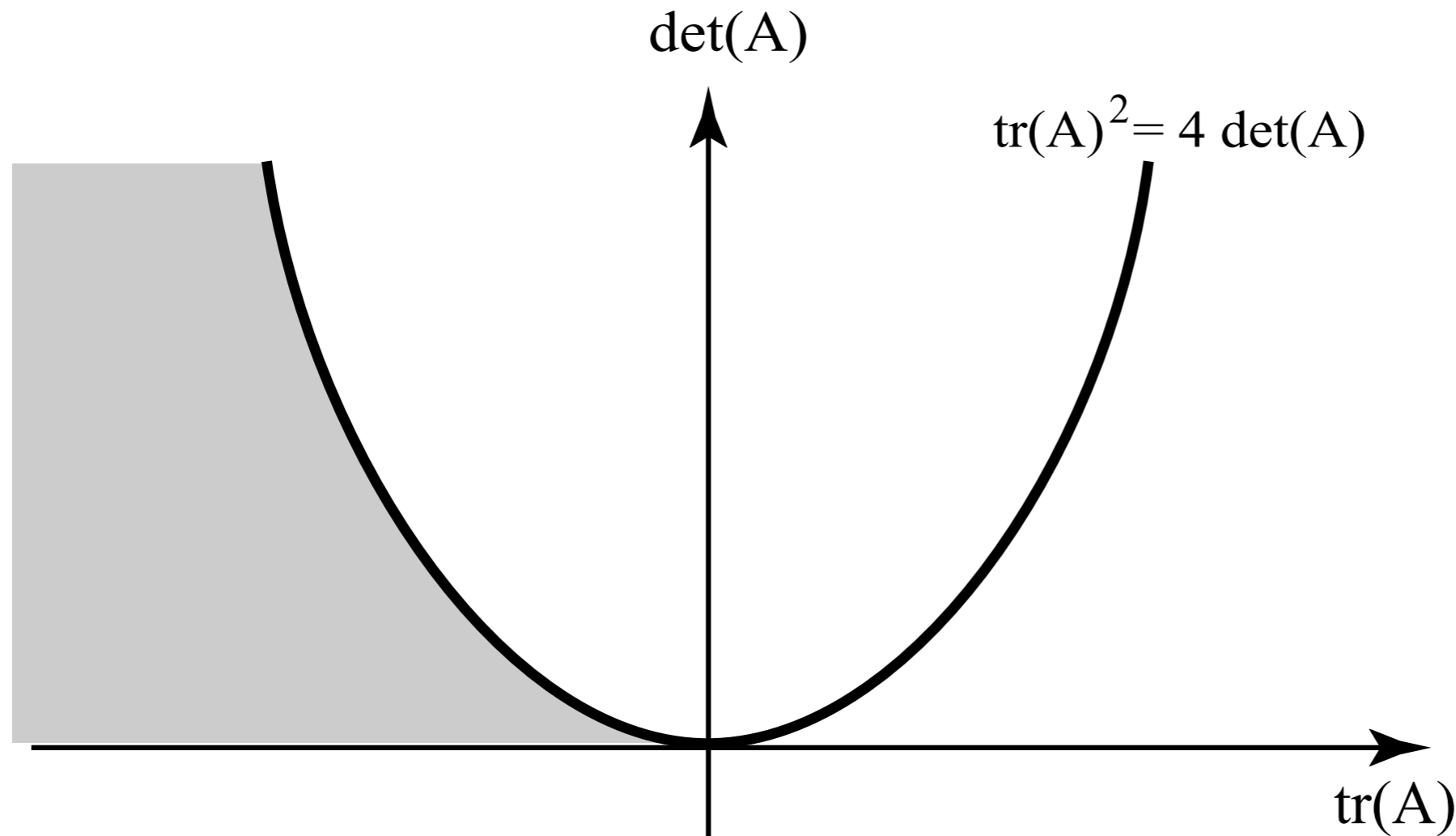


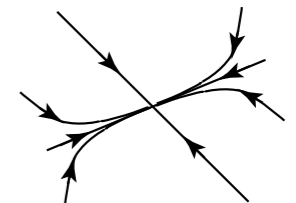
Today

- Summary of 2x2 systems
- Non-homogeneous two-tank example
- Intro to Laplace transforms

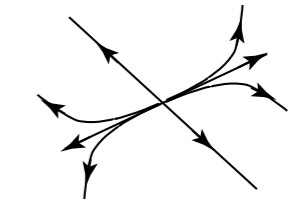
Summary - homogeneous 2x2 systems



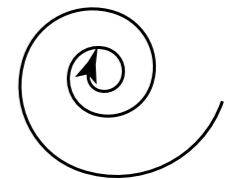
(A) stable node



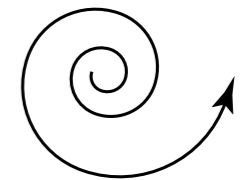
(B) unstable node



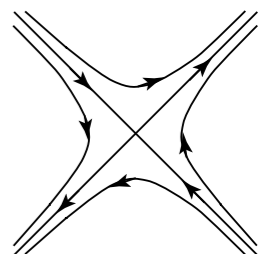
(C) stable spiral



(D) unstable spiral

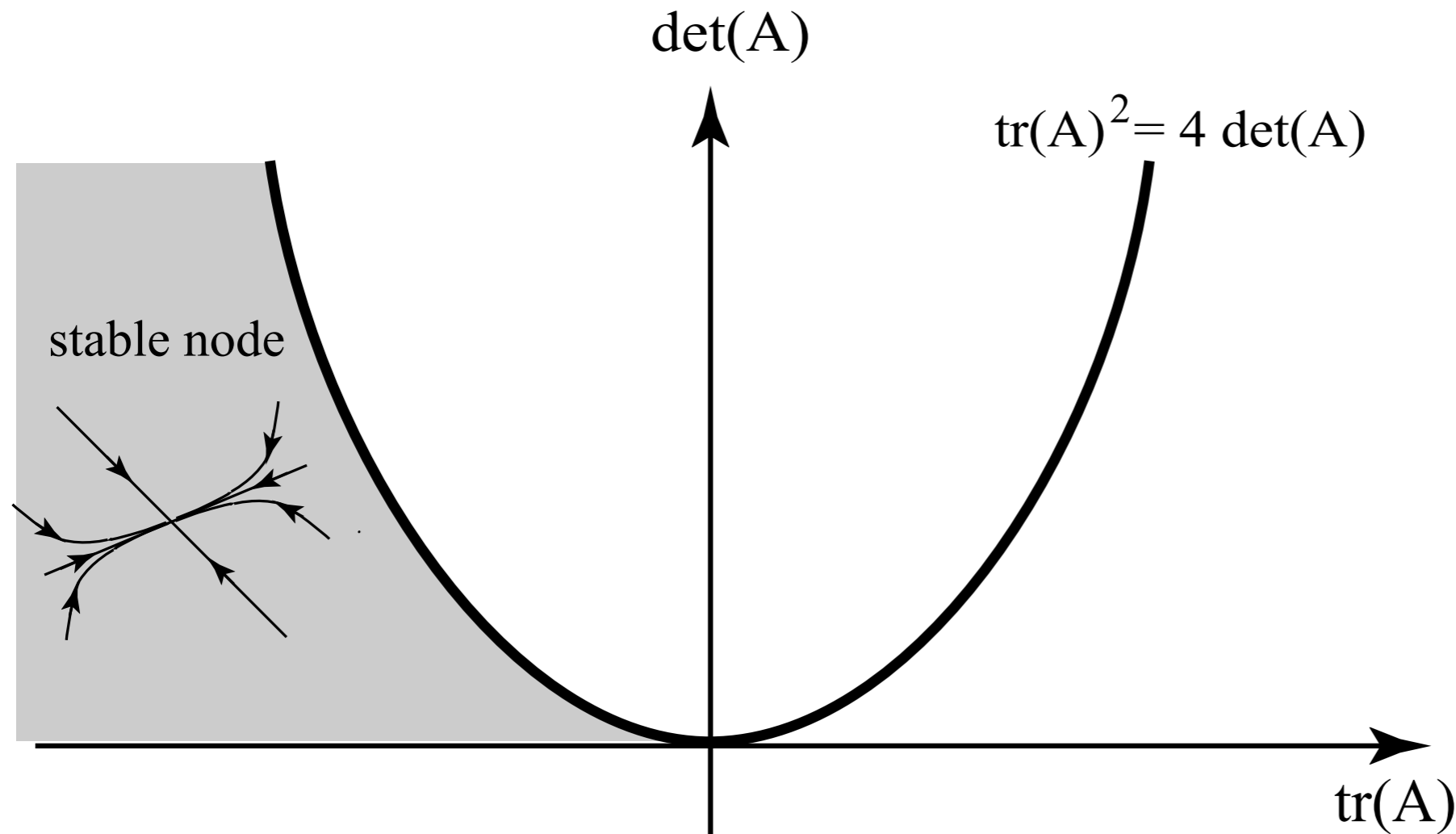


(E) saddle

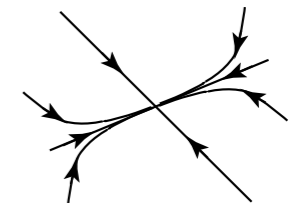


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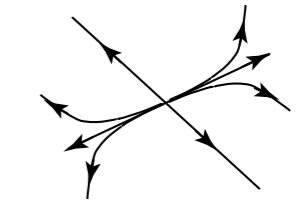
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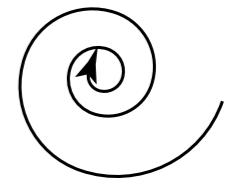
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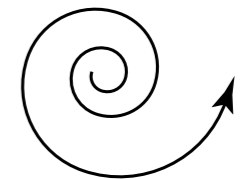
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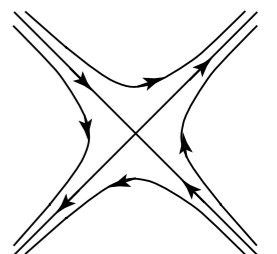
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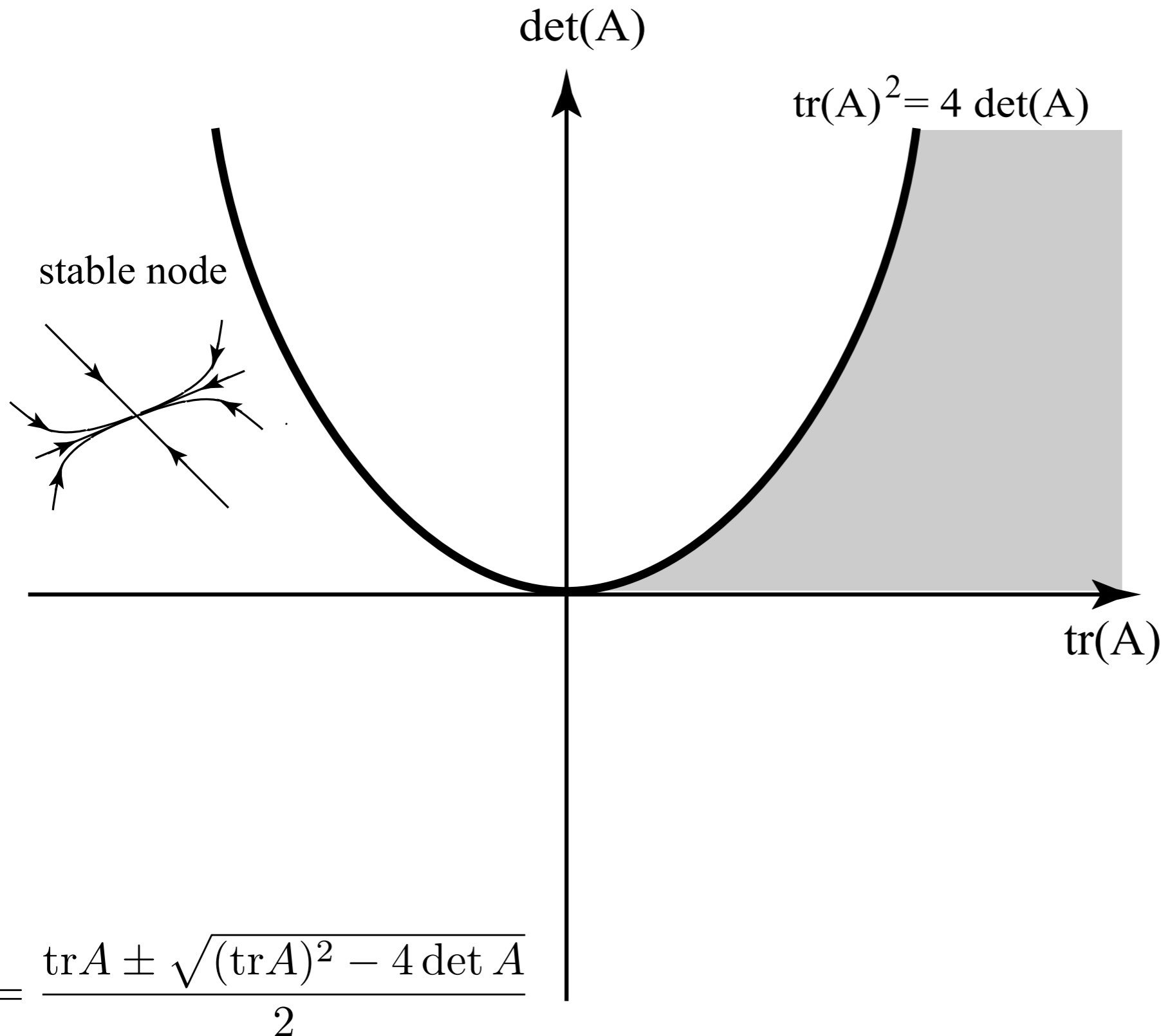


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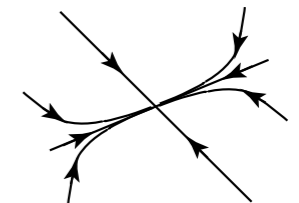


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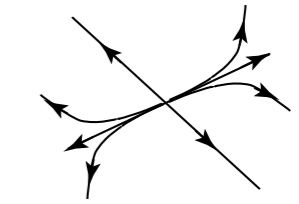
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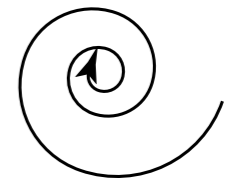
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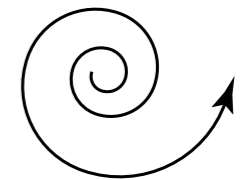
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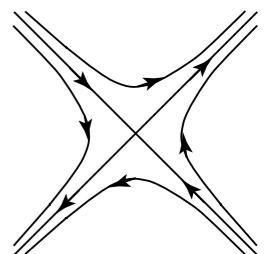
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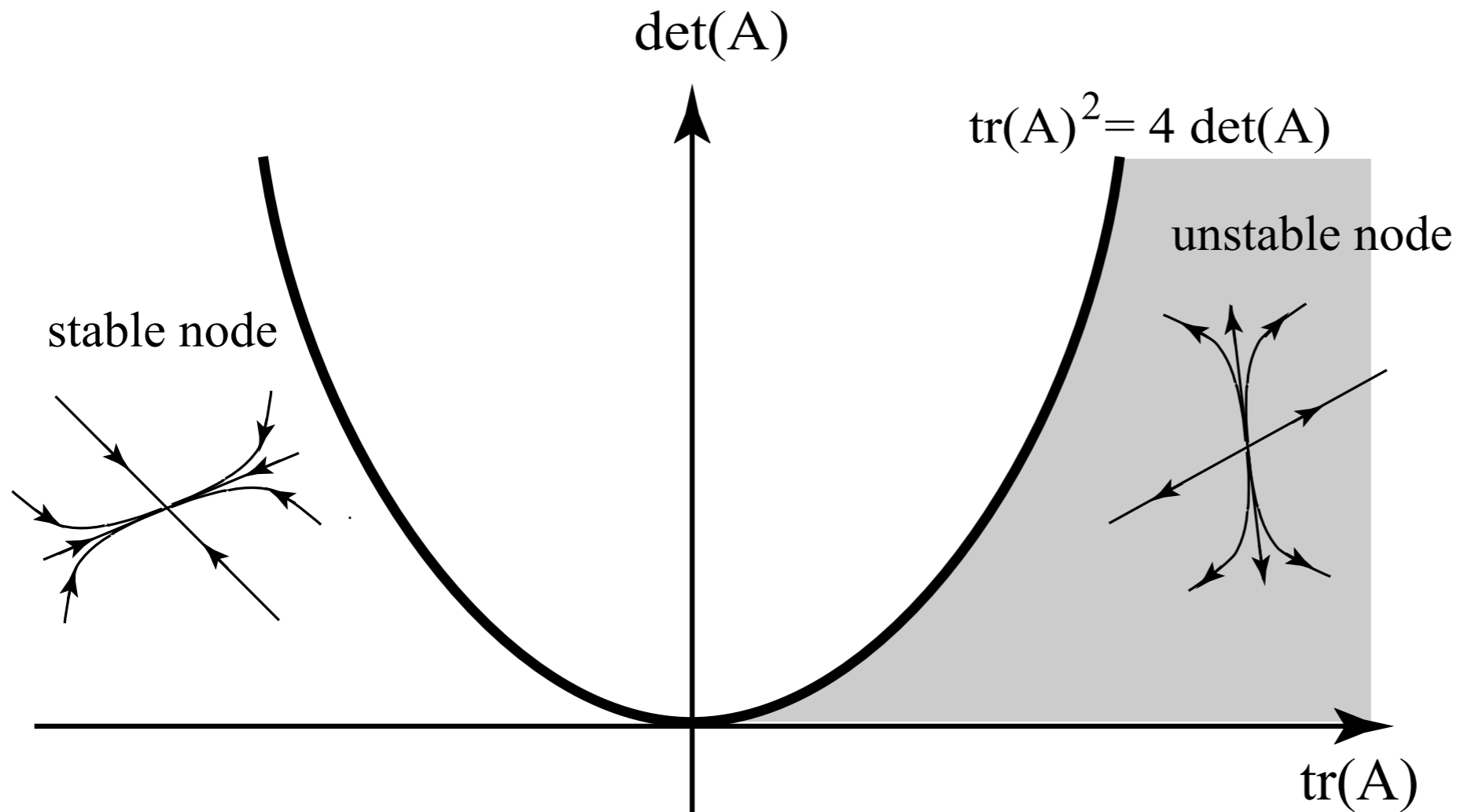
(D) unstable spiral



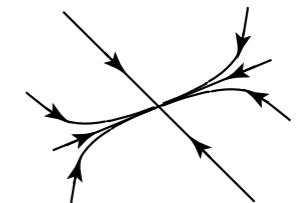
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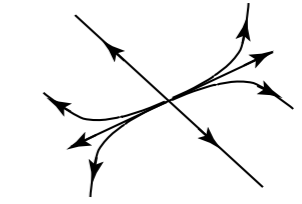
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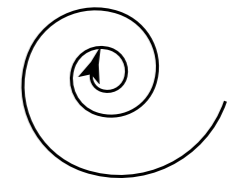
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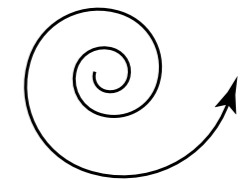
(B) unstable node



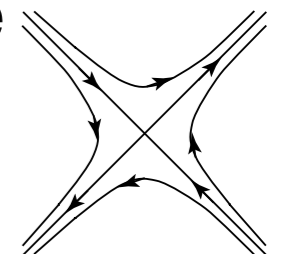
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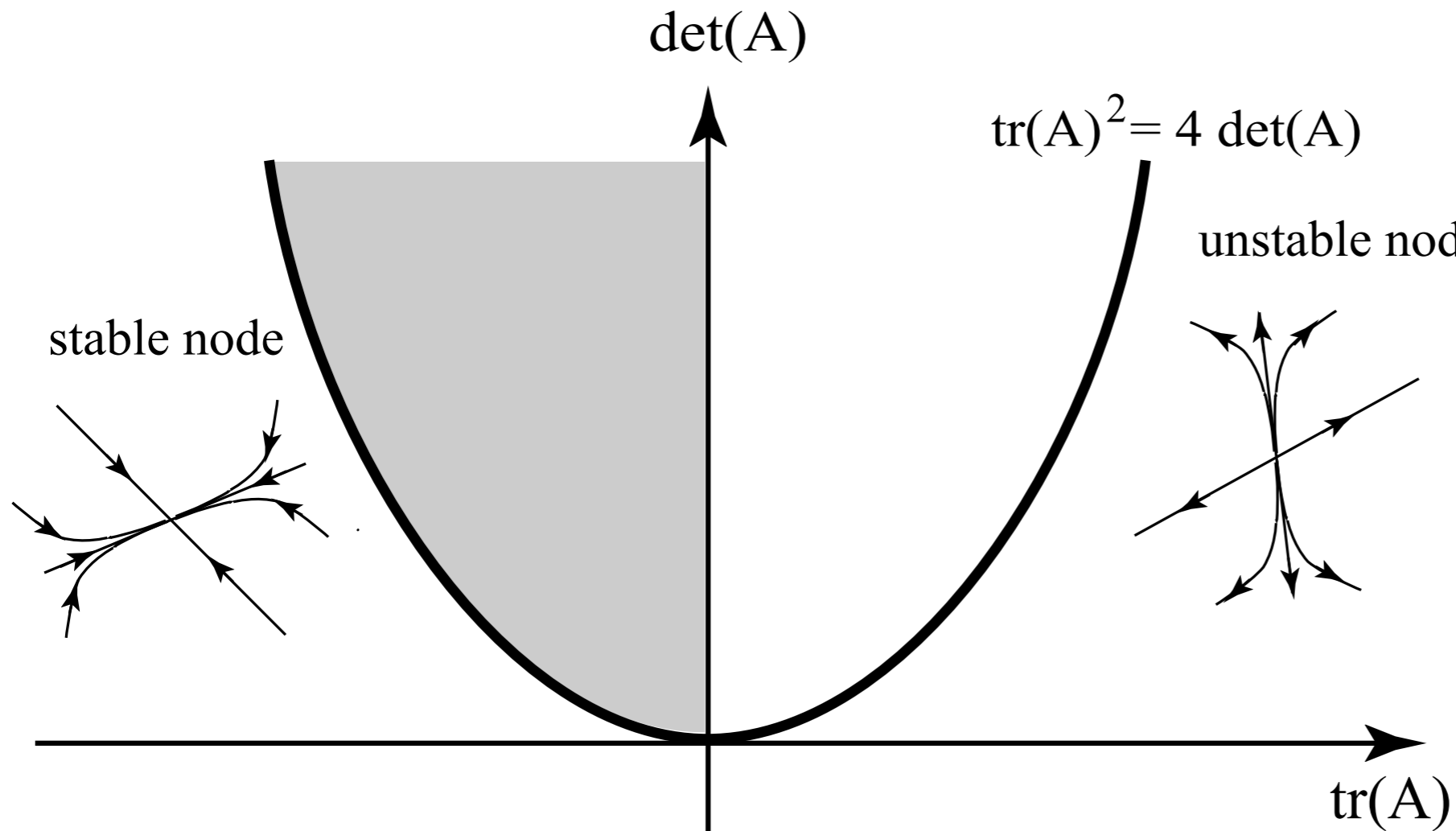


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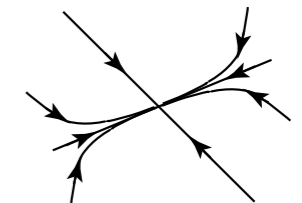


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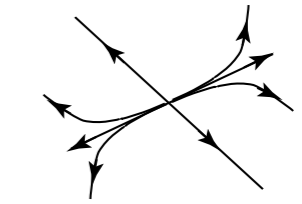
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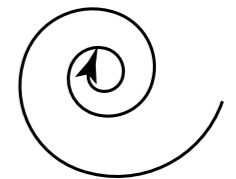
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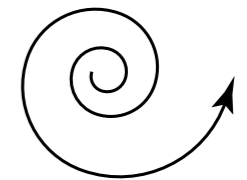
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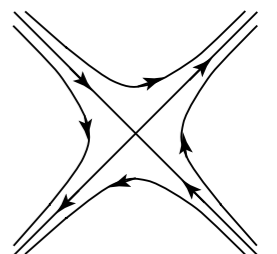
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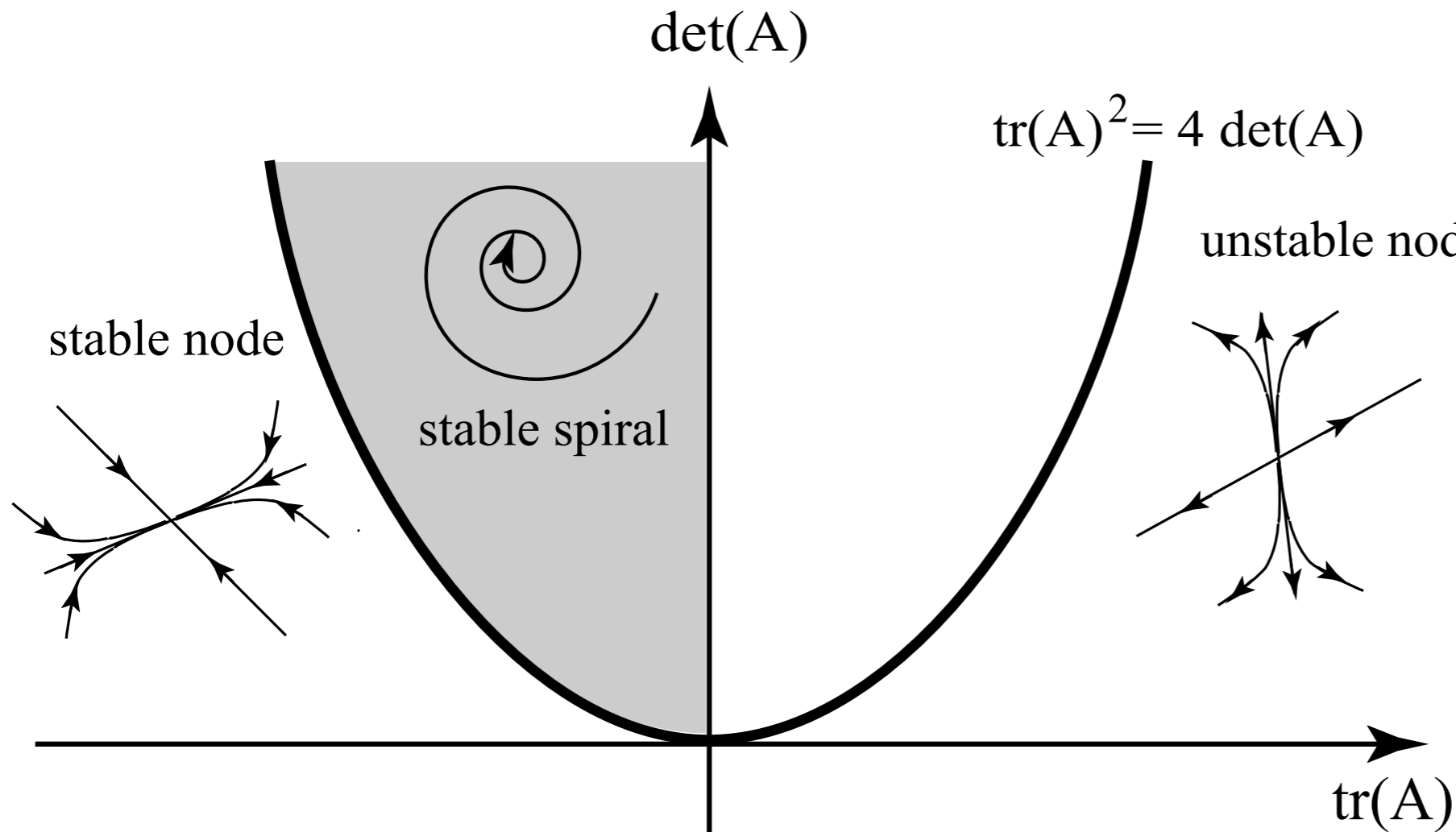


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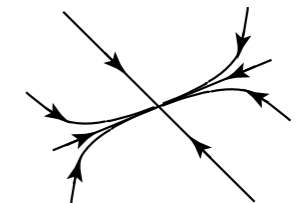


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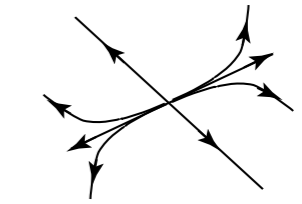
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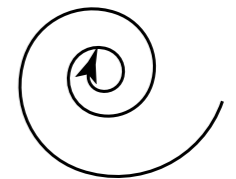
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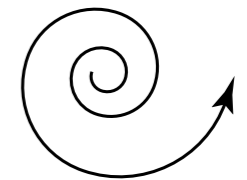
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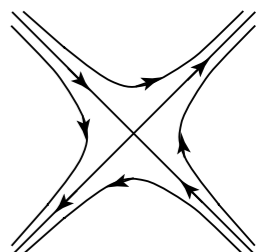
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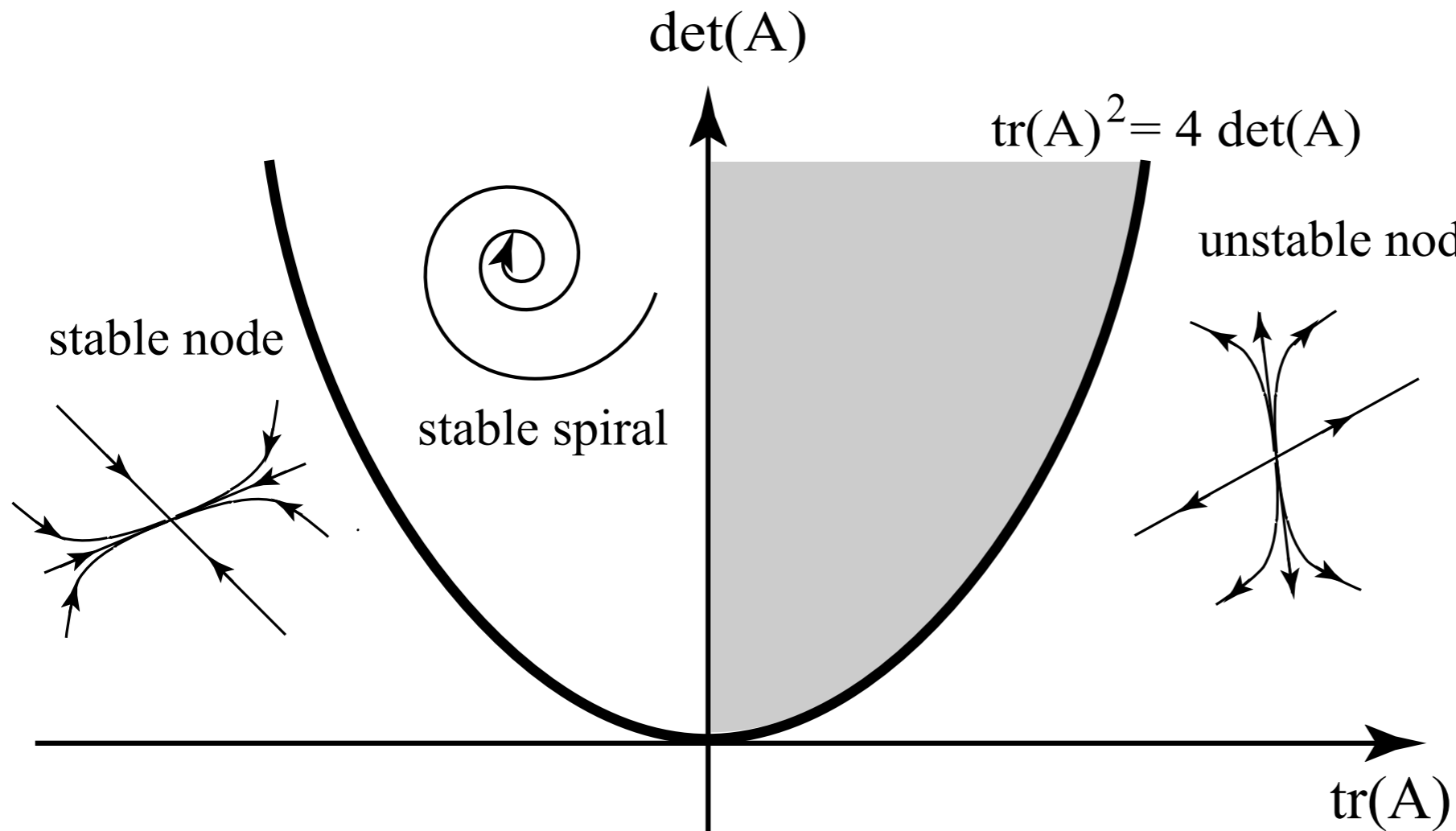


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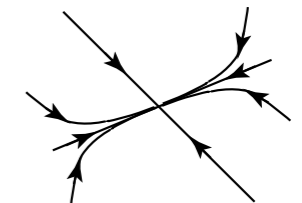


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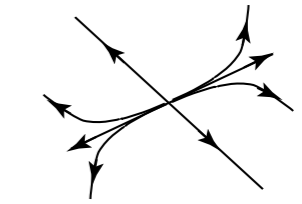
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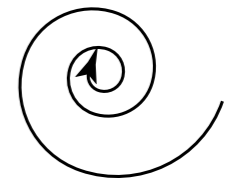
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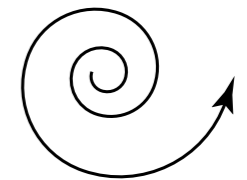
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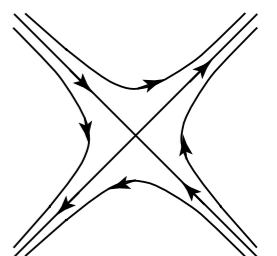
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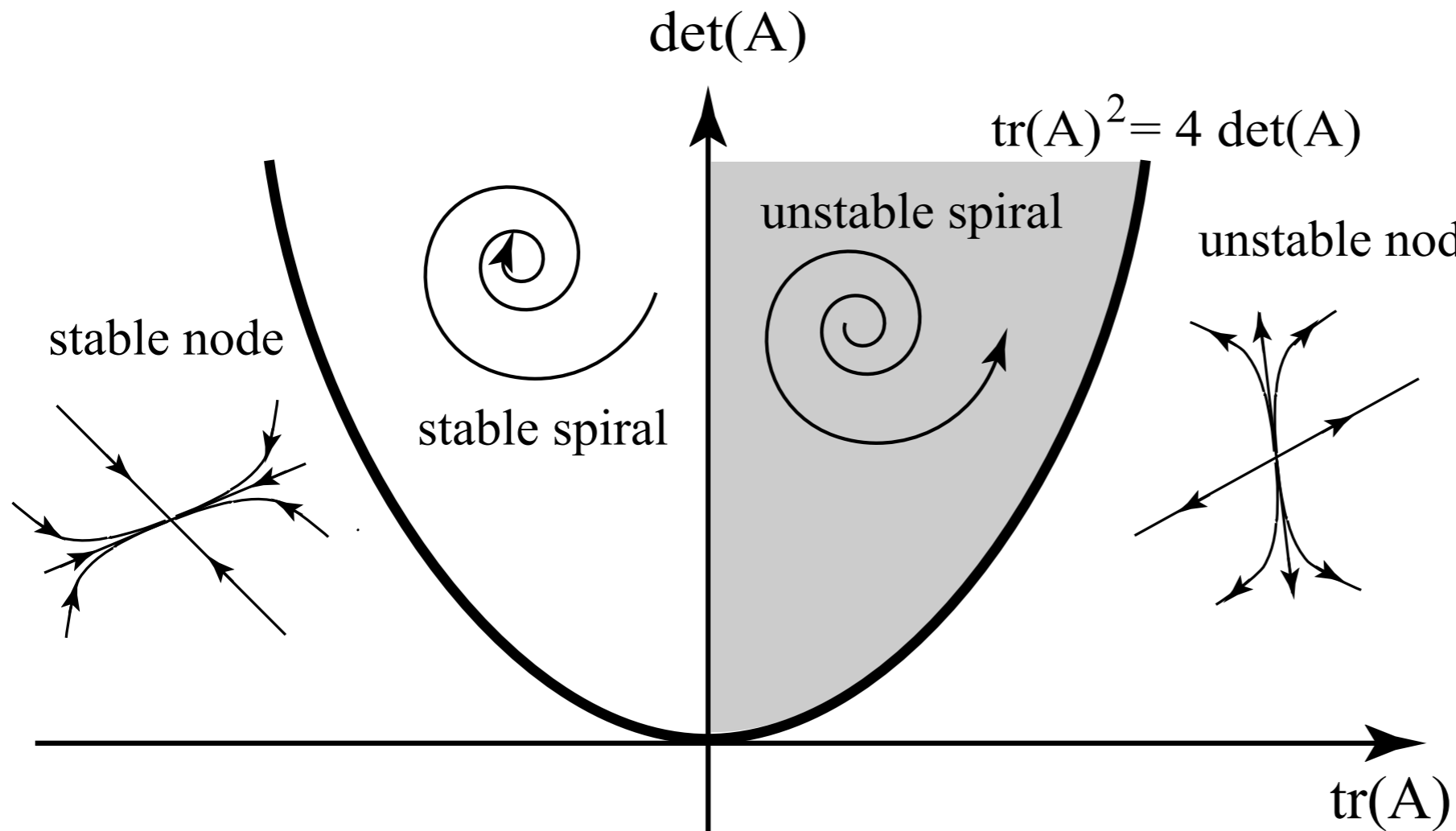


(E) saddle

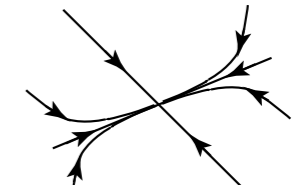


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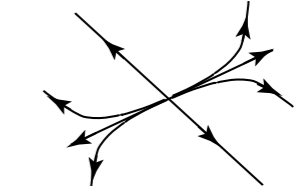
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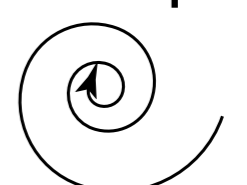
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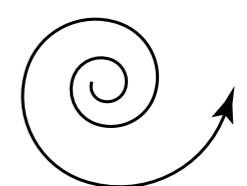
(B) unstable node



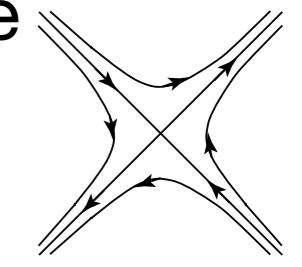
(C) stable spiral



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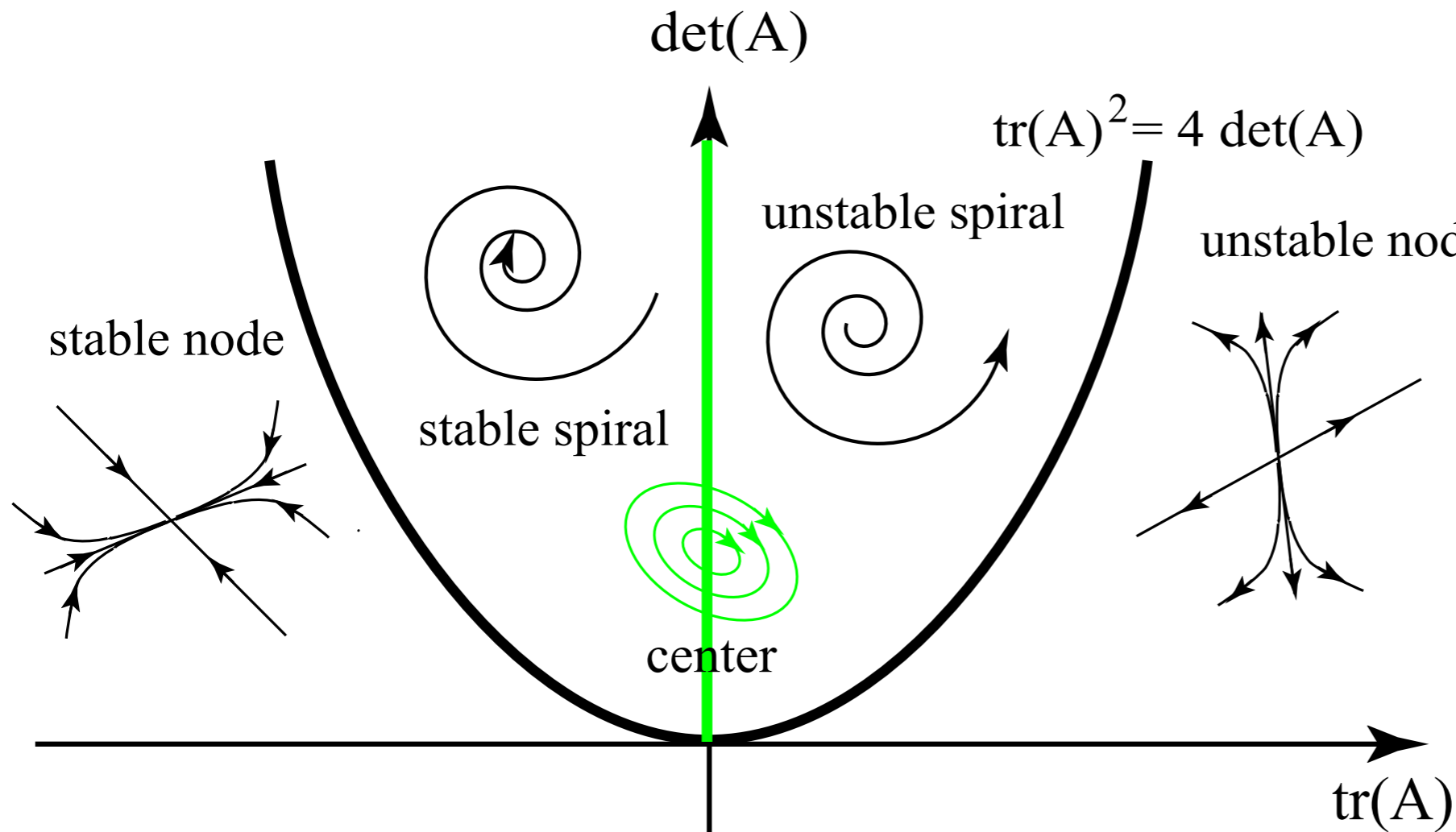


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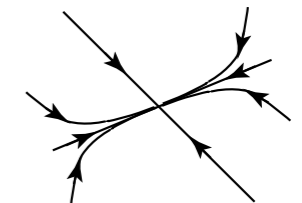


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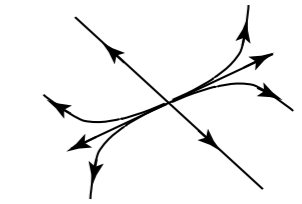
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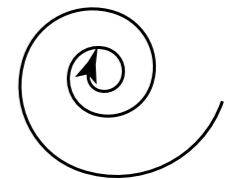
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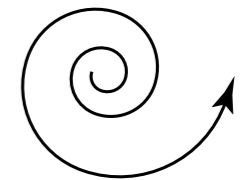
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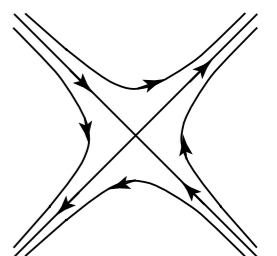
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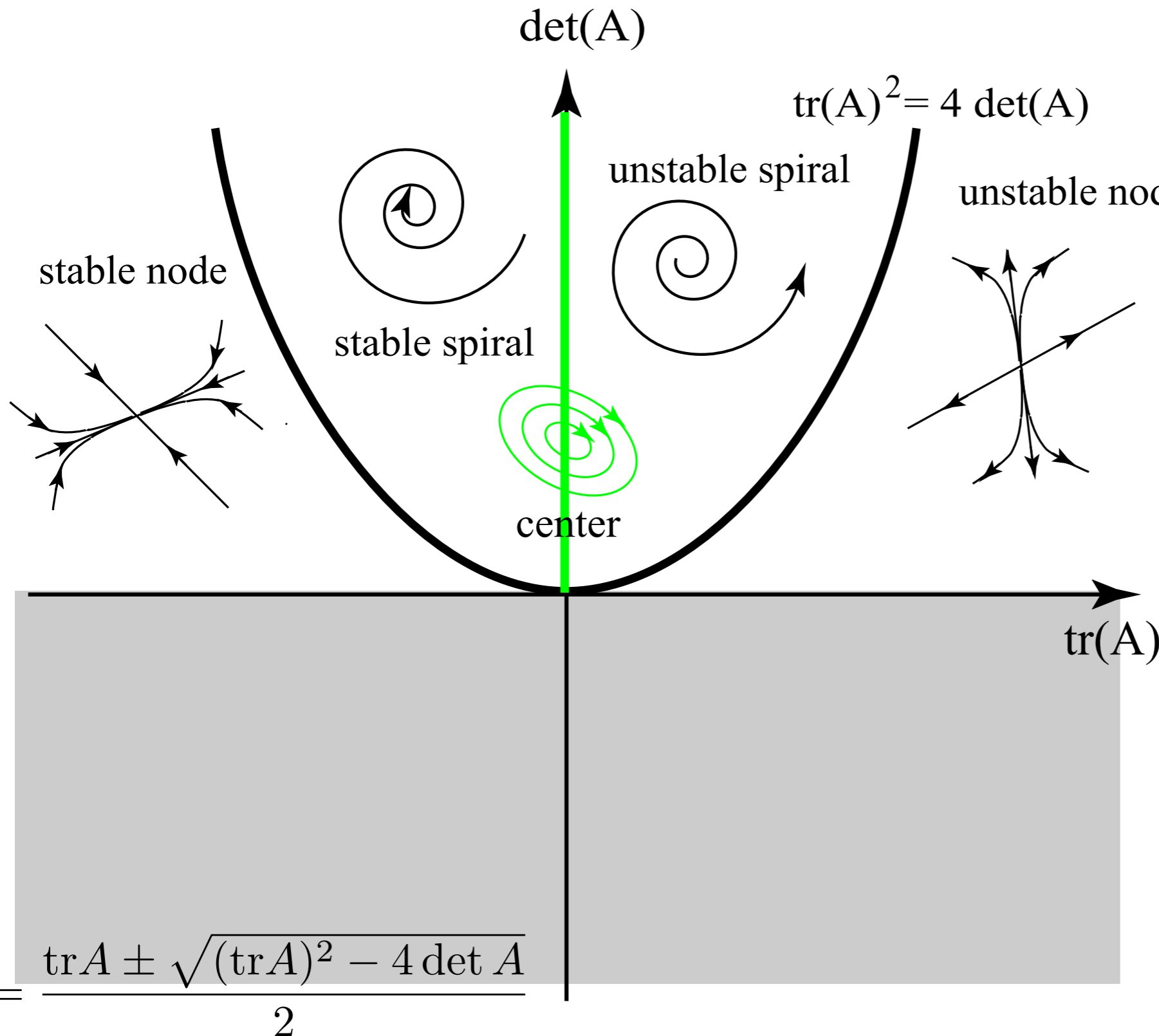


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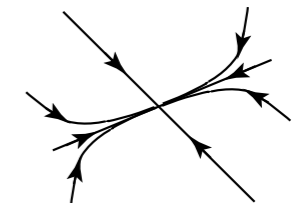


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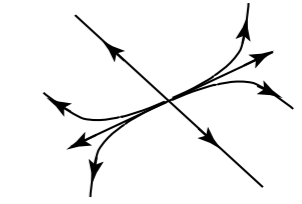
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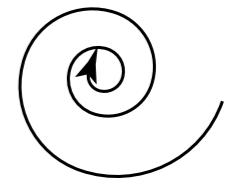
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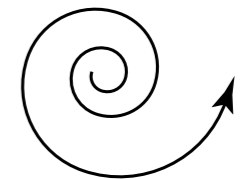
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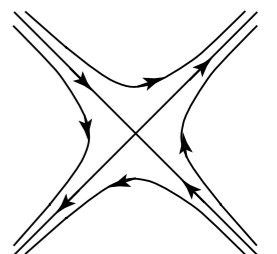
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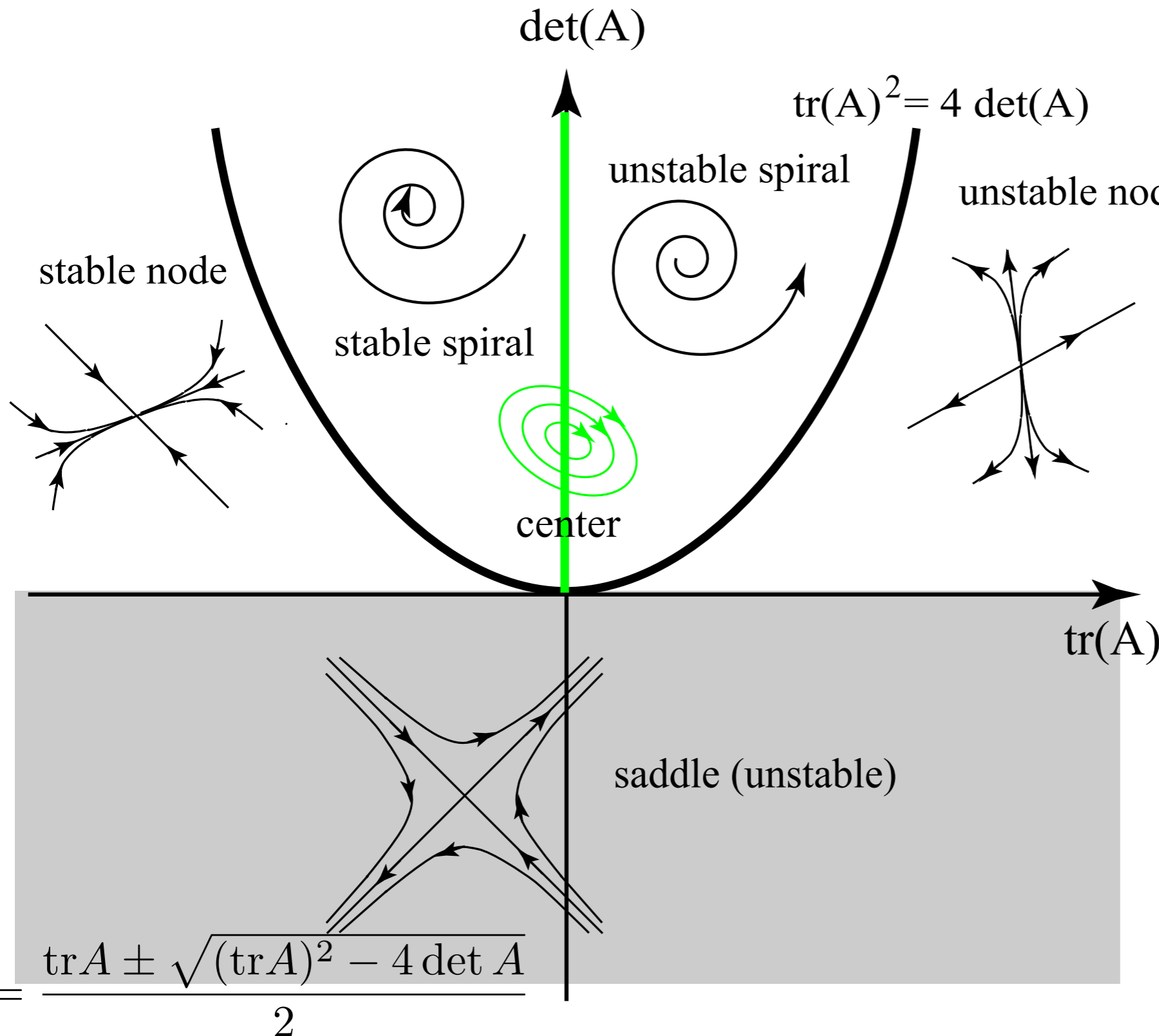
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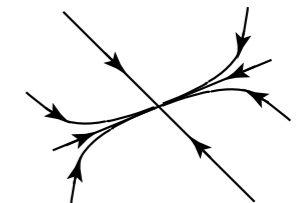
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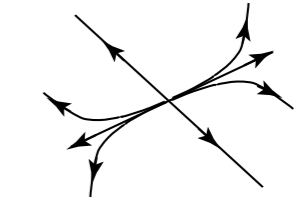
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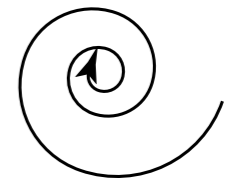
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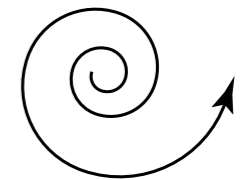
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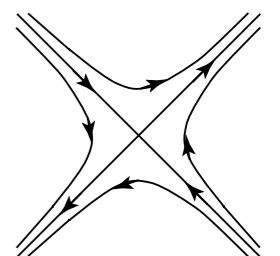
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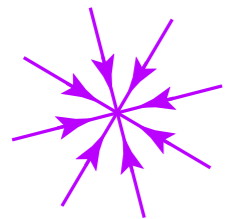


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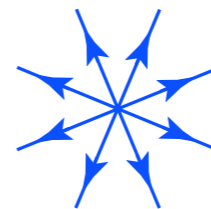


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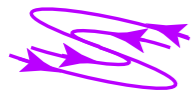
Repeated evalue cases:



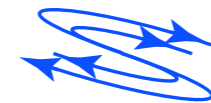
$\lambda < 0$, two indep. evector.



$\lambda > 0$, two indep. evector.

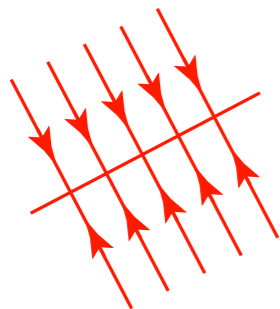


$\lambda < 0$, only one evector.

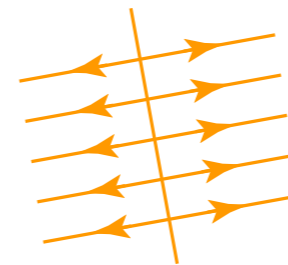


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One zero evalue (singular matrix):



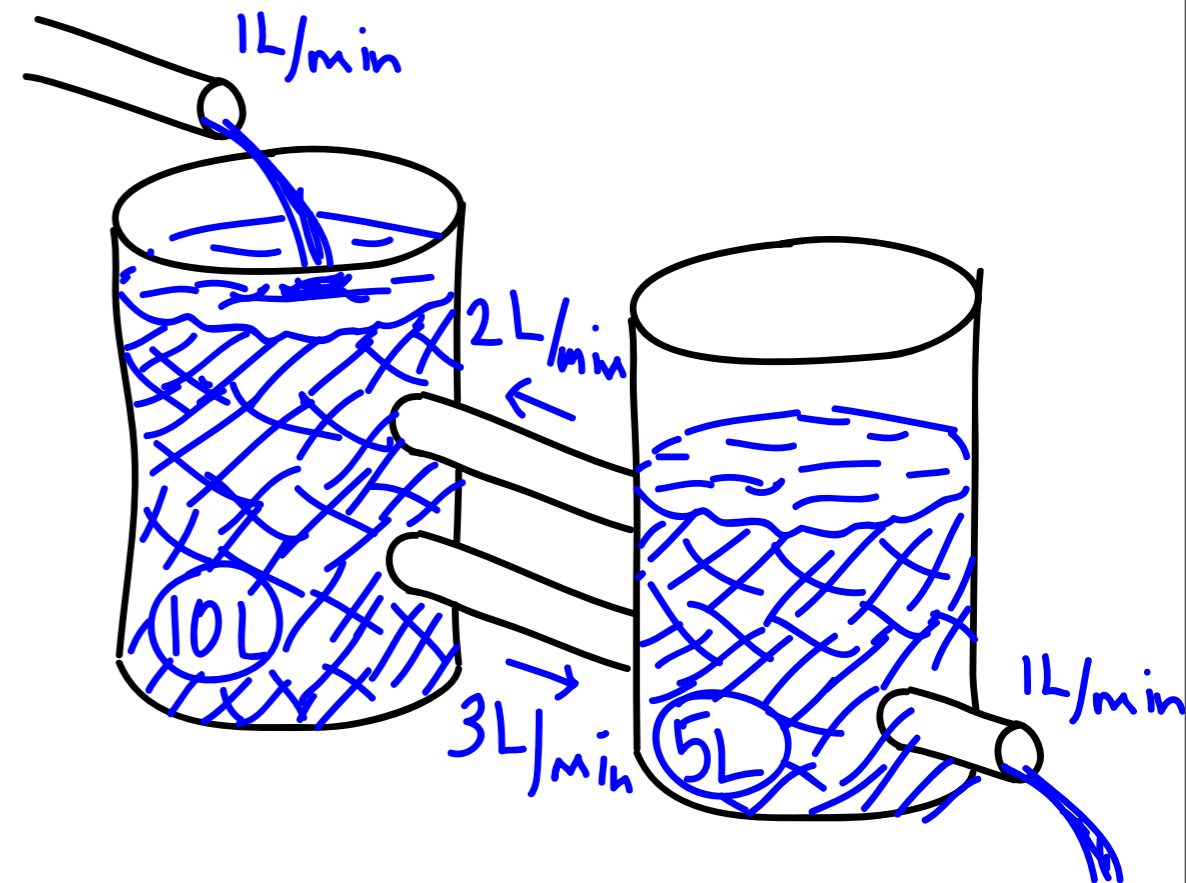
$\lambda_1 = 0$, $\lambda_2 < 0$,



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Nonhomogeneous case - example

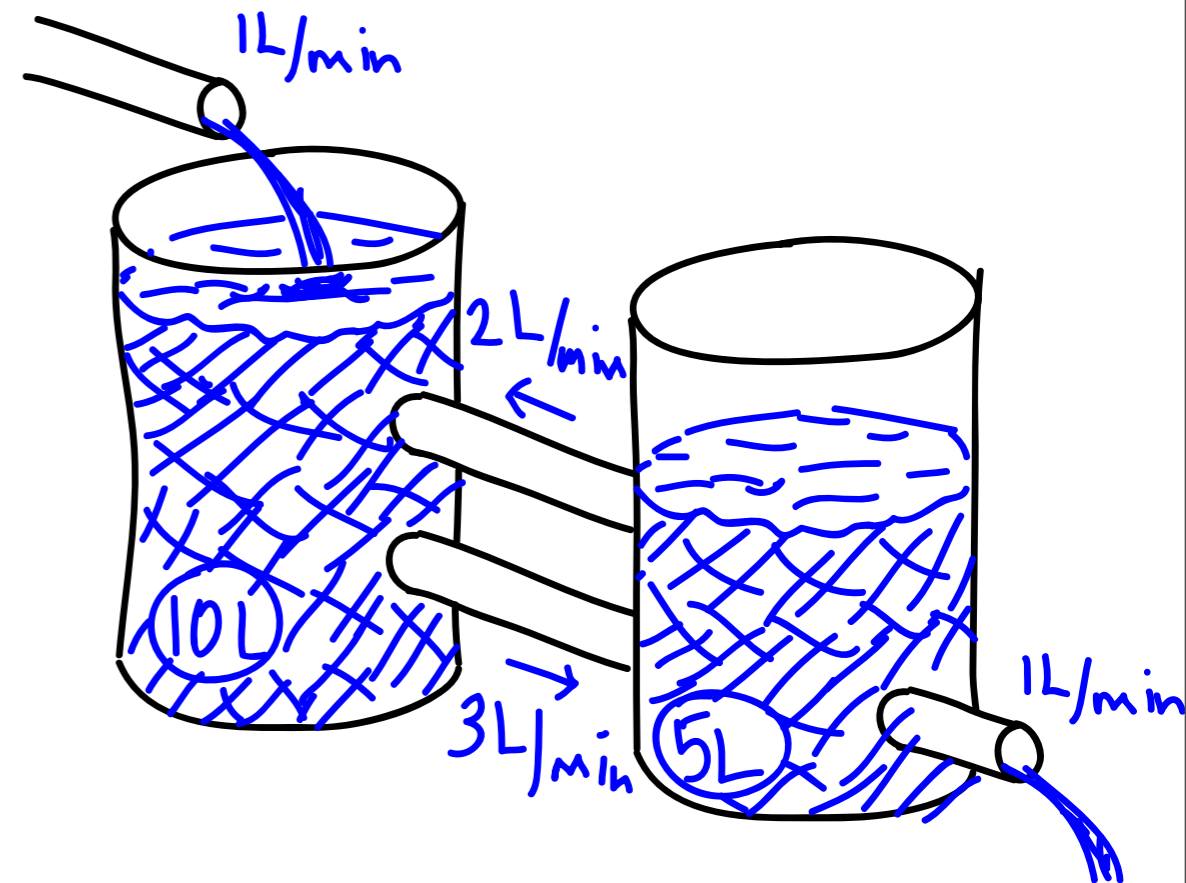
- Salt water flows into a tank holding 10 L of water at a rate of 1 L/min with a concentration of 200 g/L. The well-mixed solution flows from that tank into a tank holding 5 L through a pipe at 3 L/min. Another pipe takes the solution in the second tank back into the first at a rate of 2 L/min. Finally, solution drains out of the second tank at a rate of 1 L/min.
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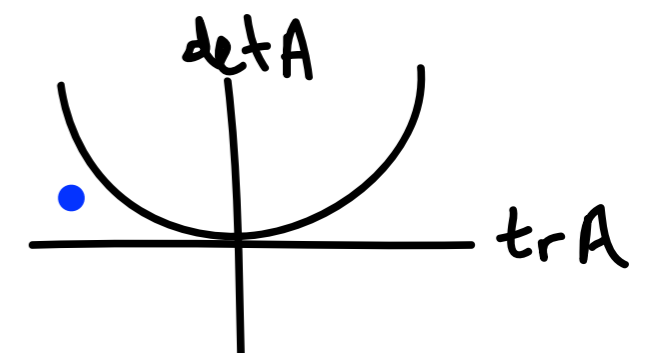
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 $\text{tr} A = -\frac{9}{10} \qquad (\text{tr} A)^2 = \frac{81}{100}$

$$\det A = \frac{9}{50} - \frac{6}{50} = \frac{3}{50} \qquad 4 \det A = \frac{12}{50}$$



Both eigenvalues
negative!

Nonhomogeneous case - example

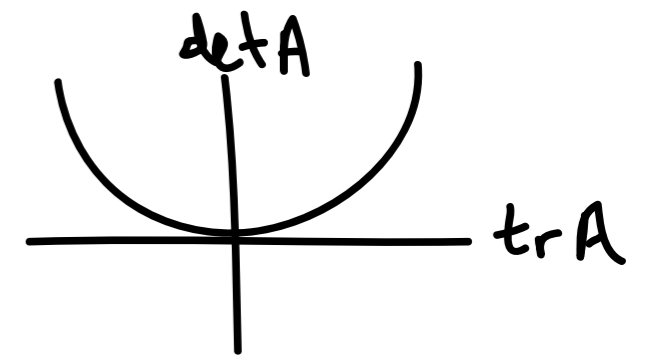
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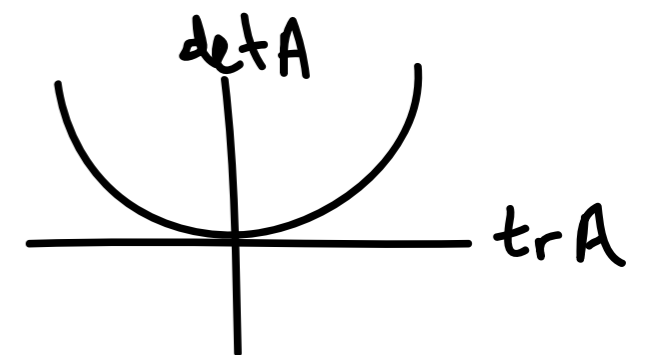
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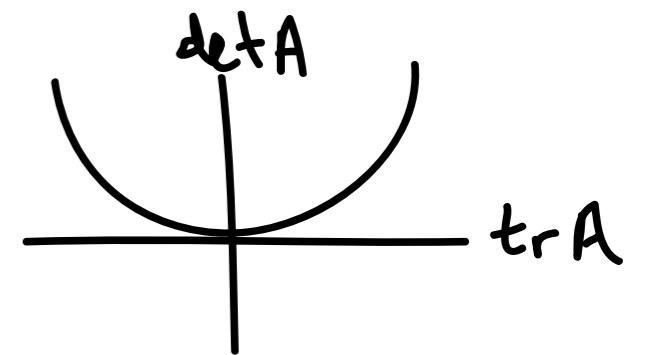
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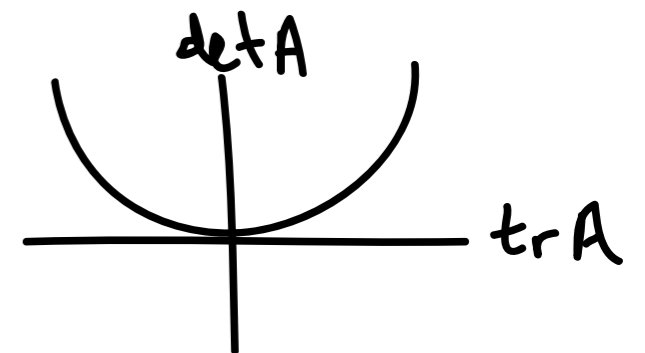
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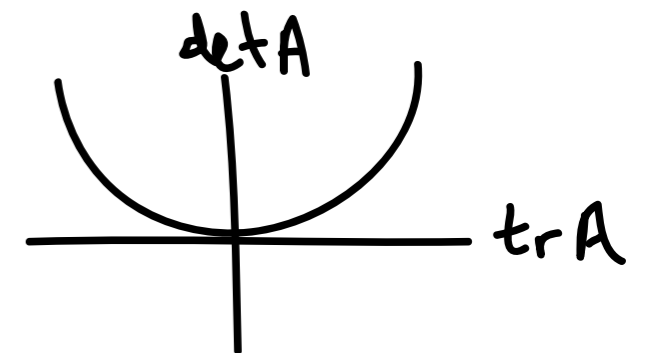
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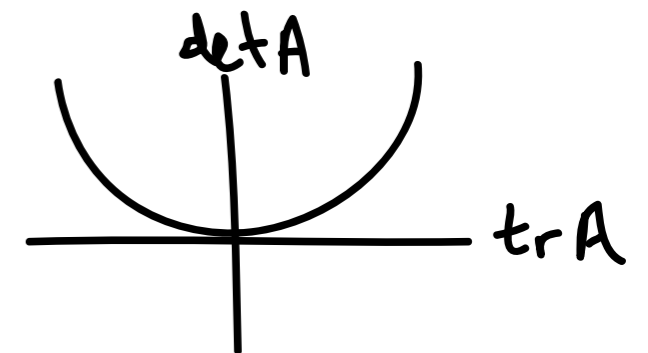
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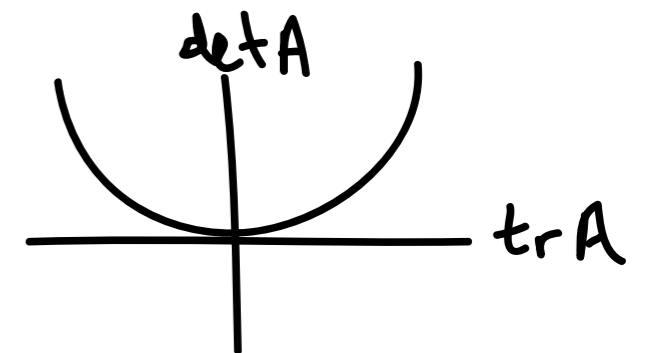
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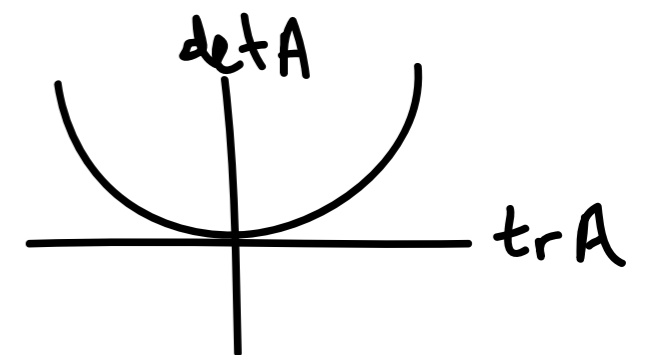
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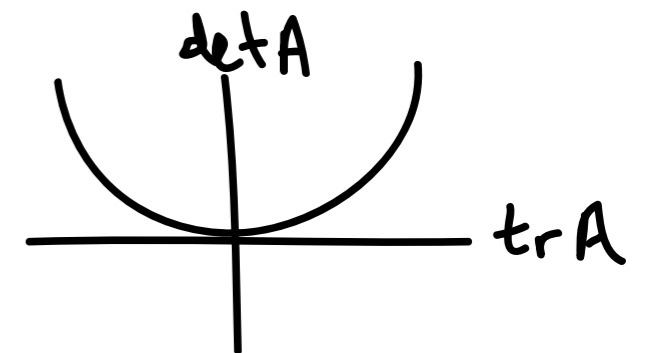
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Nonhomogeneous case - example

- A “Method of undetermined coefficients” similar to what we saw for second order equations can be used for systems.
- For this course, I’ll only test you on constant nonhomogeneous terms (like the previous example).

Laplace transforms - intro (6.1)

- Motivation for Laplace transforms:

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- These can be handled by previous techniques (modified) but it isn't pretty (solve from $t=0$ to $t=10$, use $y(10)$ as the IC for a new problem starting at $t=10$).

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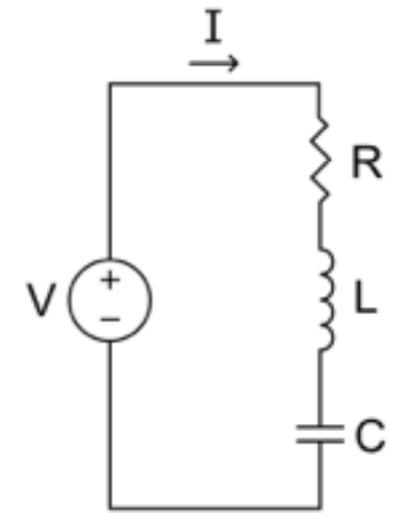
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 - Resistor, inductor and capacitor in series

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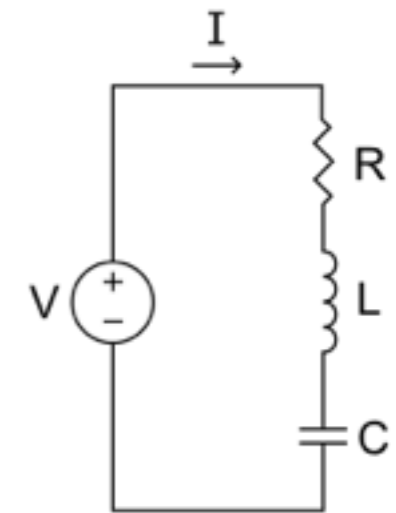


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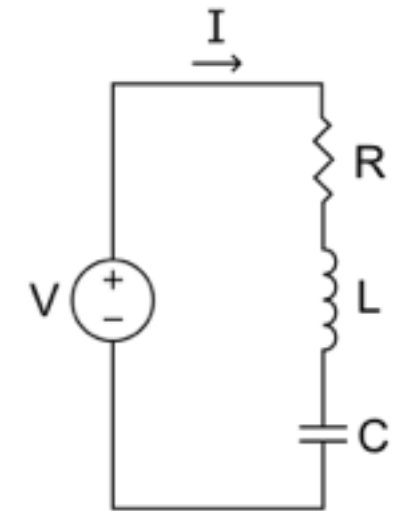
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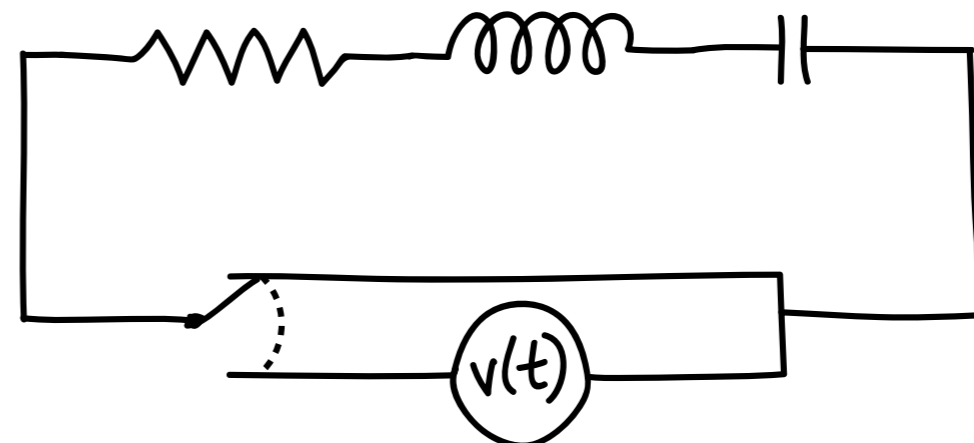
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- For $v(t) = \begin{cases} 1 & 0 < t < 10 \\ 0 & t \geq 10 \end{cases}$, the circuit has a switch that gets flipped at $t=10$.



Laplace transforms - intro (6.1)

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Unknown $y(t)$ that
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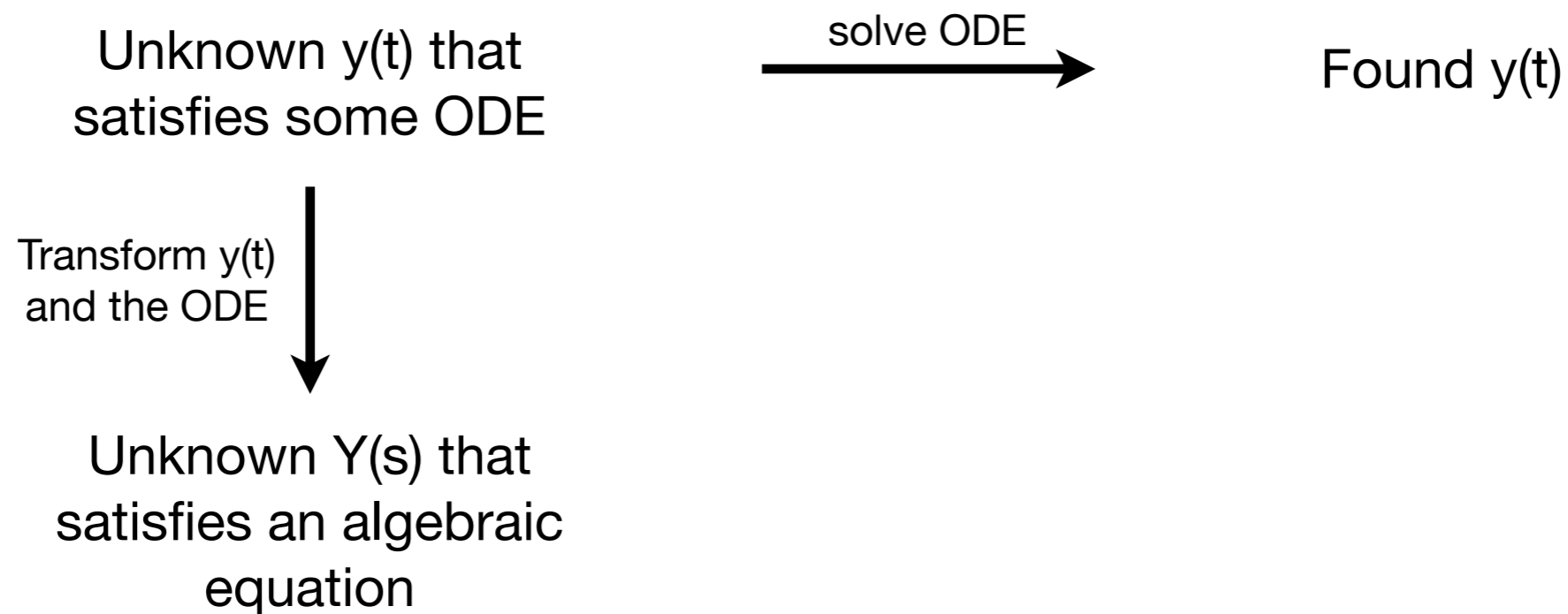


Found $y(t)$

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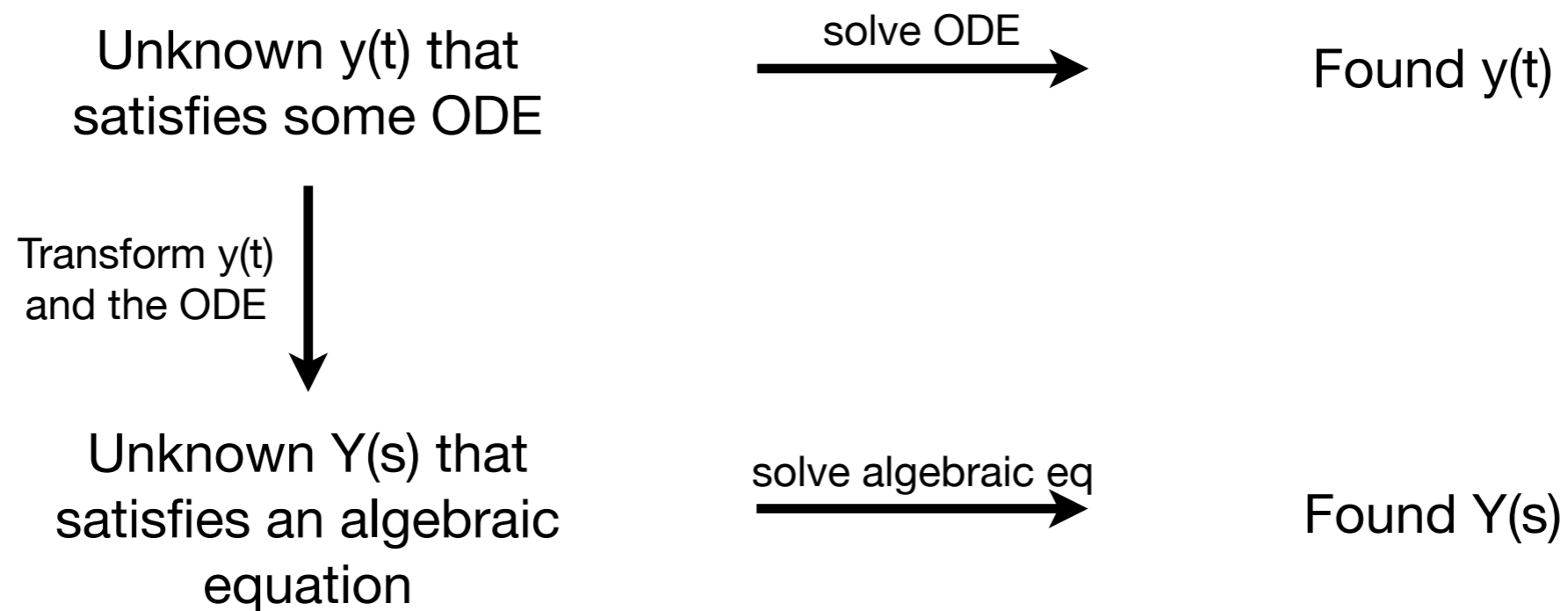
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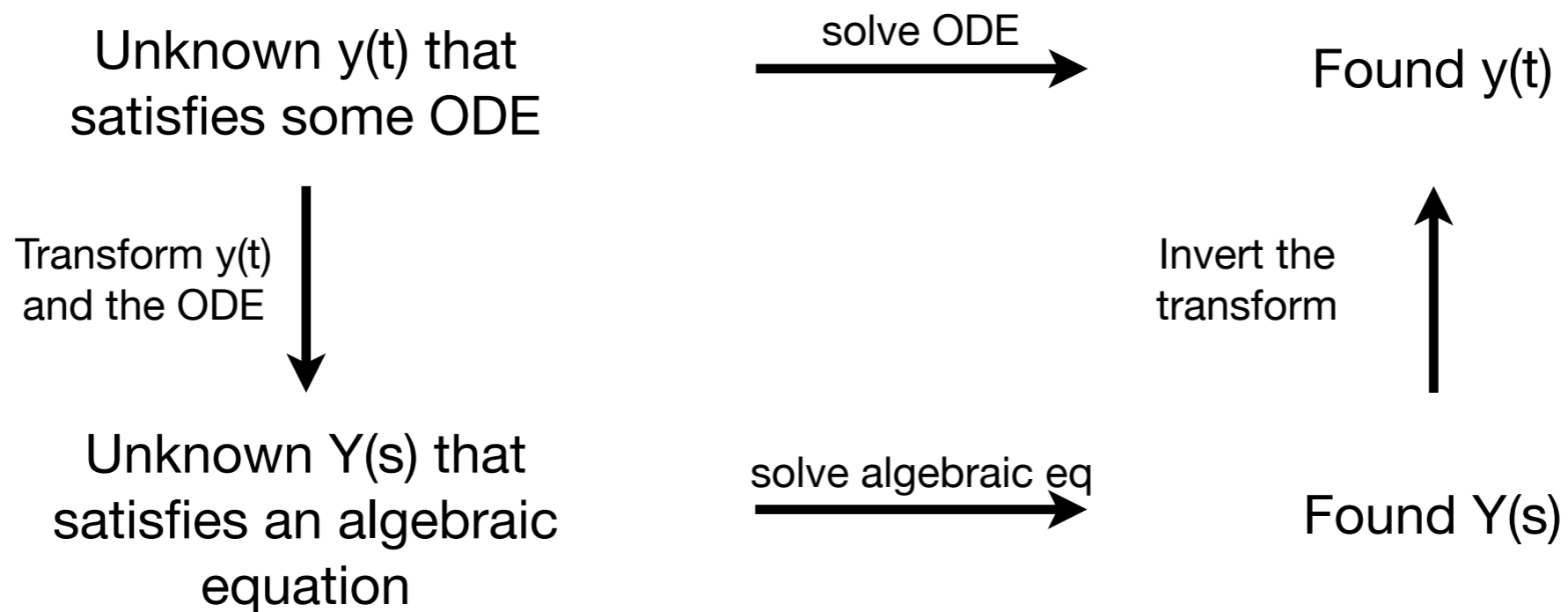
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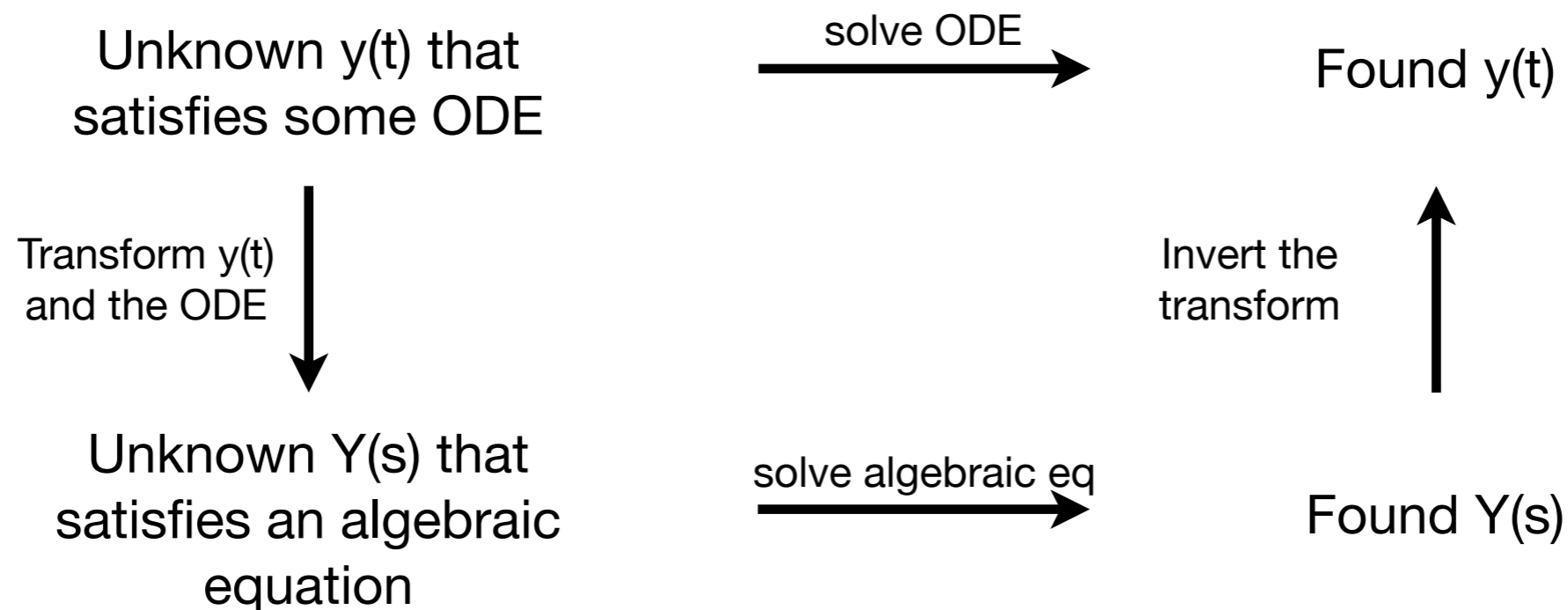
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- Laplace transform of $y(t)$: $\mathcal{L}\{y(t)\} = Y(s) = \int_0^{\infty} e^{-st} y(t) dt$

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- What is the Laplace transform of $y(t) = 3$?

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$$= -\frac{3}{s} e^{-st} \Big|_0^{\infty}$$

$$= \lim_{A \rightarrow \infty} -\frac{3}{s} e^{-st} \Big|_0^A$$

$$= -\frac{3}{s} \left(\lim_{A \rightarrow \infty} e^{-sA} - 1 \right)$$

$$= \frac{3}{s} \text{ provided } s > 0 \text{ and does not exist otherwise.}$$

Laplace transforms - examples (6.1)

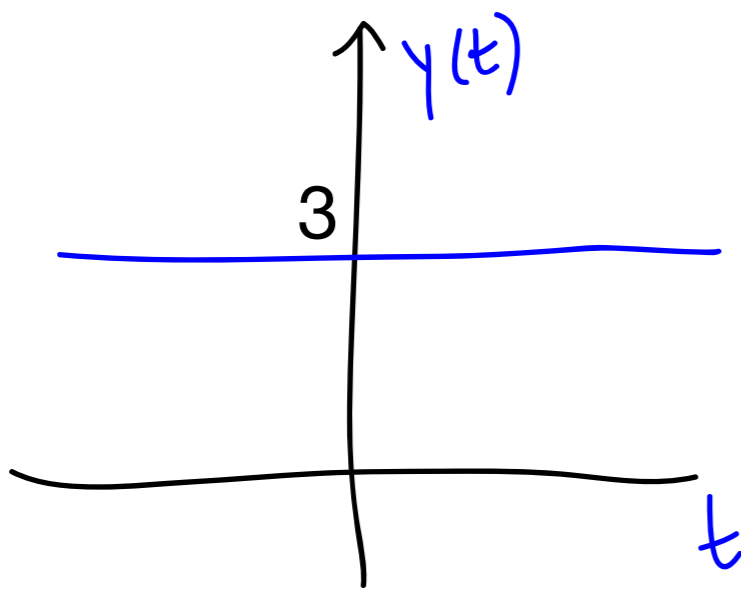
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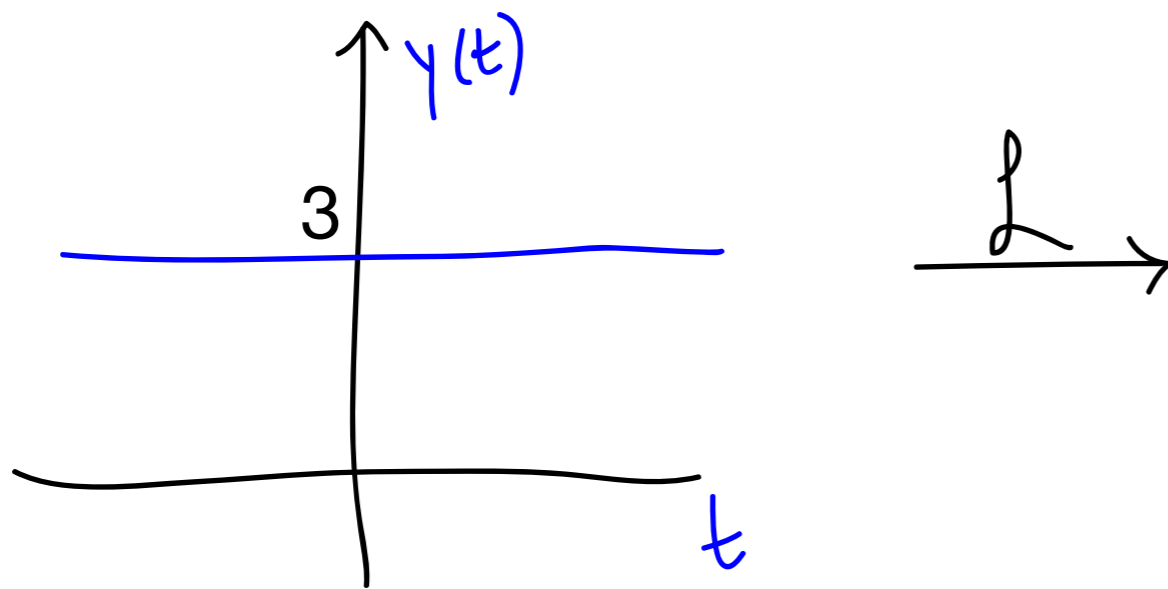
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