Separable equations

- First order ODEs of the form: $\frac{dy}{dx} = f(x)h(y)$
- Rename h(y) = 1/g(y): $\frac{dy}{dx} = \frac{f(x)}{g(y)}$
- Rewrite as $g(y)\frac{dy}{dx} = f(x)$.
- Rewrite g and f as derivatives of other functions: $G'(y)\frac{dy}{dx} = F'(x)$.
- Recognize a chain rule: $\frac{d}{dx}(G(y)) = G'(y)\frac{dy}{dx}$.
- Take antiderivatives to get $G(y) = F(x) + C$.
- Finally, solve for y if possible: $y(x) = G^{-1}(F(x) + C)$.

Separable equations

- Solve: $\frac{dy}{dx} = -\frac{x}{y}$

(A) $y(x) = x$

(B) $y(x) = \sqrt{C - x^2}$

(C) $y(x) = \sqrt{x^2 + C}$

(D) $y(x) = C - x^2$

(E) None of these (or don't know)

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$$y\frac{dy}{dx} = -x$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + D$$

$$y^2 = -x^2 + C$$

Does (B) cover all possible initial conditions?

Separable equations

$$y(x) = \sqrt{C - x^2}$$

- y(0)=2 ----> C=4
- y(1)=1 ----> C=2
- y(1)=-2 ----> C=?
- General solution: $y = \pm\sqrt{C - x^2}$
- Or express implicitly: $y^2 = -x^2 + C$
- To satisfy an IC, must choose a value for C *and* choose + or - .

Separable equations

- Solve: $\frac{dy}{dt} = \frac{1}{\cos(y)}$

(A) $y(t) = \sin(t)$

(B) $y(t) = \arcsin(t + C)$

(C) $\sin(y) = t + C$

(D) $y(t) = \arcsin(t) + C$

(E) $y(t) = \arccos(t + C)$

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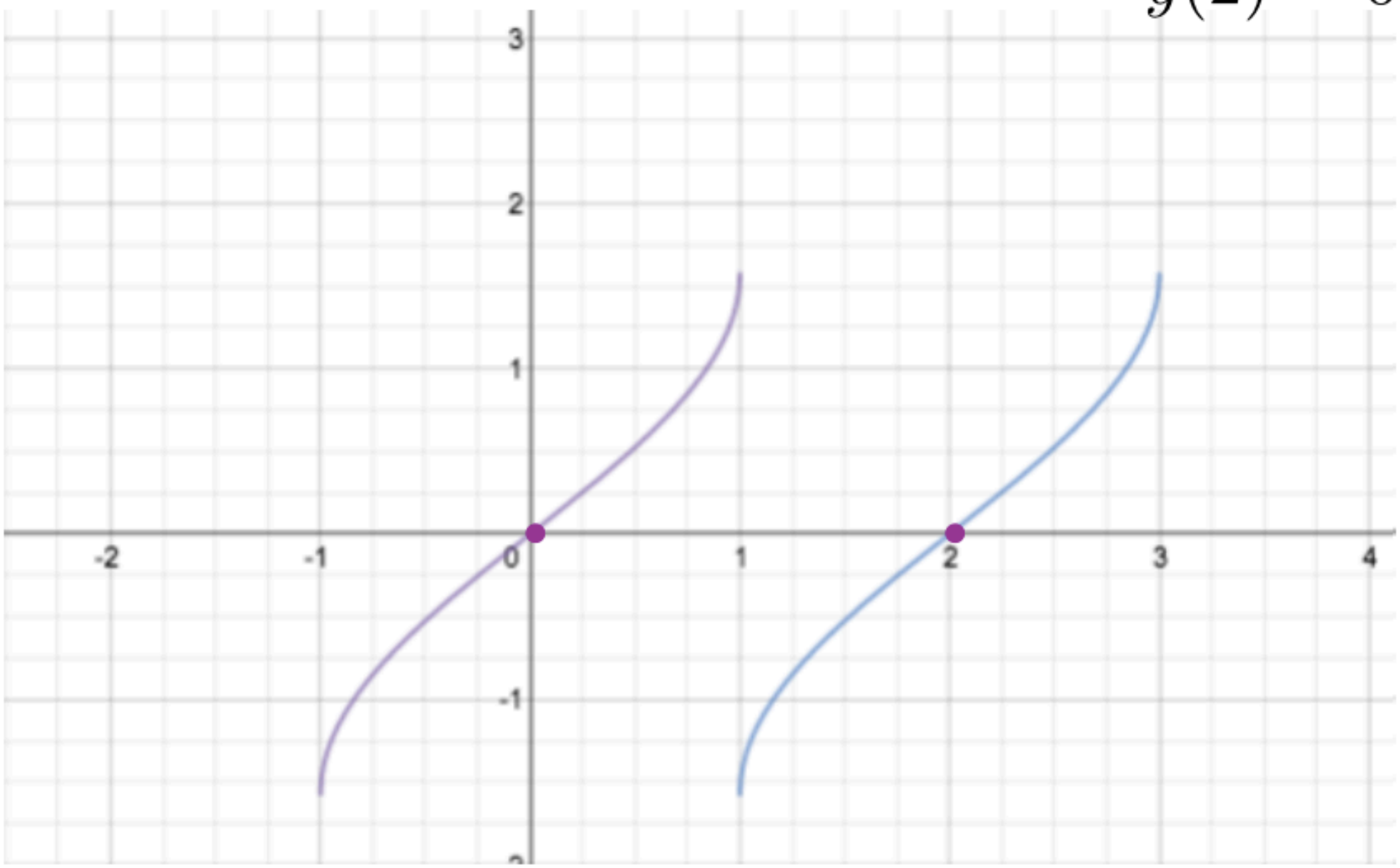
Separable equations

$$y(t) = \arcsin(t + C)$$

with IC $y(0) = 0$ $\quad C = 0$

$y(2) = 0$ $\quad C = -2$



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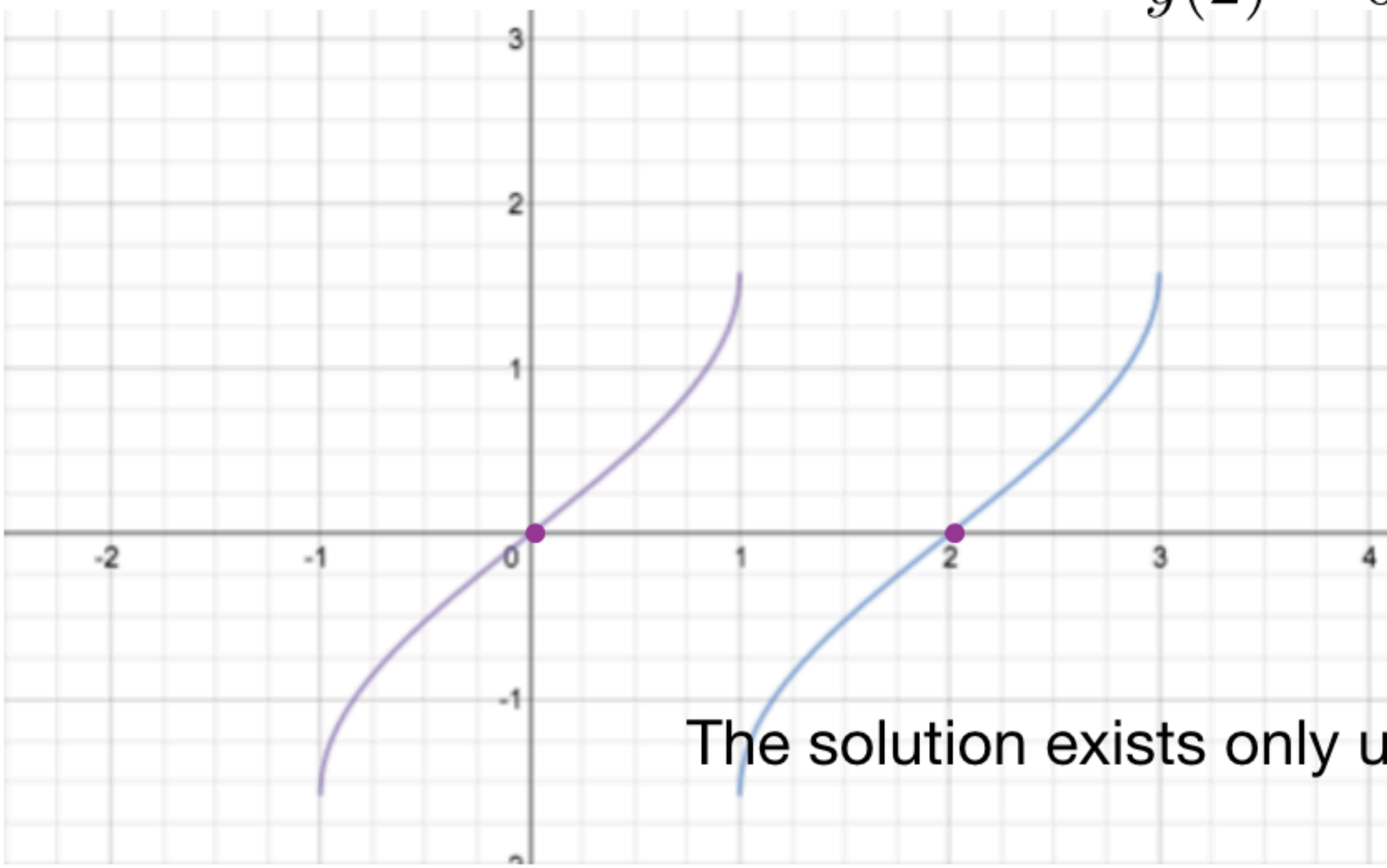

The solution exists only up until t=3.

Separable equations

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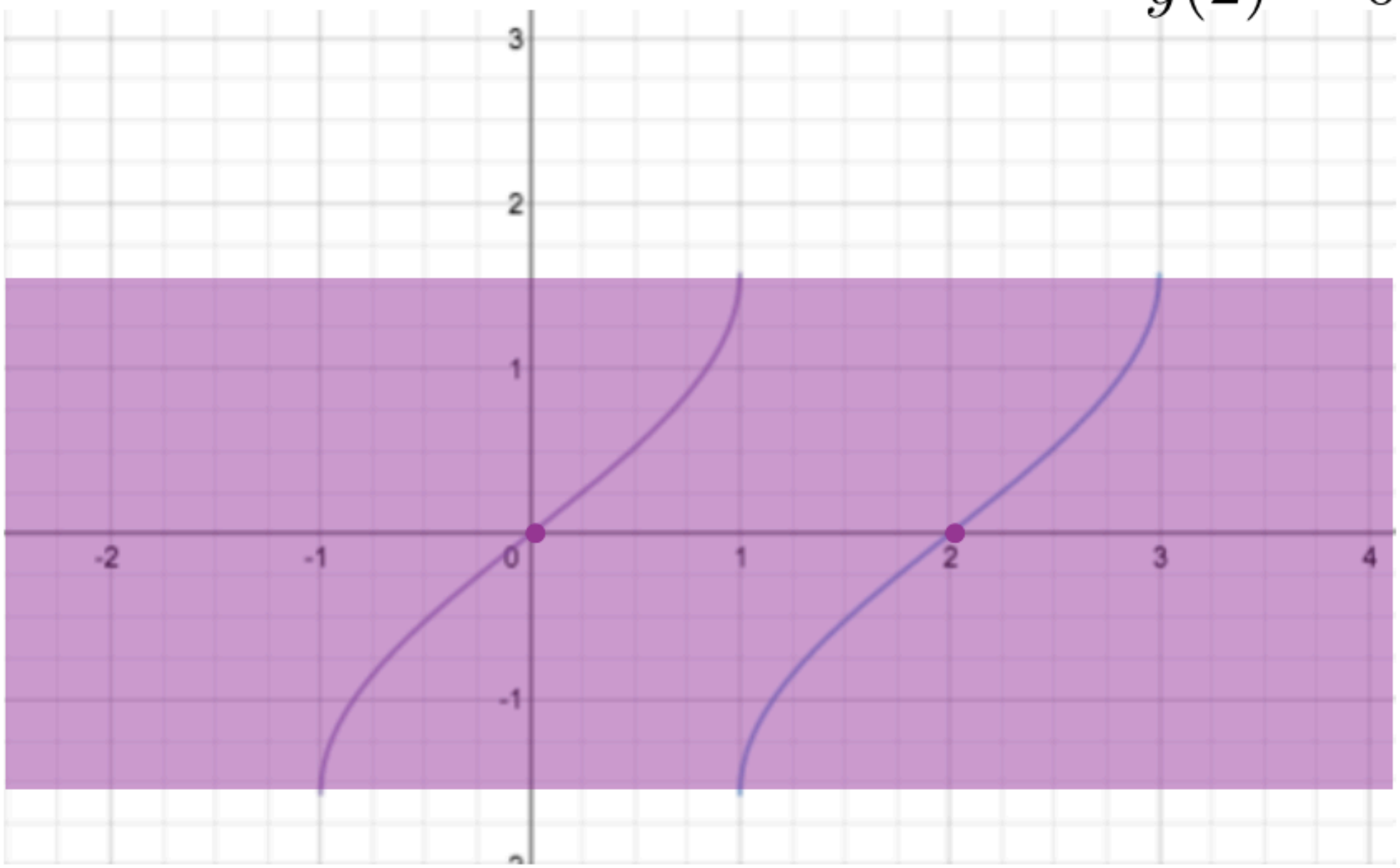


Separable equations

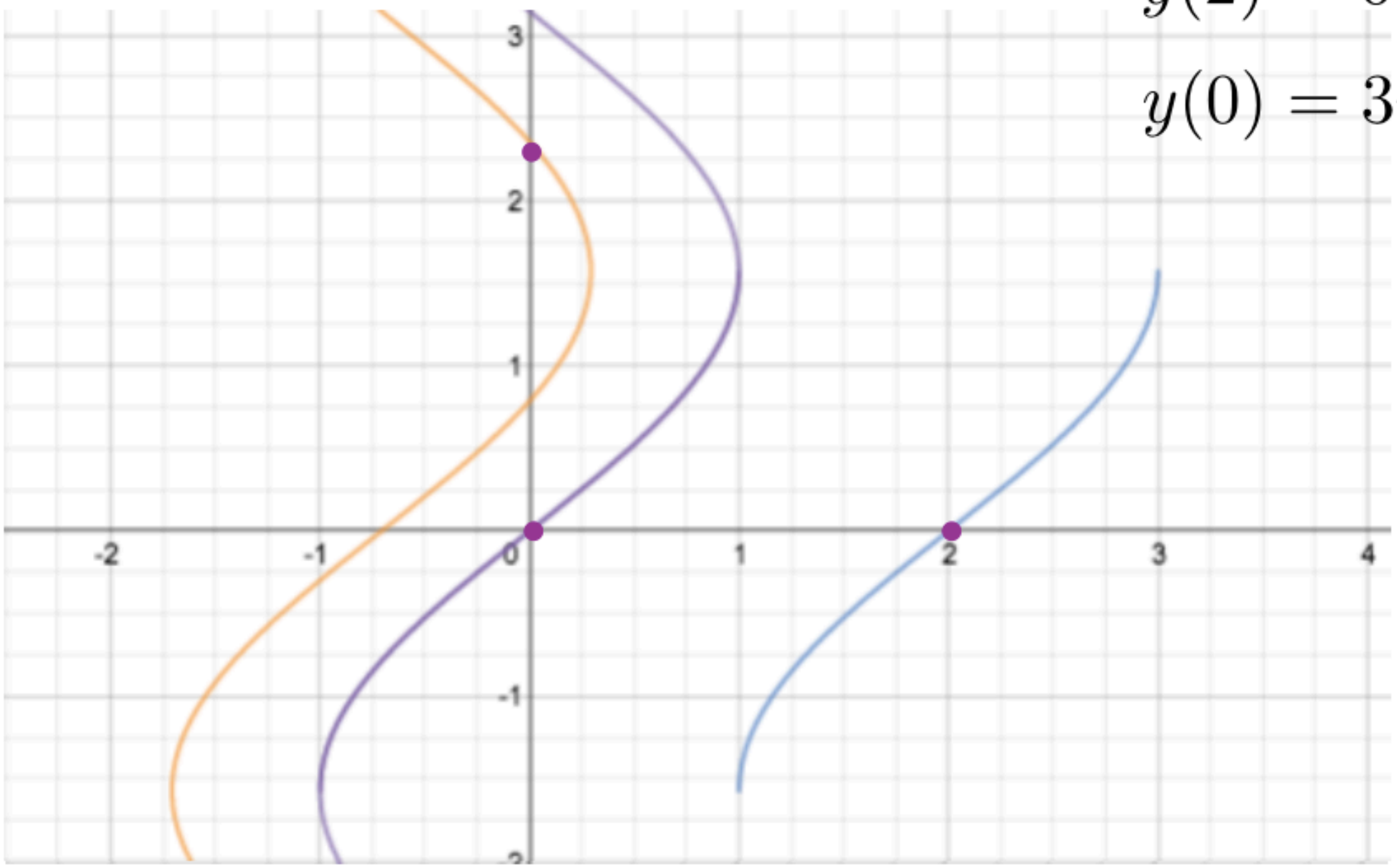
$$\sin(y) = t + C$$

with IC

$y(0) = 0 \quad C = 0$

$y(2) = 0 \quad C = -2$

$y(0) = 3\pi/4 \quad C = \frac{1}{\sqrt{2}}$



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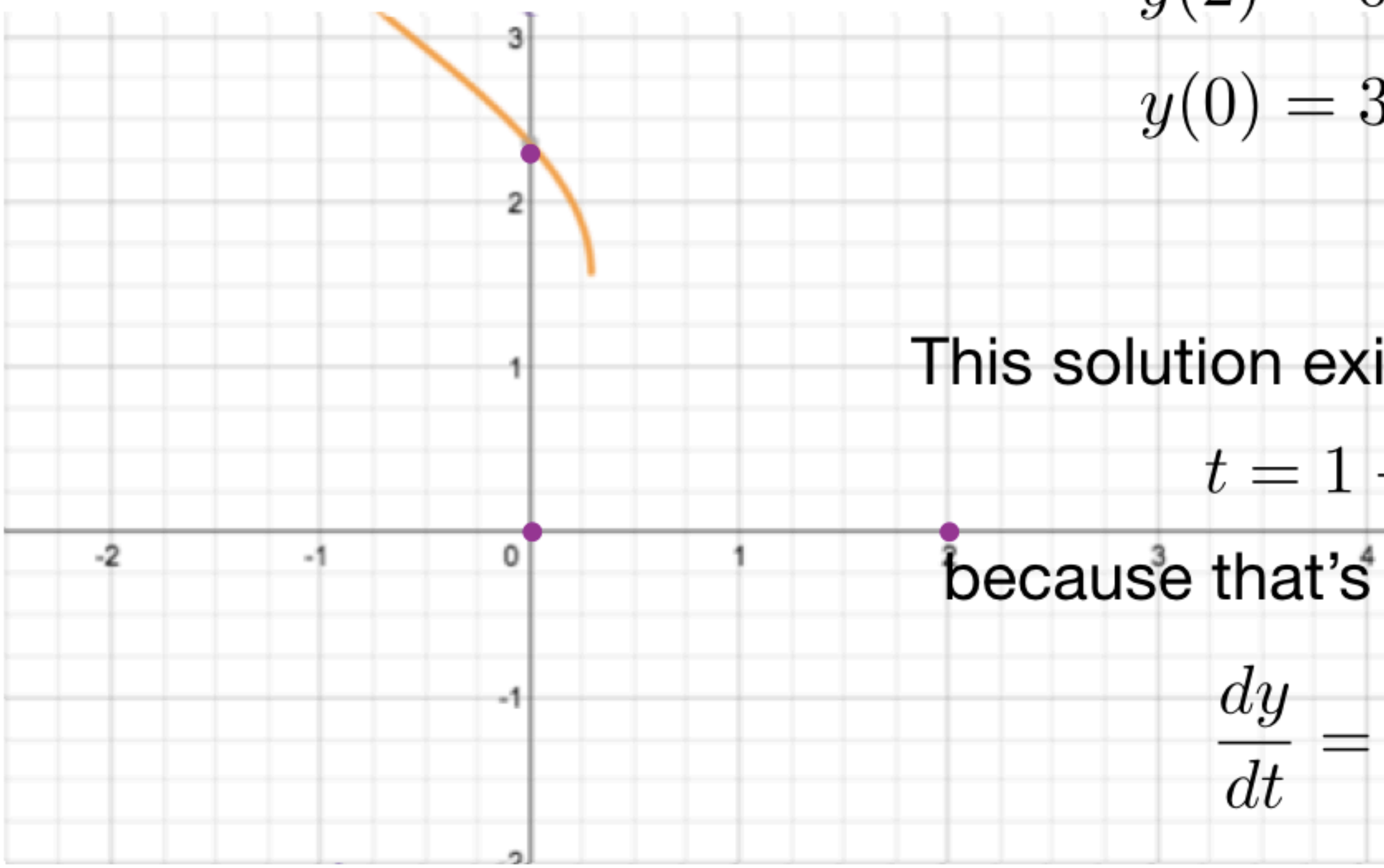


This solution exists only up until

$$t = 1 - 1/\sqrt{2}$$

because that's when y = π/2.

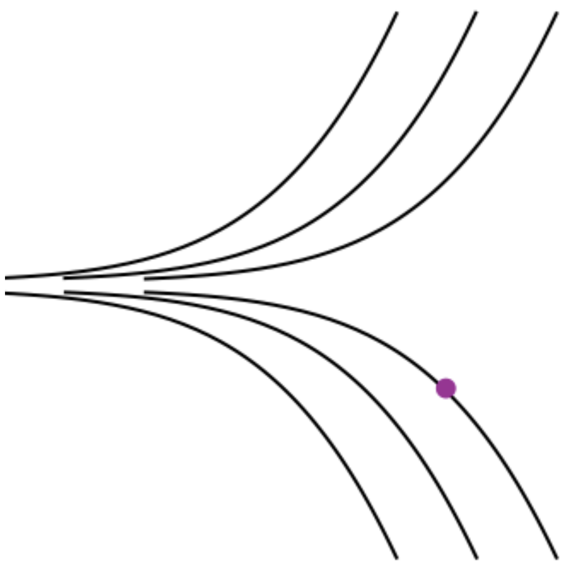
$$\frac{dy}{dt} = \frac{1}{\cos(y)}$$

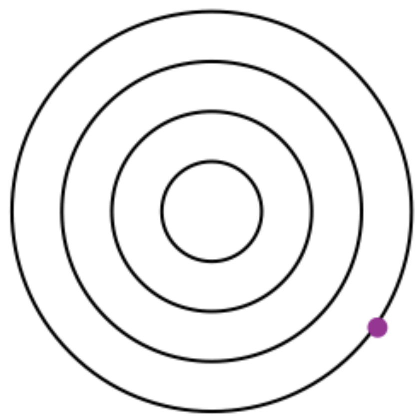
When do we get this kind of problem with ICs?

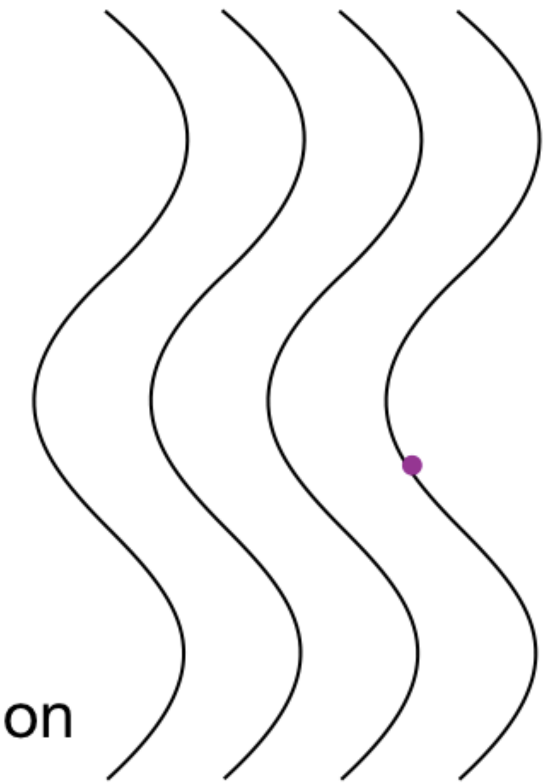
$y = Ce^{3t}$

$y^2 = -x^2 + C$

$\sin(y) = t + C$







Choose C so that the function goes through the IC.

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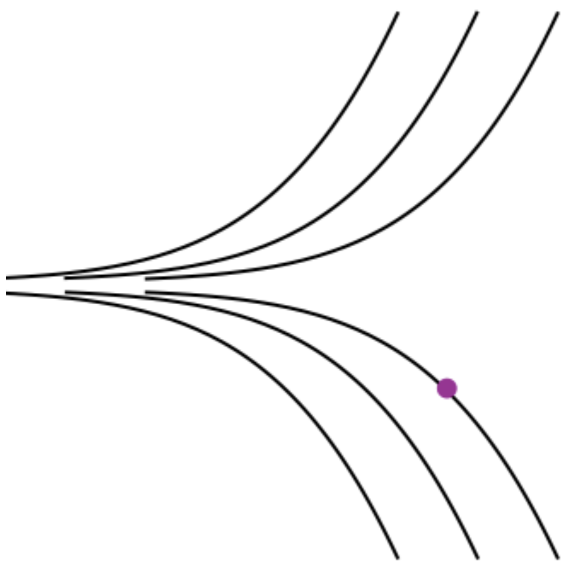
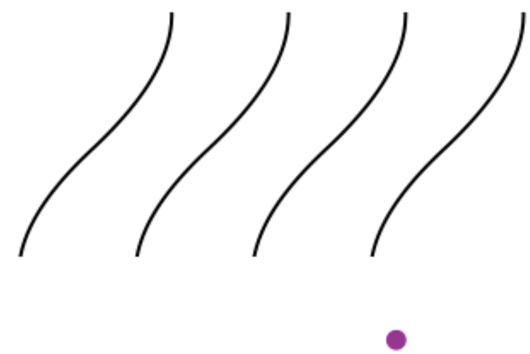


$$y^2 = -x^2 + C$$

$$y(x) = \sqrt{C - x^2}$$

$$\sin(y) = t + C$$

$$y(t) = \arcsin(t + C)$$



Choose C so that the function goes through the IC.