

MATH 256-201 – Midterm 1 – January 31, 2017.

Surname: _____

Given name: _____

Tutorial TA (circle one): Thomas Jummy Sarai Colin Xiaowei Dhananjay

This midterm has 6 pages including a blank page at the end for rough work. Answers must be justified and work must be shown. If a box is provided, place your answer in it.

1. [5 pts] Classify each of the following equations as linear (L) or non-linear (NL). Give the order of the equation. For any linear equation, state whether it is homogeneous (H) or non-homogeneous (NH); put a “–” for non-linear equations. **For any non-linear equation, circle all terms that render it non-linear.**

Equation	L/NL	Order	H/NH
$t^2y'' + 4ty' + 2y = 0$			
$y' + 3y = \cos(2t)$			
$y'' + xy' + y = \frac{ye^x}{x^2 + 1}$			

2. [5 pts] A tank initially contains m_0 kg of salt and a volume V litres of water. Saltwater with a concentration of c_0 kg/litre enters a tank at the rate r litres/minute. The solution is mixed and drains from the tank at the same rate (r litres/minute). Write down an Initial Value Problem (that is, **a differential equation and an initial condition**) for the mass of salt $m(t)$ in the tank as a function of time. You DO NOT need to solve it.

Do not write in these boxes - for marking purposes only.

1:

2:

3. Consider the equation

$$\frac{dy}{dx} = \frac{\sqrt{x}}{y}.$$

(a) [4 pts] Find the general solution to the equation.

General solution:

(b) [2 pts] What is the particular solution that solves the initial condition $y(1) = -\frac{2}{\sqrt{3}}$?

$y(x) =$

4. [4 pts] Find the general solution to the equation $y'' + 4y = 0$.

$y_h(x) =$

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3:

4:

5. [6 pts] For each proposed $f(t)$, give the form of the particular solution that you would use to carry out the Method of Undetermined Coefficients to solve the equation $y'' + 4y = f(t)$.

$f(t)$	$y_p(t)$
t^2	
$e^t \sin(2t)$	
$\cos(2t)$	

6. [6 pts] Find the particular solution that solves the equation $y'' + 4y = 5te^t$.

$y_p(x) =$

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5:

6:

7. For each of the following pairs of functions, show that they are either dependent or independent. Note that a Wronskian of zero does not ensure dependence - you have to show that a non-trivial linear combination of the functions adds to zero.

(a) **[3 pts]** $f(t) = e^t$ and $g(t) = te^t$.

(b) **[3 pts]** $f(x) = \ln(x^2)$ and $g(x) = \ln(x^3)$.

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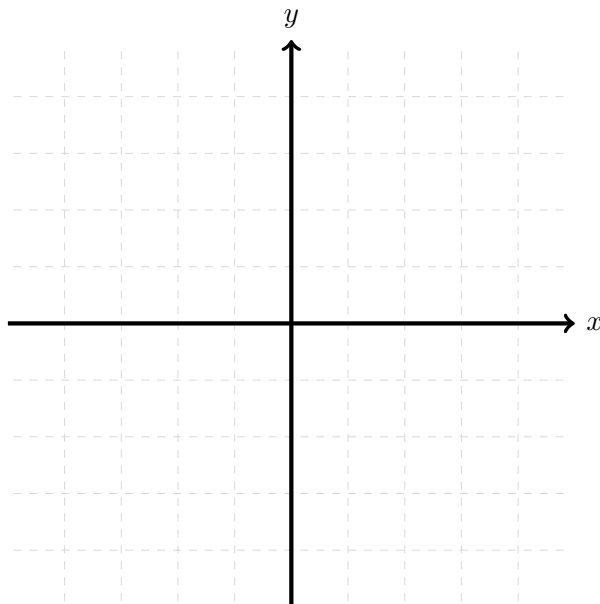
7:

8. (a) [4 pts] Find the general solution to the equation

$$y' - \frac{1}{x}y = \frac{1}{x}.$$

$$y(x) = \boxed{}$$

- (b) [**2 pts**] Sketch integral curves of the equation (i.e. solutions) for a few characteristic values of the arbitrary constant.



- (c) [1 pts] Give an example of an initial condition that cannot be solved.

Unsolvable IC: $y \left(\begin{bmatrix} \end{bmatrix} \right) = \begin{bmatrix} \end{bmatrix}$

Do not write in these boxes - for marking purposes only.

8:

Work on this page will not be marked unless there is a note on a previous page indicating that this page should be checked.