Today

- Lots of midterm review questions.

- Reminders:
  - Midterm 2 is on Thursday.
  - Assignment 8 due Thursday evening.
  - Midterm 2 will be returned in tutorial on Monday. No quiz. The TA can go over solutions if there is time but they will not answer questions about marking (other than addition errors). Talk to me.

Review problems

- A 1500 kg car hits a narrow speed bump. The suspension has a damping coefficient of 7500 kg/s and a spring constant of 9000 kg/s^2. The vertical position of the car satisfies an ODE. Write down this ODE. (There is one parameter that must be left unknown.)

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- Midterm/exam practice: solve this equation using Laplace transforms.

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- Midterm/exam practice: solve this equation using Laplace transforms.

- To think about: What happens to the response as the damping coefficient goes to infinite? (i.e. a very stiff spring)

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- Which of the following equations matches the given direction field?

(A) $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

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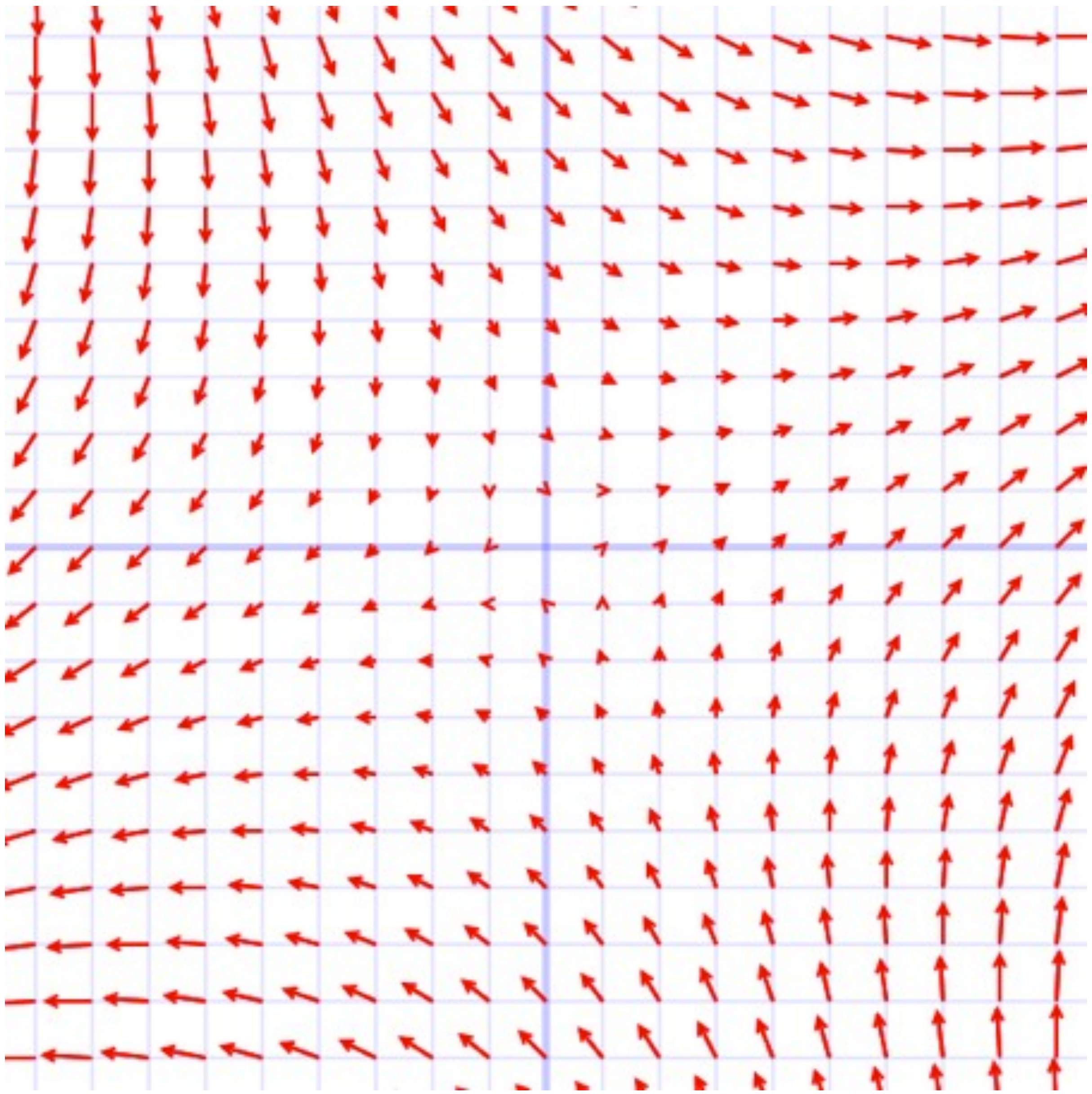
(E) Explain, please.



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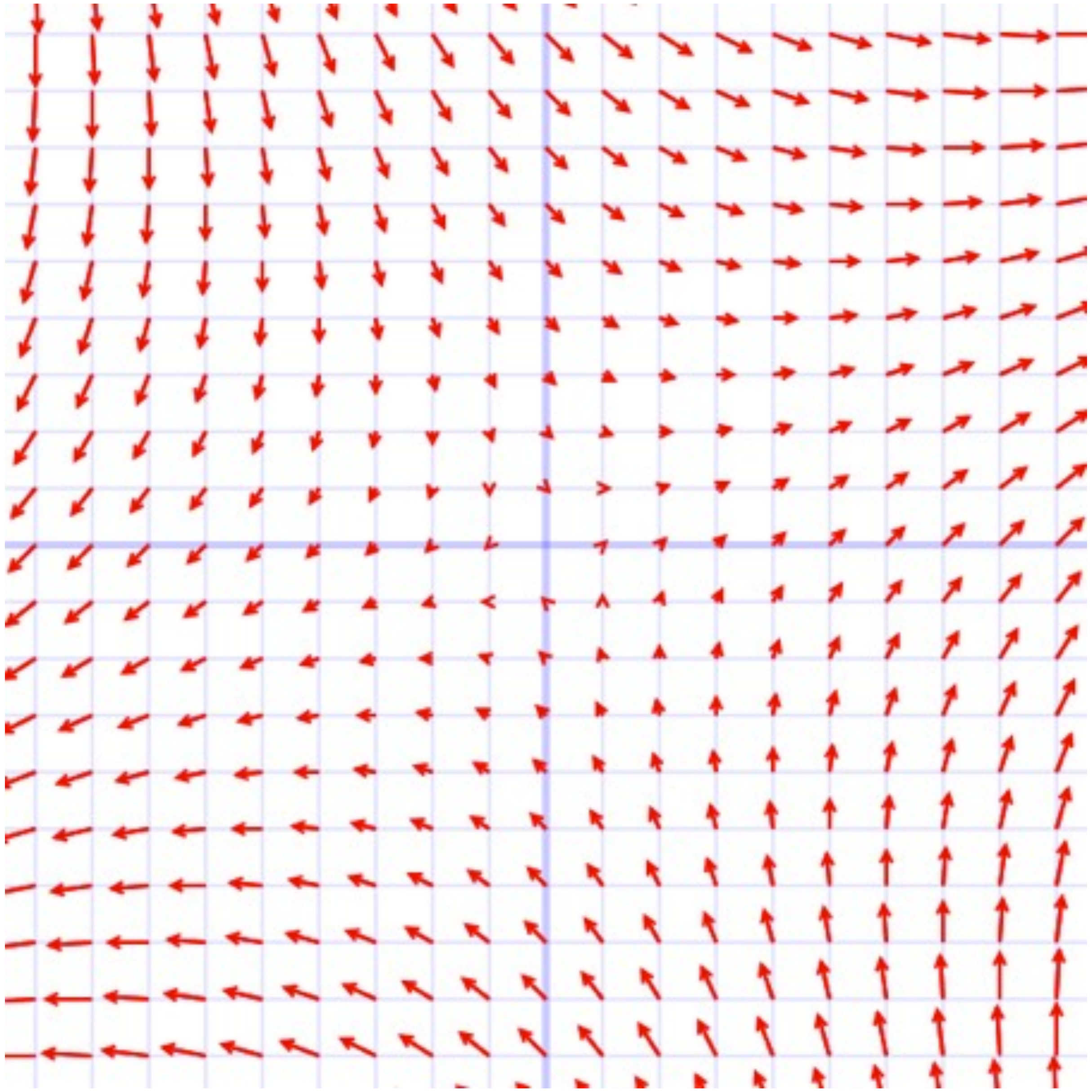

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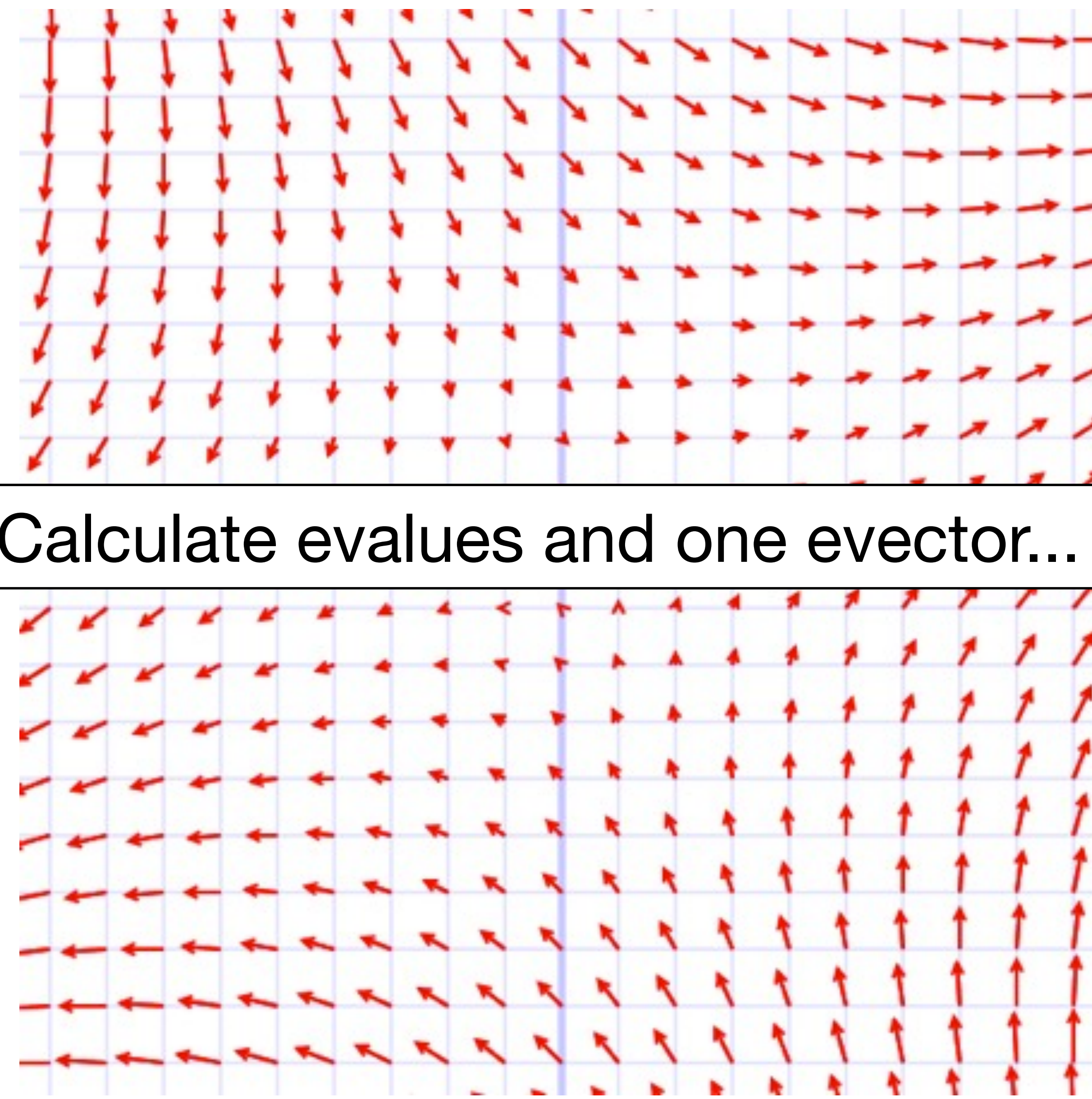

- Calculate evalues and one evector... OR

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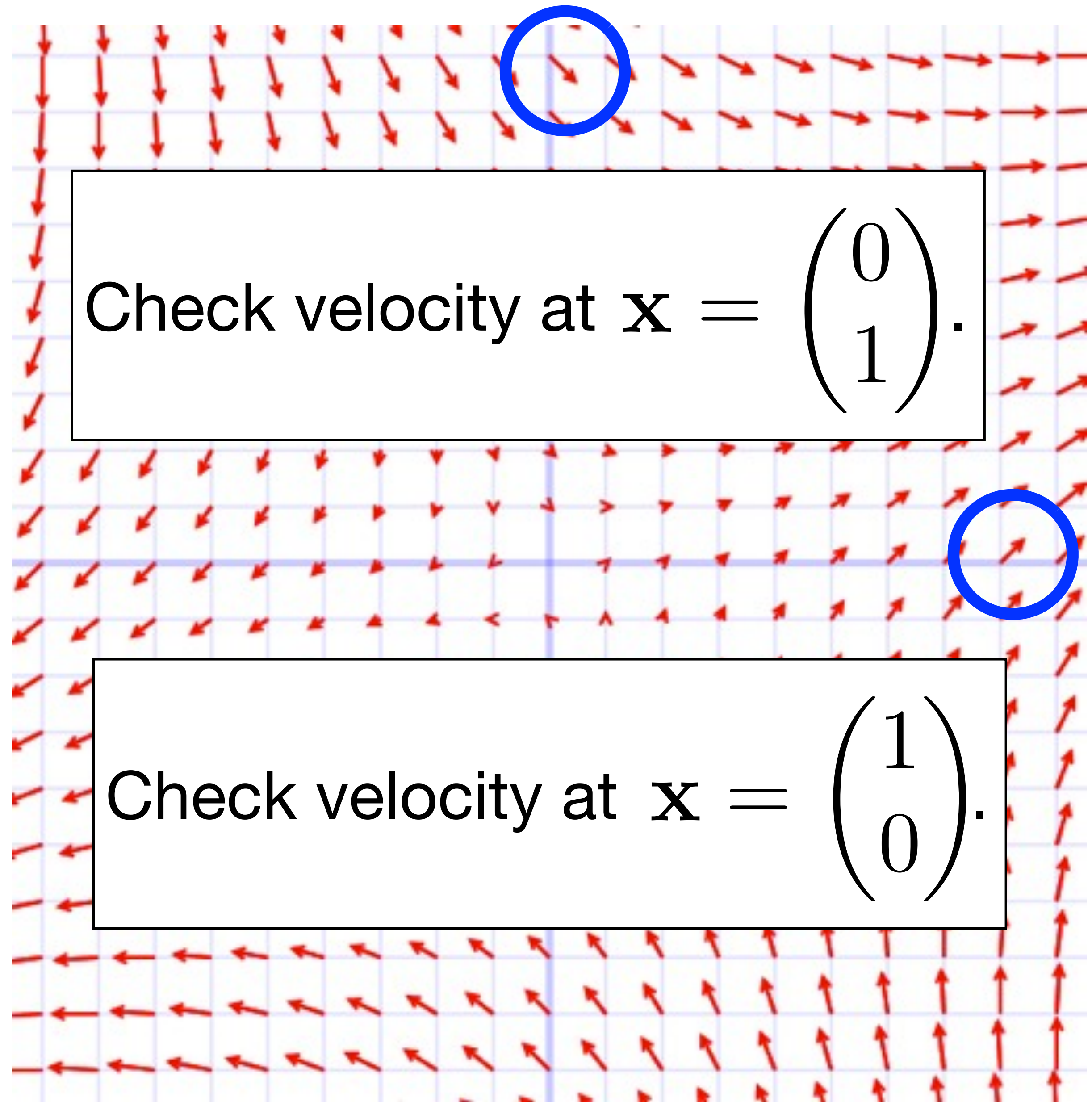

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Review problems

- A mass-spring system is at rest. At t=3, a linearly increasing force is applied until the force reaches F_0 = 10 N at t=8. After that moment, the force remains constant at that level (F_0). Write down the forcing function for this scenario.

  (A) $2t(u_3(t) - u_8(t))$

  (B) $2u_3(t)(t-3) - 2u_8(t)(t-8)$

  (C) $2u_3(t)(t-3) - 2u_8(t)(t-3)$

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- Midterm/exam practice: solve this equation by

  (a) (Laplace) Transforming the equation, solving for Y(s) and inverting.

  (b) Finding the transfer function and using a convolution integral.

Review problems

- Two tanks are connected by pipes. They initially contain large quantities of salt. Freshwater is added to the tanks so that the volumes of water are constant. The mass of salt in each tank is given by the system of equations

$$\frac{d}{dt}\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$$

where time is measured in minutes. How long does it take for the concentration in both tanks to decrease to less than one tenth of their original values?

(A) 1 minute
(B) 2 minutes
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Require $e^{\lambda t} < 1/10$ for both evalues $\lambda_1=-2$ & $\lambda_2=-3$.

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- Invert the function

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$$y(t) = u_2(t)g(t-2)$$

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$$\mathbf{v_{-1}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \mathbf{v_{-3}} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Review problems

- Consider the solution to the IVP

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 1/2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

For t>0, do we ever have y(t)<0?

(A) Yes.

(B) No.

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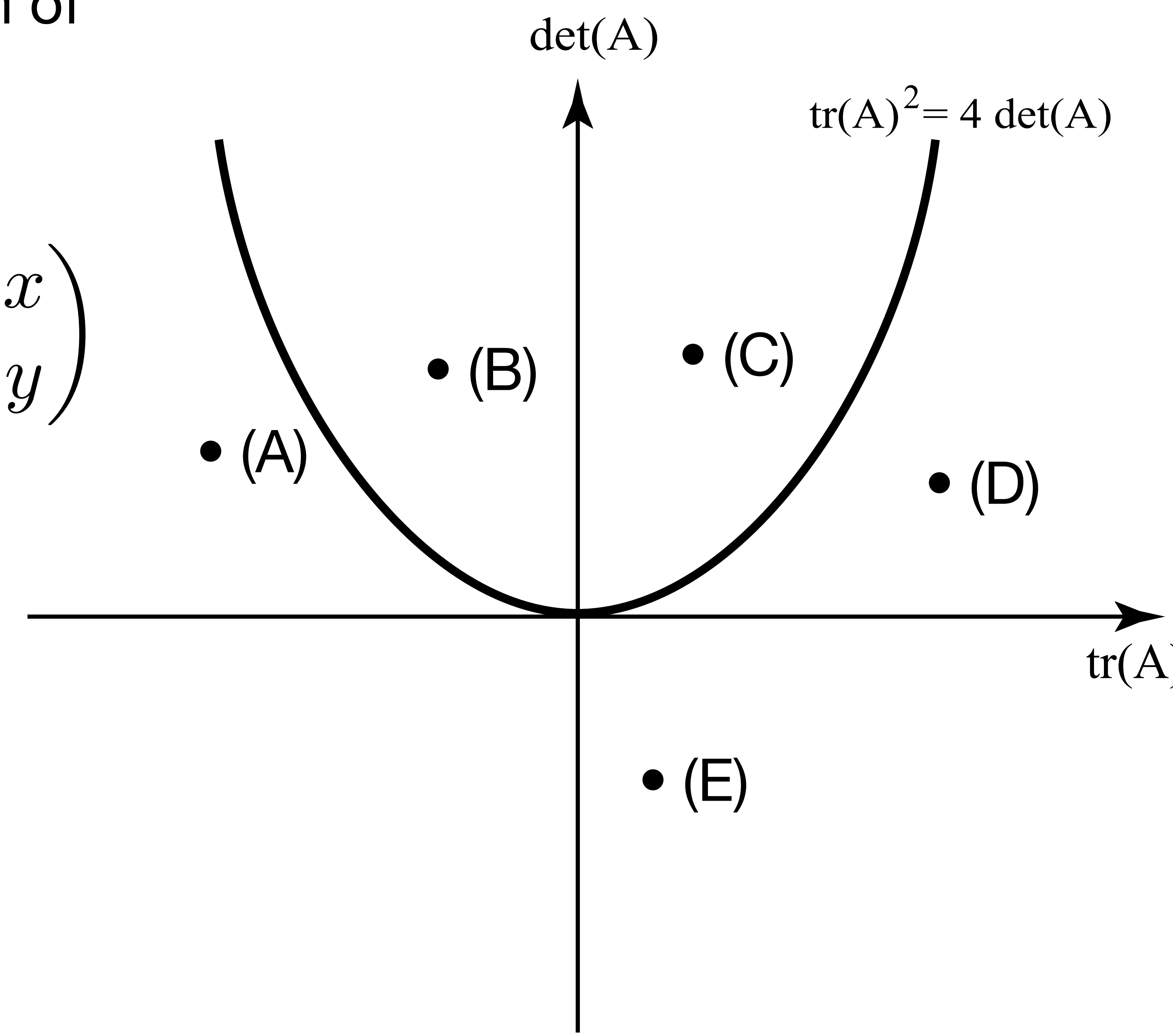

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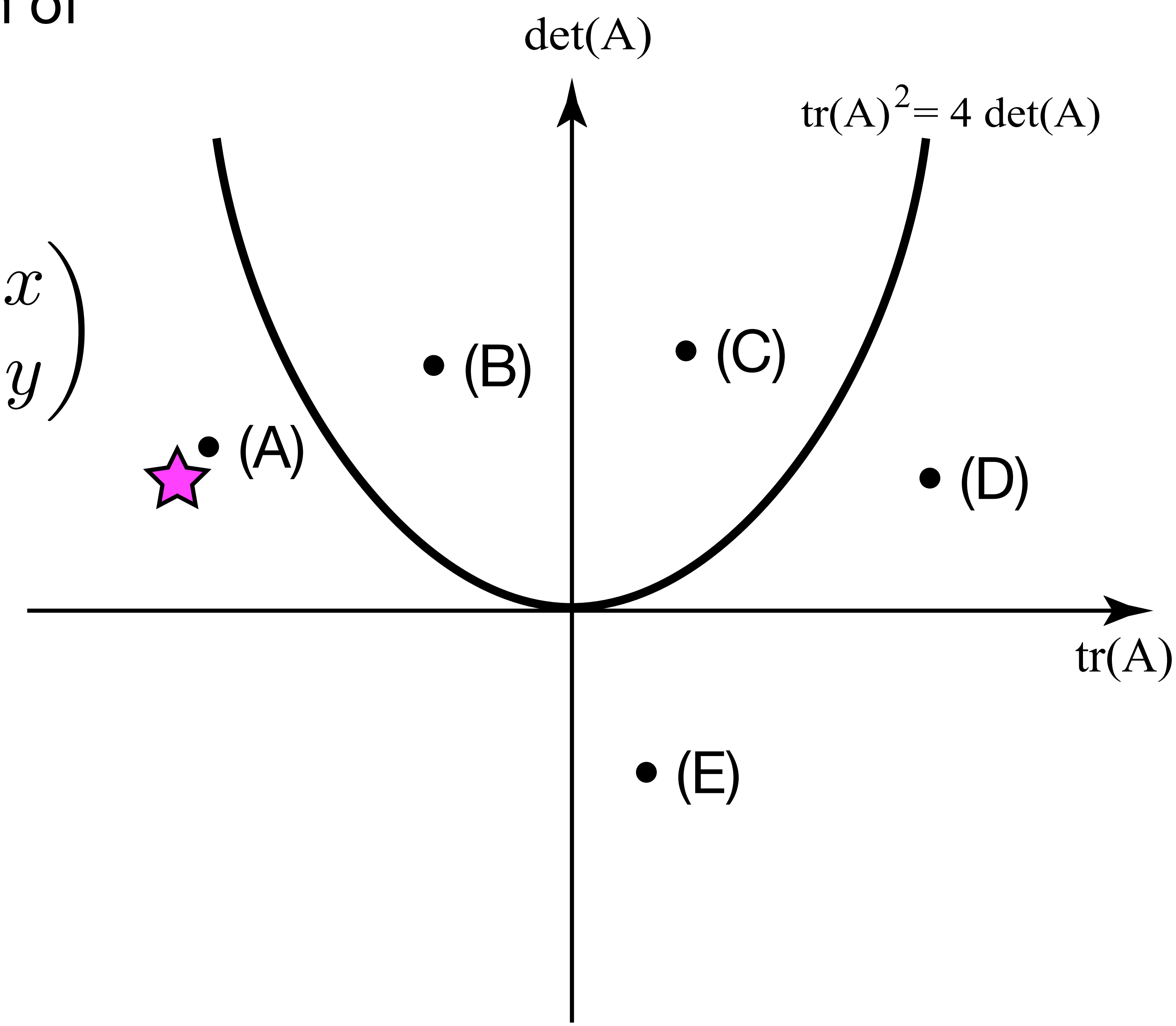

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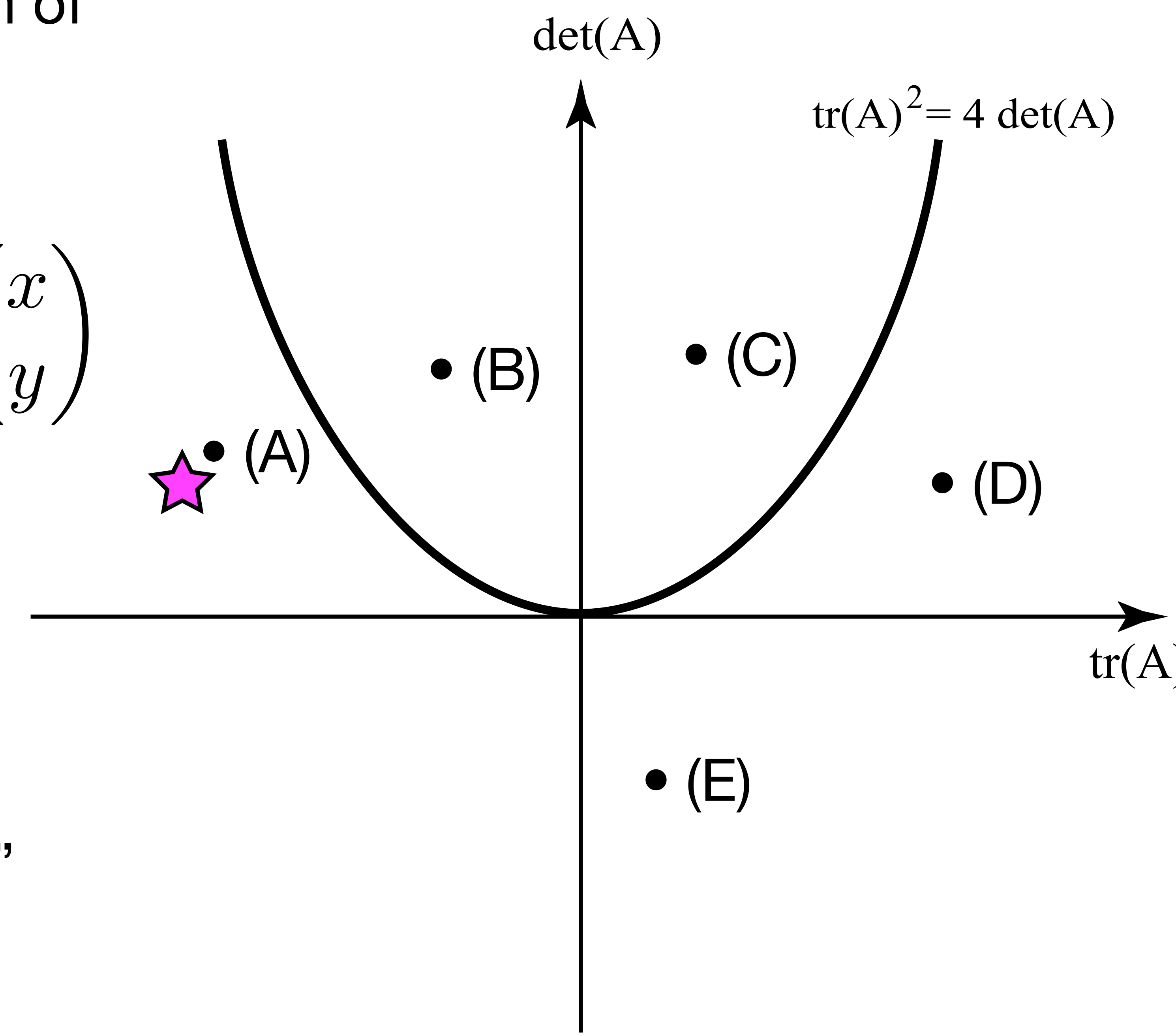
  - tr(A) = -4
  - det(A) = 3.
  - $(\mathrm{tr}(A))^2 > 4\det(A)$ so it lies below the “repeated root” parabola.



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- Plot the location of the system of equation in the tr/det plane for all possible values of α.

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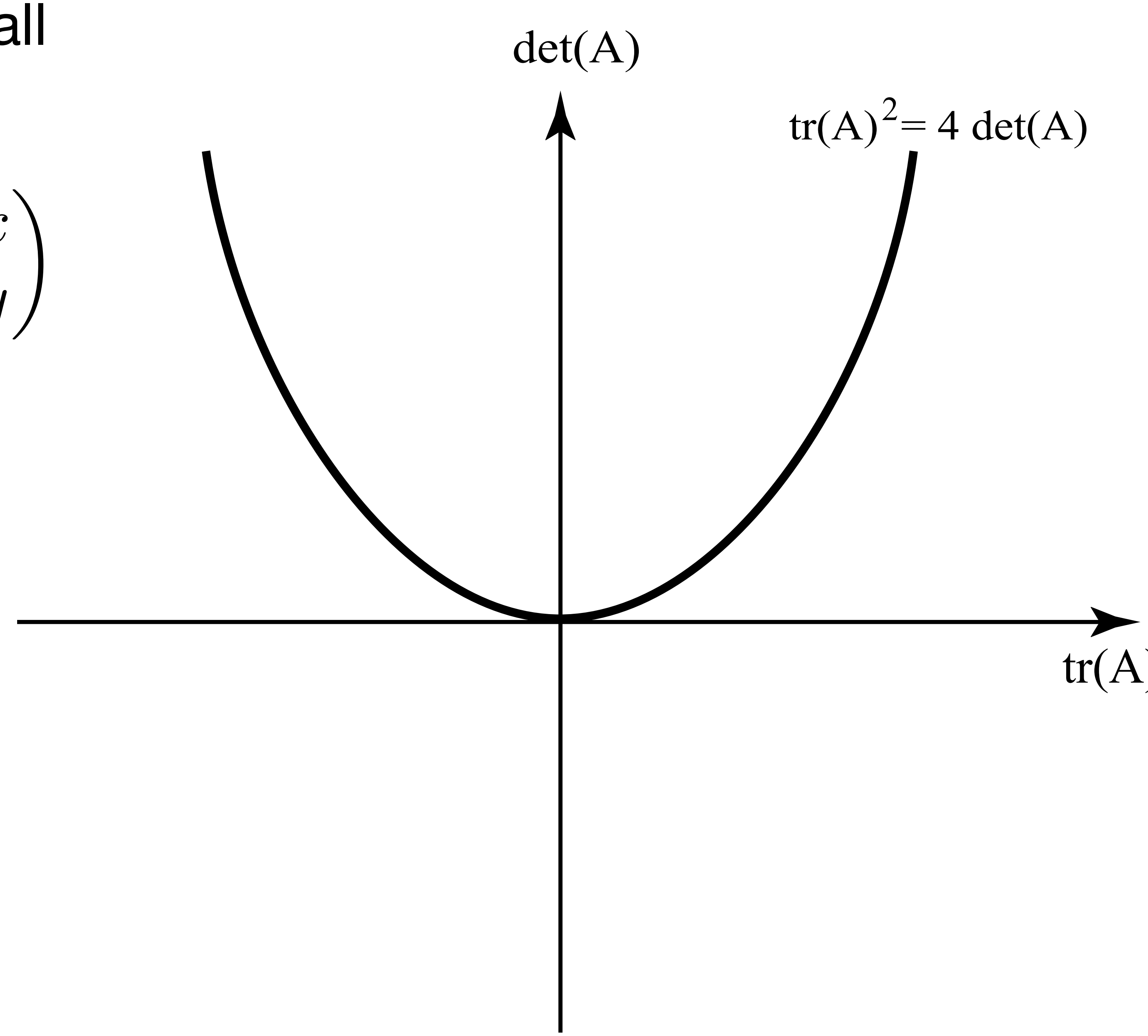

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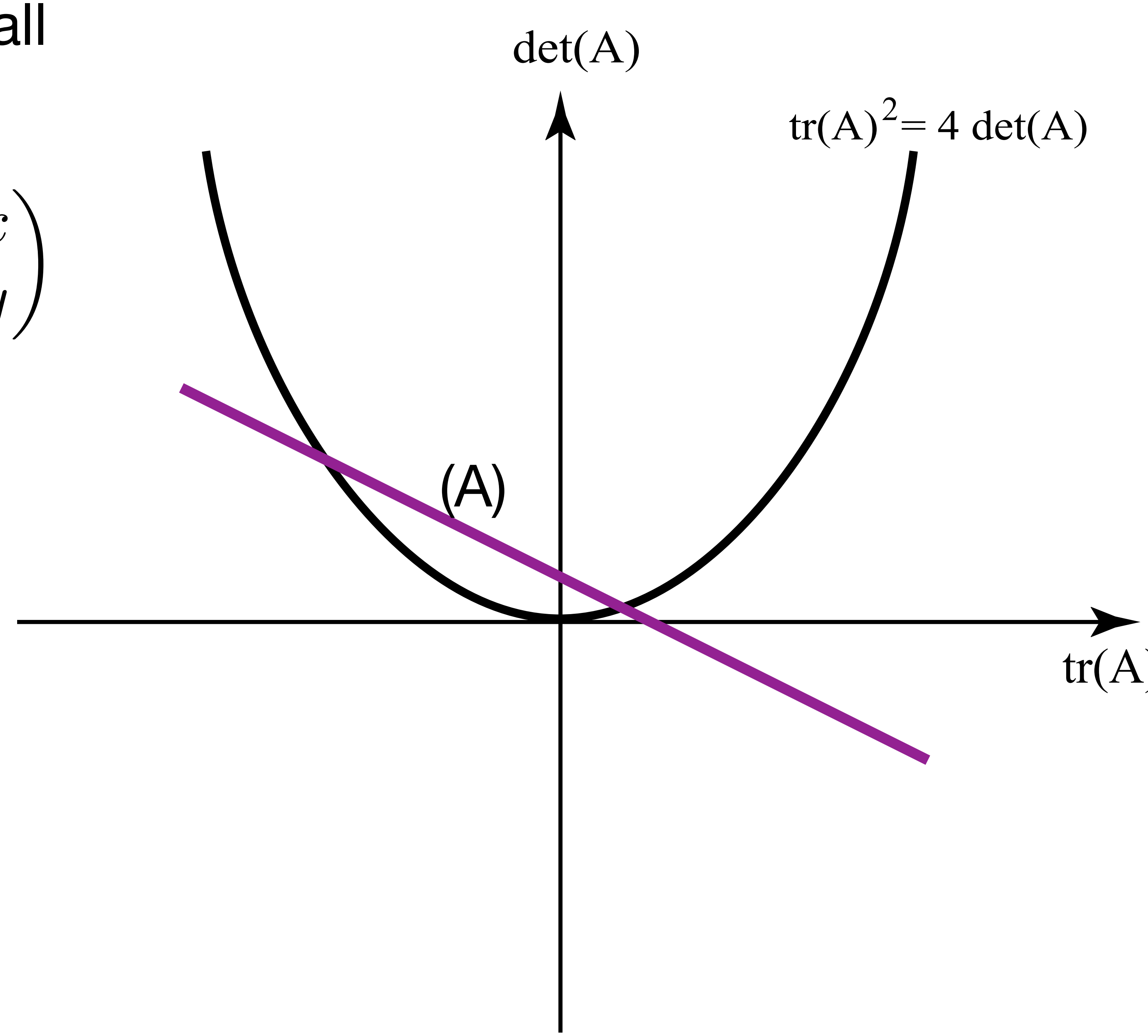

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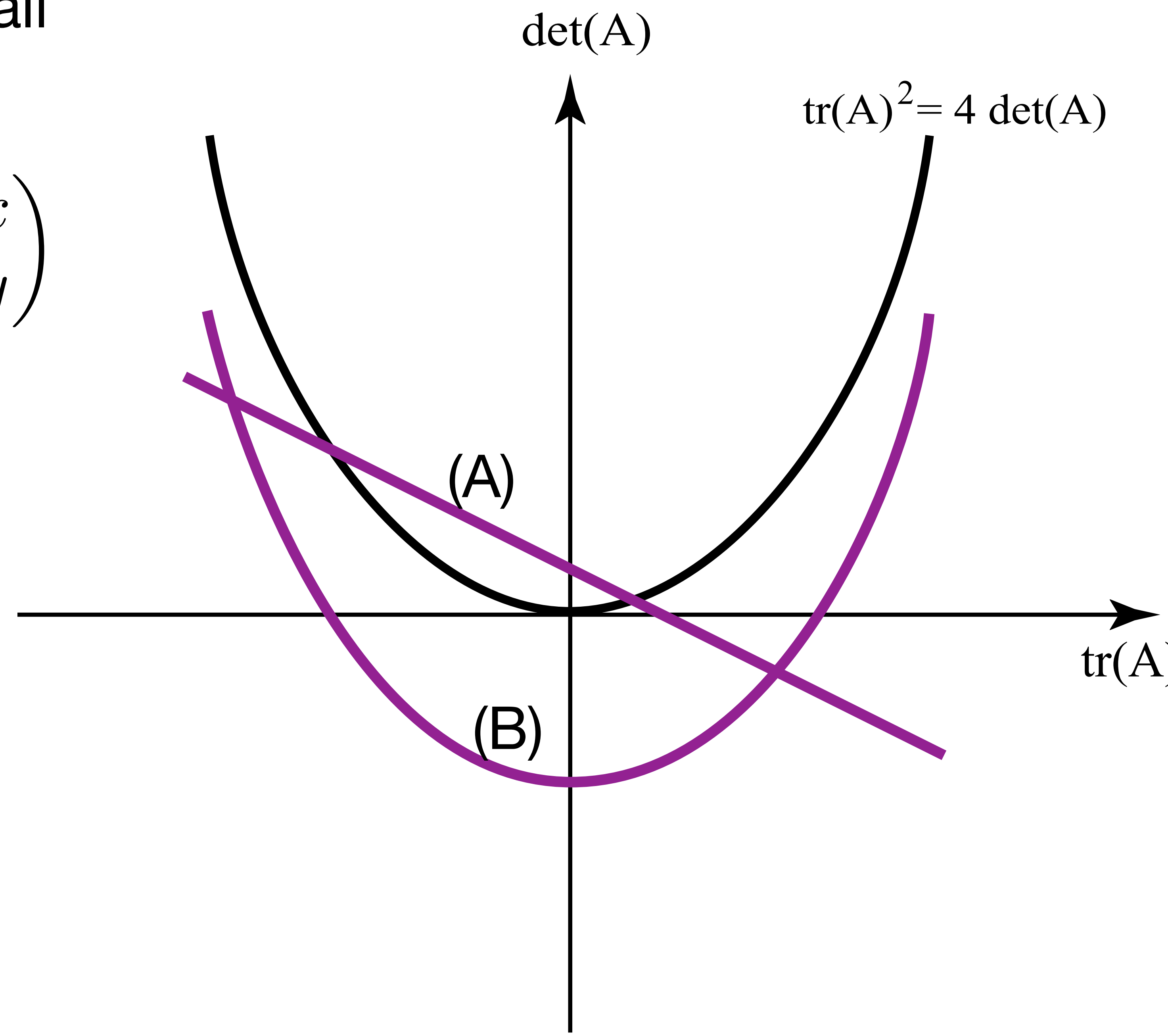

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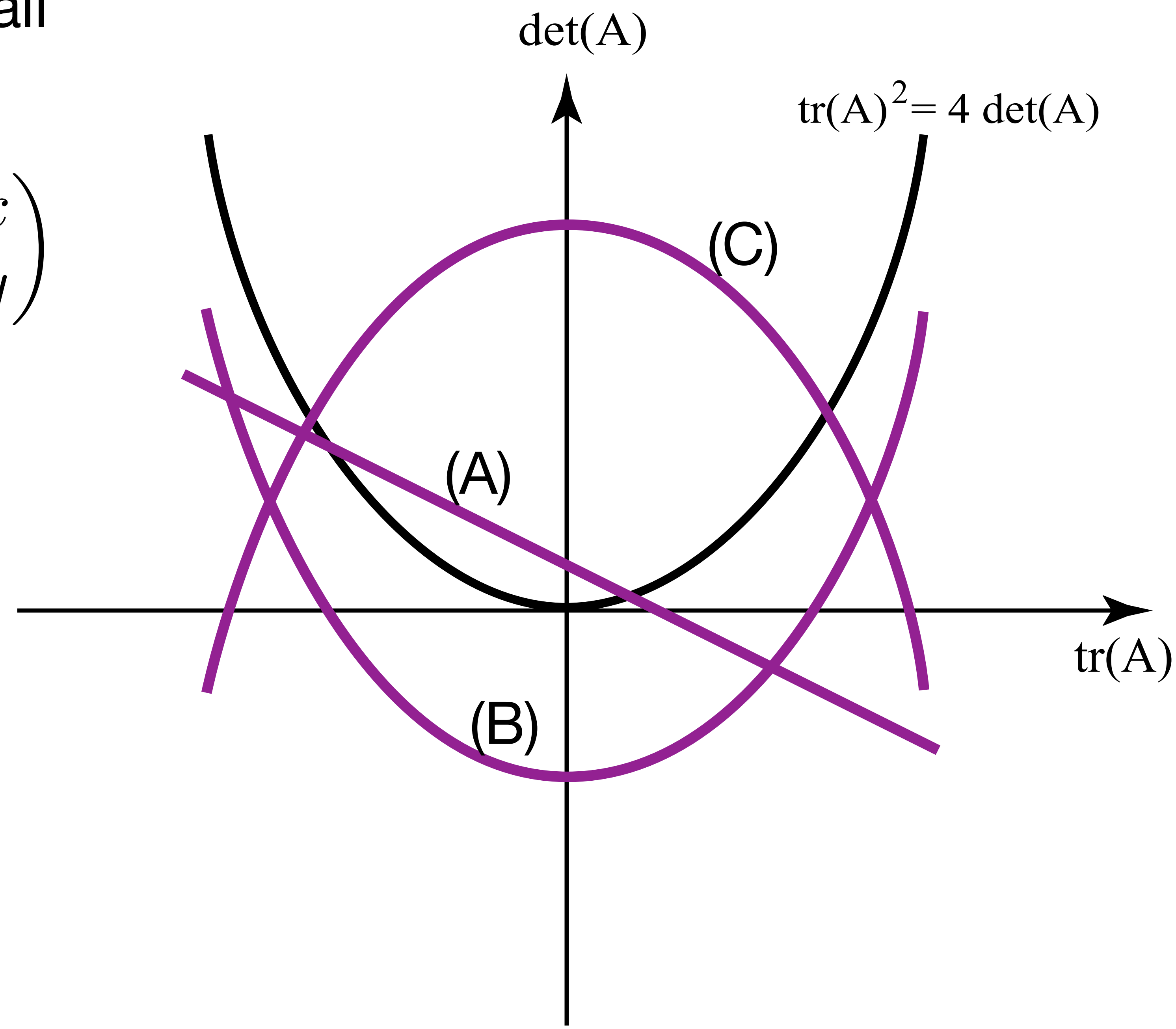

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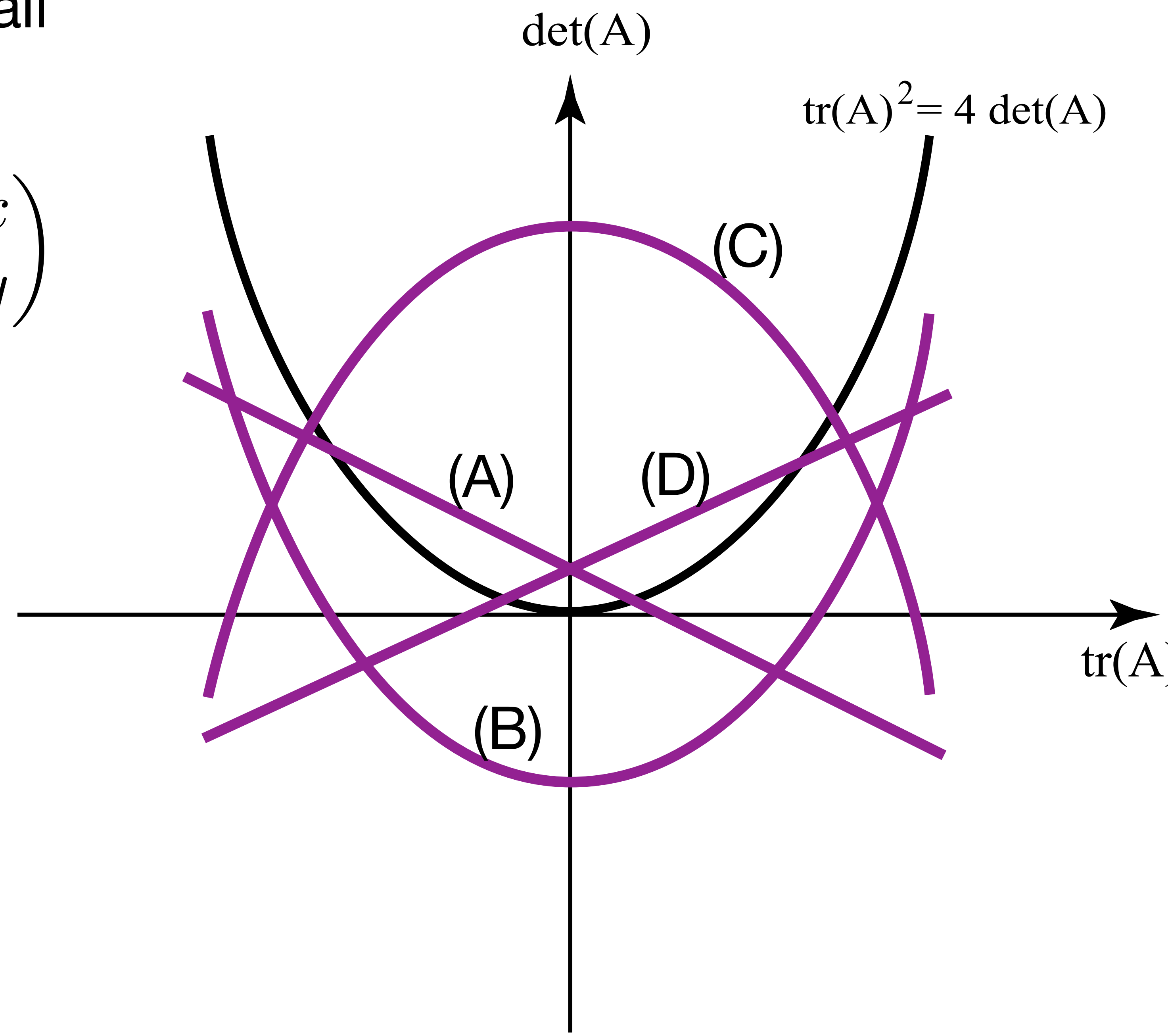

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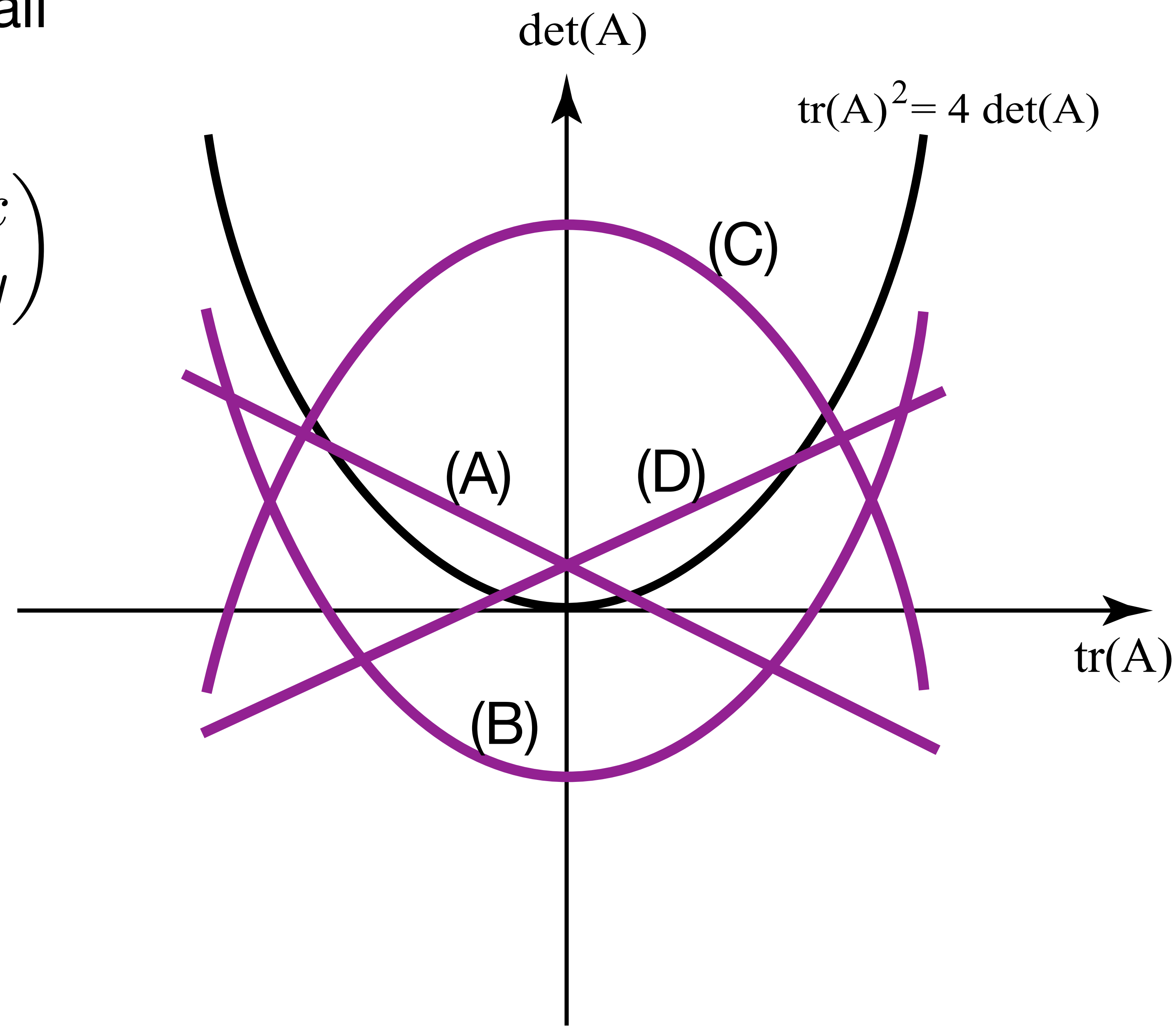

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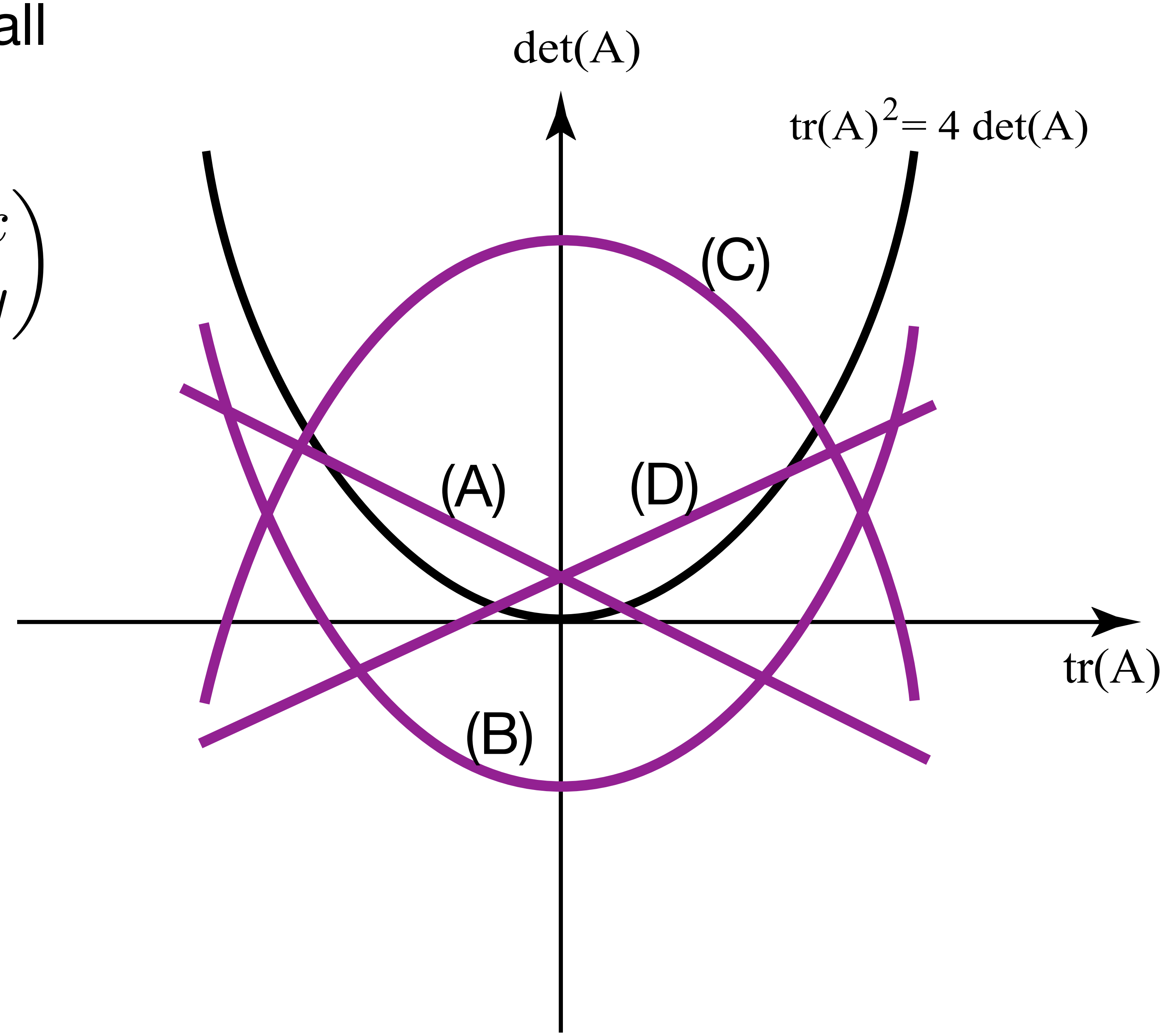

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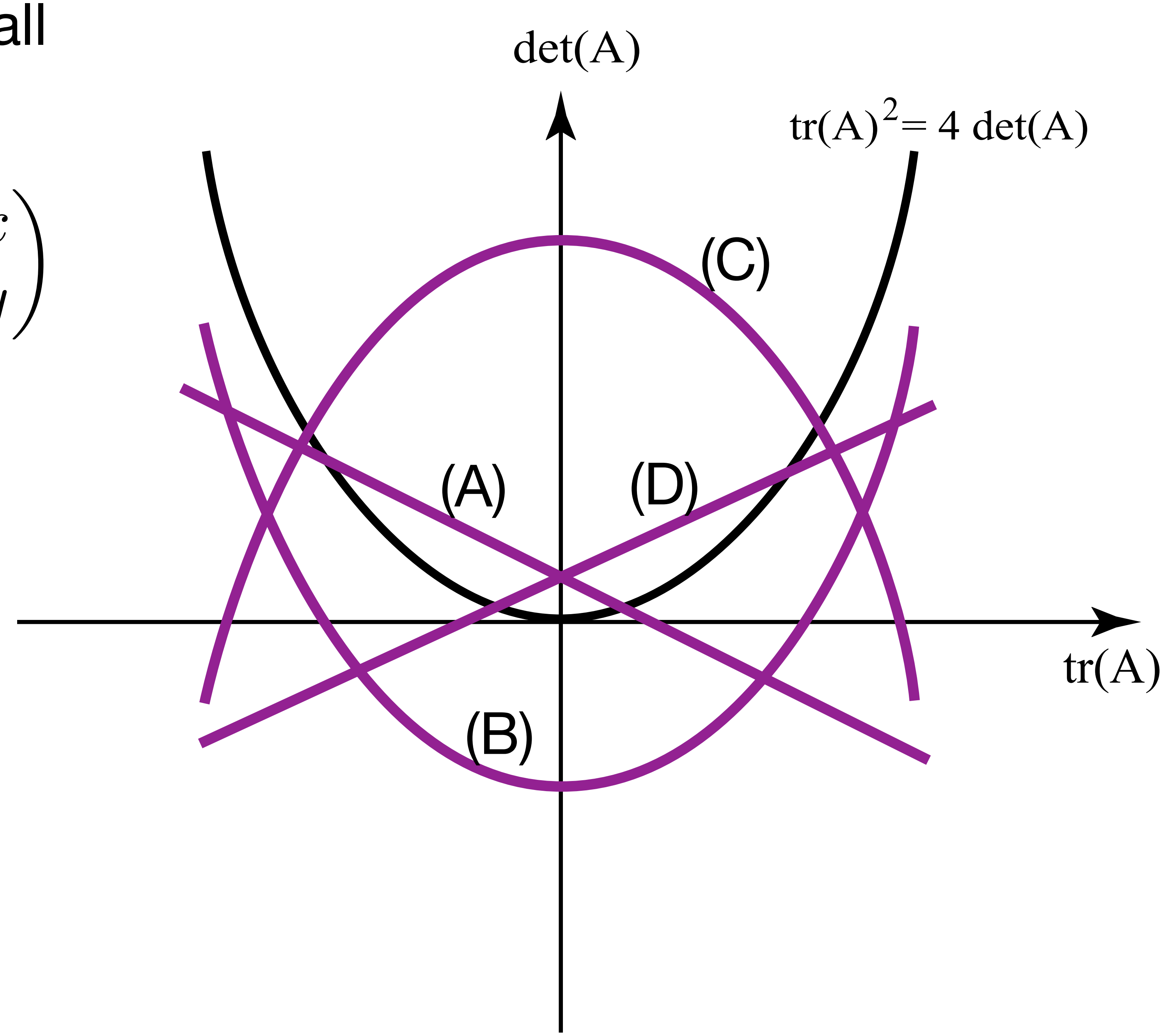

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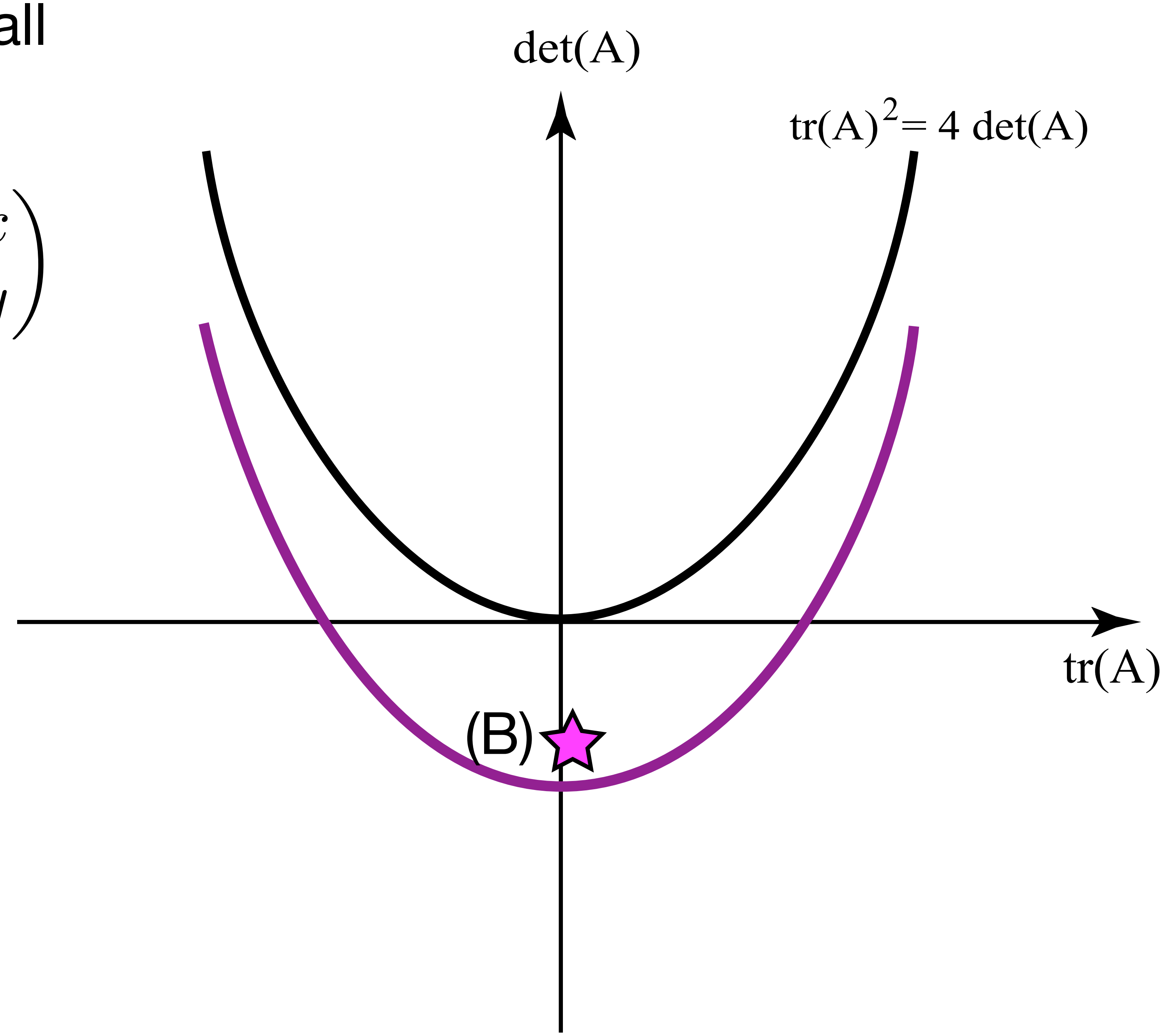

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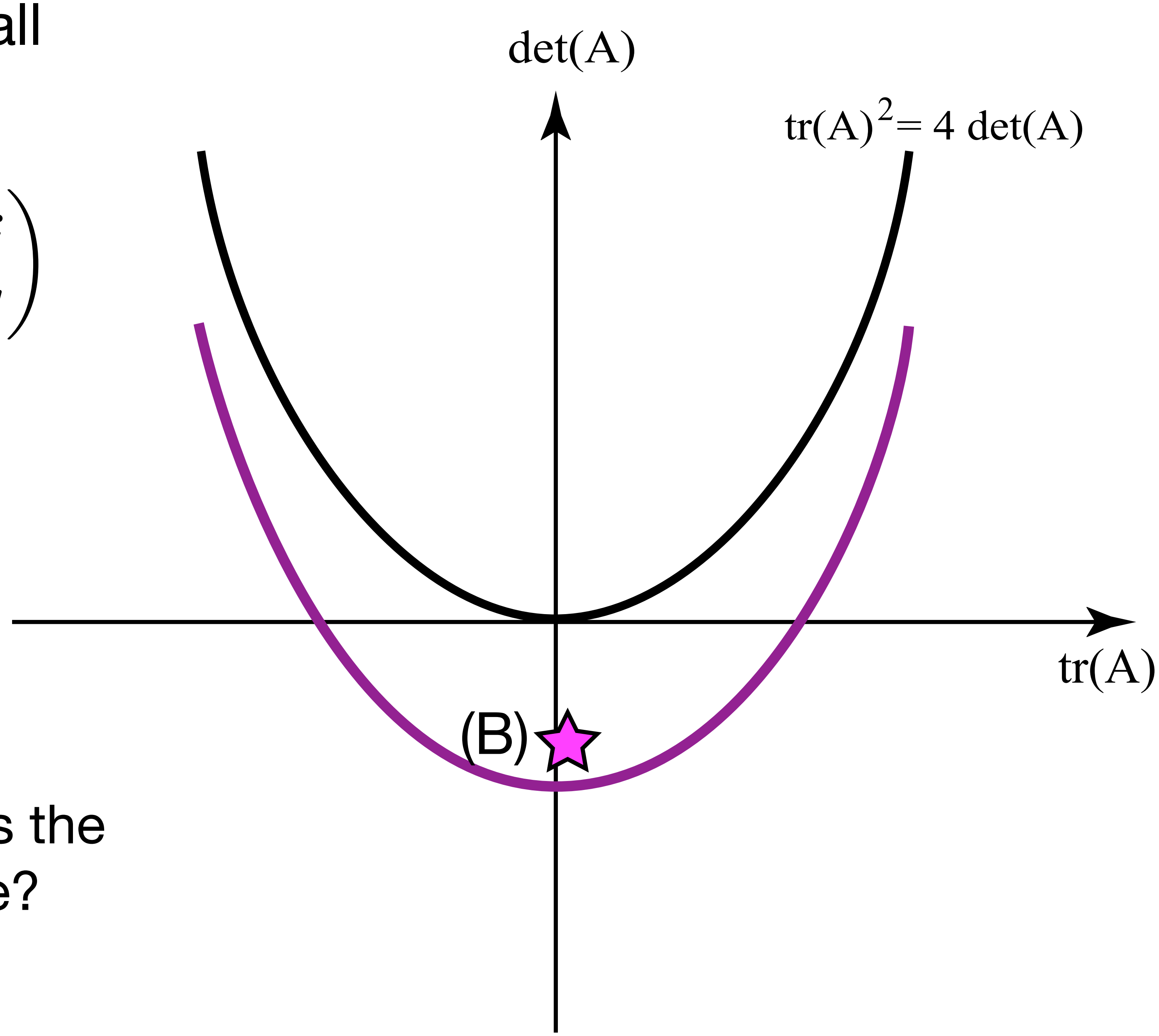

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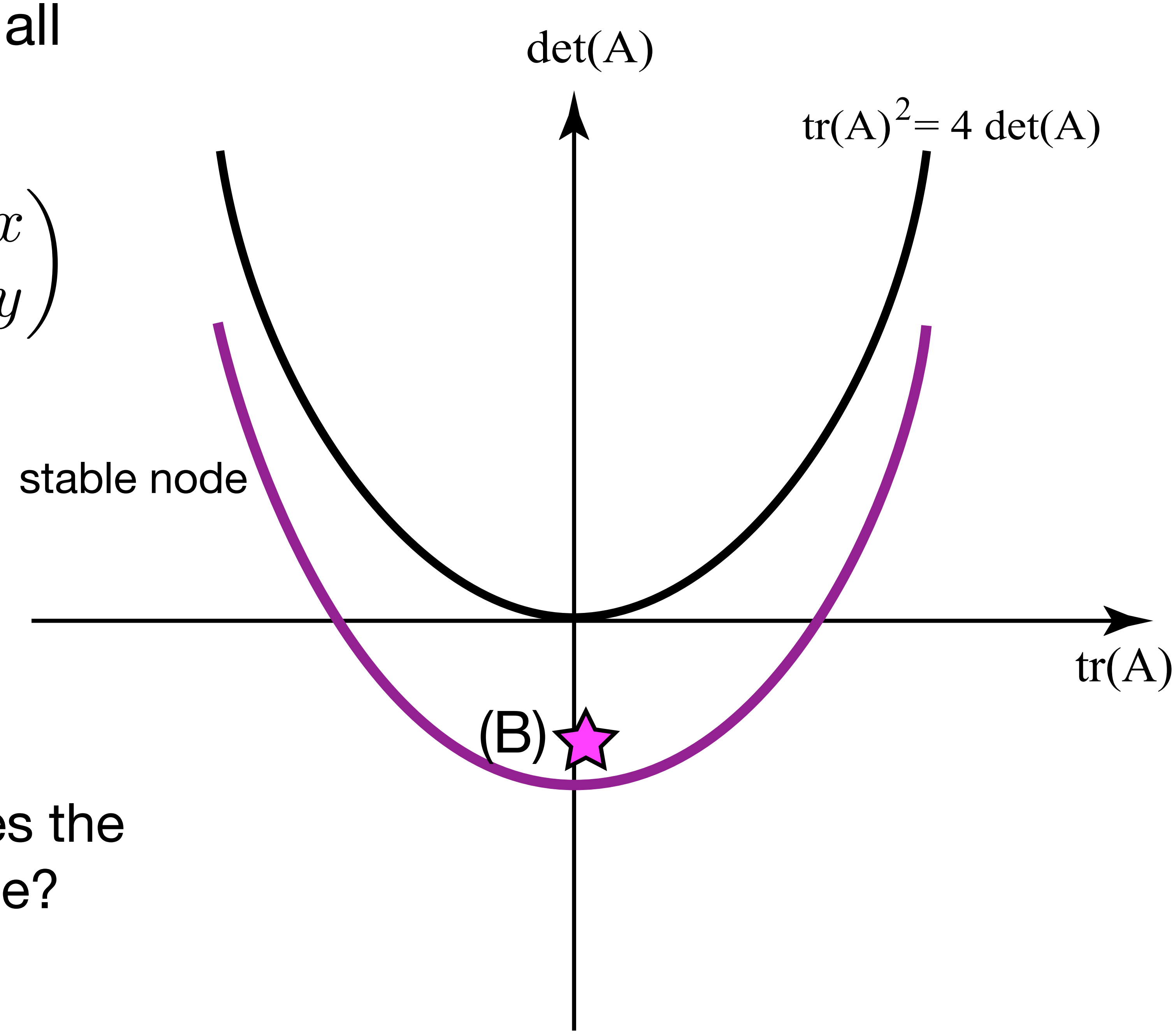

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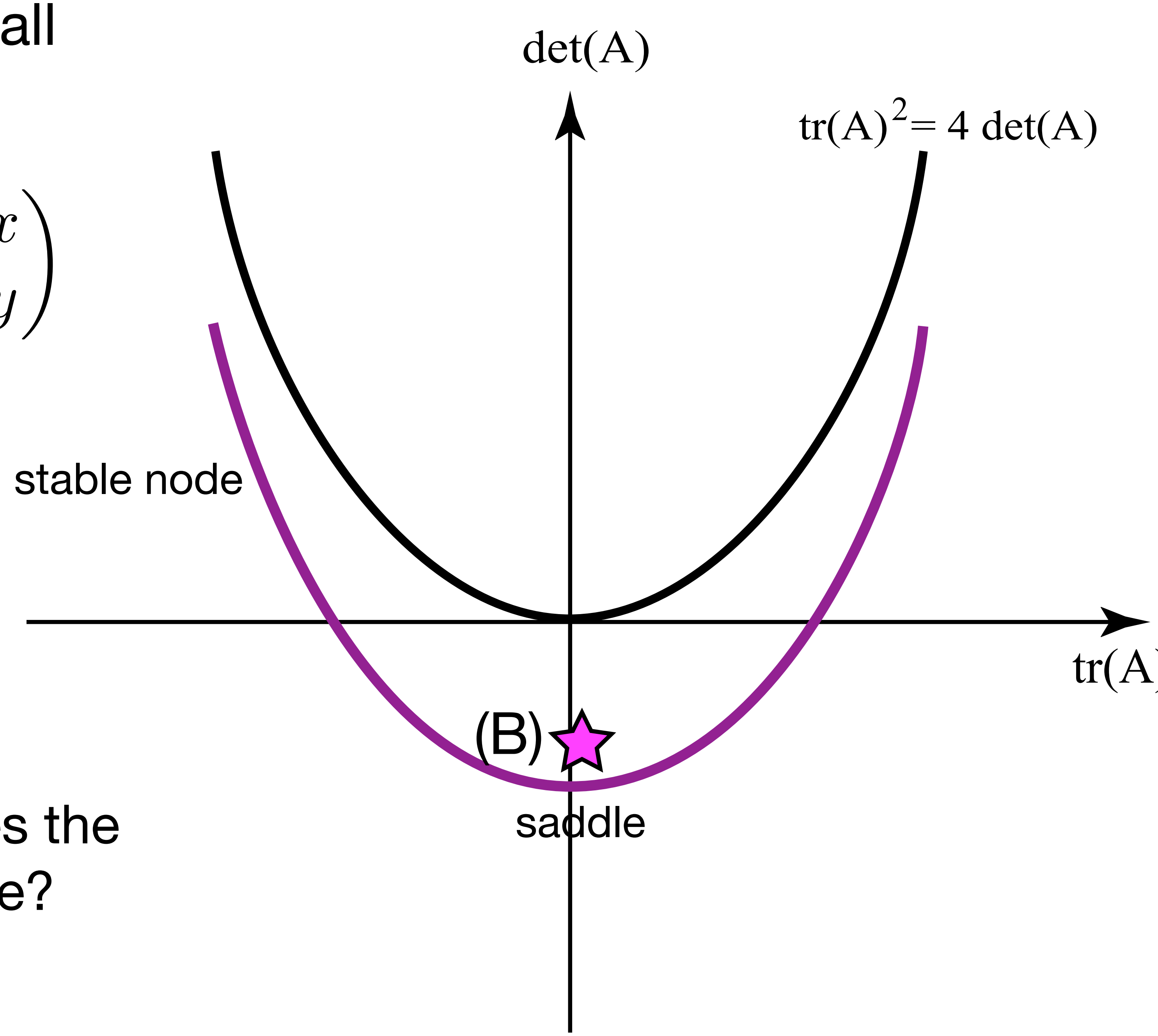

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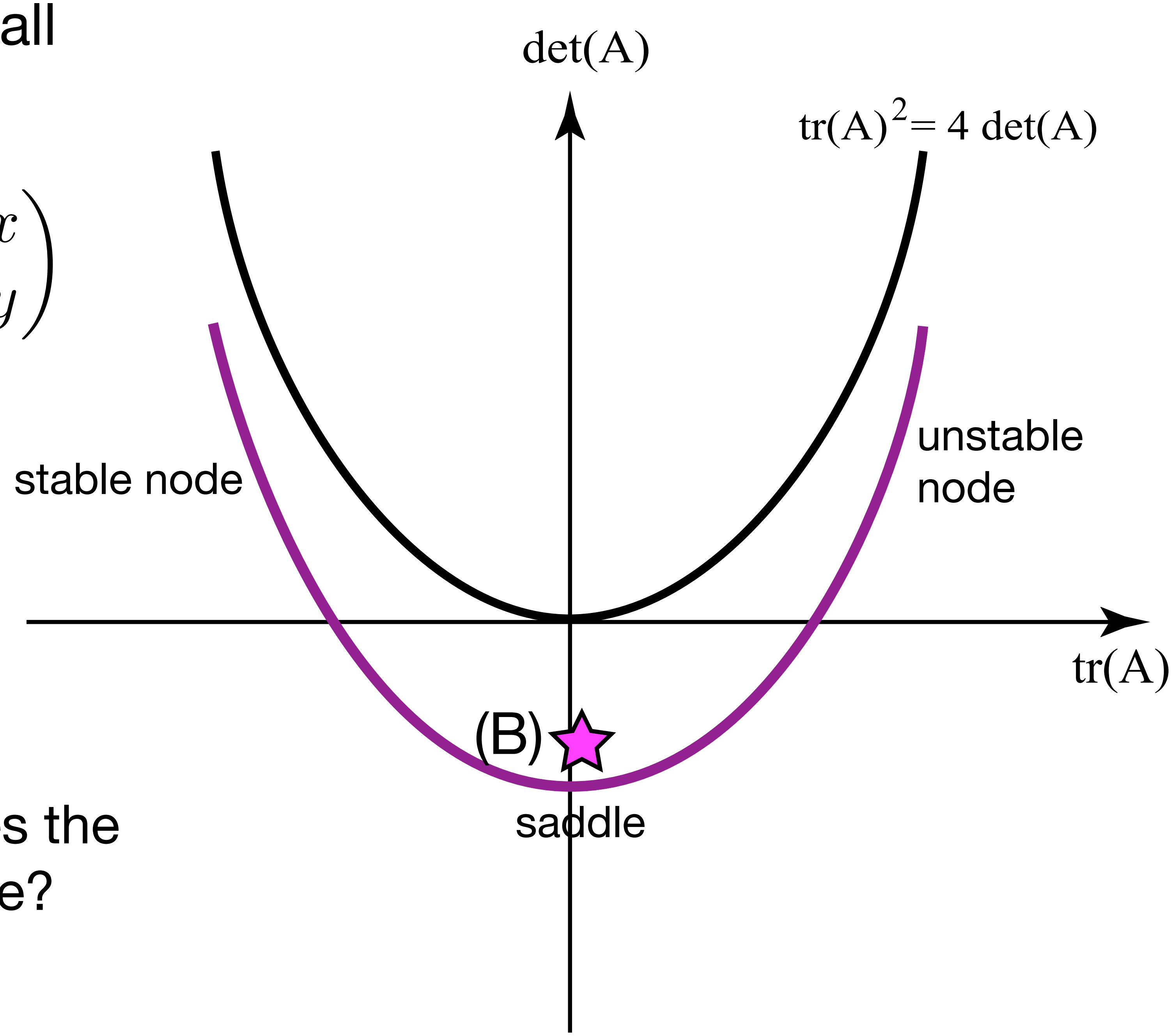

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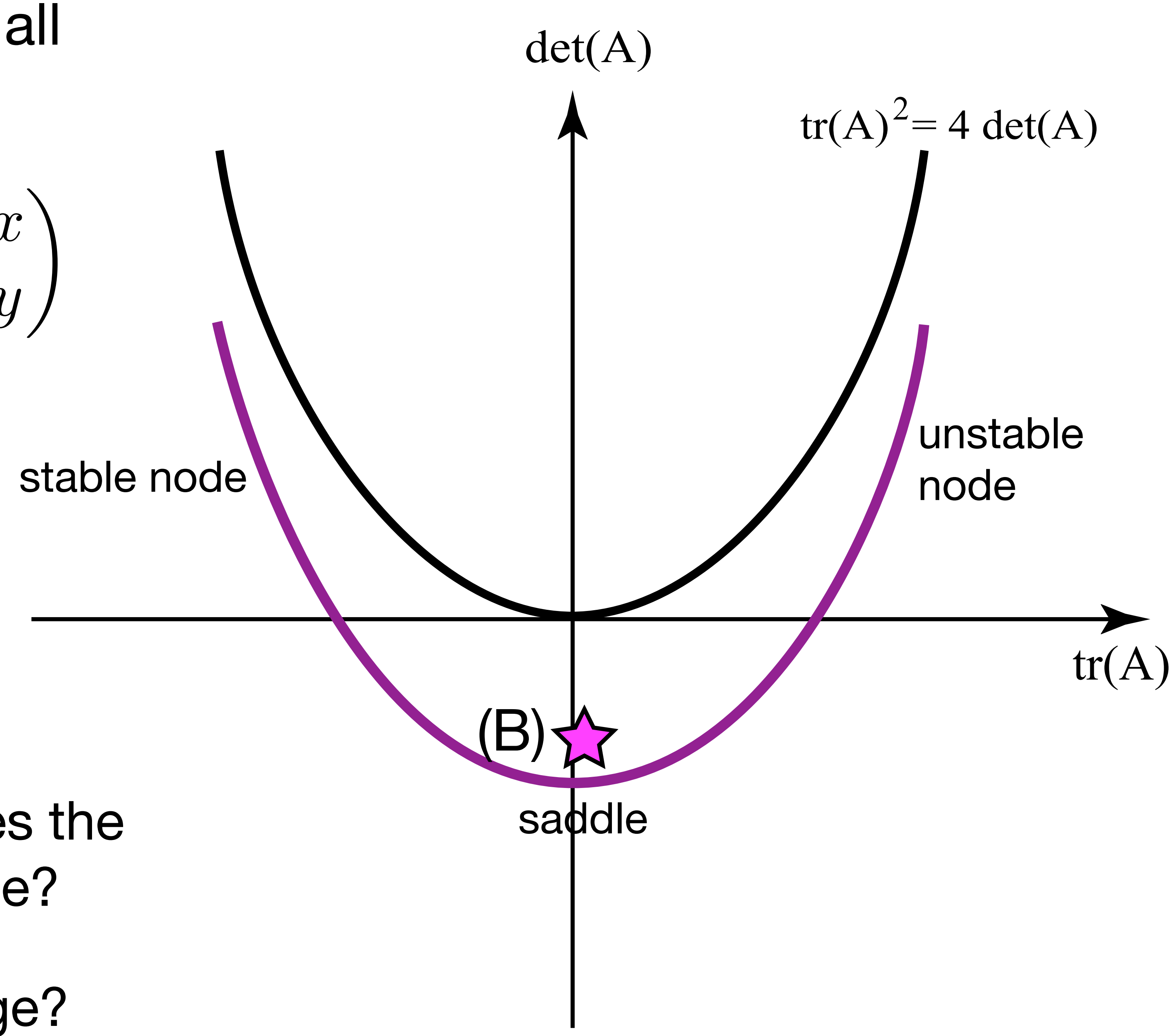

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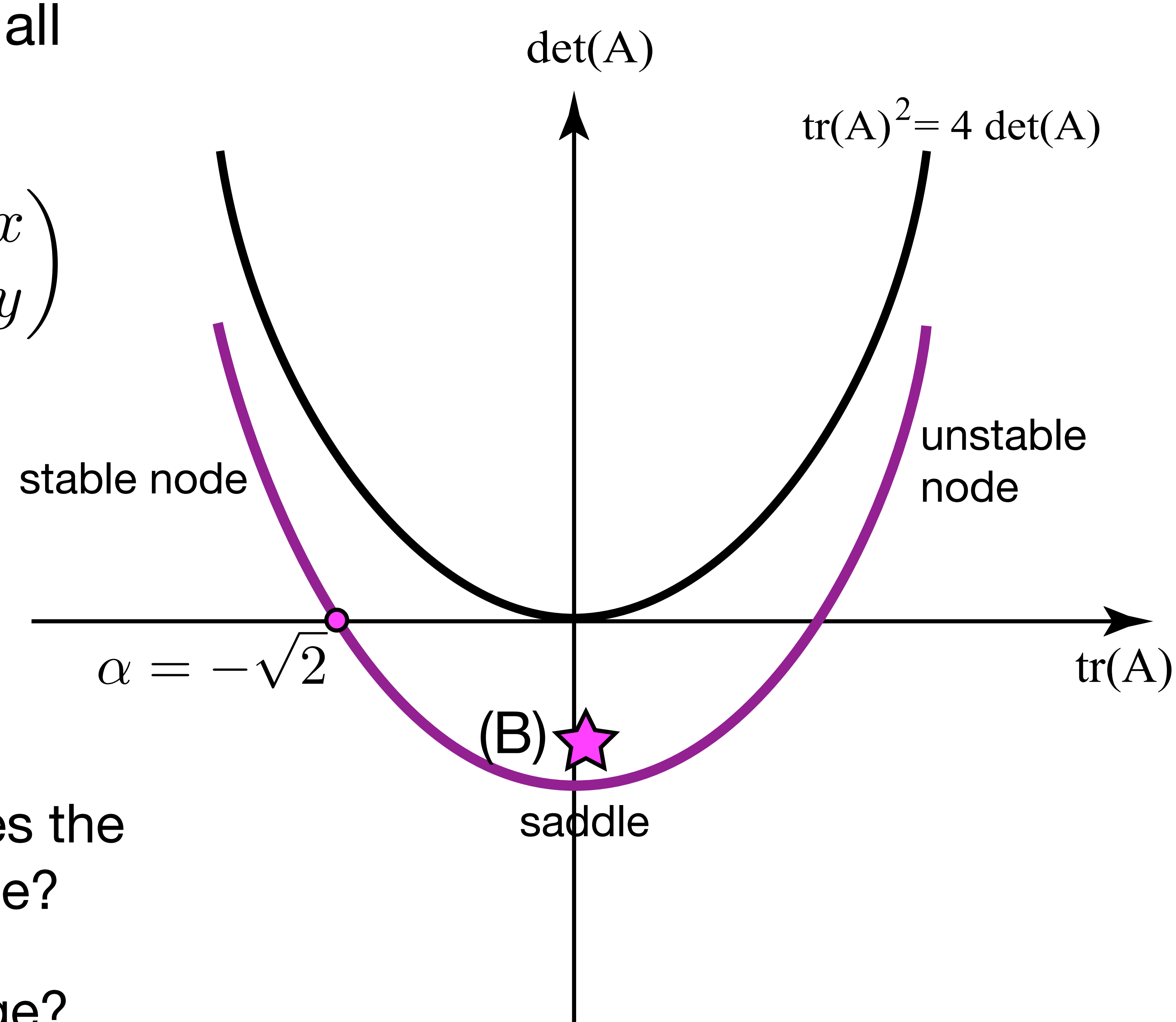

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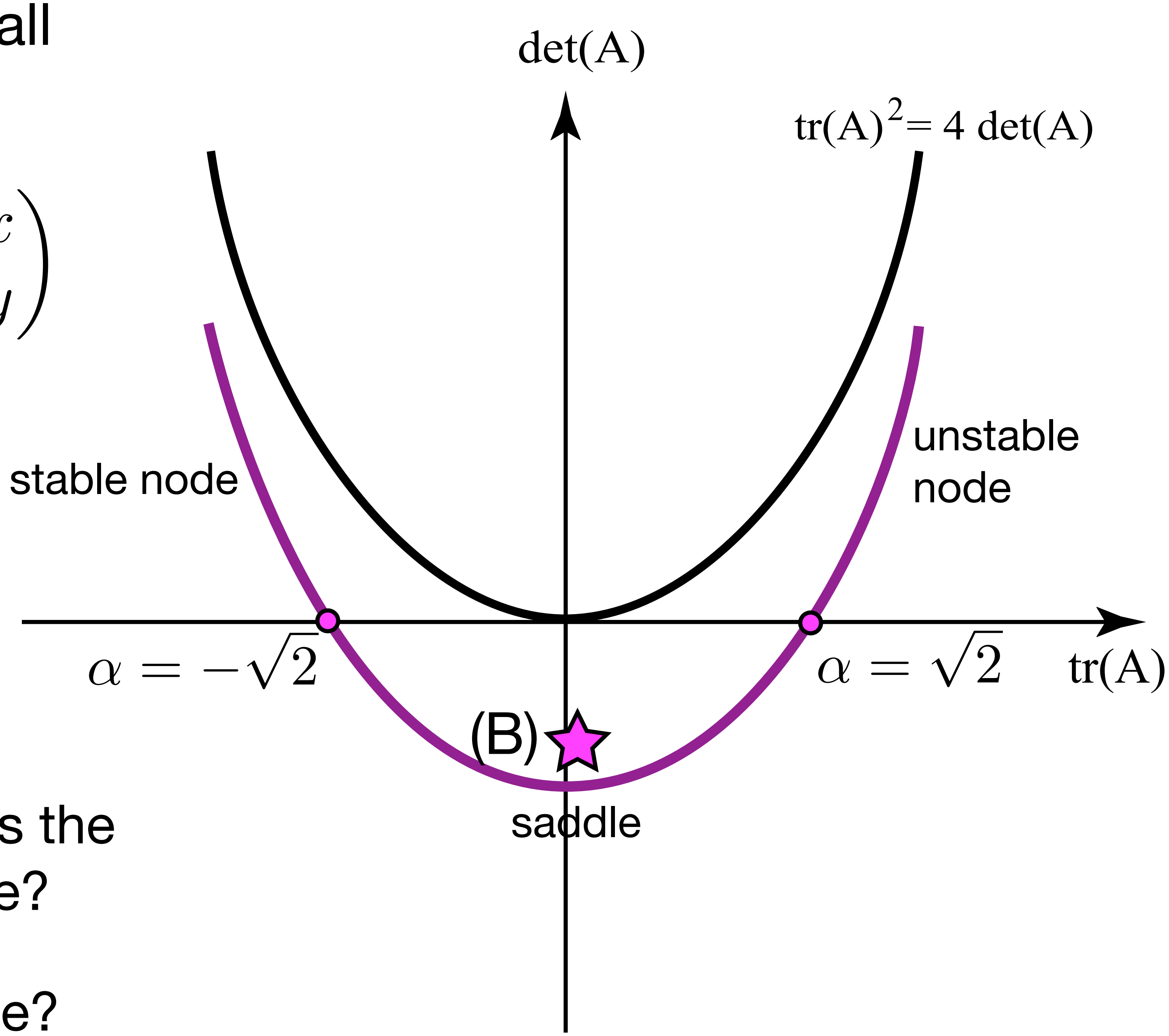

- As α goes from -∞ to ∞, how does the steady state of the system change?

- At what values of α does it change?

Review problems

- Plot the location of the system of equation in the tr/det plane for all possible values of α.

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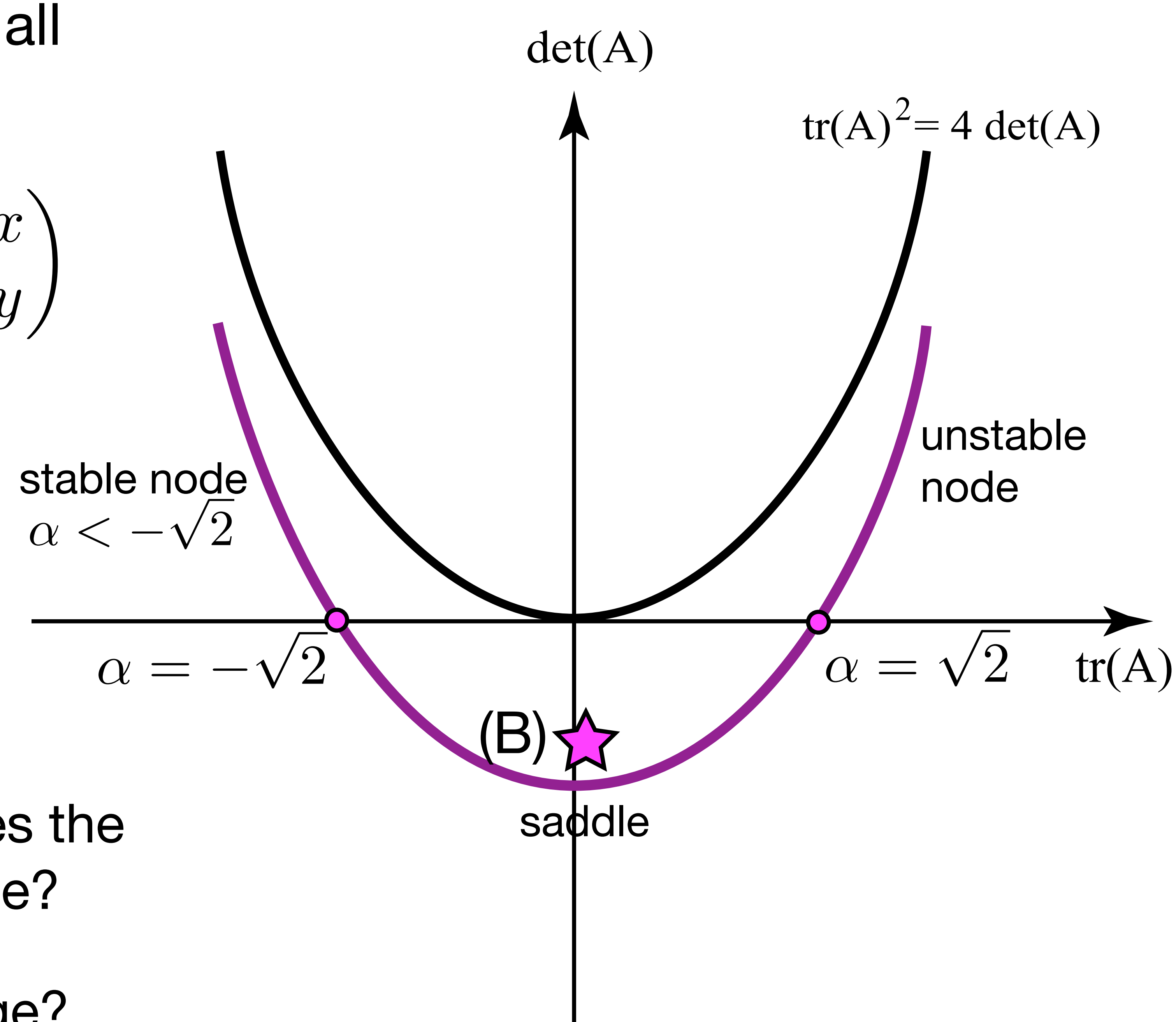

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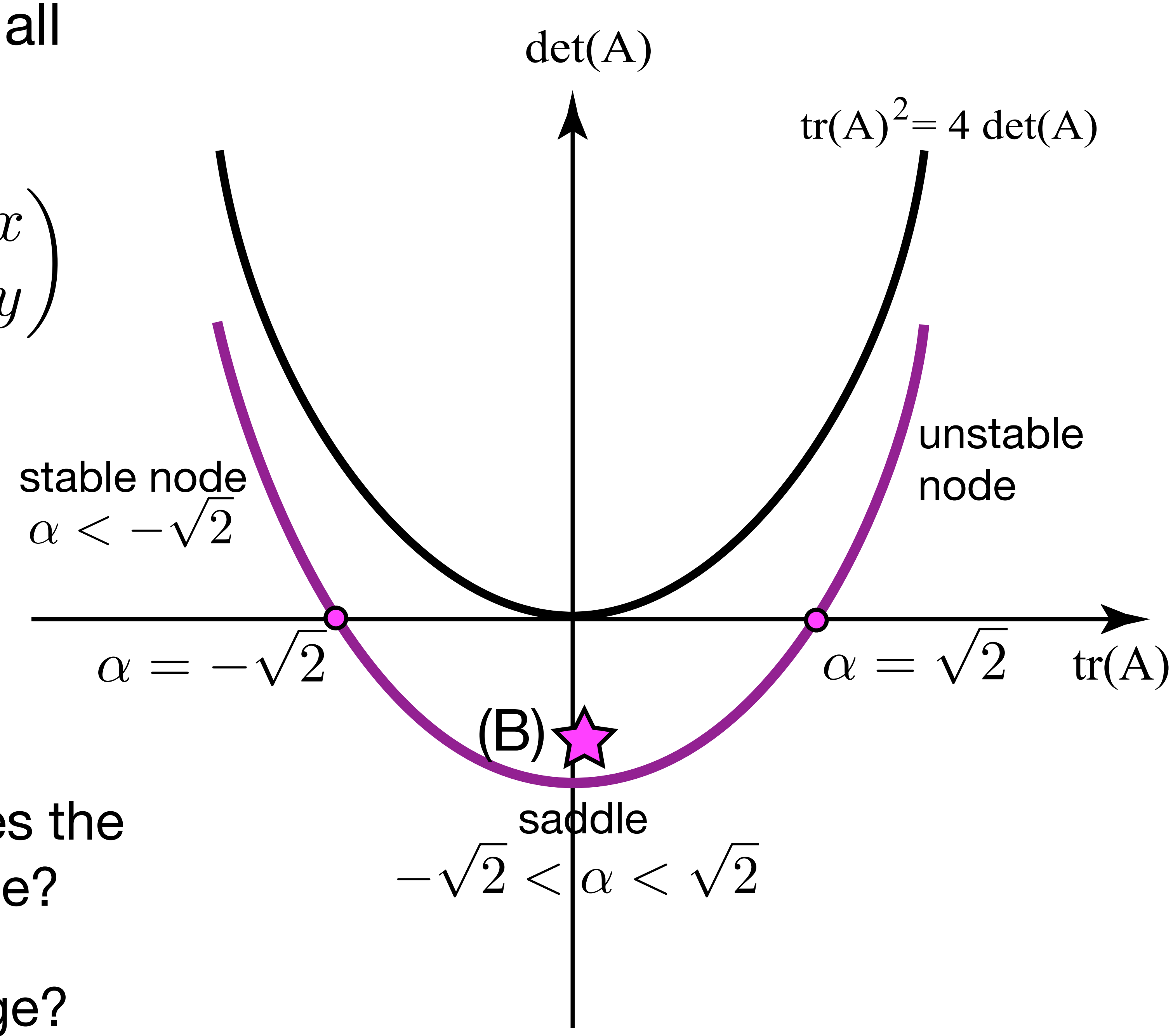

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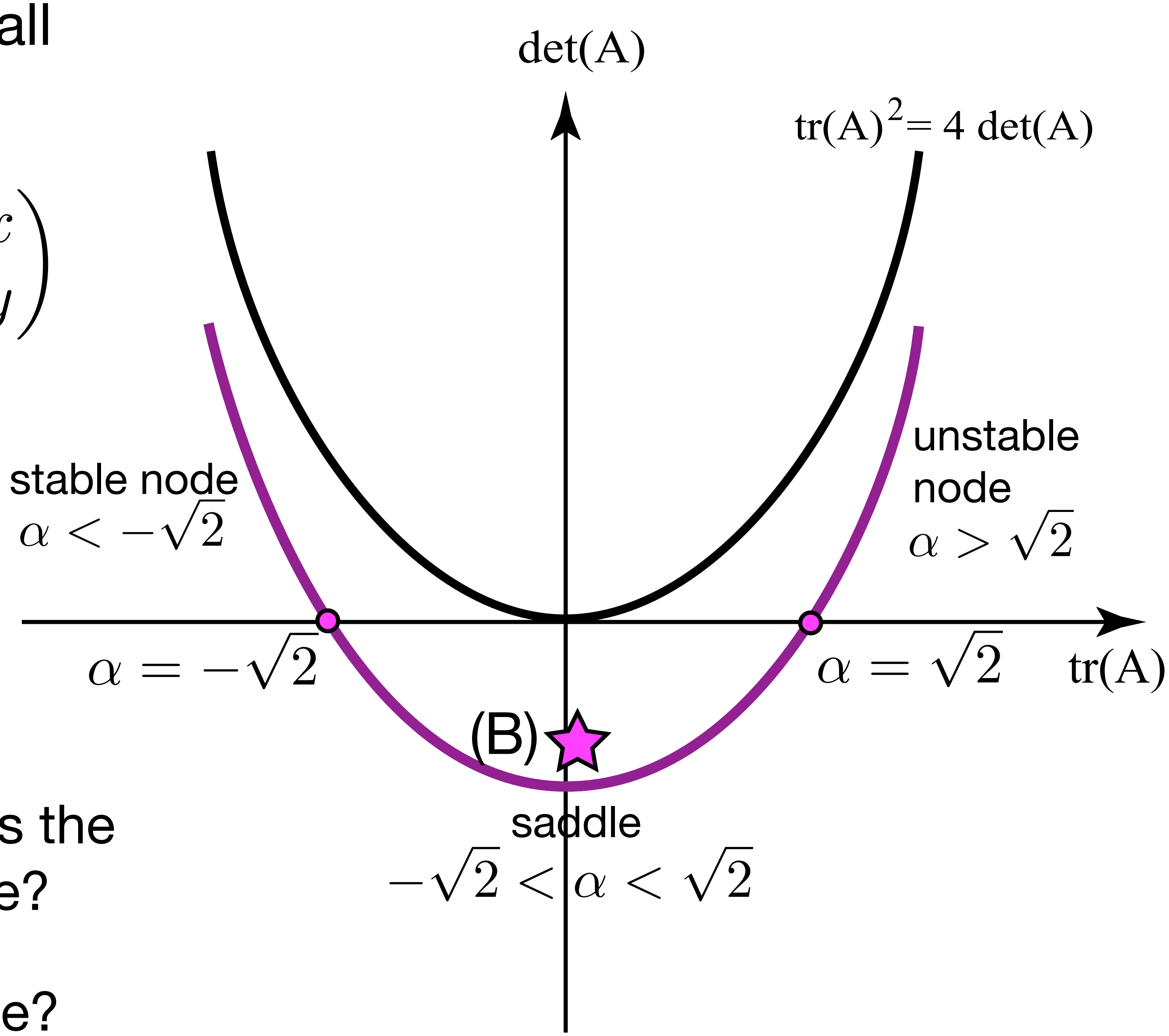

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Review problems

- What is the transfer function, H(s), for the equation

$$y'' + 4y' + 29y = u_3(t)(t-3) - u_6(t)(t-6) \text{ ?}$$

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- Midterm/exam practice: solve the equation for y(t) using convolution.

Review problems

- Invert the function

$$Y(s) = \frac{s}{s^2 + 4s + 8}$$

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$$y'' + 4y' + 8y = 0,$$
$$y(0) = 1, \; y'(0) = 0$$

Review problems

- What IVP might have lead to the transformed solution

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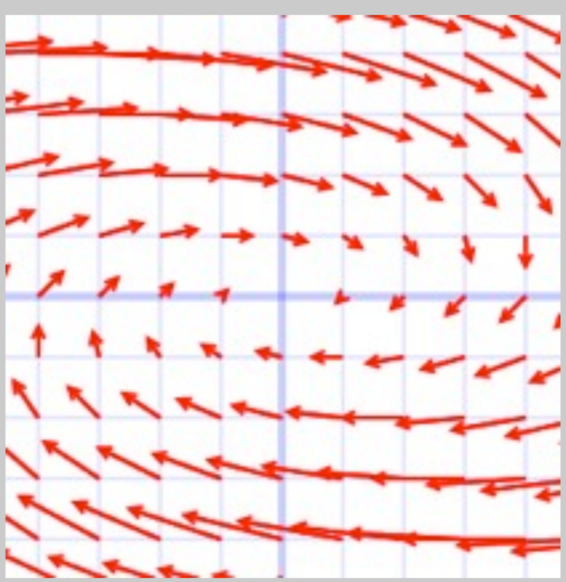
$$
\mathbf{x}(\mathbf{t}) = e^t \left(C_1 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \sin(2t) \right) + C_2 \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \cos(2t) \right) \right)
$$

(A)



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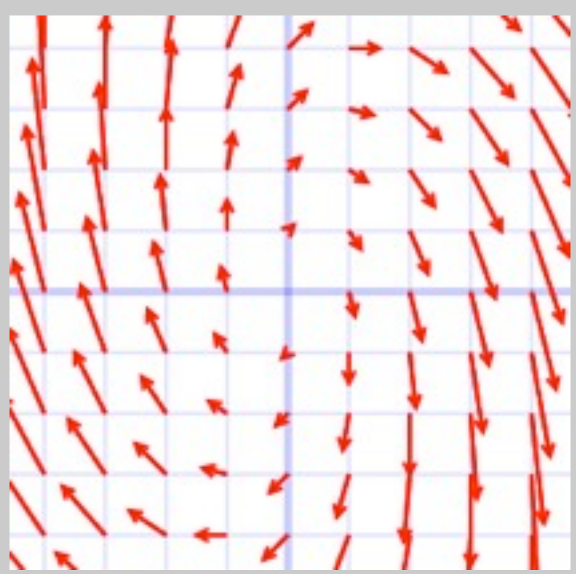

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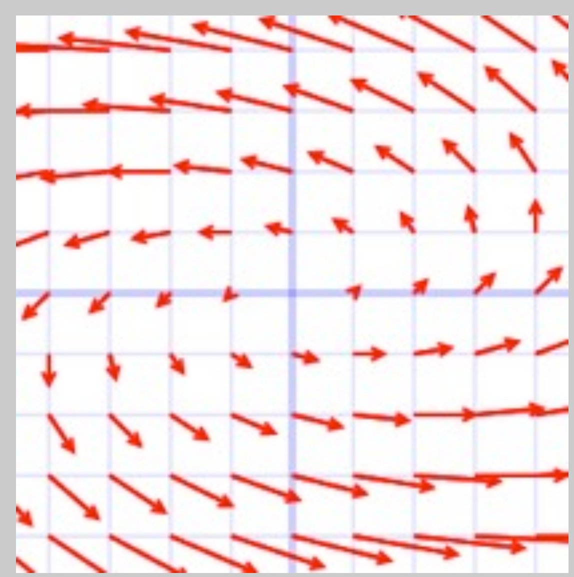

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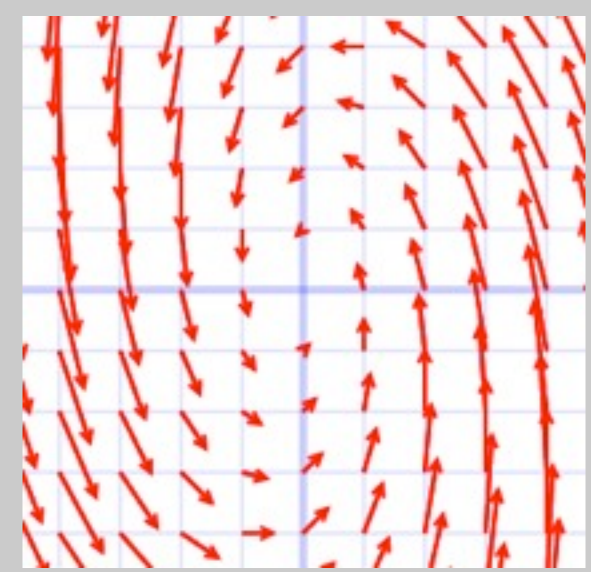

(E) Explain, please.

Review problems

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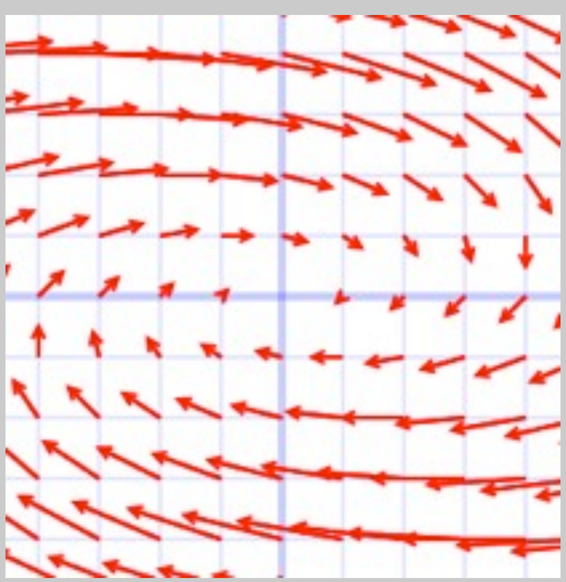
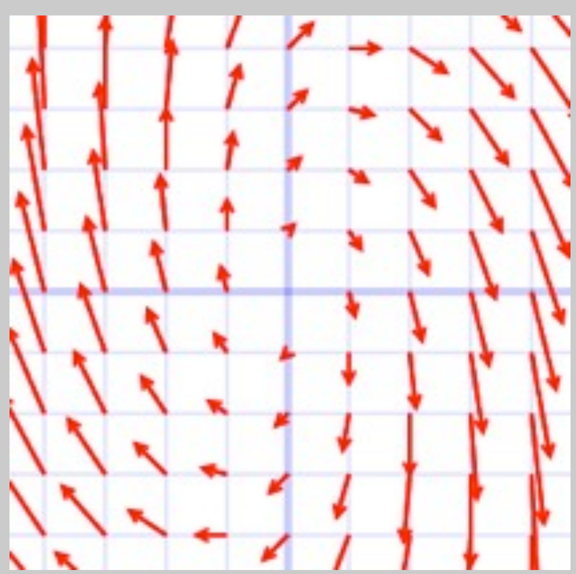


(B) ☆



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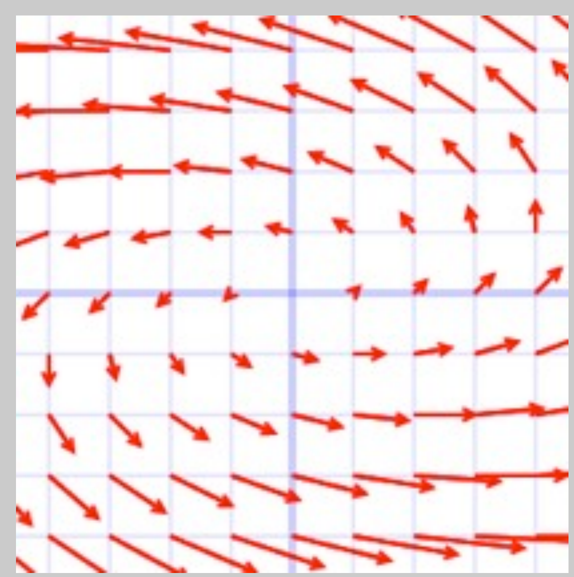

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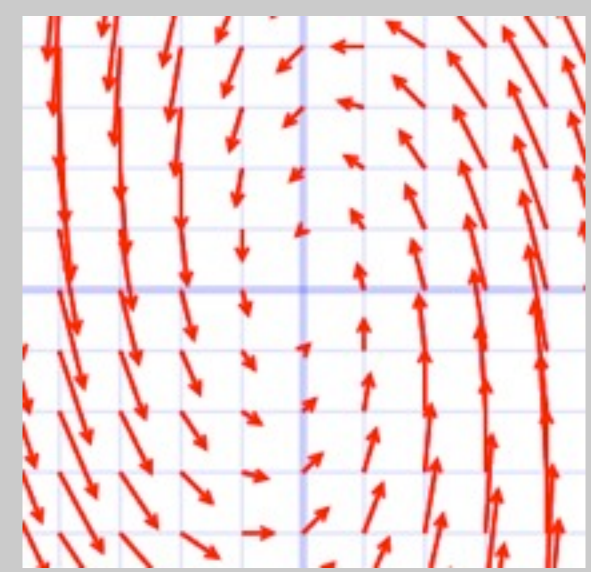

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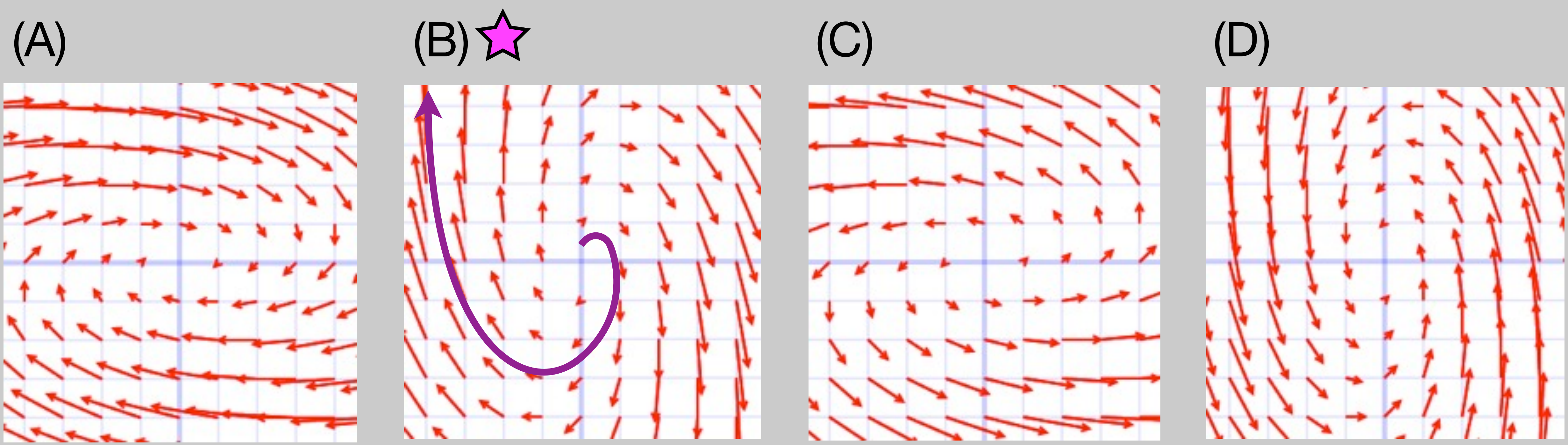

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Review problems

- A patient is given a 100 mg injection of a medication every 4 hours for weeks. The mean life of the drug in the bloodstream is 10 hours (so it is cleared at a rate 1/10 hour $^{-1}$). Sketch the amount of the drug in the patient's system as a function of time.

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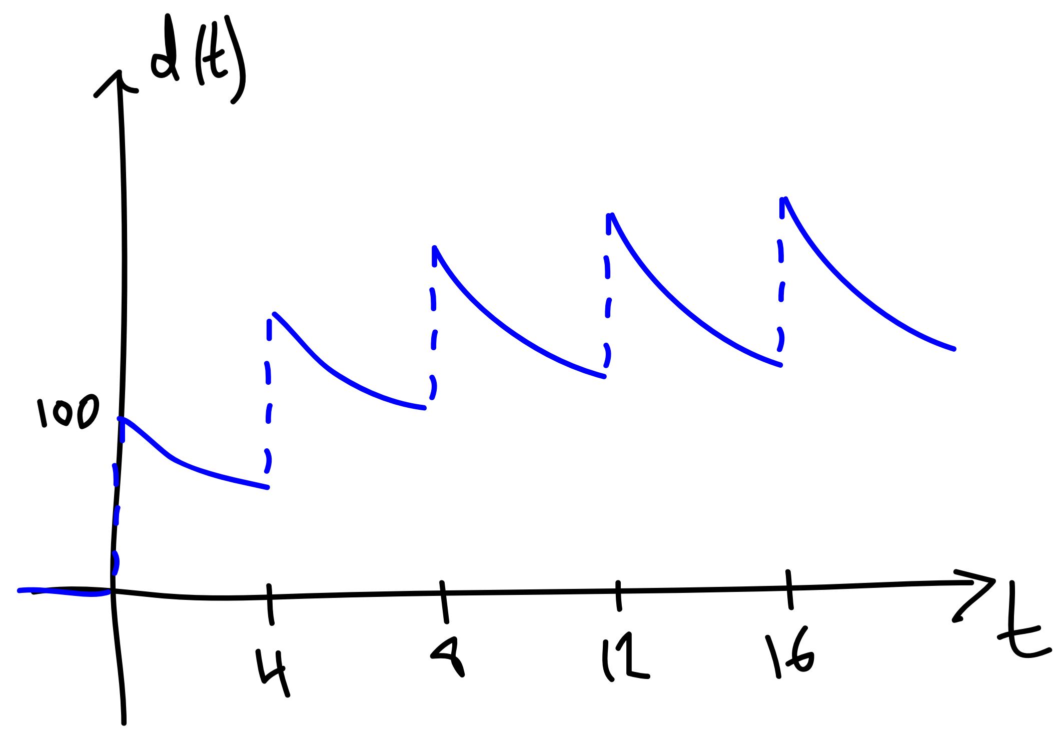

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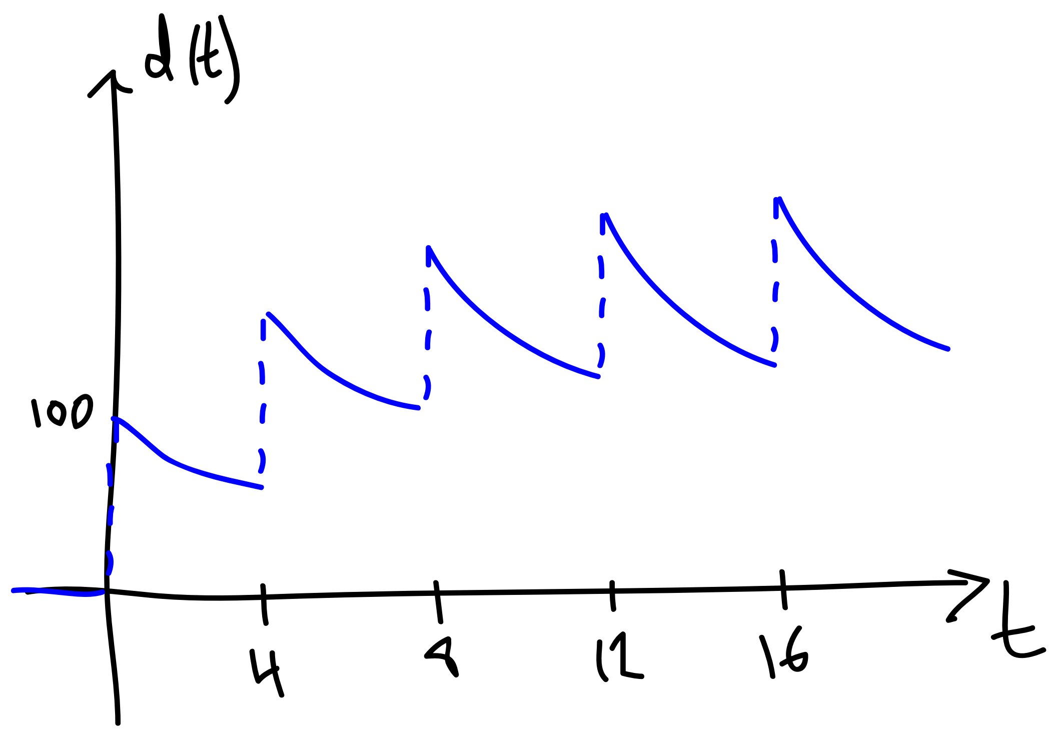
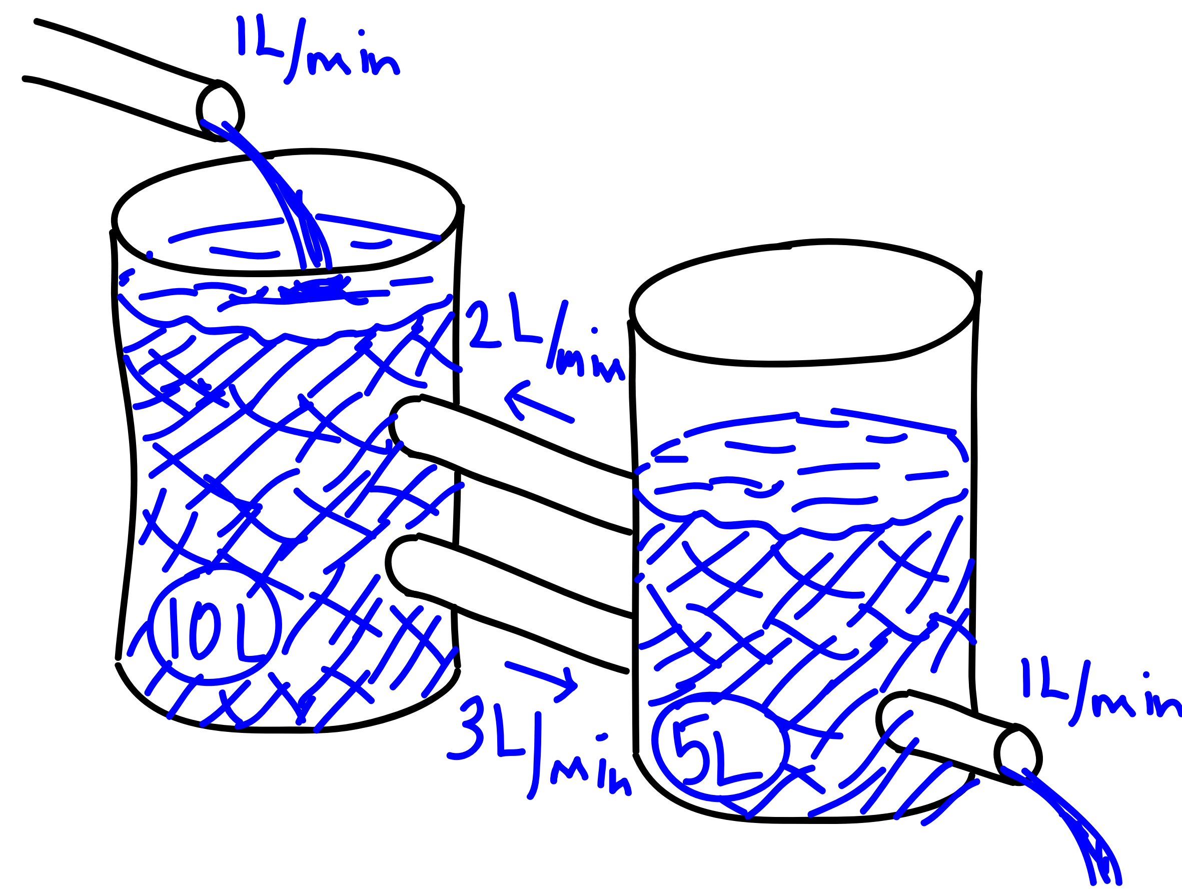


- Exercise (tricky): calculate the longterm minimum and maximum concentration.

Review problems

- Salt water flows into a tank holding 10 L of water at a rate of 1 L/min with a concentration of 200 g/L. The well-mixed solution flows from that tank into a tank holding 5 L through a pipe at 3 L/min. Another pipe takes the solution in the second tank back into the first at a rate of 2 L/min. Finally, solution drains out of the second tank at a rate of 1 L/min.
- Write down a system of equations in matrix form for the mass of salt in each tank.



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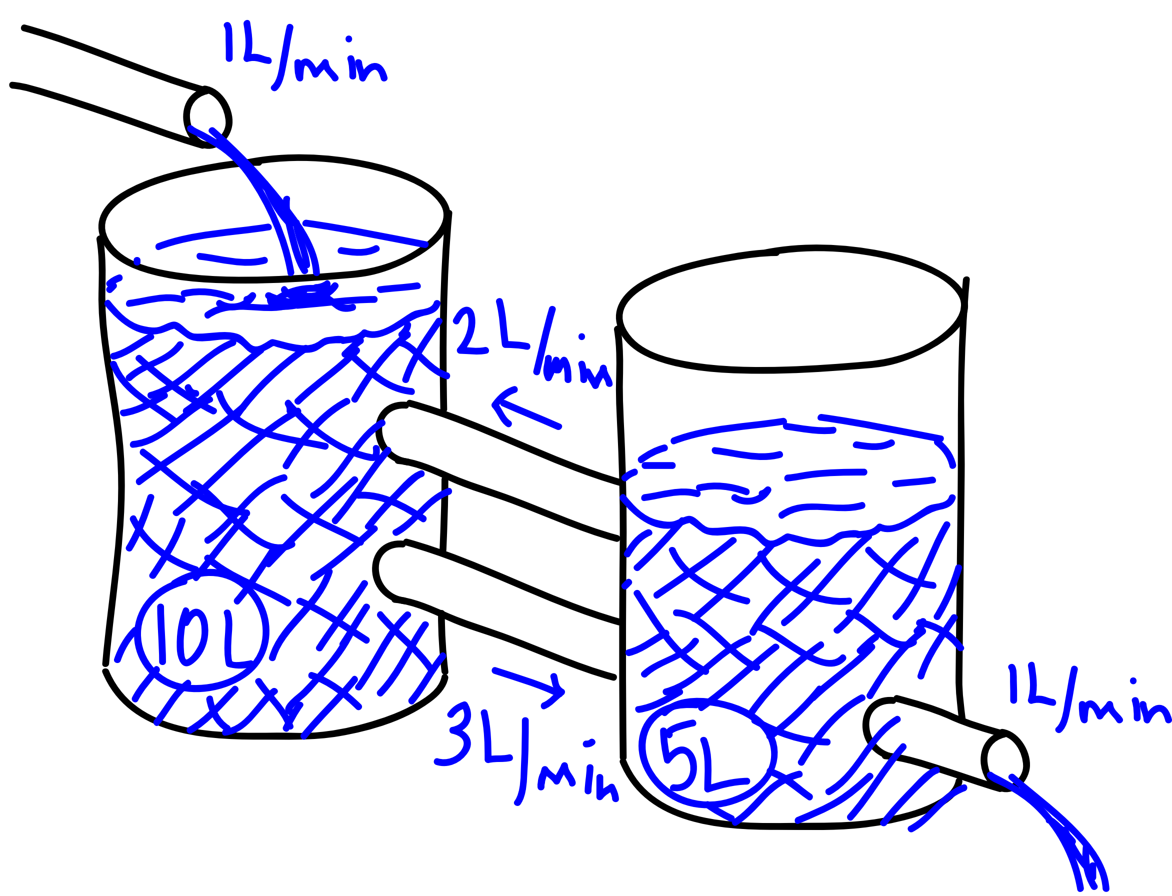

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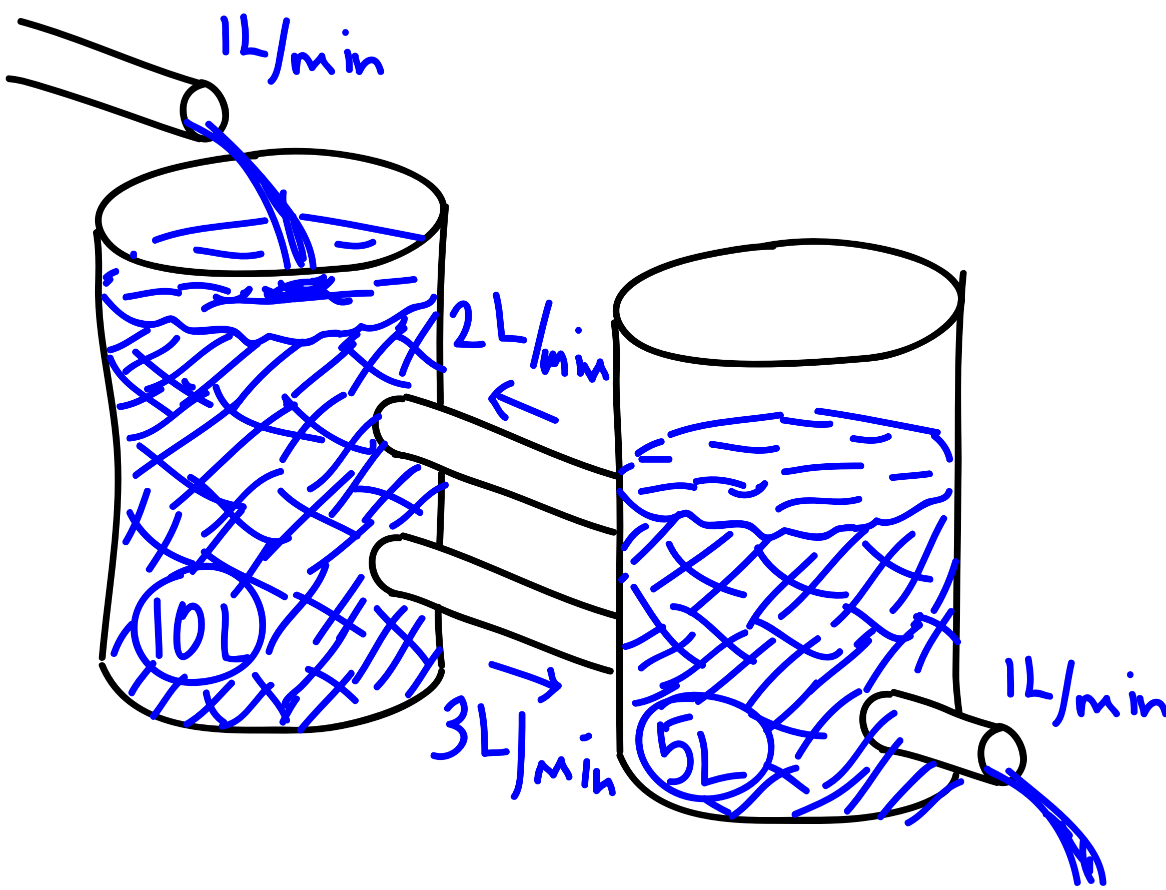
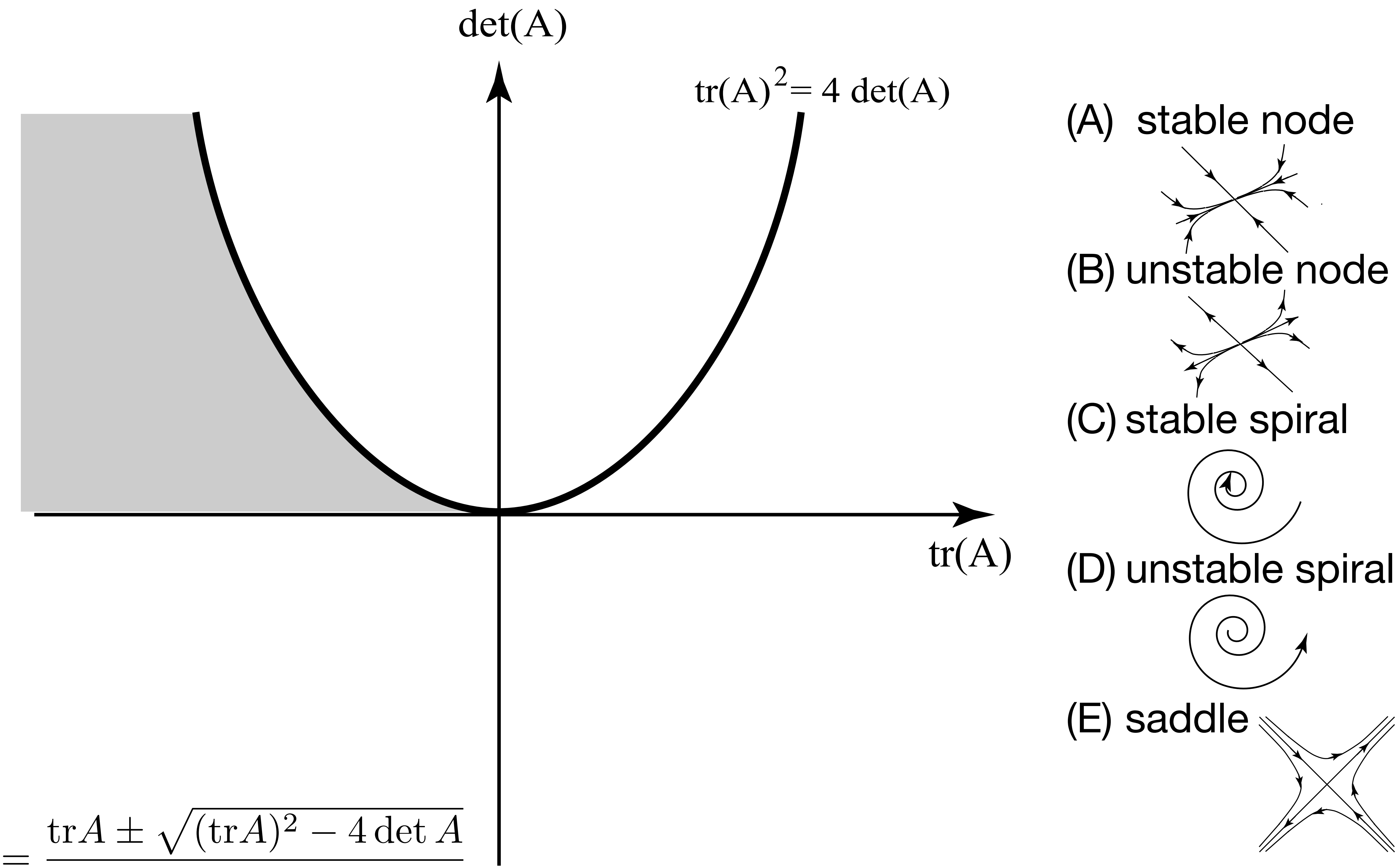


- What are $m_1(t)$ and $m_2(t)$ as $t \to \infty$?

Review problems



$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

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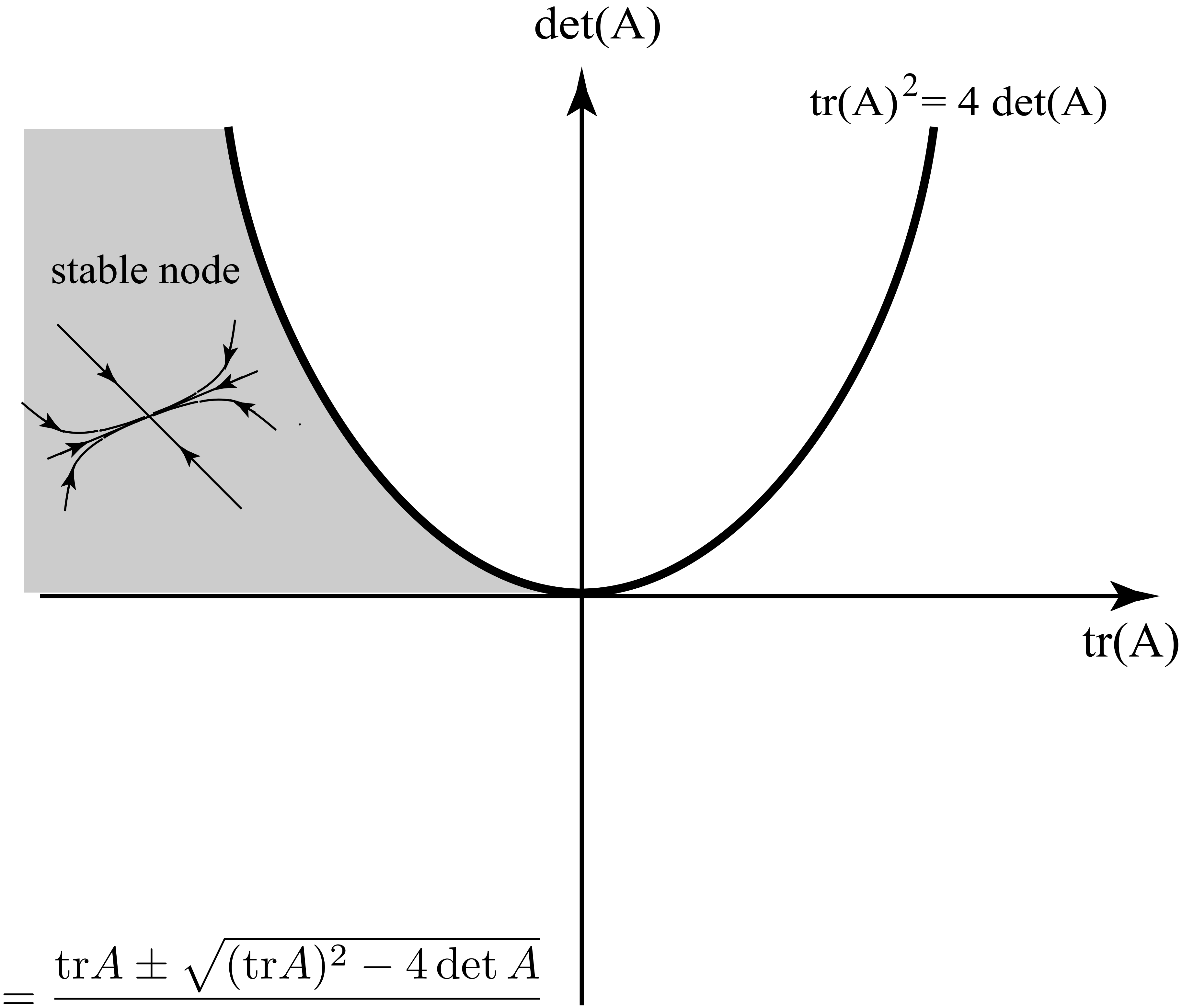
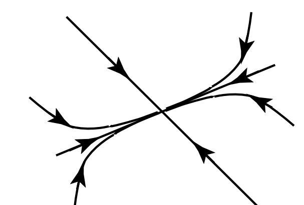


(A) stable node



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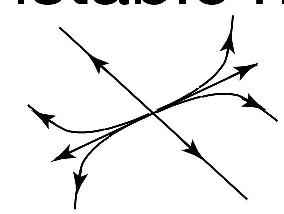
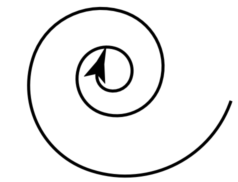
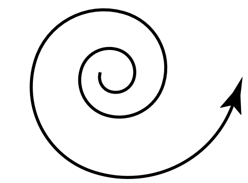


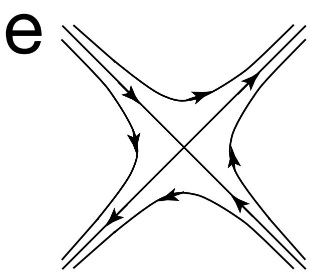
(C) stable spiral



(D) unstable spiral



(E) saddle



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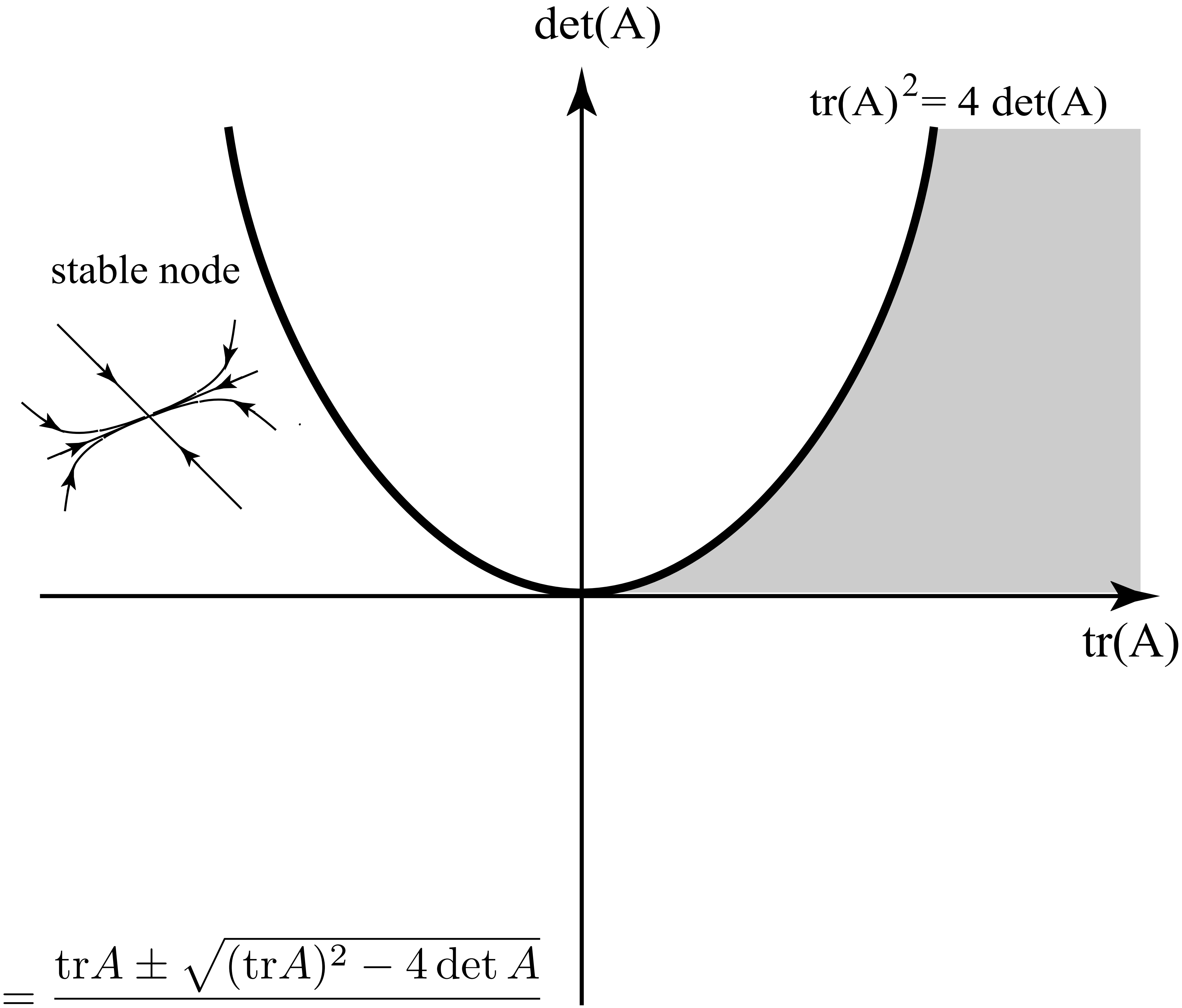

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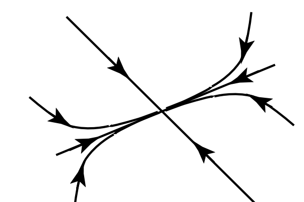

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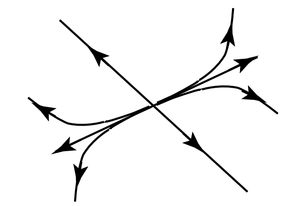

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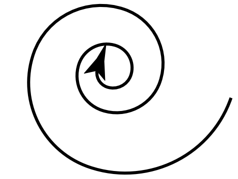

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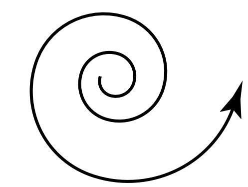

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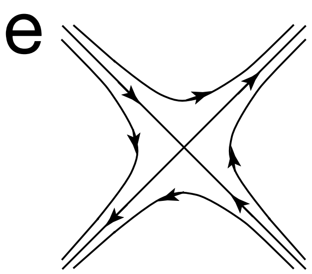

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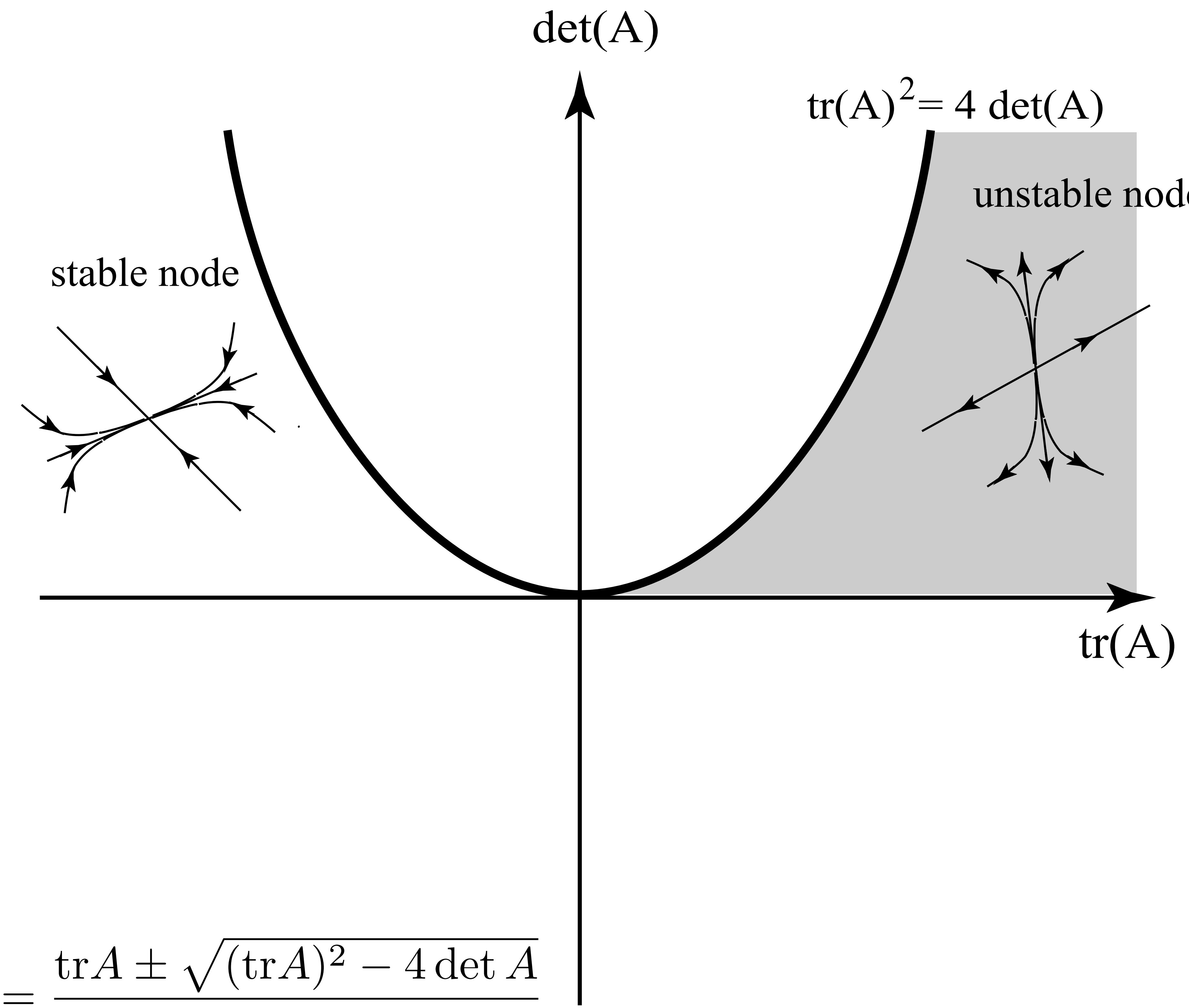
Review problems



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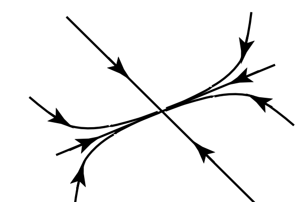

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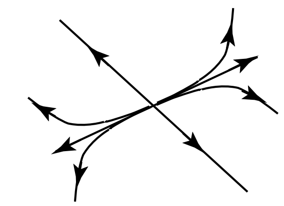

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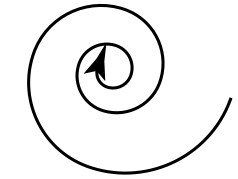

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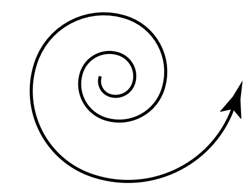

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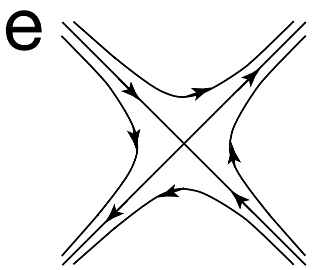

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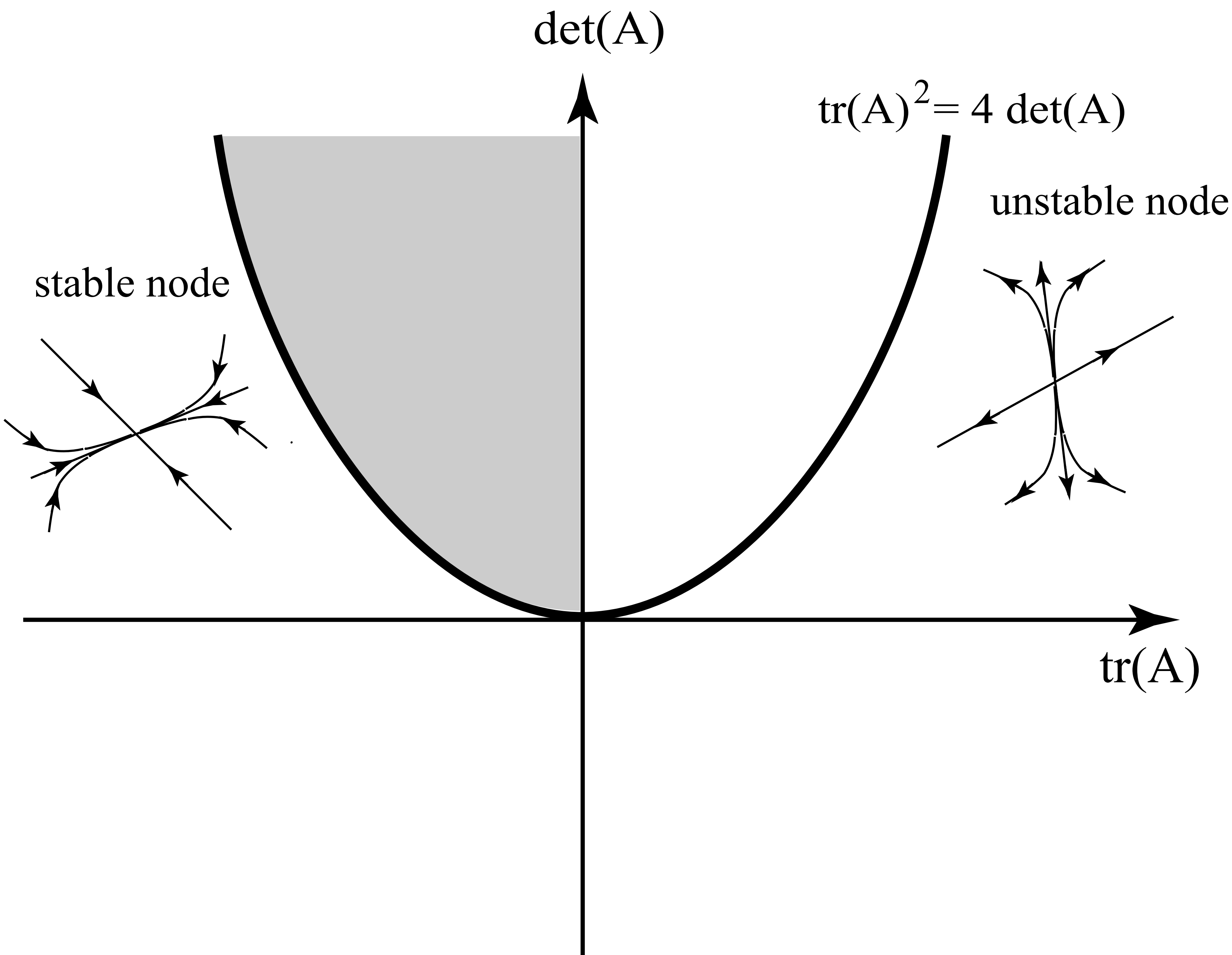
Review problems



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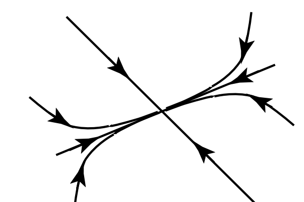

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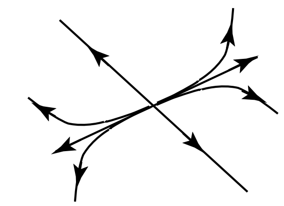

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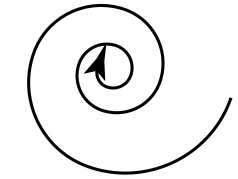

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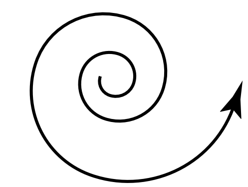

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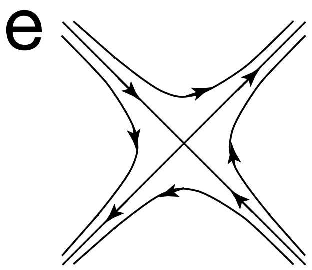

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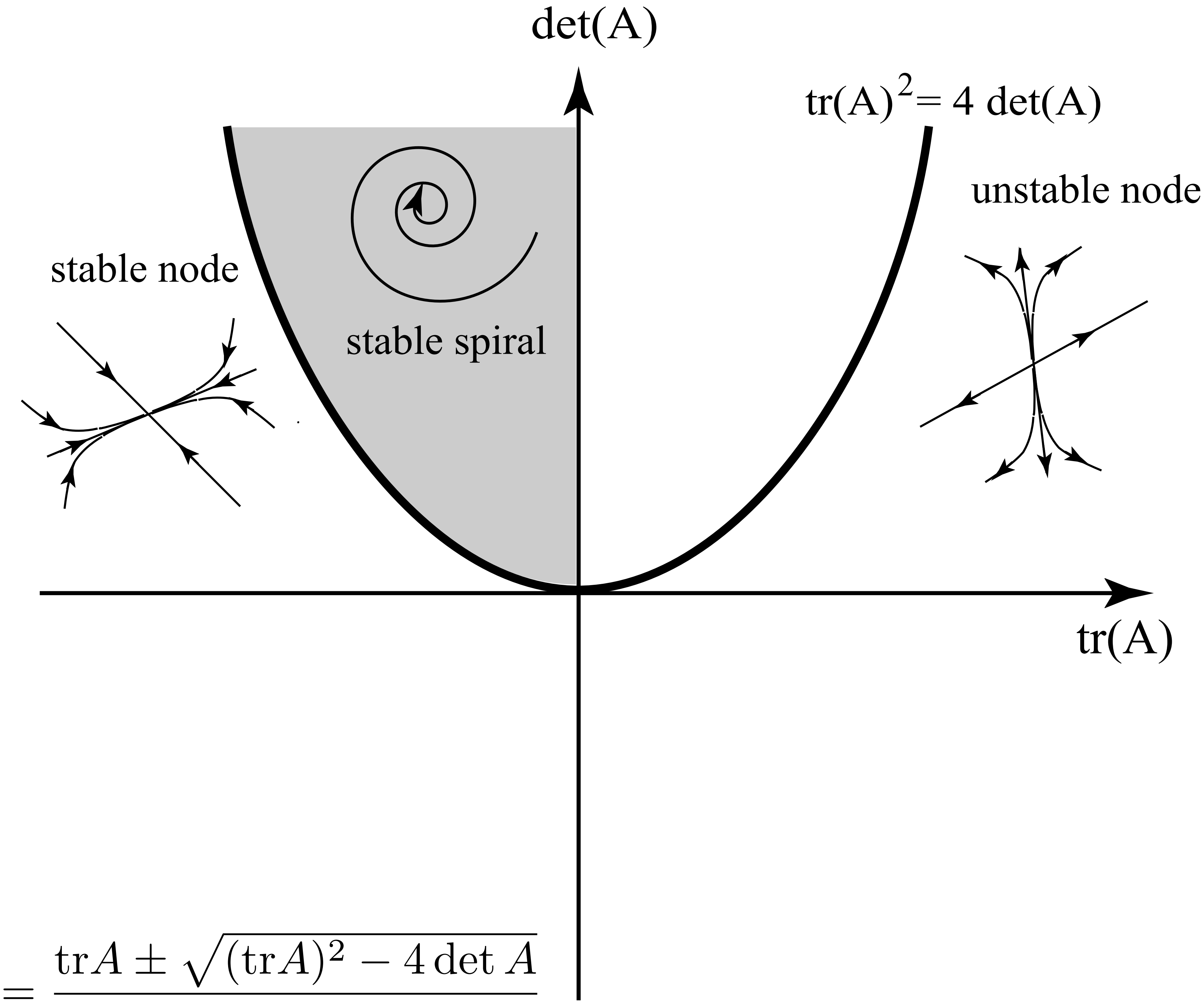

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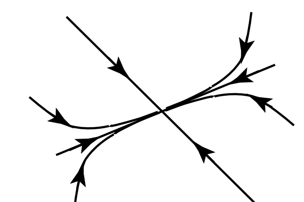

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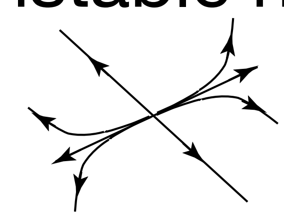

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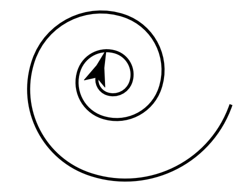
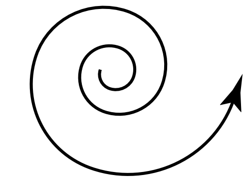
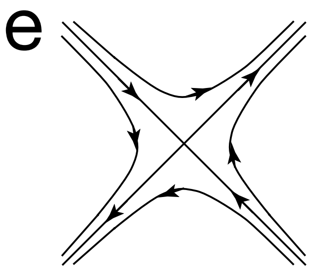


(D) unstable spiral

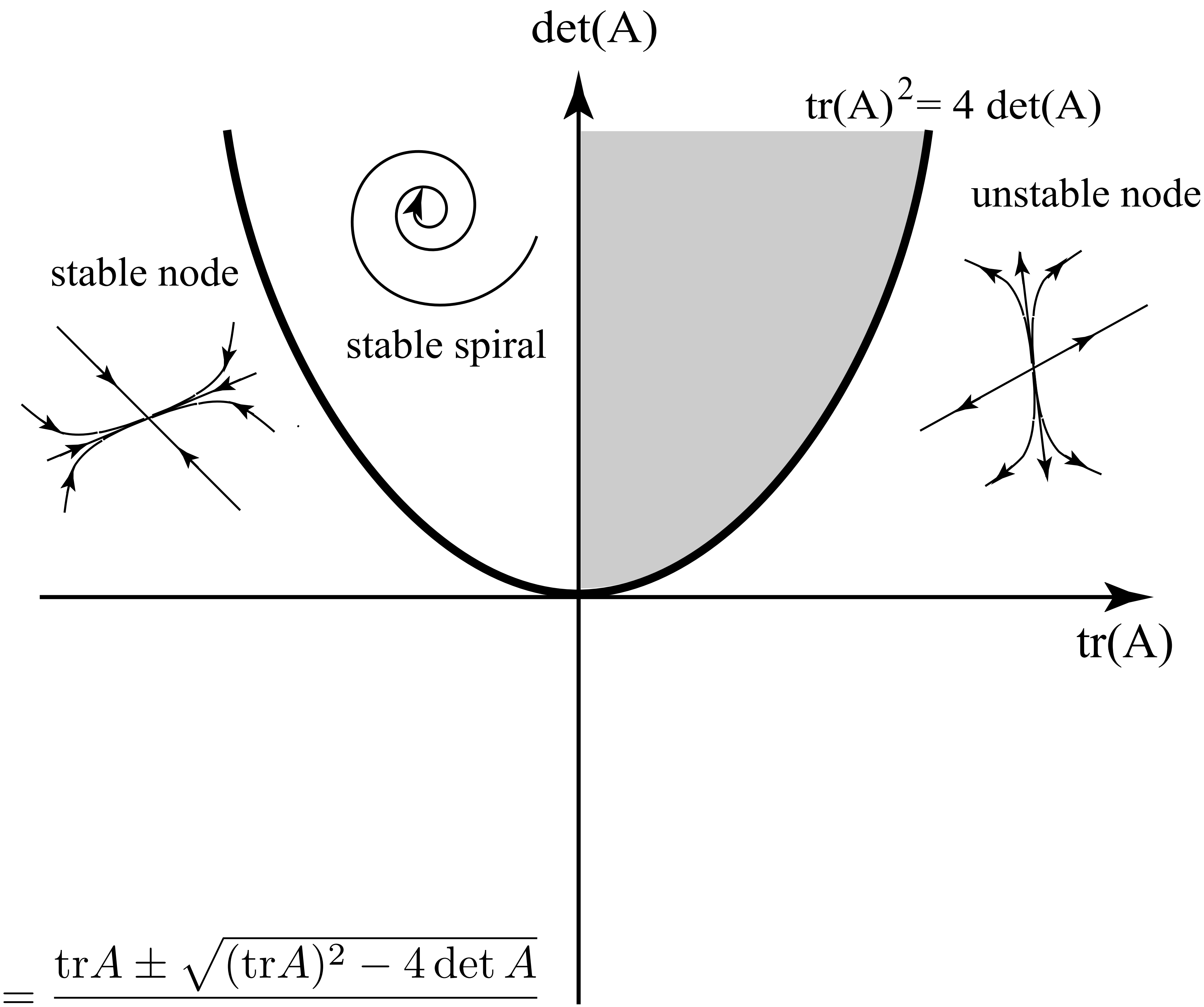


(E) saddle

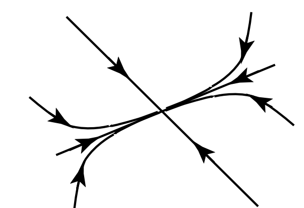


$$\lambda = \frac{\mathrm{tr} A \pm \sqrt{(\mathrm{tr} A)^2 - 4 \det A}}{2}$$

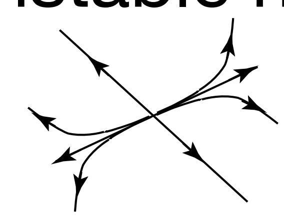
Review problems



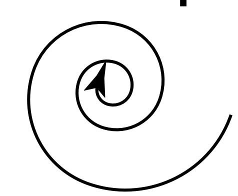
(A) stable node



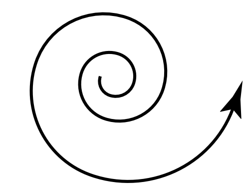
(B) unstable node



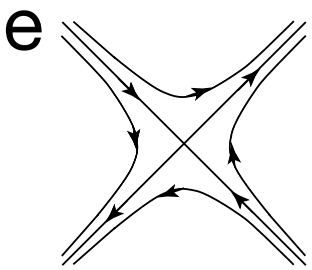
(C) stable spiral



(D) unstable spiral

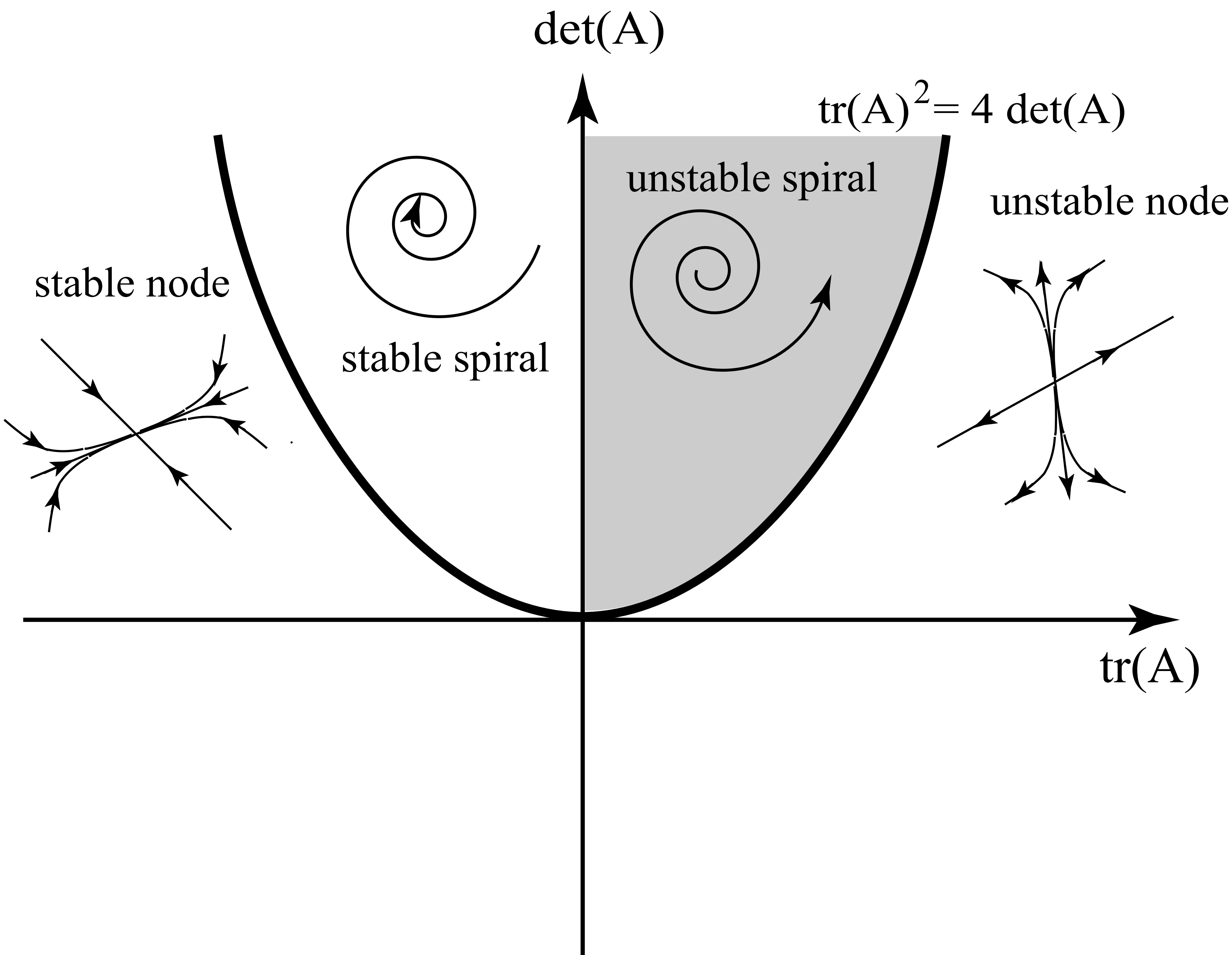


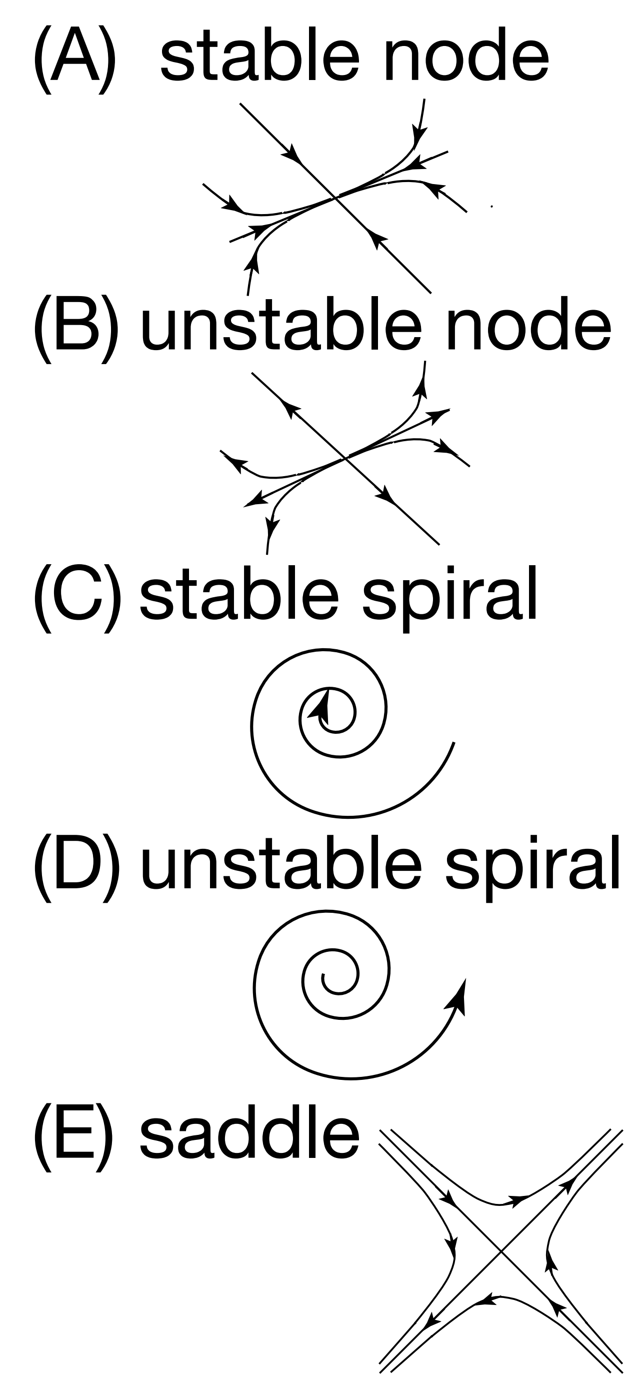
(E) saddle



$$\lambda = \frac{\mathrm{tr} A \pm \sqrt{(\mathrm{tr} A)^2 - 4 \det A}}{2}$$

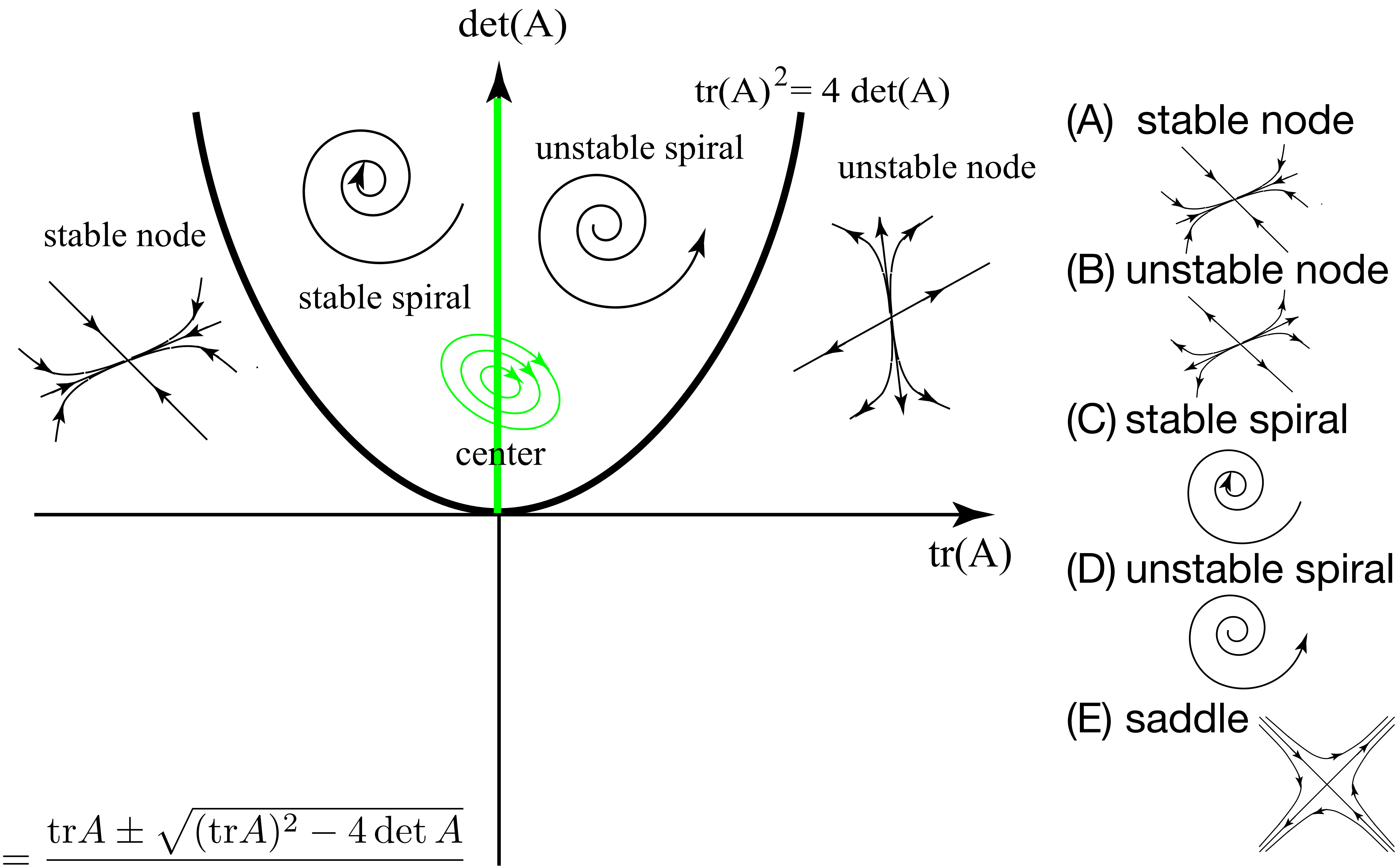
Review problems





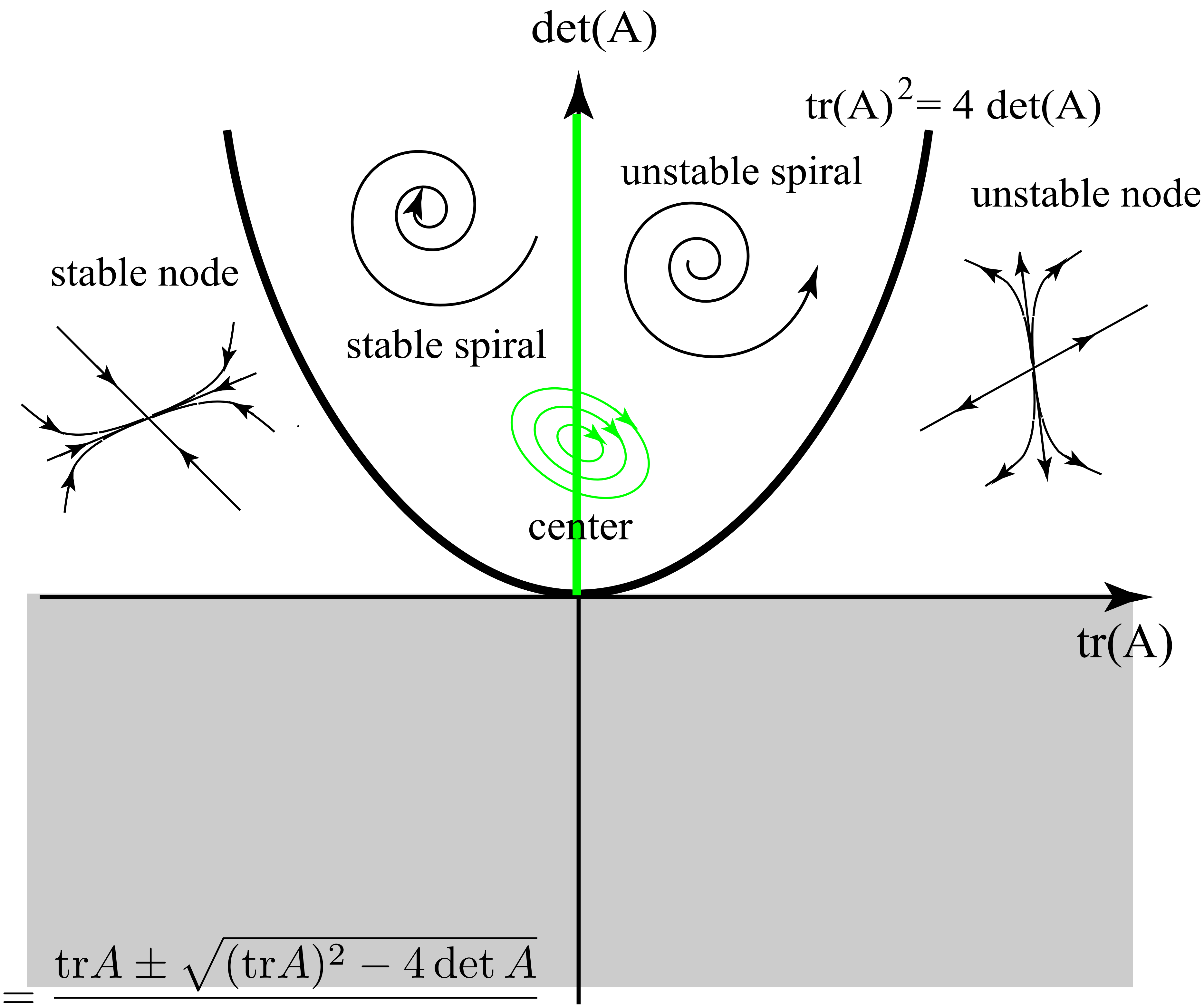
$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

Review problems



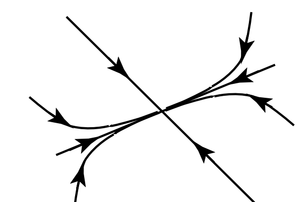
$$\lambda = \frac{\mathrm{tr} A \pm \sqrt{(\mathrm{tr} A)^2 - 4 \det A}}{2}$$

Review problems

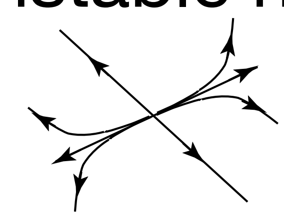


$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

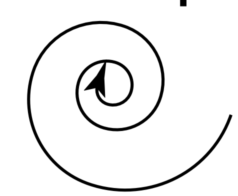
(A) stable node



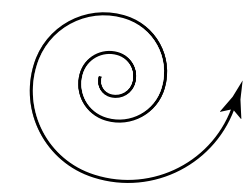
(B) unstable node



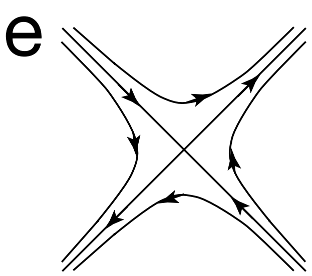
(C) stable spiral



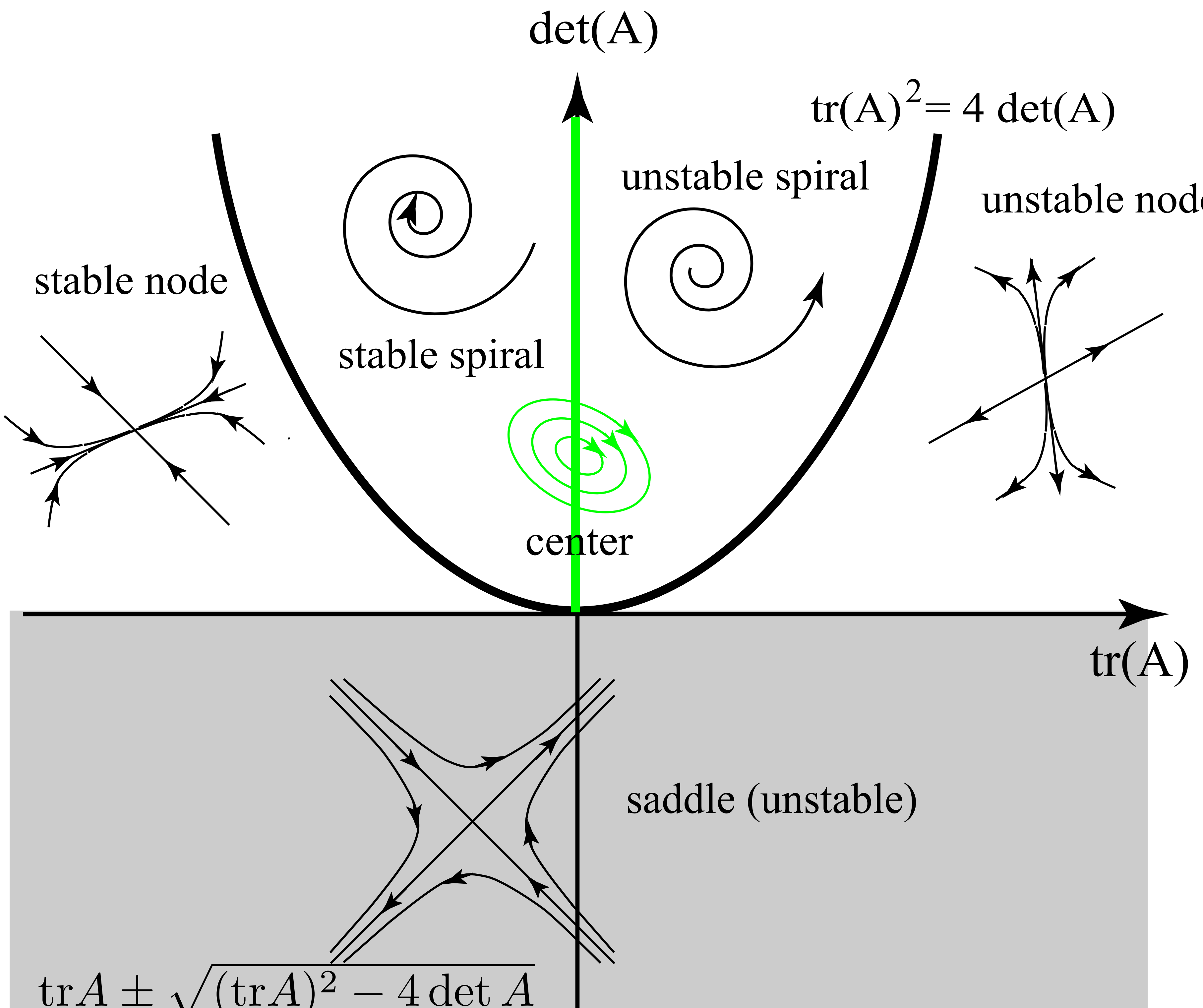
(D) unstable spiral



(E) saddle

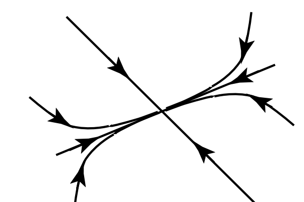


Review problems

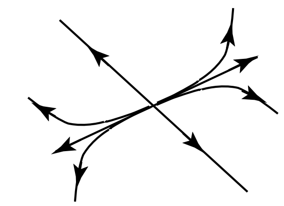


$$\lambda = \frac{\mathrm{tr}\,A \pm \sqrt{(\mathrm{tr}\,A)^2 - 4\det A}}{2}$$

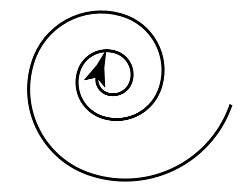
(A) stable node



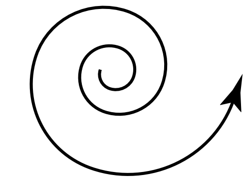
(B) unstable node



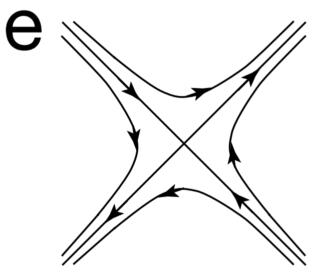
(C) stable spiral



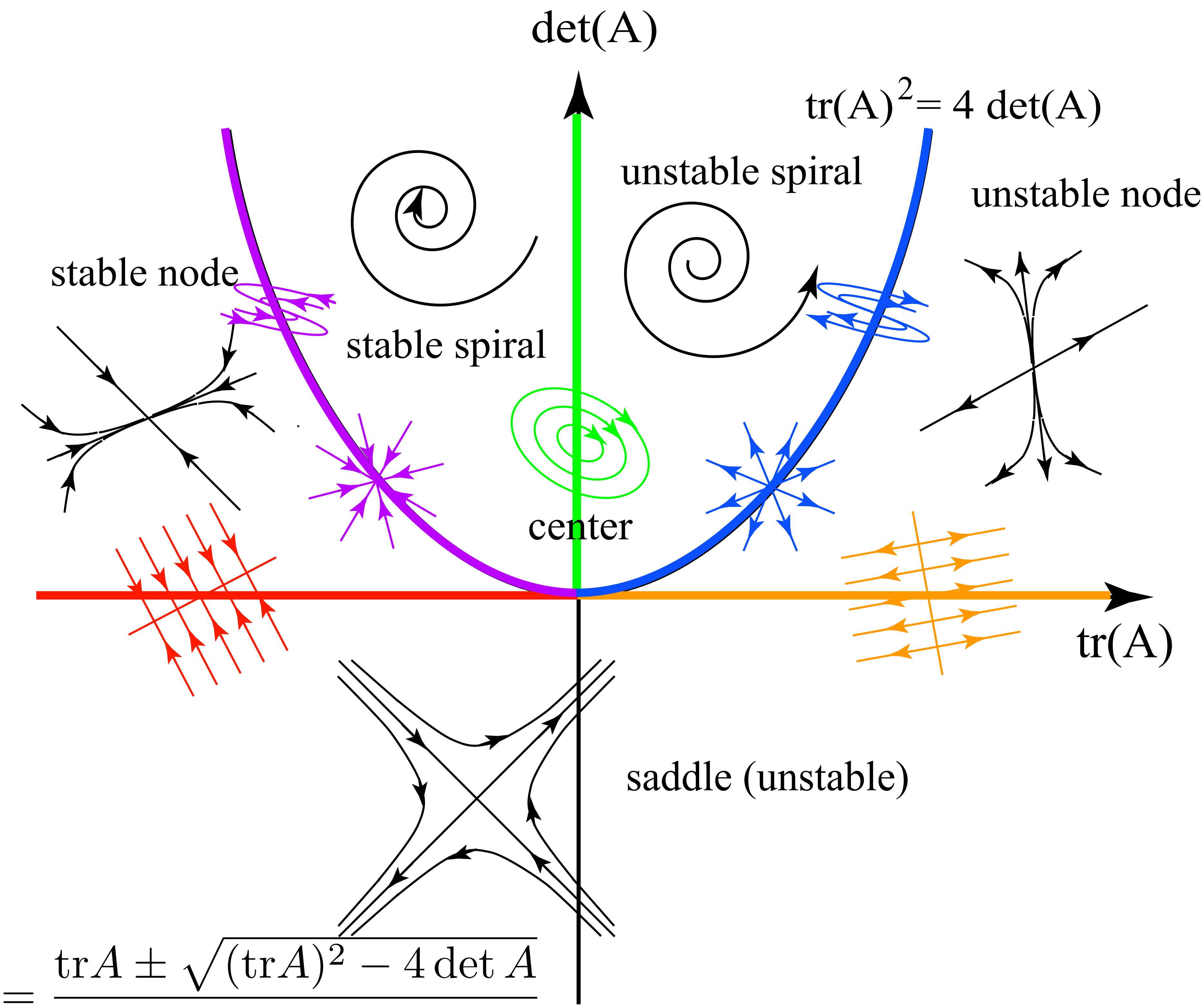
(D) unstable spiral



(E) saddle

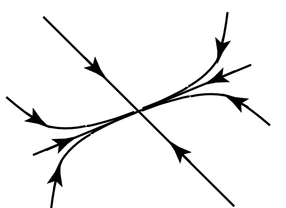


Review problems

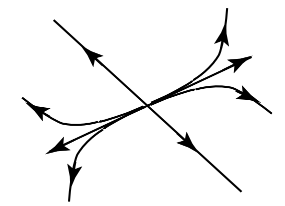


$$\lambda = \frac{\mathrm{tr} A \pm \sqrt{(\mathrm{tr} A)^2 - 4 \det A}}{2}$$

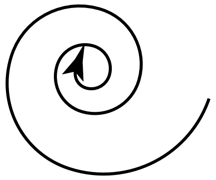
(A) stable node



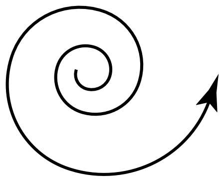
(B) unstable node



(C) stable spiral



(D) unstable spiral



(E) saddle