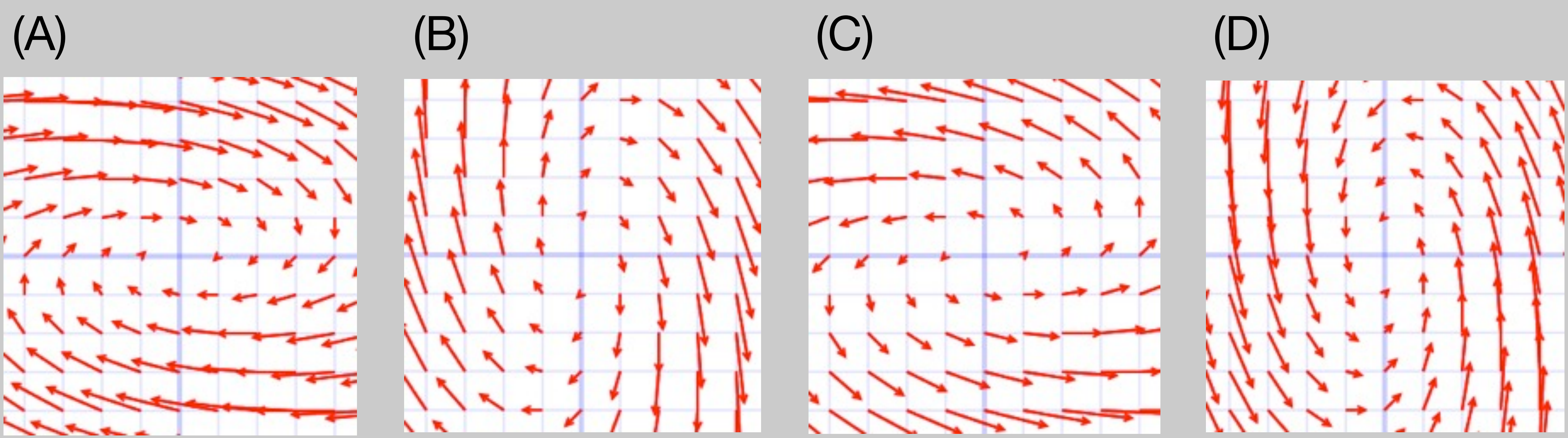
Today

- Systems with complex eigenvalue - example
- Systems with a repeated eigenvalue
- Summary of 2x2 systems with constant coefficients.

Complex eigenvalues (7.6) - example

- Back to our earlier example: $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix} \mathbf{x}$

$$\mathbf{x}(\mathbf{t}) = e^t \left(C_1 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \sin(2t) \right) + C_2 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \cos(2t) \right) \right)$$



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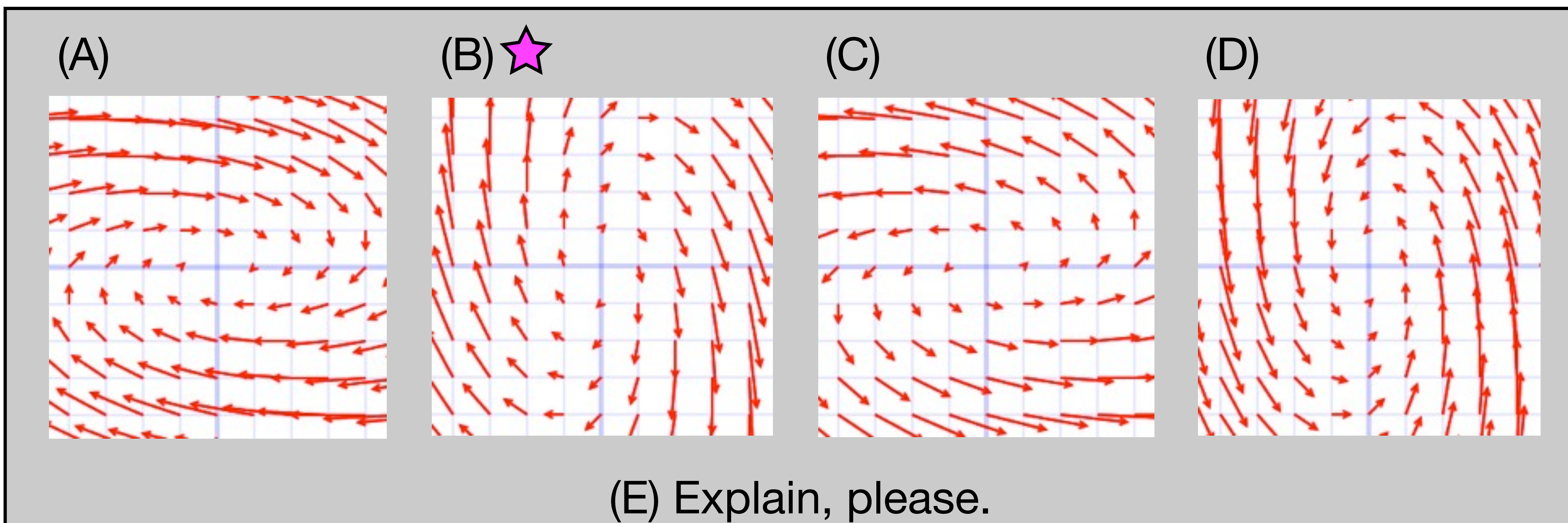

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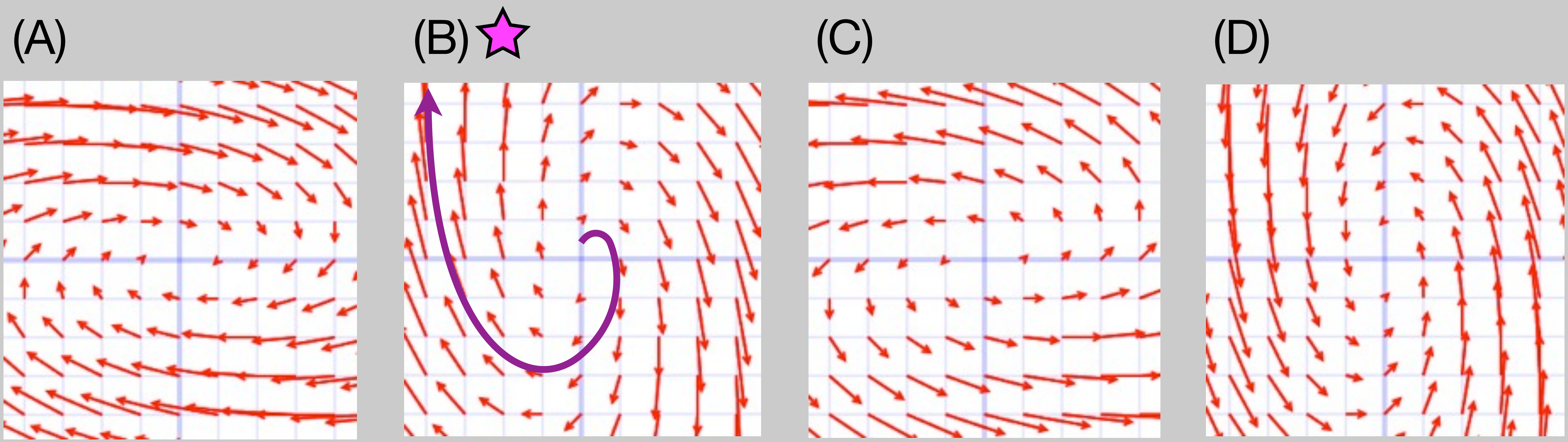

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Repeated eigenvalues

- What happens when you get two identical eigenvalues?
- Two cases:

    1. The single eigenvalue has two distinct eigenvectors.

    2. There is only one eigenvector (matrix is defective).

1. $\overline{\mathbf{x}}' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \overline{\mathbf{x}}$

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$$\det(A - \lambda I) = (\lambda - 3)^2 = 0$$

$$\lambda = 3$$

$$(A - \lambda I)\mathbf{v} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{v} = 0$$

All vectors solve this so choose any two independent vectors:

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x}(t) = C_1 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2. $\overline{\mathbf{x}}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \overline{\mathbf{x}}$

$$\det(A - \lambda I) = \lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2$$

$$(A - \lambda I)\mathbf{v} = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{v} = 0$$

$$\mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ <-- only 1 evector!

$$\mathbf{x}(t) = C_1 e^{2t}\mathbf{v} + C_2 e^{3t}(\mathbf{w} + t\mathbf{v})$$

$$(A - \lambda I)\mathbf{w} = \mathbf{v}$$

$$\mathbf{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Summary - homogeneous 2x2 systems

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- If *A* is nonsingular then $\mathbf{x}(t) = \mathbf{0}$ is the only steady state.

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  stable node

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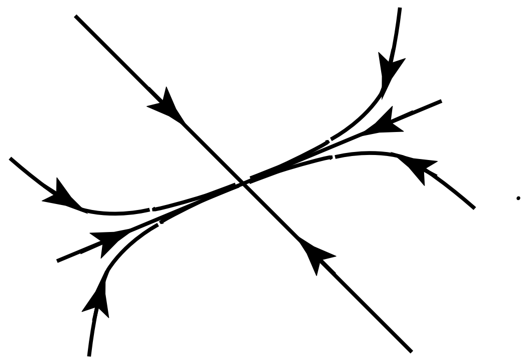

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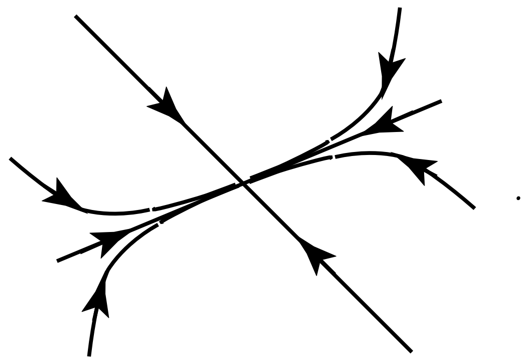

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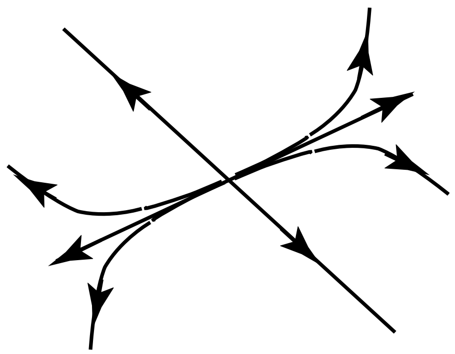

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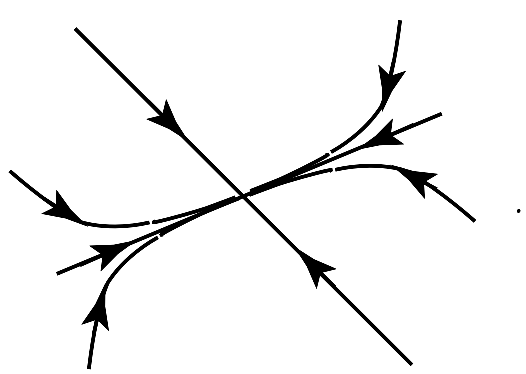

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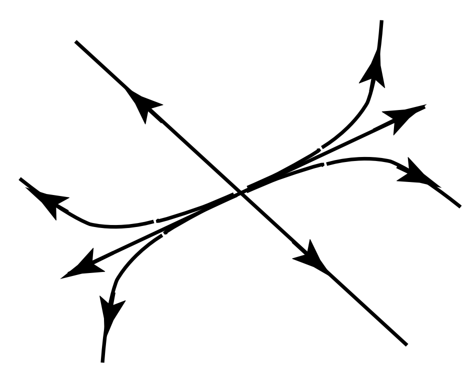

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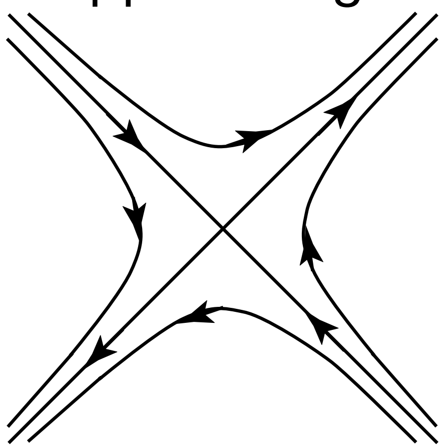

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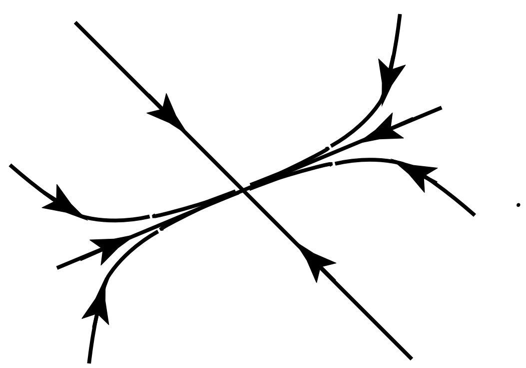

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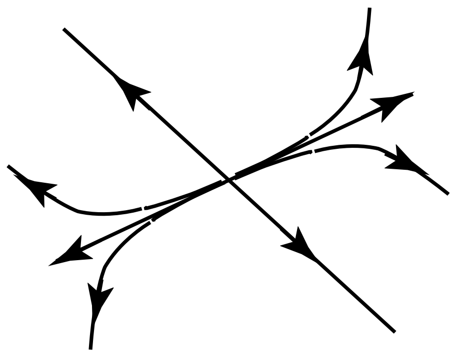

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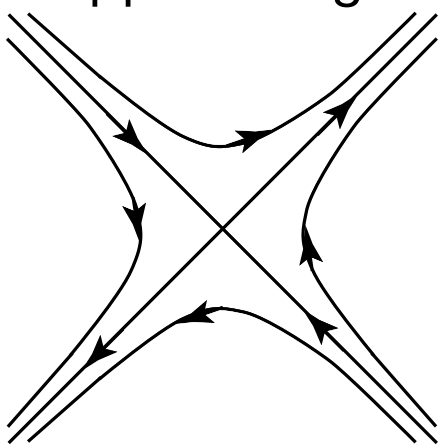

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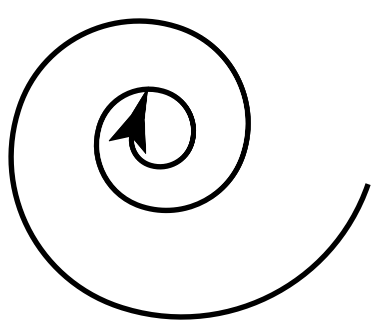

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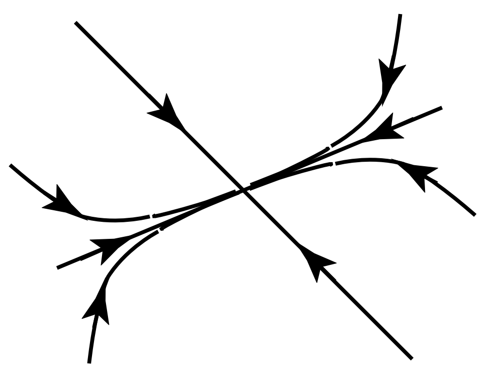

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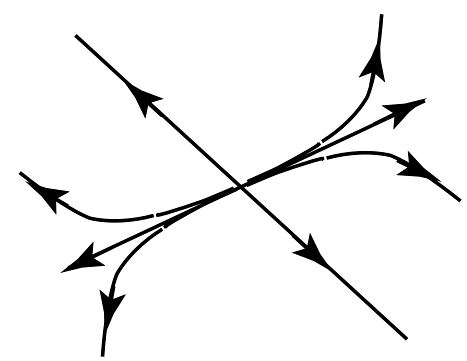

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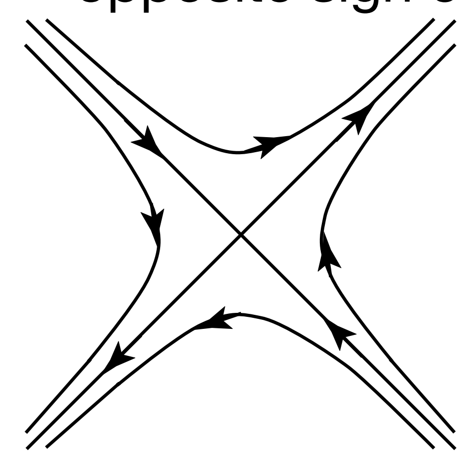

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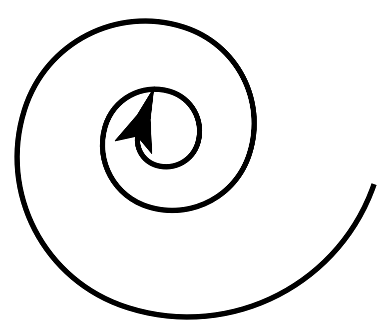

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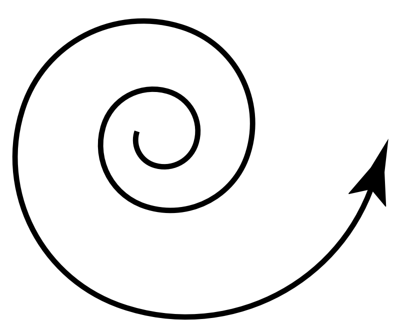

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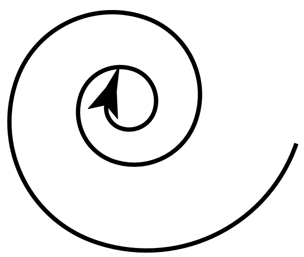

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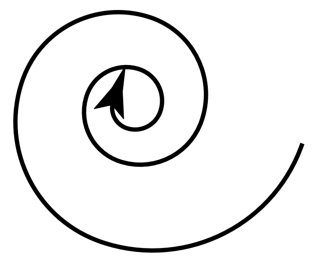

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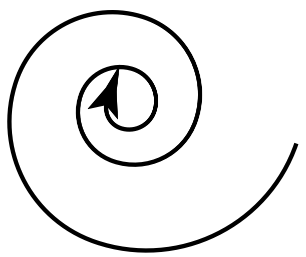

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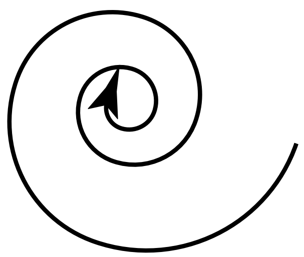

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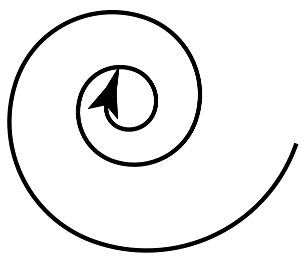

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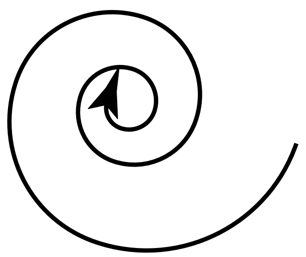

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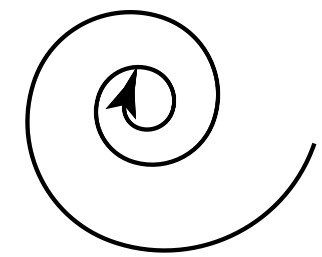

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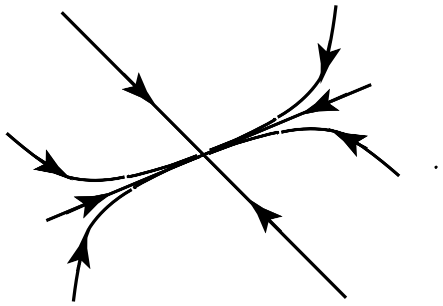

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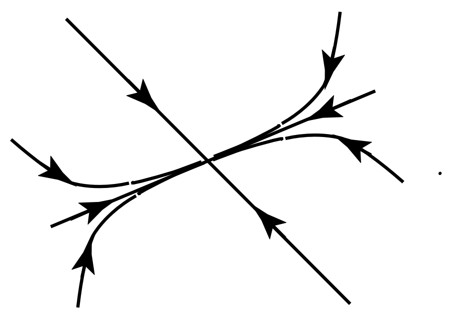

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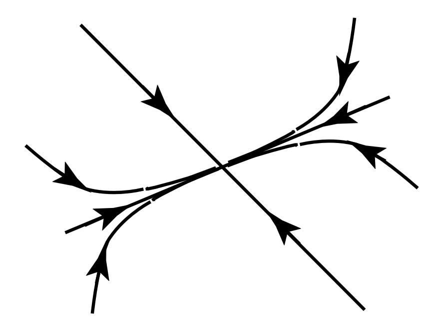

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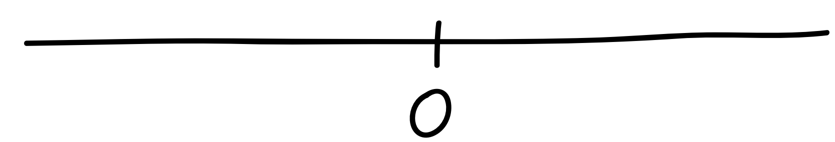

(C) $\begin{cases} \mathrm{tr}A < 0,\ \det(A) > 0 \\ (\mathrm{tr}A)^2 > 4\det A \end{cases}$

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(E) Explain, please.

$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

Summary - homogeneous 2x2 systems

- When is the origin a stable node?

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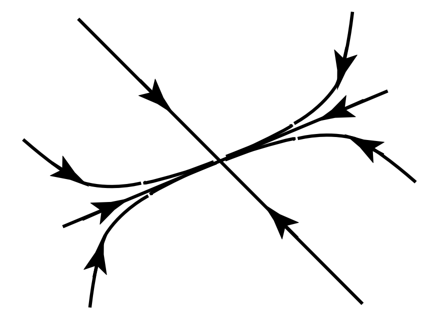

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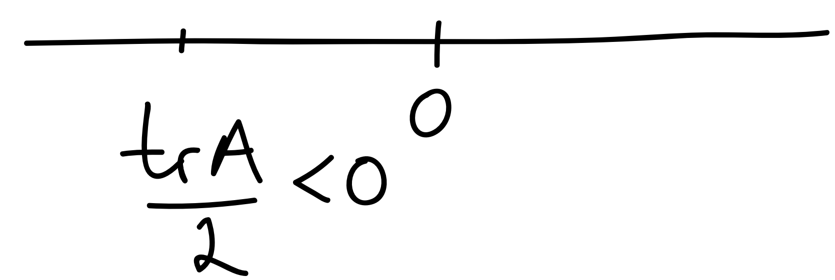

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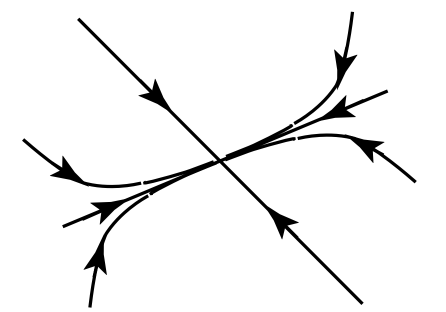

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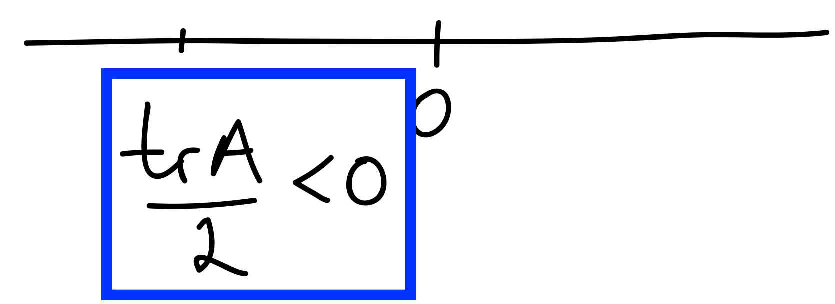

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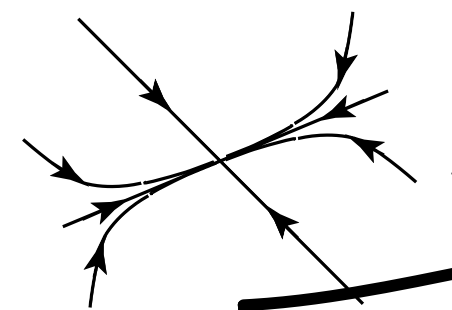

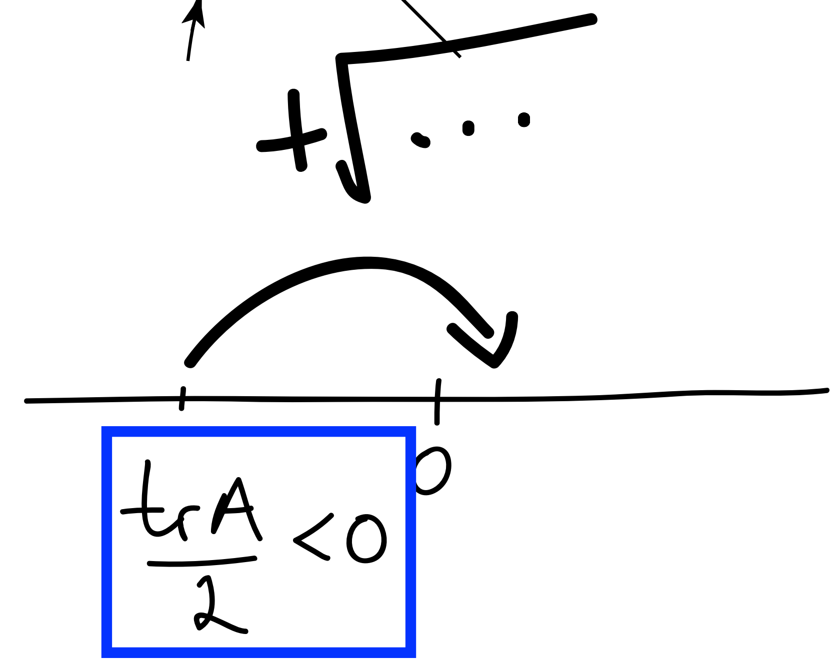

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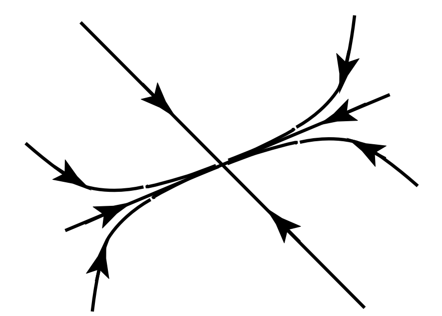

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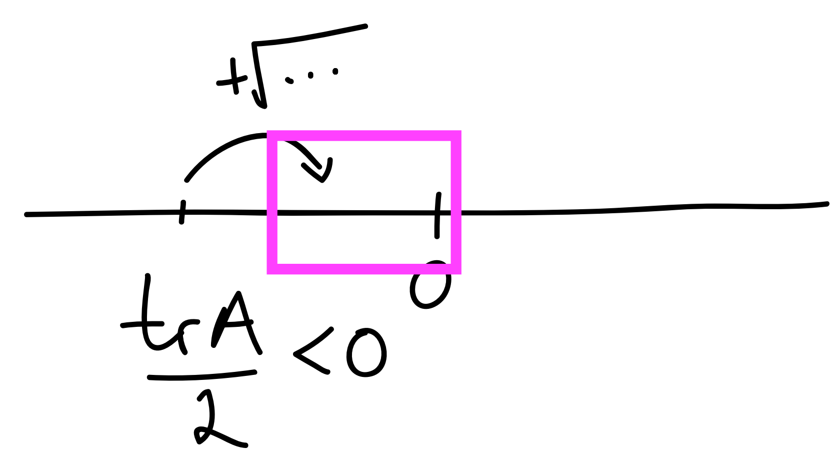

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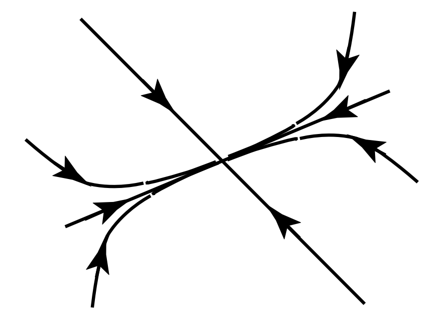

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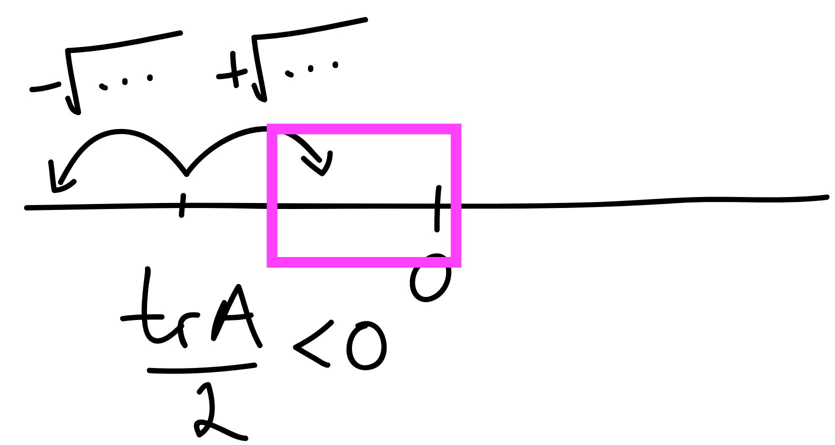

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