Today

- Reminders:
    - Pre-lecture assignment for Thursday 7 am
    - Week 1 assignment due Friday 5 pm.
- Separating variables
- Modeling tank inflow/outflow scenarios
- Existence and uniqueness (not going to test on the theory but important to know for general understanding)

Examples

- Find the general solution to

$$\frac{dy}{dt} - 3y = -4e^{-t}$$

- and plot a few of the integral curves.

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(A) $y(t) = e^{-t}$

(B) $y(t) = e^{-t} + Ce^{3t}$

(C) $y(t) = e^{-3t}$

(D) $y(t) = e^{-4t} + C$

(E) Don’t know.

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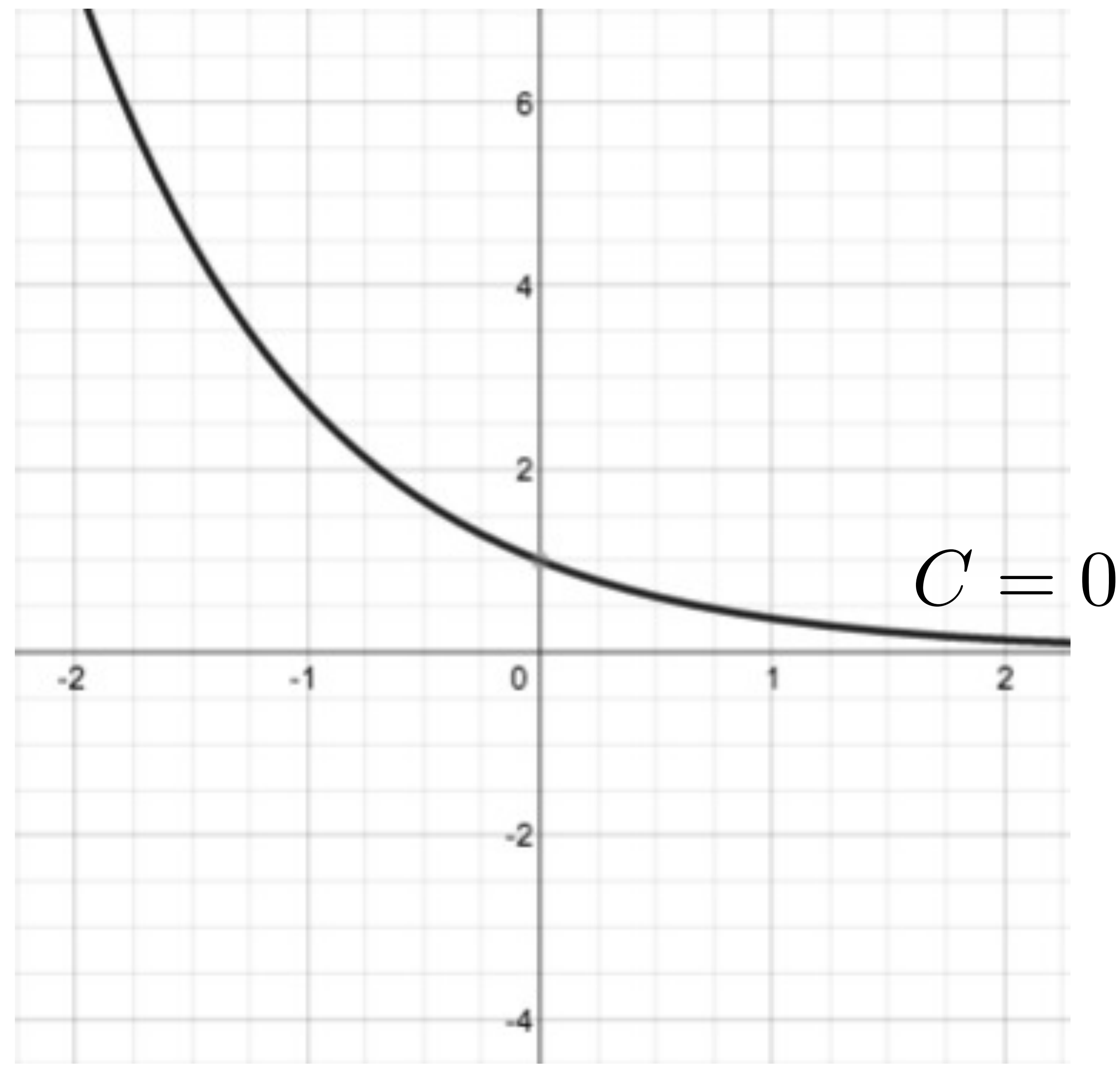

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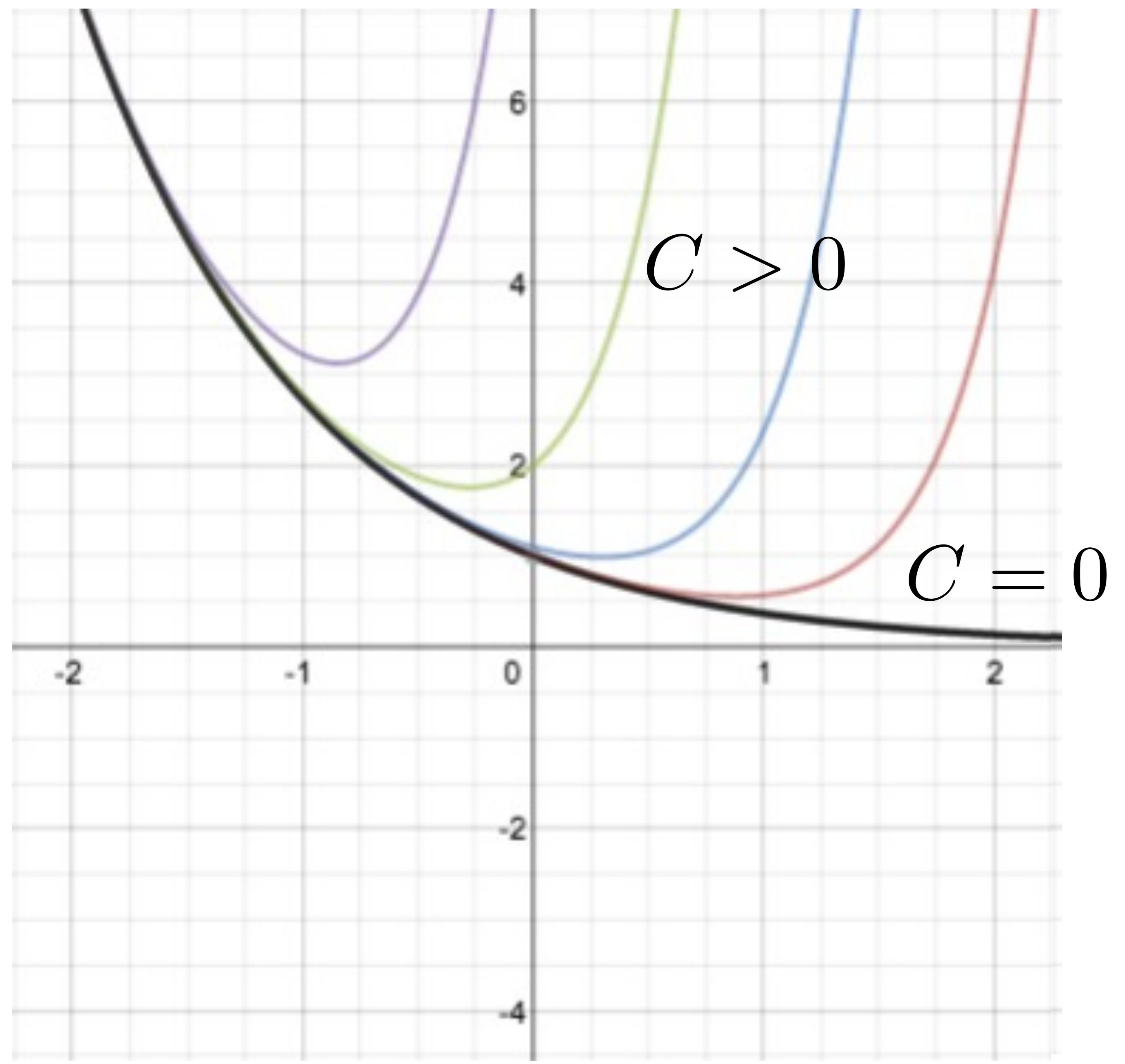

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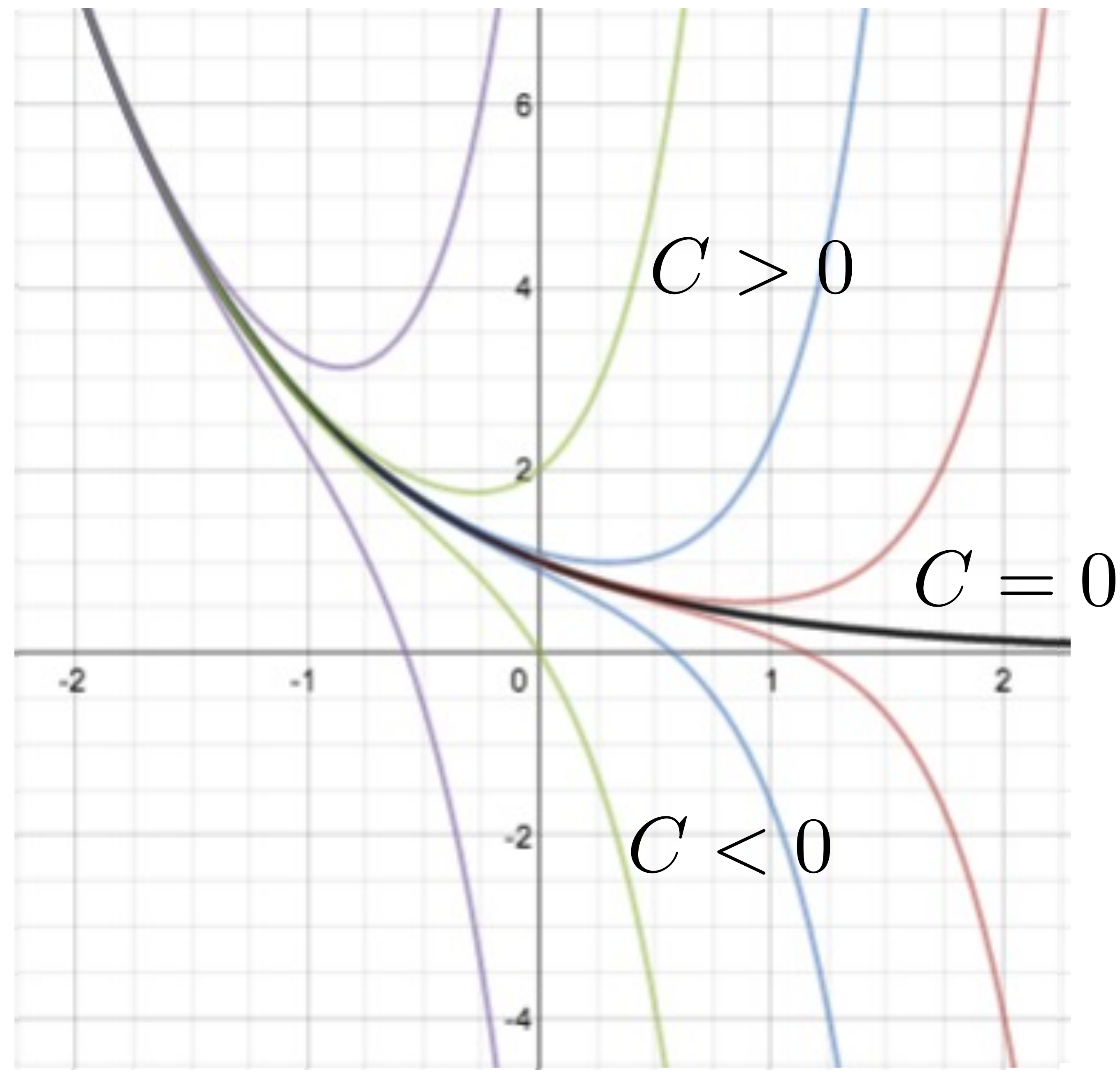

Limits at infinity

- If y(t) is a particular solution to

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- depending on C, how many different results are possible for

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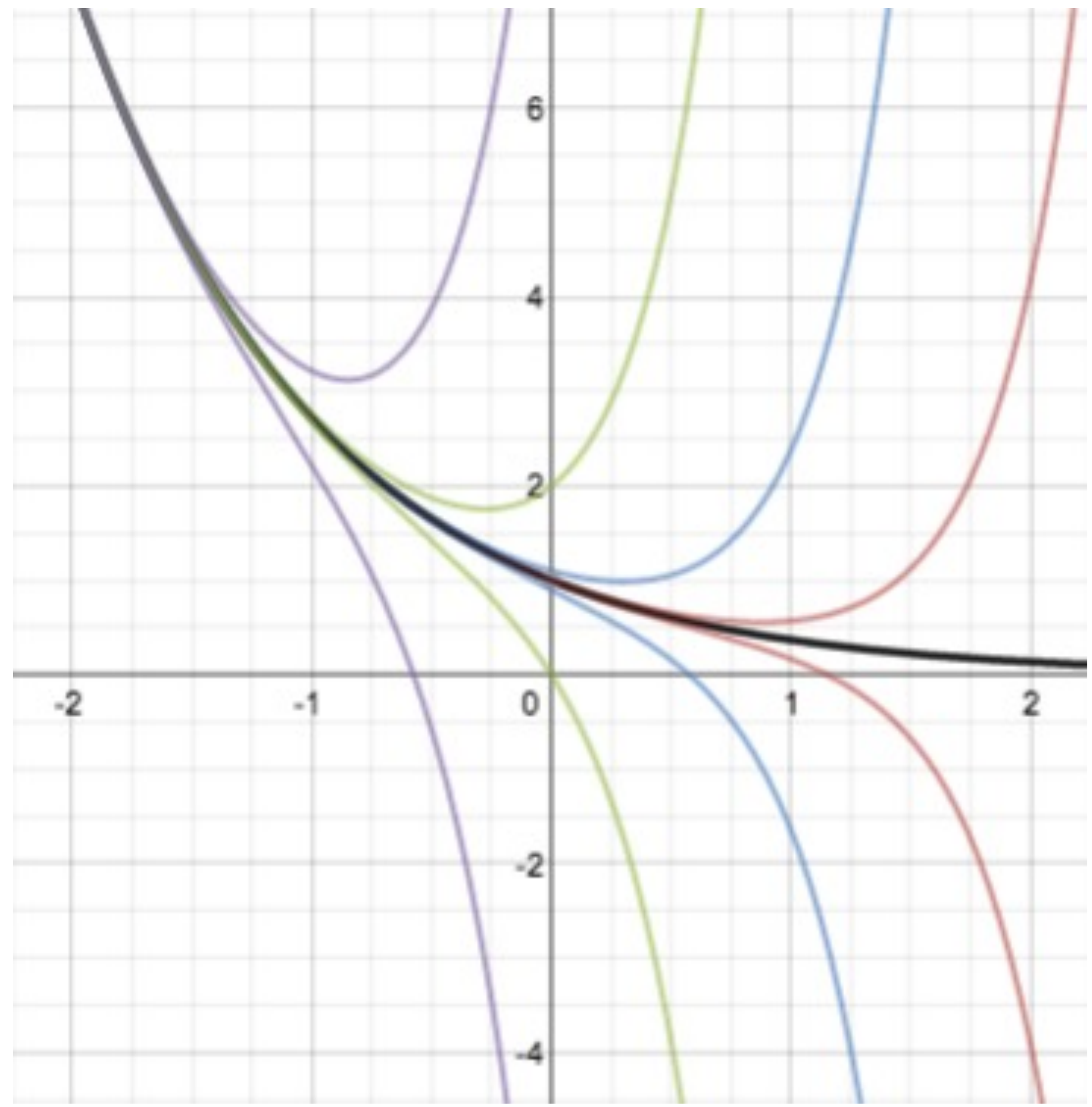

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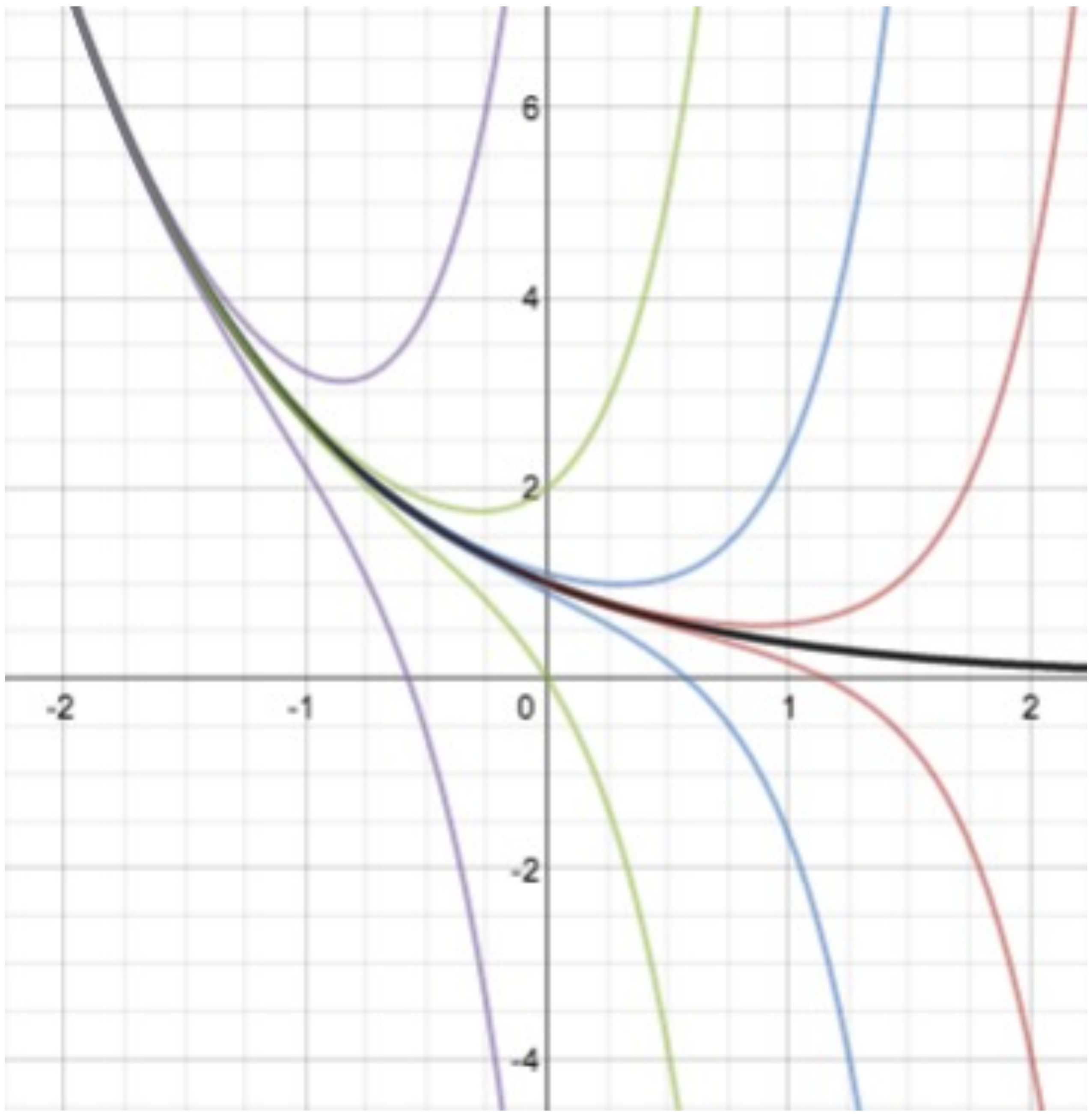

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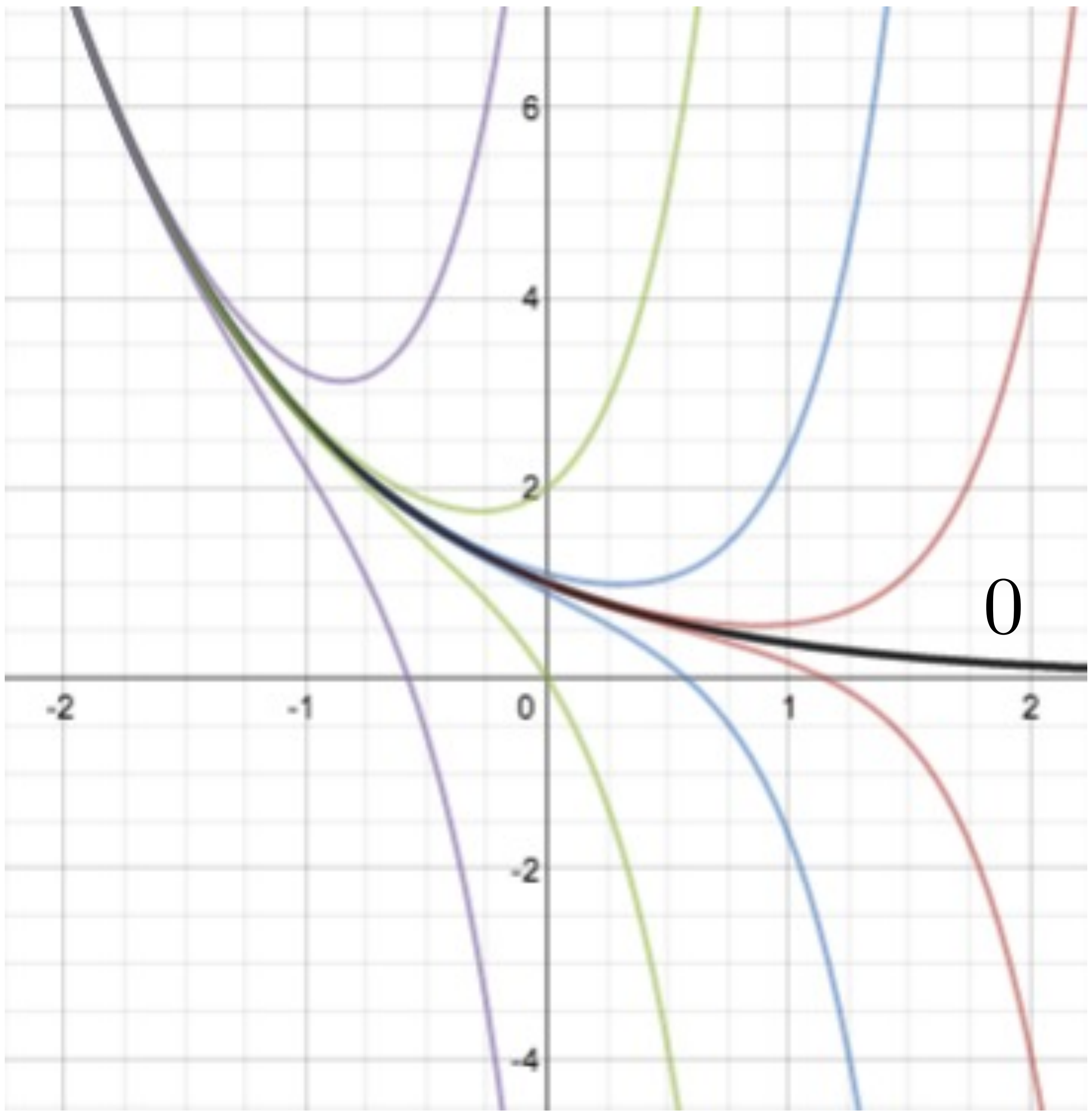

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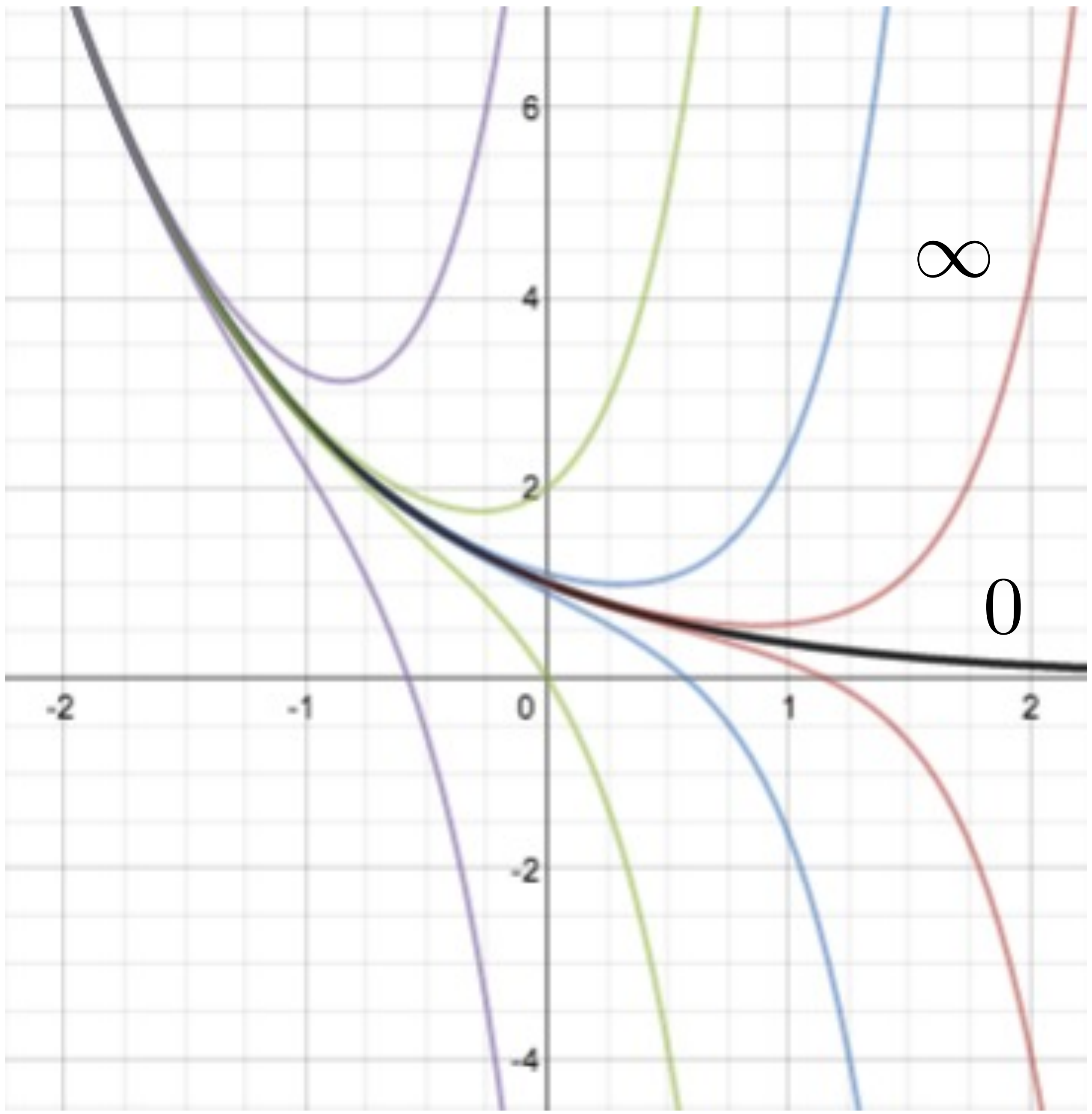

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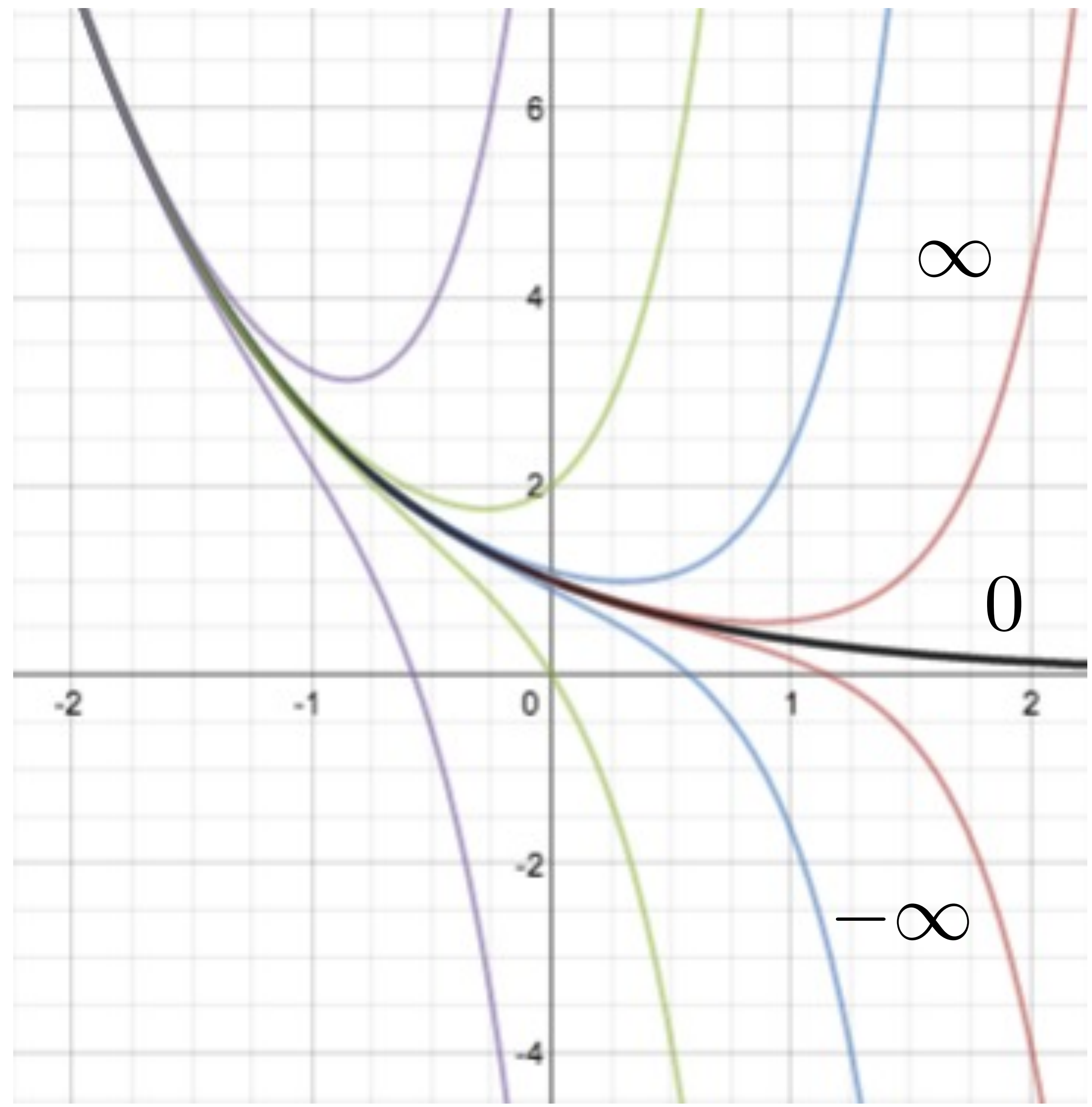

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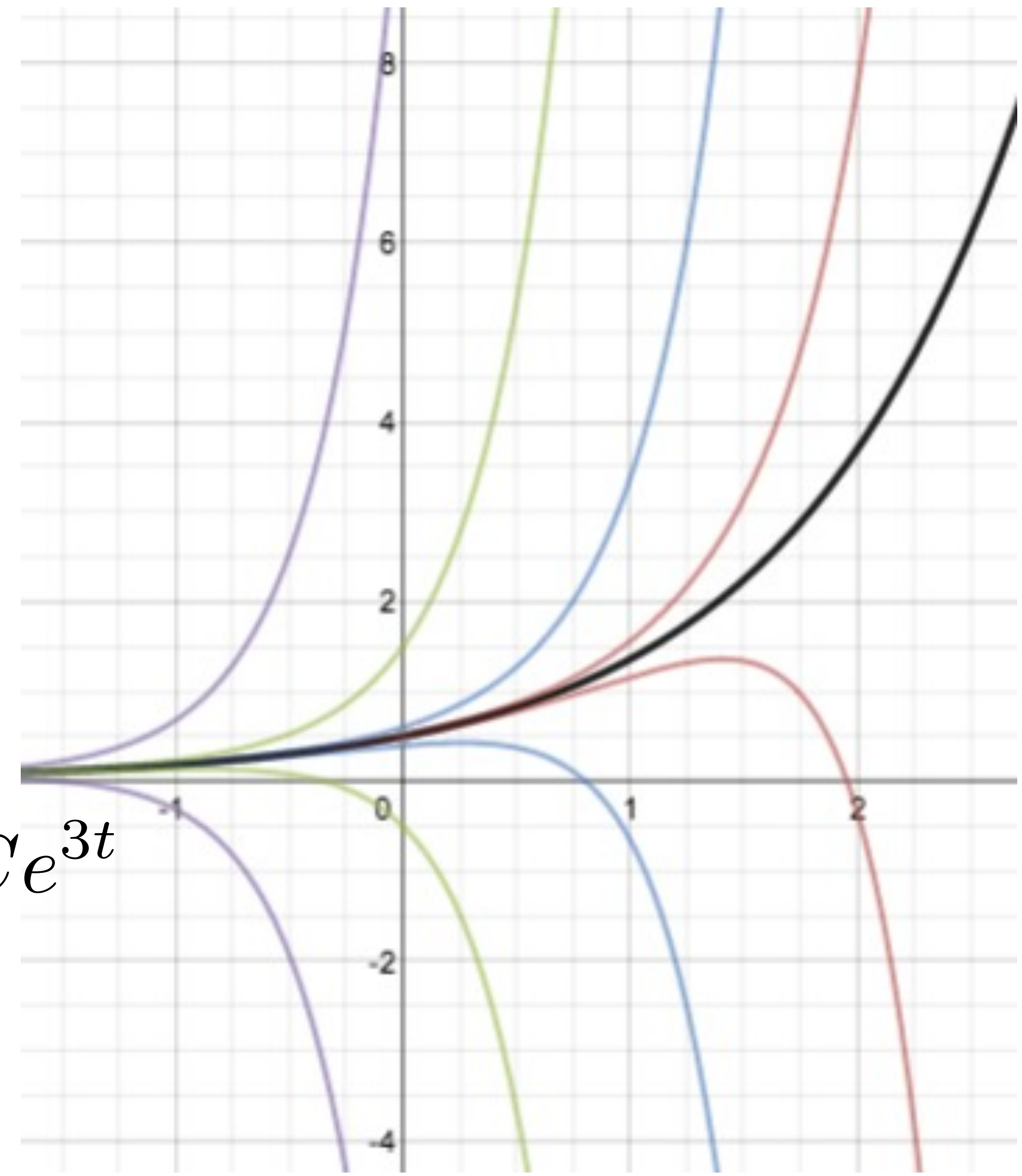

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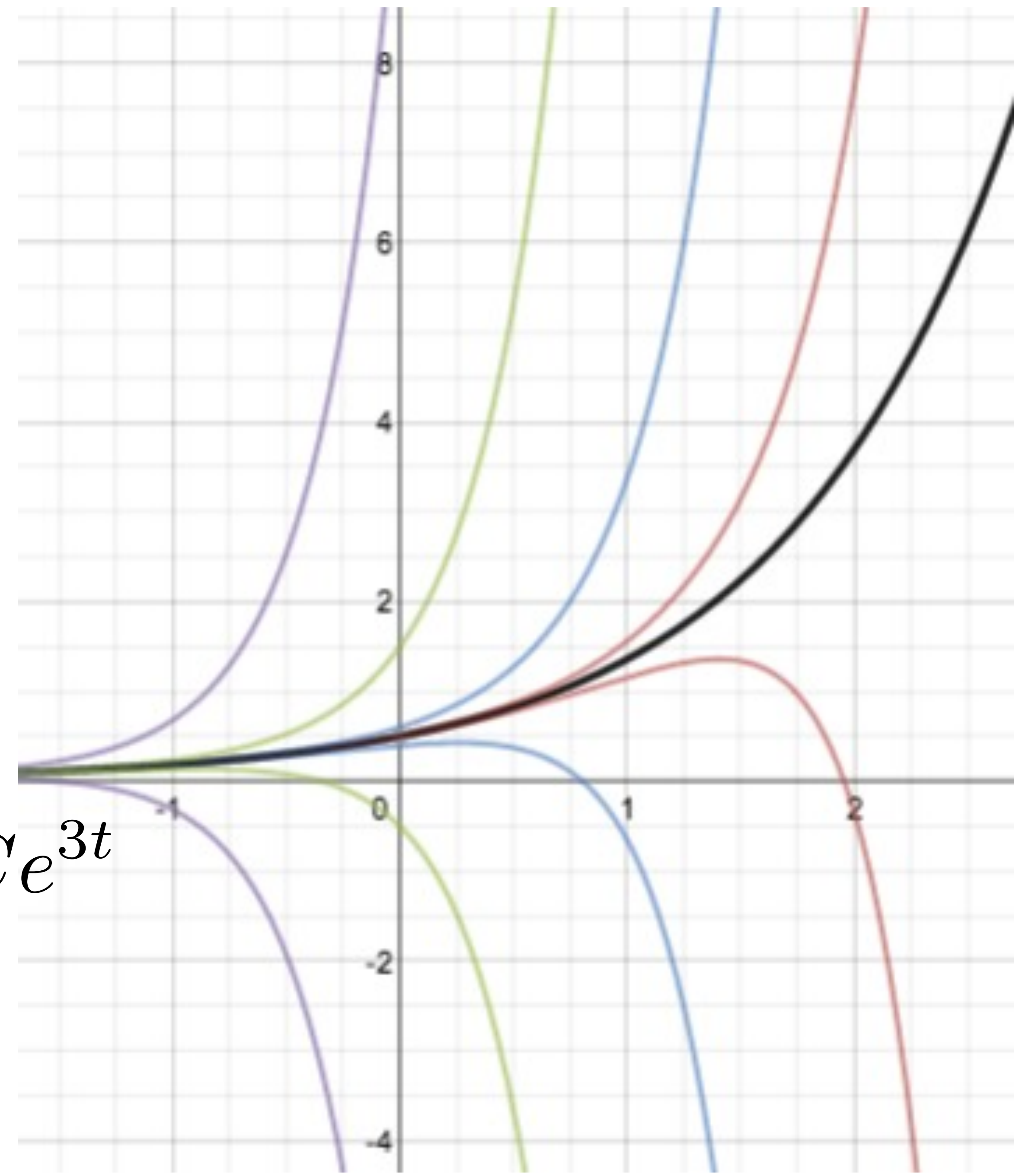

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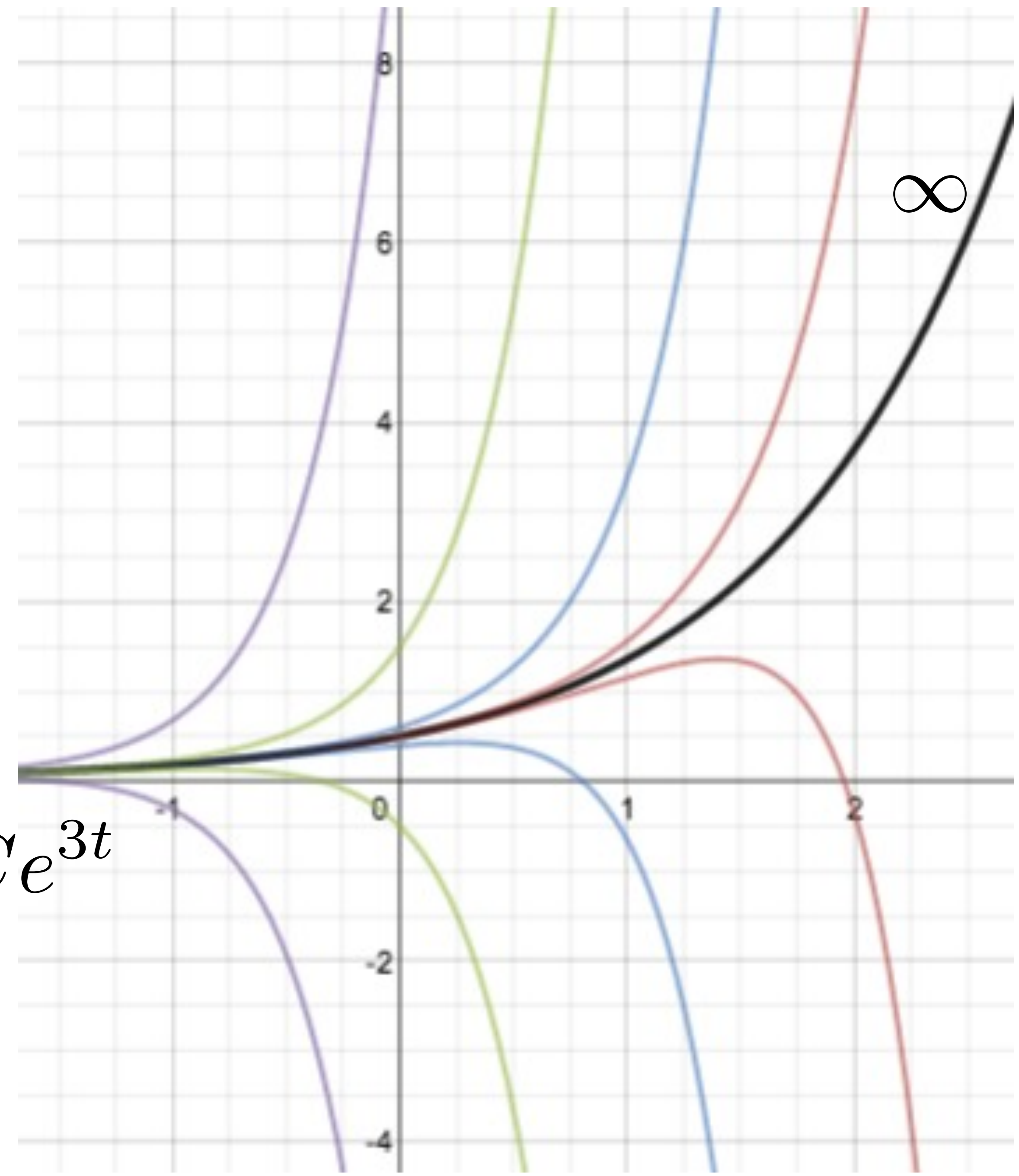

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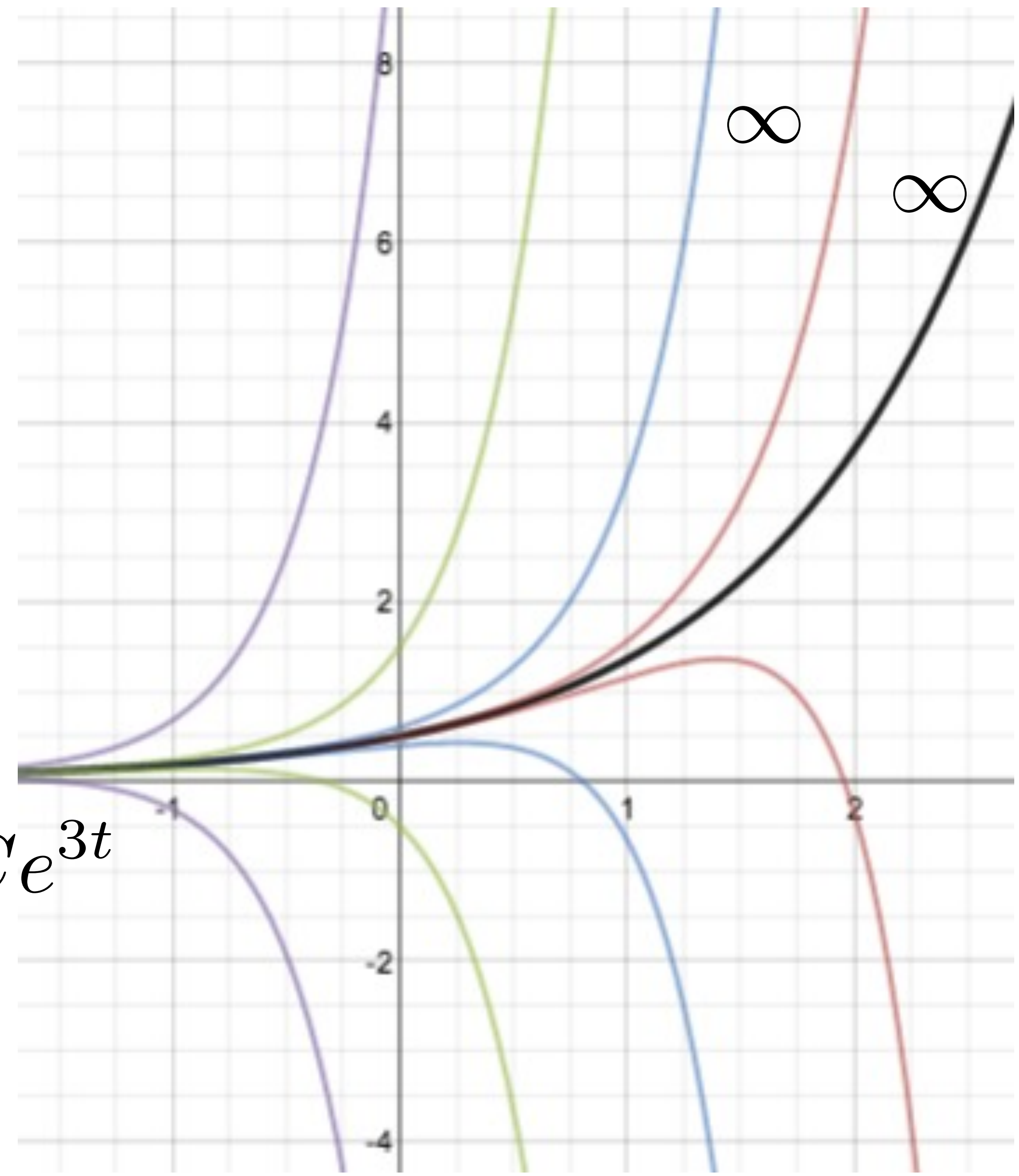

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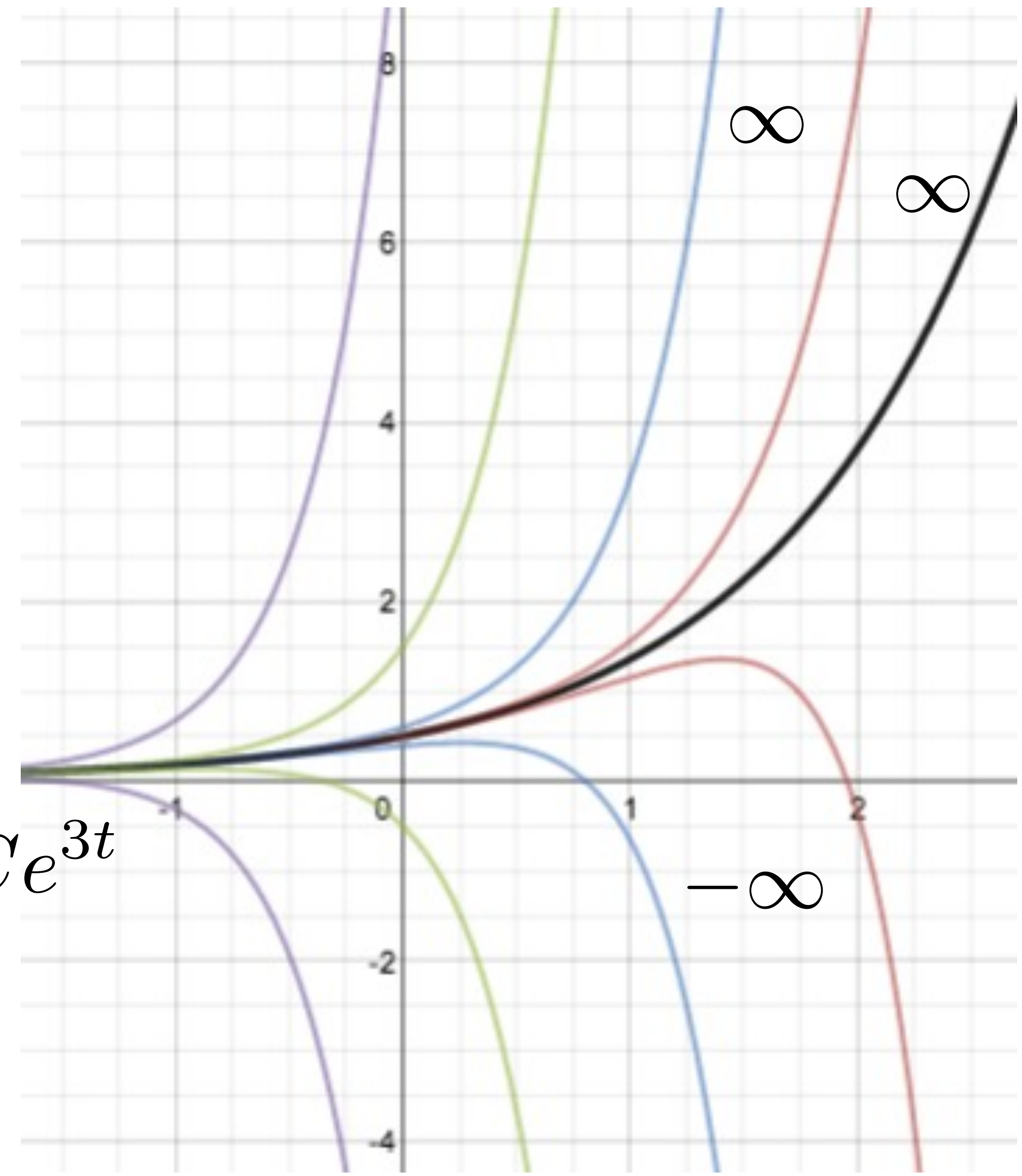

Separable equations

- What is $\frac{d}{dt}e^y$?

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(B) $e^y \frac{dy}{dt}$

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- Solve $\dfrac{dy}{dt} = e^{-y}t^2$.

(A) $y(t) = t^2e^t + C$

(B) $y(t) = \dfrac{1}{3}t^3 + C$

(C) $y(t) = \ln\left(\dfrac{1}{3}t^3\right) + C$

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- What is $\dfrac{d}{dt}e^y$?

Hint: rewrite as $e^y \dfrac{dy}{dt} = t^2$.

(D) ye^{y} … dt

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- First order ODEs of the form: $$\frac{dy}{dx} = f(x)h(y)$$

Separable equations

- First order ODEs of the form: $\dfrac{dy}{dx} = f(x)h(y)$
- Rename h(y) = 1/g(y): $\dfrac{dy}{dx} = \dfrac{f(x)}{g(y)}$
- Rewrite as $g(y)\dfrac{dy}{dx} = f(x)$.
- Rewrite g and f as derivatives of other functions: $G'(y)\dfrac{dy}{dx} = F'(x)$.
- Recognize a chain rule: $\dfrac{d}{dx}(G(y)) = G'(y)\dfrac{dy}{dx}$.
- Take antiderivatives to get $G(y) = F(x) + C$.
- Finally, solve for y if possible: $y(x) = G^{-1}(F(x) + C)$.

Separable equations

- Solve: $$\frac{dy}{dx} = -\frac{x}{y}$$

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(B) $y(x) = \sqrt{C - x^2}$

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Does (B) cover all possible initial conditions?

Separable equations

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- General solution: $y = \pm\sqrt{C - x^2}$
- Or express implicitly: $y^2 = -x^2 + C$
- To satisfy an IC, must choose a value for C *and* choose + or - .

Separable equations

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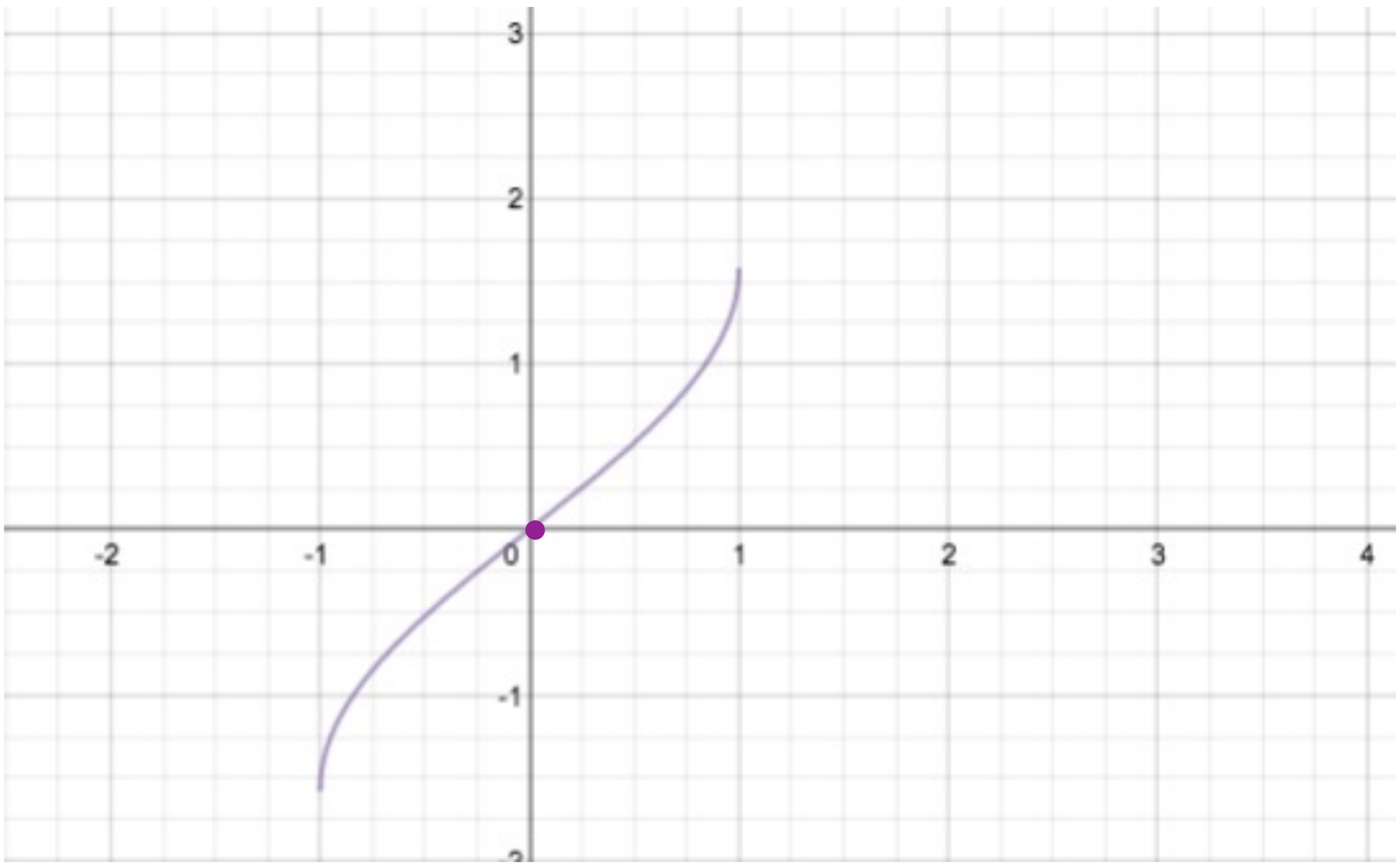
$y(t) = \arcsin(t + C)$ with IC $y(0) = 0$ $C = 0$



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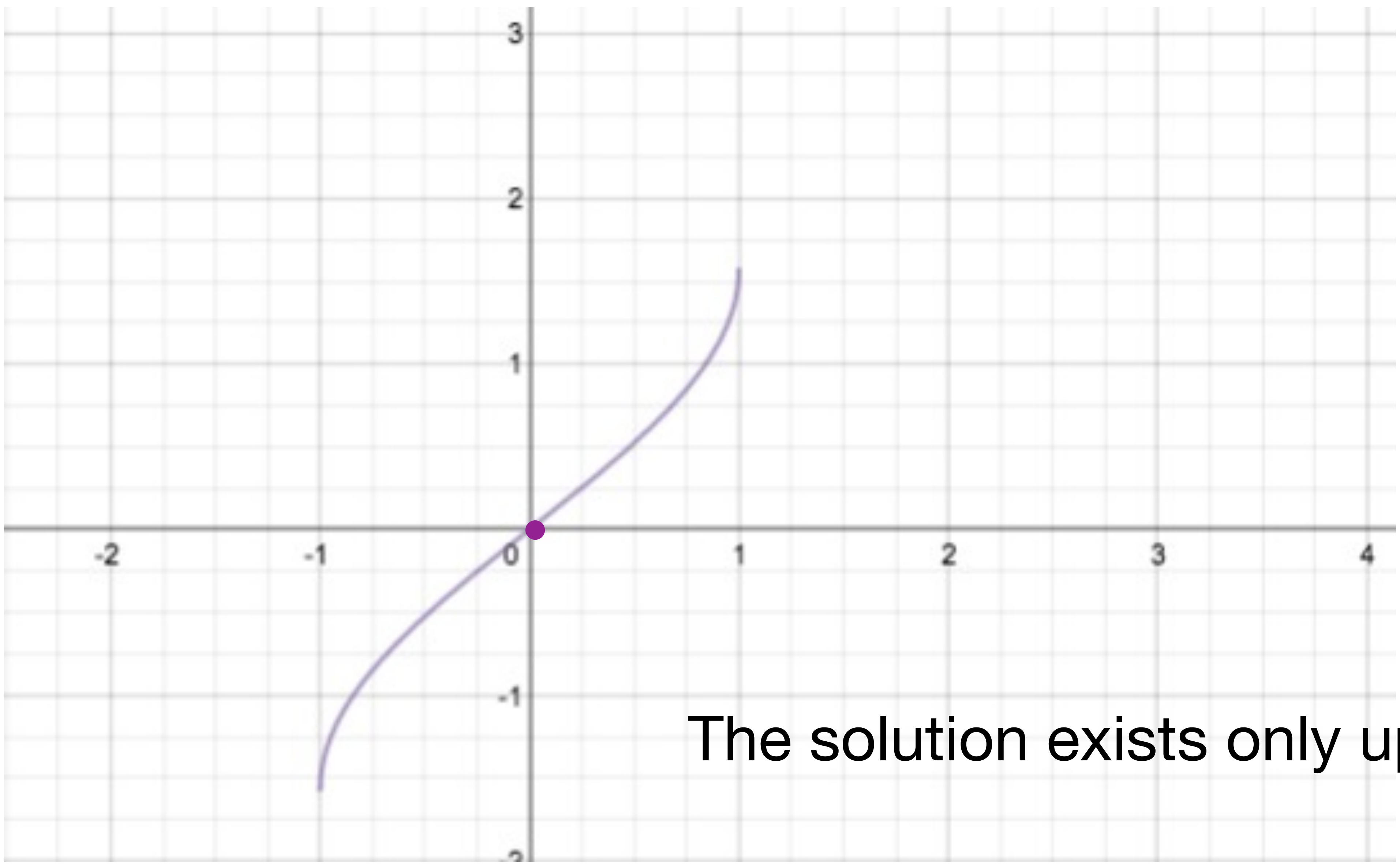
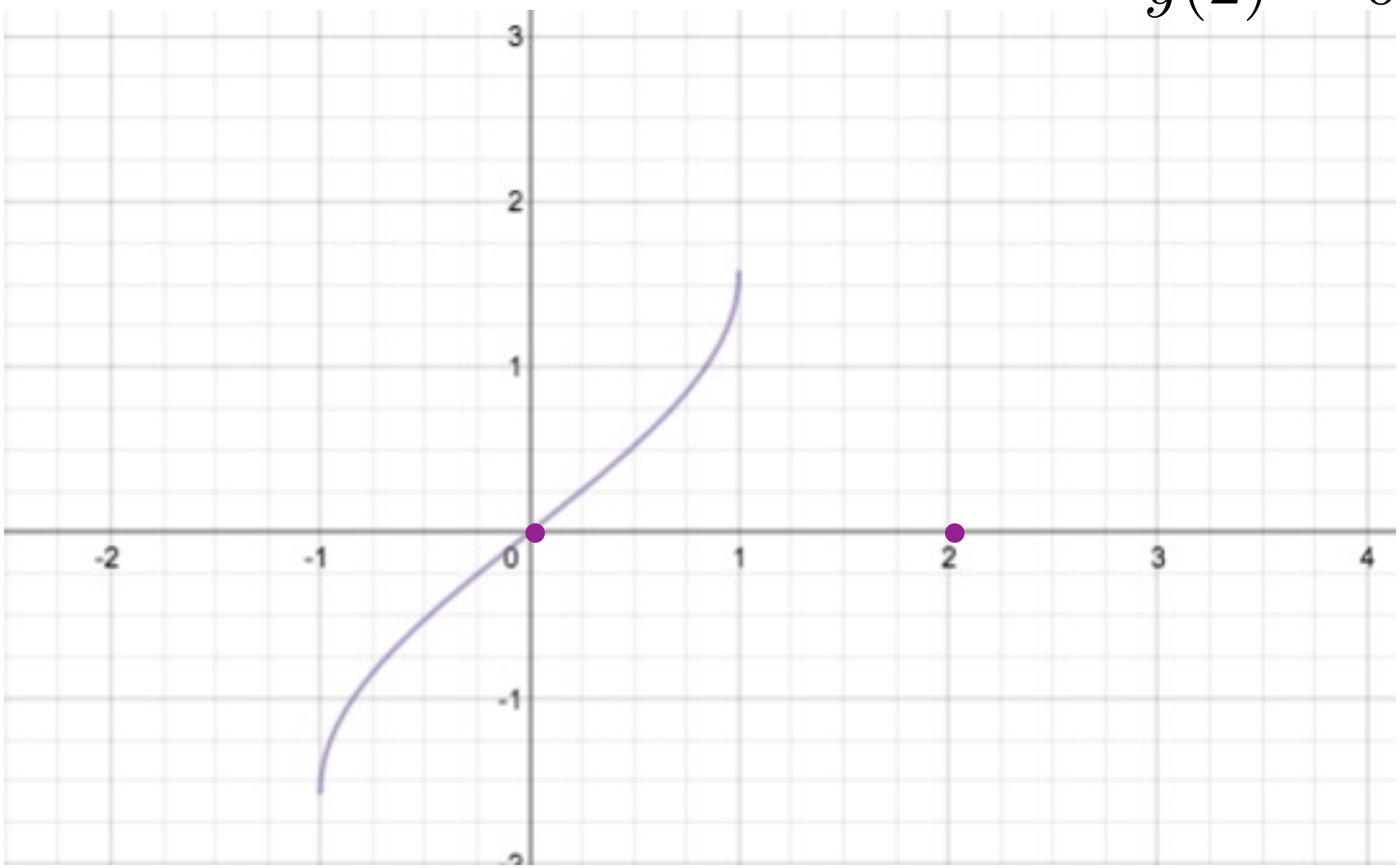


The solution exists only up until t=1.

Separable equations

$y(t) = \arcsin(t + C)$ with IC $y(0) = 0$ $\quad C = 0$

$y(2) = 0$



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$$y(t) = \arcsin(t + C)$$

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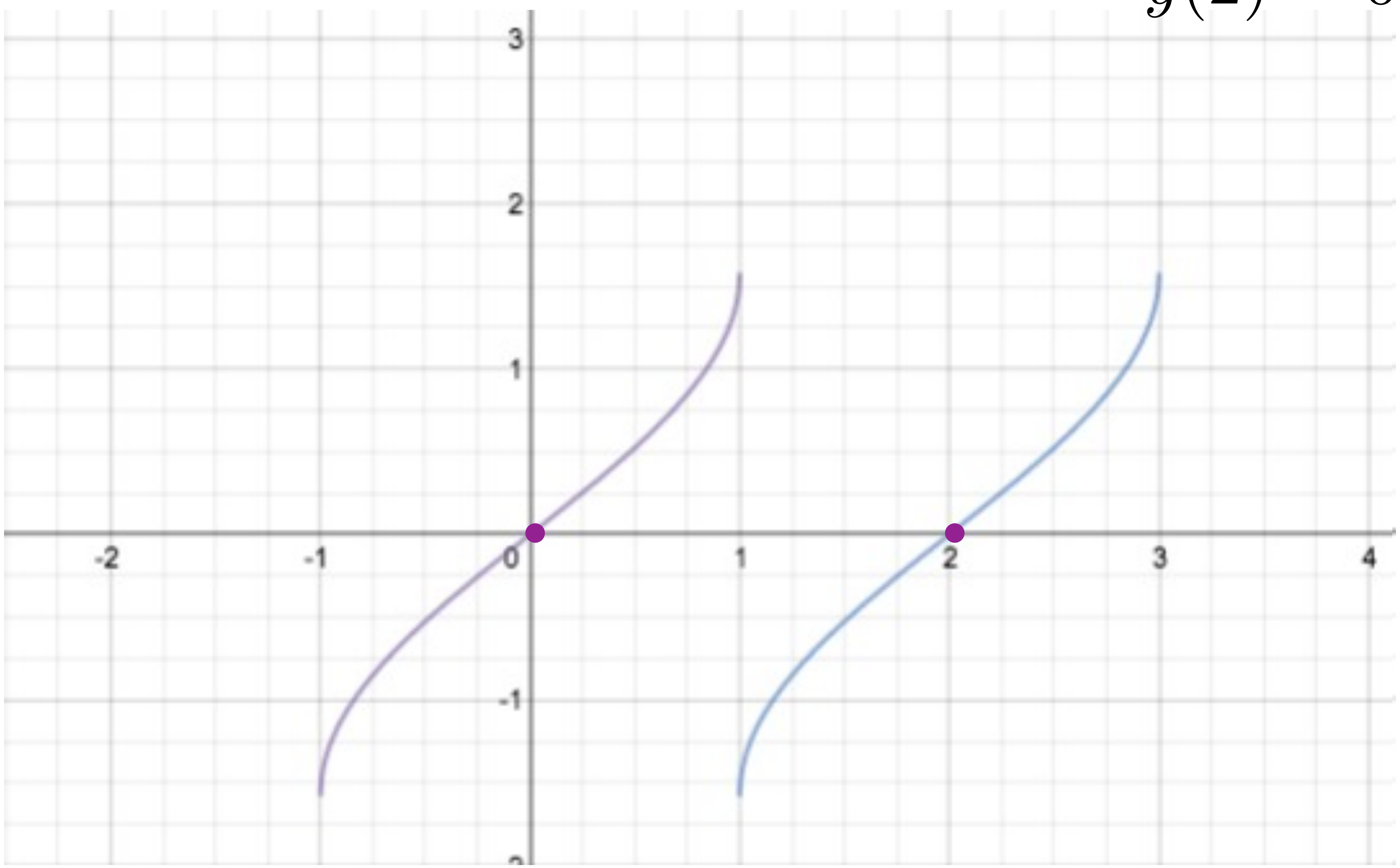

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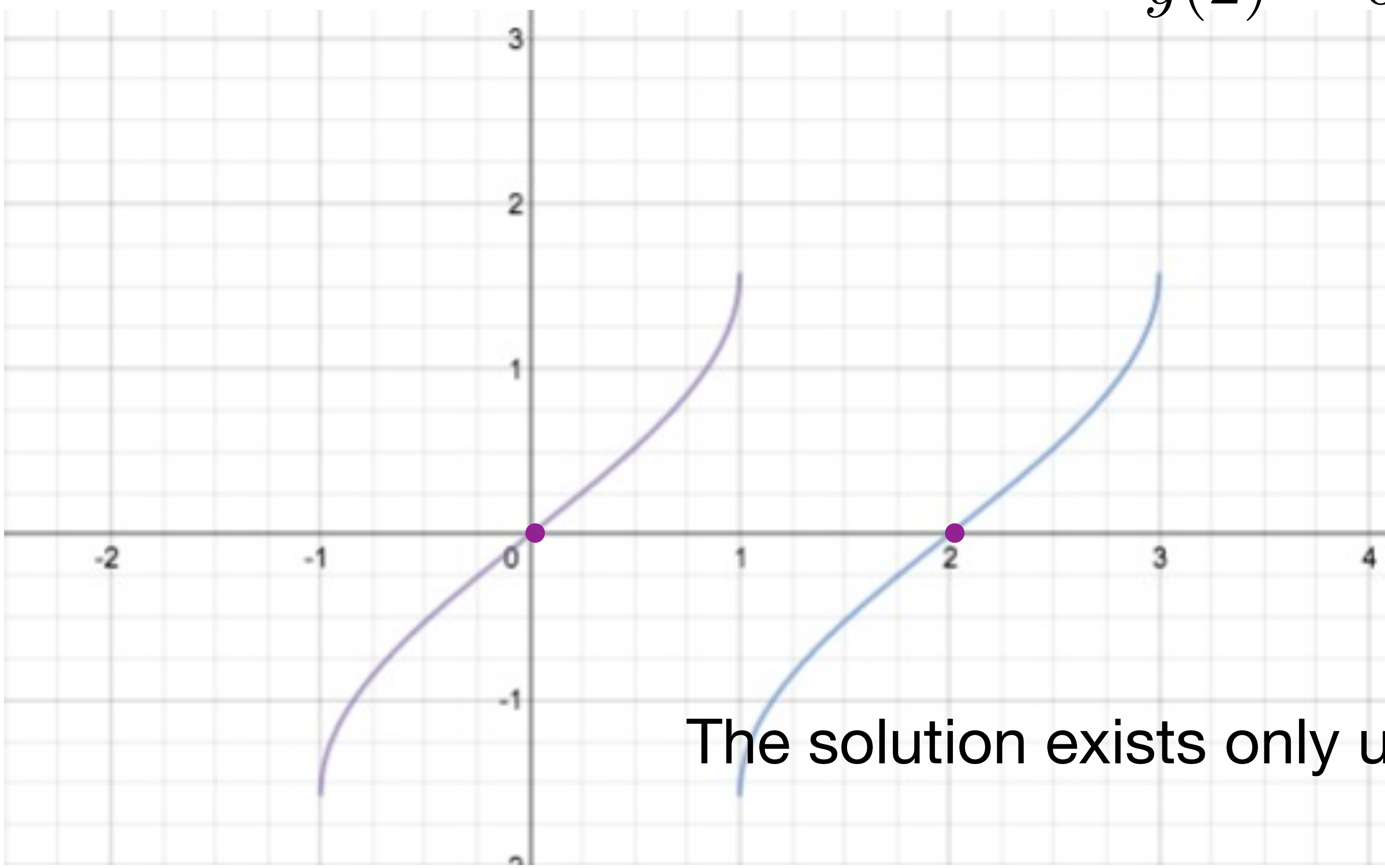

The solution exists only up until t=3.

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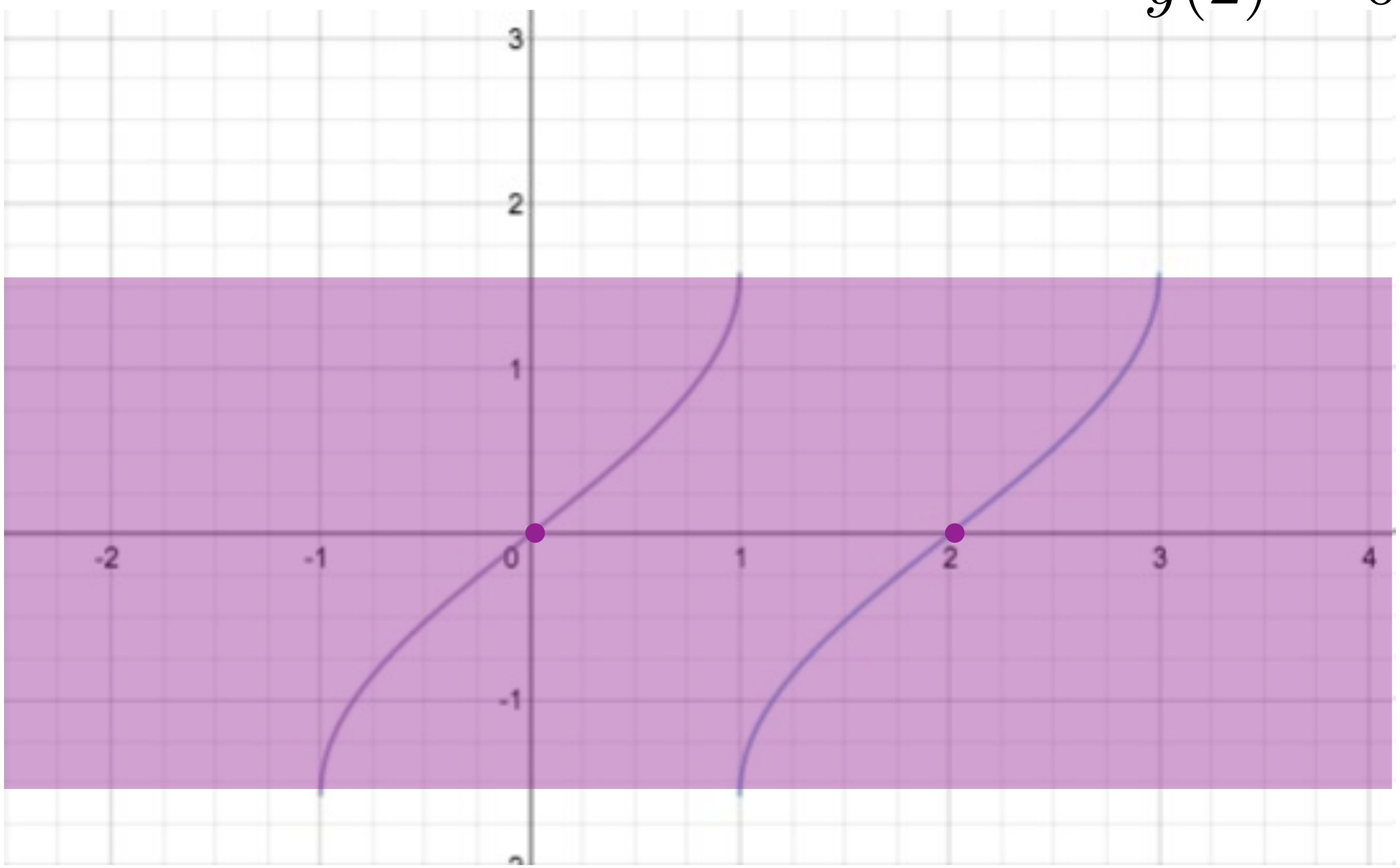

Separable equations

$$y(t) = \arcsin(t + C)$$

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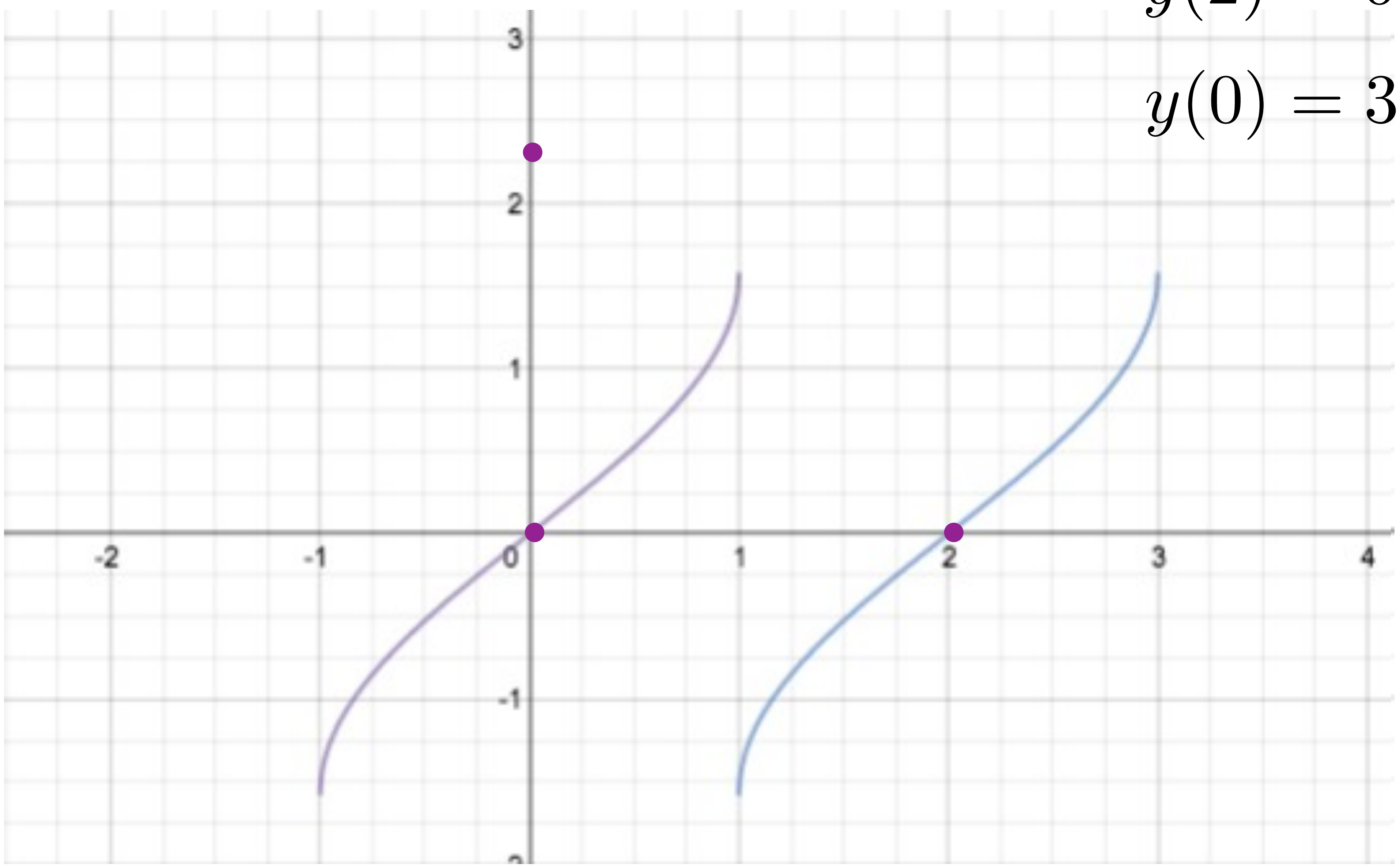
$y(0) = 3\pi/4$



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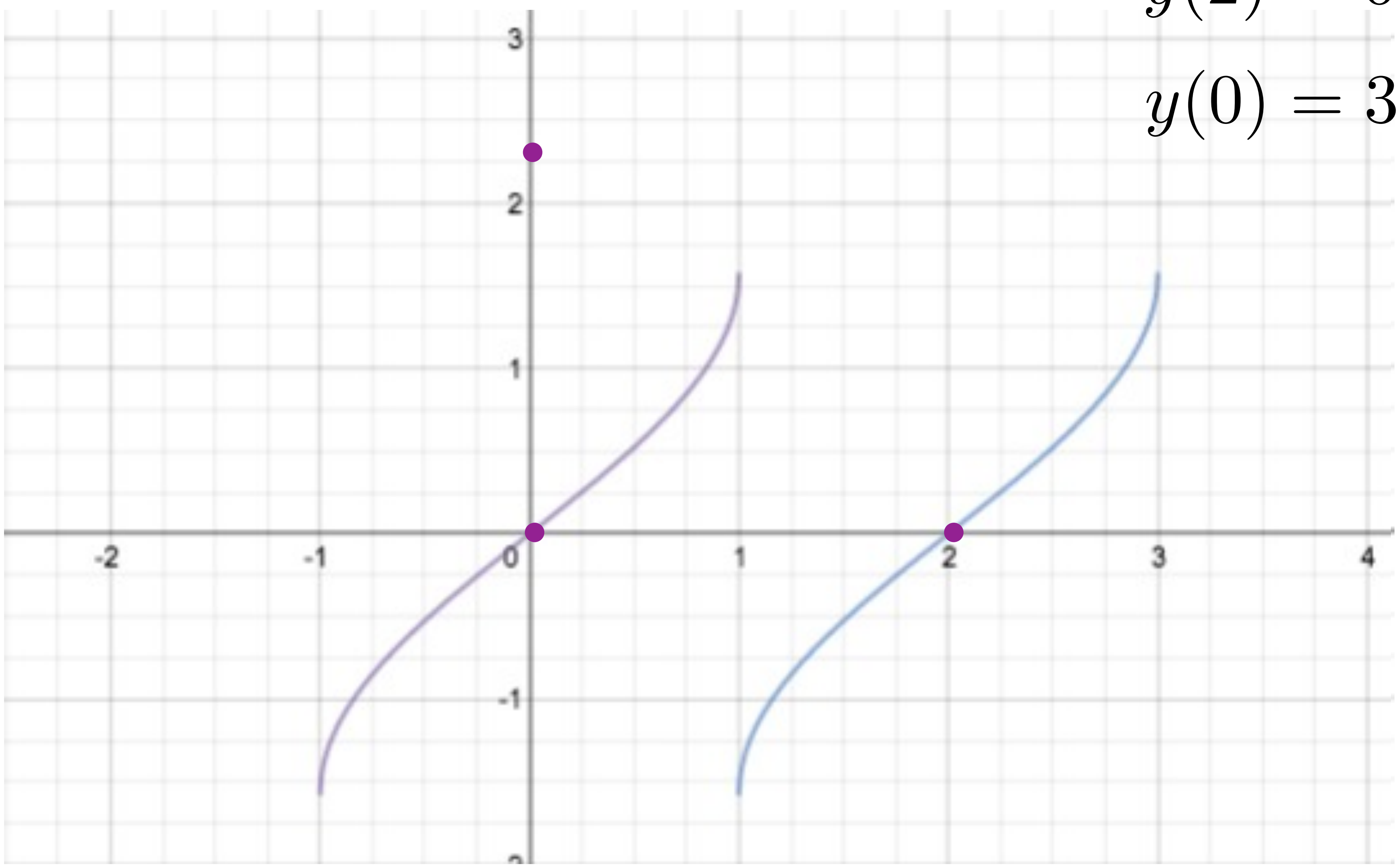

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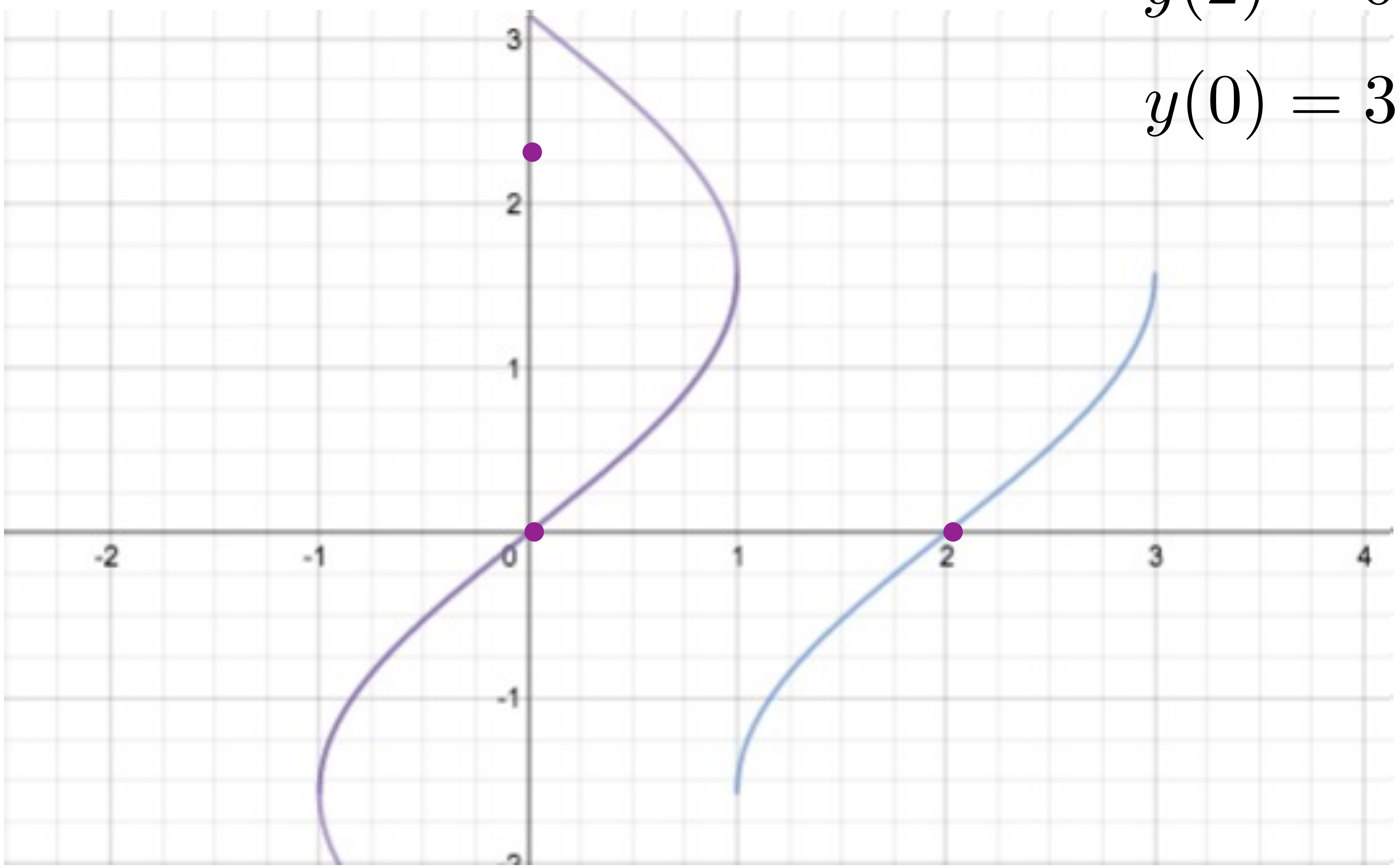

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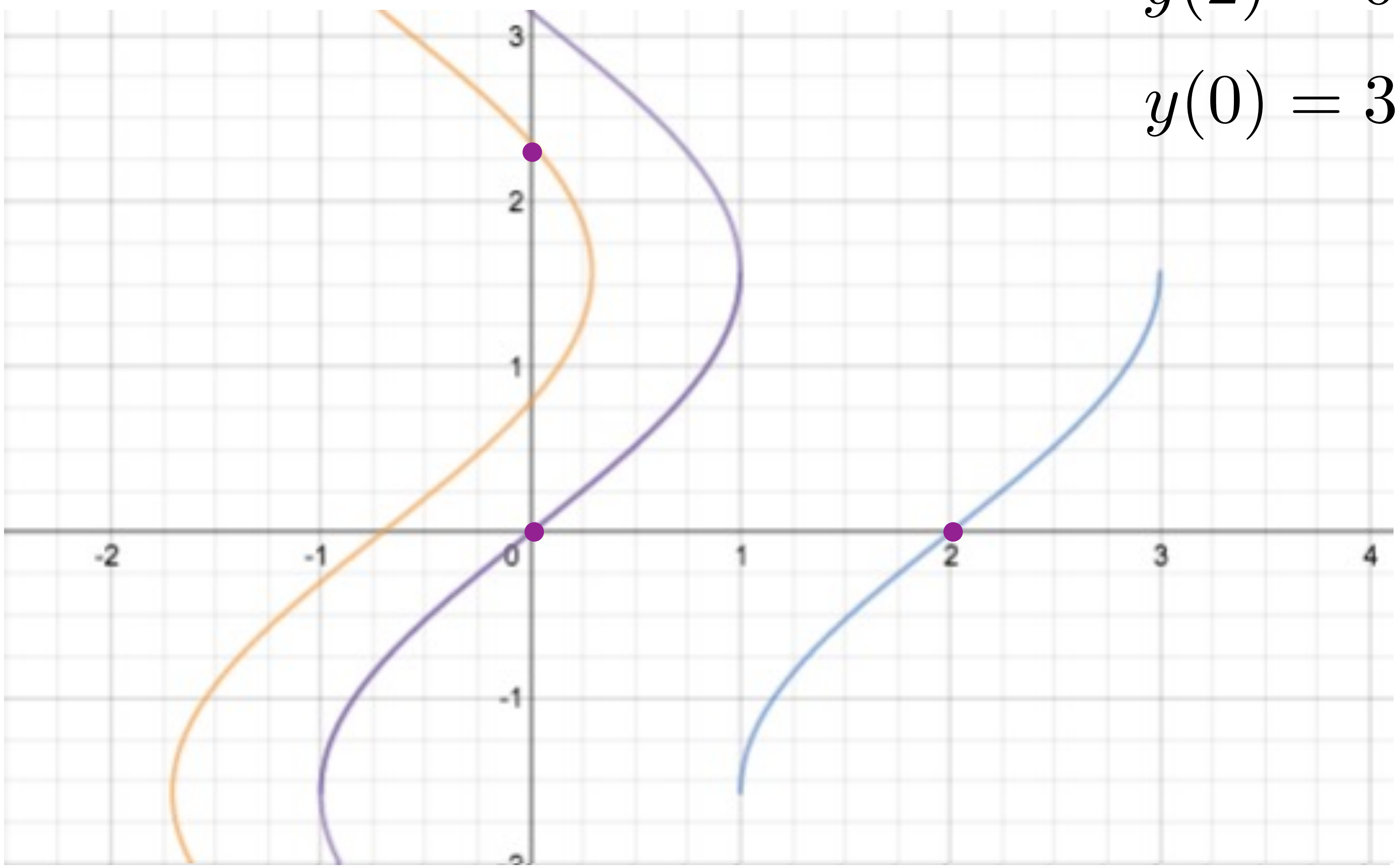

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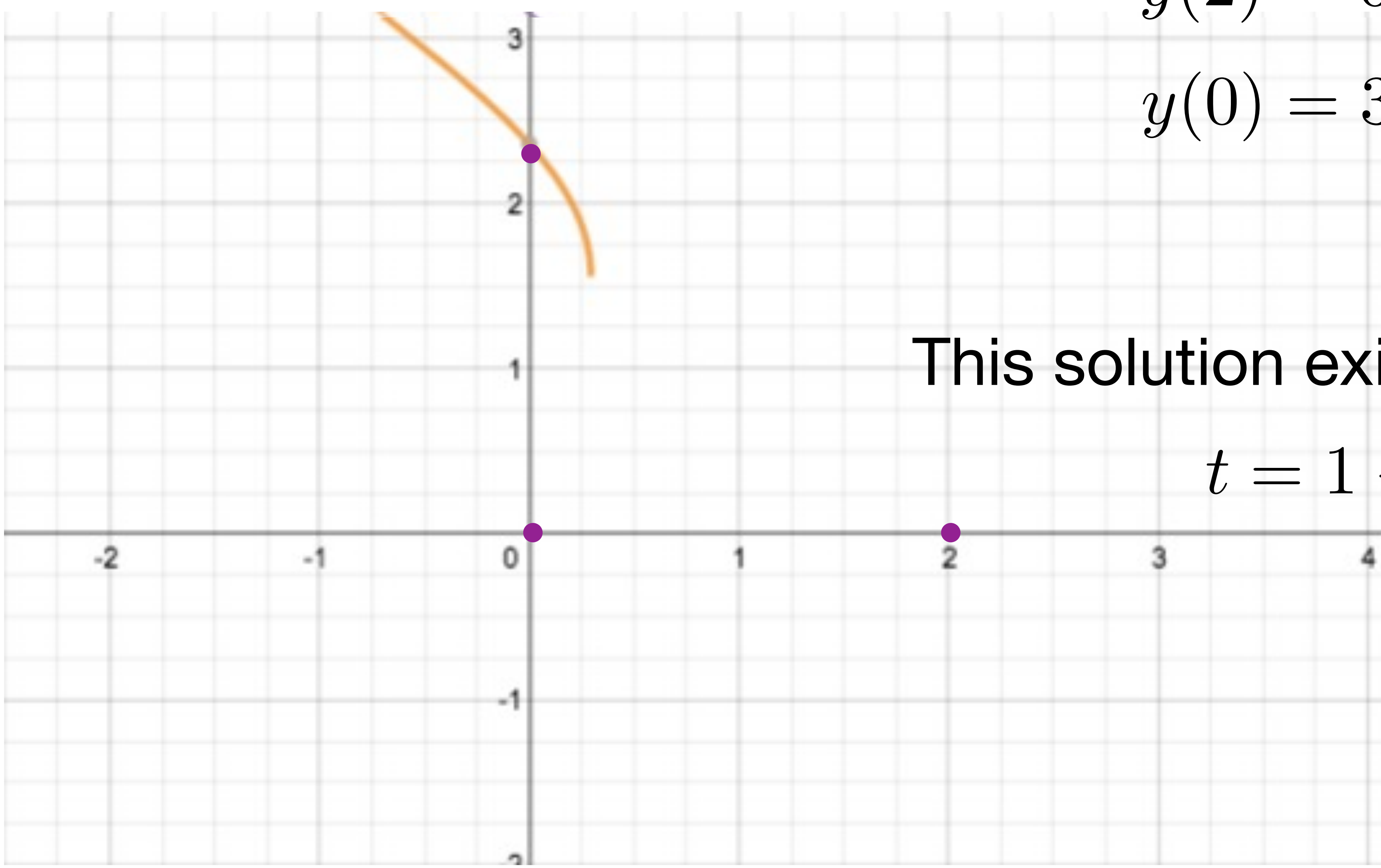

This solution exists only up until

$$t = 1 - 1/\sqrt{2}$$

Separable equations

$$\sin(y) = t + C$$

with IC

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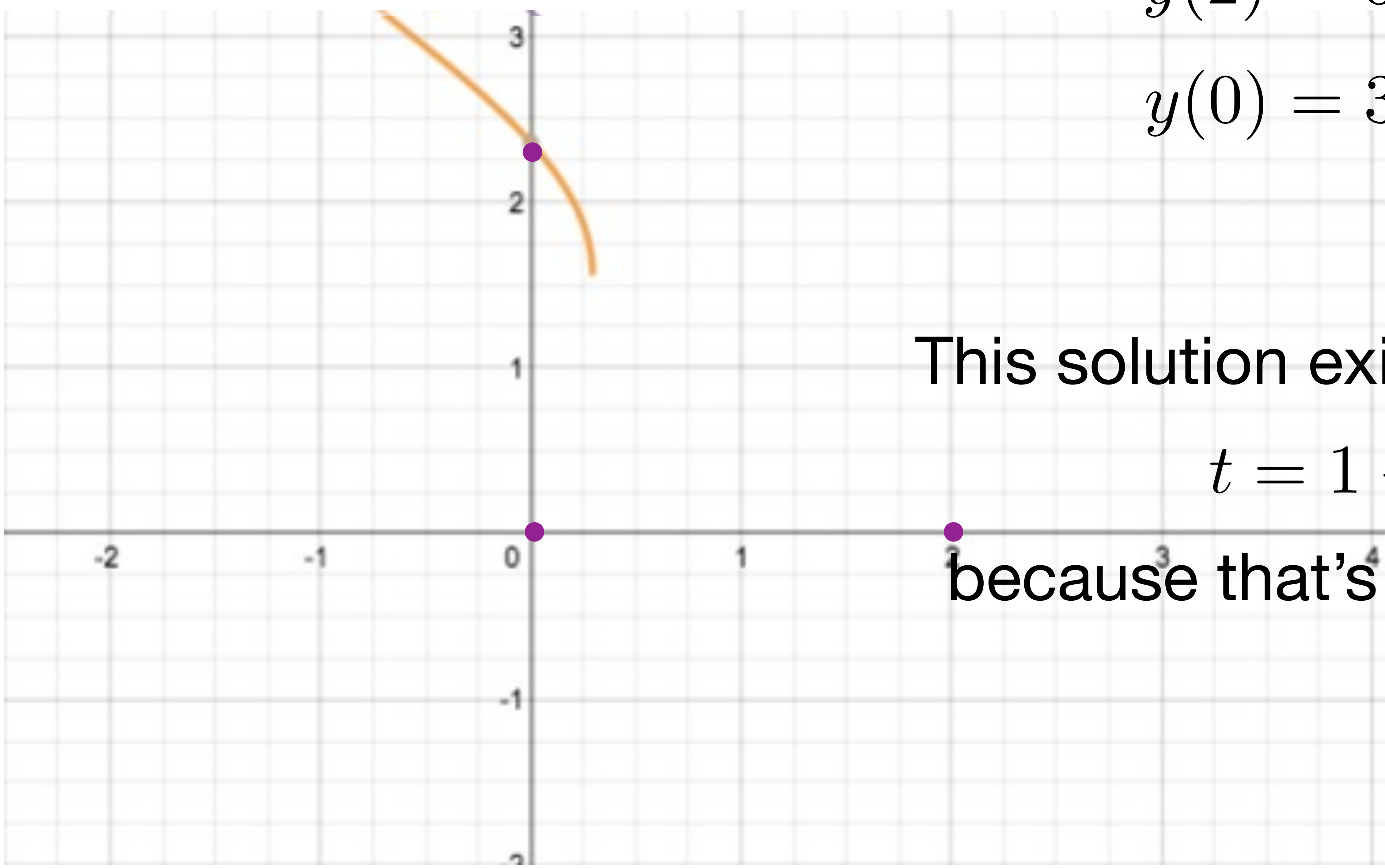
$y(0) = 3\pi/4 \qquad C = \frac{1}{\sqrt{2}}$



This solution exists only up until

$$t = 1 - 1/\sqrt{2}$$

because that's when y = π/2.

Separable equations

$$\sin(y) = t + C$$

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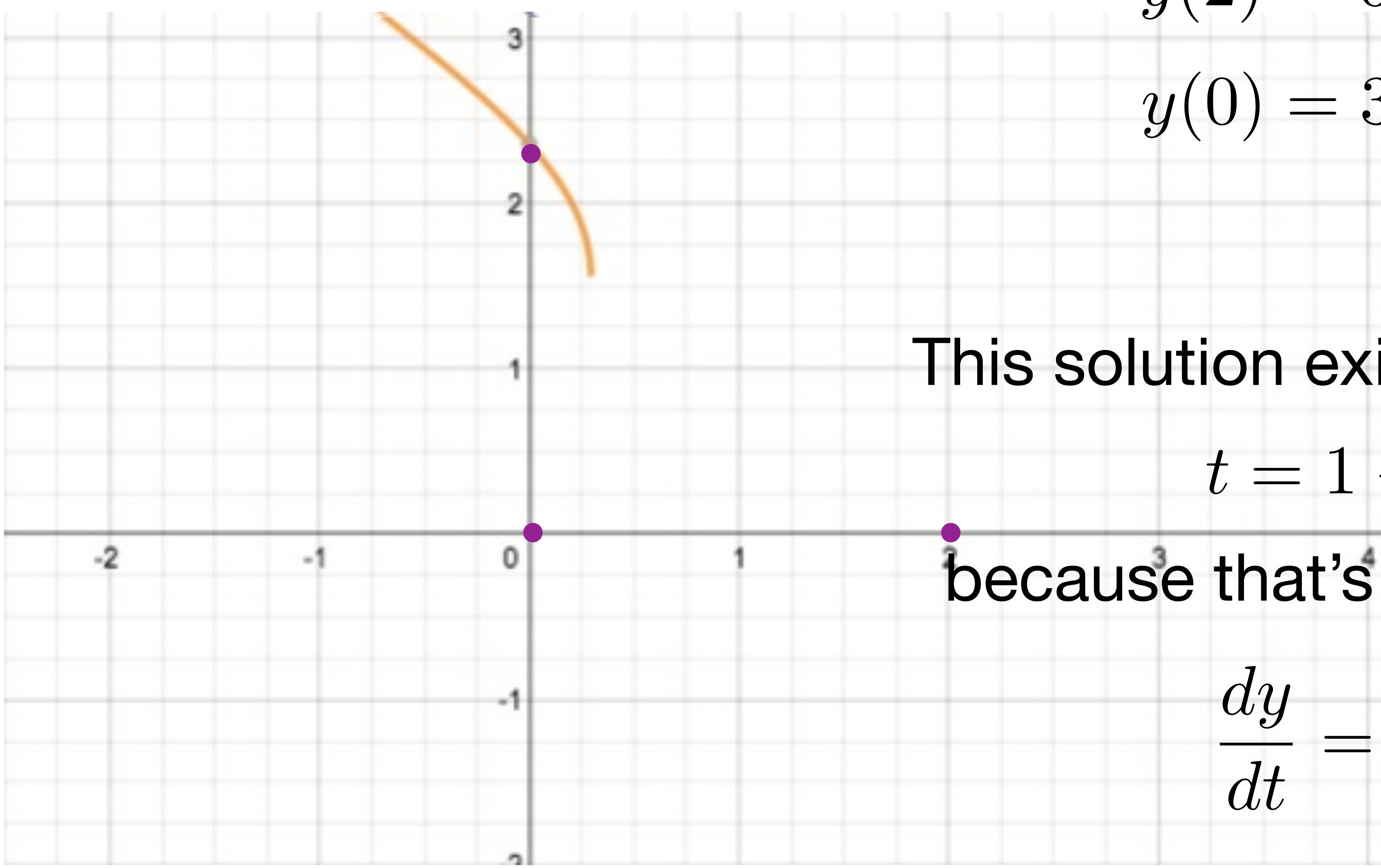

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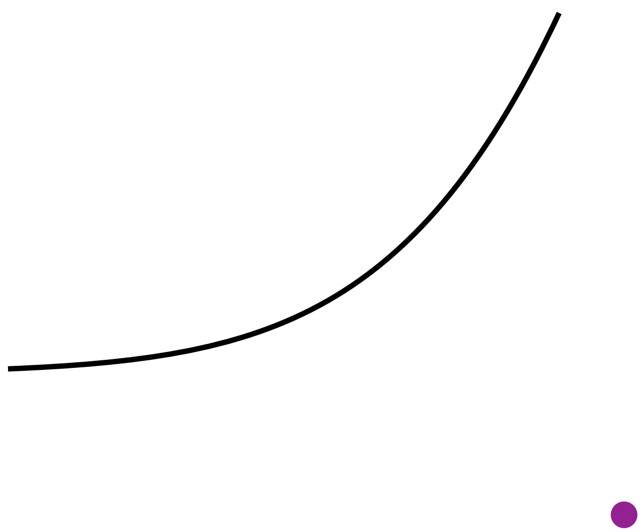

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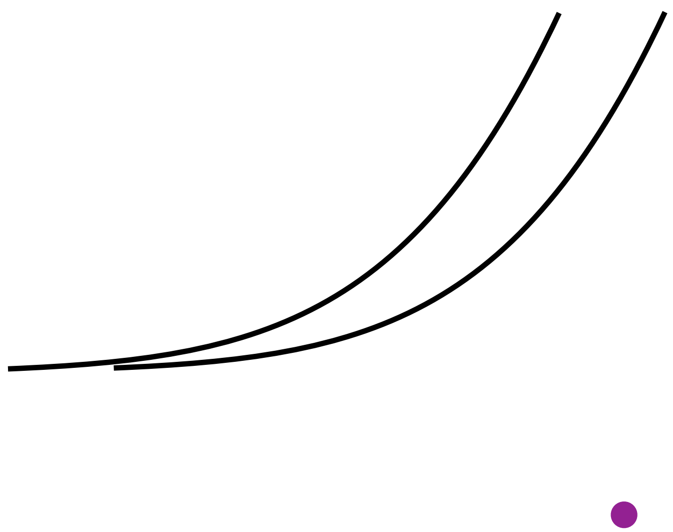

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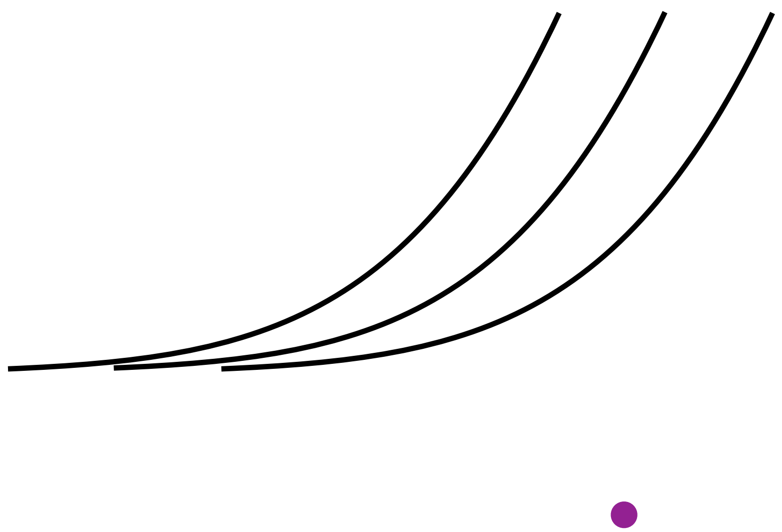

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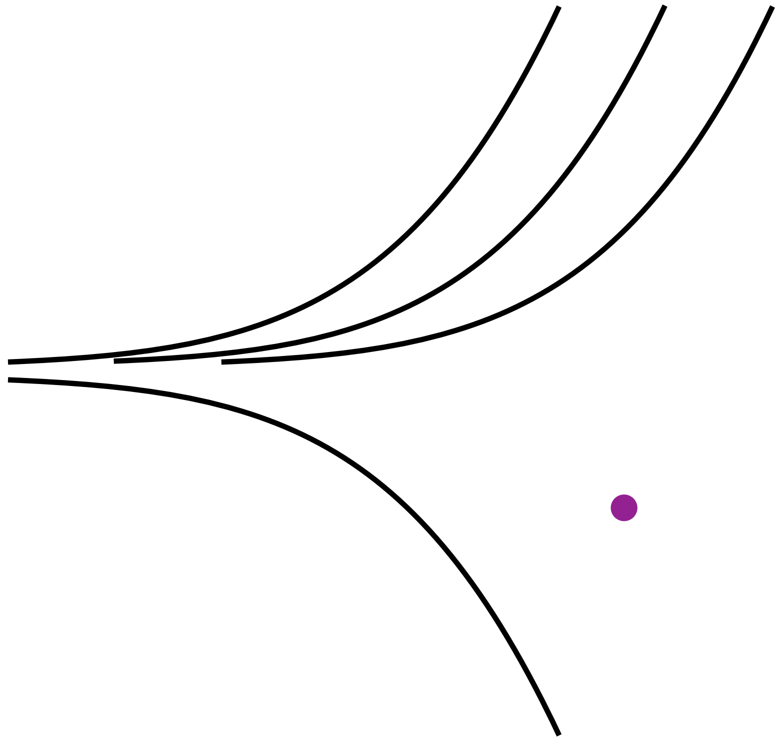

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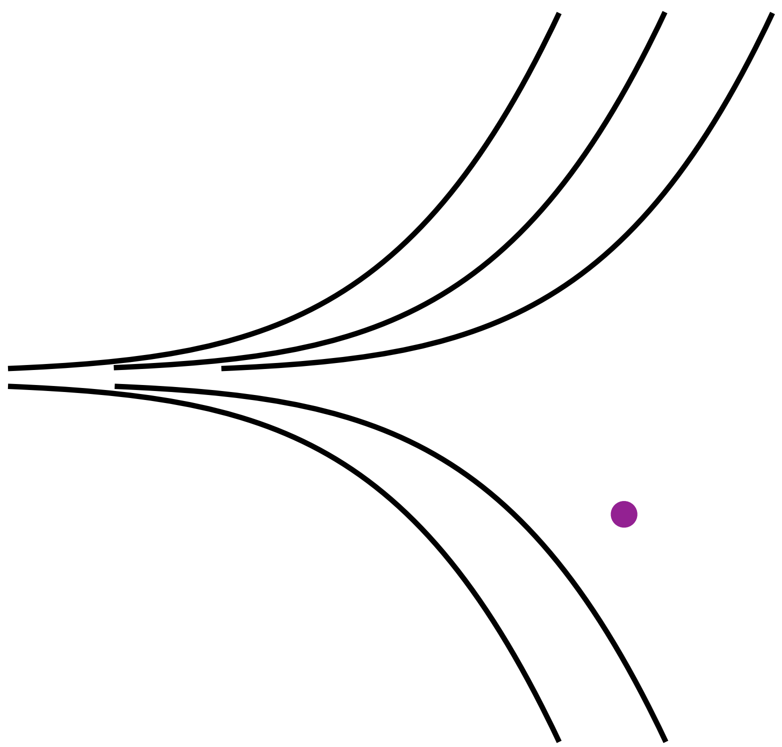

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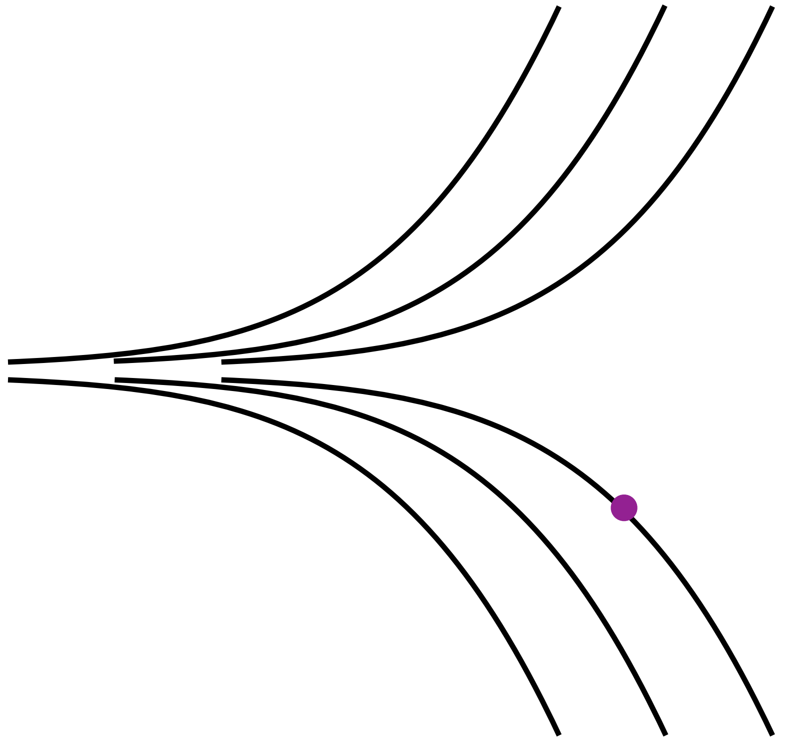

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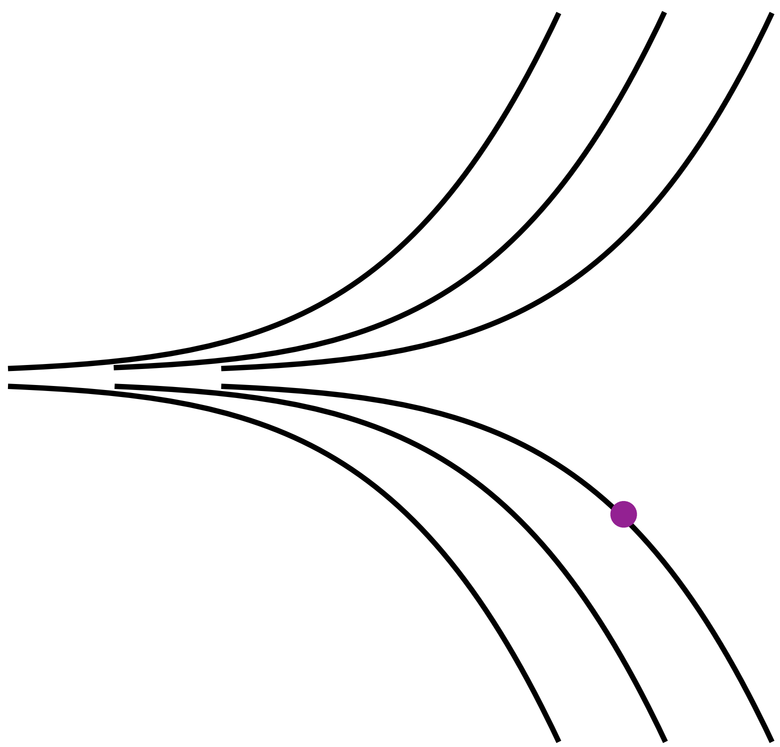

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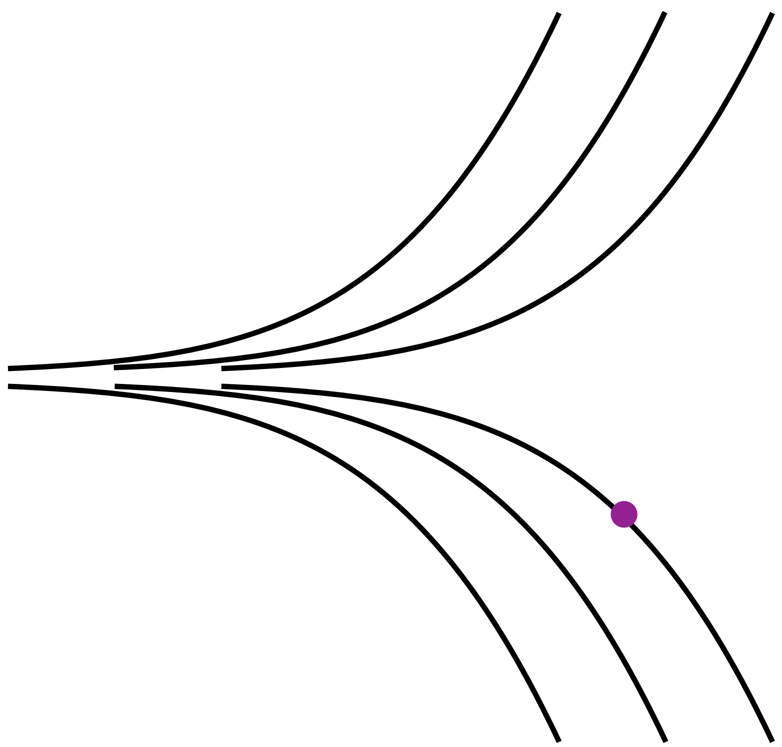

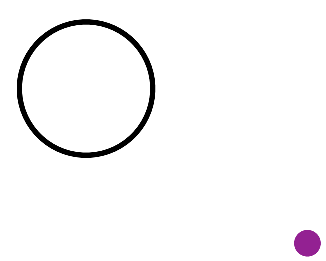

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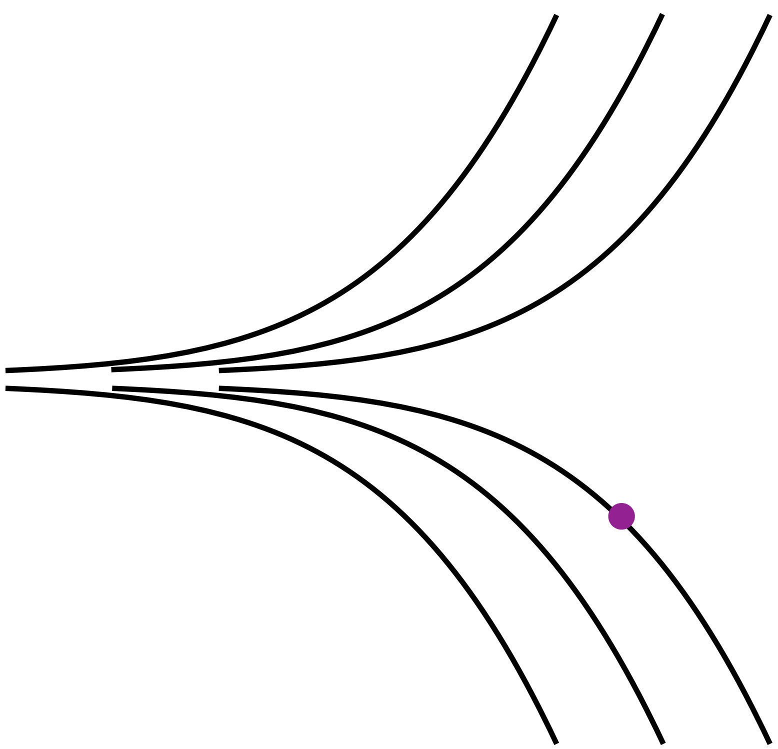

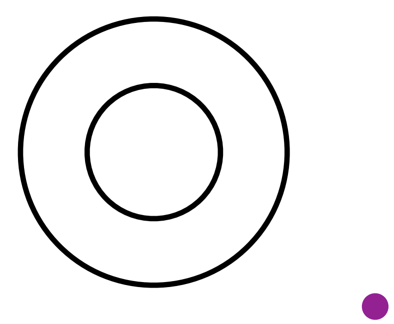

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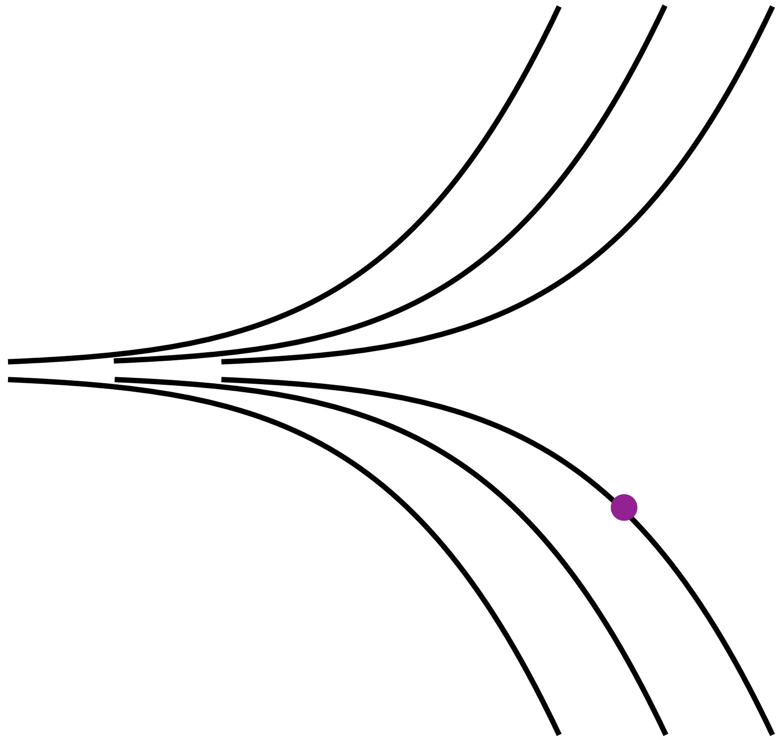

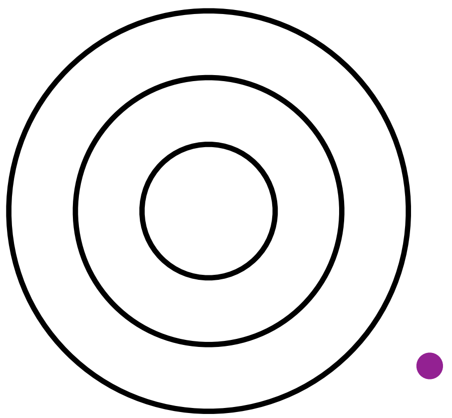

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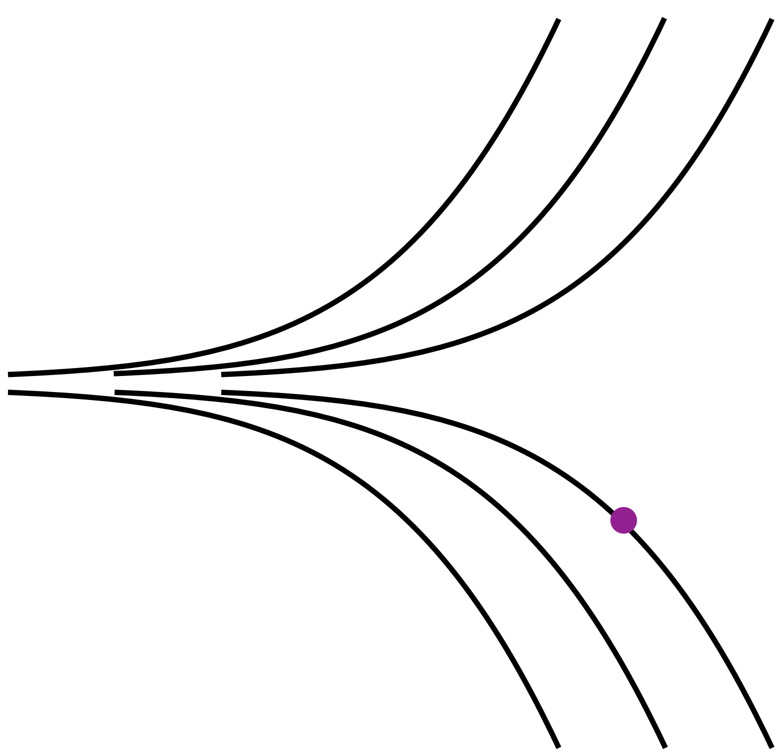

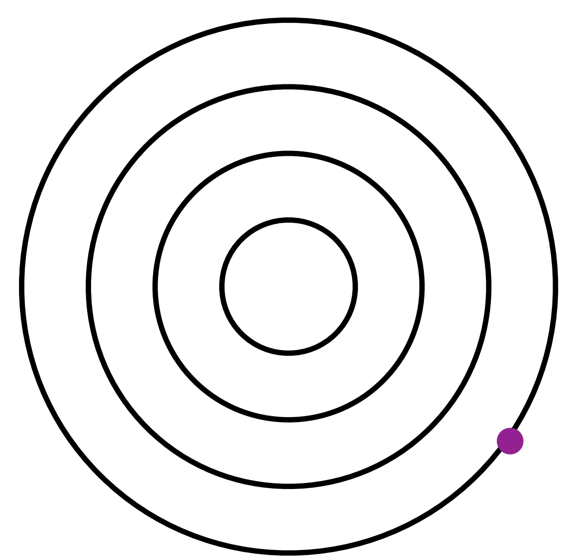

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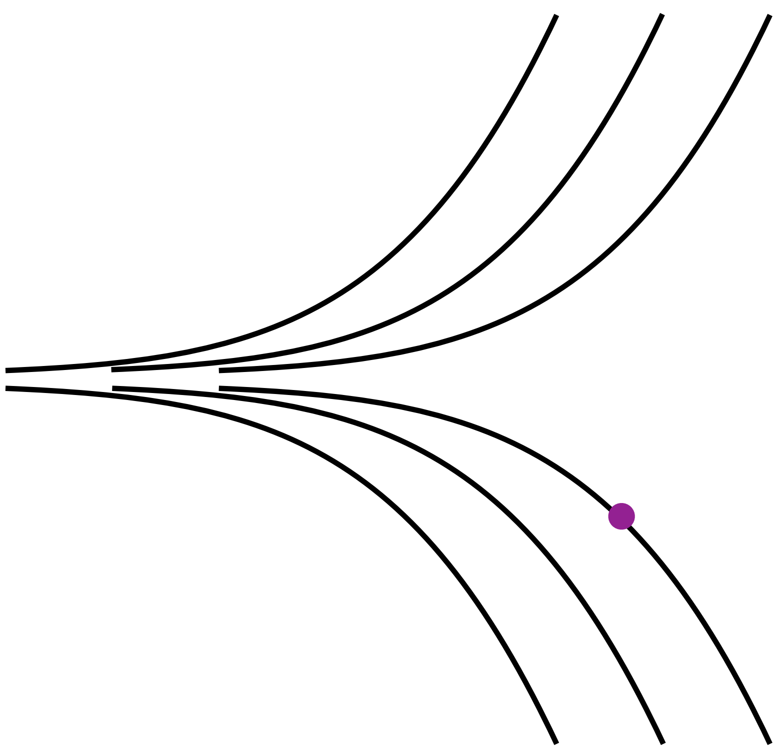

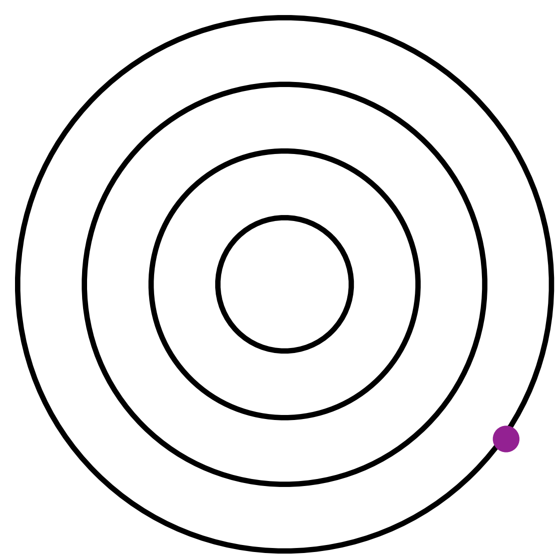

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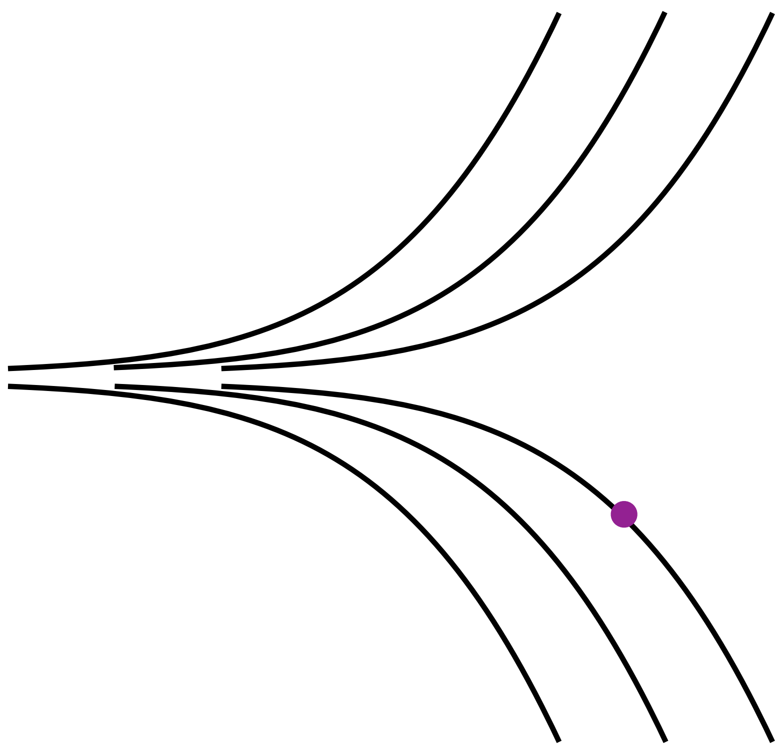

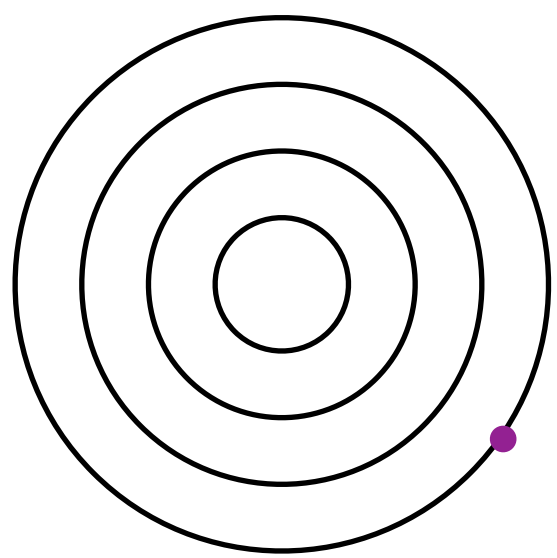

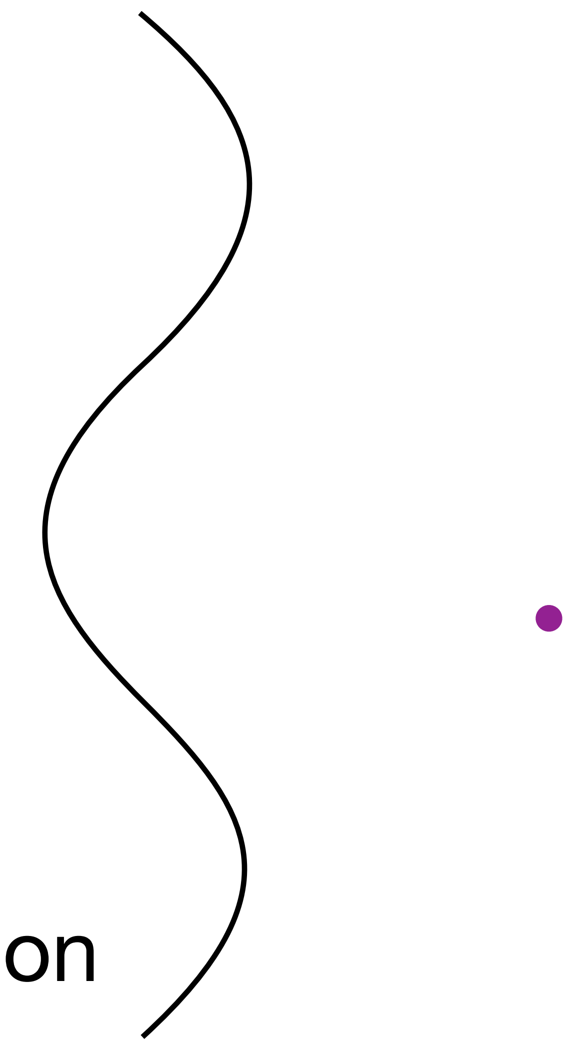

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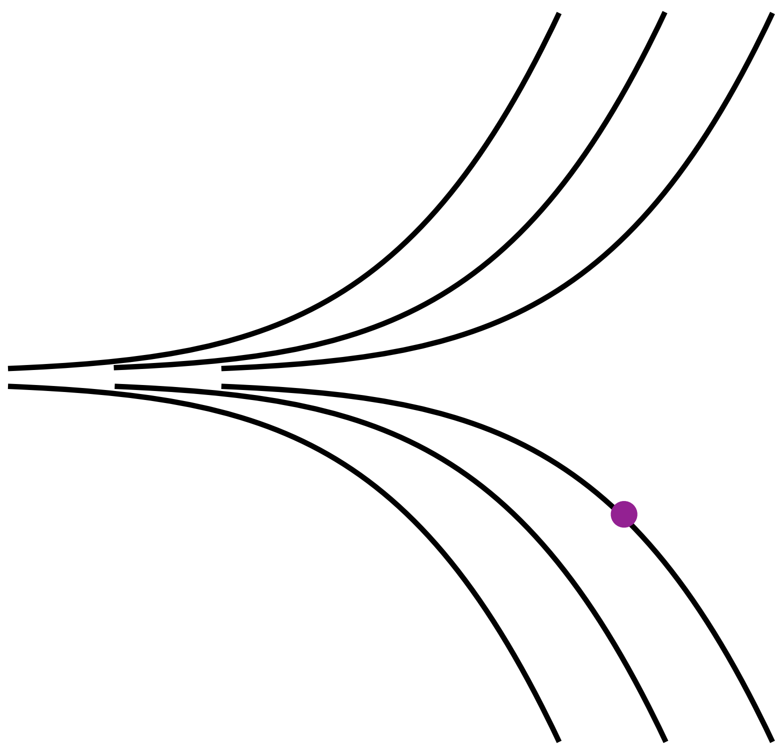

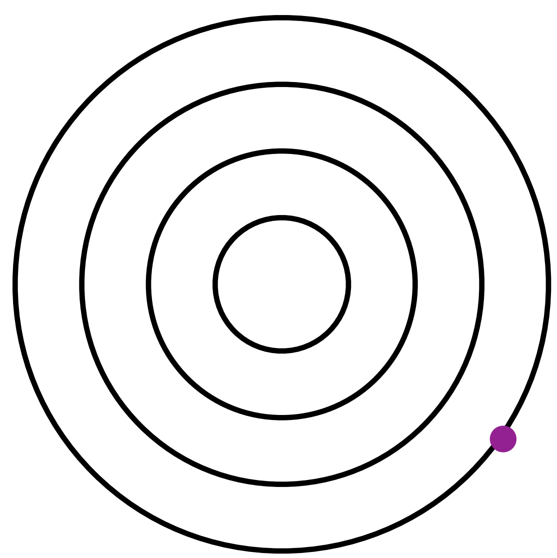

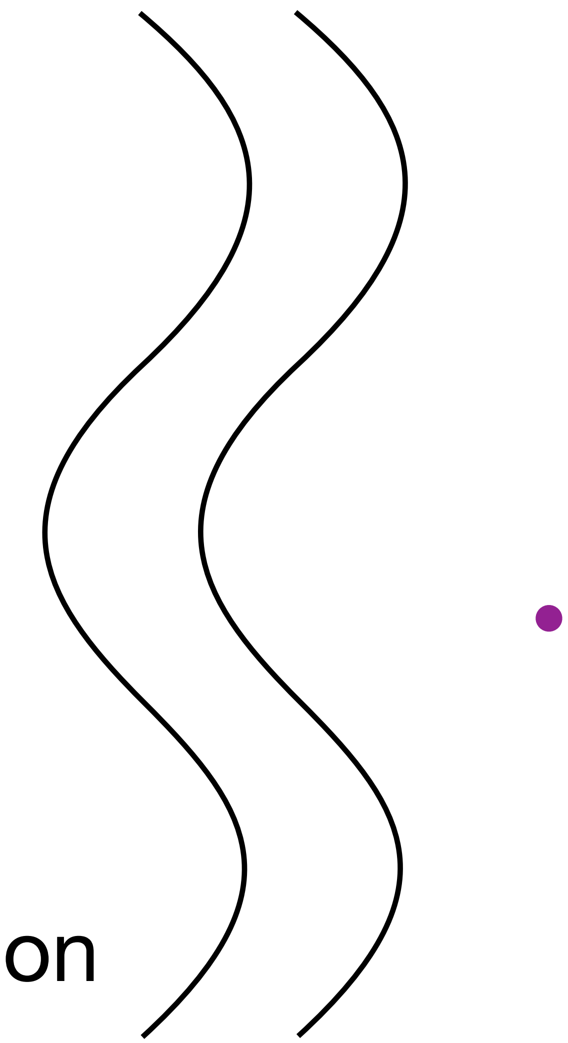

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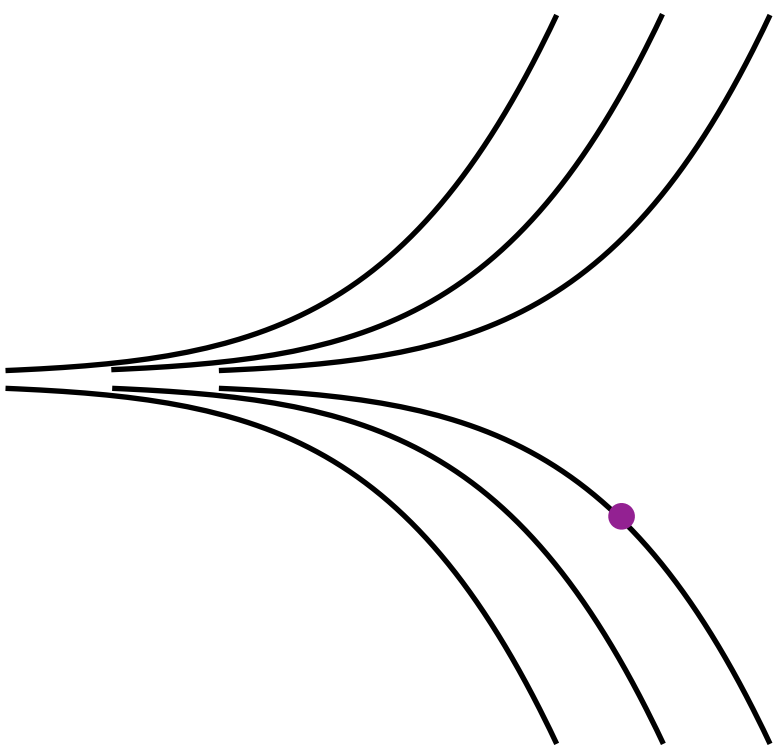

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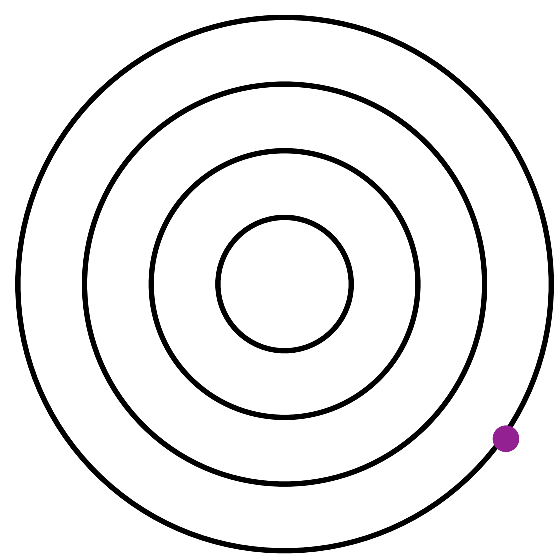

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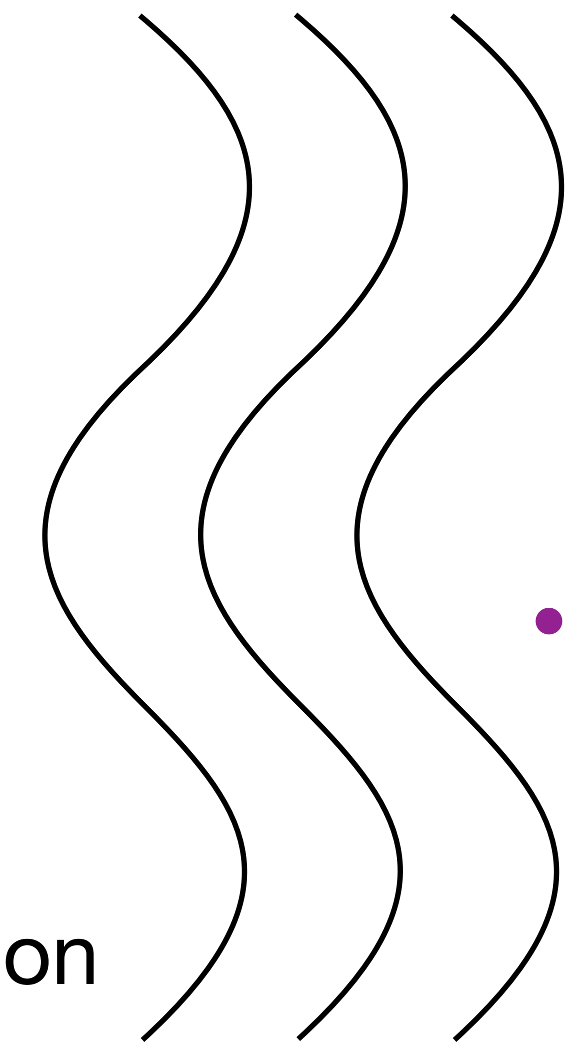

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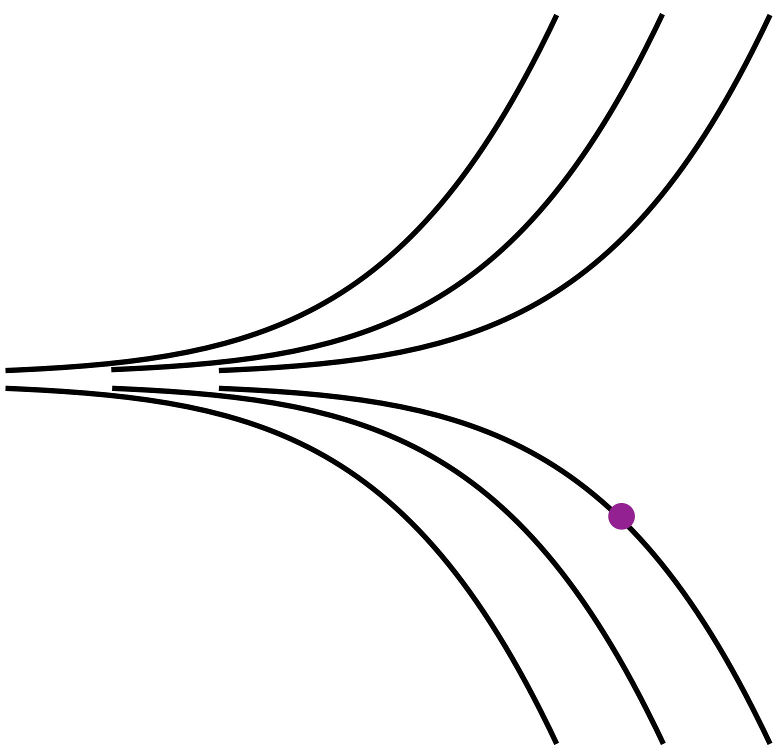

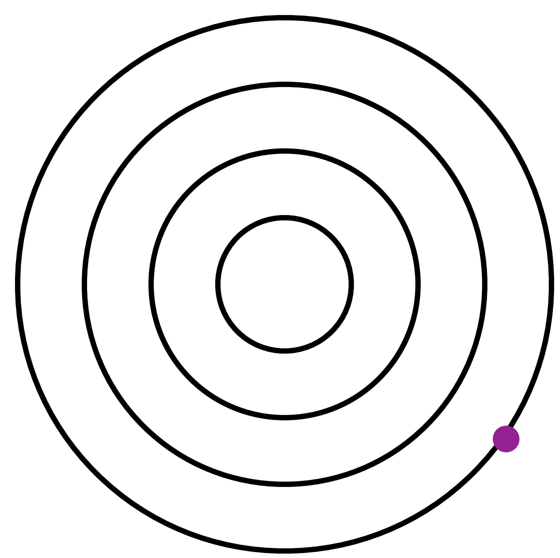

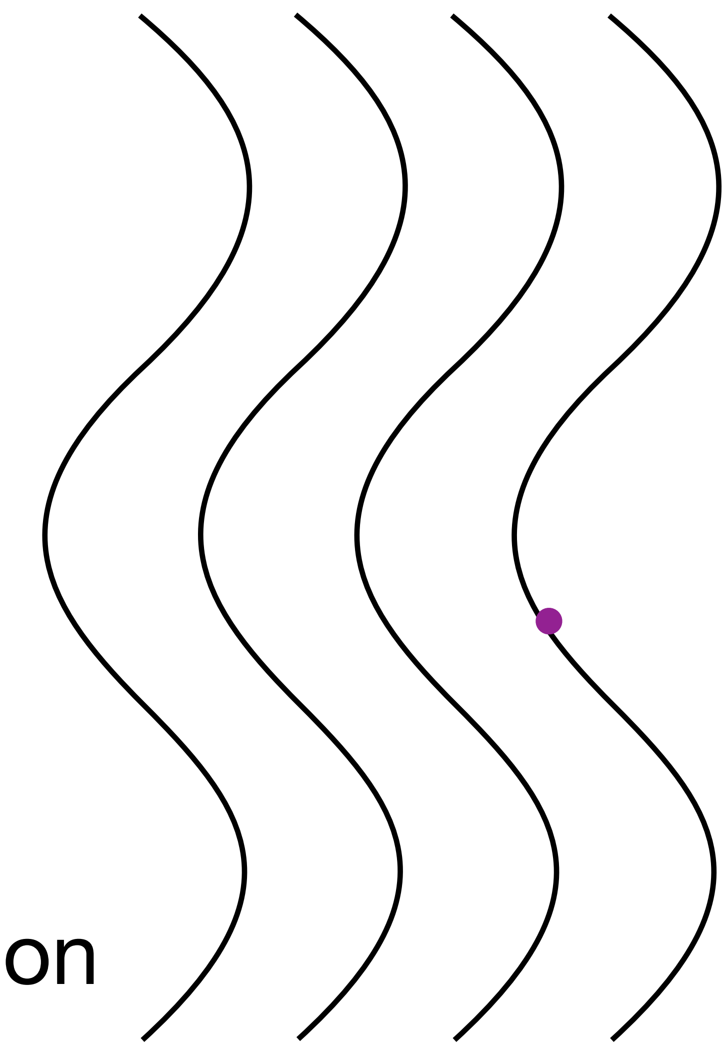

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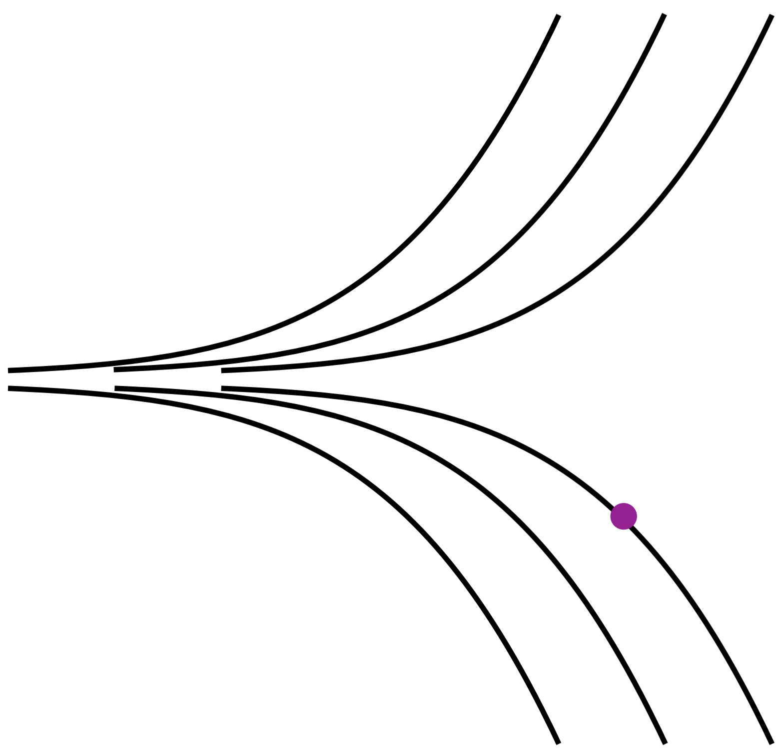

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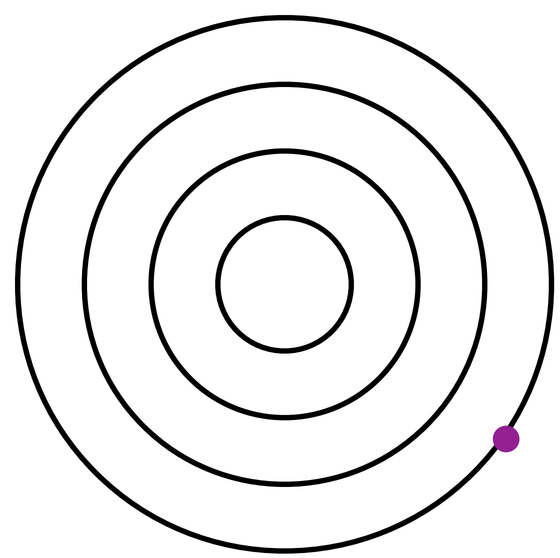

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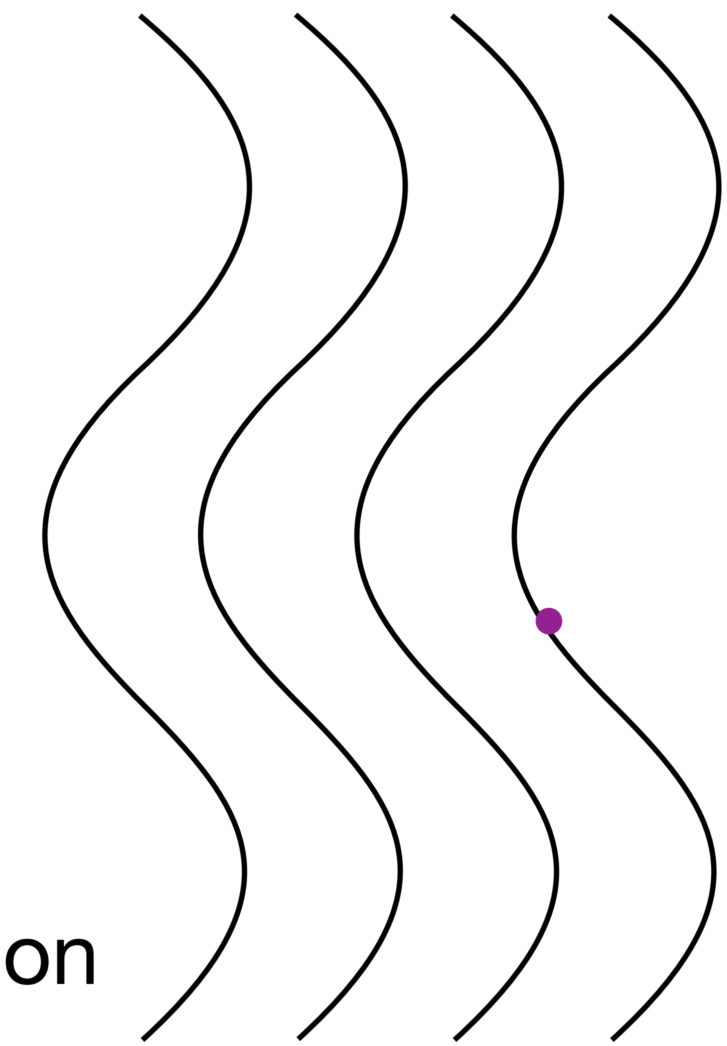

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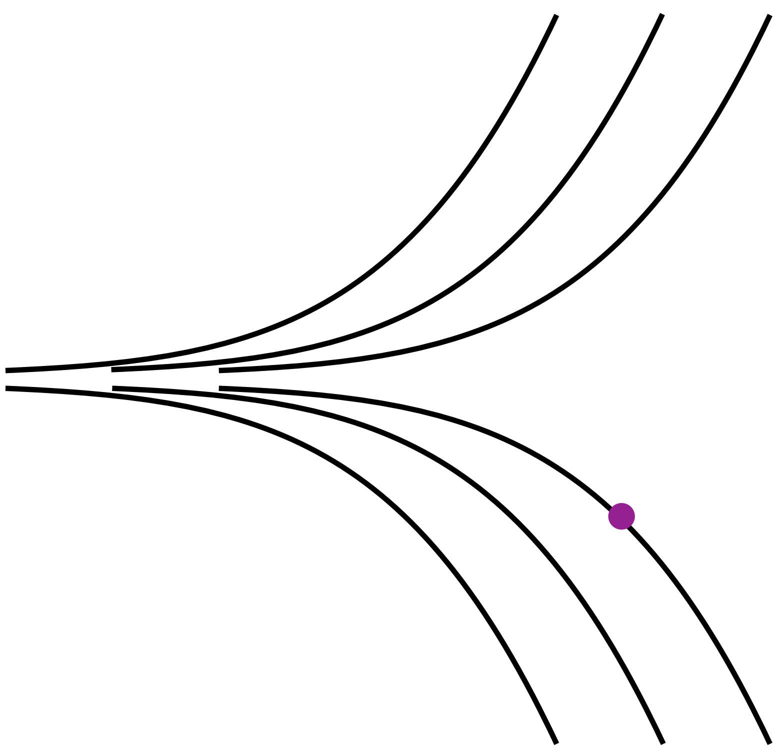

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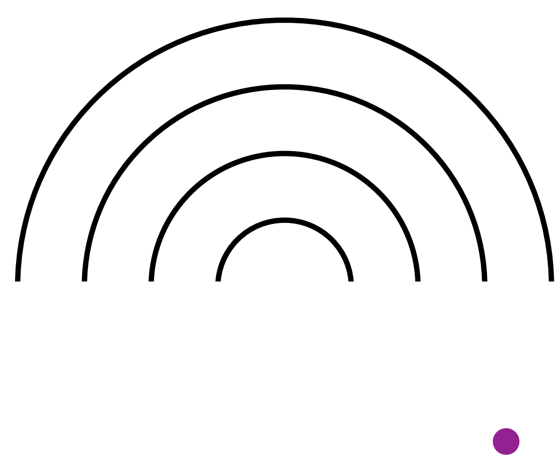

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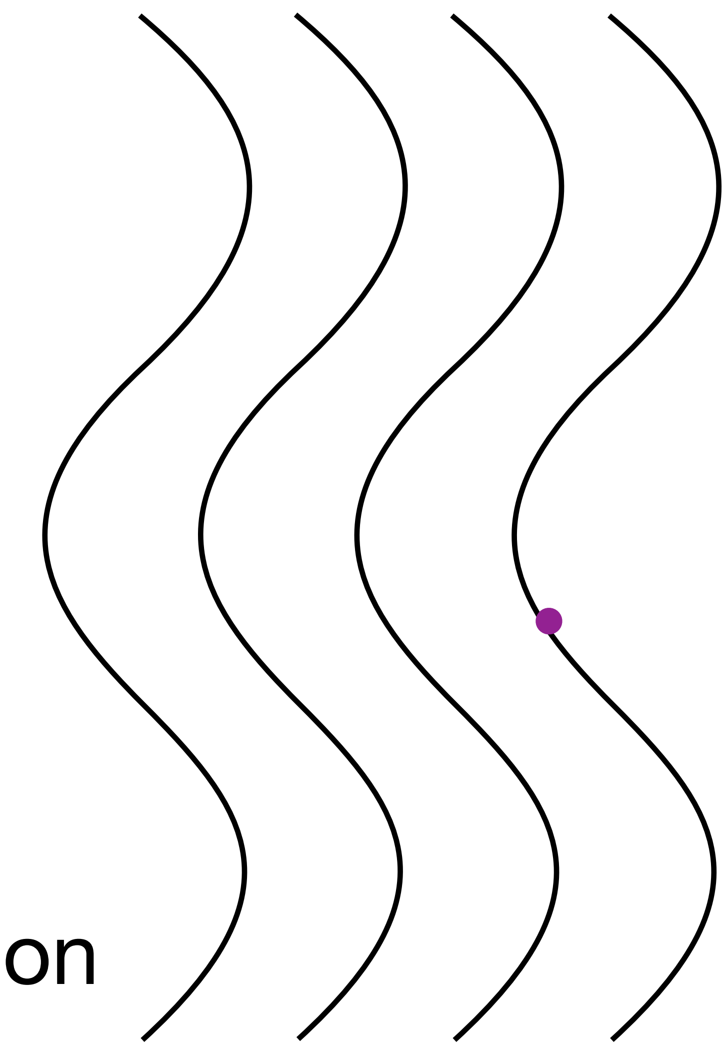

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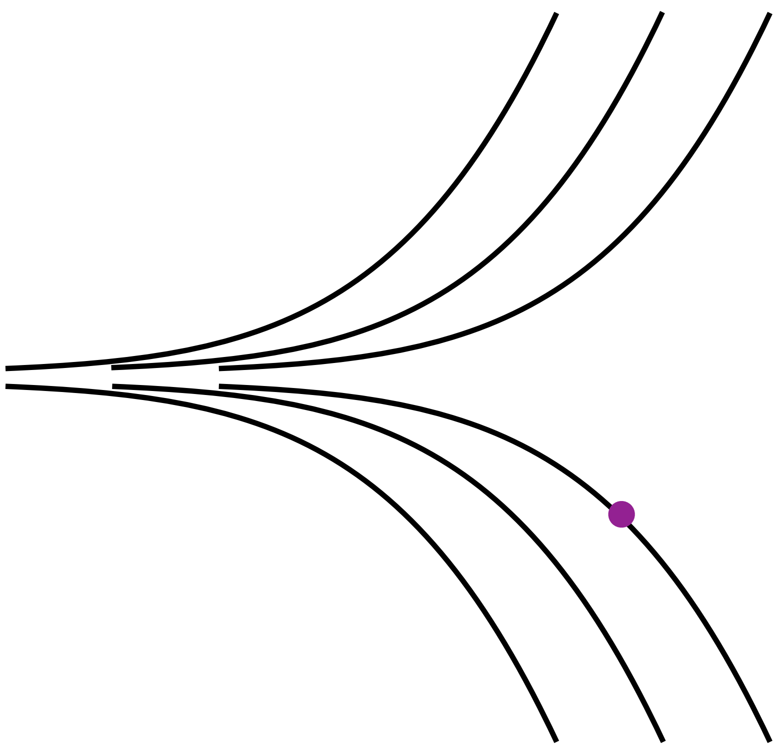

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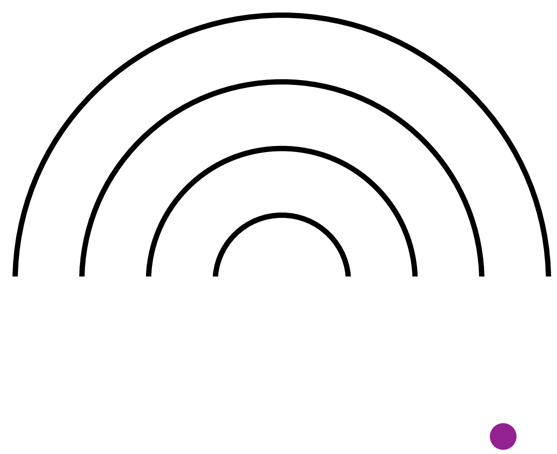

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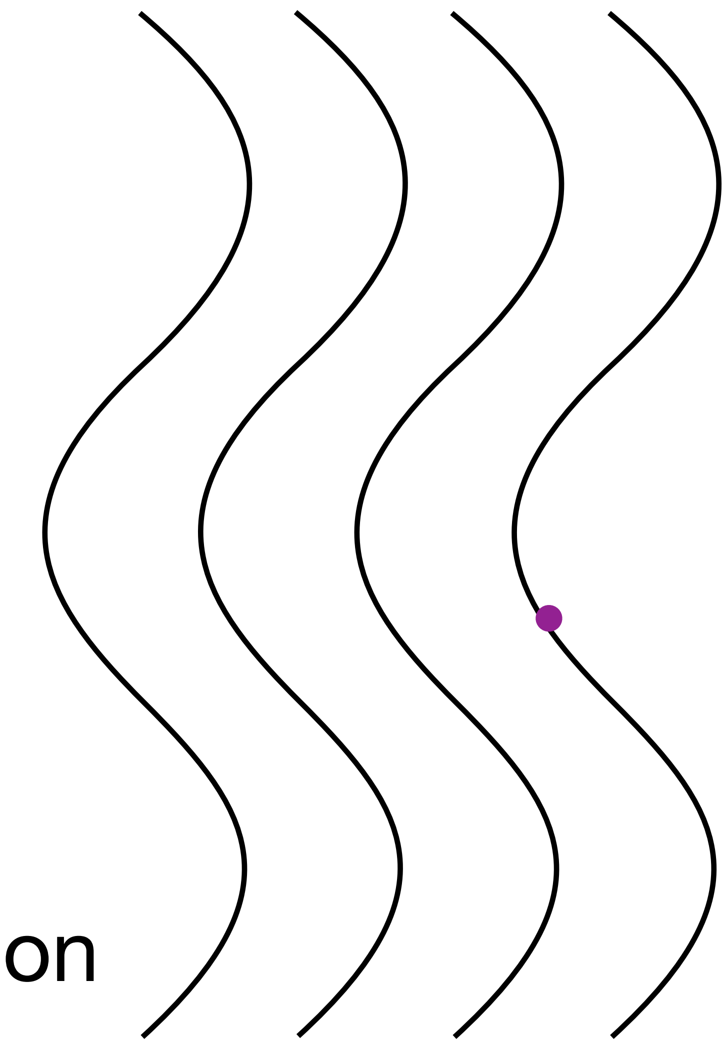

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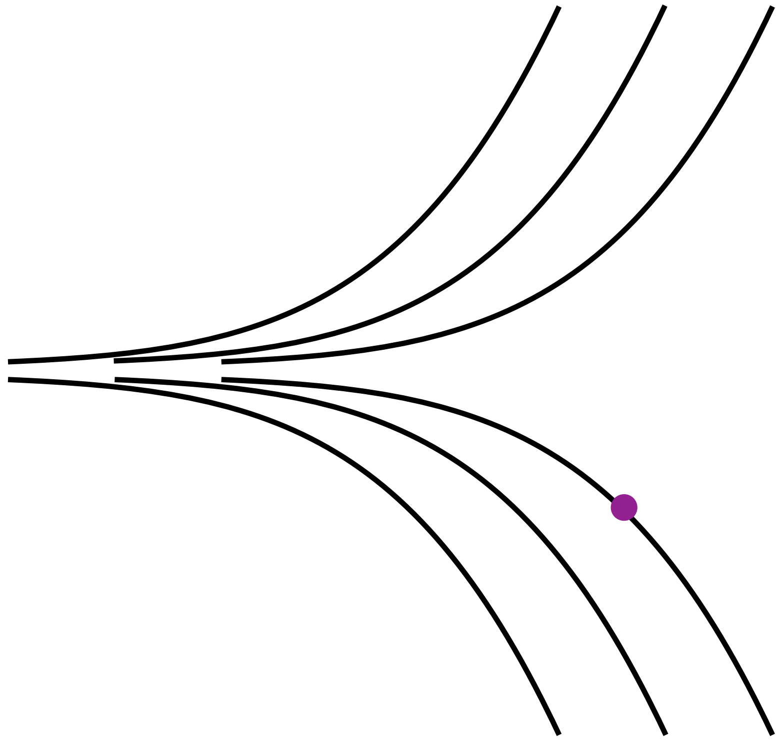

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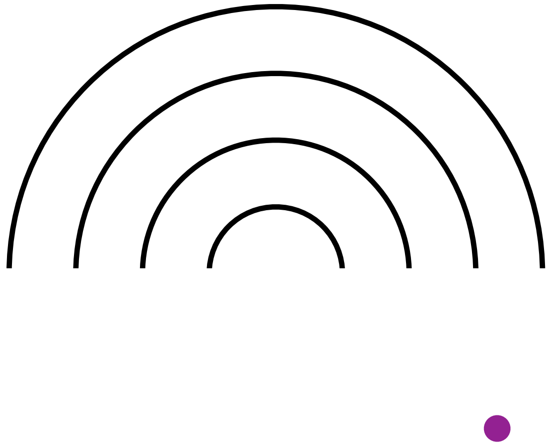

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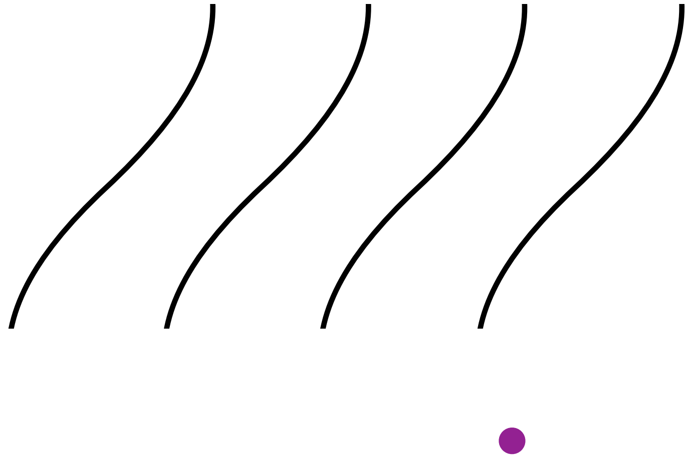

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    - Rearrange and take limit as $\Delta t \to 0$ to get an equation for q(t).

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- Freshwater flows into a tank at a rate 2 L/min. The tank starts with a concentration of 100 g / L of salt in it and holds 10 L. The tank is well mixed and the mixed water drains out at the same rate as the inflow.

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- Finally, taking a limit:

$$\boxed{\frac{dm}{dt} = -\frac{1}{5} m(t)}$$

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- m(0) = 1000 g.

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- What method could you use to solve the ODE $\dfrac{dm}{dt} = -\dfrac{1}{5}m(t)$?

  (A) Integrating factors.

  (B) Separating variables.

  (C) Just knowing some derivatives.

  (D) All of these.

  (E) None of these.

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To think about: what is the most general equation that can be solved using (A) and (B)?

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Answer to (b)?

$$\lim_{t\to\infty} m(t) = 0$$

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- Saltwater with a concentration of 200 g/L flows into a tank at a rate 2 L/min. The tank starts with no salt in it and holds 10 L. The tank is well mixed and the mixed water drains out at the same rate as the inflow.

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(B) $m' = 400 - 2m, \quad m(0) = 200$

(C) $m' = 400 - m/5, \quad m(0) = 0$

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- What happens when $m < 2000$? ---> $m' > 0$.
- What happens when $m > 2000$? ---> $m' < 0$.
- Limiting mass: 2000 g (Long way: solve the eq. and let $t \to \infty$.)