Today

- The geometry of homogeneous and nonhomogeneous matrix equations
- Solving nonhomogeneous equations
  - Method of undetermined coefficients

Second order, linear, constant coeff, **non**homogeneous (3.5)

- Our next goal is to figure out how to find solutions to nonhomogeneous equations like this one:

$$y'' - 6y' + 8y = \sin(2t)$$

- But first, a bit more on the connections between matrix algebra and differential equations . . .

Some connections to linear (matrix) algebra

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$$z = L[y] = \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y$$

- This one is linear because

$$L[cy] = cL[y]$$
$$L[y+z] = L[y] + L[z]$$

Note: y, z are functions of t and c is a constant.

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$$L[y] = 0$$

- A non-homogeneous differential equation has the form

$$L[y] = g(t)$$

Solutions to homogeneous matrix equations

- The matrix equation $A\overline{x} = \overline{0}$ could have (depending on A)

  (A) no solutions.

  (B) exactly one solution.

  (C) a one-parameter family of solutions.

  (D) an n-parameter family of solutions.

Choose the answer that is incorrect.

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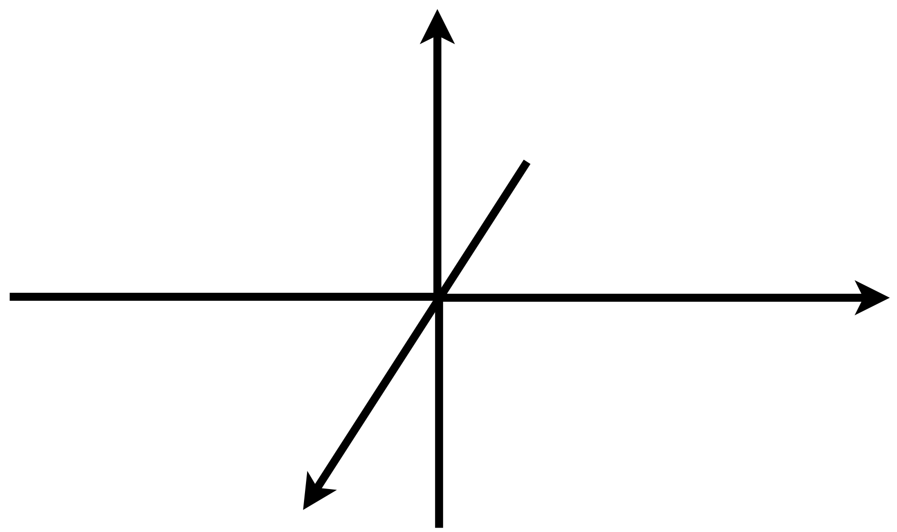

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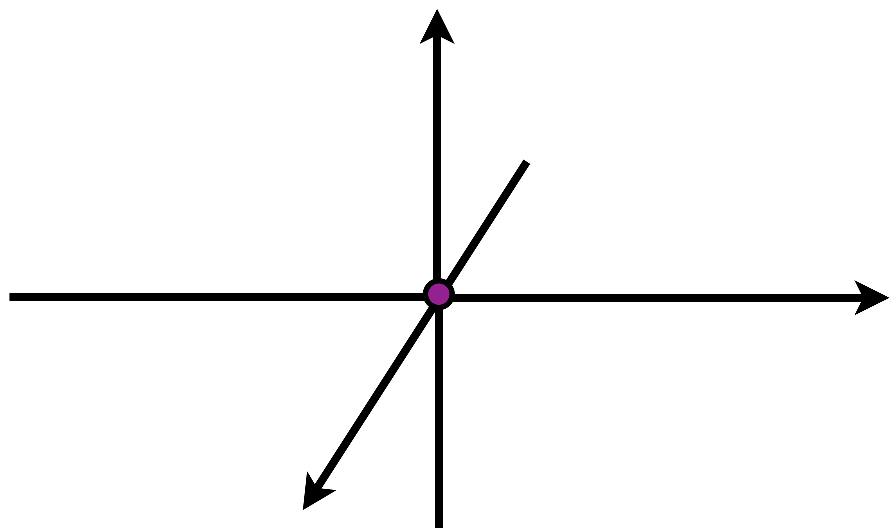

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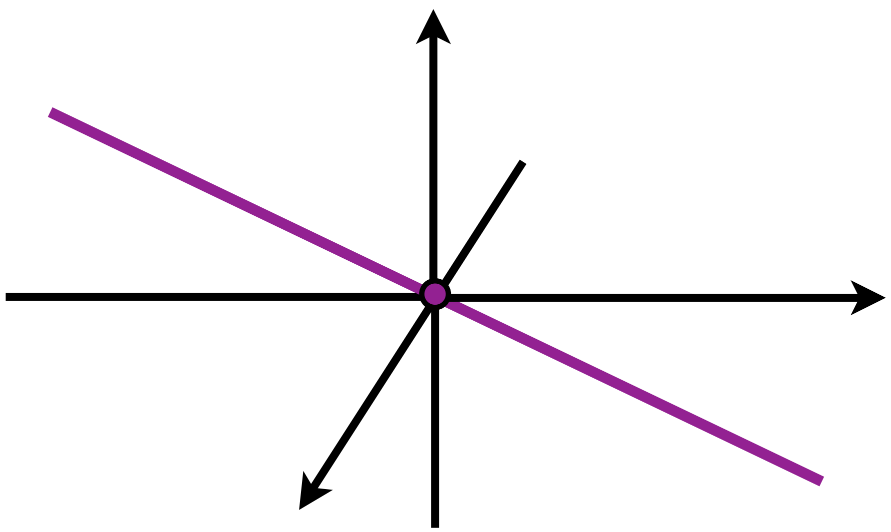

Possibilities:

$$\overline{x} = C \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

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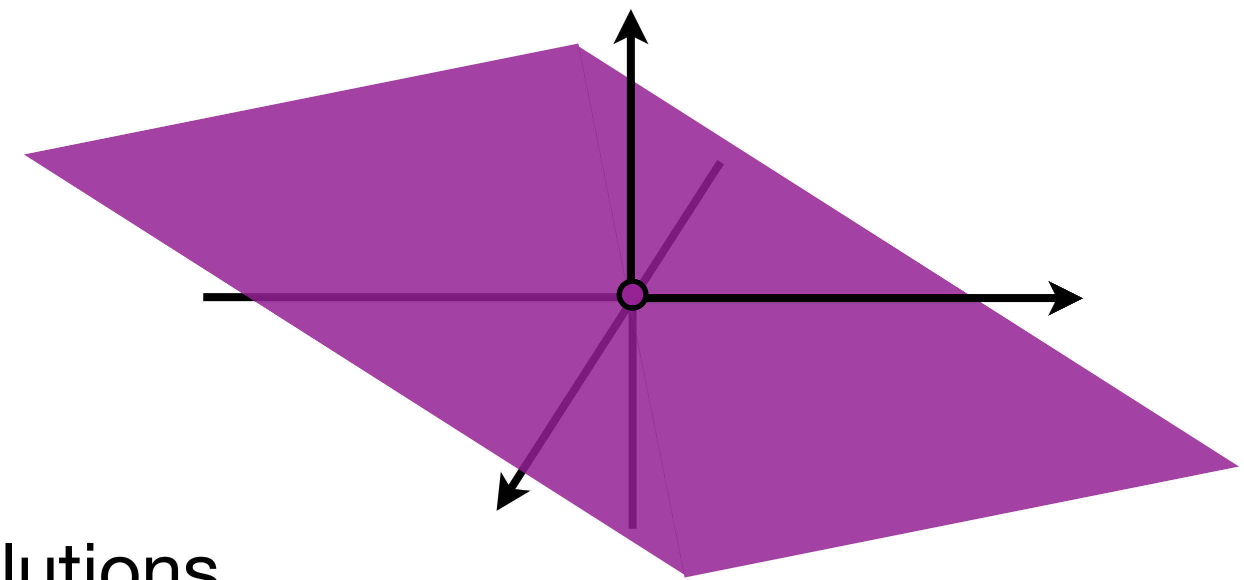

Possibilities:

$$\overline{x} = C_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

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- Example 1. Solve the equation $A\overline{x} = \overline{0}$ where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & -2 \\ 2 & 1 & 1 \end{pmatrix}$$

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In this case, only two of them really matter.

- so $x_1 - \frac{1}{3}x_3 = 0$ and $x_2 + \frac{5}{3}x_3 = 0$ and x_3 can be whatever (because it doesn't have a leading one).

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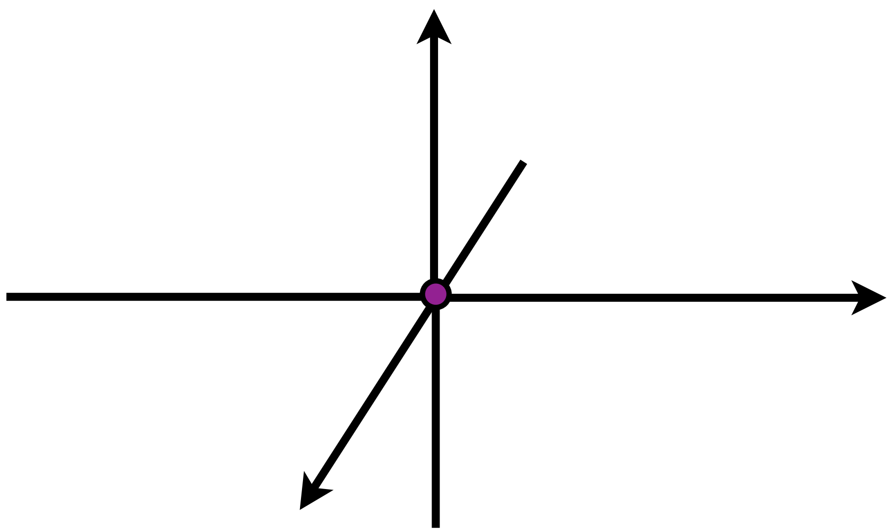

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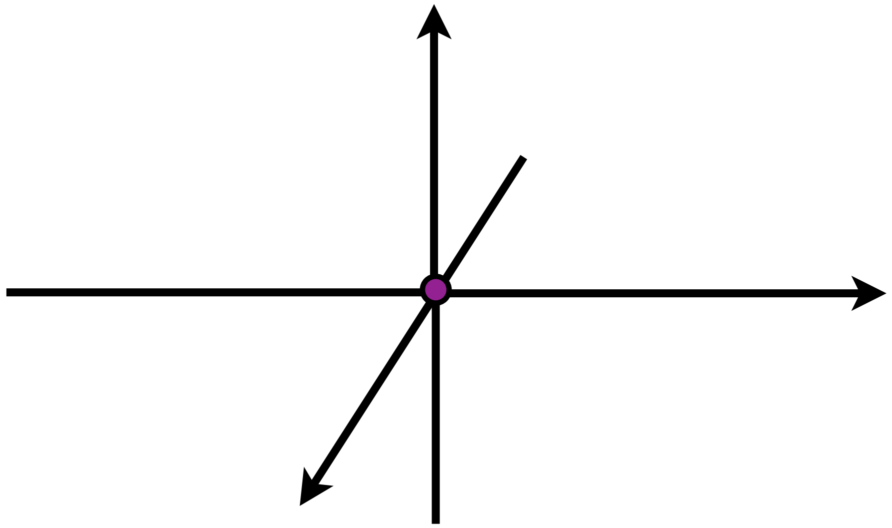

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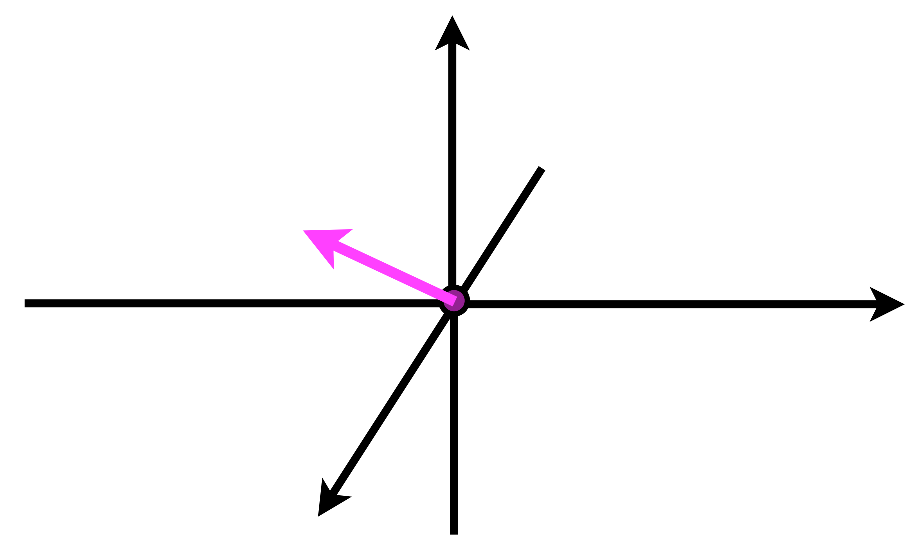

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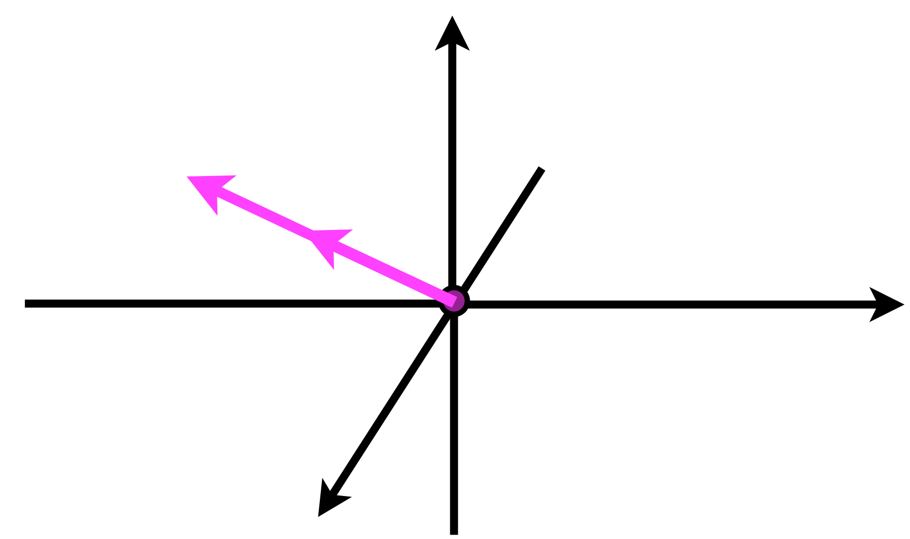

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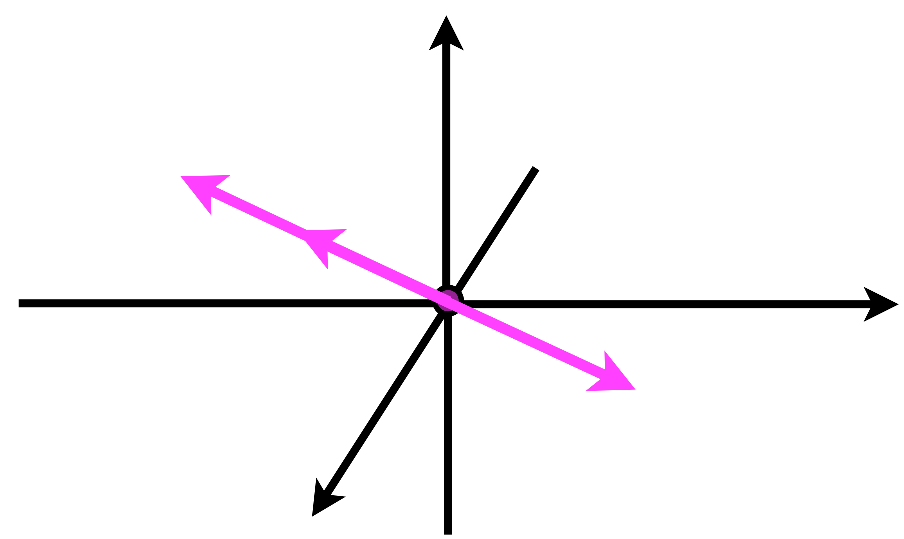

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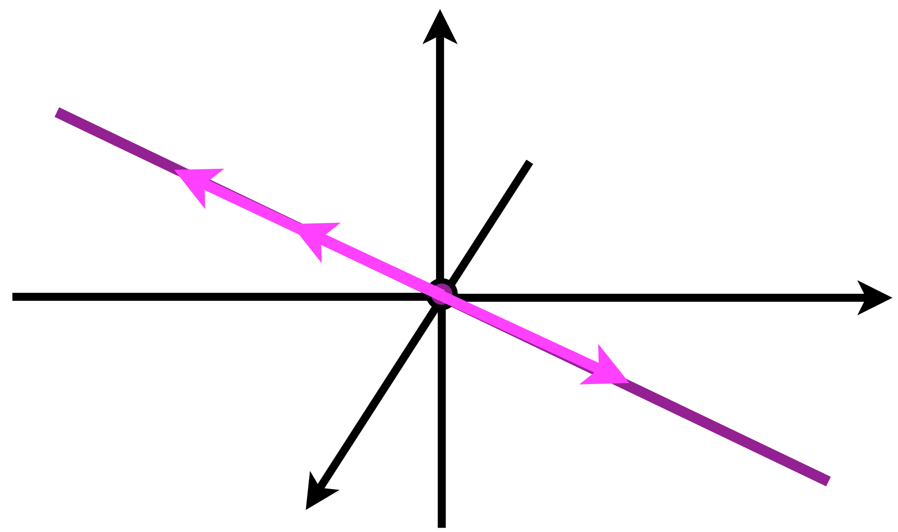

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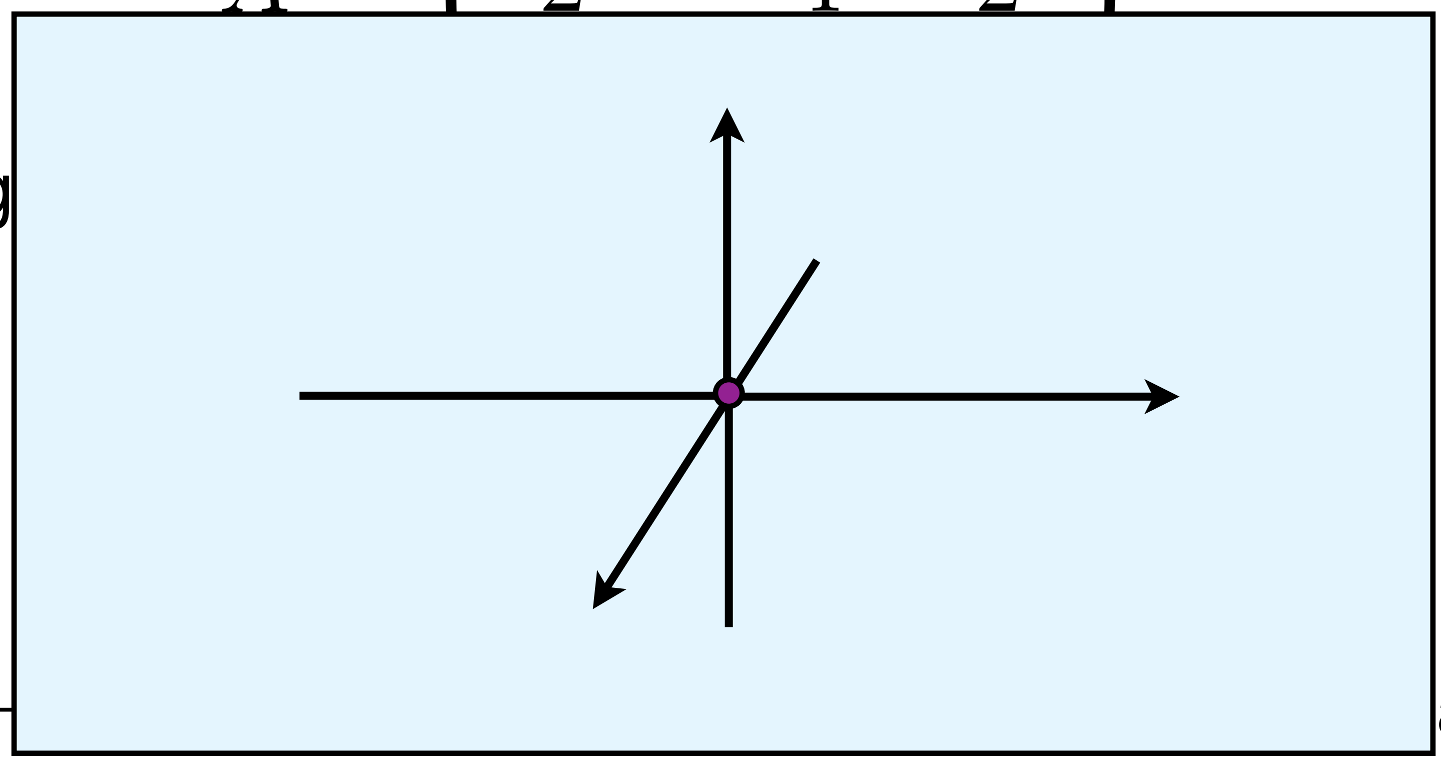

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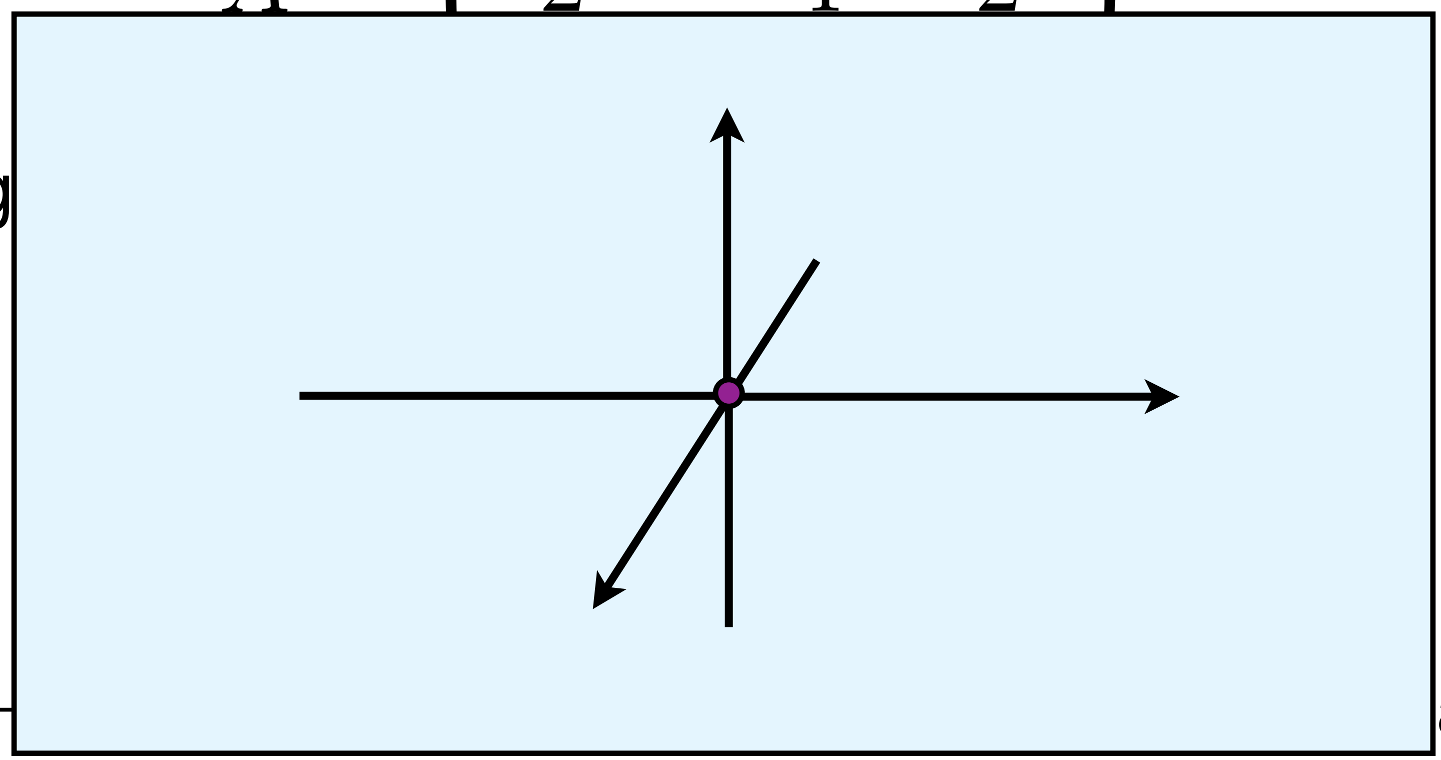

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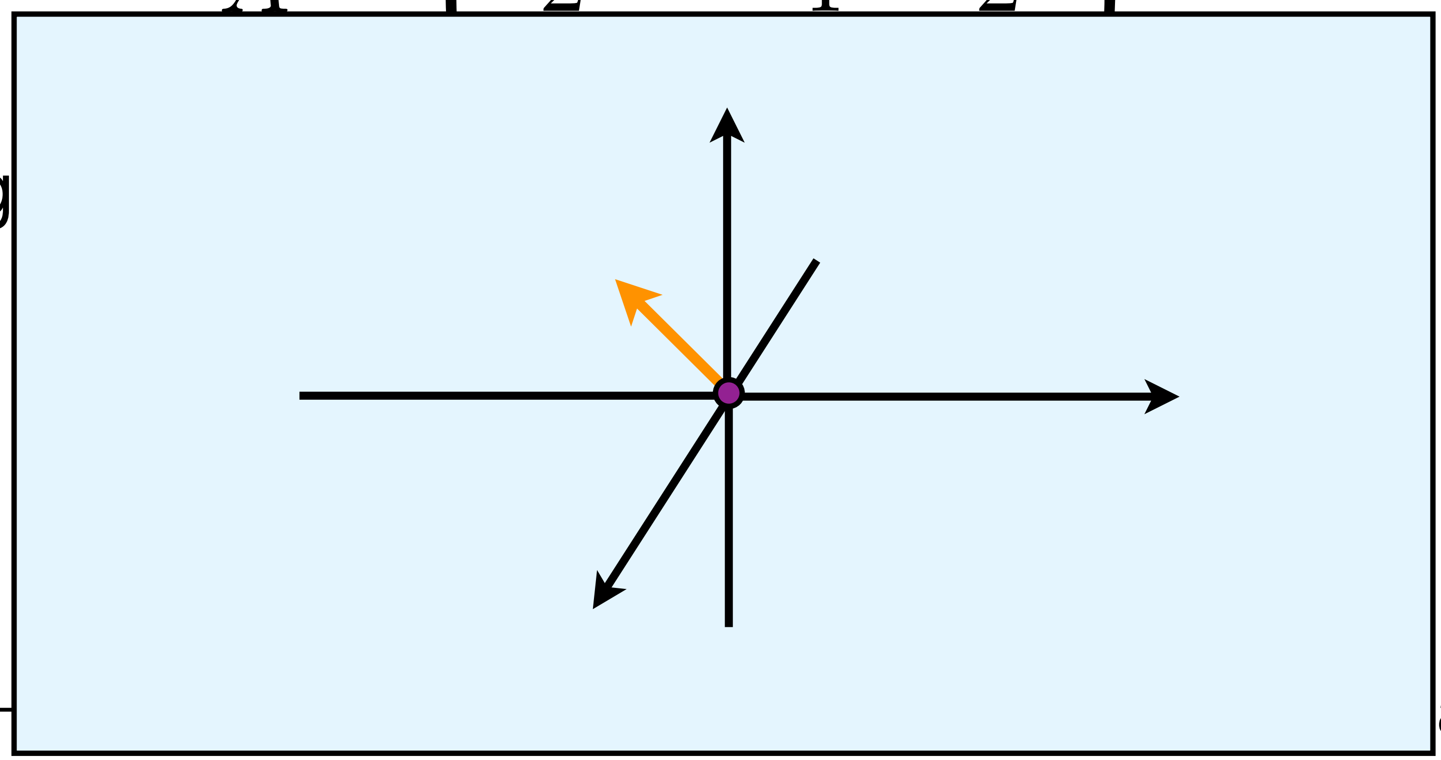

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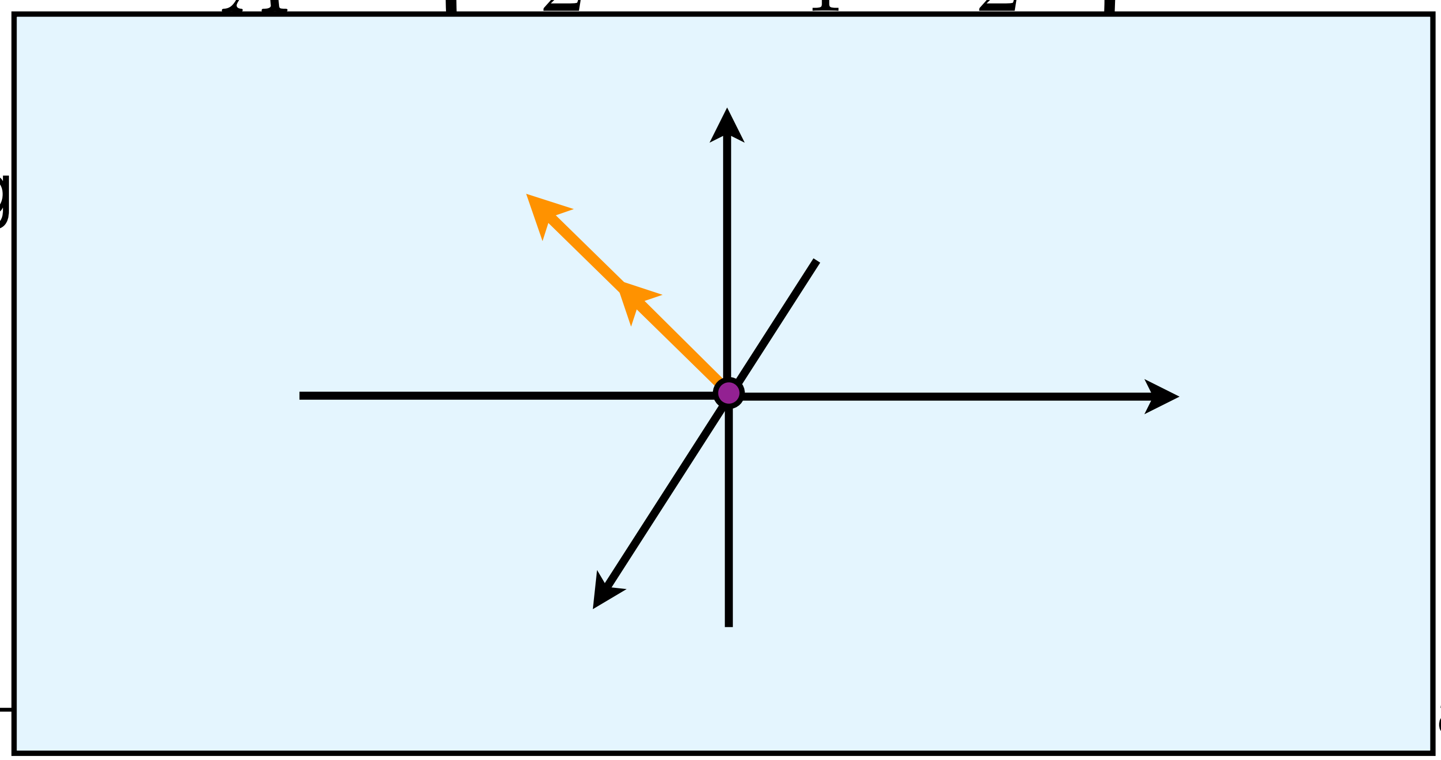

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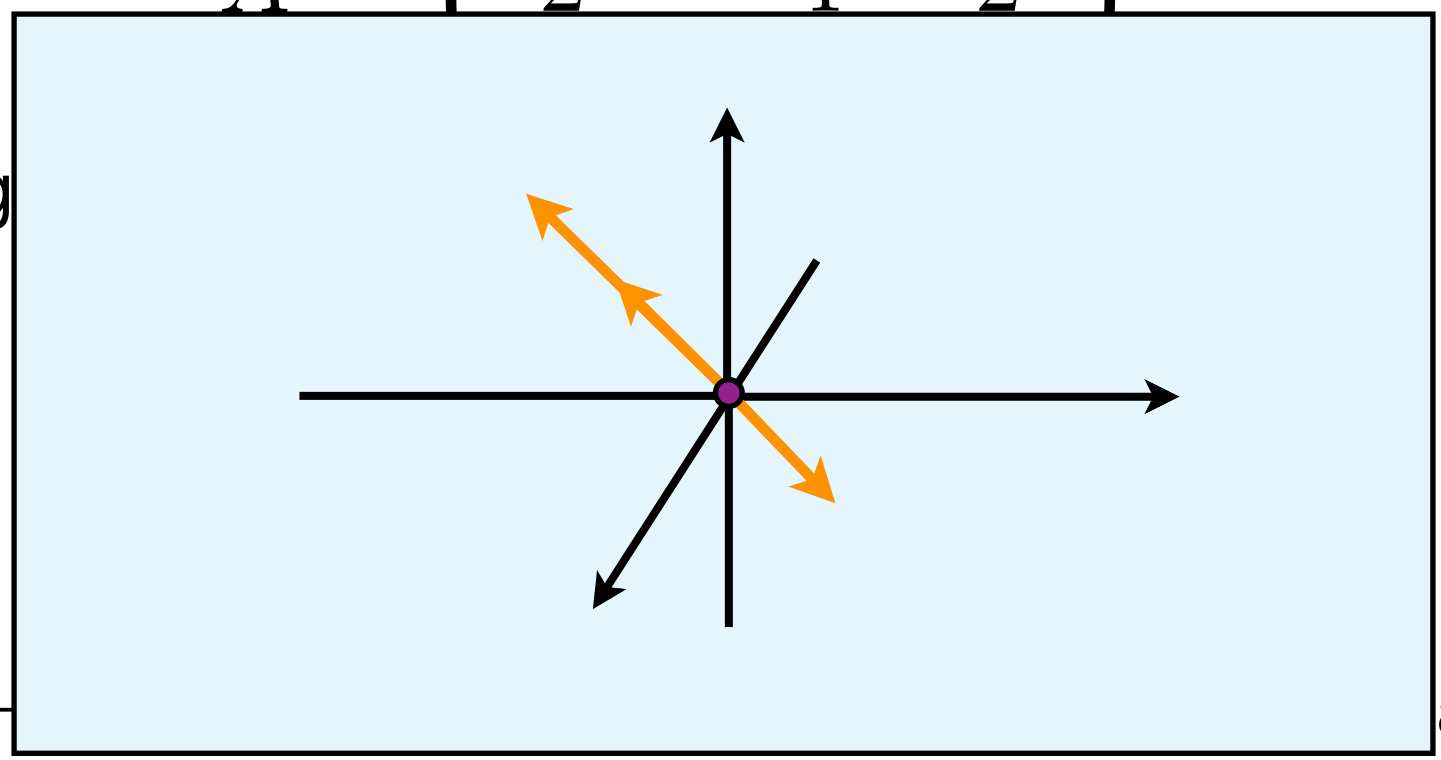

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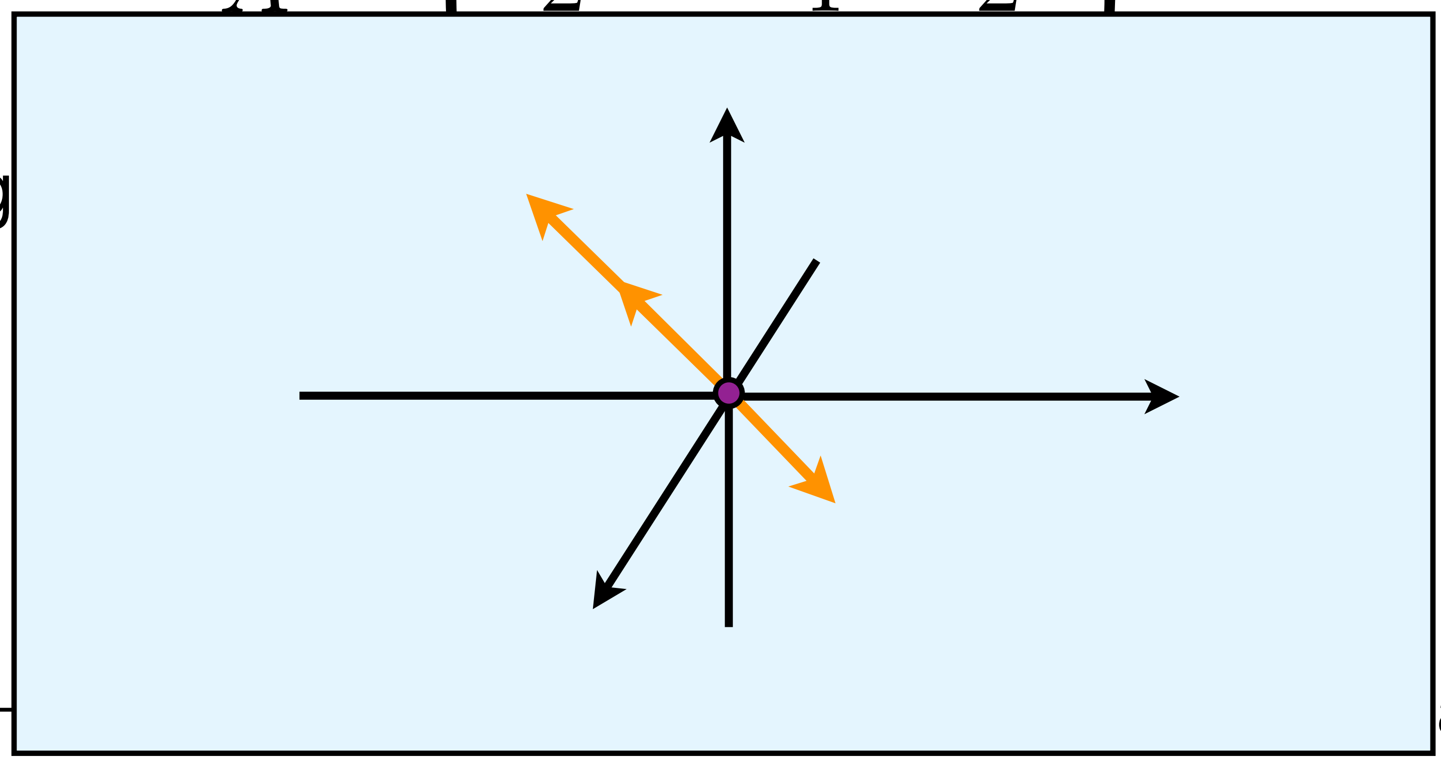

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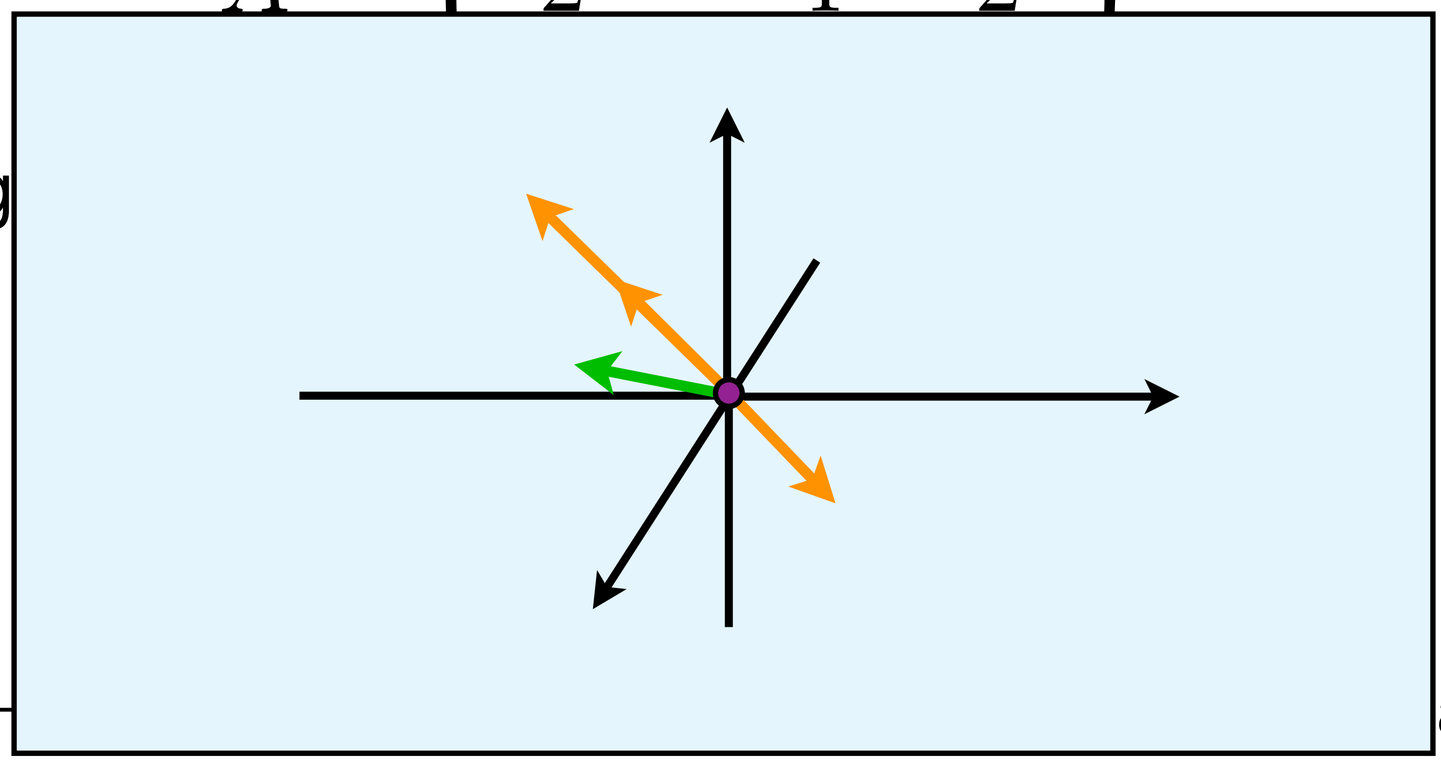

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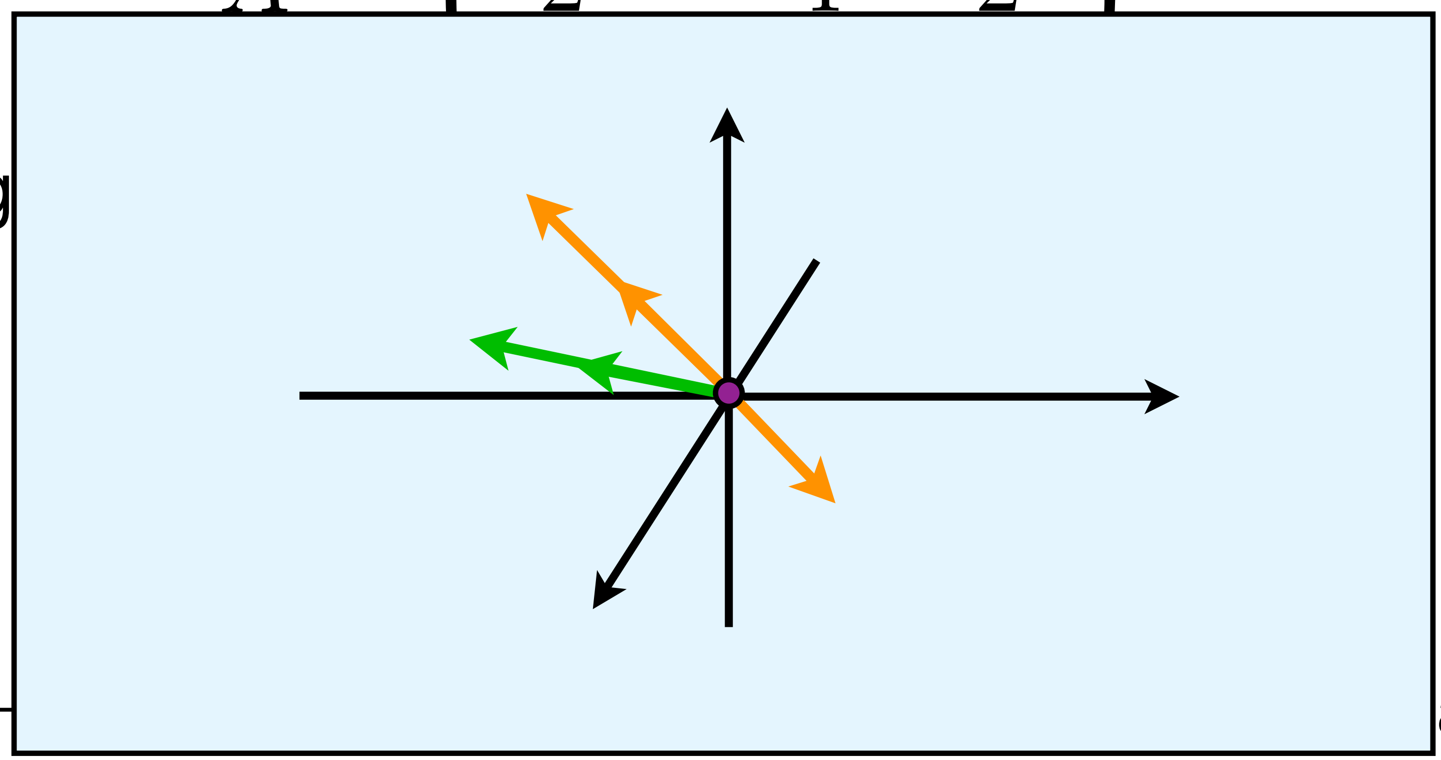

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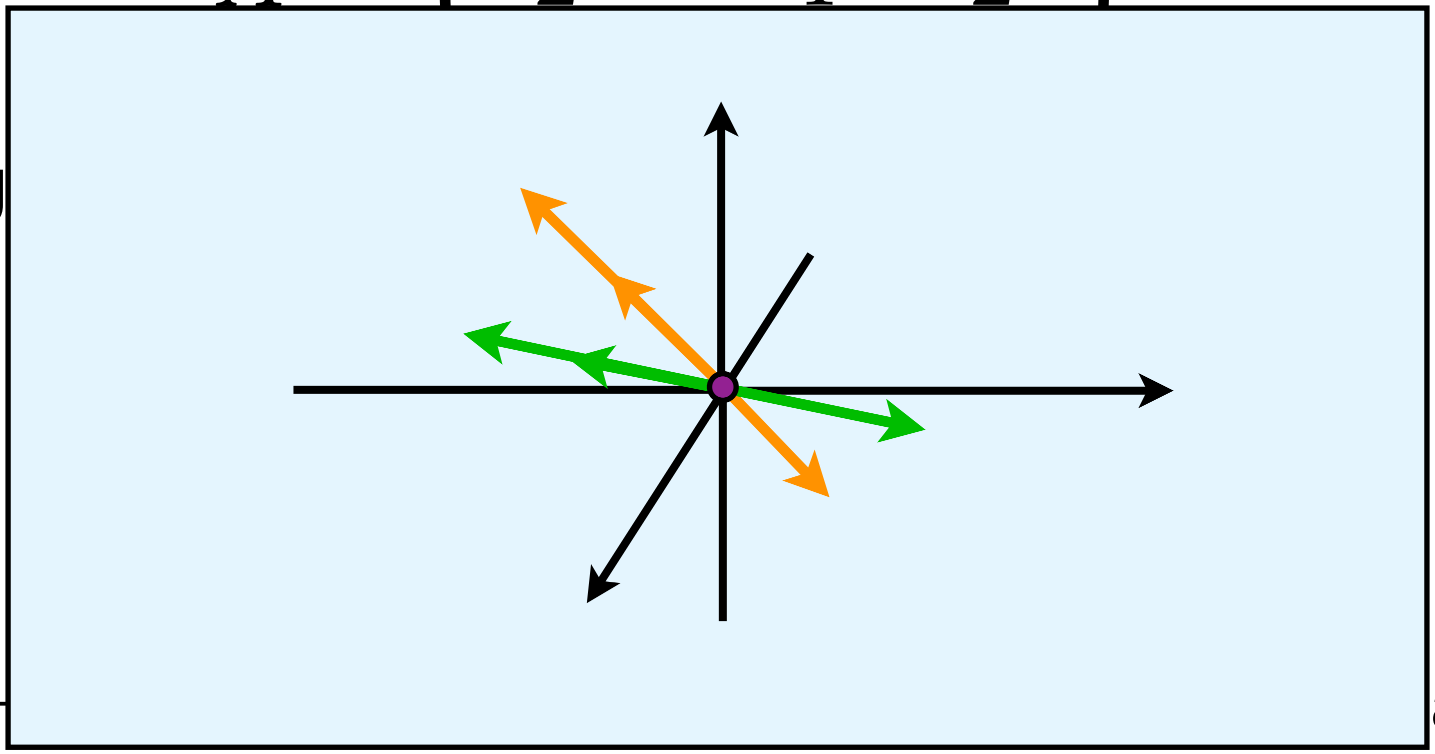

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- so $x_1 - 2x_2 +$ atever.

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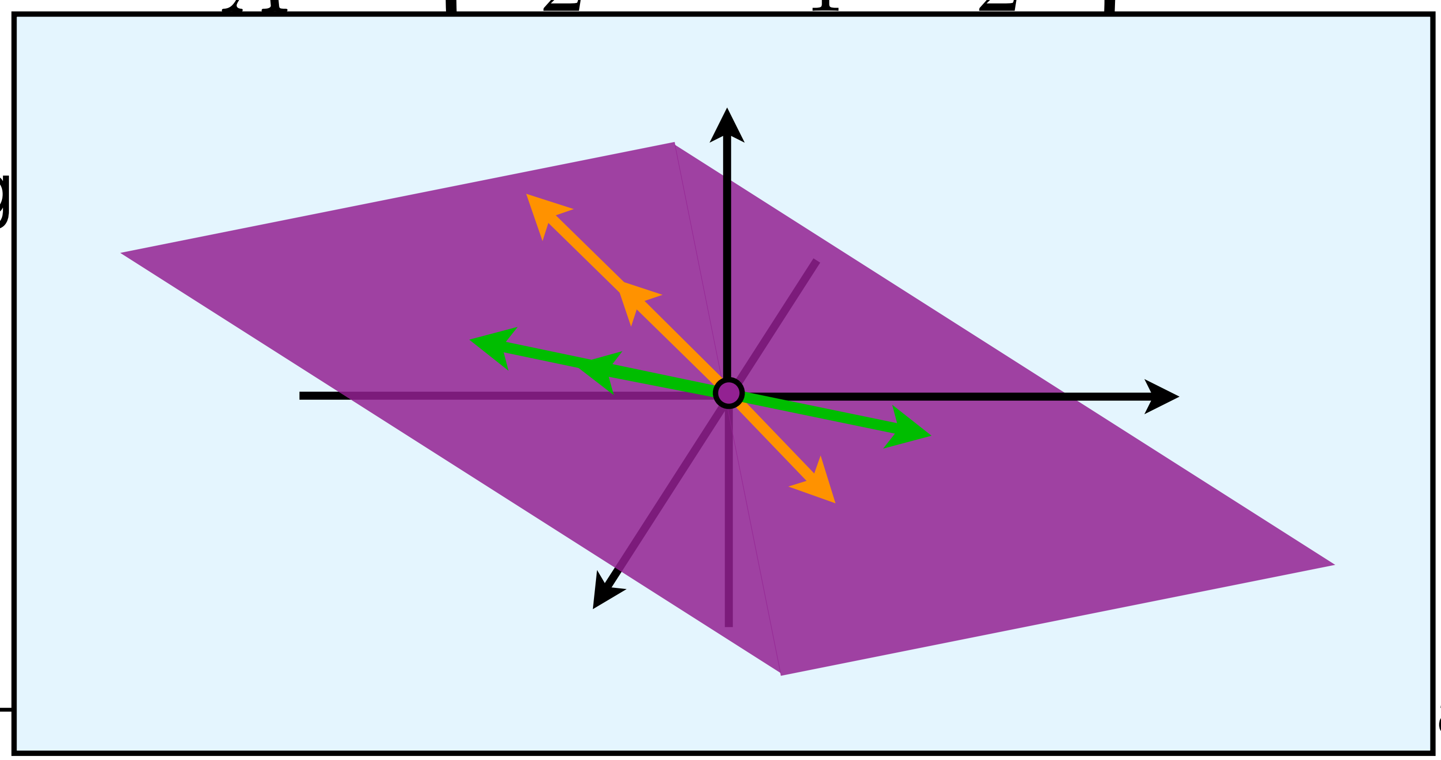

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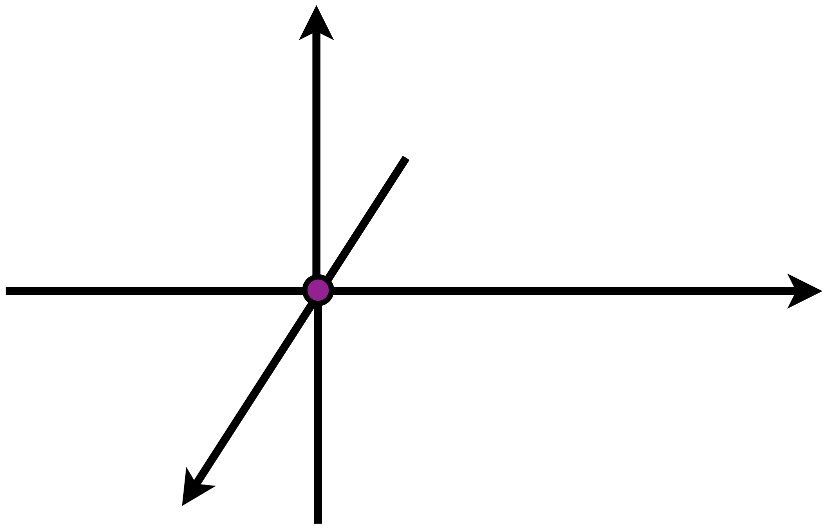

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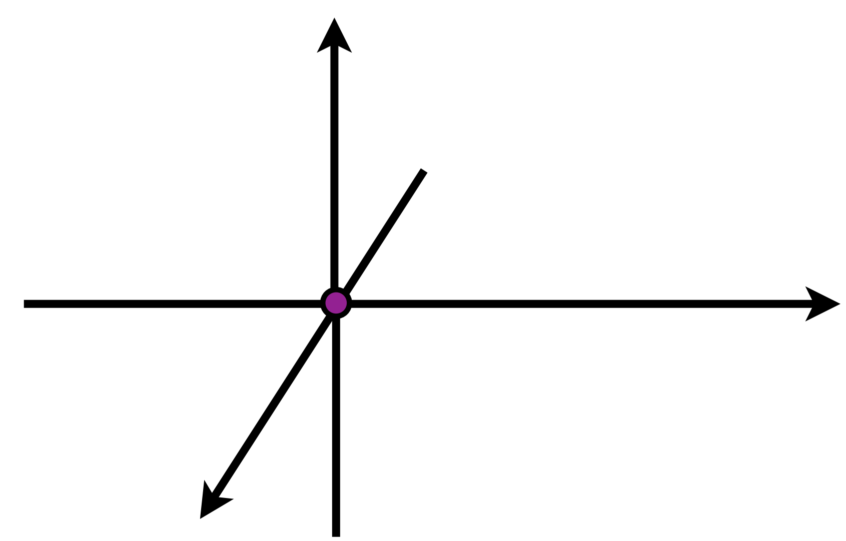

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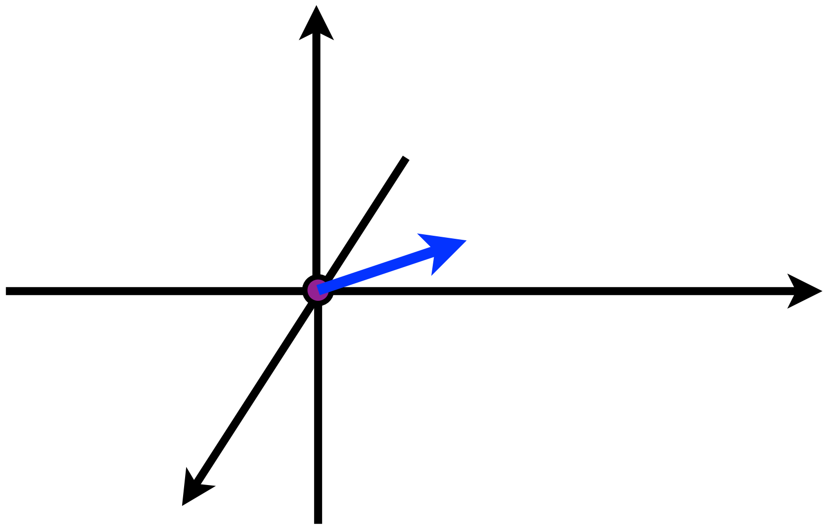

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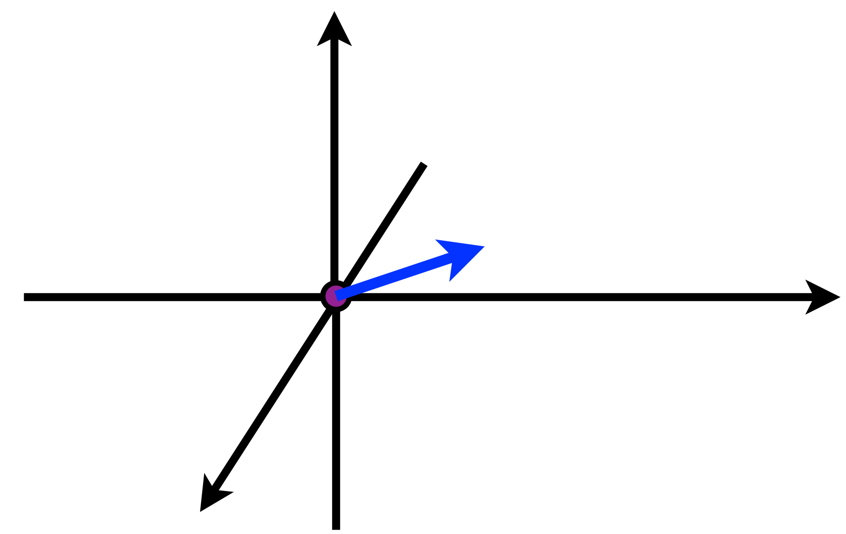

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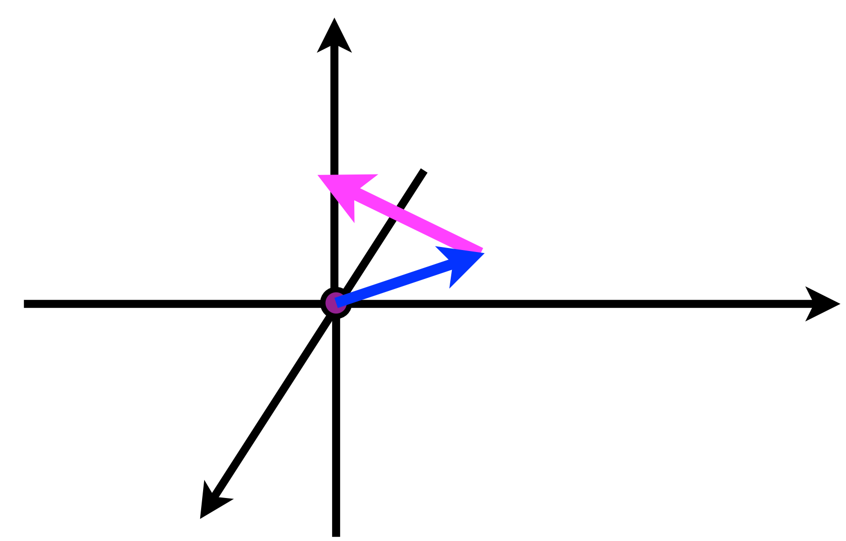

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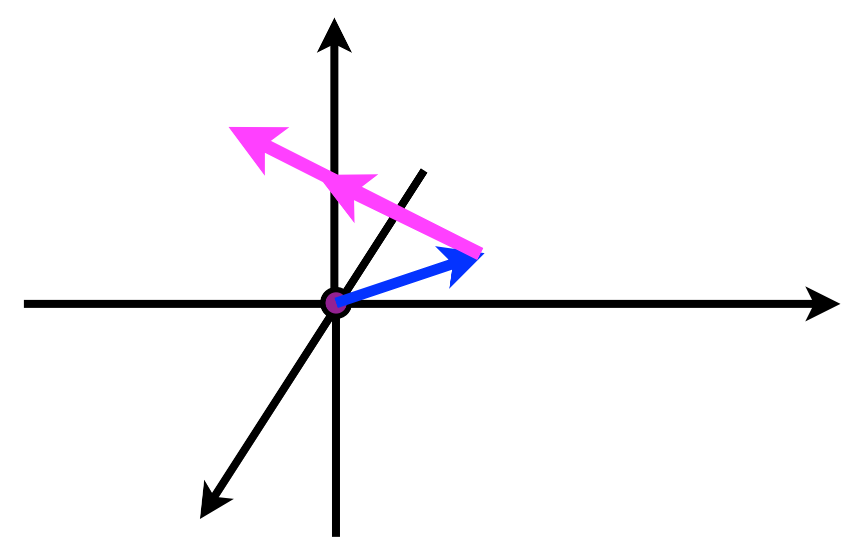

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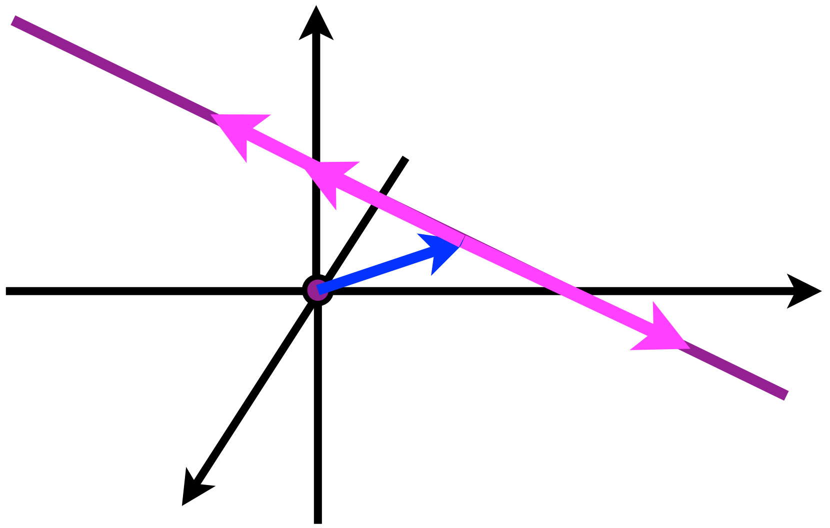

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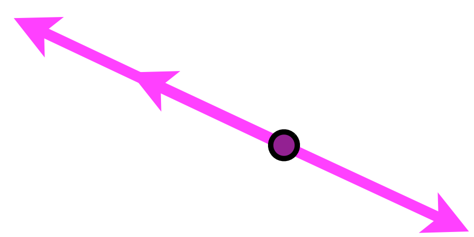

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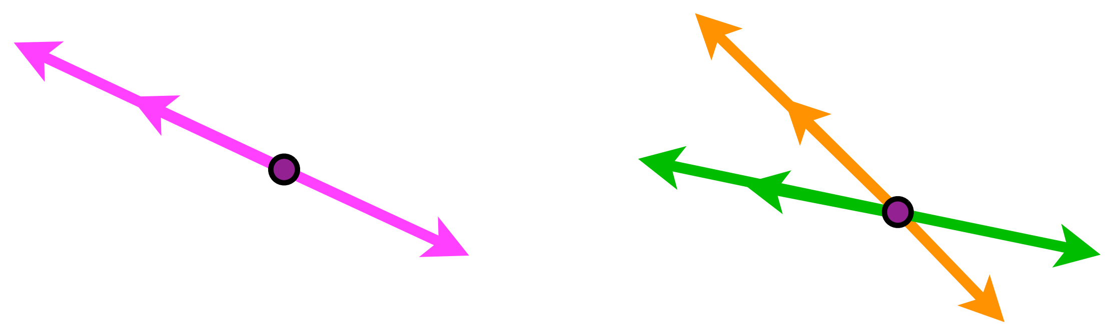

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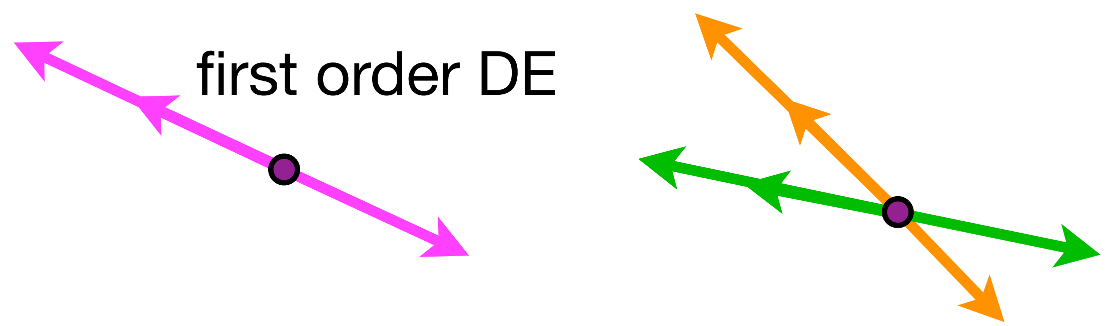

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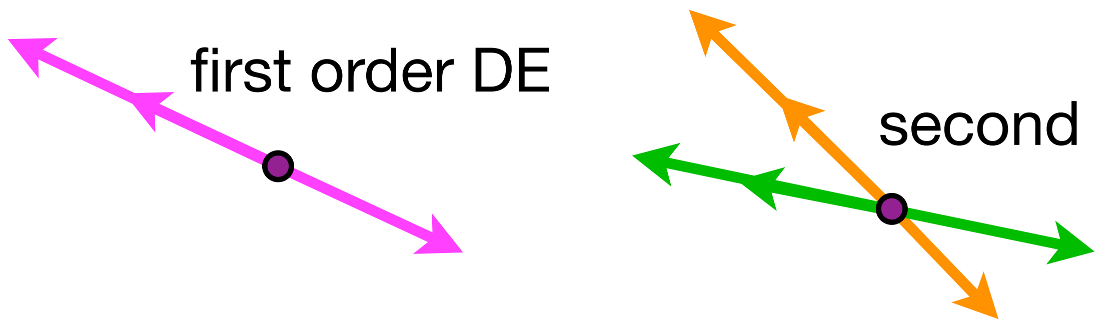

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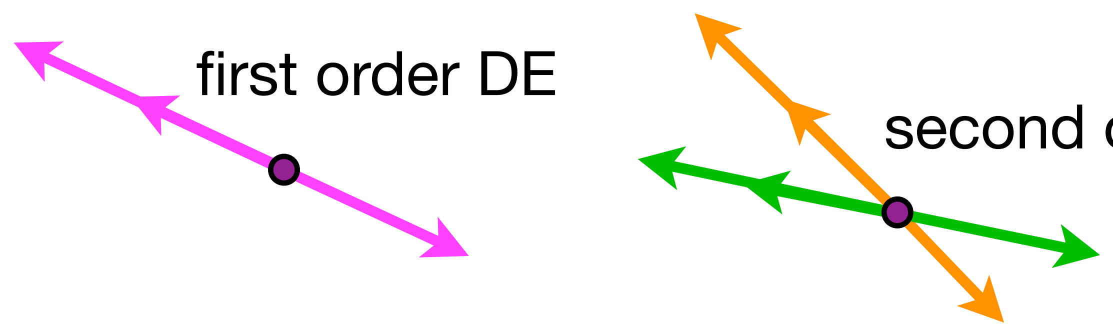

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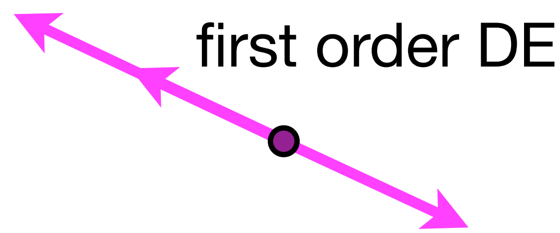

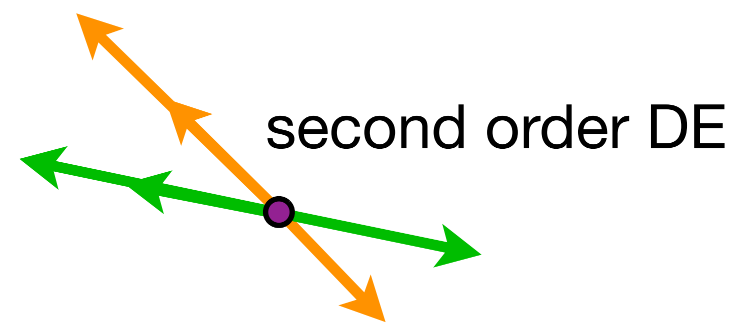

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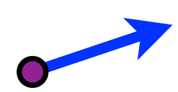

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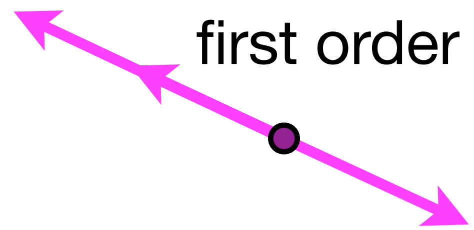
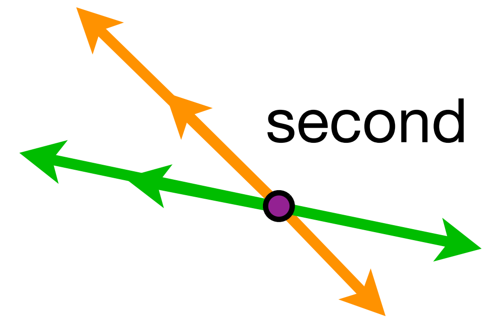


first order DE



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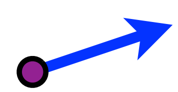

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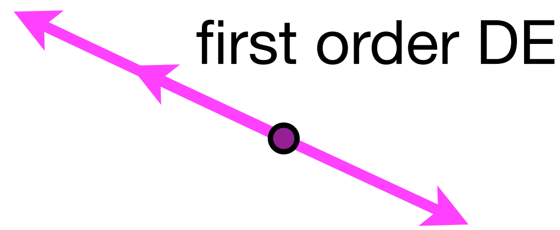

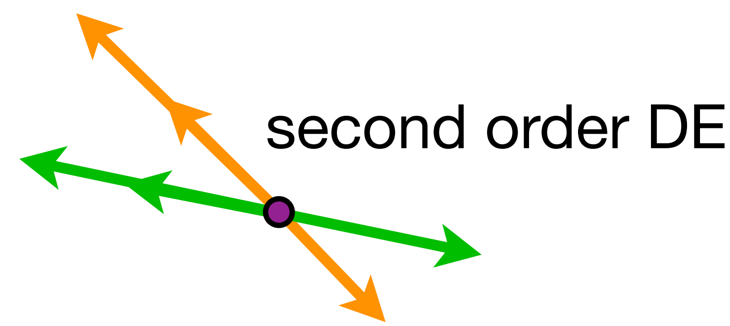

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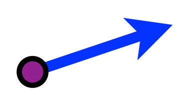

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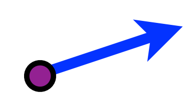

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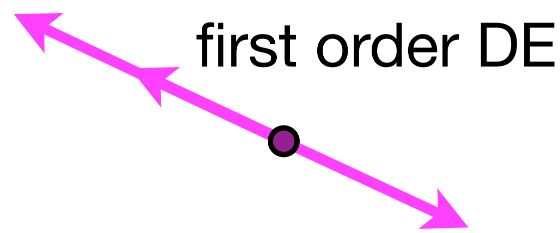

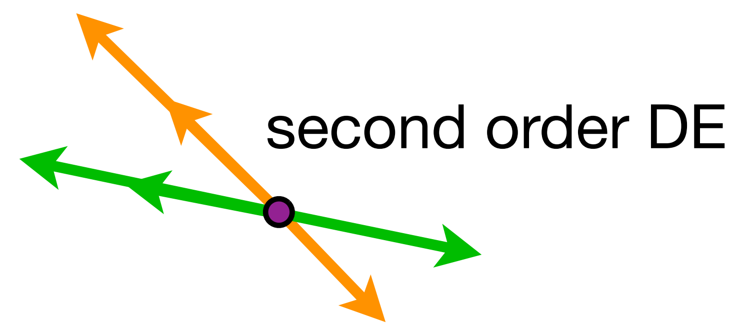

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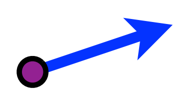

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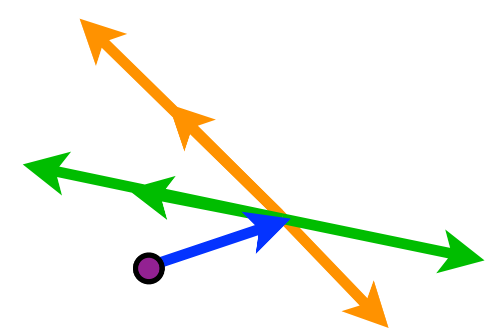

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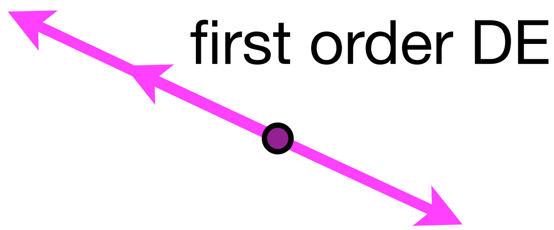

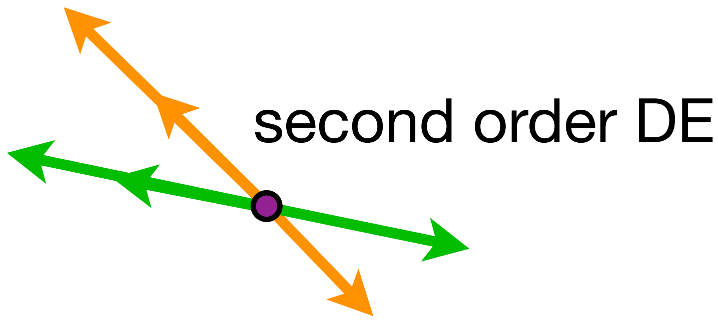

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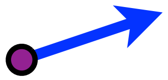

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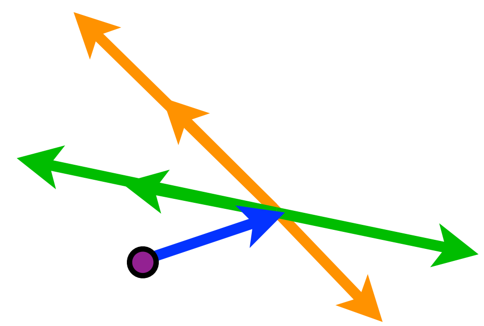

- For step 2, try “Method of undetermined coefficients”...

Method of undetermined coefficients (3.5)

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- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.

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Method of undetermined coefficients (3.5)

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  - Step 1: Solve the associated homogeneous equation
$$y'' + 2y' - 3y = 0.$$
$$y_h(t) = C_1 e^t + C_2 e^{-3t}$$
  - Step 2: What do you have to plug in to $L[\ \cdot\ ]$ to get e^{2t} out?
    - Try $y_p(t) = Ae^{2t}$.
    - $L[y_p(t)] = L[Ae^{2t}] =$

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        - Try $y_p(t) = Ae^{2t}$.
        - $L[y_p(t)] = L[Ae^{2t}] = \begin{cases} \text{(A) } 5e^{2t} & \text{(C) } 4e^{2t} \\ \text{(B) } 5Ae^{2t} & \text{(D) } 4Ae^{2t} \end{cases}$

Method of undetermined coefficients (3.5)

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    - $L[y_p(t)] = L[Ae^{2t}] =$

(A) $5e^{2t}$ (C) $4e^{2t}$

(B) $5Ae^{2t}$ (D) $4Ae^{2t}$

Method of undetermined coefficients (3.5)

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    - A is an undetermined coefficient (until you determine it).

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.

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Method of undetermined coefficients (3.5)

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$$L[C_1 e^t + C_2 e^{-3t}] = 0$$

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    - We know that, for any C_1 and C_2,

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    - We also know that

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    - Finally, by linearity, we know that

$$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$

Method of undetermined coefficients (3.5)

- Example 4. Define the operator $L[y] = y'' + 2y' - 3y$. Find the general solution to $L[y] = e^{2t}$. That is, $y'' + 2y' - 3y = e^{2t}$.
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    - So what's left to do to find our general solution? Pick A =?

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    $$L[C_1 e^t + C_2 e^{-3t} + Ae^{2t}] = 0 + 5Ae^{2t}$$
    - So what's left to do to find our general solution? Pick A = 1/5.

Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the solution to the associated homogeneous equation?

(A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

(B) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t)$

(C) $y_h(t) = C_1 e^{2t} + C_2 t e^{2t}$

(D) $y_h(t) = C_1 \cos(2t) + C_2 \sin(2t) + e^t$

(E) Don't know.

Method of undetermined coefficients (3.5)

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Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
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    (C) $y_p(t) = Ae^t$

    (D) $y_p(t) = Ate^t$

    (E) Don't know

Method of undetermined coefficients (3.5)

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    (E) Don't know

Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
  - What is the value of A that gives the particular solution (Ae^t) ?

    (A) A = 1

    (B) A = 3

    (C) A = 1/3

    (D) A = -1/3

    (E) Don't know.

Method of undetermined coefficients (3.5)

- Example 5. Find the general solution to the equation $y'' - 4y = e^t$.
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    (D) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t} + e^t$

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Method of undetermined coefficients (3.5)

- Example 7. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the solution to the associated homogeneous equation?

    ☆ (A) $y_h(t) = C_1 e^{2t} + C_2 e^{-2t}$

    (B) $y_h(t) = C_1 \cos(2t) + C$ ...

    (C) $y_h(t) = C$ ...

    ... $C_1 e^{2t} + C_2 e^{-2t} + e^t$

    (E) Don't know.

Same as the last example

Method of undetermined coefficients (3.5)

- Example 7. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

    (A) $y_p(t) = Ae^{2t}$

    (B) $y_p(t) = Ae^{-2t}$

    (C) $y_p(t) = Ate^{2t}$

    (D) $y_p(t) = Ae^{t}$

    (E) $y_p(t) = Ate^{t}$

- General rule: when your guess at y_p makes LHS=0, try multiplying it by t.

Method of undetermined coefficients (3.5)

- Example 7. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0 \ !$$

(B) $y_p(t) = Ae^{-2t}$

(C) $y_p(t) = Ate^{2t}$

(D) $y_p(t) = Ae^{t}$

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$$(Ae^{2t})'' - 4Ae^{2t} = 0 \ !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^{t}$$

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$$(Ae^{2t})'' - 4Ae^{2t} = 0 \ !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^t$$
$$e^{-t}y' - e^{-t}y = 1$$

- General rule: when your guess at y_p makes LHS=0, try multiplying it by t.

Method of undetermined coefficients (3.5)

- Example 7. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the form of the particular solution?

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(E) $y_p(t) = Ate^{t}$

$$(Ae^{2t})'' - 4Ae^{2t} = 0\ !$$

- Simpler example in which the RHS is a solution to the homogeneous problem.

$$y' - y = e^{t}$$
$$e^{-t}y' - e^{-t}y = 1$$
$$y = te^{t} + Ce^{t}$$

- General rule: when your guess at y_p makes LHS=0, try multiplying it by t.

Method of undetermined coefficients (3.5)

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Method of undetermined coefficients (3.5)

- Example 7. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the value of A that gives the particular solution (Ate^{2t})?

    (A) A = 1

    (B) A = 4

    (C) A = -4

    (D) A = 1/4

    (E) A = -1/4

Method of undetermined coefficients (3.5)

- Example 7. Find the general solution to the equation $y'' - 4y = e^{2t}$.
  - What is the value of A that gives the particular solution (Ate^{2t})?

(A) A = 1

$$(Ate^{2t})' = Ae^{2t} + 2Ate^{2t}$$

(B) A = 4

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$$(Ate^{2t})' = Ae^{2t} + 2Ate^{2t}$$

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$$(Ate^{2t})' = Ae^{2t} + 2Ate^{2t}$$

$$(Ate^{2t})'' = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t}$$
$$= 4Ae^{2t} + 4Ate^{2t}$$

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$$(Ate^{2t})' = Ae^{2t} + 2Ate^{2t}$$

$$\begin{aligned}(Ate^{2t})'' &= 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t} \\ &= 4Ae^{2t} + 4Ate^{2t}\end{aligned}$$

$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) =$$

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$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) = 4Ae^{2t}$$

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Need: $= e^{2t}$

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$$(Ate^{2t})'' = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t}$$

$$= 4Ae^{2t} + 4Ate^{2t}$$

$$\left(Ate^{2t}\right)'' - 4\left(Ate^{2t}\right) = 4Ae^{2t}$$

Need: $= e^{2t}$

Method of undetermined coefficients (3.5)

- Example 8. Find the general solution to $y'' - 4y = \cos(2t)$.
  - What is the form of the particular solution?

(A) $y_p(t) = A\cos(2t)$

(B) $y_p(t) = A\sin(2t)$

(C) $y_p(t) = A\cos(2t) + B\sin(2t)$

(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

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(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Challenge: What small change to the DE makes (D) correct?

Method of undetermined coefficients (3.5)

- Example 8. Find the general solution to $y'' + y' - 4y = \cos(2t)$.
  - What is the form of the particular solution?

(A) $y_p(t) = A\cos(2t)$

(B) $y_p(t) = A\sin(2t)$

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(D) $y_p(t) = t(A\cos(2t) + B\sin(2t))$

(E) $y_p(t) = e^{2t}(A\cos(2t) + B\sin(2t))$

Method of undetermined coefficients (3.5)

- Example 9. Find the general solution to $y'' - 4y = t^3$.
  - What is the form of the particular solution?

    (A) $y_p(t) = At^3$

    (B) $y_p(t) = At^3 + Bt^2 + Ct$

    (C) $y_p(t) = At^3 + Bt^2 + Ct + D$

    (D) $y_p(t) = At^3 + Be^{2t} + Ce^{-2t}$

    (E) Don't know.

Method of undetermined coefficients (3.5)

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(D) $y_p(t) = At^3 + Be^{2t} + Ce^{-2t}$

(E) Don't know.

waste of time including solution to homogeneous eq.

Method of undetermined coefficients (3.5)

- When RHS is sum of terms:

$$y'' - 4y = \cos(2t) + t^3$$

$$y_p(t) = A\cos(2t) + B\sin(2t) + Ct^3 + Dt^2 + Et + F$$

Method of undetermined coefficients (3.5)

- Example 10. Find the general solution to $y'' + 2y' = e^{2t} + t^3$.
  - What is the form of the particular solution?

(A) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt$

(B) $y_p(t) = Ae^{2t} + Bt^3 + Ct^2 + Dt + E$

(C) $y_p(t) = Ae^{2t} + (Bt^4 + Ct^3 + Dt^2 + Et)$

(D) $y_p(t) = Ae^{2t} + Be^{-2t} + Ct^3 + Dt^2 + Et + F$

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$y_p(t) = Ae^{2t} + t(Bt^3 + Ct^2 + Dt + E)$

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For each wrong answer, for what DE is it the correct form?

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  - Include all functions that are part of the $g(t)$ family (e.g. cos **and** sin)
  - If part of the $g(t)$ family is a solution to the homogeneous (h-)problem, use $t \times$ ($g(t)$ family).
  - If $t \times$ (part of the $g(t)$ family), is a solution to the h-problem, use $t^2 \times$ ($g(t)$ family).
  - For sums, group terms into families and include a term for each.
  - For products of families, use the above rules and multiply them.
  - If your guess includes a solution to the h-problem, you may as well remove it as it won't survive L[ ] so you won't be able to determine its undetermined coefficient.