## MATH 256 – Midterm 1 – February 2, 2016.

Last name: \_

\_\_\_\_\_ First name: \_\_\_\_\_ Student #: \_\_\_\_\_

Place a box around each answer so that it is clearly identified. Point values are approximate and may differ slightly in the final marking scheme.

1. [3 pts] Consider the equation y'' + 6y' + ky = 0. Place (a), (b) and (c) in the boxes below to correctly complete the sentences.



2. [5 pts] Find the solution to the equation y' = x/y subject to the initial condition y(0) = -2.

3. [4 pts] Show that  $\sin(t)$  and  $\cos(t)$  are independent functions but that  $\sin(t)$  and  $\cos(t + \frac{\pi}{2})$  are not independent. Recall that to show dependence, you must find constants  $C_1$  and  $C_2$  that make a linear combination of the functions zero.

2:

3:

4. [4 pts] Find the general solution to the equation tw' - w = 0.

5. [6 pts] Use Reduction of Order to find a second solution to the equation  $t^2y'' - 3ty' + 3y = 0$  given that  $y_1(t) = t$ . Along the way, you should find yourself faced with the equation tw' - w = 0. You may refer to your answer to the previous problem at that point.

Do not write in these boxes - for marking purposes only.

5:

6. [5 pts] The equation for the motion of a mass spring system is y'' + 3y' + 2y = f(t). For each f(t) in the table below, give the form of the particular solution. The undetermined coefficients do not need to be calculated.



7. [4 pts] Brij is developing a bioremediation process to clean up sewage spills. He sets up a trial experiment in the fountain on University Boulevard in which he pours sewage into the water at a rate of 30 litres per hour. The sewage contains bacteria at a concentration of 4 grams of bacteria per litre. Brij also adds a bacteria-eating algae to the water that can filter the water at a rate of 10 litres per hour, removing all bacteria from filtered water. The fountain initially holds 30000 litres of water. Write down a differential equation that describes the change in mass of bacteria in the fountain.

Do not write in these boxes - for marking purposes only.

- 8. The differential equation y' + f(t)y = g(t) has the general solution  $y(t) = (C \cos(t))/t$  where C is an arbitrary constant.
  - (a) **[1 pt]** What is the general solution to the equation y' + f(t)y = 0?

(b) **[2 pts]** What is f(t)?

(c) [2 pts] What is g(t)?

Do not write in these boxes - for marking purposes only. 8		Total:		(out of 36)
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