MATH 256-201

Tutorial 6 Worksheet

Feb 27, 2017

Surname: ____________________ Given name: ____________________

Student number: ____________________

1. Find the general solution to the following linear system of differential equations

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} \mathbf{x} \tag{1}$$

2. Consider the following system

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix} \mathbf{x} + \mathbf{b}, \quad \text{where} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

   (a) Determine the general solution when $b_1 = 1$ and $b_2 = 2$.

(b) In part (a), b_1 and b_2 were specially chosen in that the row reduced form of the matrix equation had a full row of zeros (including the RHS) and therefore a solution. Write down an equation for b_1 and b_2 that ensures this will happen.

(c) Now consider the case with $b_1 = 3$ and $b_2 = 3$. Because this $\mathbf{b}$ does not satisfy the equation from part (b), a different form for your particular guess is needed. By analogy with second order systems, we guess $\mathbf{x_p} = t\mathbf{v} + \mathbf{w}$. Now we take the following steps to find out the general solution for this case:

i. Plugging this $\mathbf{x_p}$ into the system of ODEs, we find that we must have $\mathbf{v} = t\mathbf{A}\mathbf{v} + \mathbf{A}\mathbf{w} + \mathbf{b}$ for all t. This requires that $\mathbf{A}\mathbf{v} = 0$ and $\mathbf{A}\mathbf{w} = \mathbf{v} - \mathbf{b}$;

ii. $\mathbf{A}\mathbf{v} = 0$ has a whole family of solutions. In fact, since 0 is an eigenvalue of $\mathbf{A}$, $\mathbf{v}$ should be a corresponding eigenvector;

iii. Next consider the equation $\mathbf{A}\mathbf{w} = \mathbf{v} - \mathbf{b}$ which we must solve for $\mathbf{w}$. Notice that for any vector $\mathbf{w}$, $\mathbf{A}\mathbf{w}$ will always have a second component equal to twice its first component. Thus, to be able to solve $\mathbf{A}\mathbf{w} = \mathbf{v} - \mathbf{b}$, we must make sure that $\mathbf{v} - \mathbf{b}$ has second component equal to twice its first component. Find the vector $\mathbf{v}$ from the family of solutions to $\mathbf{A}\mathbf{v} = 0$ that does this. Then find $\mathbf{w}$ and write down the general solution for this case.