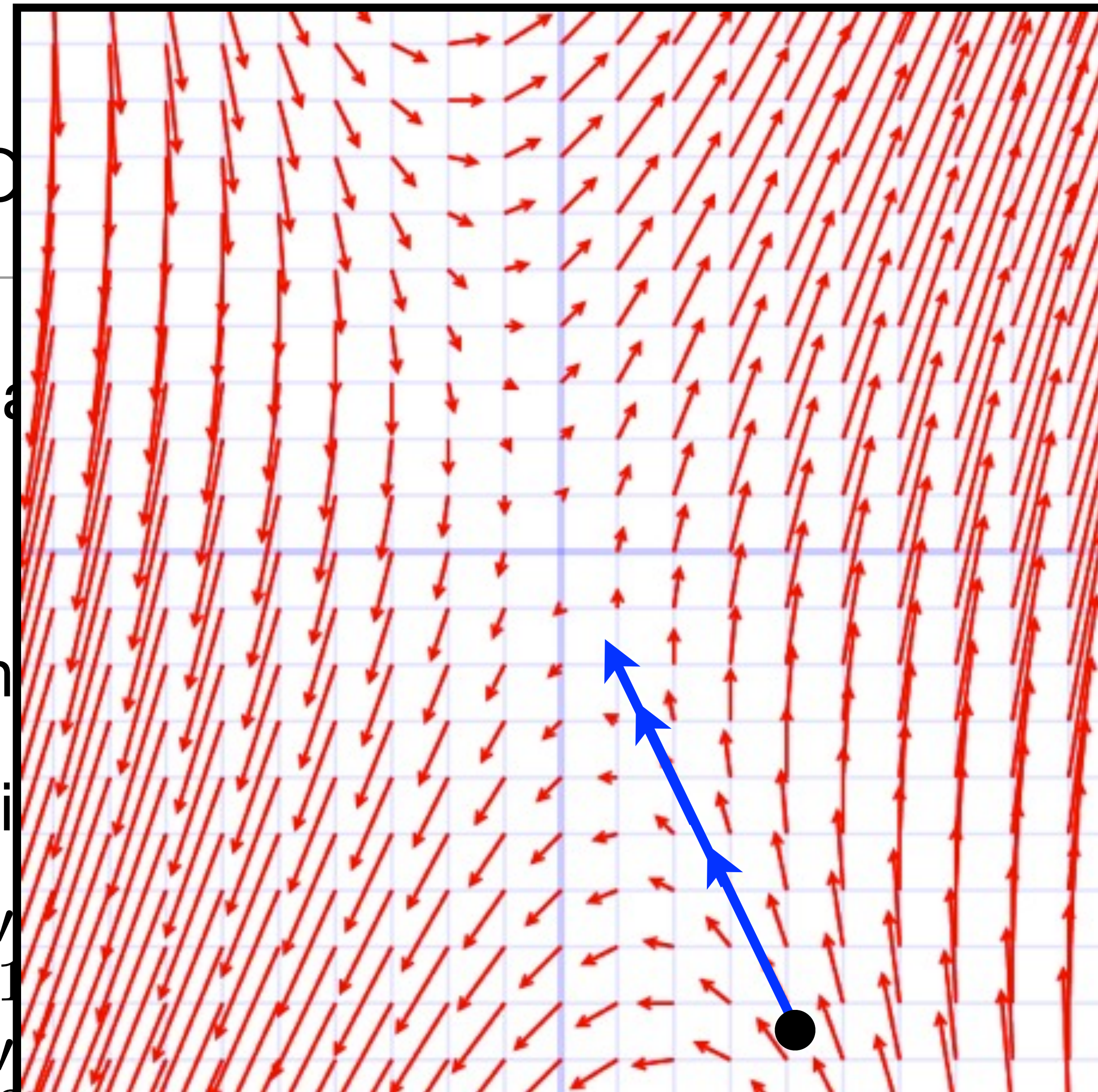

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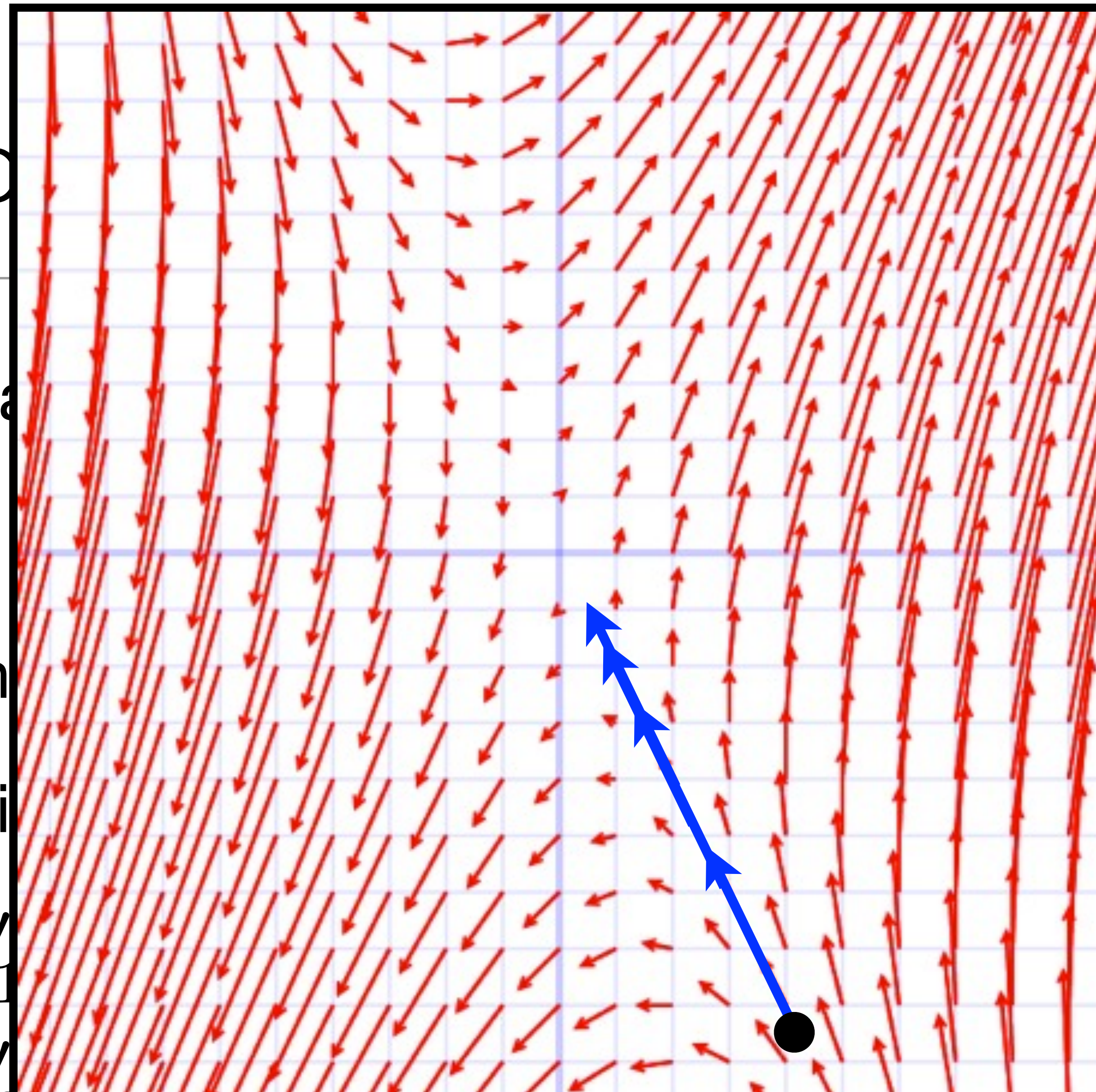

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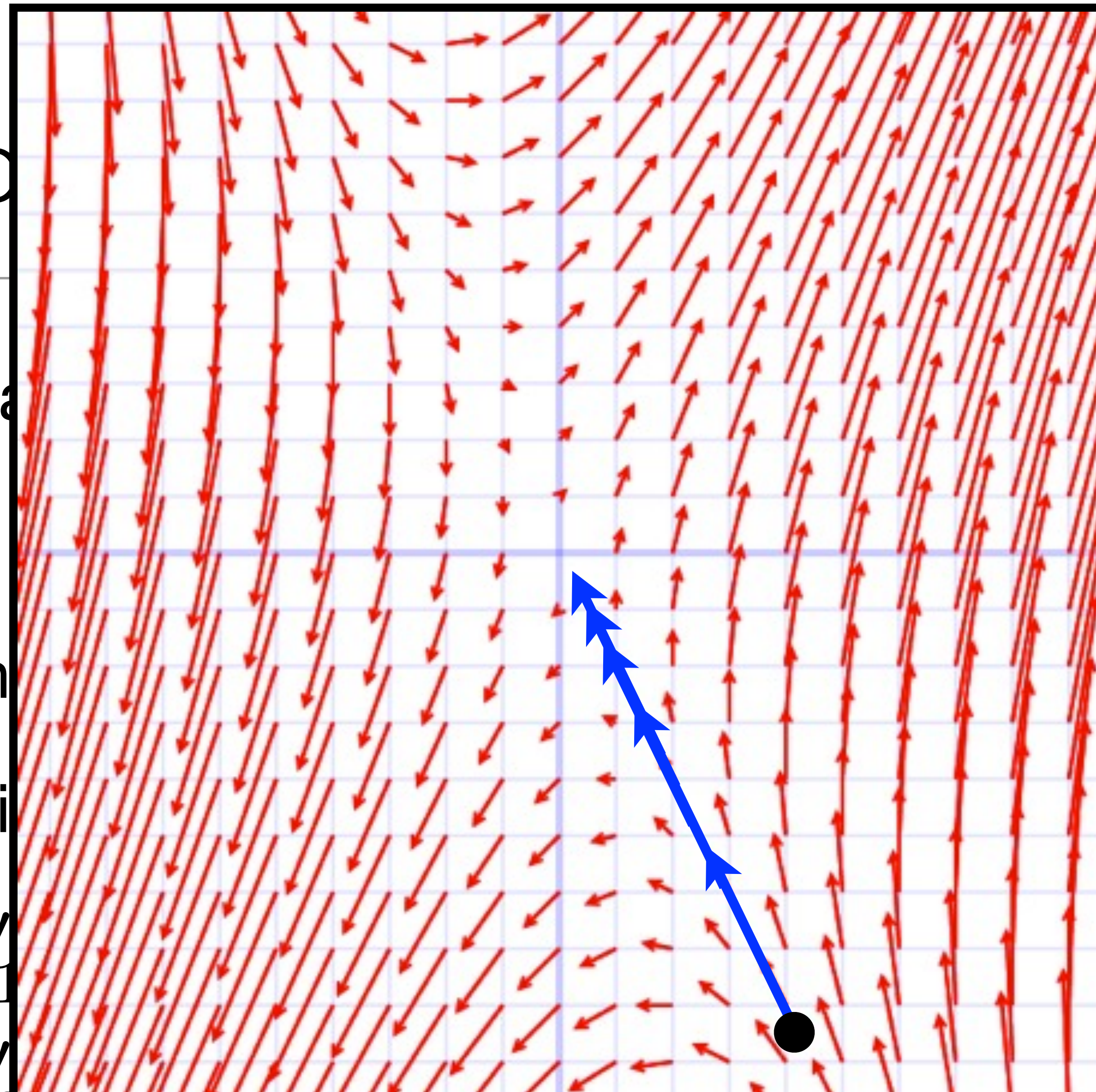

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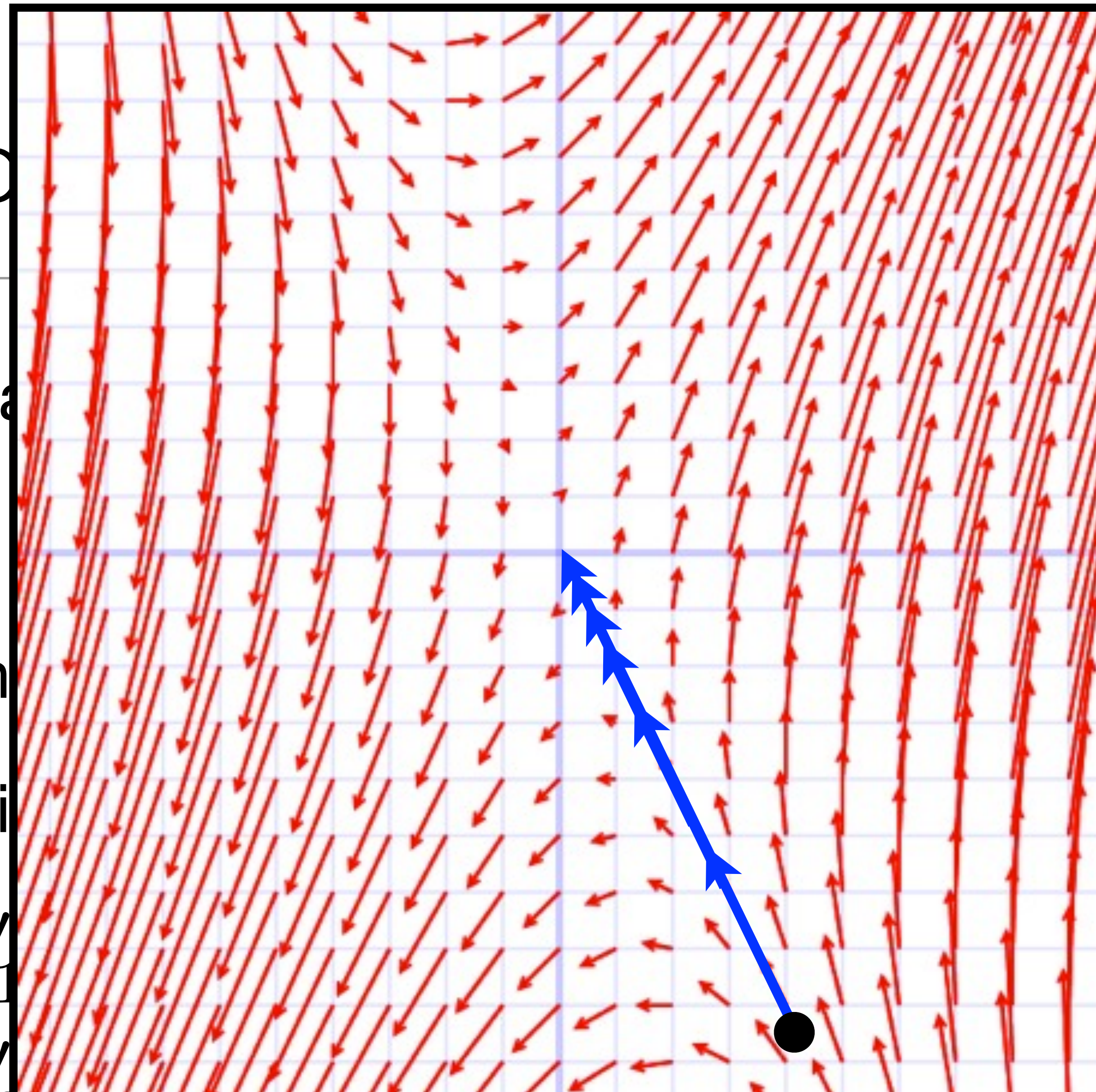

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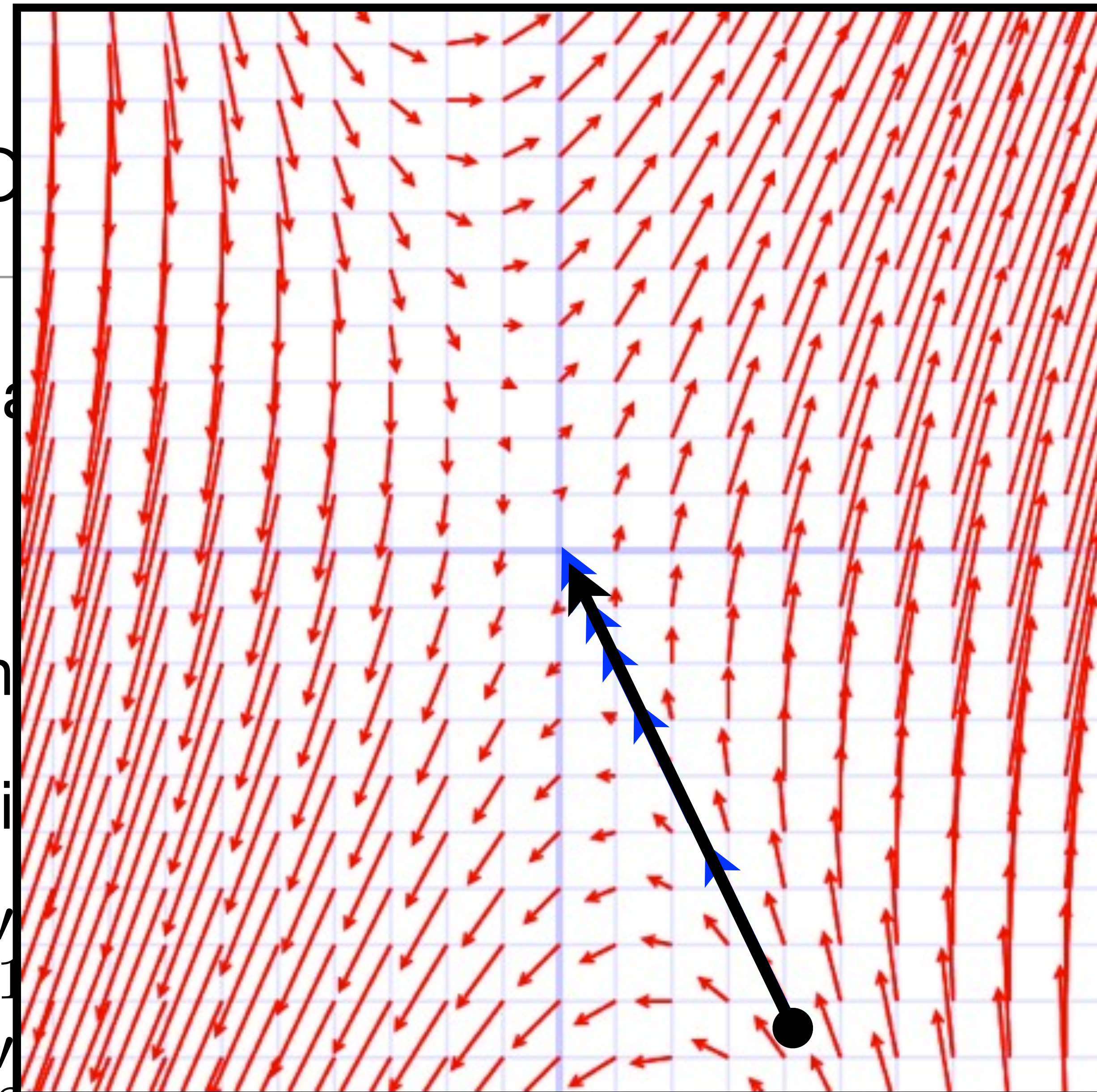

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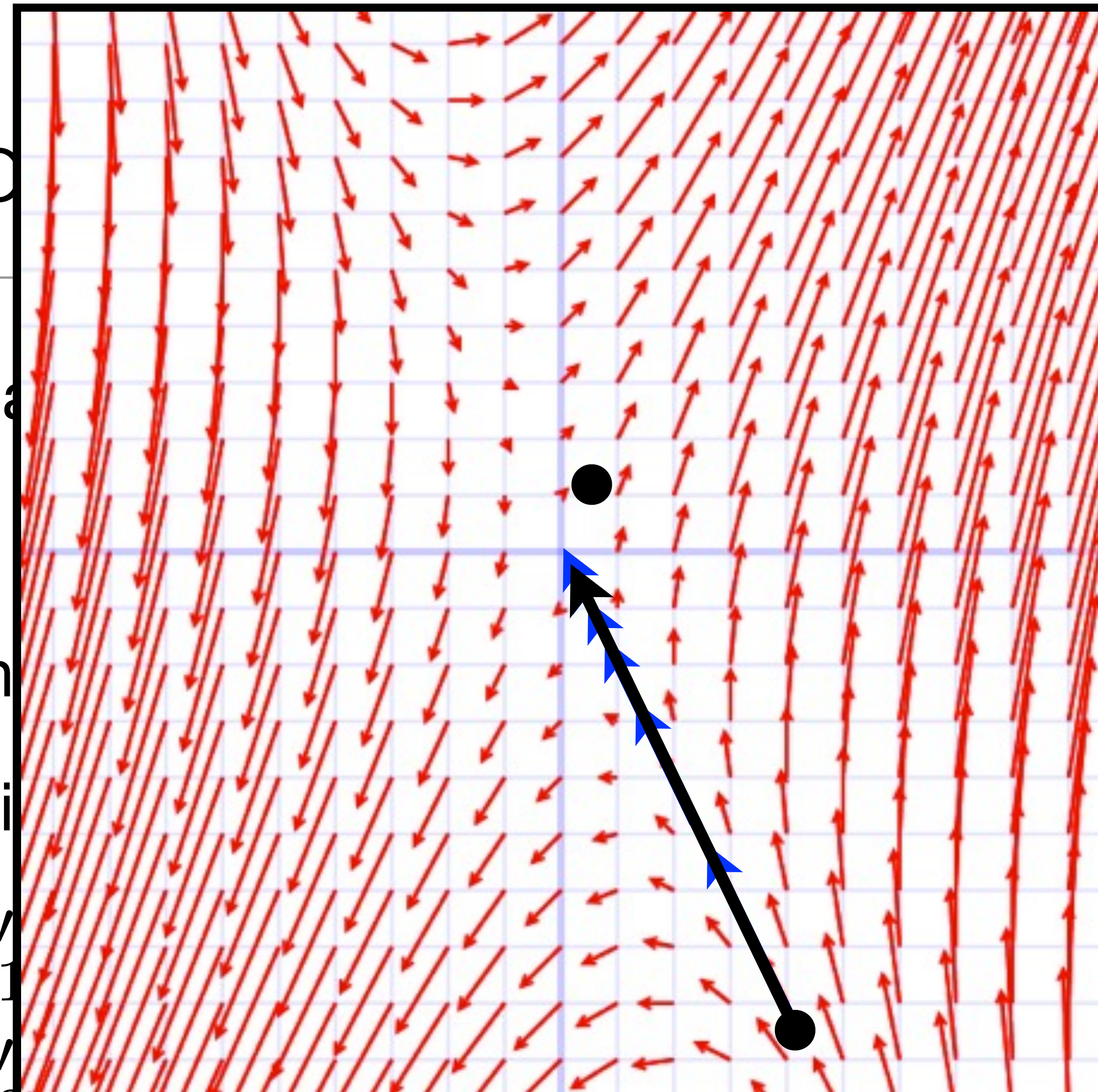

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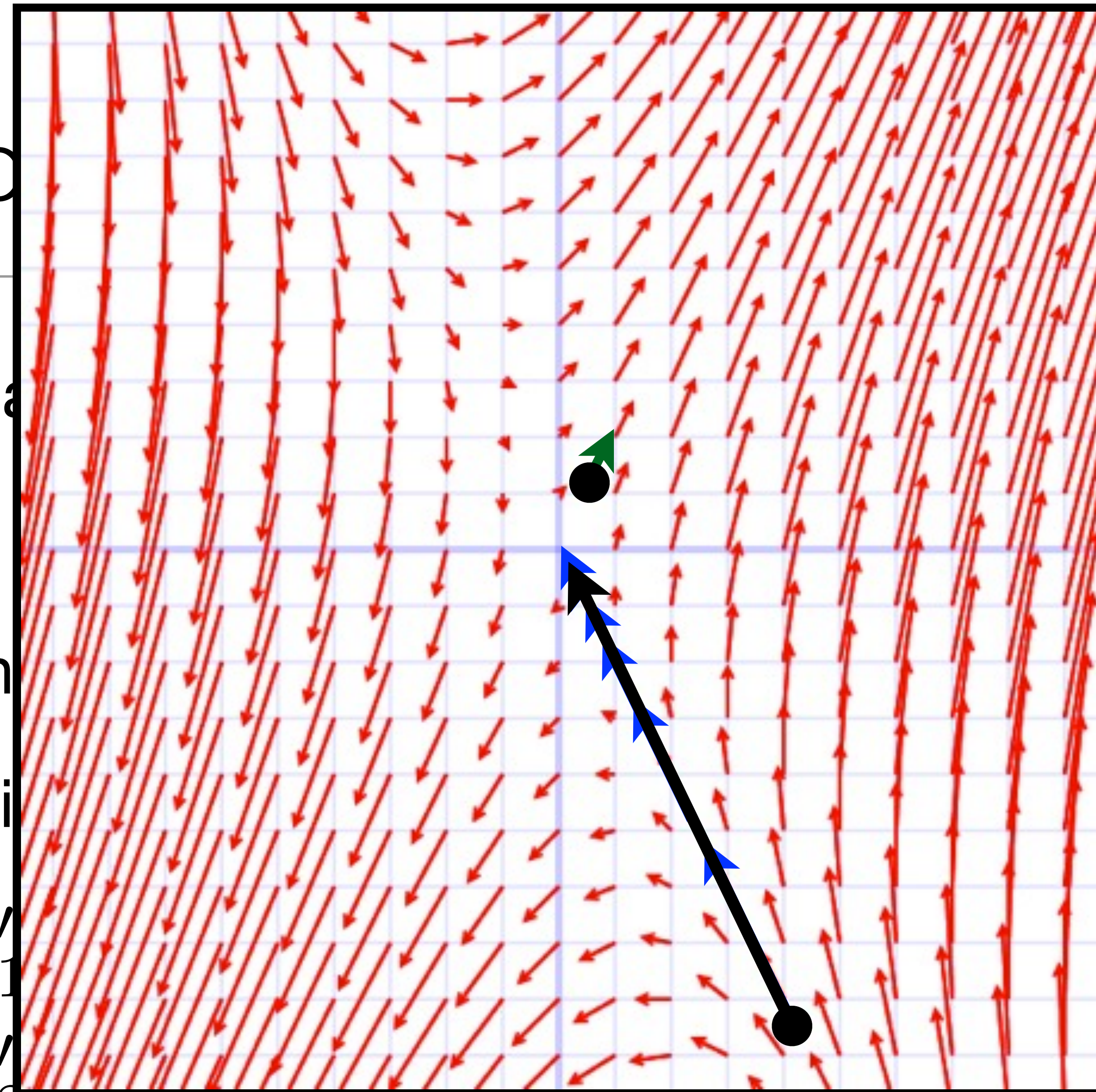

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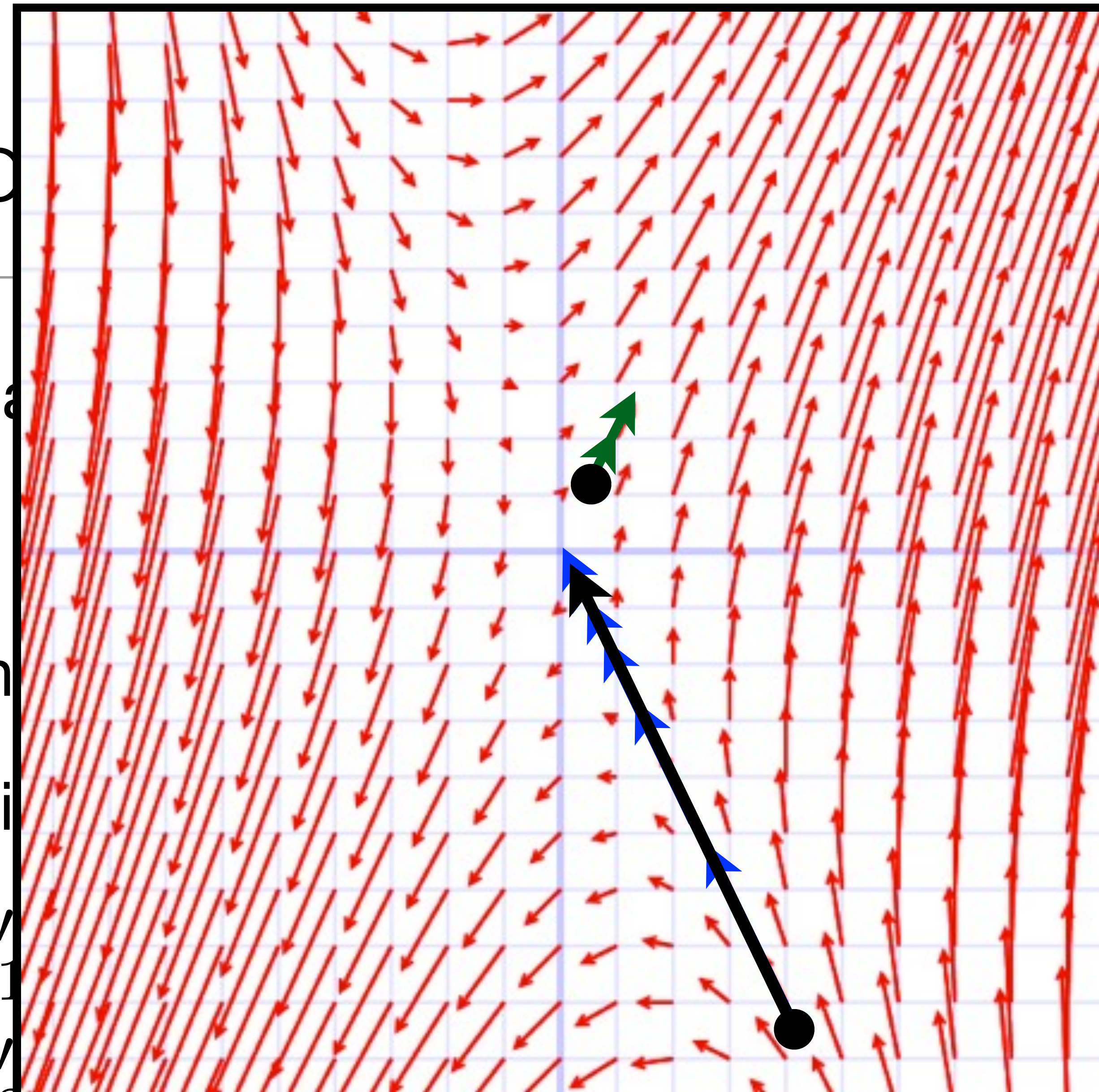

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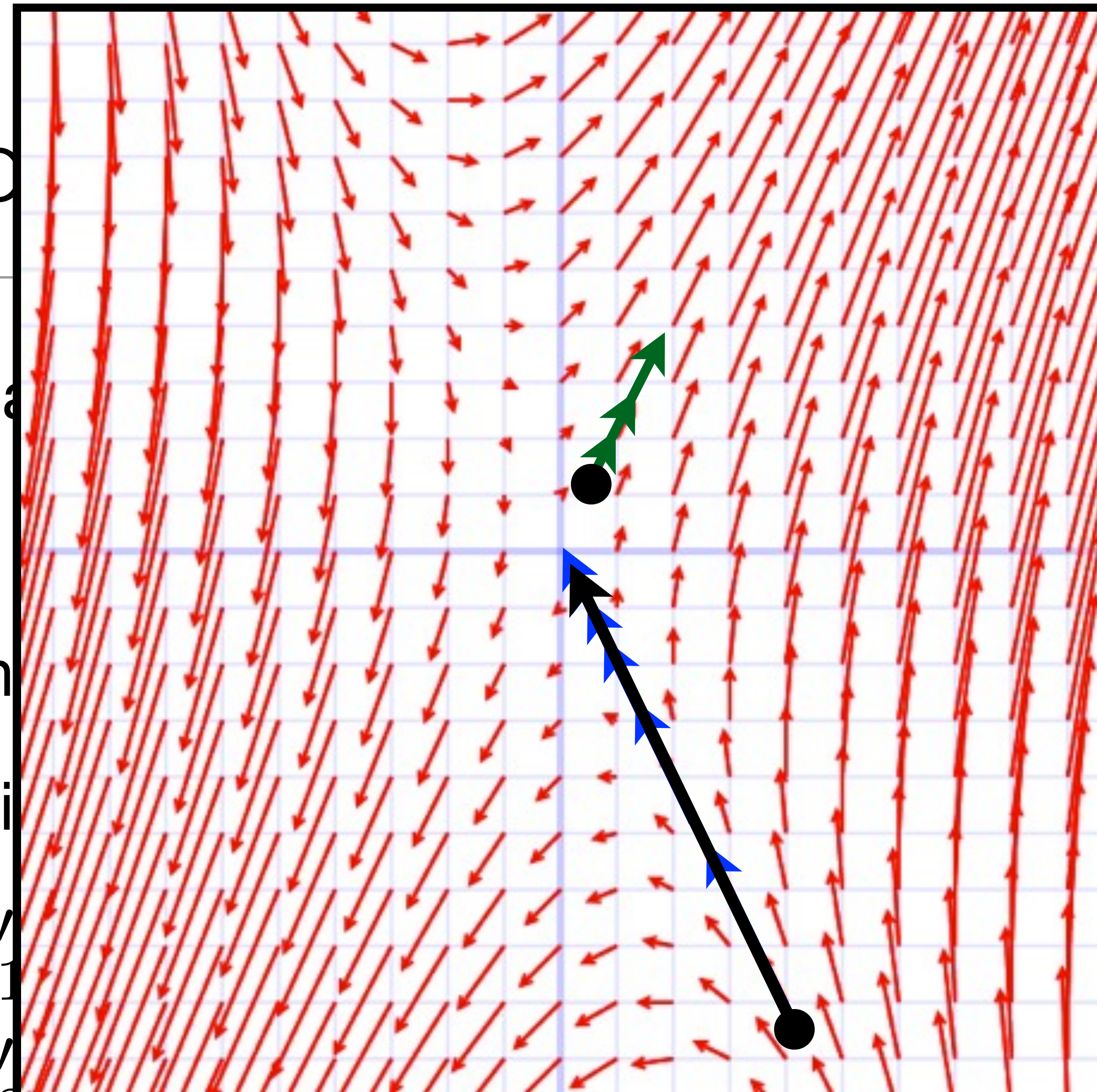

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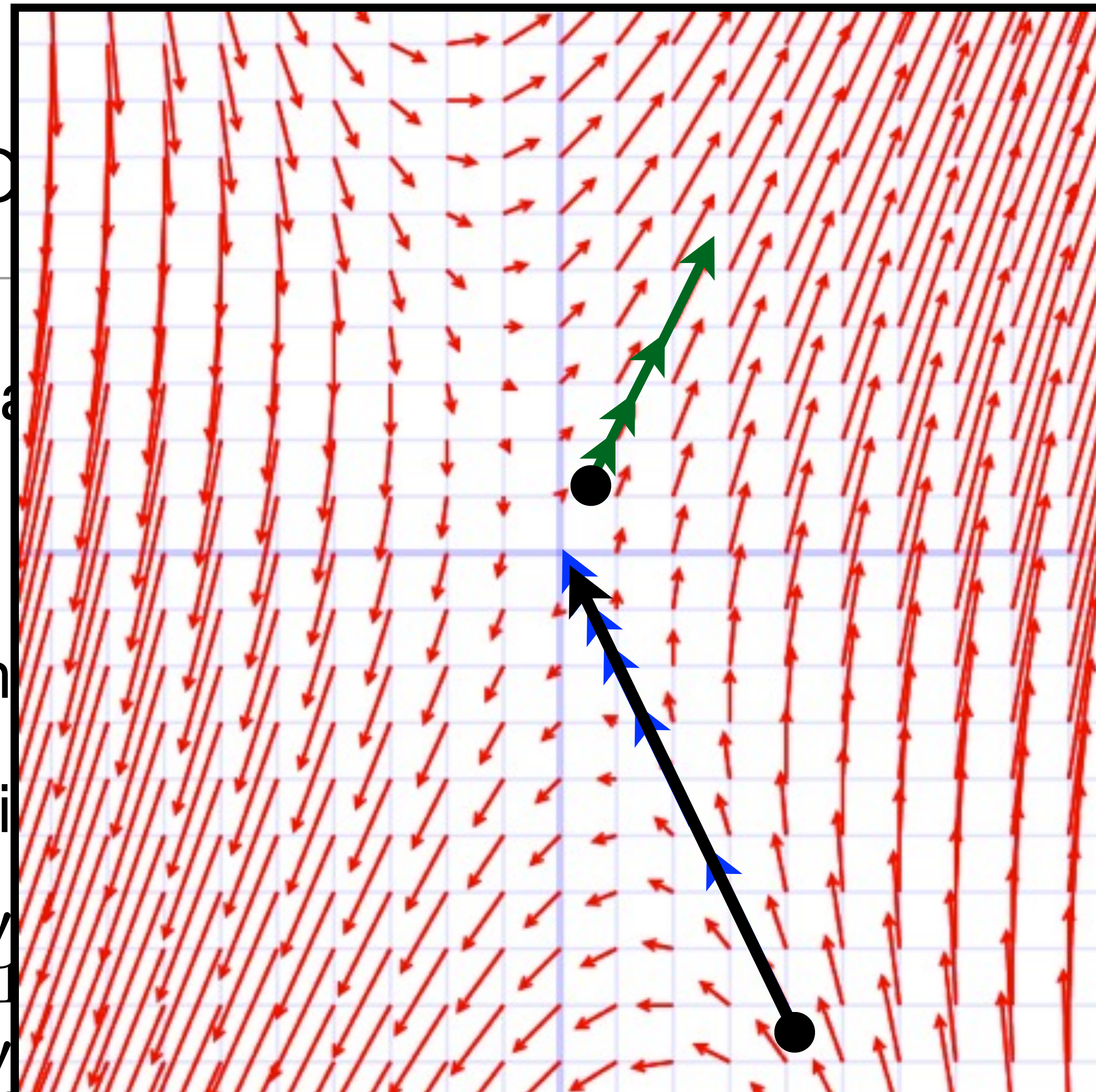

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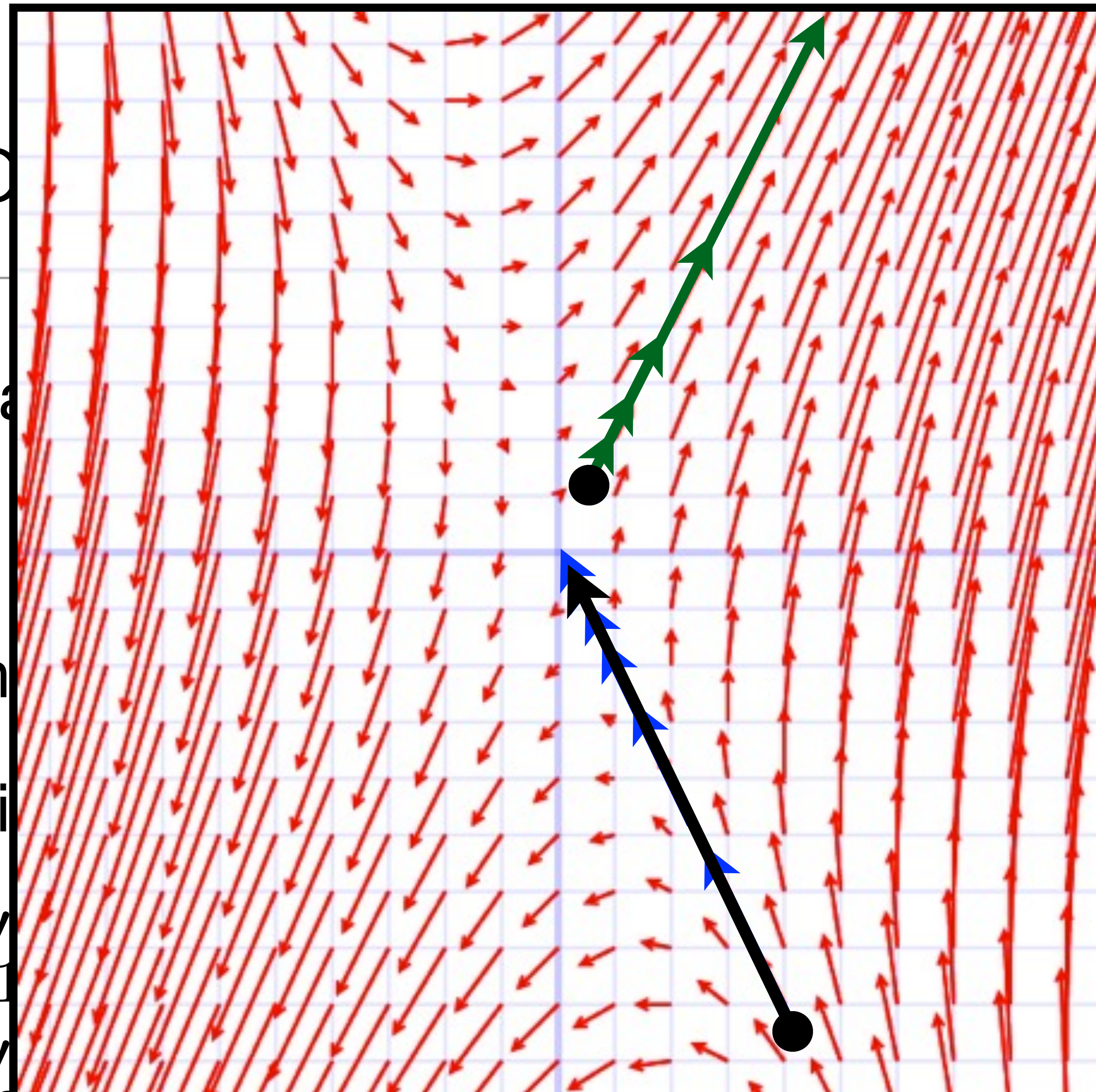

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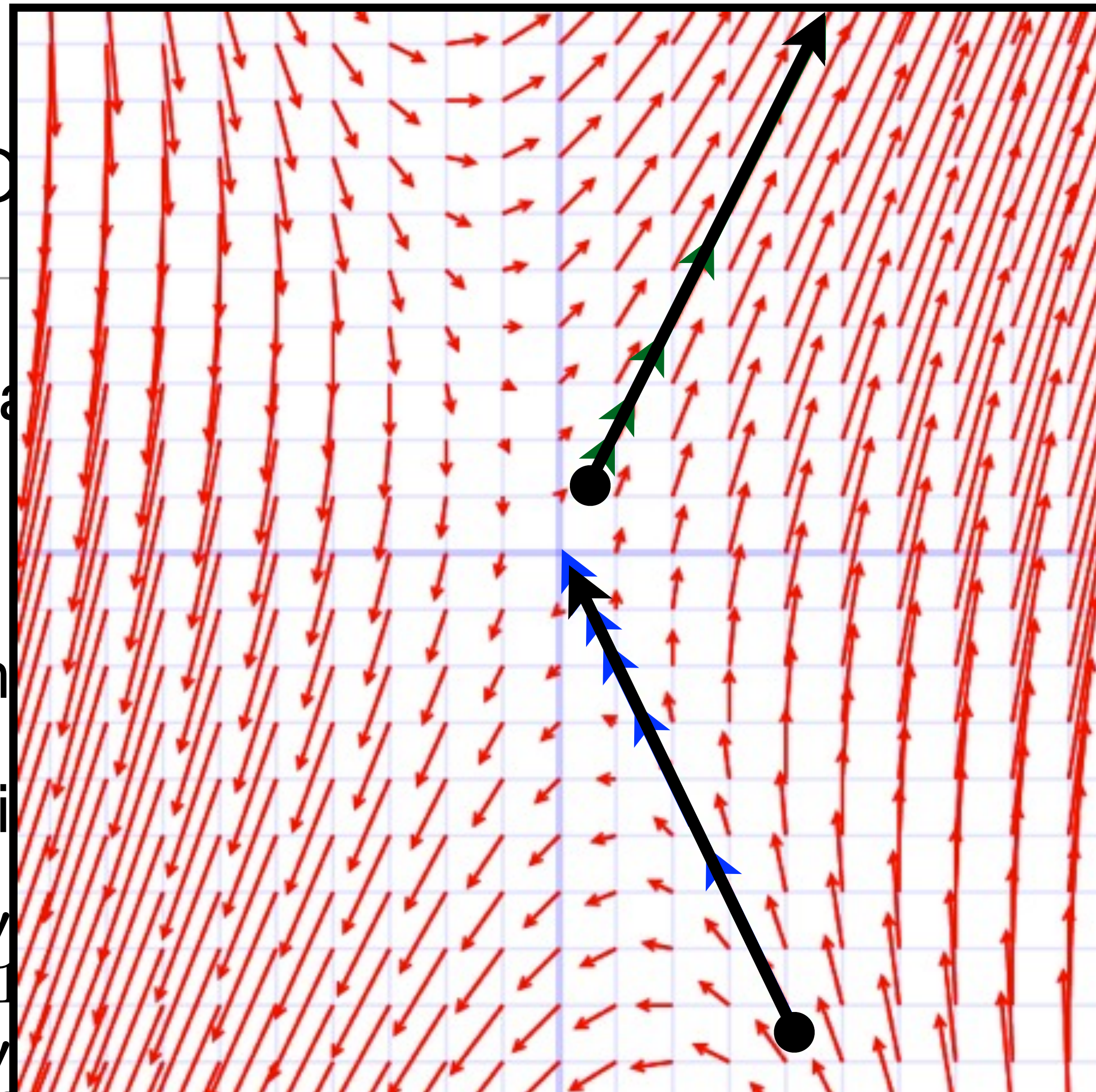

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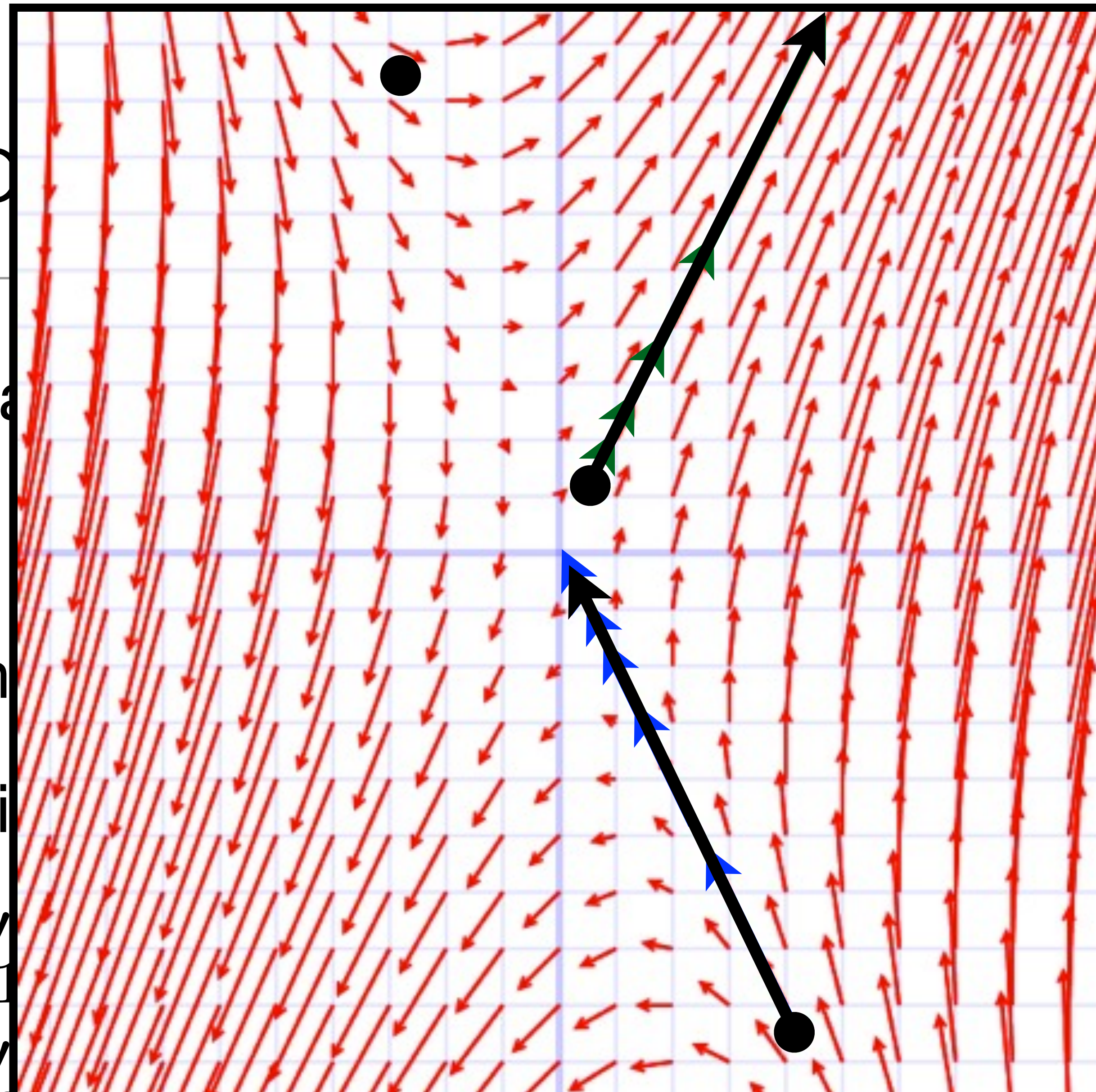

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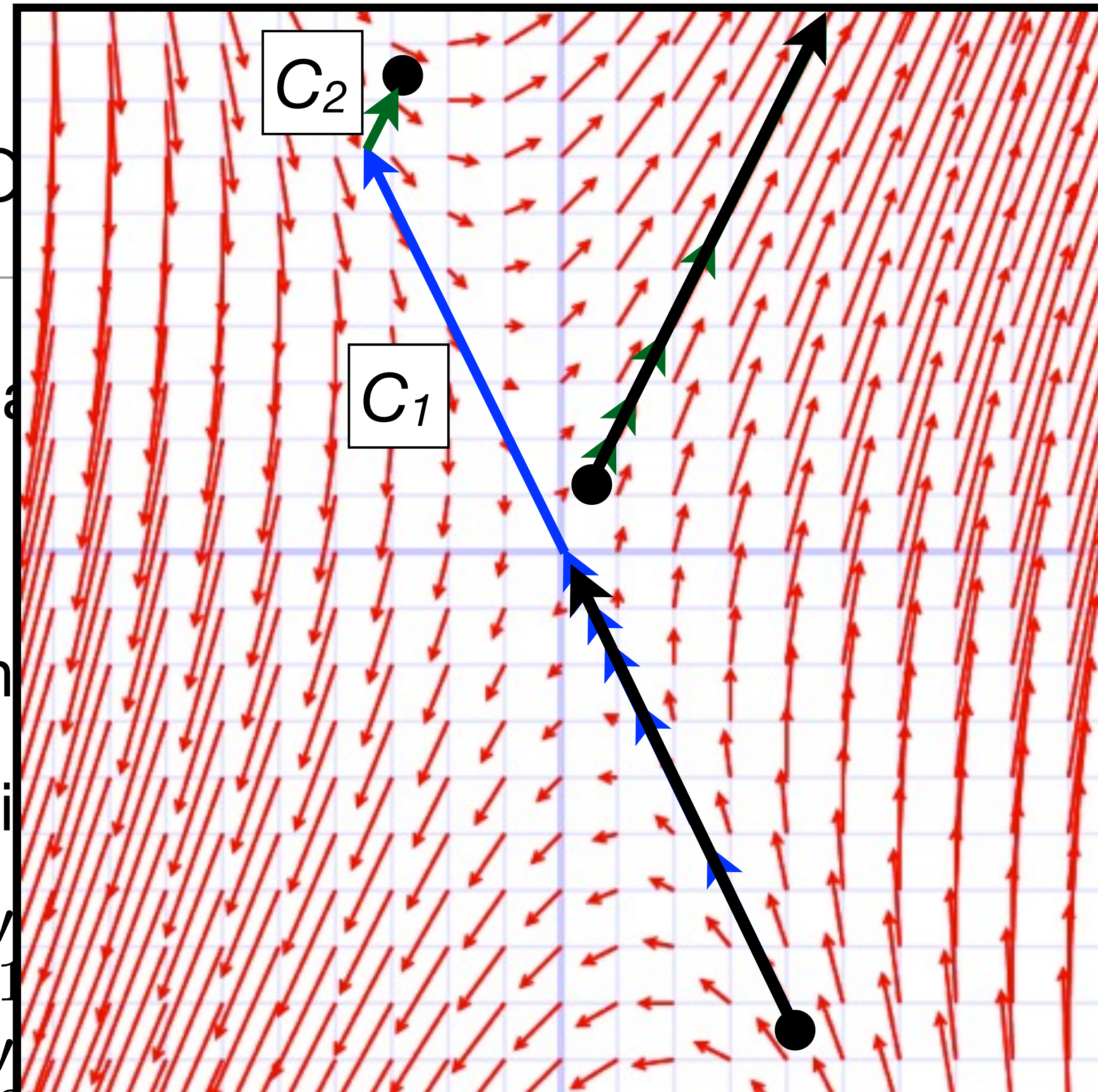

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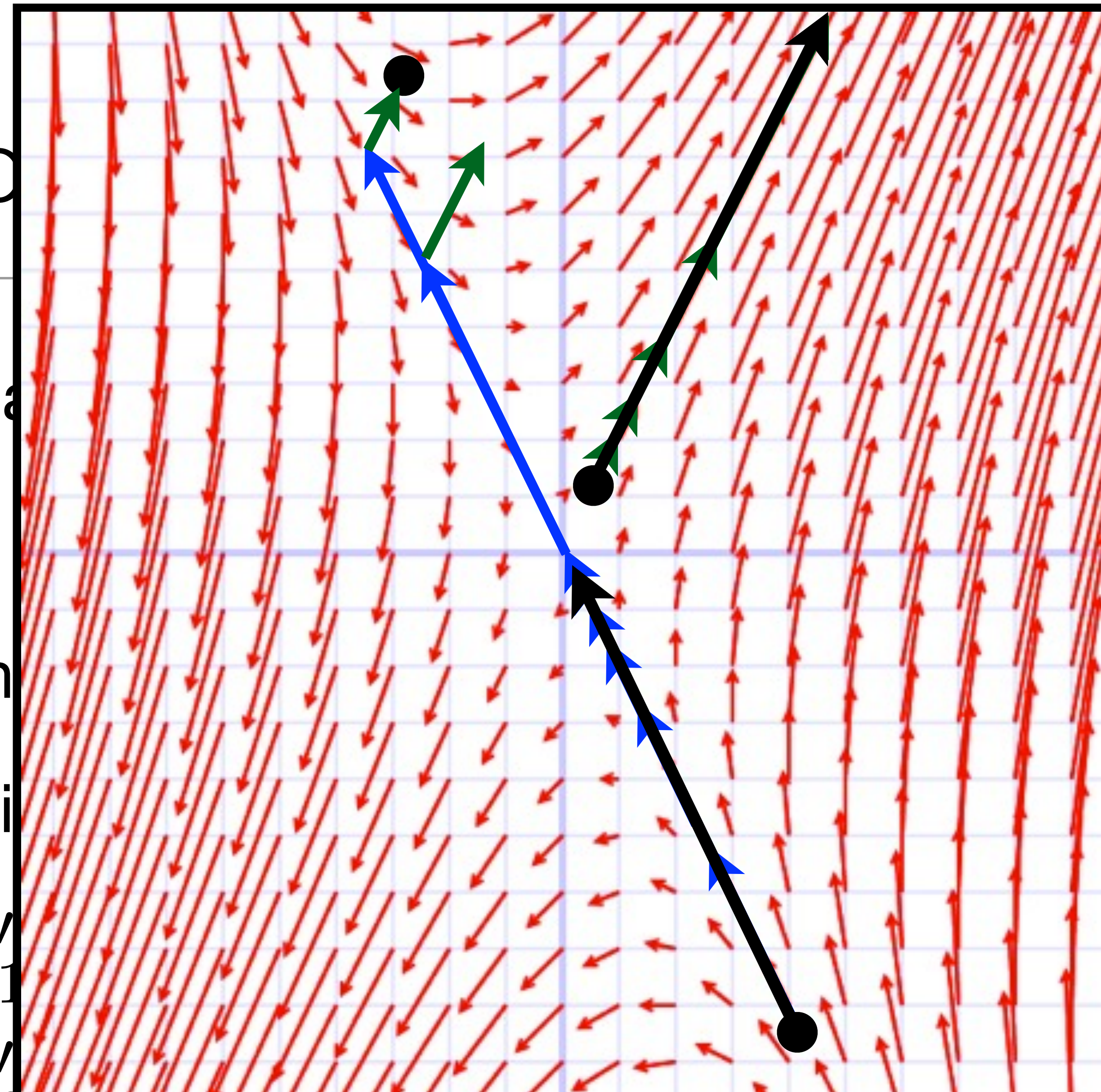

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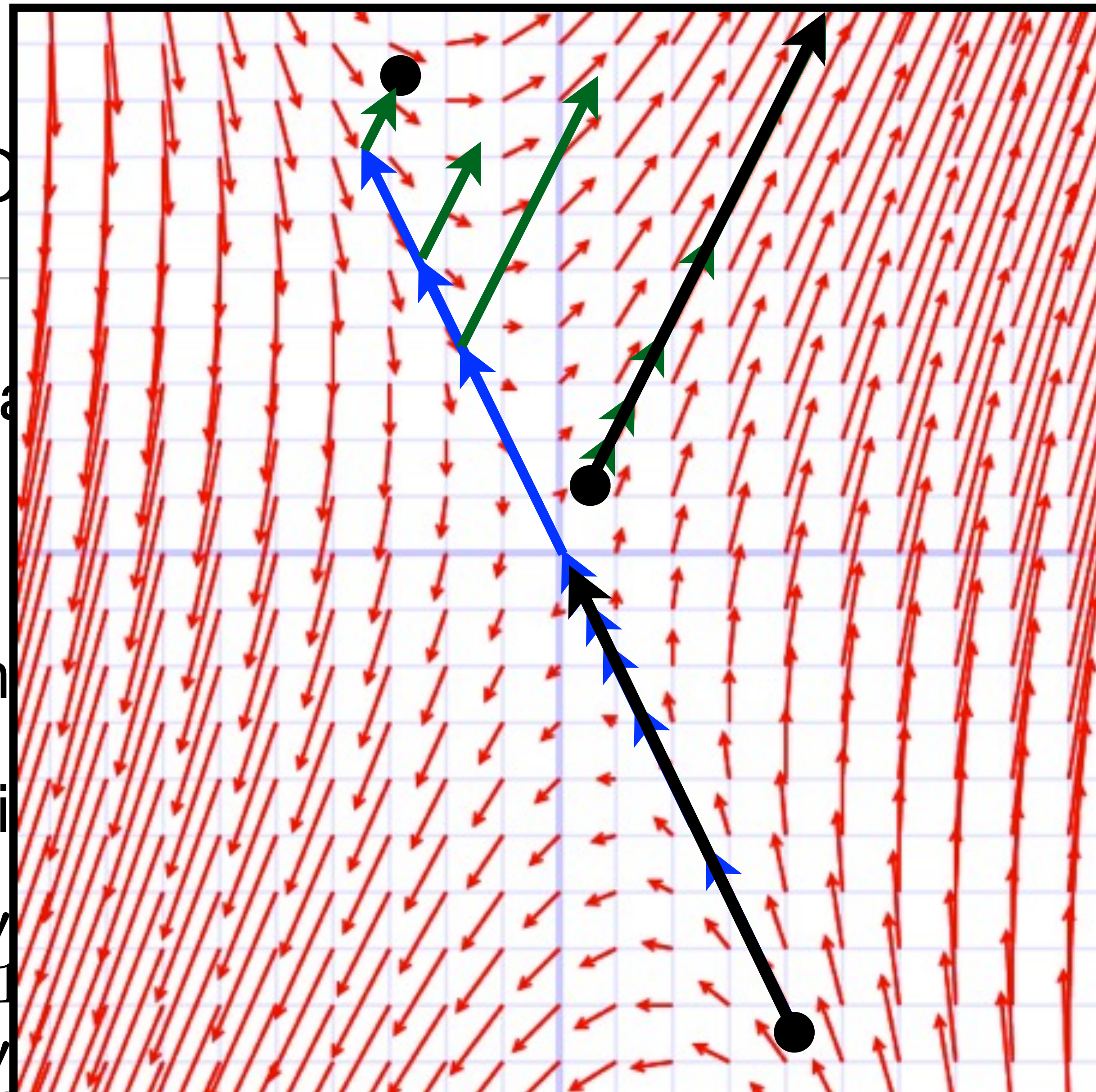

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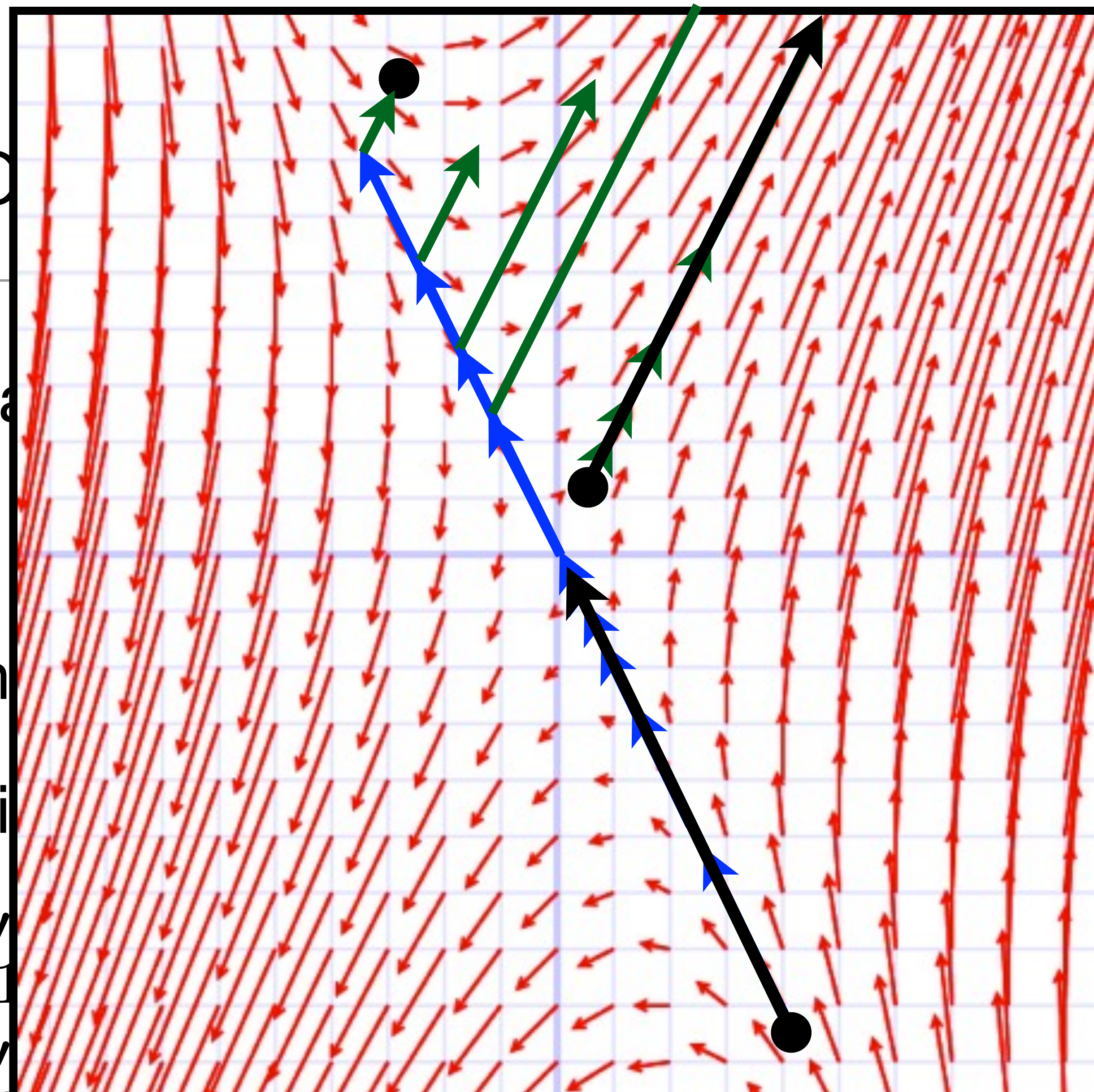

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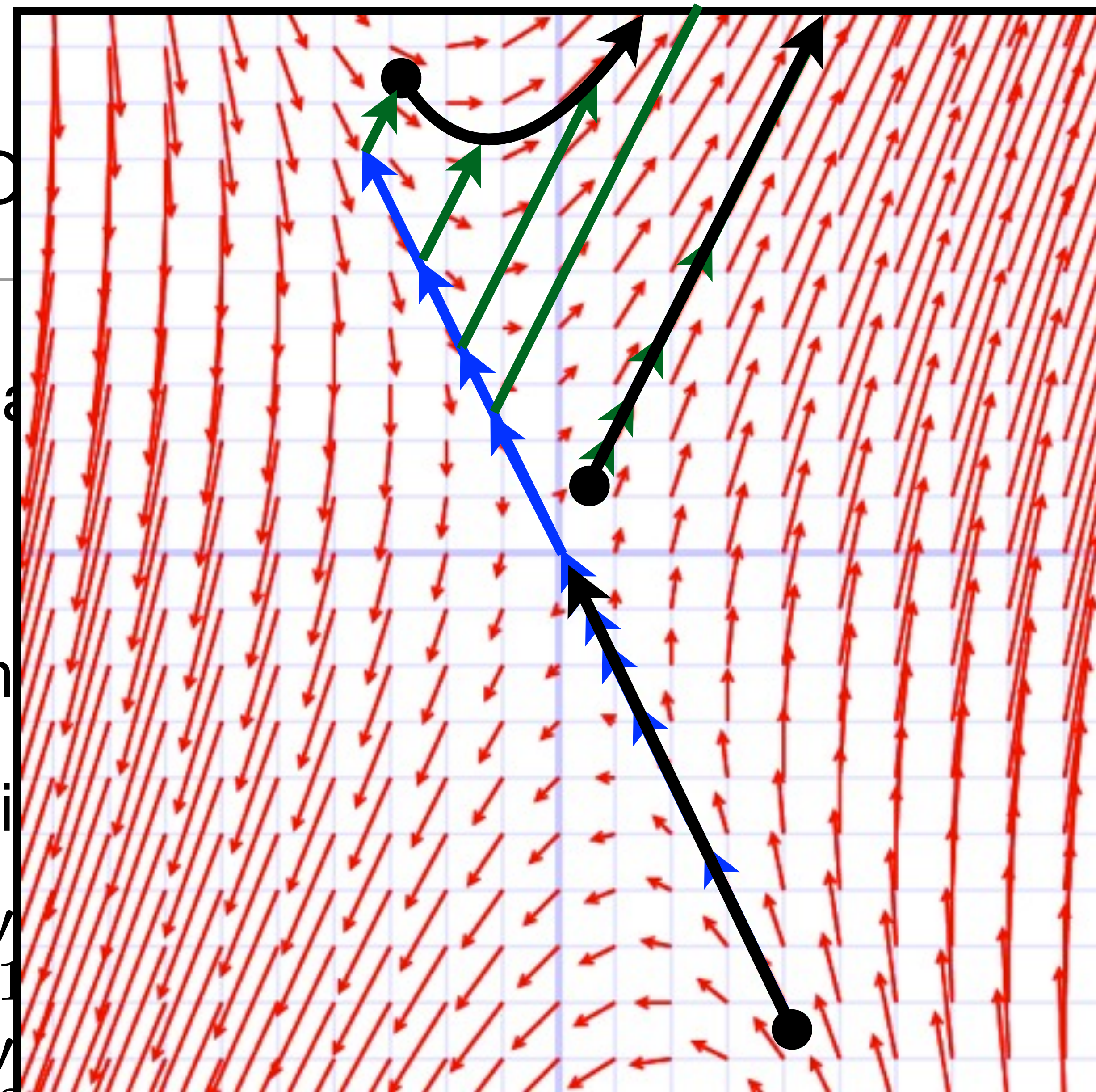

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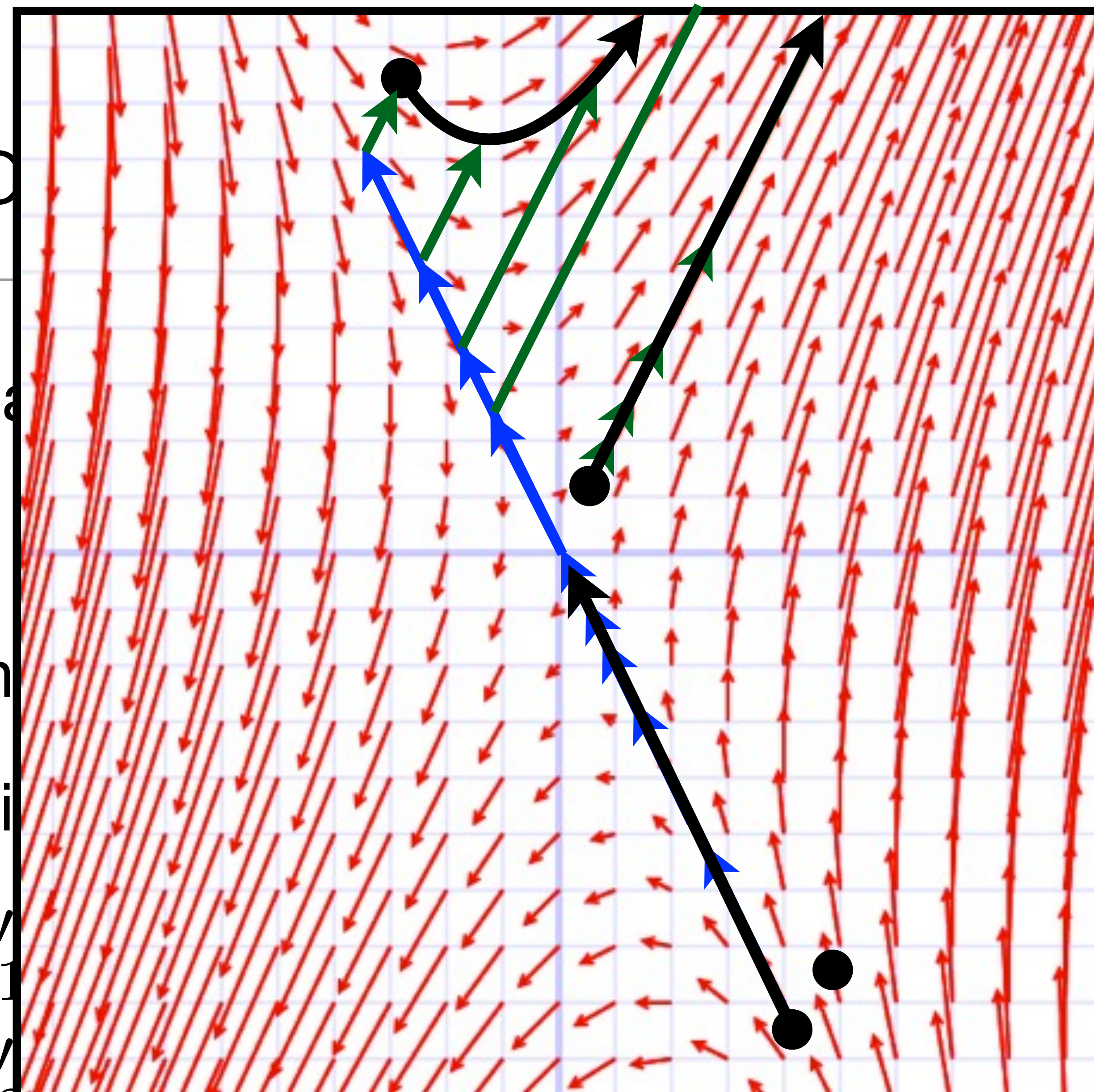

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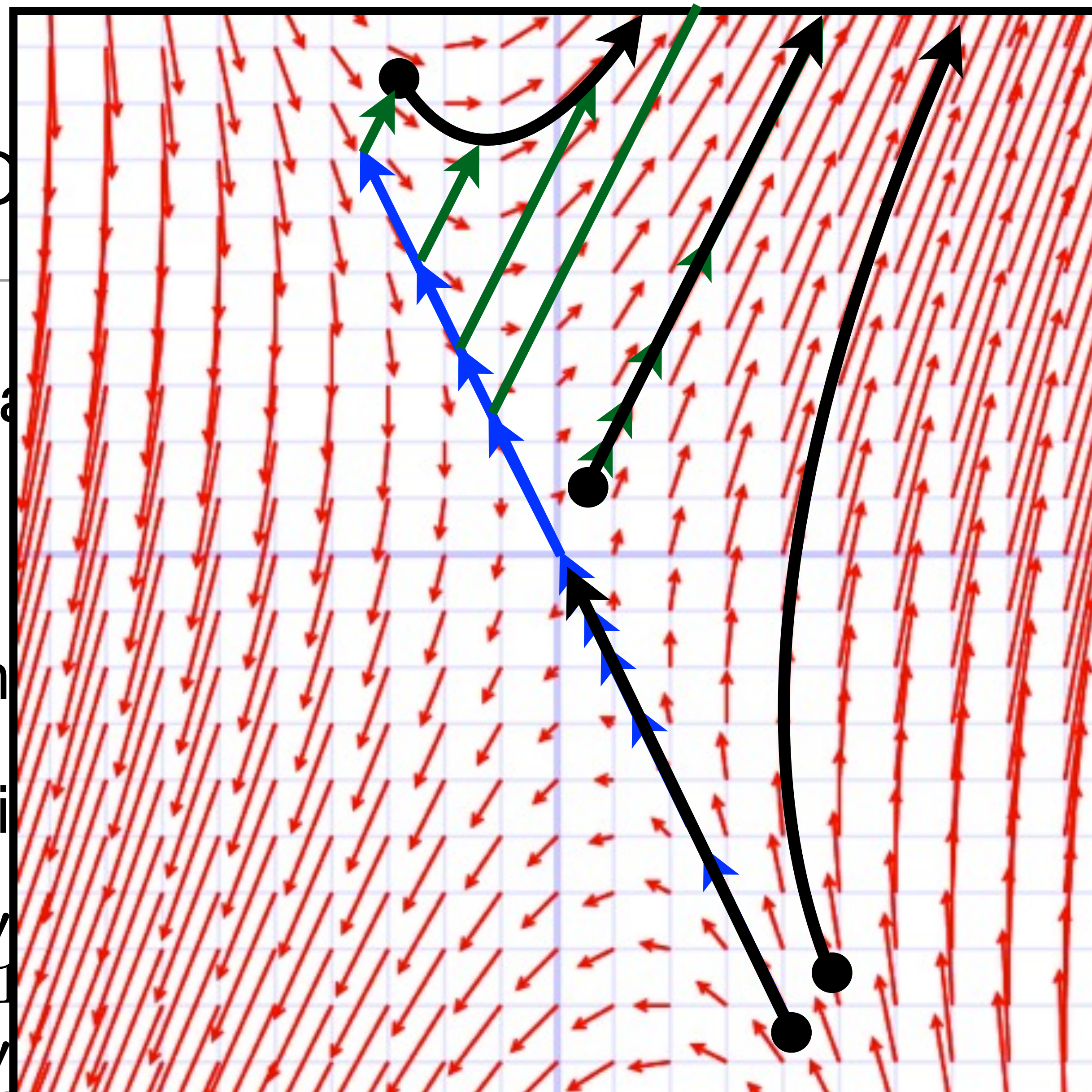

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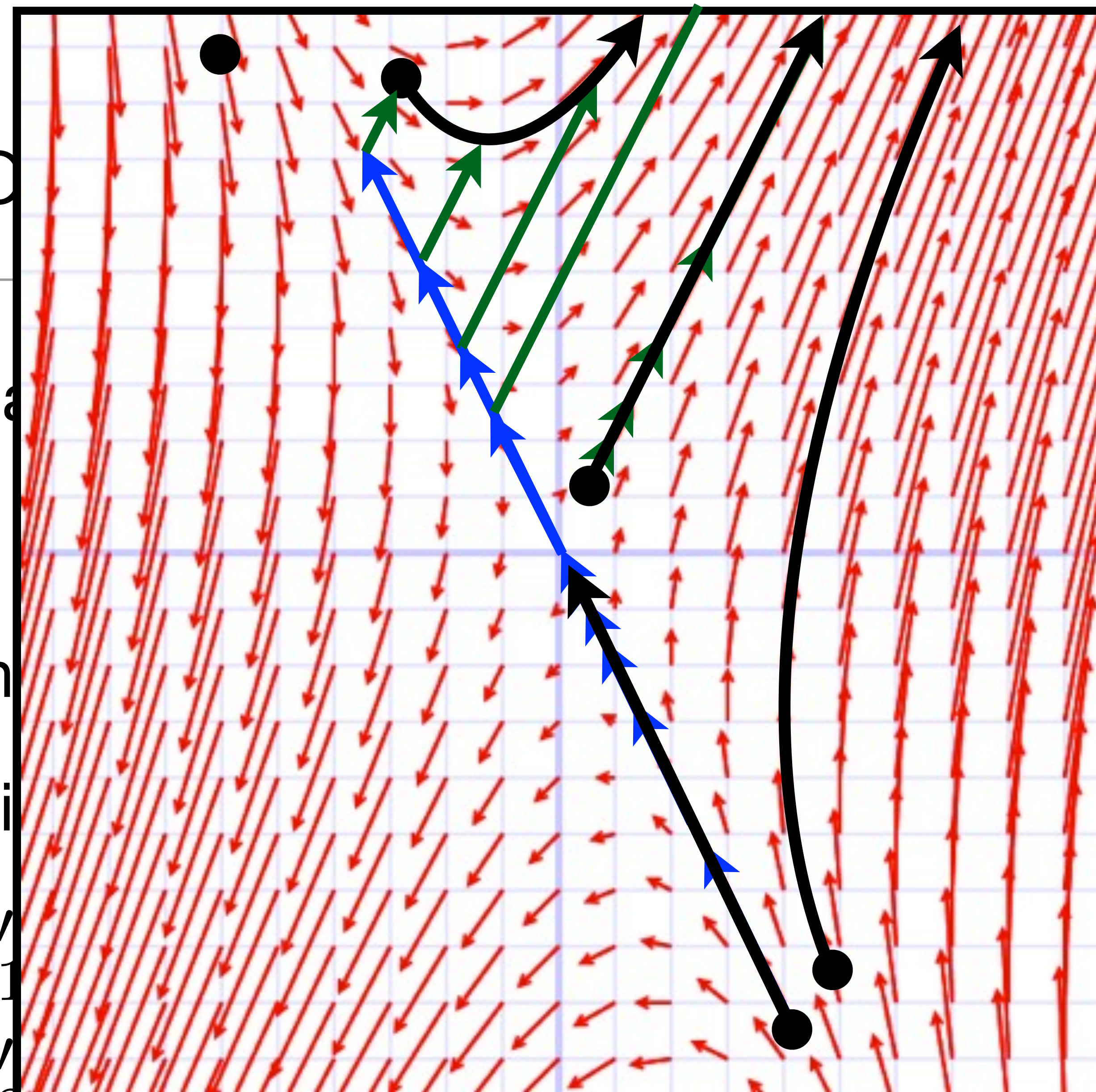

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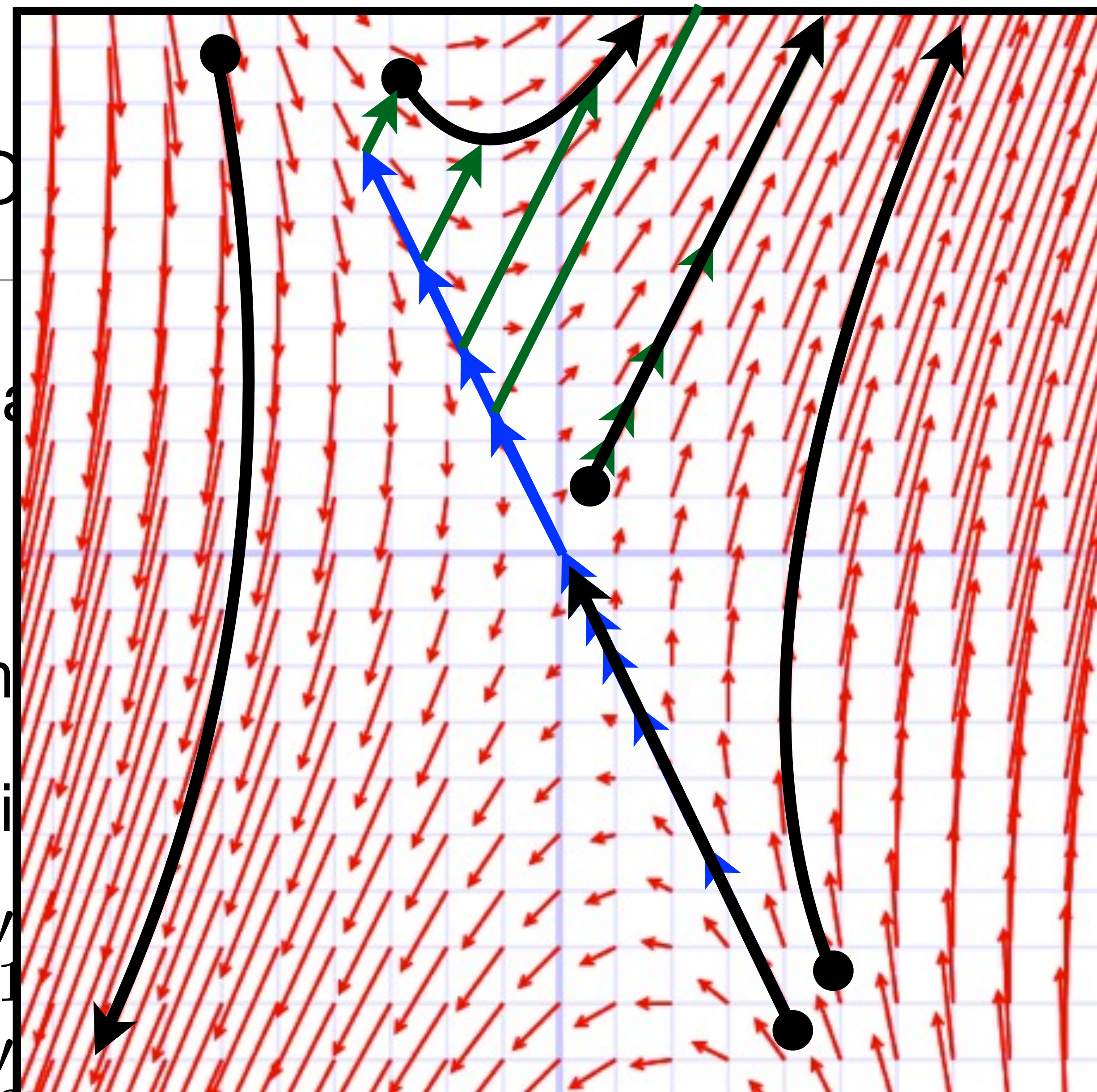

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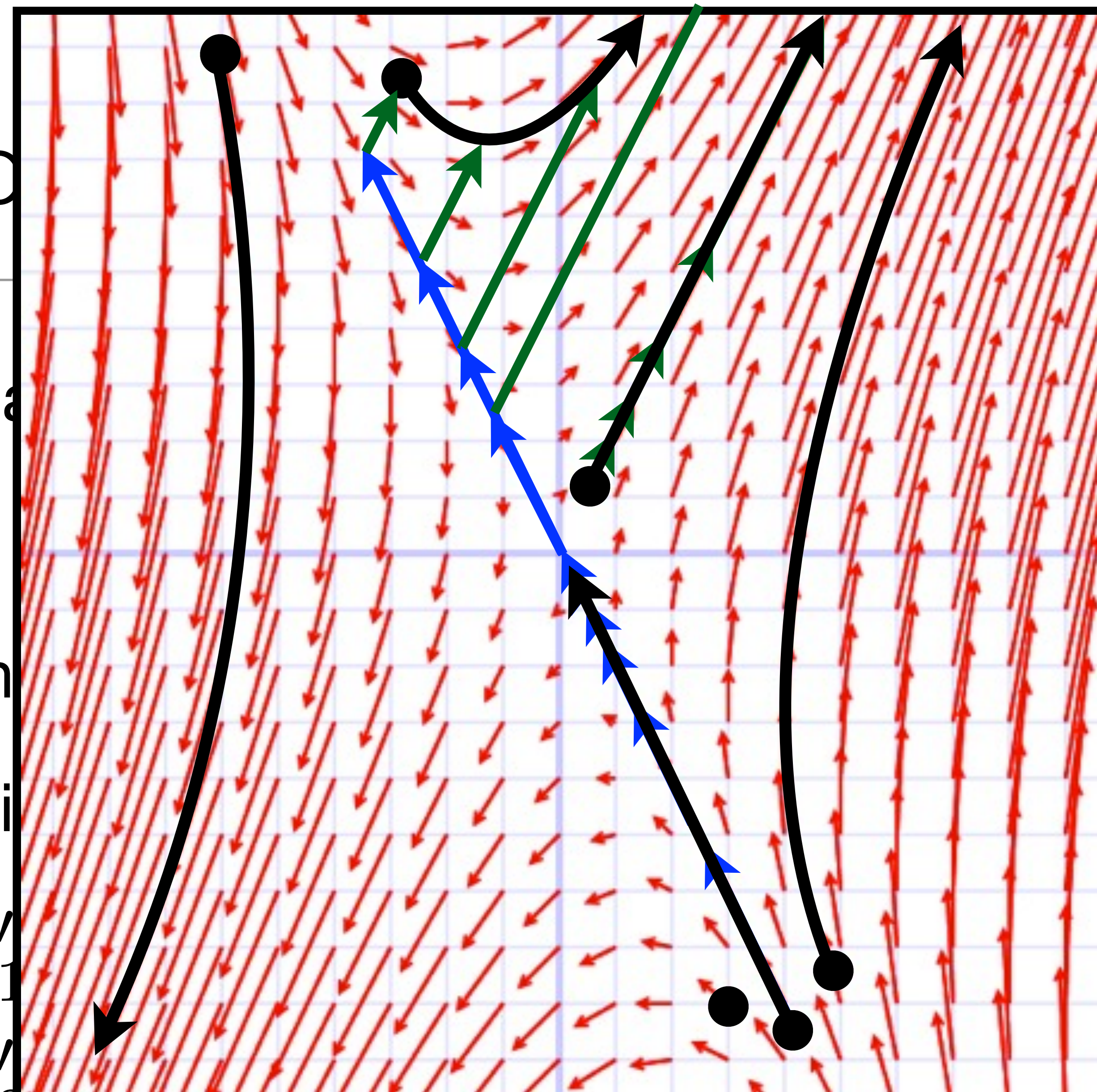

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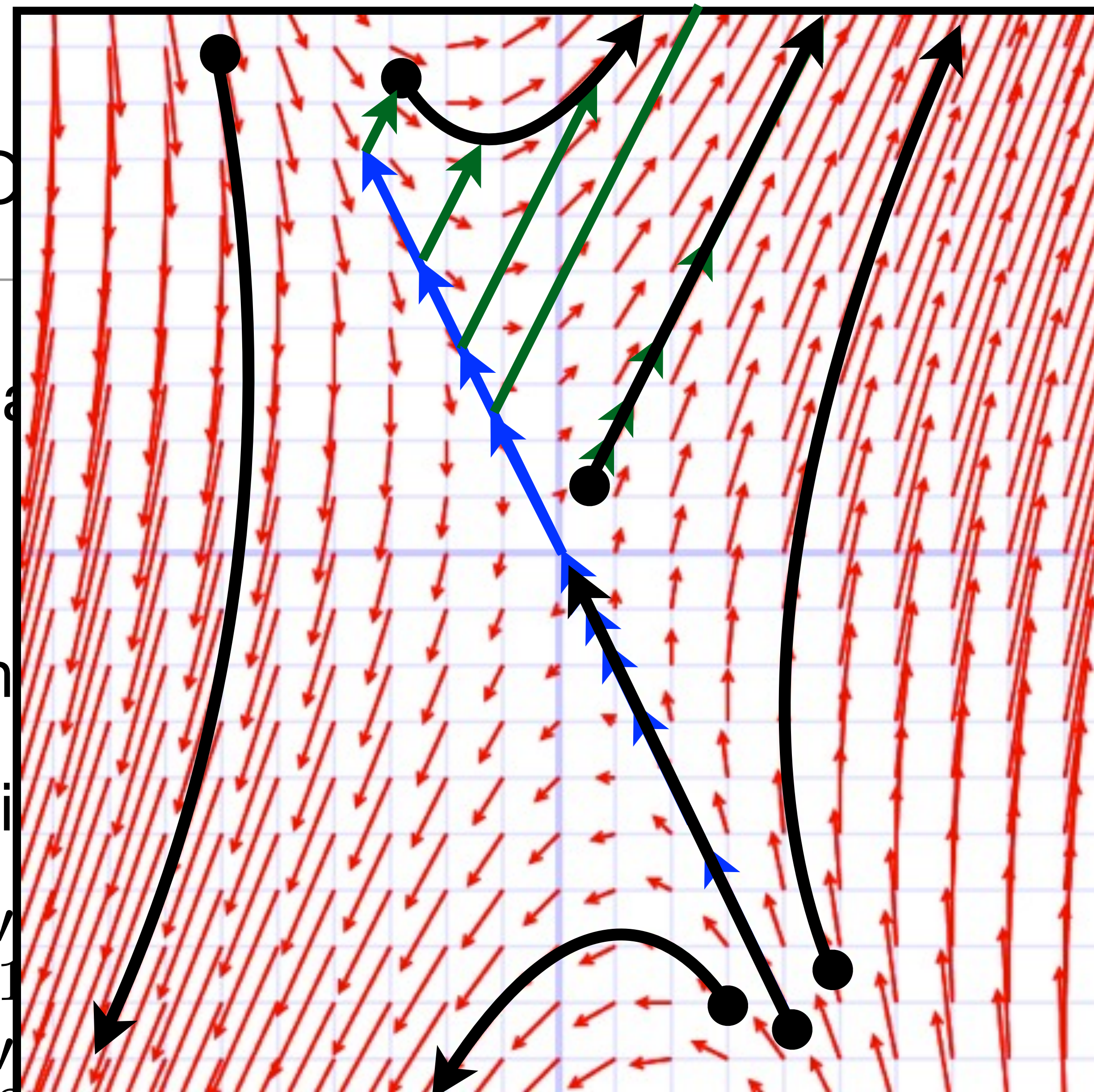

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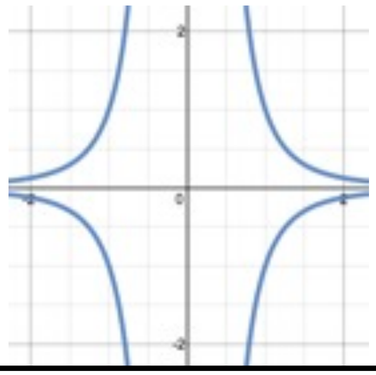

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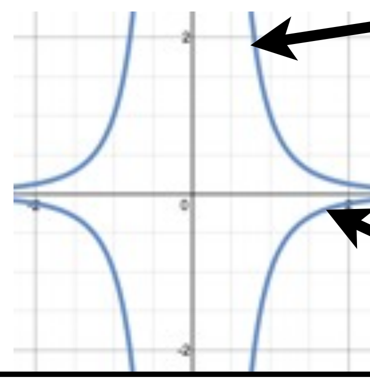

far from x_2 axis

close to x_1 axis

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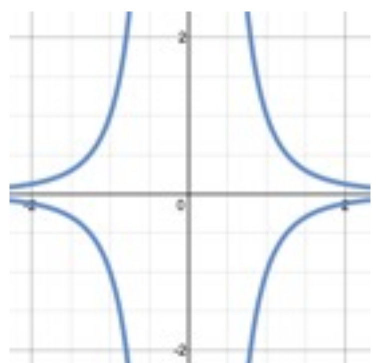

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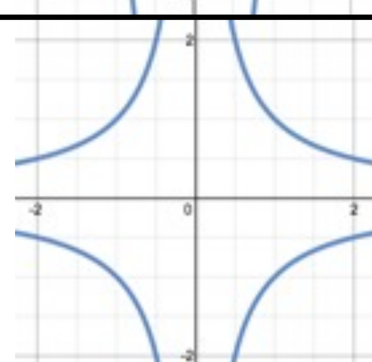

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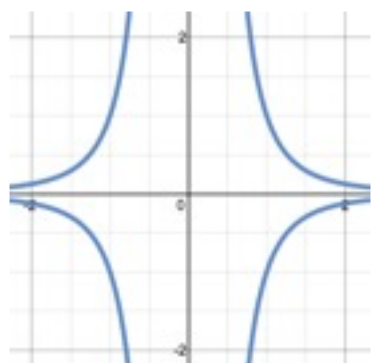

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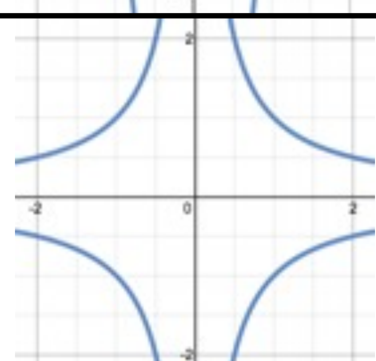

$$\lambda_2 = -\frac{1}{3}\lambda_1$$

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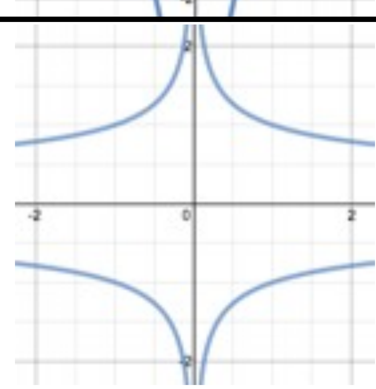

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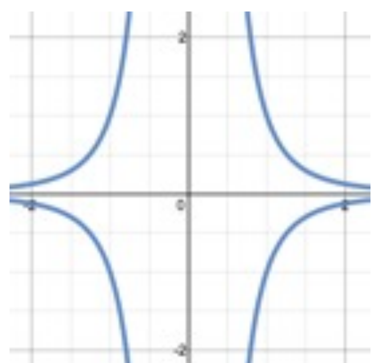

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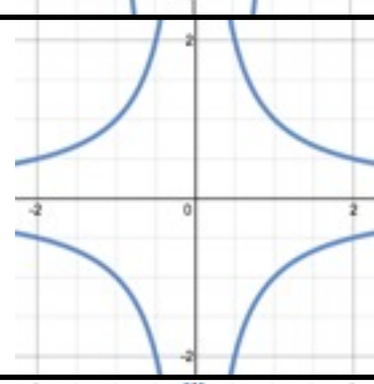

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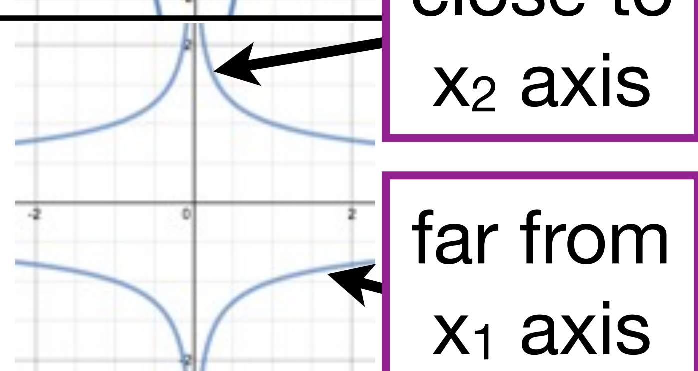

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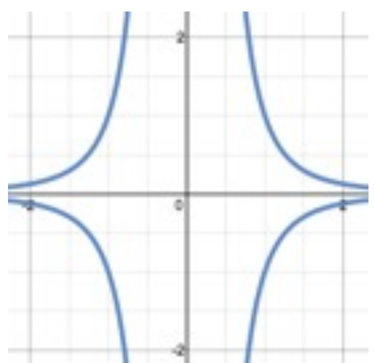

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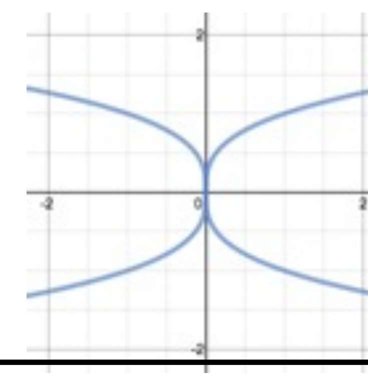

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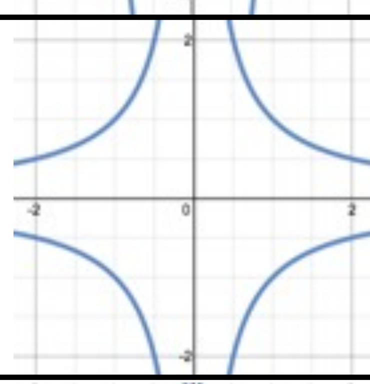

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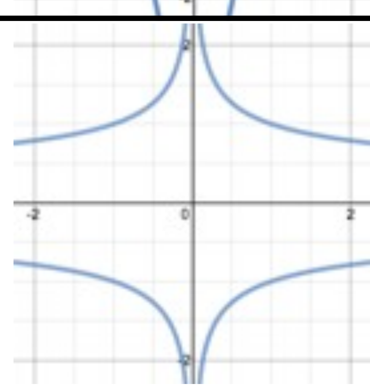

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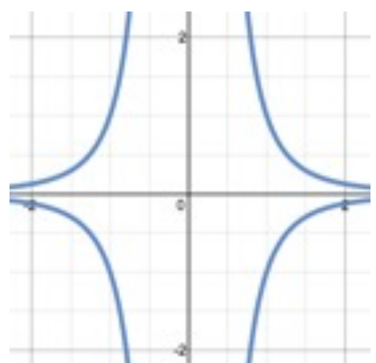

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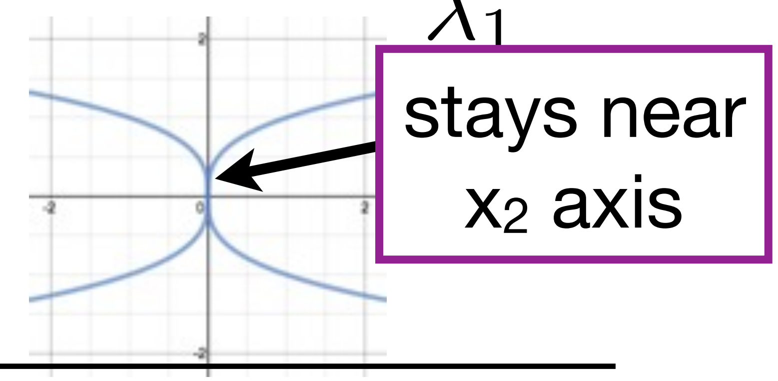

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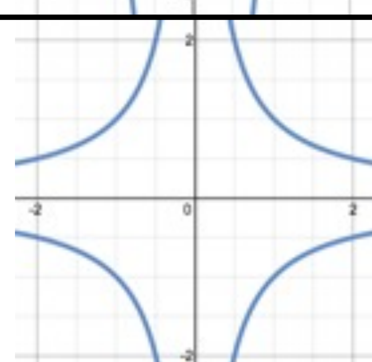

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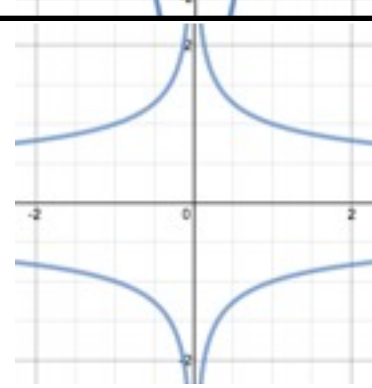

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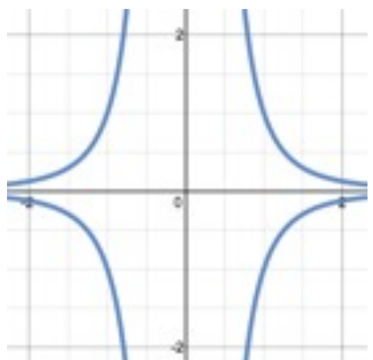

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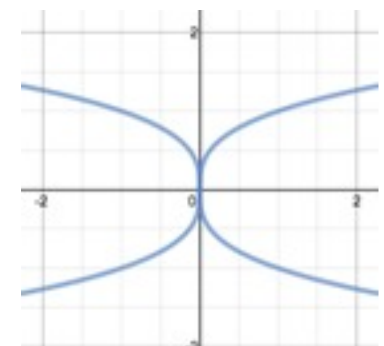

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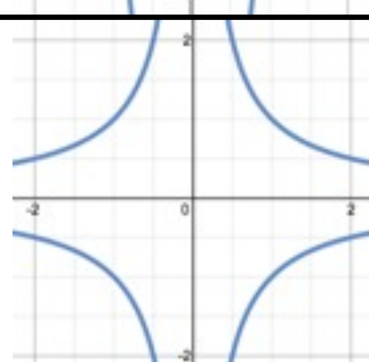

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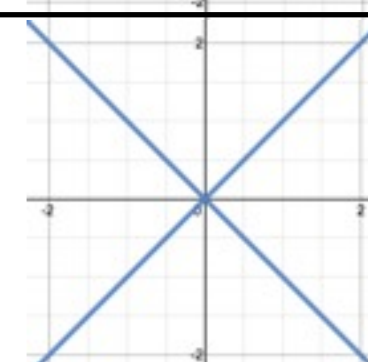

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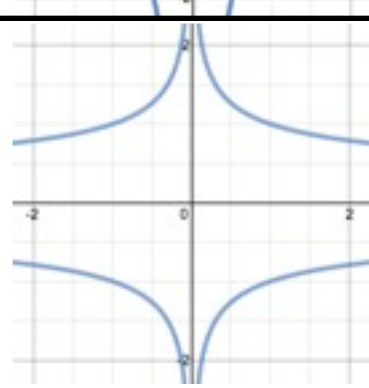

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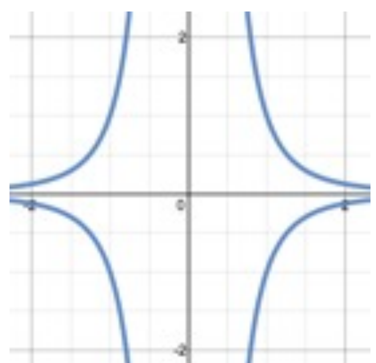

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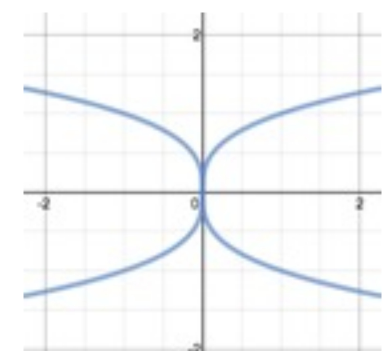

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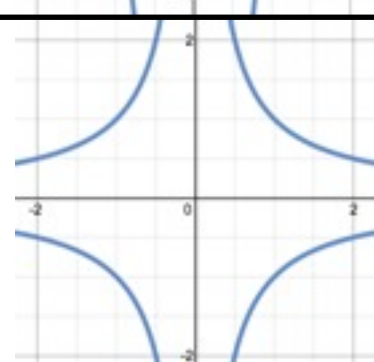

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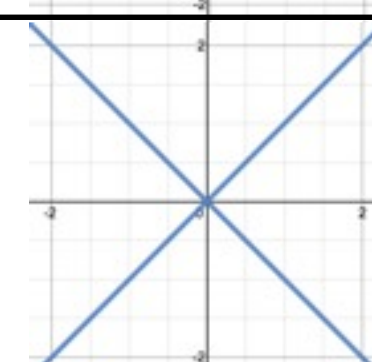

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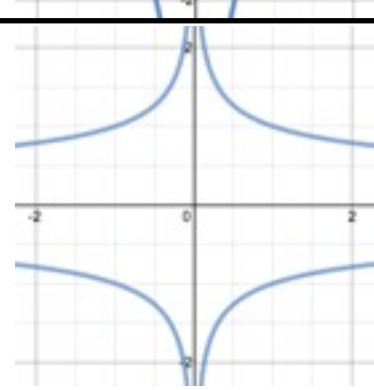

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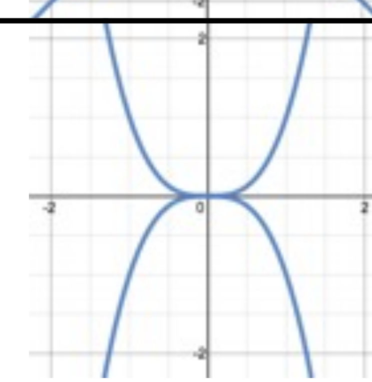

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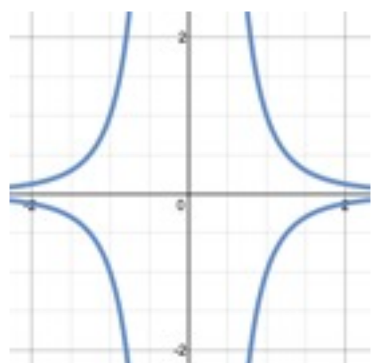

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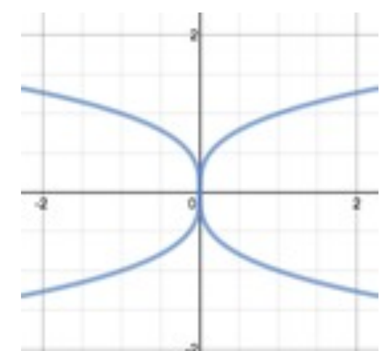

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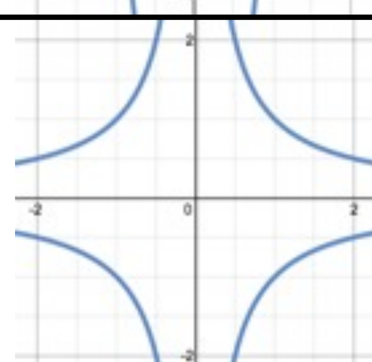

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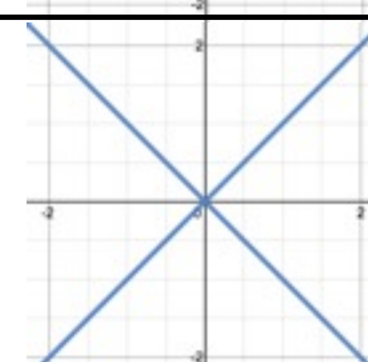

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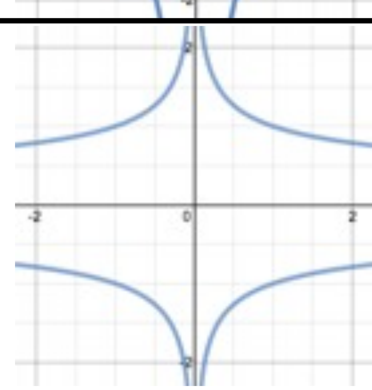

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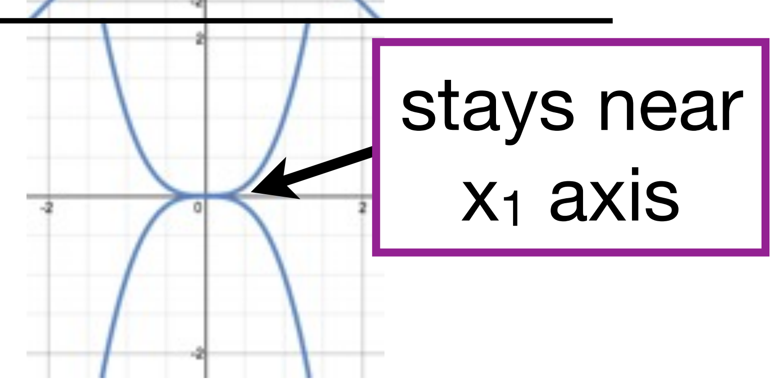

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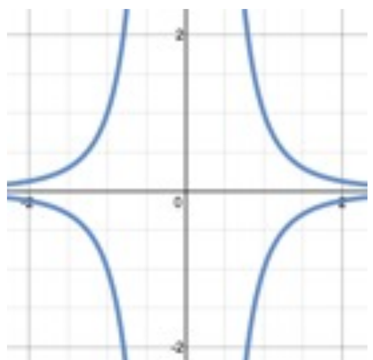

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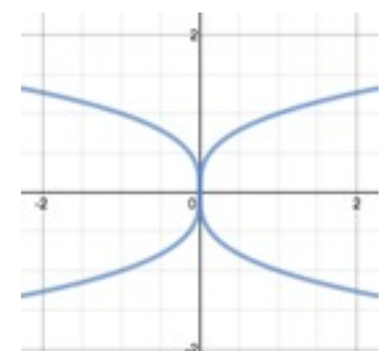

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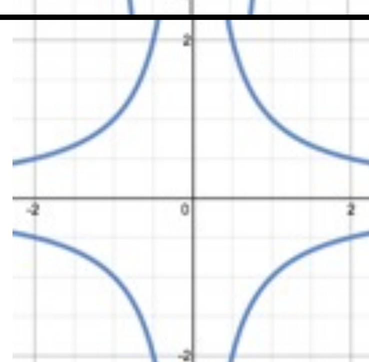

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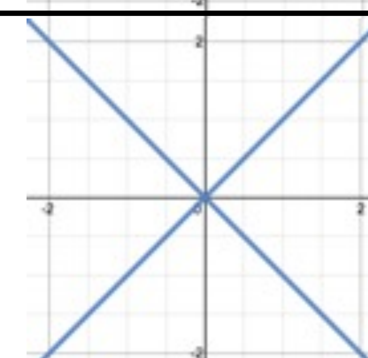

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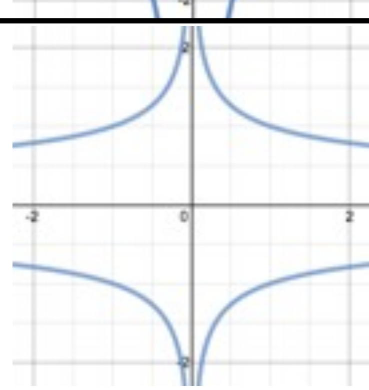

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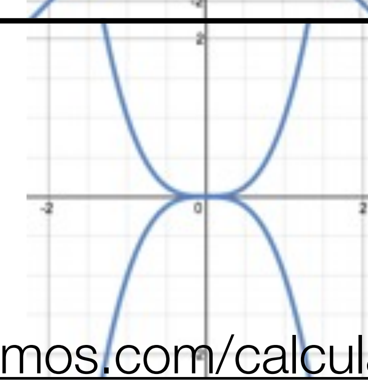

https://www.desmos.com/calculator/c4rhrgotmo

Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

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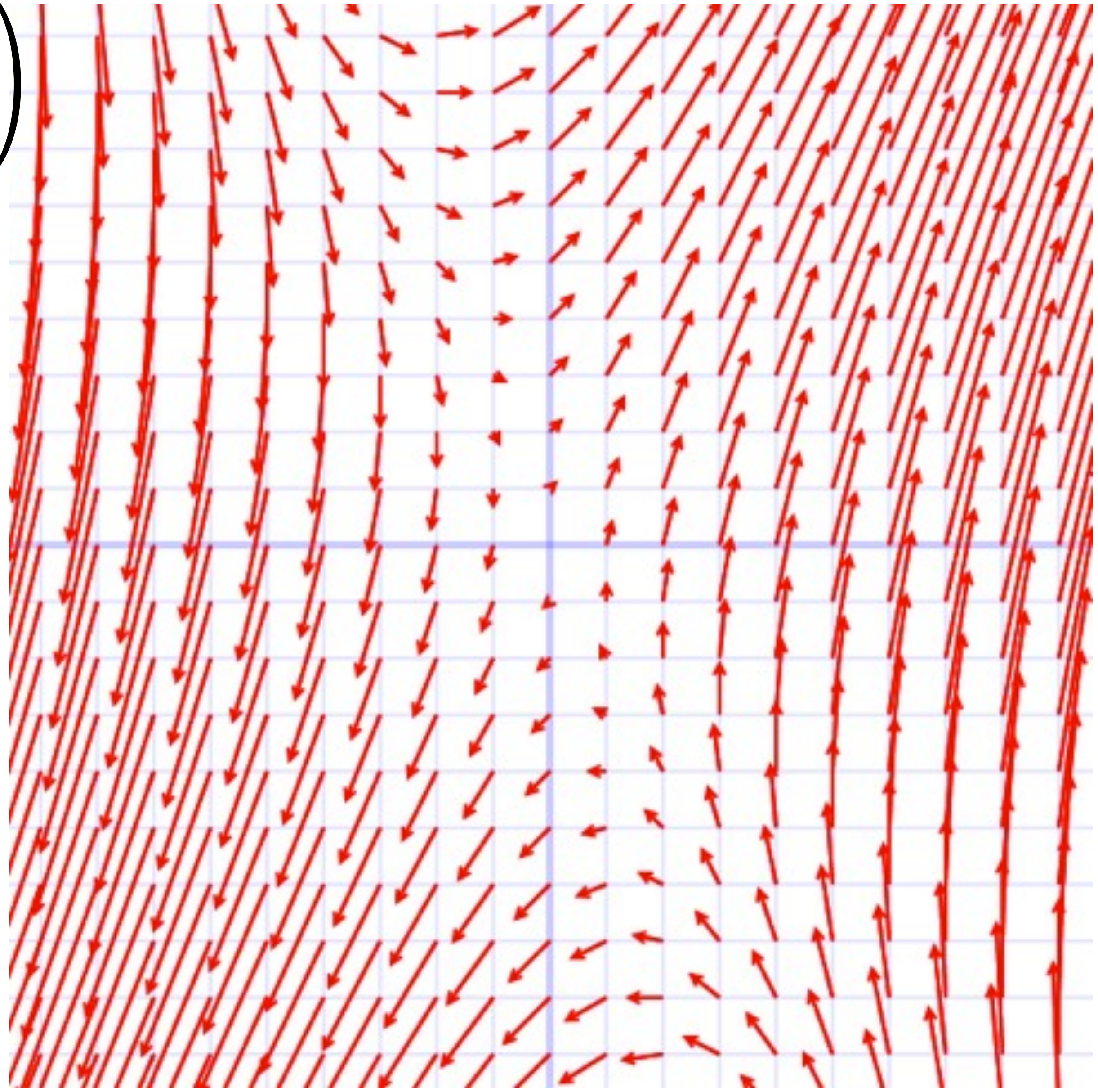

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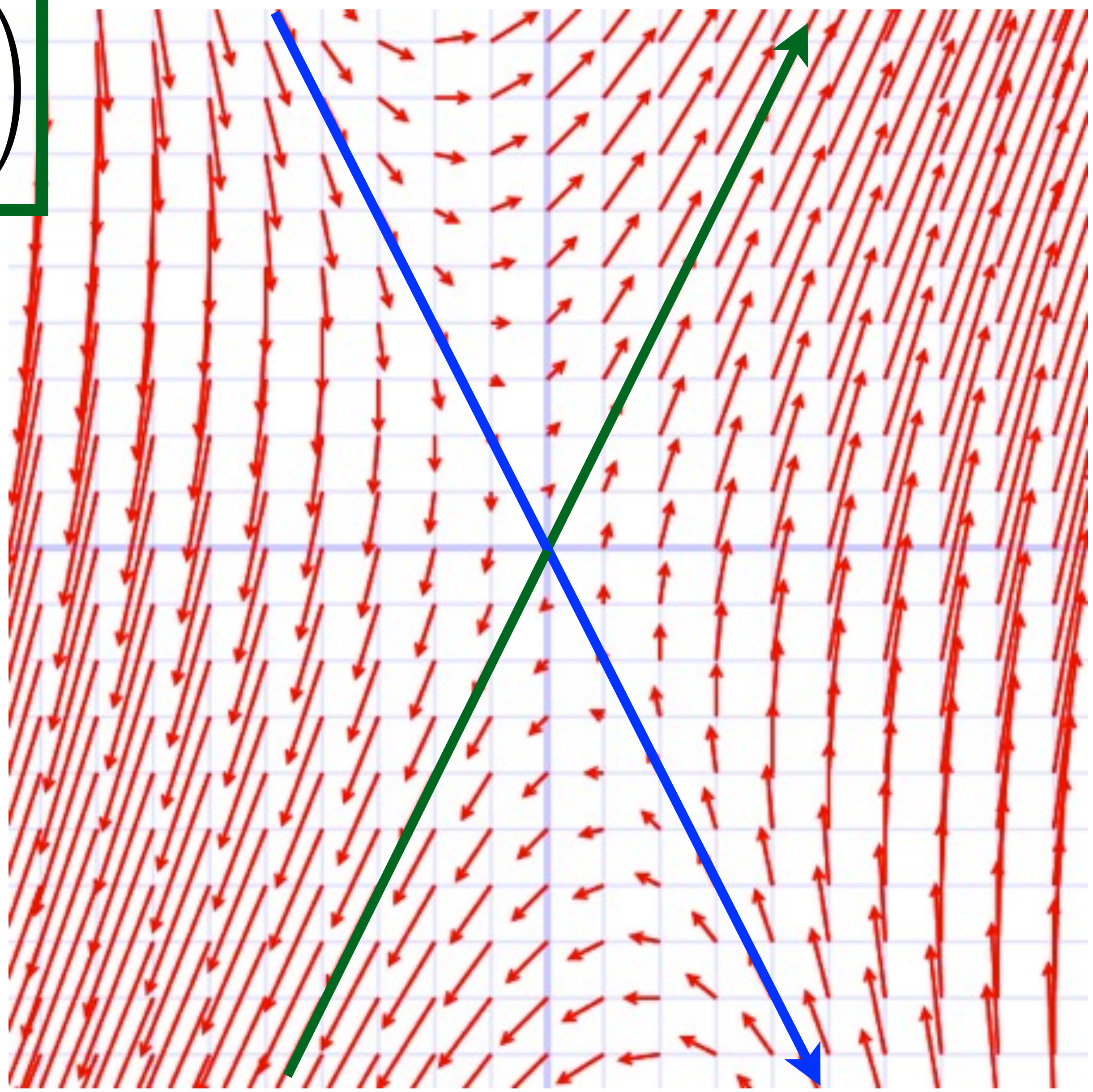

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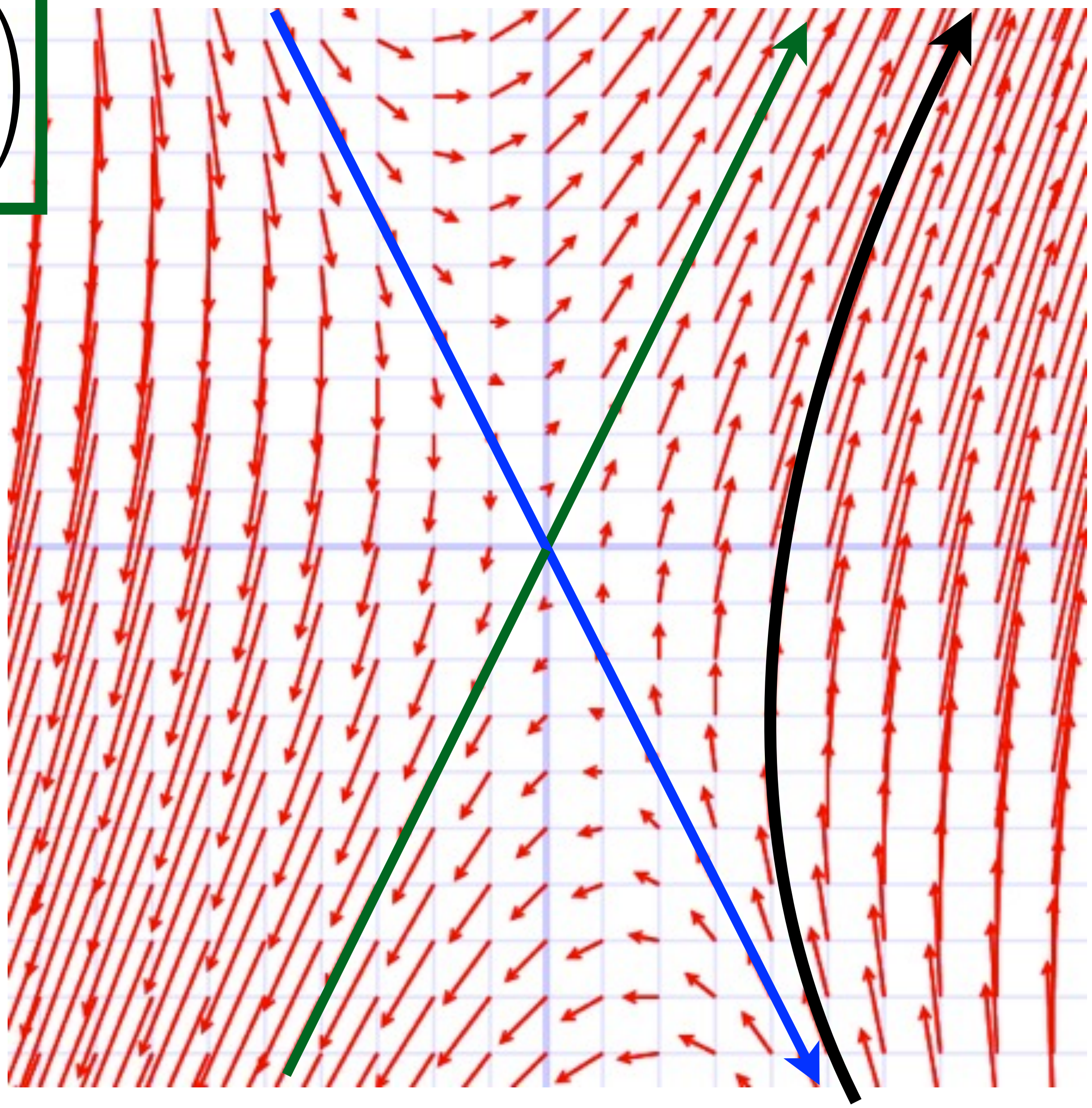

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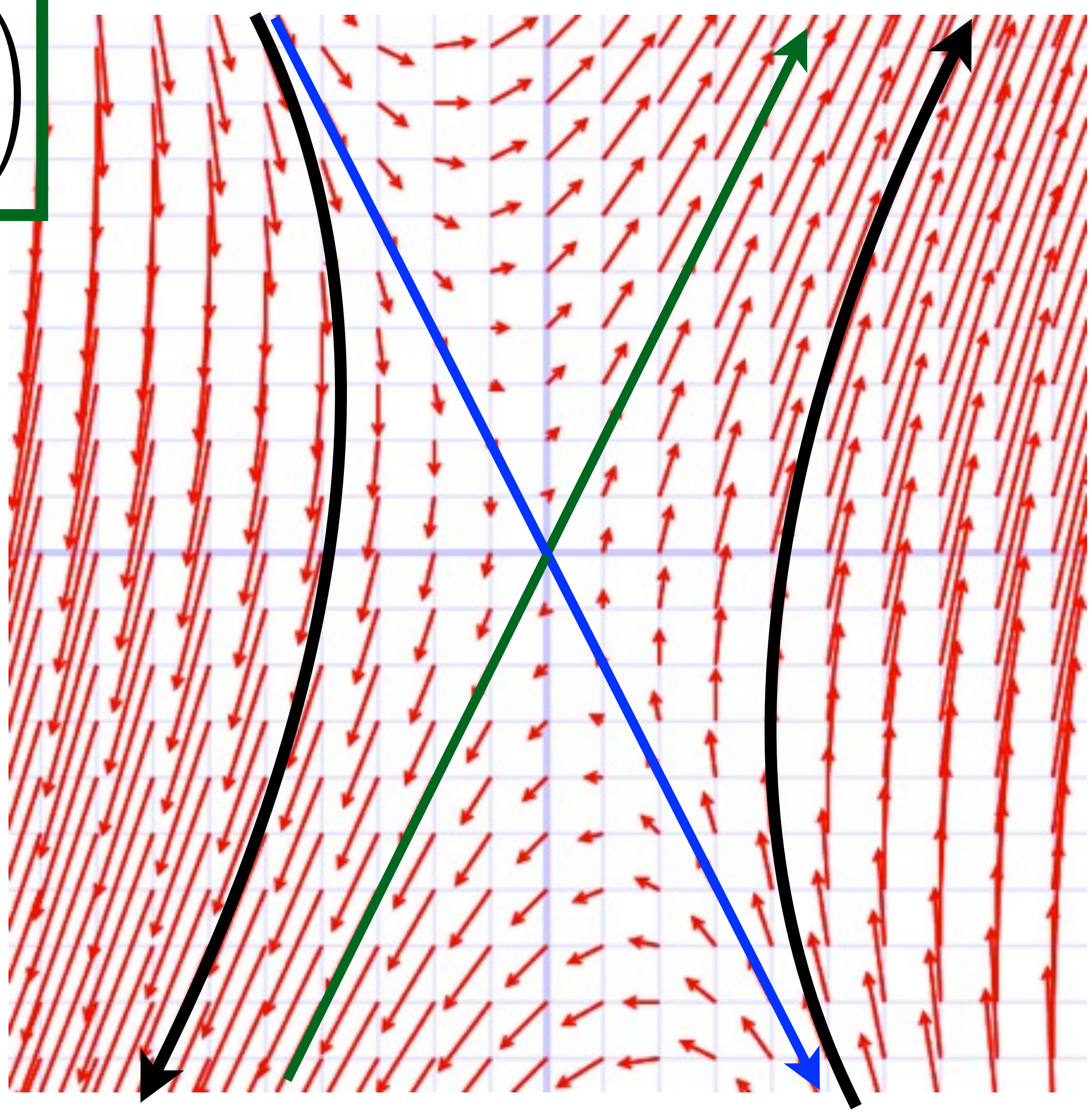

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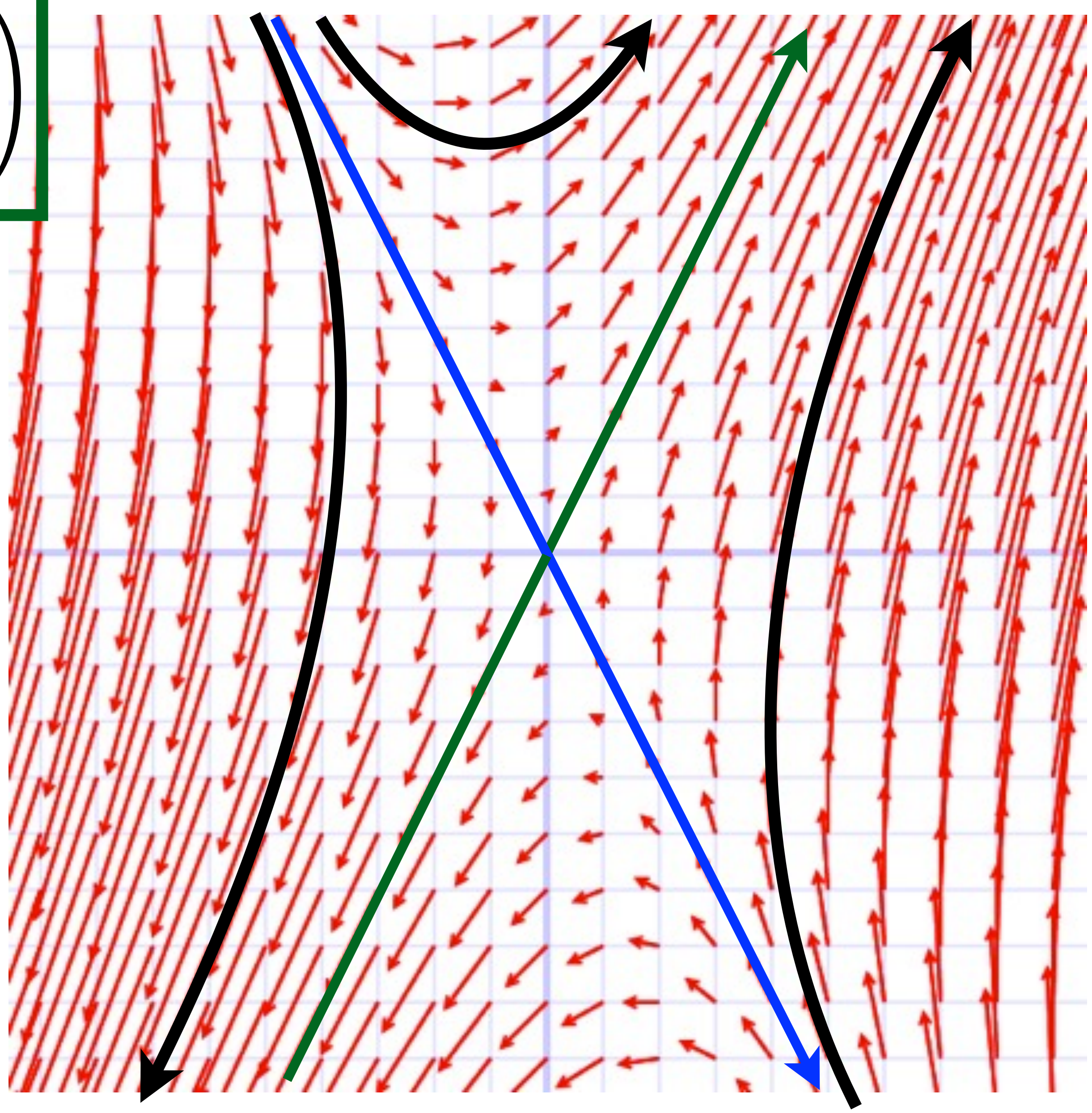

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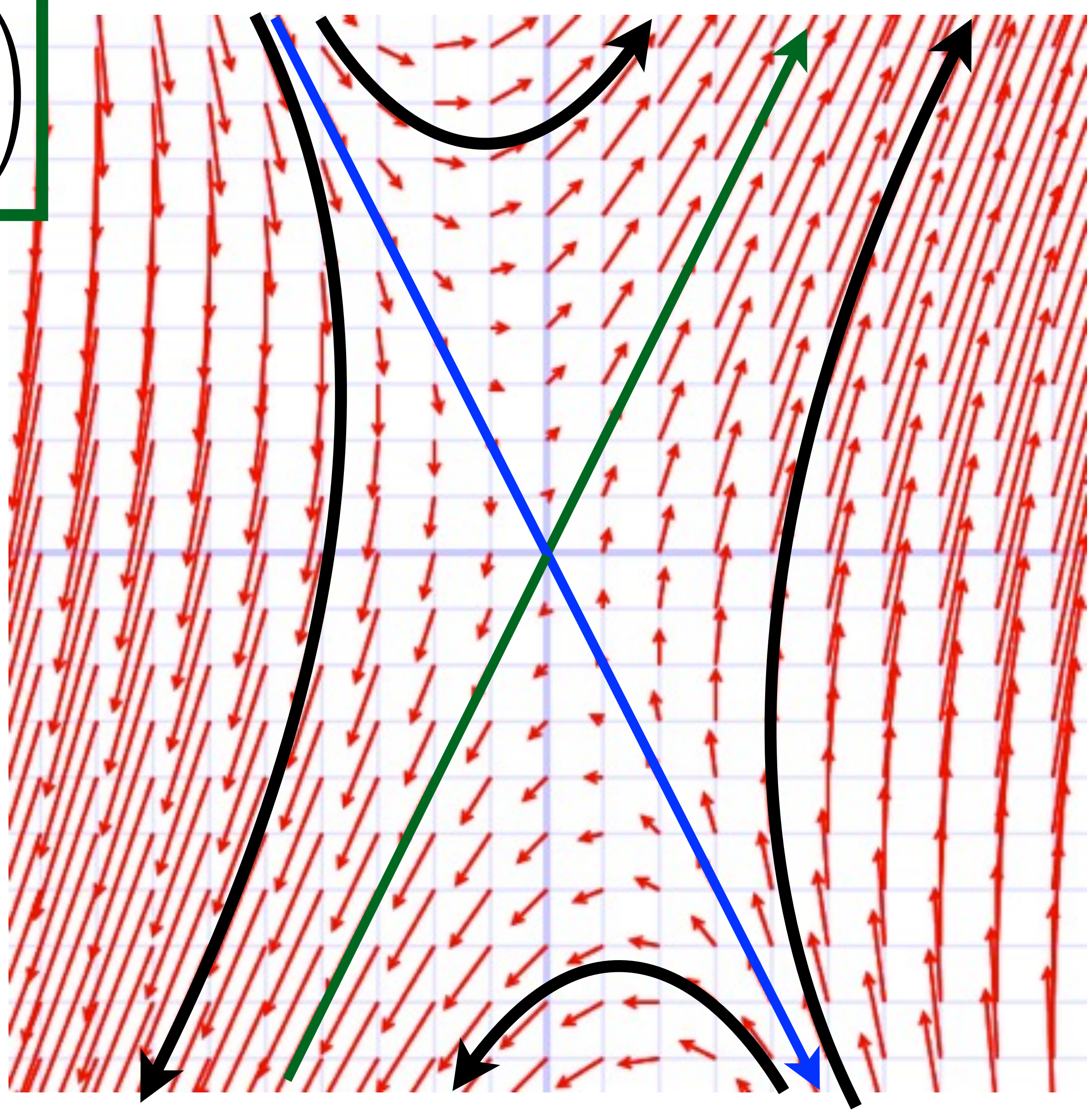

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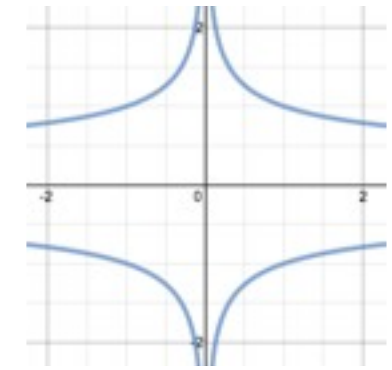

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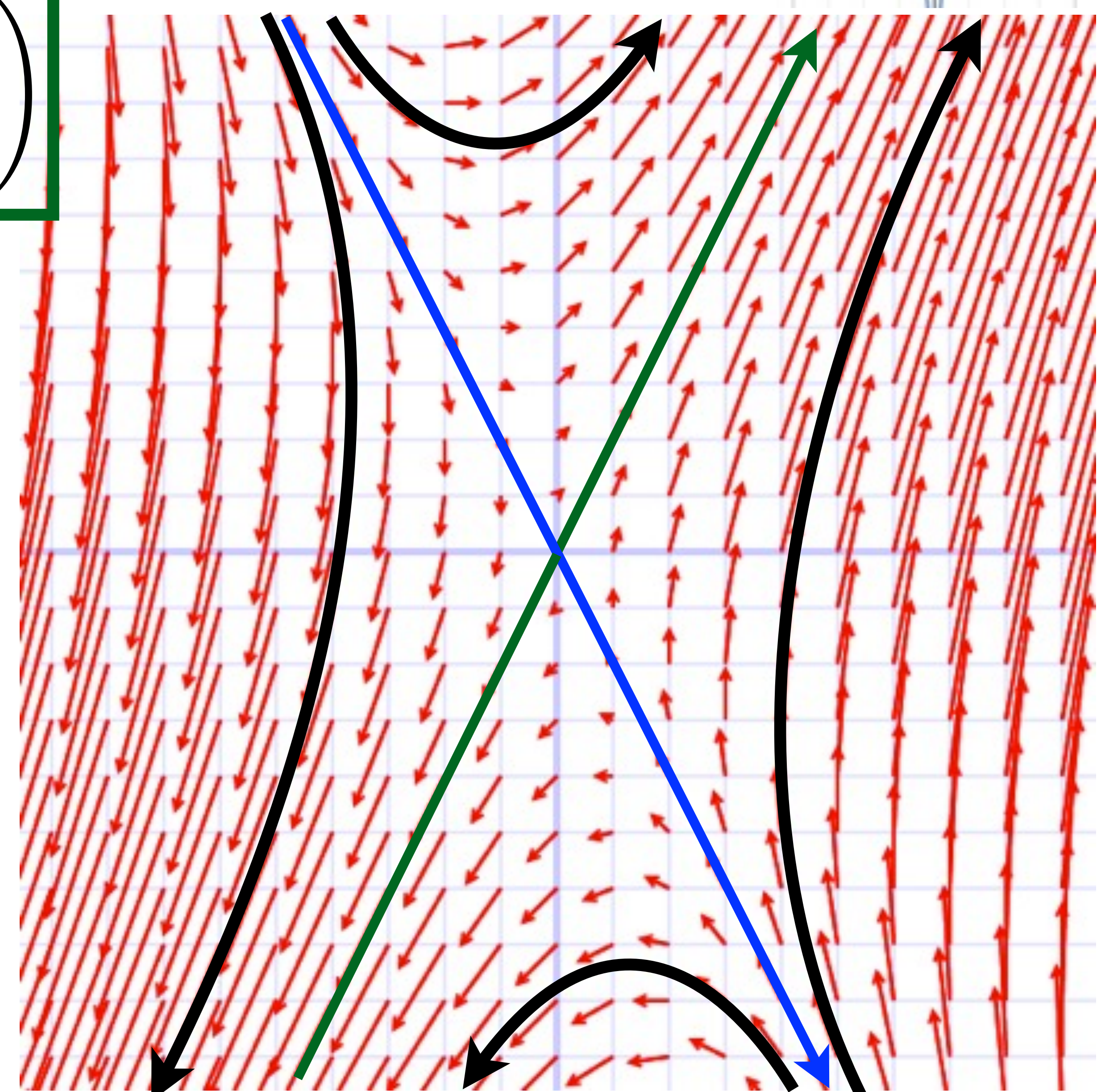

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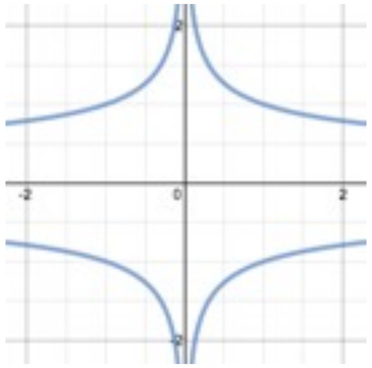

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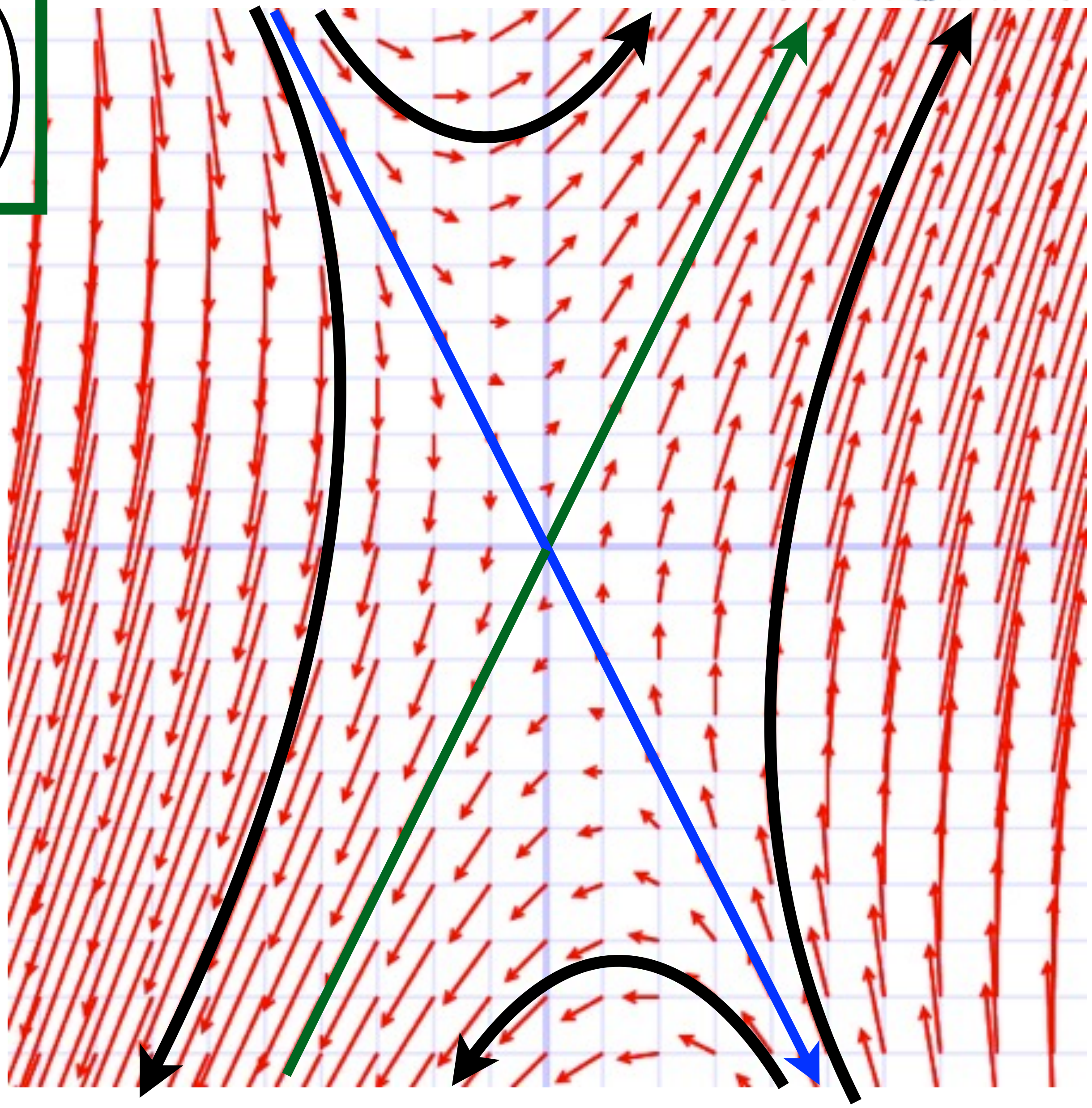


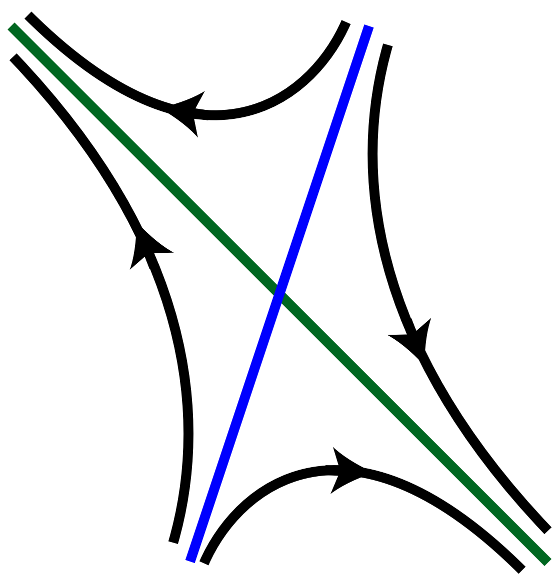
- Going forward in time, the blue component shrinks slower than the green component grows so solutions appear closer to blue "axis" than to green "axis"

Shapes of solution curves in the phase plane

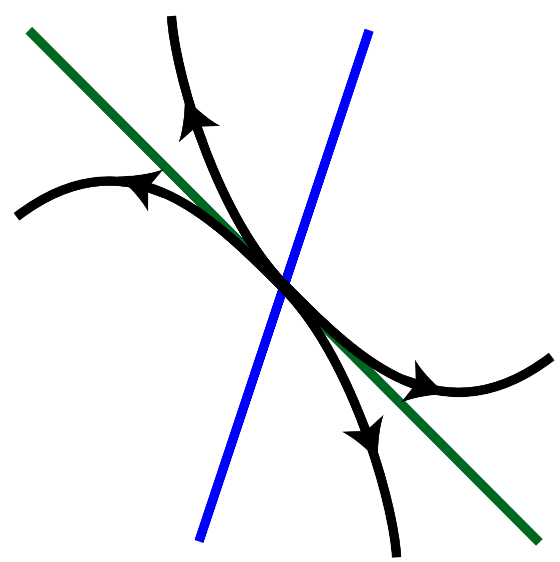
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ ?}$$

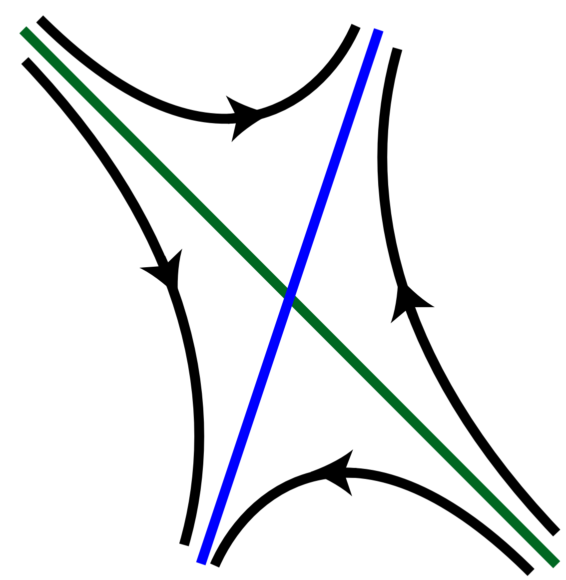
(A)



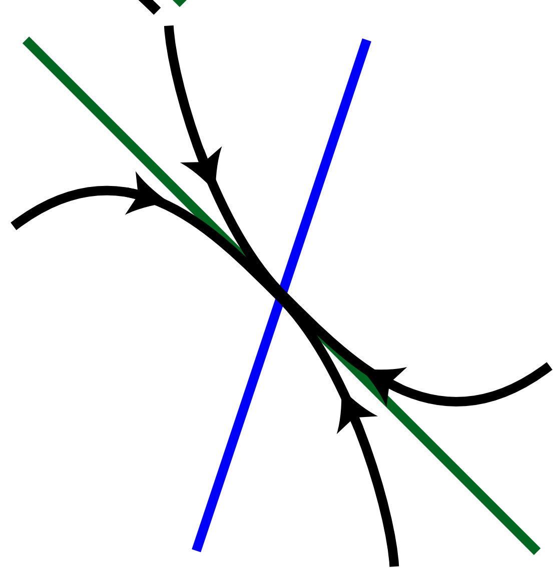
(B)



(C)



(D)



(E) Explain, please.

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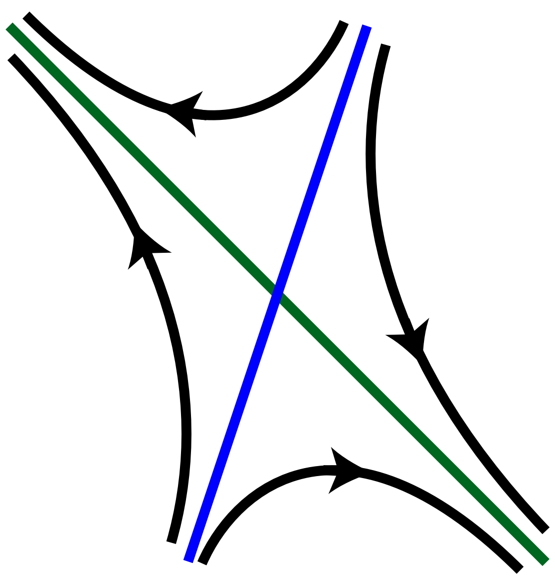
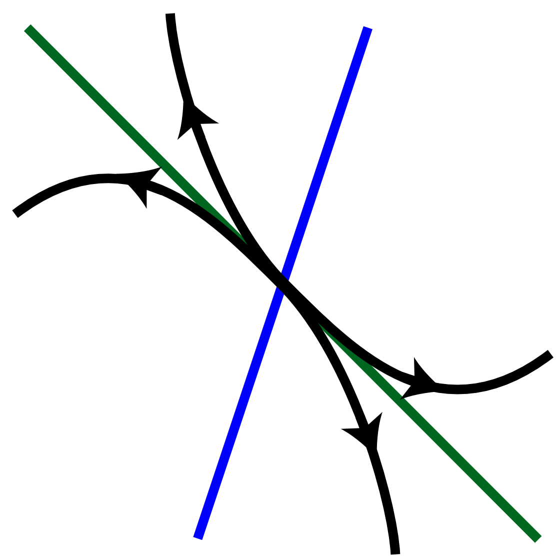
$$\mathbf{x} = C_1 e^{3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ ?}$$

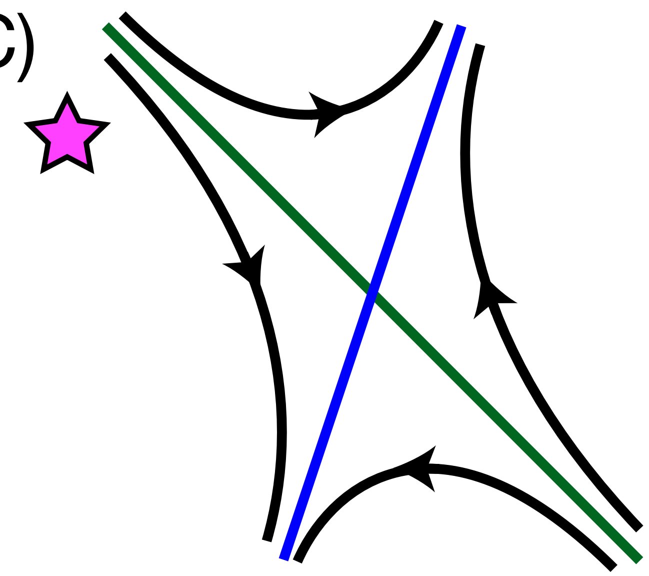
(A)



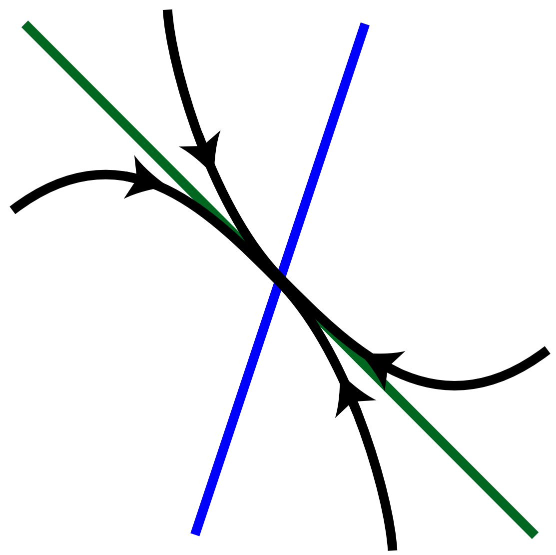
(B)



(C)



(D)



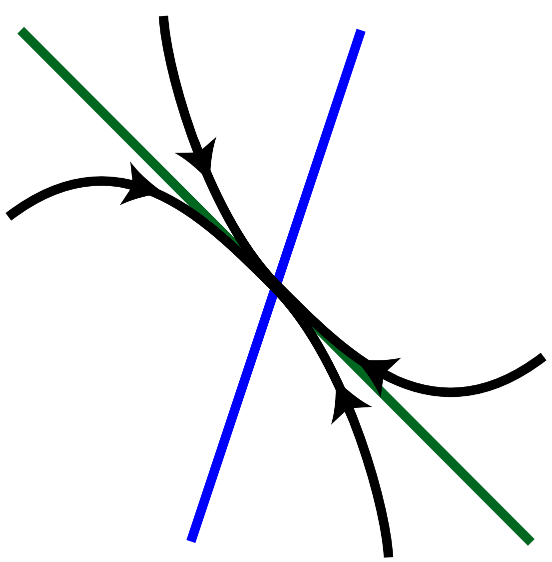
(E) Explain, please.

Shapes of solution curves in the phase plane

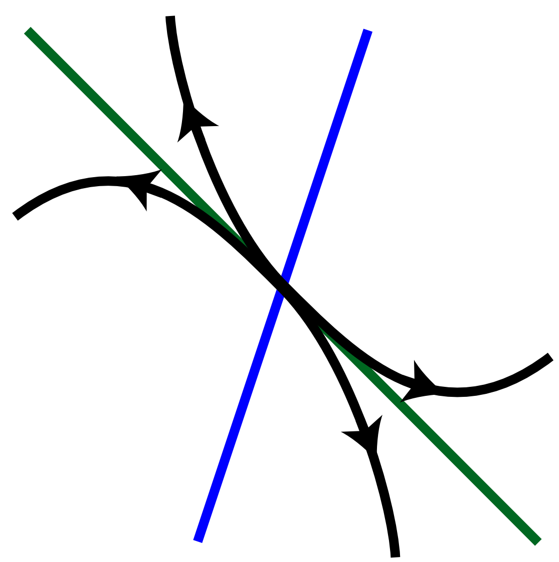
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ ?}$$

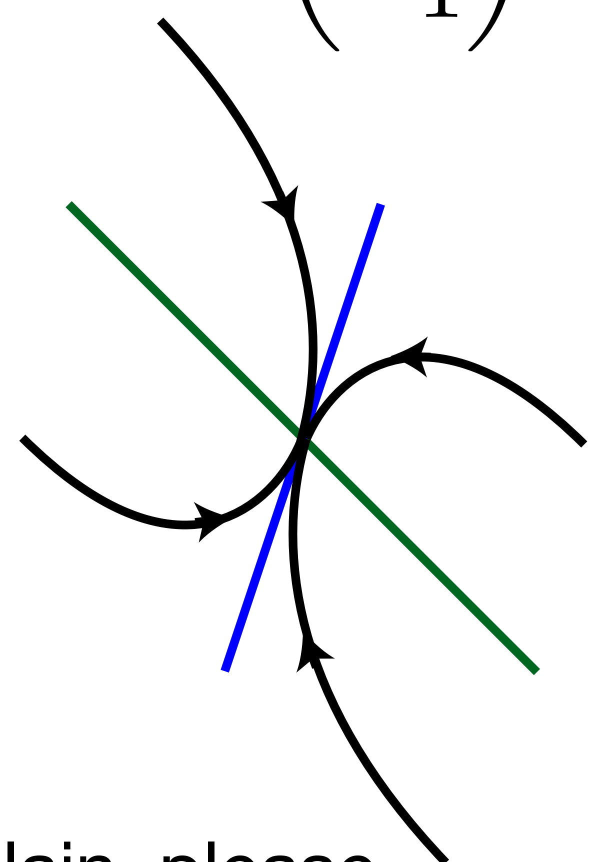
(A)



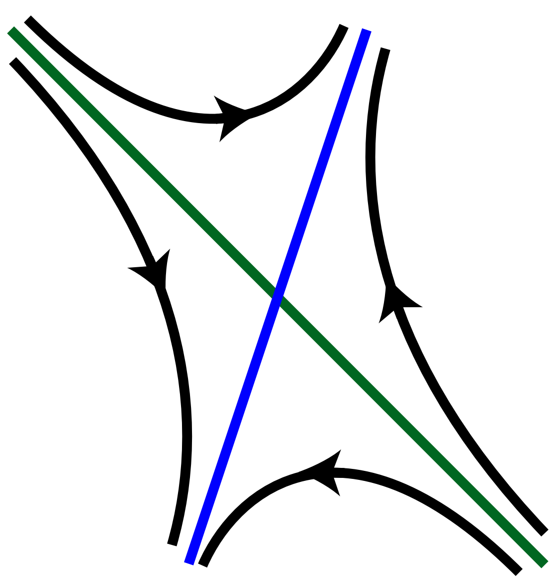
(B)



(C)



(D)

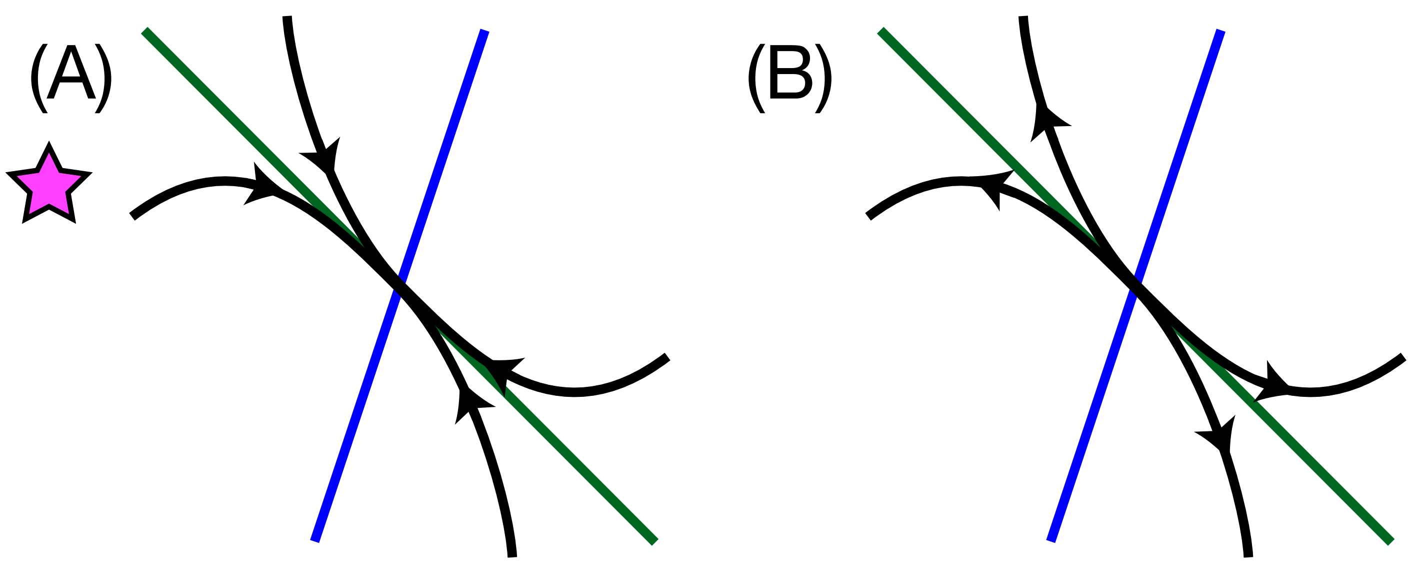


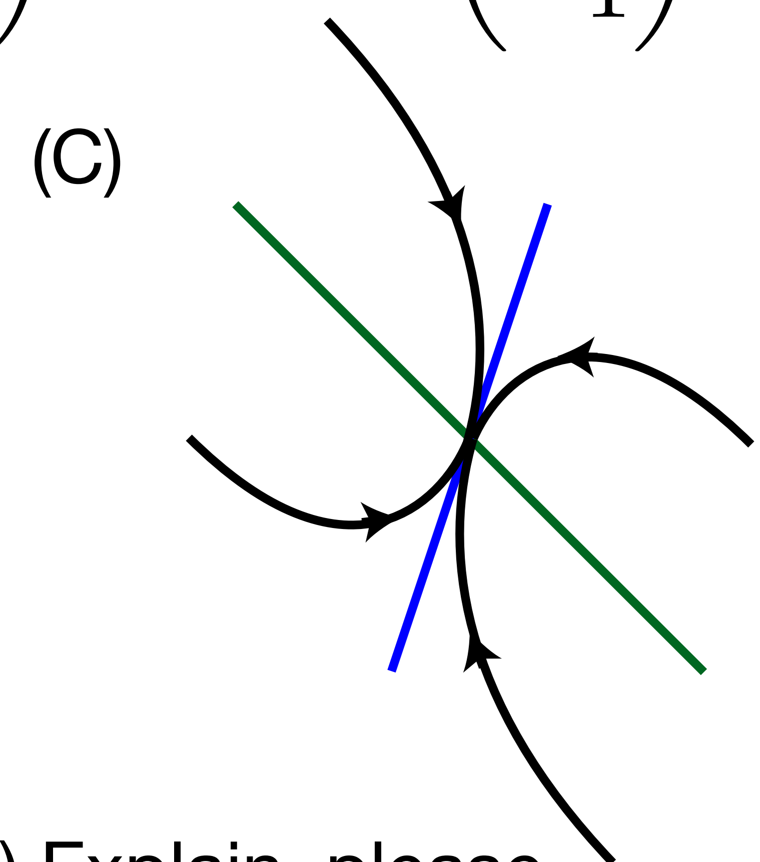
(E) Explain, please.

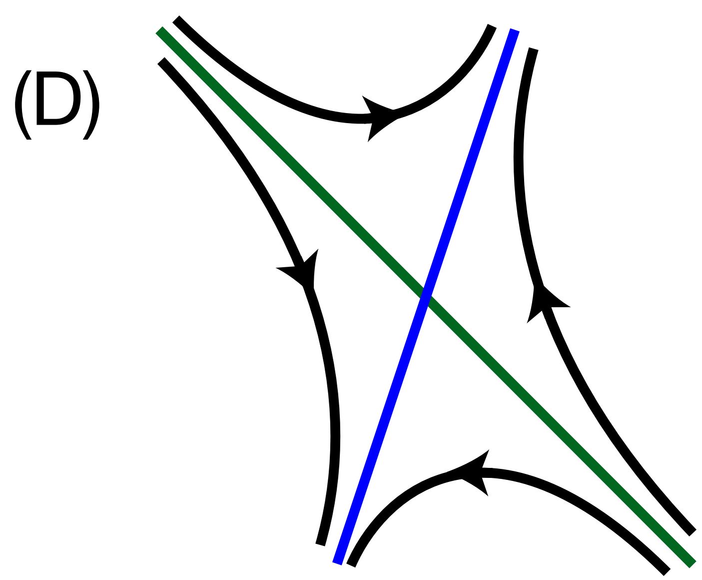
Shapes of solution curves in the phase plane

- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{-3t}\begin{pmatrix}1\\3\end{pmatrix} + C_2 e^{-t}\begin{pmatrix}1\\-1\end{pmatrix}\ ?$$







(E) Explain, please.