Today

- Systems with complex eigenvalues - how to figure out rotation
- Systems with a repeated eigenvalue
- Summary of 2x2 systems with constant coefficients.

Direction of rotation in complex eigenvalue case

$$x' = x - 8y$$
$$y' = 8x + y$$

(A) Solutions decay to zero exponentially.

(B) Solutions grow exponentially.

(C) Solutions rotate clockwise.

(D) Solutions rotate counterclockwise.

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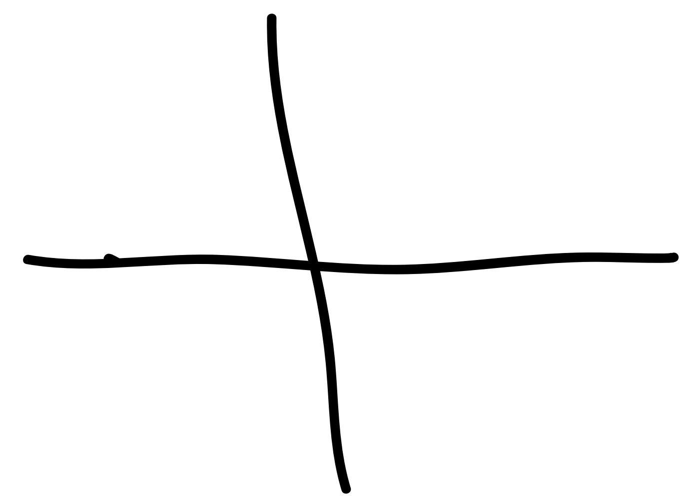

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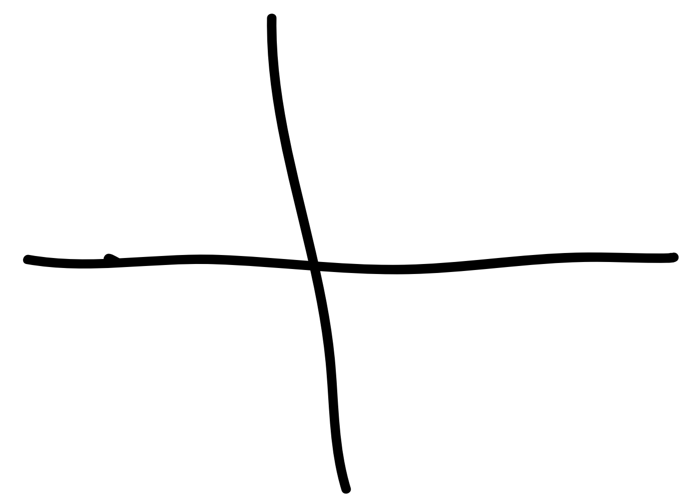

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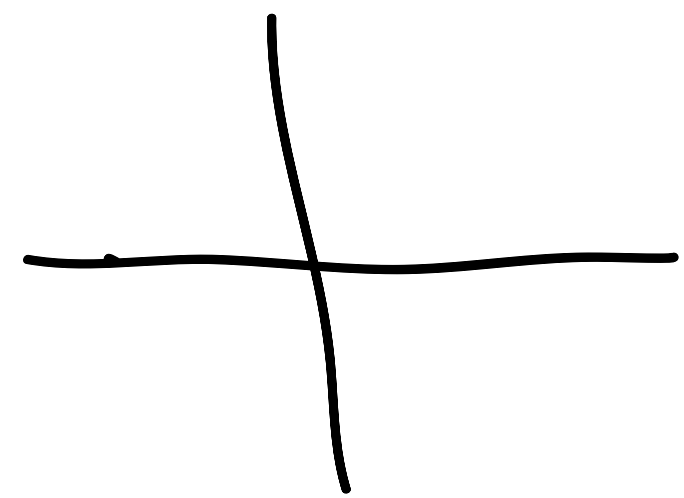

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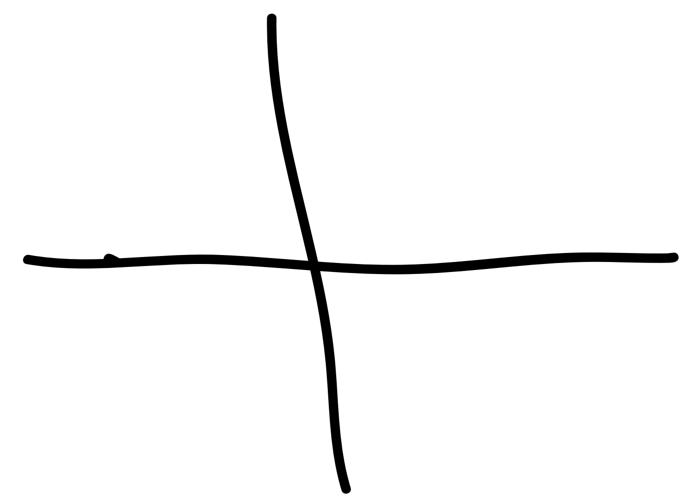

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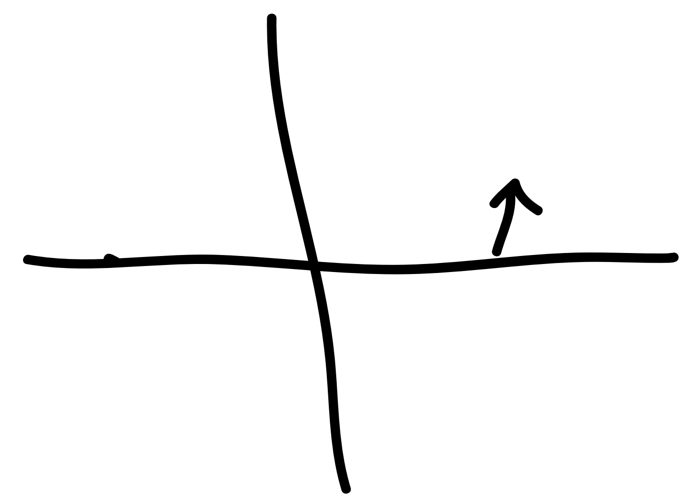

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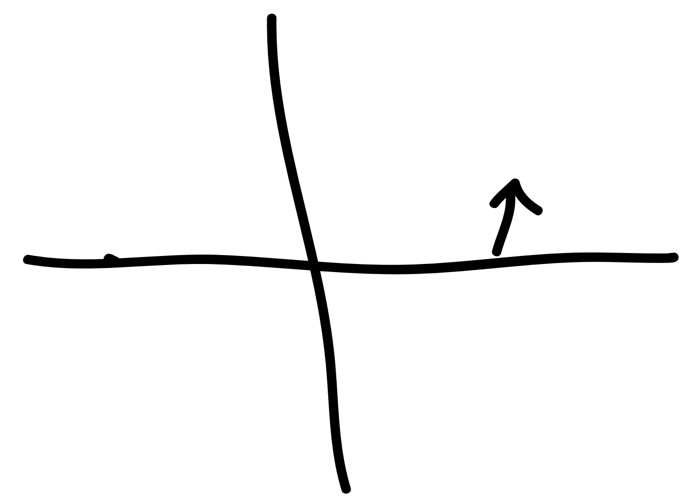

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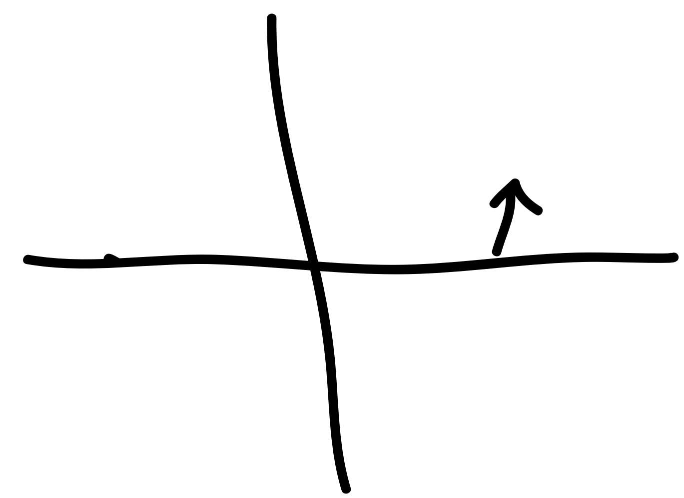

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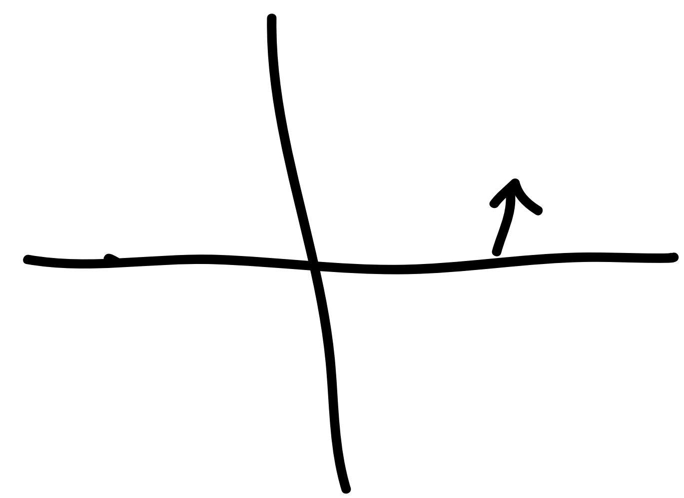

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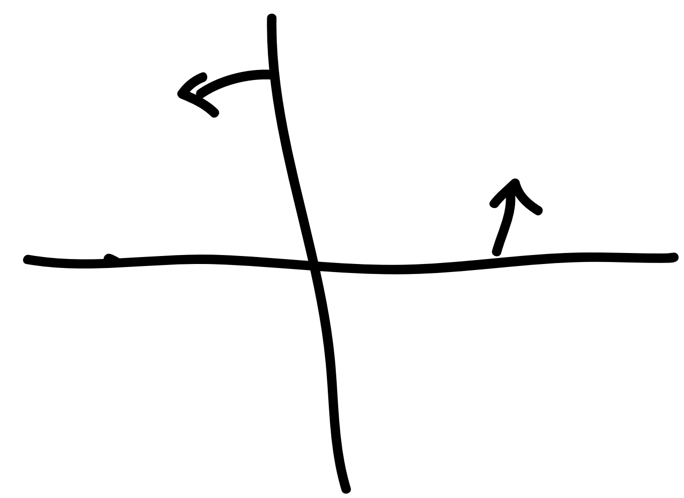

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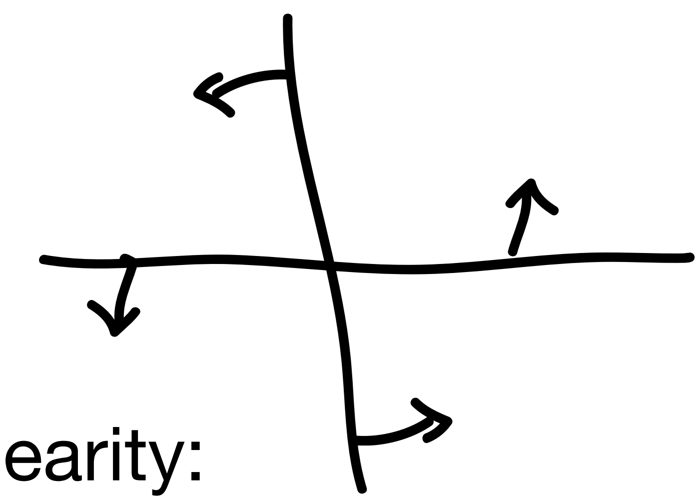

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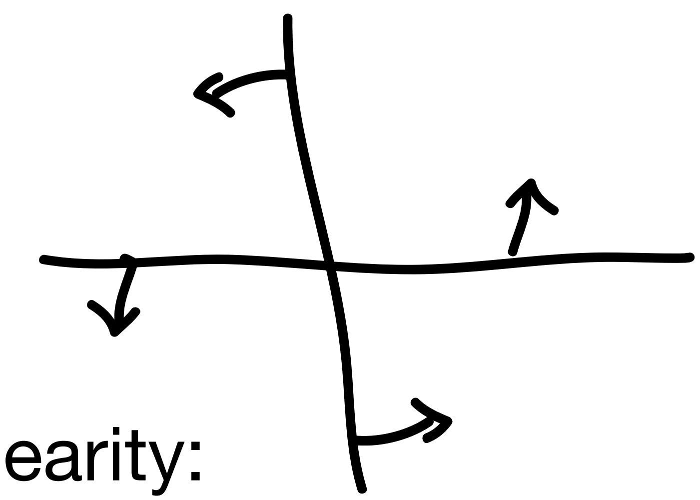

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Counterclockwise rotation!

Repeated eigenvalues

- What happens when you get two identical eigenvalues?

- Two cases:

  1. The single eigenvalue has two distinct eigenvectors.

  2. There is only one eigenvector (matrix is defective).

1. $\overline{\mathbf{x}}' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \overline{\mathbf{x}}$ 2. $\overline{\mathbf{x}}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \overline{\mathbf{x}}$

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Repeated eigenvalues

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$$\det(A - \lambda I) = (\lambda - 3)^2 = 0$$

$$\lambda = 3$$

$$(A - \lambda I)\mathbf{v} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{v} = 0$$

All vectors solve this so choose any two independent vectors:

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x}(t) = C_1 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2. $\overline{\mathbf{x}}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \overline{\mathbf{x}}$

$$\det(A - \lambda I) = \lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2$$

$$(A - \lambda I)\mathbf{v} = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{v} = 0$$

$\mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ <-- only 1 evector!

$$\mathbf{x}(t) = C_1 e^{2t} \mathbf{v} + C_2 e^{2t} (\mathbf{w} + t\mathbf{v})$$

$$(A - \lambda I)\mathbf{w} = \mathbf{v}$$

$\mathbf{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ <-- called "generalized evector"

Systems of ODEs - steps for solving (2x2)

- Find evalues (λ) and evectors (**v**) or generalized evectors (**w**) of A:
  - Distinct real - $\mathbf{x}(t) = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$

    where λ and $\mathbf{v_i}$ solve (A - λI) $\mathbf{v_i}$ =0.
  - Complex - $\mathbf{x}(\mathbf{t}) = e^{\alpha t}\left[C_1\left(\mathbf{a}\cos(\beta t) - \mathbf{b}\sin(\beta t)\right)\right.$

    $$\left.+C_2\left(\mathbf{a}\sin(\beta t) + \mathbf{b}\cos(\beta t)\right)\right]$$

    where $\lambda_1 = \alpha + \beta i$ and $\mathbf{v_1} = \mathbf{a} + \mathbf{b}i$.
  - Repeated with two eigenvectors (diagonal matrices only) -

    $$\mathbf{x}(t) = C_1 e^{\lambda t}\mathbf{v_1} + C_2 e^{\lambda t}\mathbf{v_2}$$
  - Repeated with one eigenvector - $\mathbf{x}(t) = C_1 e^{\lambda t}\mathbf{v} + C_2 e^{\lambda t}(\mathbf{w} + t\mathbf{v})$

    where λ and **v** solve (A - λI) **v** = **0** and **w** solves (A - λI) **w** = **v**.

Steady state - two notions

- Forced mass-spring systems - long term behaviour after transient dies down.
    - If you don't start right on the SS, a transient decays exponentially so eventually only y_p remains.
    - SS can be oscillation (not constant).
- Constant solutions of a system of ODEs (discussed in the next slides).
    - Transient may decay or grow exponentially.
    - Always constant solutions!

Summary - homogeneous 2x2 systems

Steady states - constant solutions (set $x'=0$ and solve $Ax=0$).

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    - It is also an eigenvector associated with eigenvalue $\lambda = 0$.
- If *A* is nonsingular then $\mathbf{x}(t) = \mathbf{0}$ is the only steady state.

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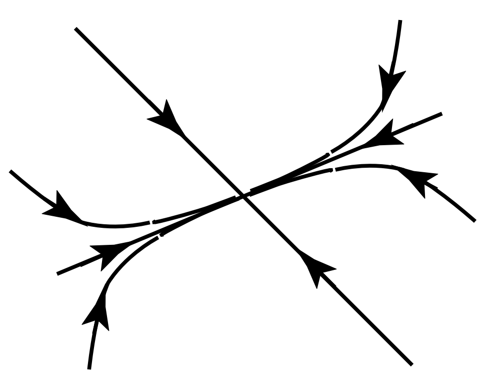
stable node

- real negative evalues



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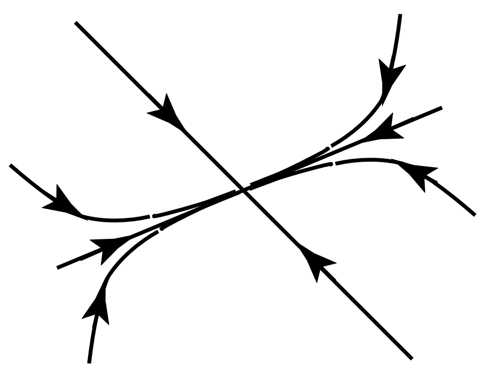

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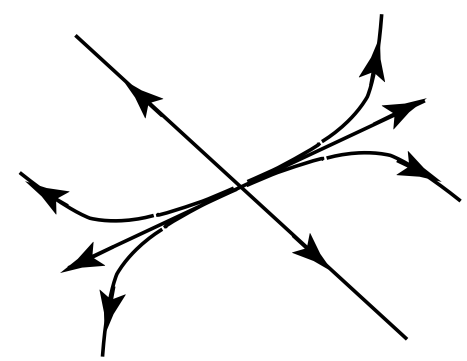

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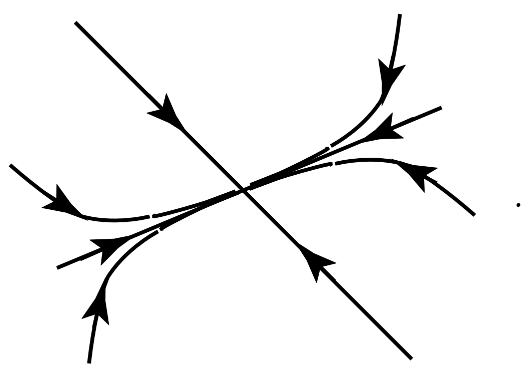

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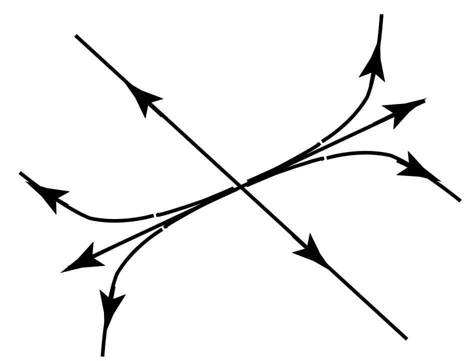

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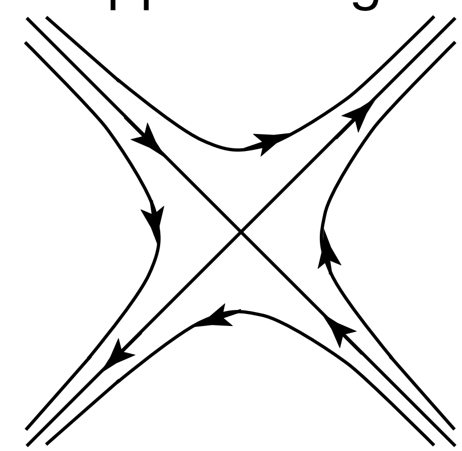

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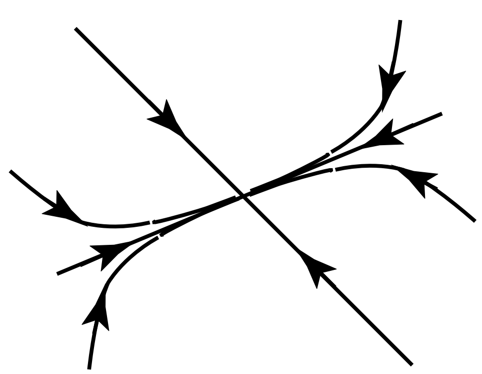

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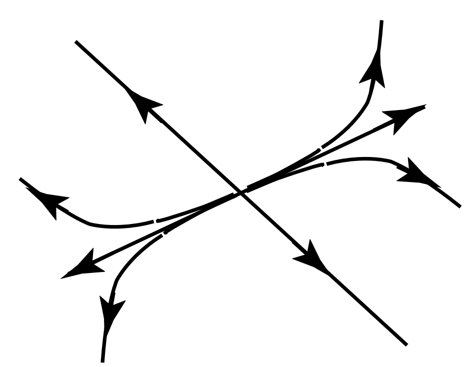

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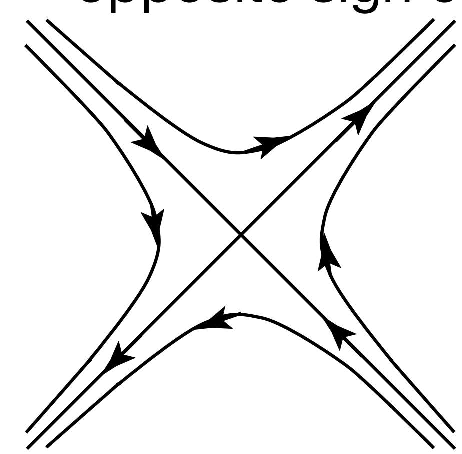
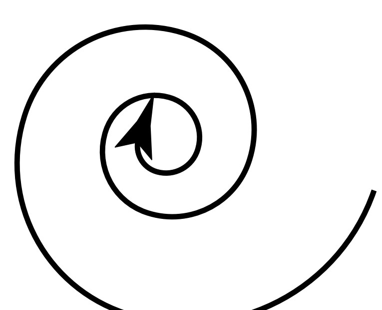


stable spiral

- complex evalues, negative real part



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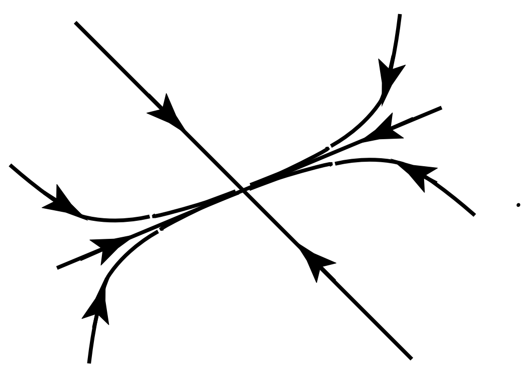

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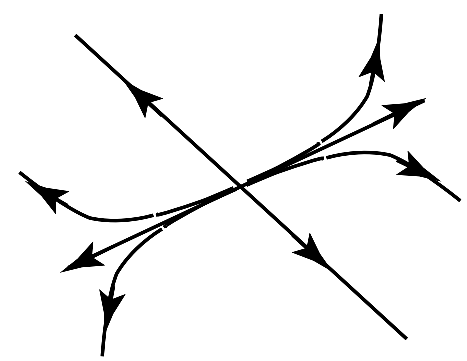

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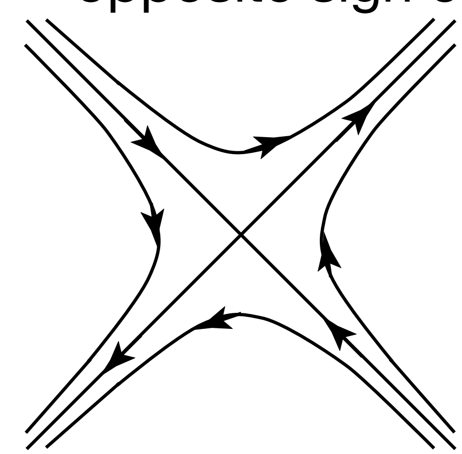

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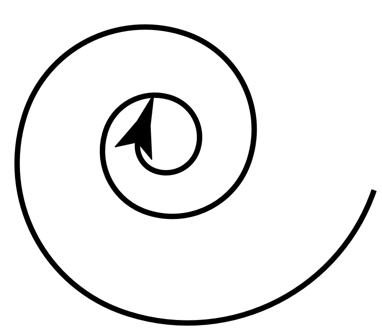

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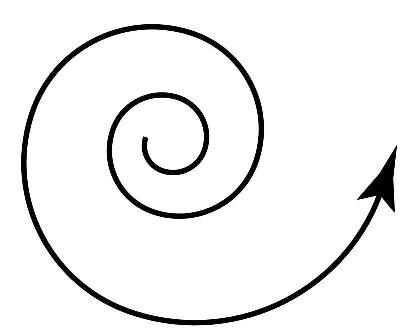

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&= \lambda^2 - \mathrm{tr}(A)\lambda + \det(A) \\
&= 0
\end{aligned}$$

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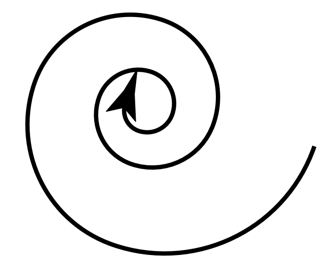
- When do the solutions spiral IN to the origin?

$$\lambda^2 - \mathrm{tr}A\lambda + \det A = 0$$



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(E) Explain, please.

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$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

Summary - homogeneous 2x2 systems

- When do the solutions spiral IN to the origin?

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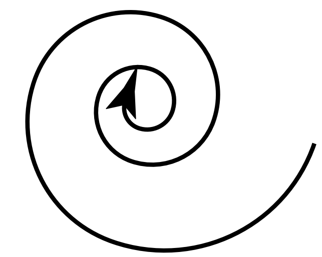

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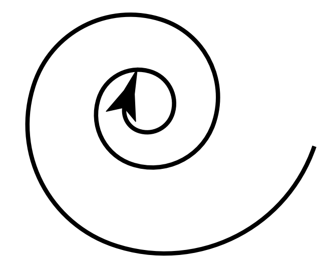

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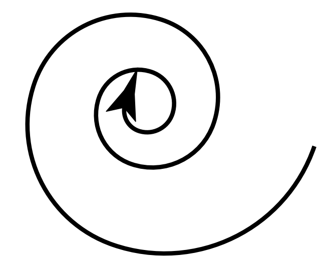

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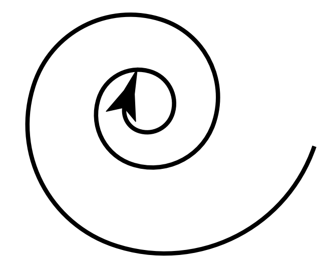

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ensures complex evalue

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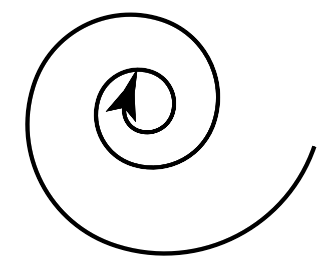

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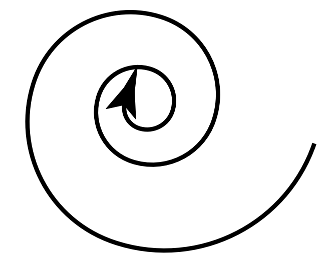

ensures negative real part

☆(A) $\begin{cases} \mathrm{tr}A < 0 \\ (\mathrm{tr}A)^2 < 4\det A \end{cases}$ $\quad \lambda = \boxed{\dfrac{\mathrm{tr}A}{2}} \pm \boxed{\dfrac{\sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}}$

ensures complex evalue

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Summary - homogeneous 2x2 systems

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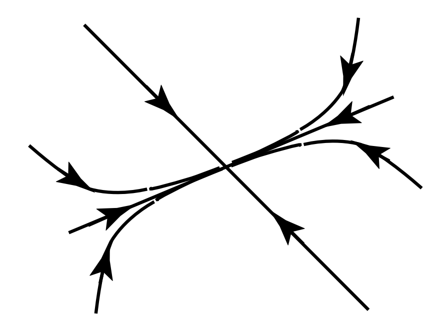

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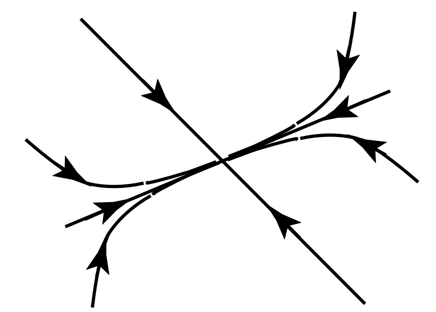

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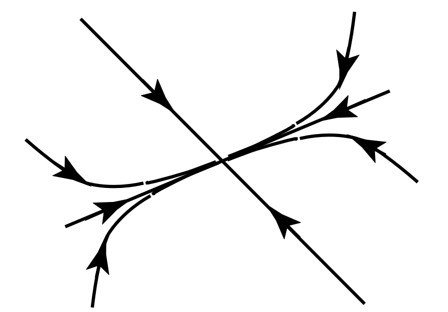

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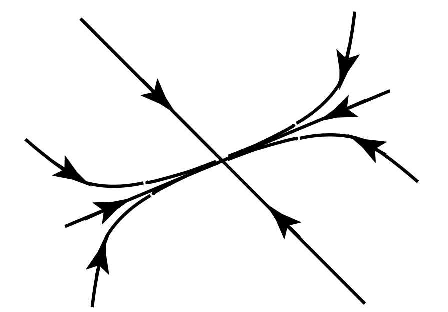

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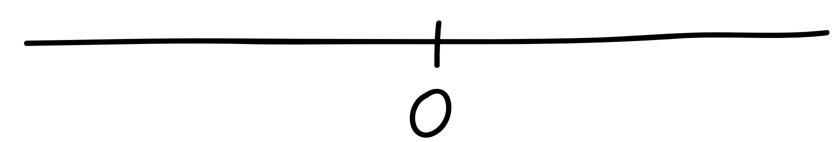

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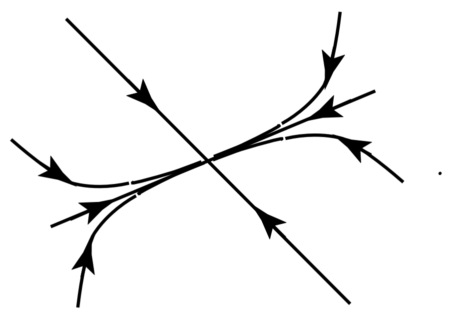

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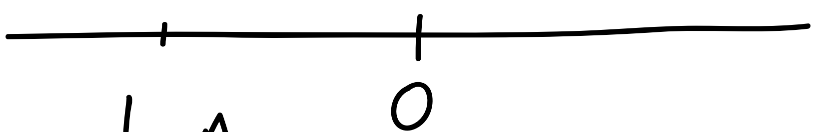

$\frac{\text{tr}A}{2} < 0$

(C) $\begin{cases} \text{tr}A < 0, \ \det(A) > 0 \\ (\text{tr}A)^2 > 4\det A \end{cases}$

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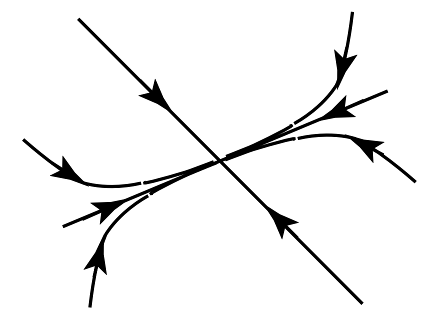

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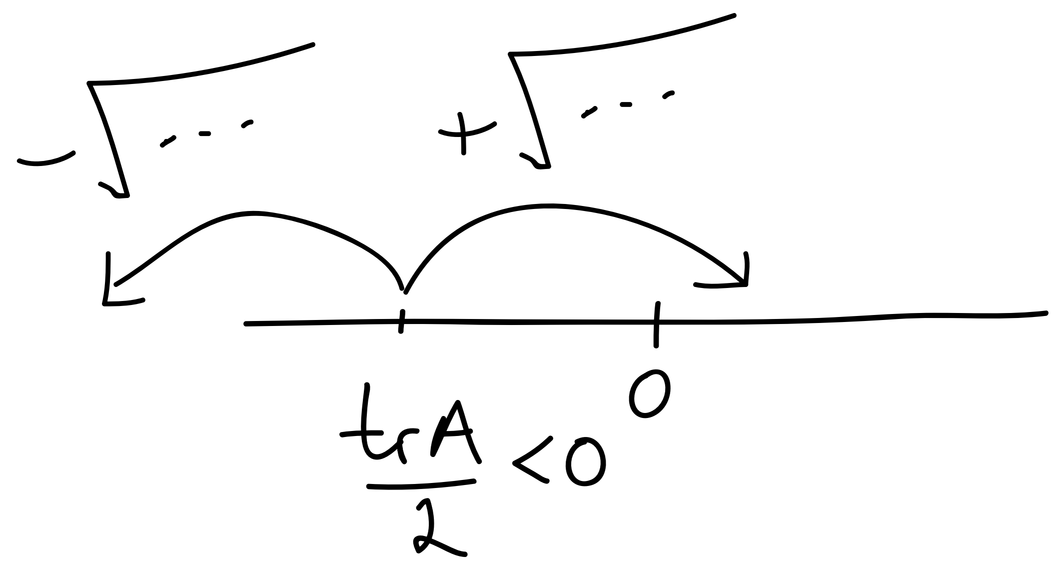

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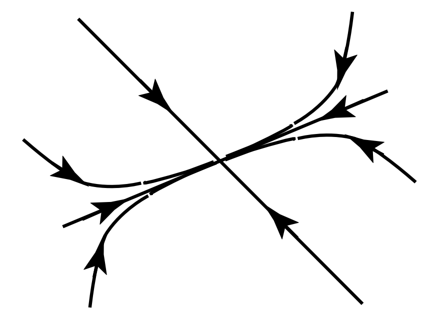

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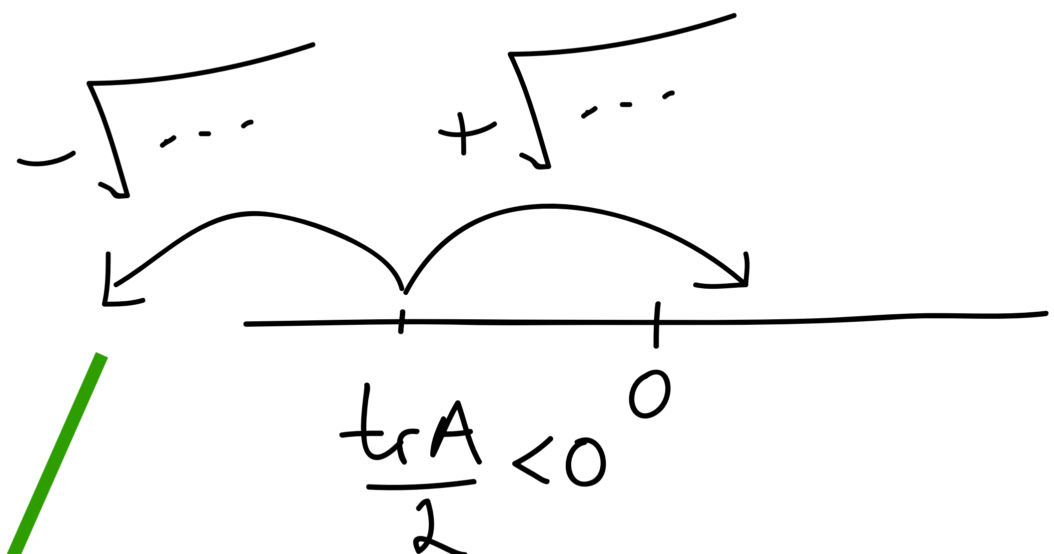

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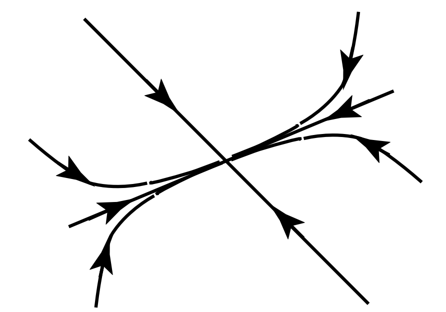

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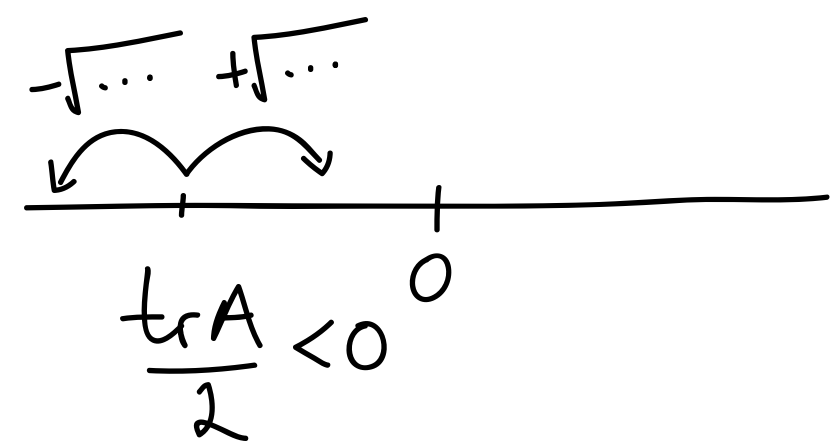

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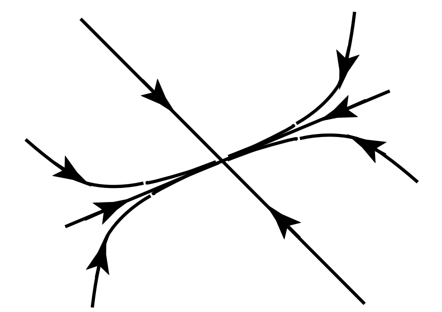

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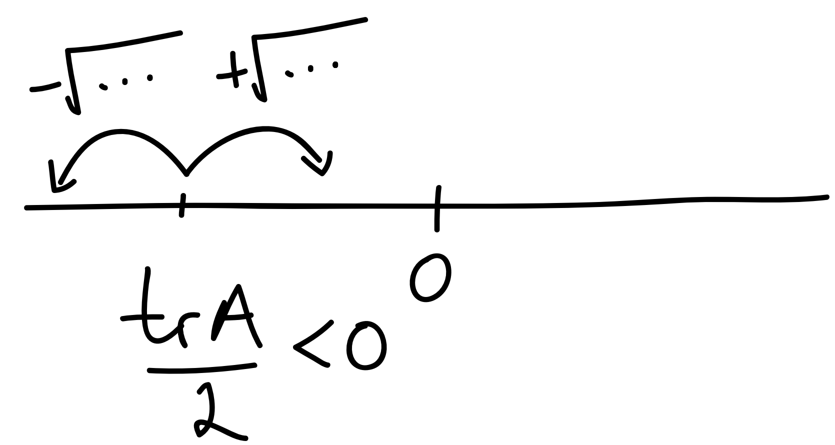

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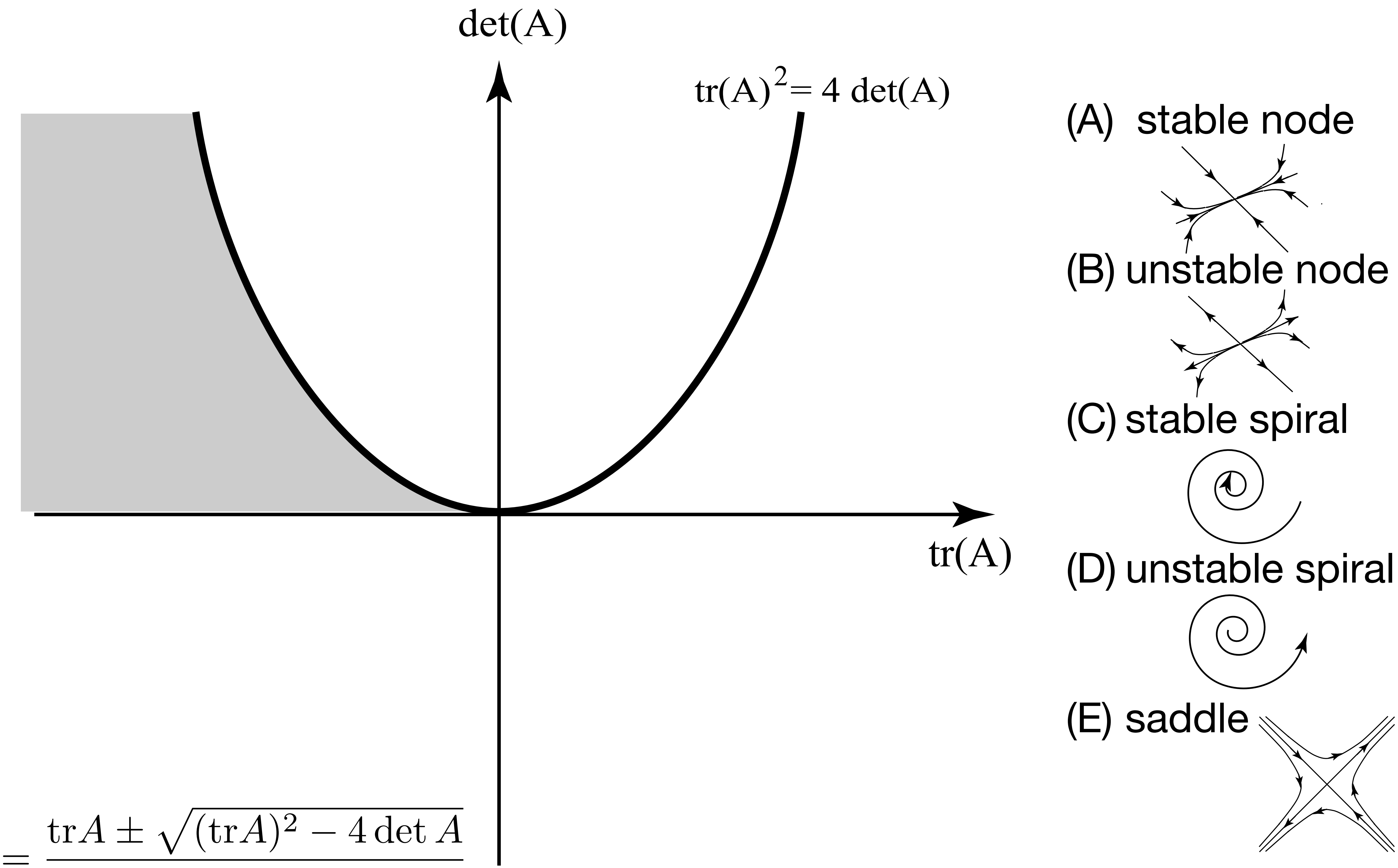

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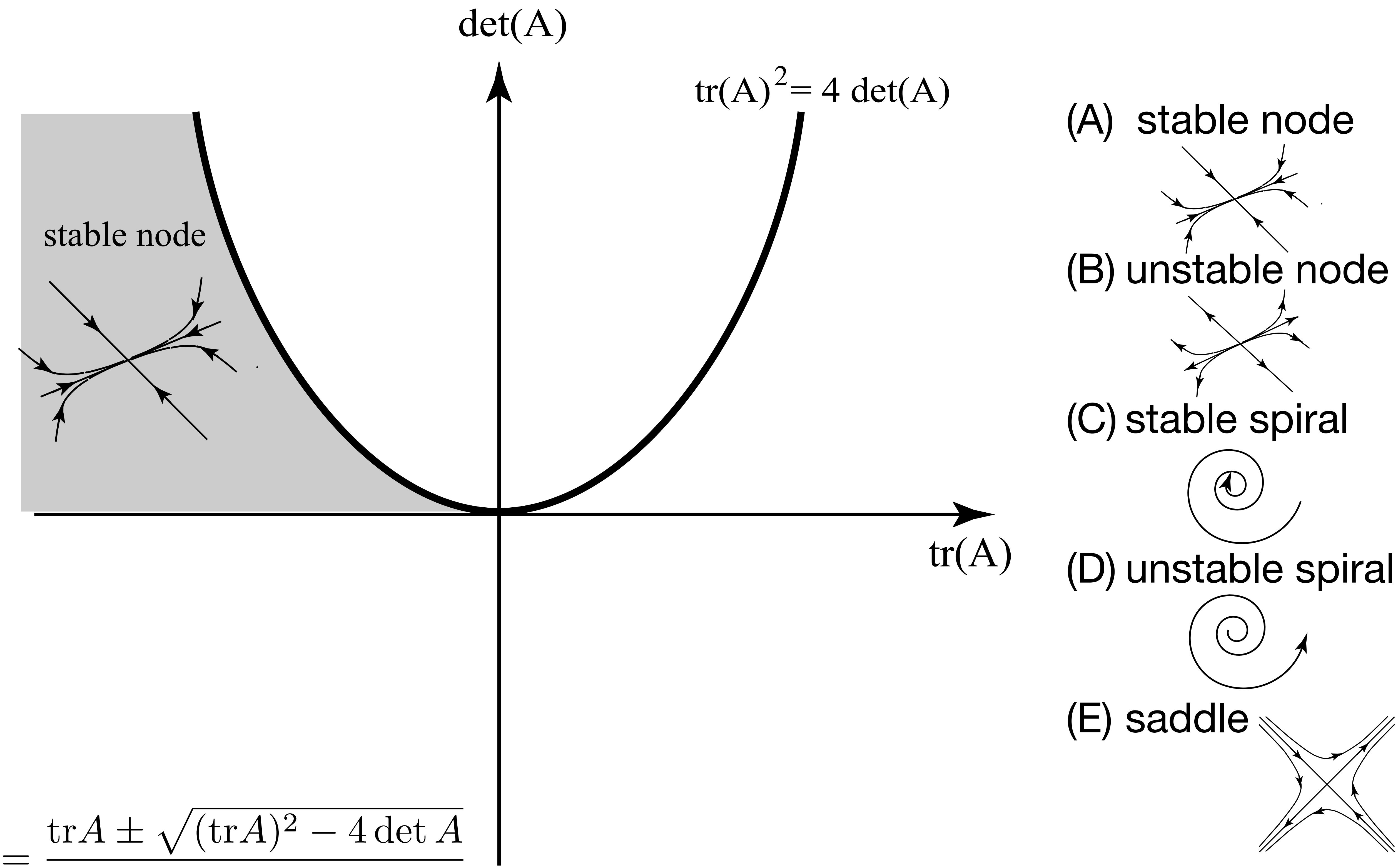

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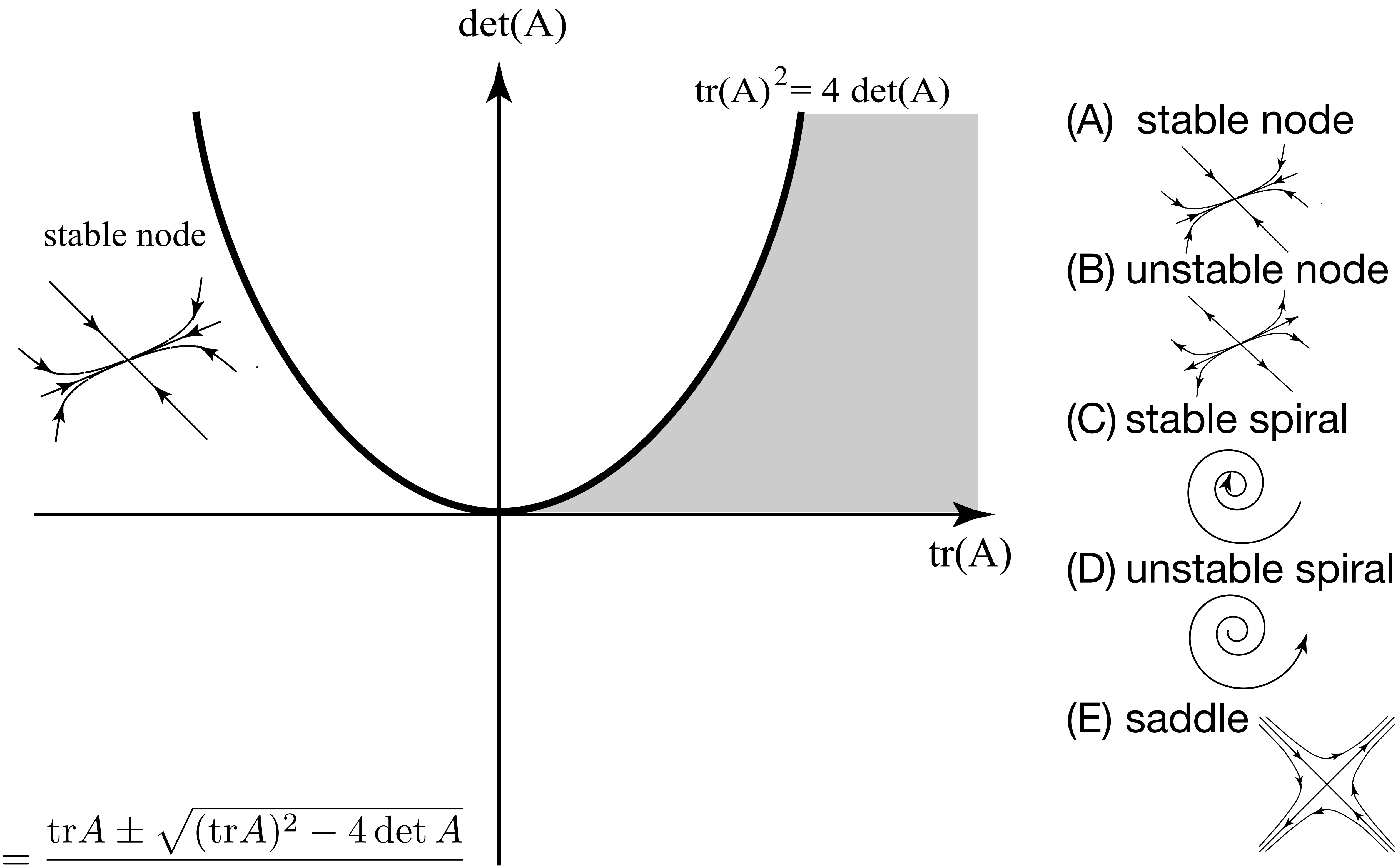

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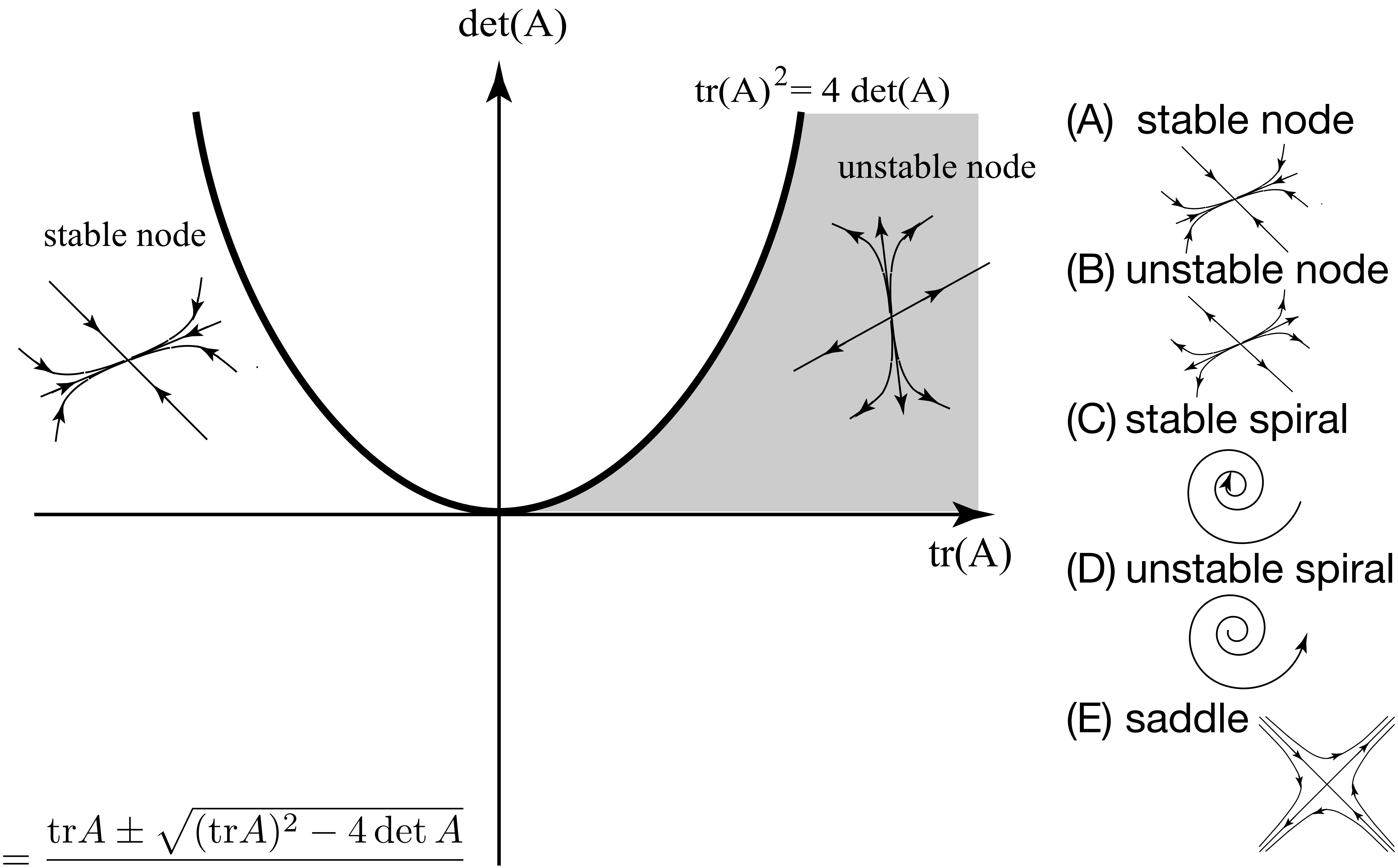

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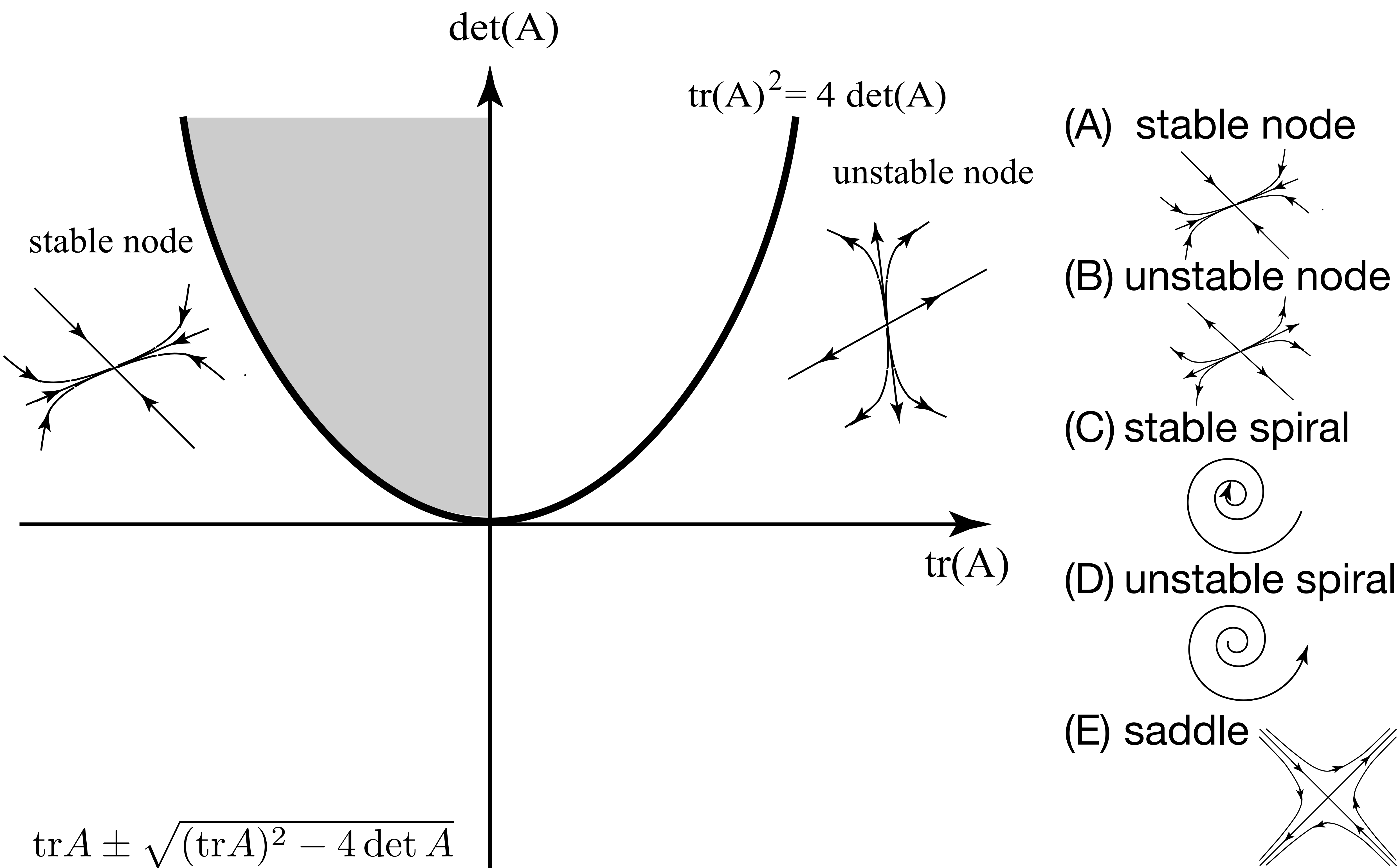

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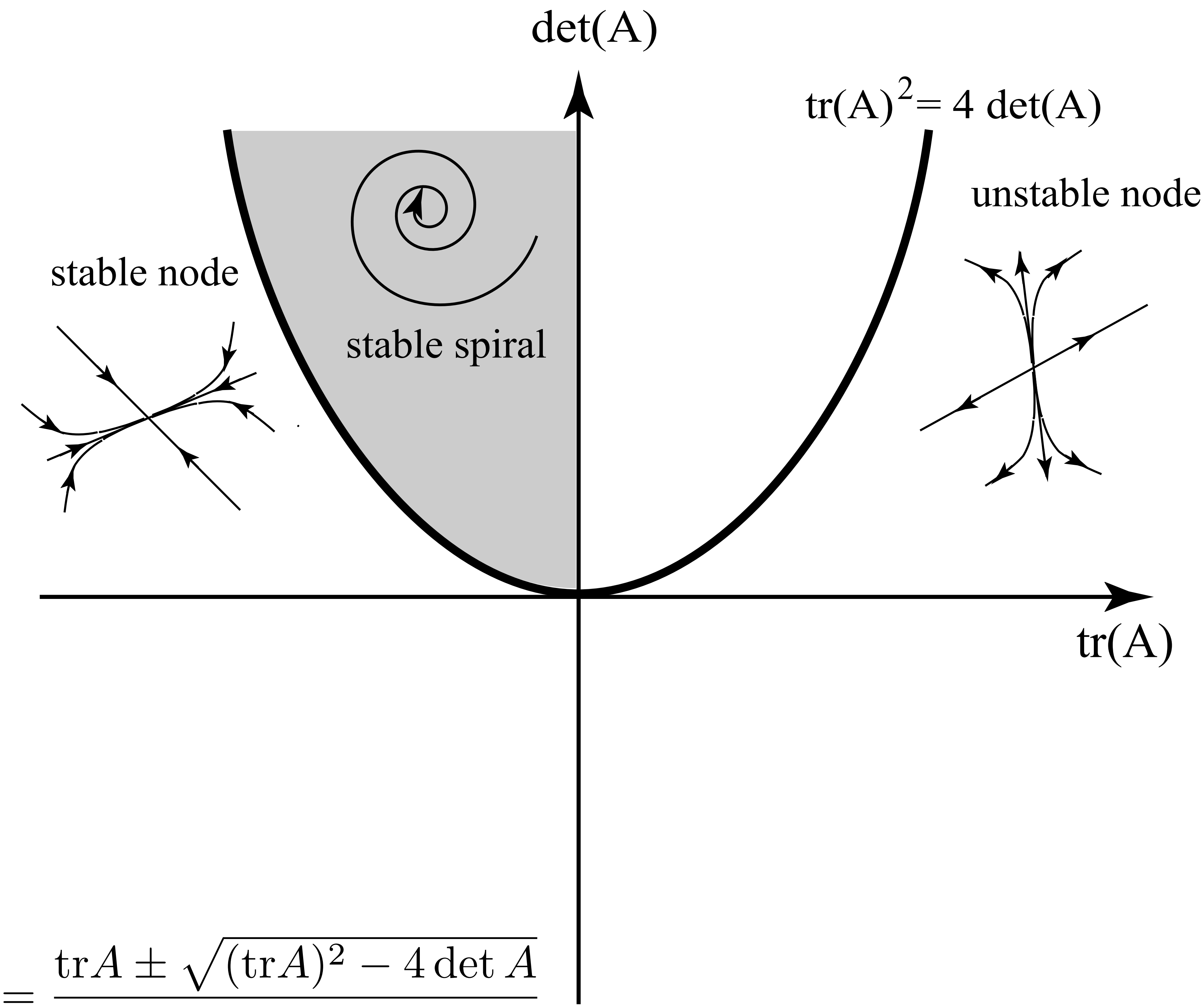
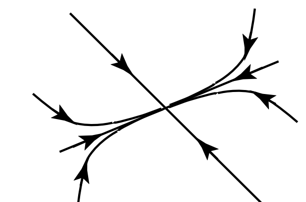
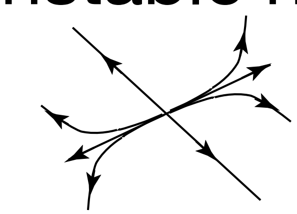


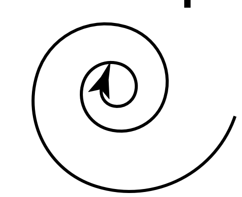
(A) stable node



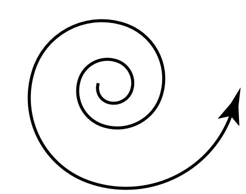
(B) unstable node



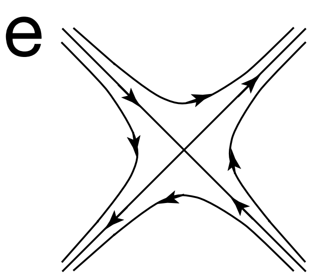
(C) stable spiral



(D) unstable spiral



(E) saddle



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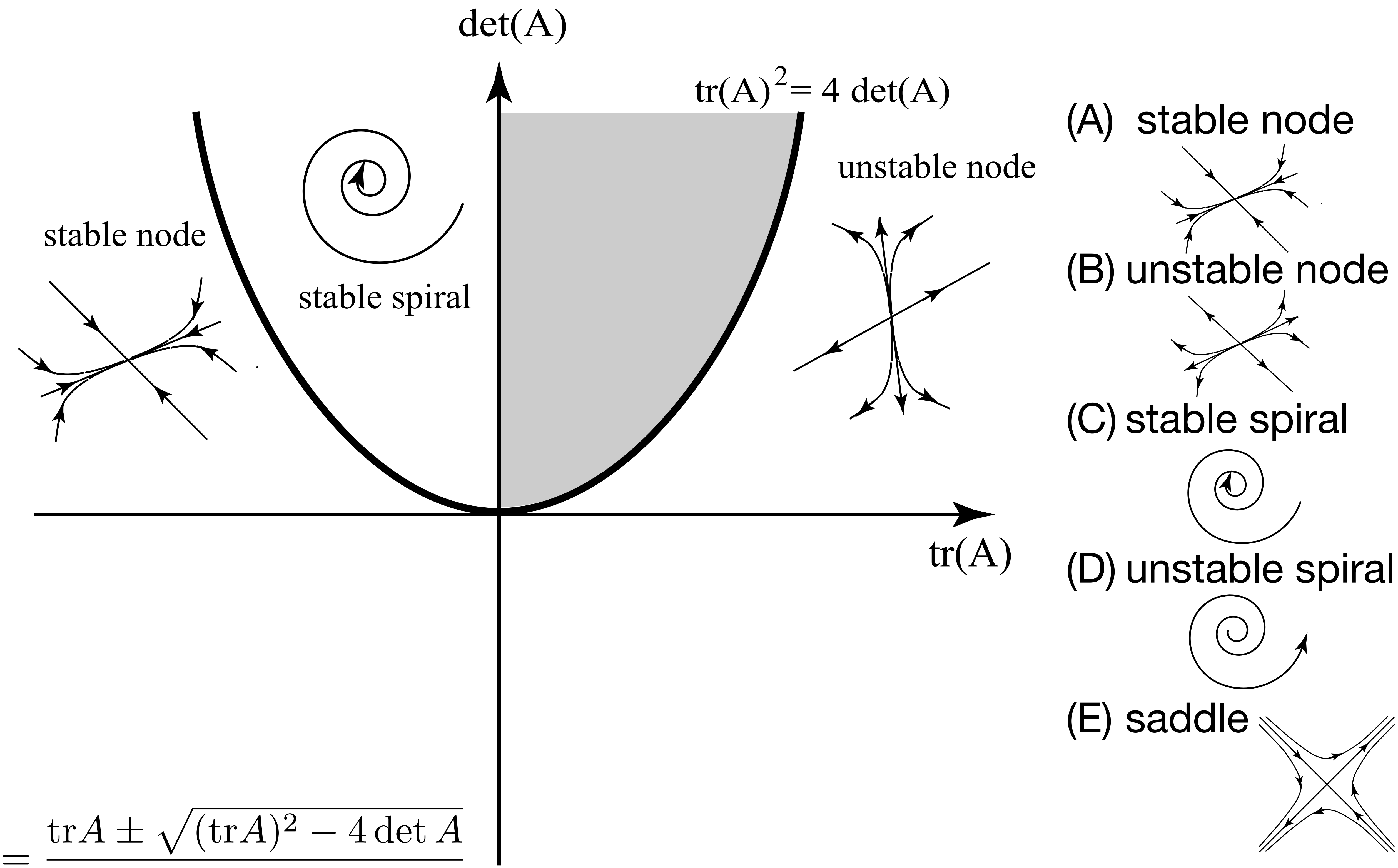
Summary - homogeneous 2x2 systems



$$\lambda = \frac{\mathrm{tr} A \pm \sqrt{(\mathrm{tr} A)^2 - 4 \det A}}{2}$$

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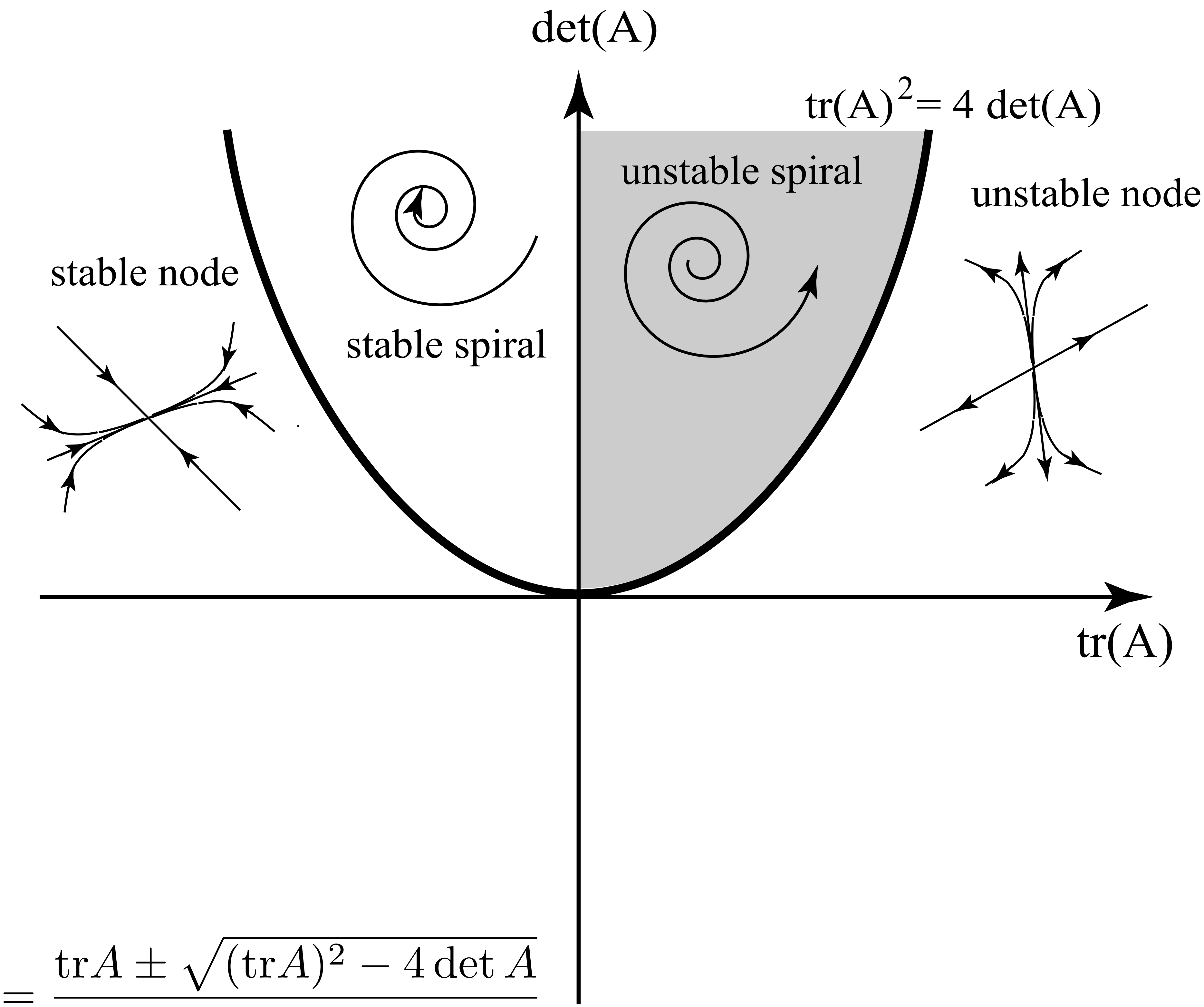
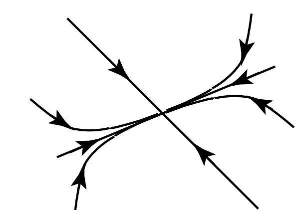
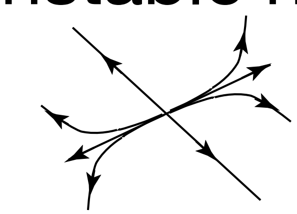


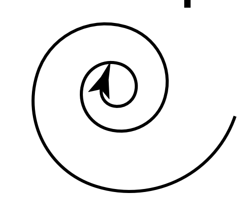
(A) stable node



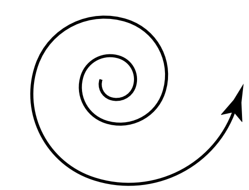
(B) unstable node



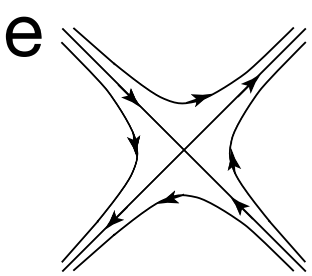
(C) stable spiral



(D) unstable spiral



(E) saddle



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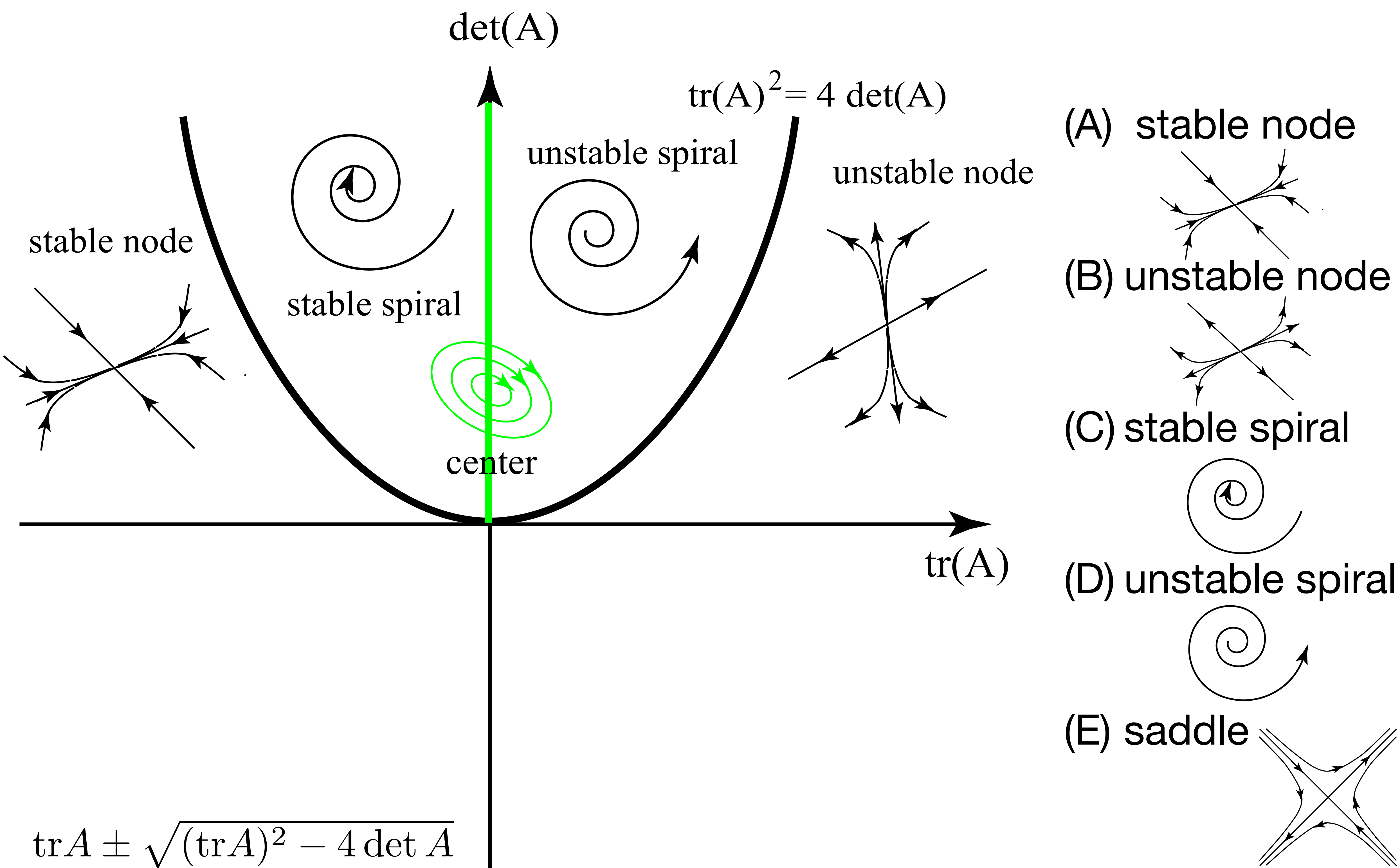

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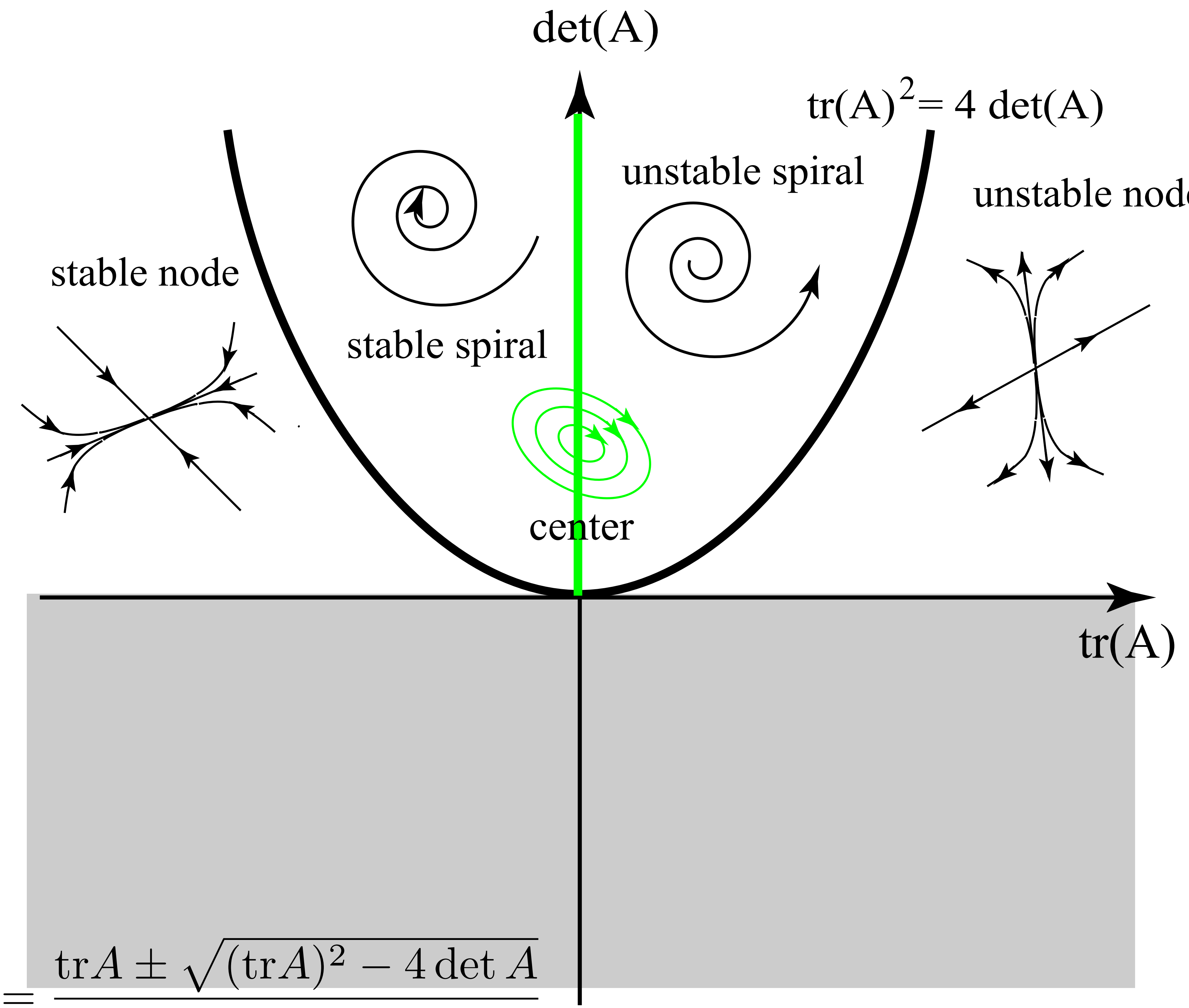

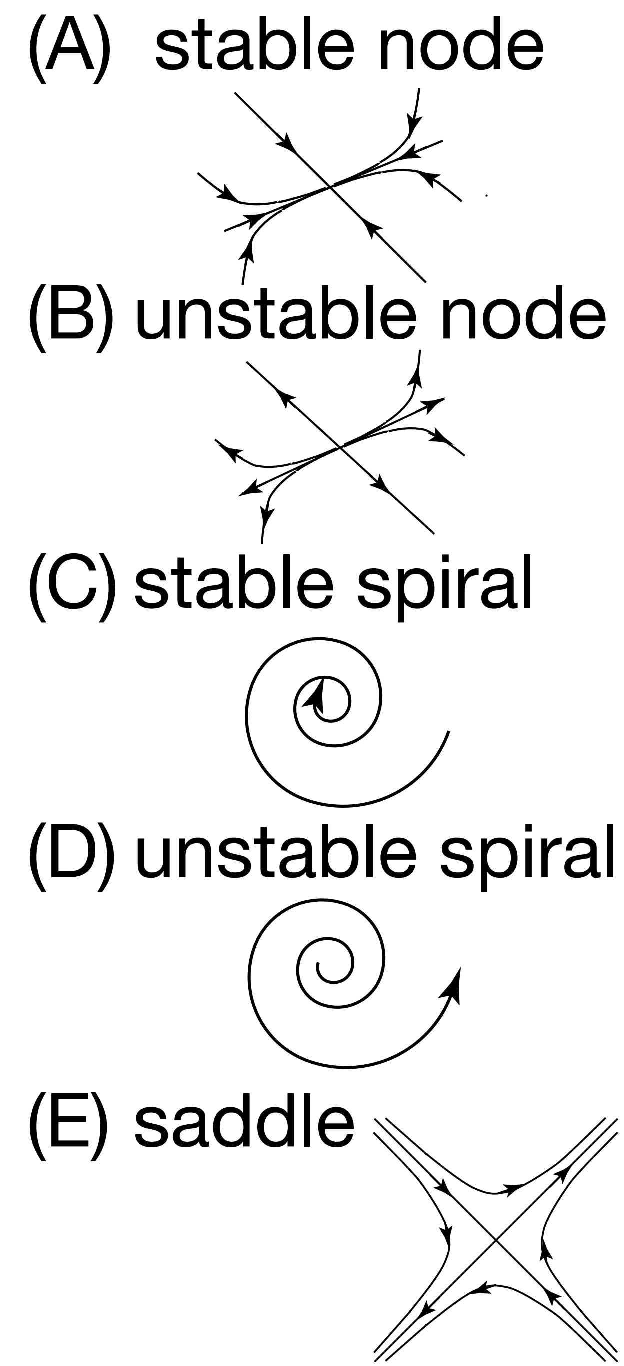

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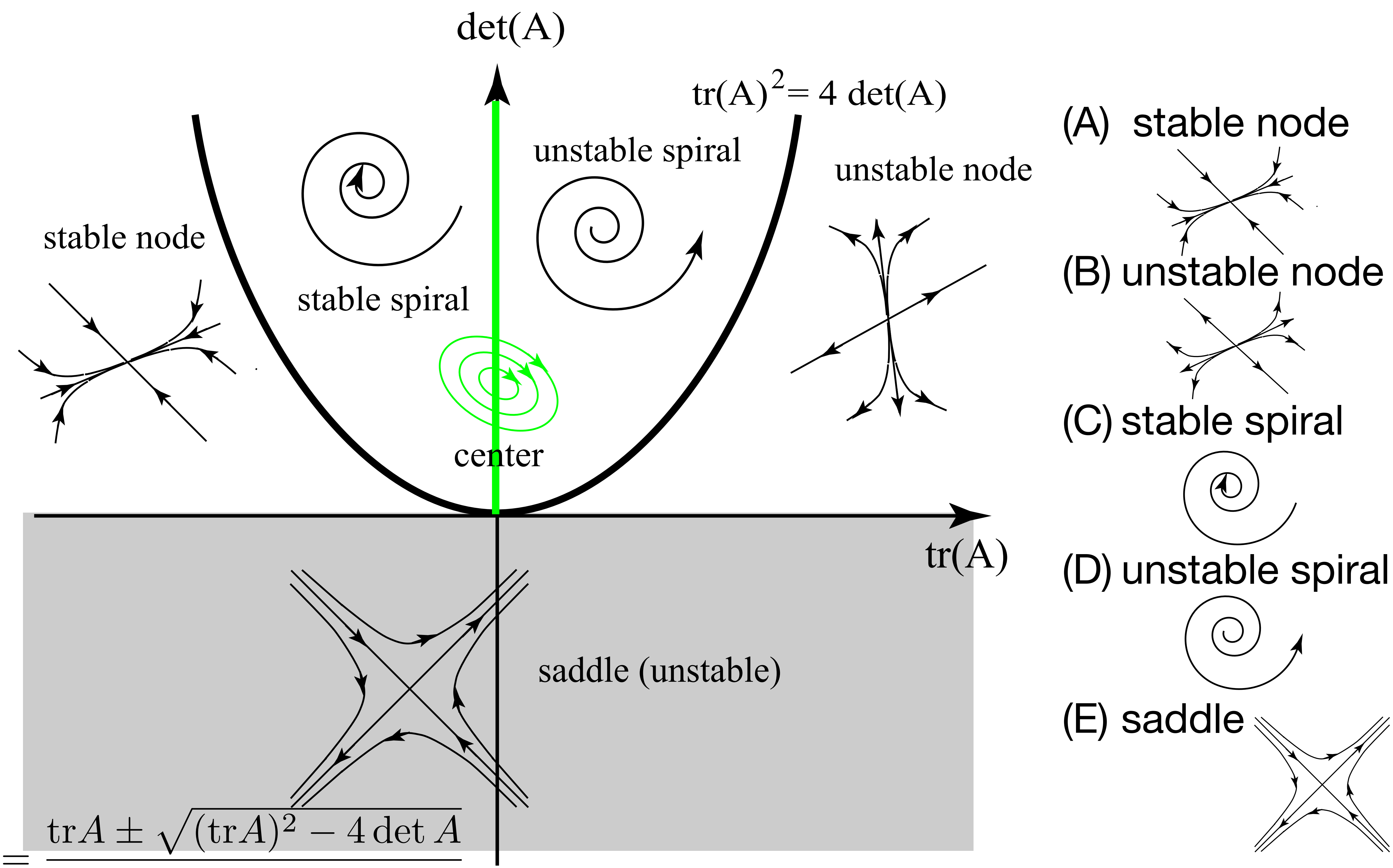

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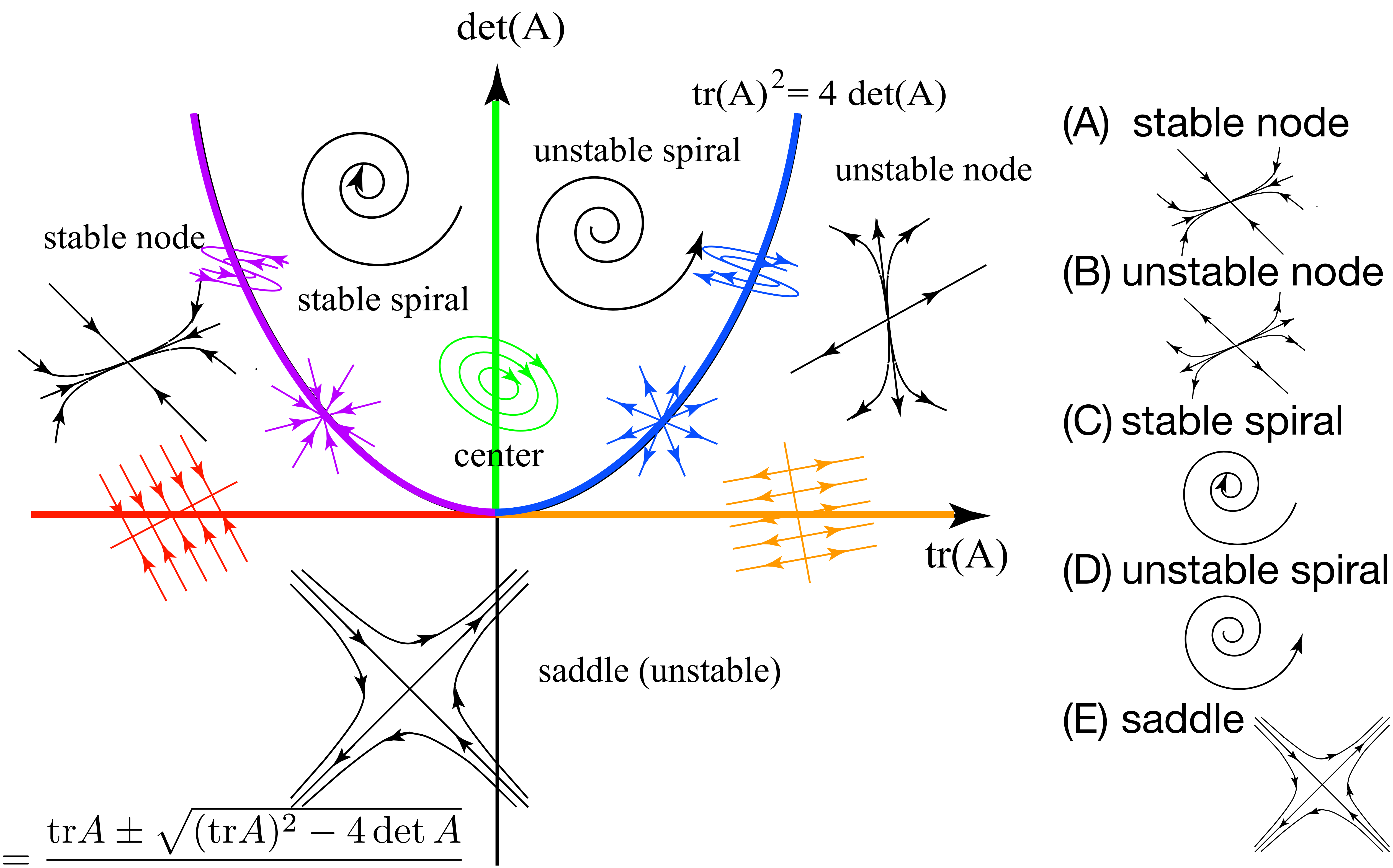
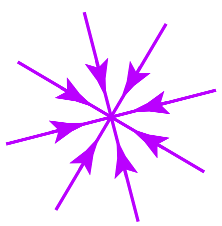


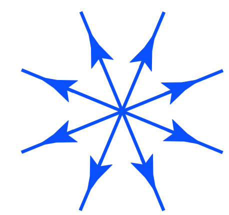
$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4 \det A}}{2}$$

Summary - homogeneous 2x2 systems

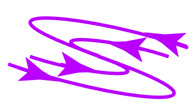
Repeated evalue cases:



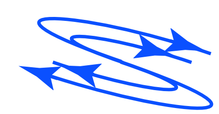
$\lambda<0$, two indep. evectors.



$\lambda>0$, two indep. evectors.

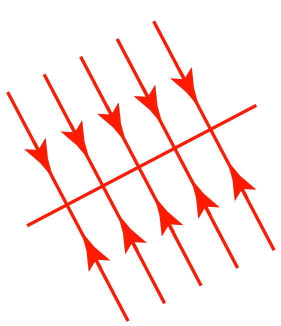


$\lambda<0$, only one evector.

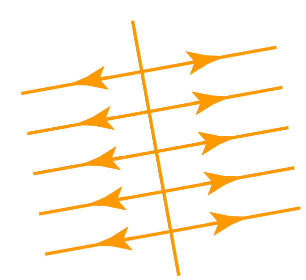


$\lambda>0$, only one evector.

One zero evalue (singular matrix):



$\lambda_1=0$, $\lambda_2<0$,



$\lambda_1=0$, $\lambda_2>0$,