Today

- Fourier series for Heat / Diffusion equation

Fourier series (Method Undetermined Coefficients)

- Back to our ODE, what do we choose for the w_n if f(t) has period T? Keep in mind that we want all the functions involved to have period T.

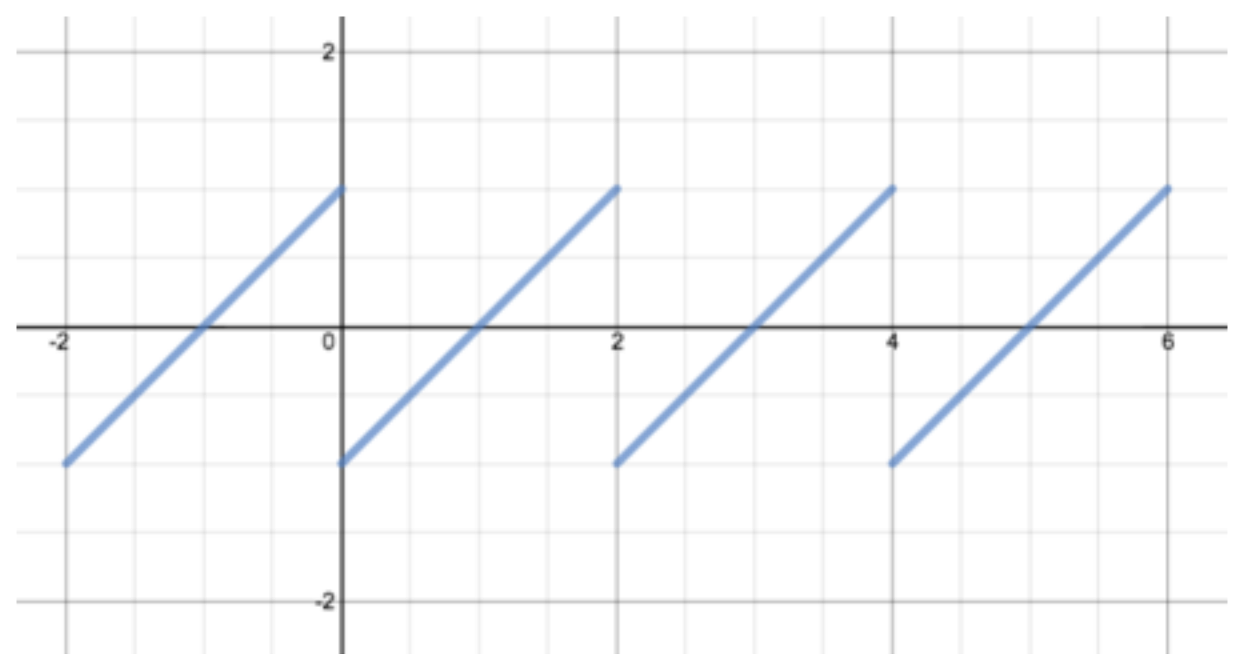
$$ay'' + by' + cy = f(t) \stackrel{?}{=} A_0 + \sum_{n=1}^{N} a_n \cos(\omega_n t) + \sum_{n=1}^{N} b_n \sin(\omega_n t)$$

(A) $w_n = \pi / T$

(B) $w_n = 2\pi / T$

(C) $w_n = n\pi / T$

(D) $w_n = 2\pi n / T$



(E) Don’t know. Explain please.

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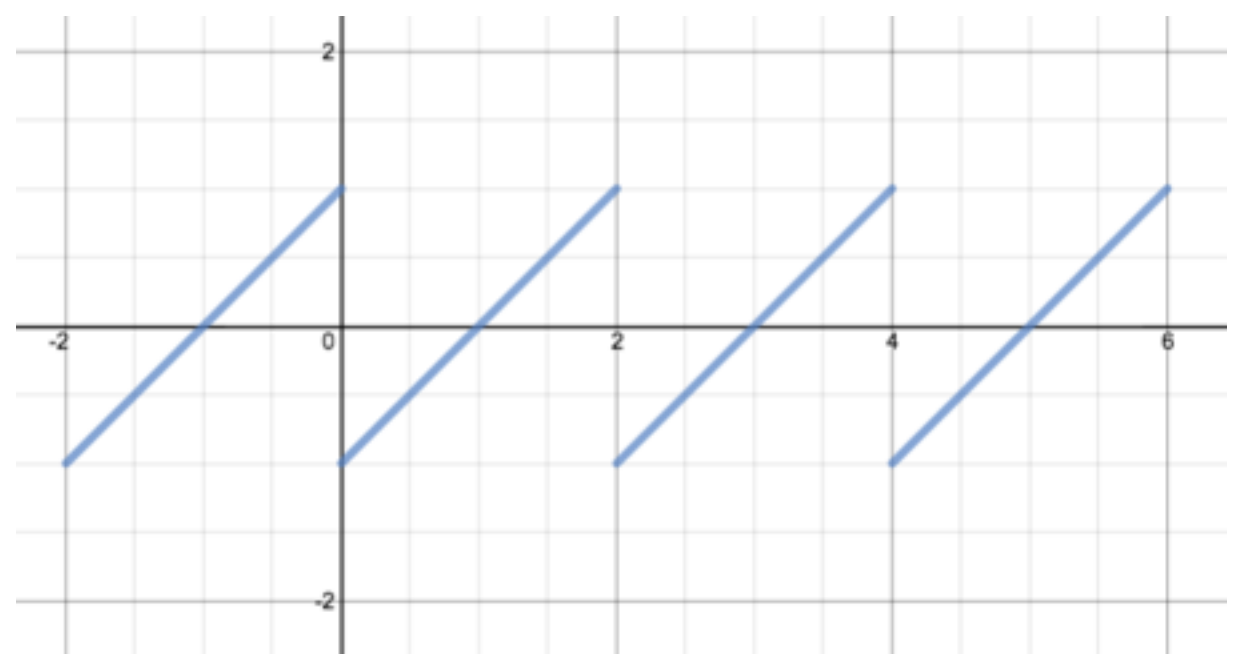

Calculate FS on doc cam.

Fourier series (Heat/Diffusion equation)

- When we talk about the Heat/Diffusion equation, we'll need to satisfy conditions at x=0 and x=L (ends of a heated rod or a pipe filled with solution):

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$$u(x) = A_0 + \sum_{n=1}^{N} a_n \cos(\omega_n x) + \sum_{n=1}^{N} b_n \sin(\omega_n x)$$

(A) $w_n = \pi / L$

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- Here, the function is not periodic on [0,L] but rather [-L,L]!!

Fourier series (Heat/Diffusion equation)

- Want to find Fourier series coefficients A_0, a_n, b_n, that make

$$u(x) \approx A_0 + \sum_{n=1}^{N} a_n \cos(\omega_n x) + \sum_{n=1}^{N} b_n \sin(\omega_n x)$$

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- This integral is zero when

(A) g is even, h is odd.

(B) g is even, h is even.

(C) g is odd, h is odd.

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Fourier series (Heat/Diffusion equation)

- Define $v_0(x) = 1$

Fourier series (Heat/Diffusion equation)

- Define $v_0(x) = 1 \quad v_n(x) = \cos\left(\frac{n\pi x}{L}\right) \quad n = (0,)1, 2, 3, \ldots$

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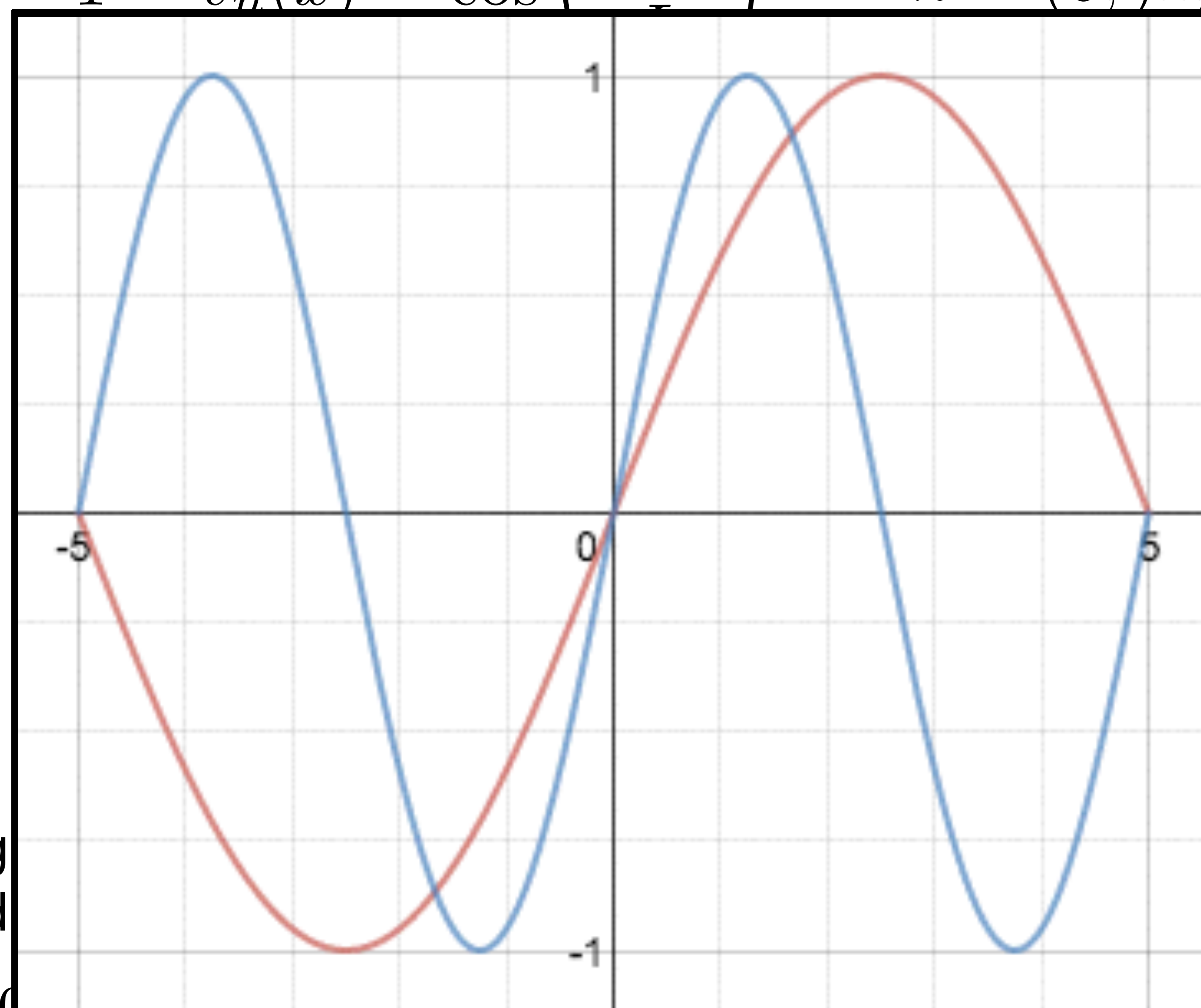

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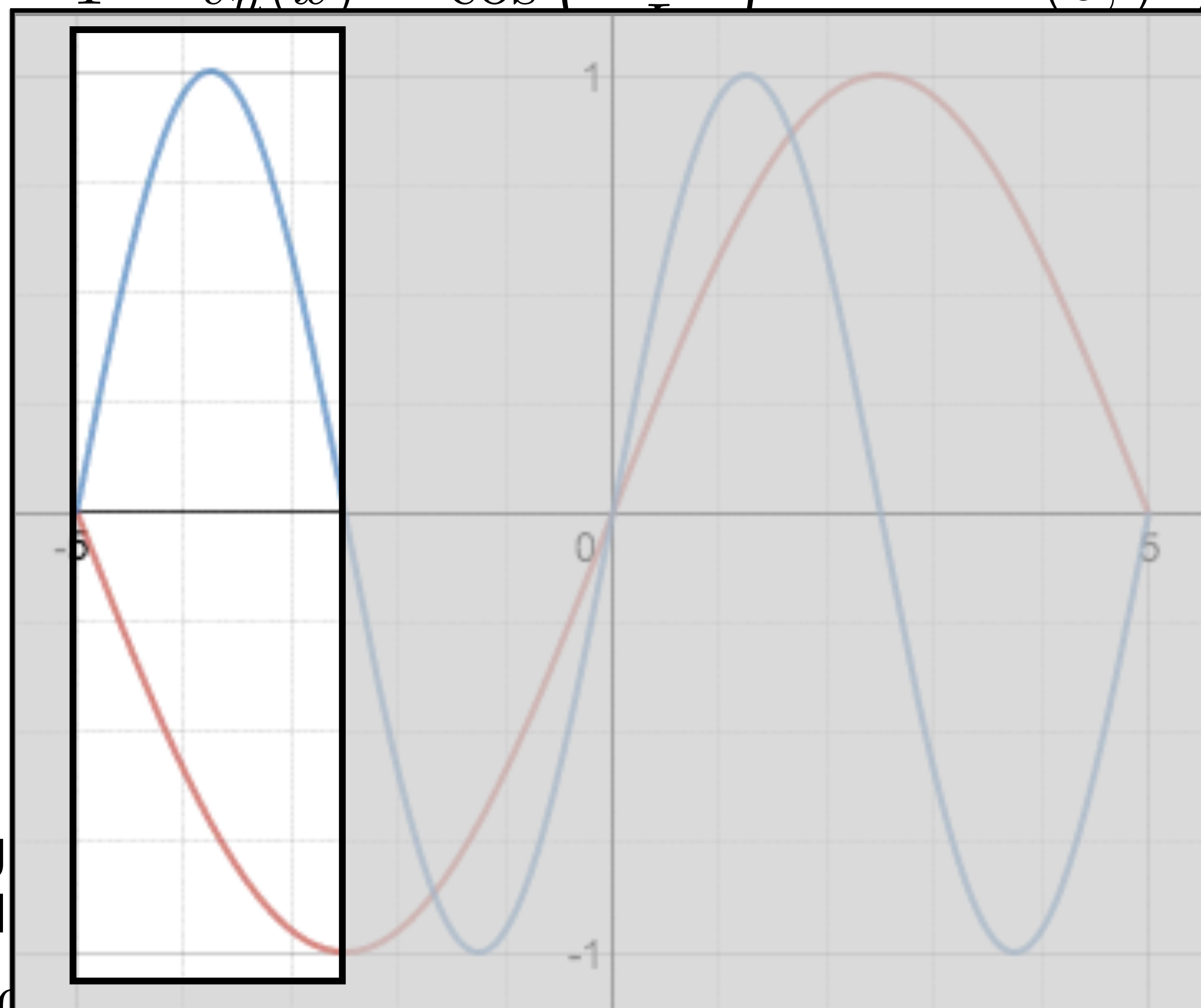

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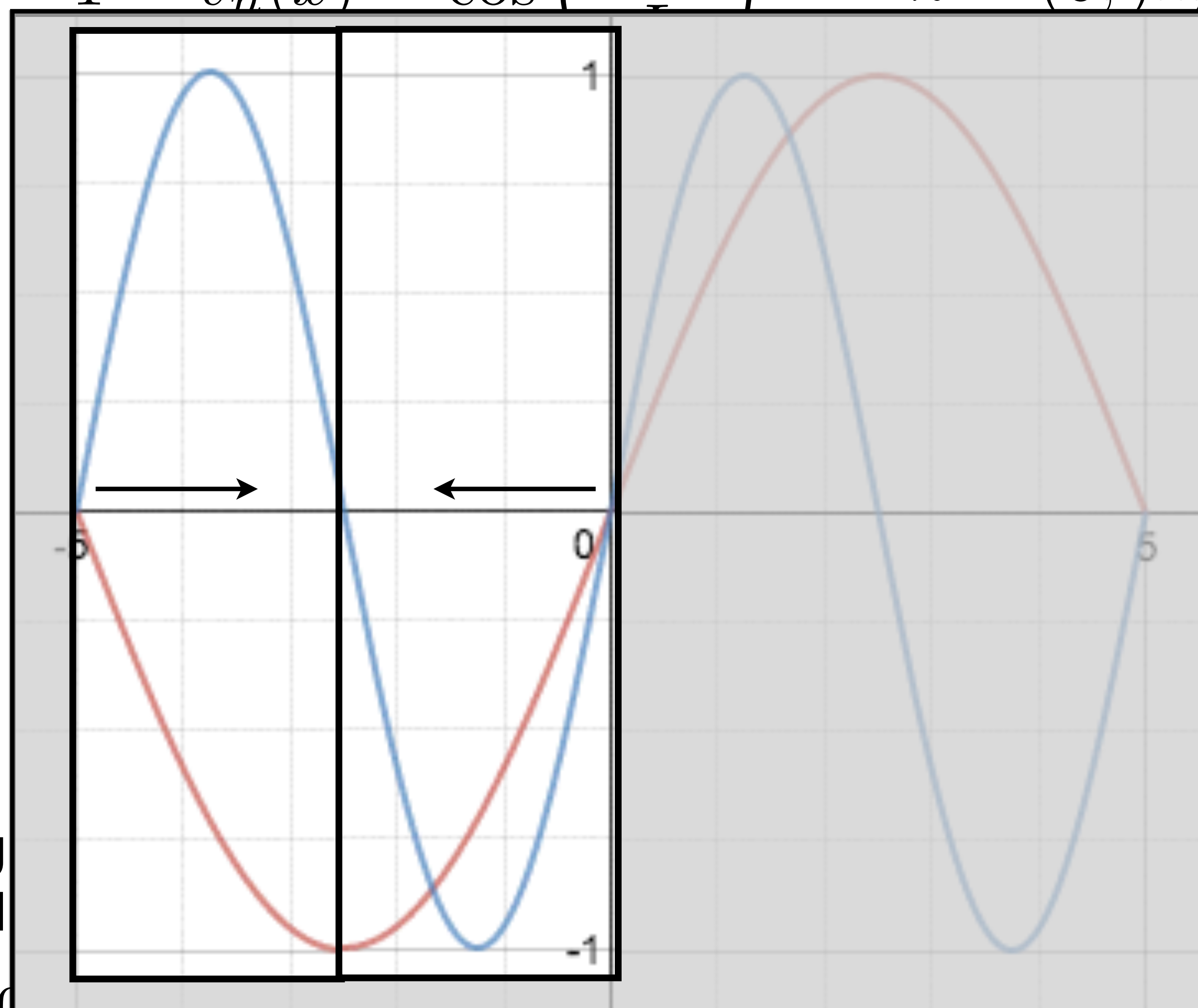

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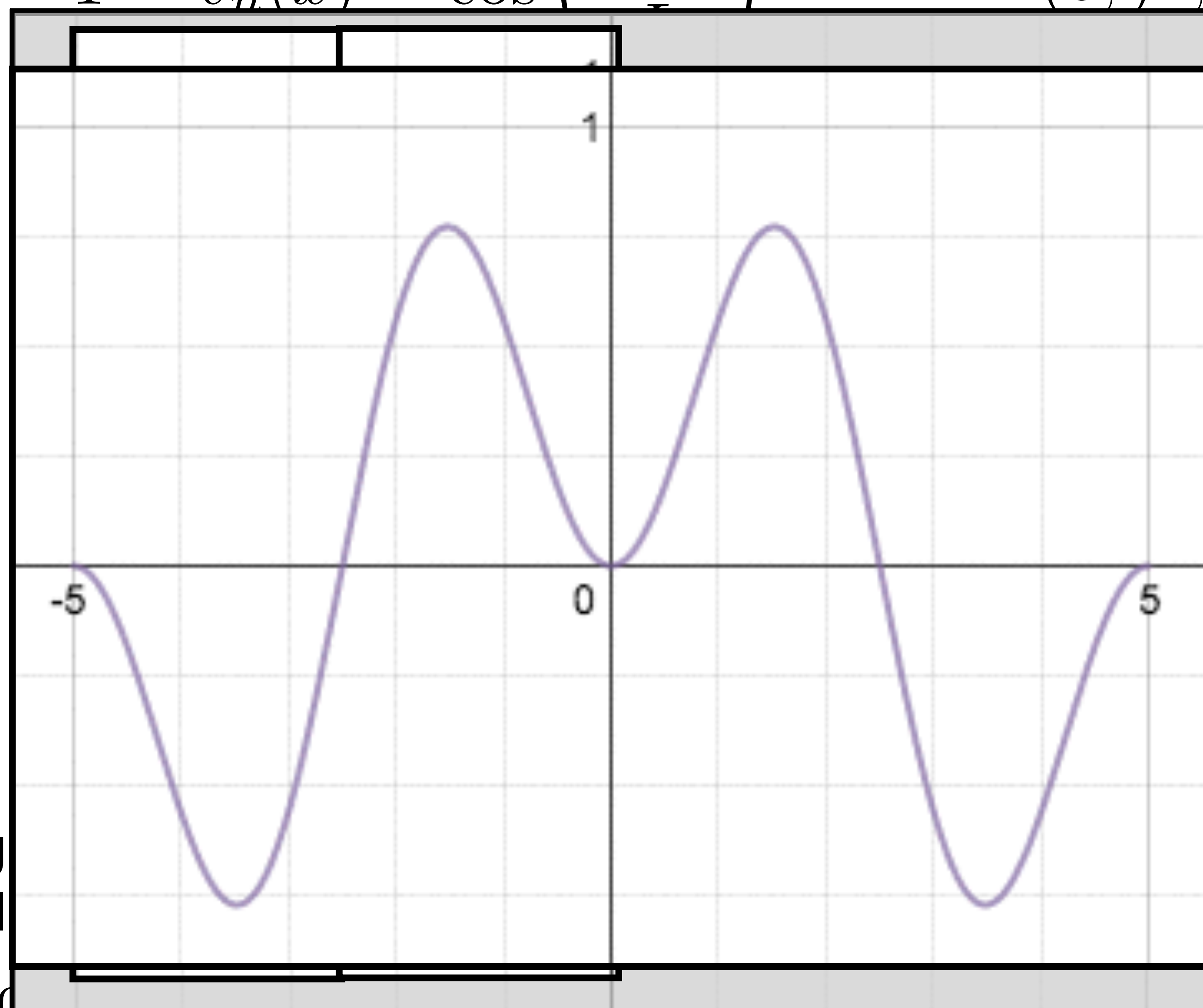

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$$v_n \circ v_n = \int_{-L}^{L} \cos^2\left(\frac{n\pi x}{L}\right)\ dx = L$$

$$g(x) \circ h(x) = \int_{-L}^{L} g(x)h(x)\ dx$$

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- Define a function $f_{FS}(x)$ on the interval [-L,L] by choosing coefficients A_0, a_n and b_n and setting

$$f_{FS}(x) = A_0 + a_1 \cos\left(\frac{\pi x}{L}\right) + a_2 \cos\left(\frac{2\pi x}{L}\right) + \cdots$$
$$+ b_1 \sin\left(\frac{\pi x}{L}\right) + b_2 \sin\left(\frac{2\pi x}{L}\right) + \cdots$$

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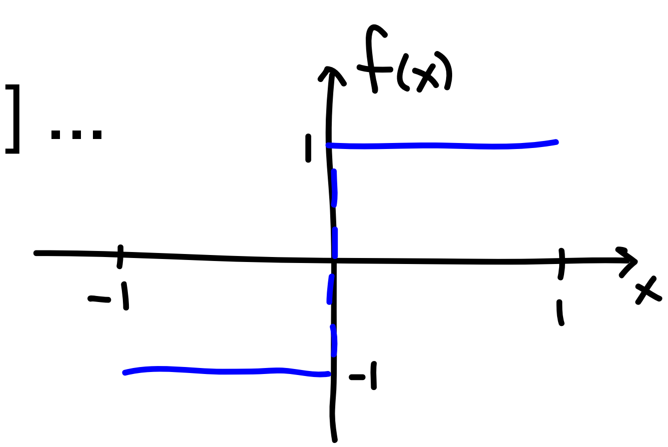
- Let's check for $f(x) = 2u_0(x)-1$ on the interval [-1,1] ...



Fourier series

- Calculate the coefficients of the Fourier series of a function:

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$$v_n(x) = \cos\left(\frac{n\pi x}{L}\right)$$
$$w_n(x) = \sin\left(\frac{n\pi x}{L}\right)$$

$$f_{FS}(x) = \frac{a_0}{2} v_0(x) + a_1 v_1(x) + a_2 v_2(x) + \cdots$$
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$$f_{FS}(x) \circ v_n(x) =$$

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$$f_{FS}(x) \circ v_n(x) = \frac{a_0}{2} \cancelto{0}{v_0(x) \circ v_n(x)} + a_1 \cancelto{0}{v_1(x) \circ v_n(x)} + a_2 \cancelto{0}{v_2(x) \circ v_n(x)} + \cdots$$
$$+ b_1 \cancelto{0}{w_1(x) \circ v_n(x)} + b_2 \cancelto{0}{w_2(x) \circ v_n(x)} + \cdots$$

$$= a_n v_n(x) \circ v_n(x) = a_n L$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f_{FS}(x) \cos\left(\frac{n\pi x}{L}\right)\, dx$$

Fourier series (Heat/Diffusion equation)

- Find the Fourier series for f(x) = $2u_0(x)-1$ on the interval [-1,1].

$$f_{FS}(x) = A_0 + a_1 \cos\left(\frac{\pi x}{L}\right) + a_2 \cos\left(\frac{2\pi x}{L}\right) + \cdots$$
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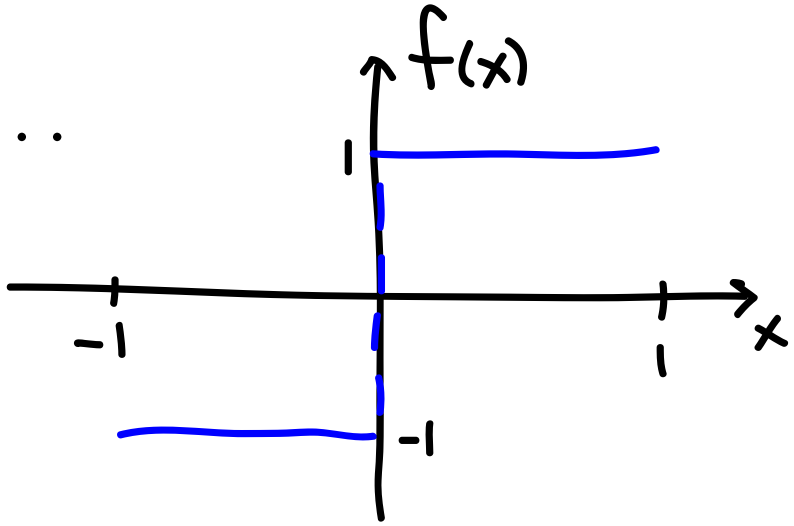

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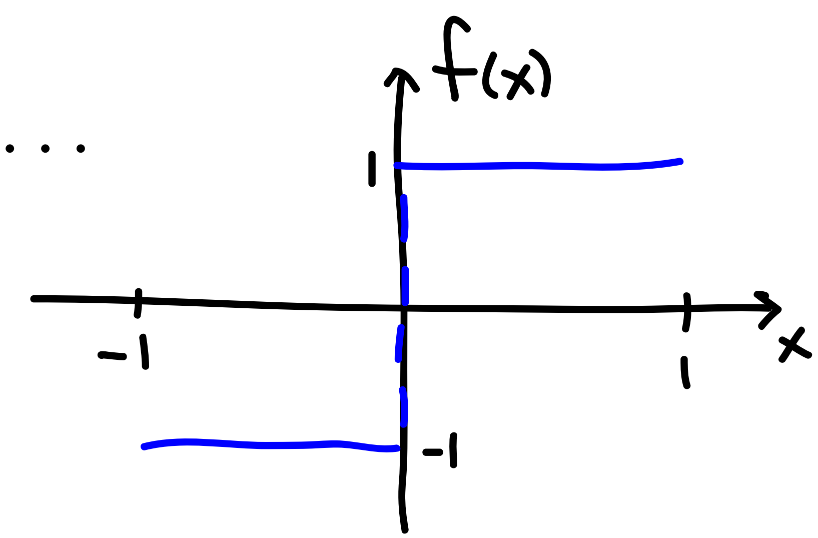

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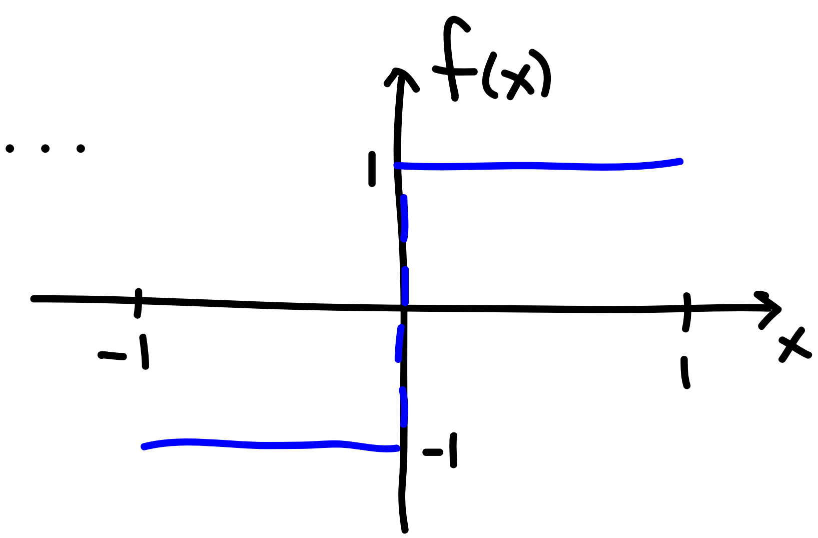

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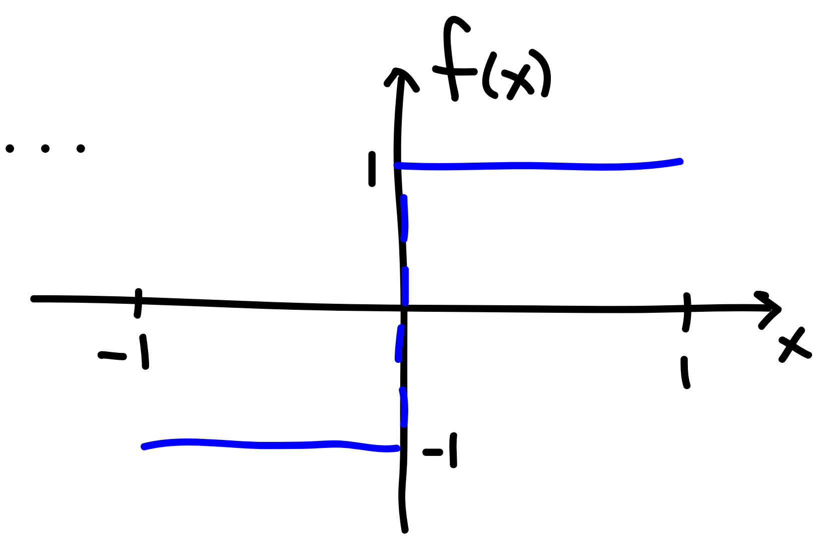

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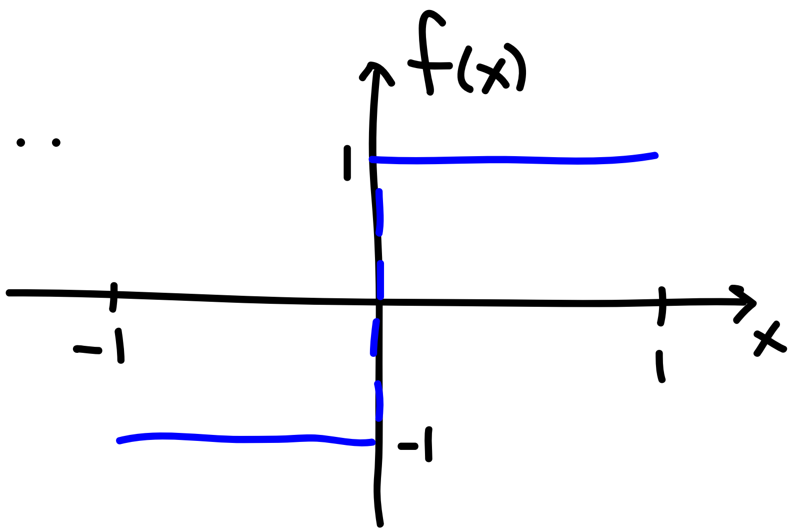

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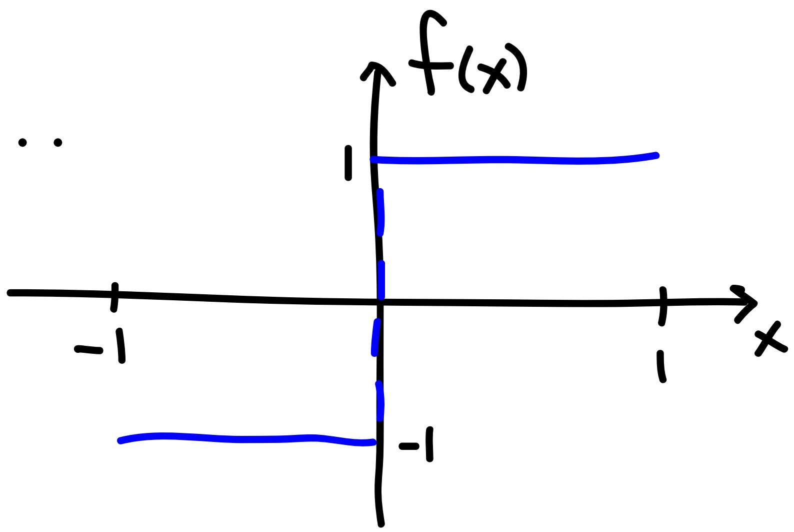

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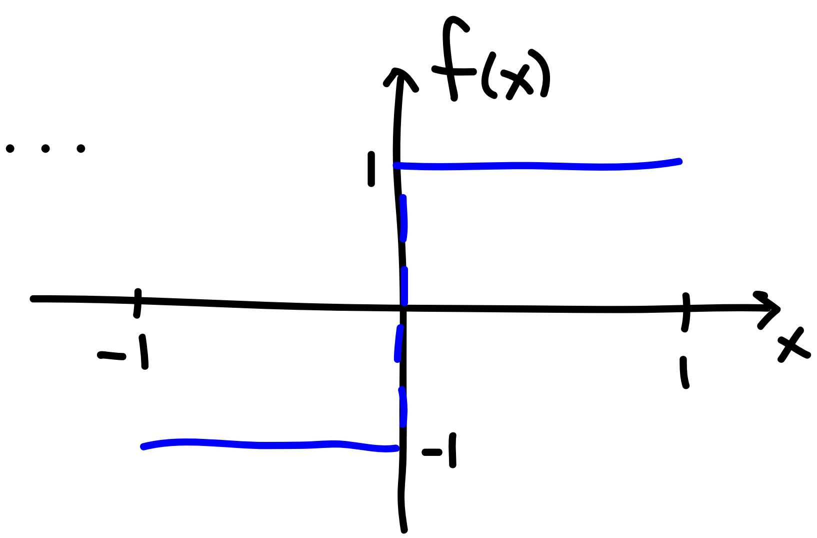

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A_0 is the average value of f(x)!

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