

# Welcome to MATH 256

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Differential equations (for Chemical and Biological Engineering students)

Instructor:

Prof. Eric Cytrynbaum

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[wiki.math.ubc.ca/mathbook/M256](http://wiki.math.ubc.ca/mathbook/M256)

Office: MATX 1215

Office hours: Tues 11:30 am - 1 pm, Thurs 3:30 - 4:30 pm.

# Course goals

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- **Primary:** Learn to solve ordinary and partial differential equations (mostly linear first and second order DEs).
- **Secondary:** Learn to use DEs to model physical, chemical, biological systems (really just an intro to this skill).

# Prerequisites

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- First year calculus (MATH 100/101).
- Linear algebra (MATH 152).
- Multivariable calculus (MATH 200 or 253).
- Talk to me if you aren't sure that you're prepared for this course.

# Tools we'll be using this term

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- WeBWork for homework assignments.
- Piazza for online discussion.
- Clickers for in-class responses.
- Cell phones and facebook for getting distracted during lectures and while studying.

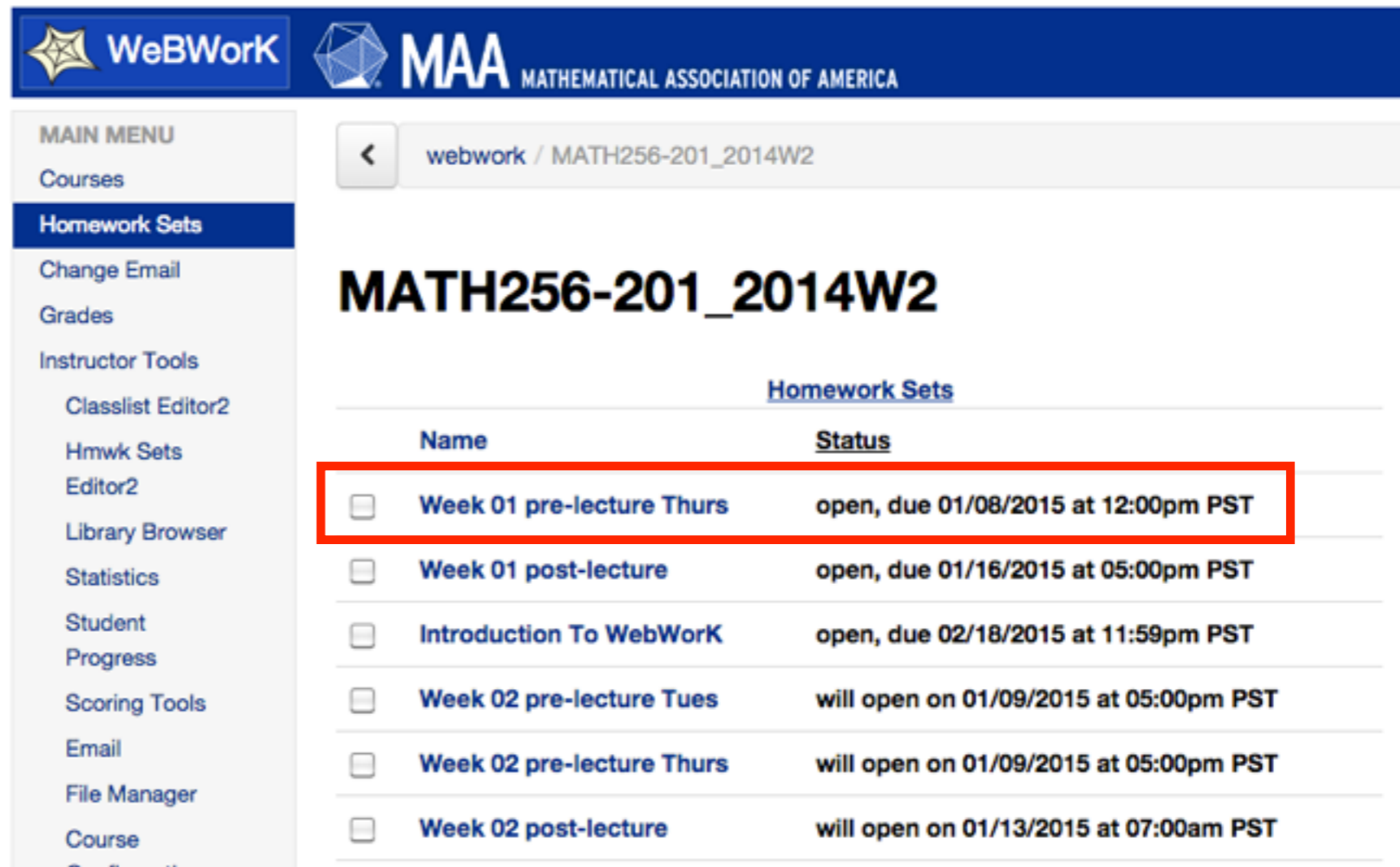
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# WeBWork

- Online homework system.
- [https://webwork.elearning.ubc.ca/webwork2/MATH256-201\\_2014W2](https://webwork.elearning.ubc.ca/webwork2/MATH256-201_2014W2)
- Log in using your CWL.



The screenshot displays the WeBWork interface for the course MATH256-201\_2014W2. The top navigation bar includes the WeBWork logo and the MAA (Mathematical Association of America) logo. The left sidebar contains a 'MAIN MENU' with options such as 'Courses', 'Homework Sets', 'Change Email', 'Grades', 'Instructor Tools', 'Classlist Editor2', 'Hmwk Sets Editor2', 'Library Browser', 'Statistics', 'Student Progress', 'Scoring Tools', 'Email', 'File Manager', and 'Course Configuration'. The main content area shows the course title 'MATH256-201\_2014W2' and a list of homework sets under the heading 'Homework Sets'. The first homework set, 'Week 01 pre-lecture Thurs', is highlighted with a red box. The status of this homework set is 'open, due 01/08/2015 at 12:00pm PST'.

Name	Status
<input type="checkbox"/> Week 01 pre-lecture Thurs	open, due 01/08/2015 at 12:00pm PST
<input type="checkbox"/> Week 01 post-lecture	open, due 01/16/2015 at 05:00pm PST
<input type="checkbox"/> Introduction To WebWork	open, due 02/18/2015 at 11:59pm PST
<input type="checkbox"/> Week 02 pre-lecture Tues	will open on 01/09/2015 at 05:00pm PST
<input type="checkbox"/> Week 02 pre-lecture Thurs	will open on 01/09/2015 at 05:00pm PST
<input type="checkbox"/> Week 02 post-lecture	will open on 01/13/2015 at 07:00am PST

# Why WeBWorK?

---

- Automated marking (instant feedback).
- Free for students (unlike hw systems provided by textbook companies).
- Stable, open source, widely used at UBC and many other universities.
- Frees up TA resources for things like the Math Learning Centre (<http://www.math.ubc.ca/~MLC/>)

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- Have you used WeBWorK previously? (A) Yes. (B) No.



# Piazza

- Online discussion forum.
- Sign up at <https://piazza.com>

The screenshot shows the Piazza interface for a course named MATH 256. The top navigation bar includes the Piazza logo, the course name, and tabs for Q & A, Course Page, and Manage Class. A user profile for Eric Cytrynbaum is visible in the top right. Below the navigation bar, there are filters for Unread, Updated, Unresolved, and Following. A search bar and a 'New Post' button are also present. The main content area displays a pinned post titled 'Welcome to Piazza!' from the instructor, Eric Cytrynbaum, dated 12:37PM. The post text reads: 'Welcome to Piazza! We'll be using Piazza as our online discussion forum this term. The quicker you begin asking questions on Piazza (rather than via emails), the quicker you'll benefit from the collective knowledge of your classmates and instructors. We encourage you to ask questions when you're struggling to understand a concept—you can even do so anonymously.' Below the post, there is an 'edit' button, a 'good note' button, and a '0' count. A section for 'followup discussions' is also visible, with a text input field for composing a new discussion.

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- Have you used Piazza previously? (A) Yes. (B) No.
- Would you prefer having a facebook page for the course? (A) Yes. (B) No.

# Clickers

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- Personal response system.
- Register your clicker at <https://connect.ubc.ca>

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- Active learning - you should be thinking and doing during class.
- My goal is to make clicker Qs that many of you get wrong - they help us to target what you don't understand yet.
- Points are for (thinking and then) clicking, not for getting answers correct.
- I don't look at the results on an individual basis so they are effectively anonymous.

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# More info online...



## Navigation

- [MATH 256 Home](#)
- [Course schedule](#)
- [Lecture slides](#)
- [Pre-lecture resources](#)
- [WeBWork](#)
- [Piazza](#)
- [Instructors' site](#)

 [Log in](#)

Page

View

## MATH 256 - Differential Equations

### Course description

This course serves as an introduction to differential equations with a focus on solution techniques, transforms and modeling. Topics include linear ordinary differential equations, Laplace transforms, Fourier series and separation of variables for linear partial differential equations.

This website is the course website for MATH 256 taught in 2014W Term 2.

### Course details

- [Instructor information](#)
- [Marking scheme](#)
- [Important dates](#)
- [Course schedule](#)
- [Other course information](#)
- [Lecture slides](#)
- [Solutions](#)
- [Pre-lecture resources](#) - links to websites and videos that will help you do the pre-lecture assignments.
- [General resources](#) - including links to old course websites, old assignments, suggested practice problems etc.



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- Simple model to predict how fast he'll go, how long it will take etc.



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- Flaws with this model?
- $g$  is not constant...
- ...but  $6371 \text{ km} \approx 6411 \text{ km}$  so not bad.
- $k$  is not constant either (depends on air density) - this is significant!

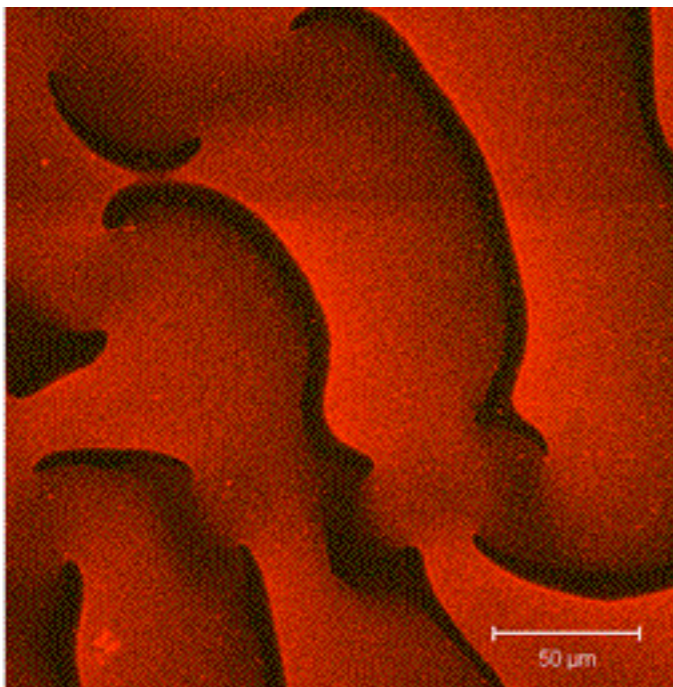


# A bacterial cell division regulator

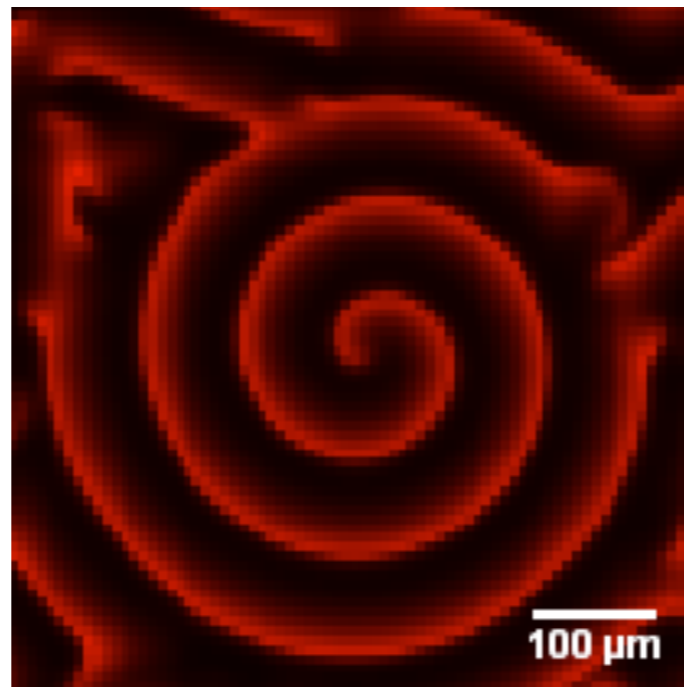
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- Two interacting bacterial proteins that undergo complicated dynamics.
- Differential equation model help understand how they work.

Experiment



Model



$$\frac{\partial u}{\partial t} = u - uv + D \frac{\partial^2 u}{\partial x^2}$$

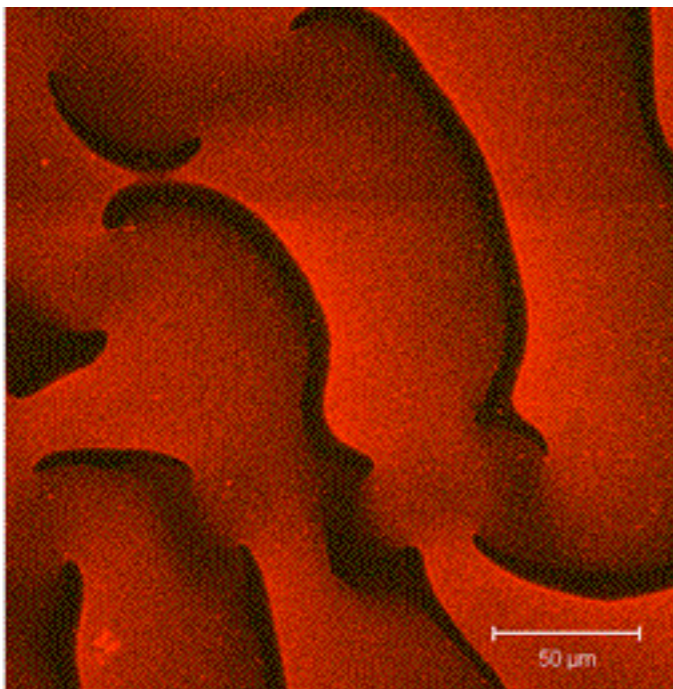
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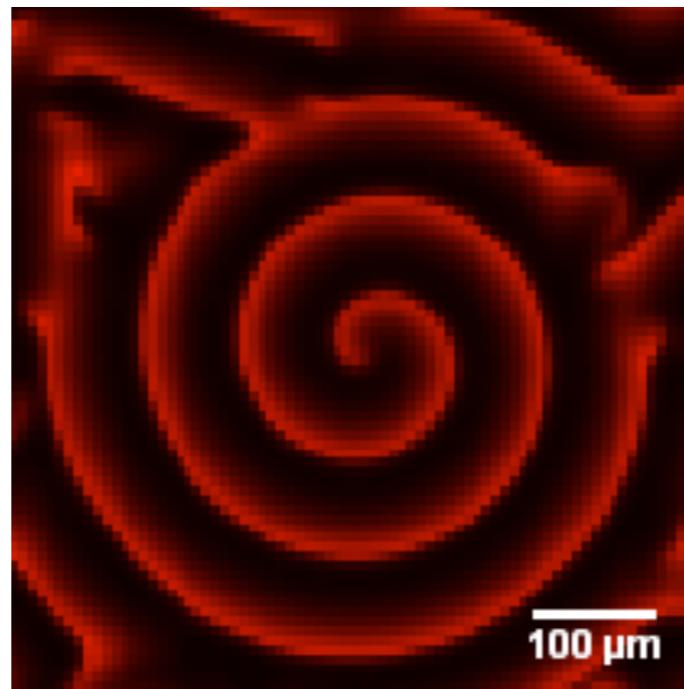
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- **Arbitrary constant** - a constant that does not appear in the DE but arises while solving the equation (usually at an integration step).
- **A particular solution** - a solution with no arbitrary constants in it.
- **The general solution** - a solution with one or more arbitrary constants that encompass ALL possible solutions to the DE.

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---

- Plug it in and make sure it satisfies the equation.

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A cylindrical bucket has a hole in the bottom. If  $h(t)$  is the height of the water at any time  $t$  in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket ( $h(0) = 4$ ), what is the solution to this differential equation?

- A.  $h(t) = (2 - 3t)^2$
- B.  $h(t) = \sqrt{16 - 2t}$
- C.  $h(t) = (3 - 3t)^2$
- D.  $h(t) = 4 - 6t^2$

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## Method of integrating factors (Section 2.1)

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$$\frac{d}{dt} (t^2 y(t)) =$$

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(B)  $t^2 \frac{dy}{dt}$

(C)  $2ty$

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- Given that  $\frac{d}{dt} (t^2 y(t)) = t^2 \frac{dy}{dt} + 2ty$

- if you're given the equation  $t^2 \frac{dy}{dt} + 2ty = 0$

- you can rewrite it as  $\frac{d}{dt} (t^2 y(t)) = 0$

- so the solution is  $t^2 y(t) = C$  or equivalently  $y(t) = \frac{C}{t^2}$ .

arbitrary constant  
that appeared at an  
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# Method of integrating factors (Section 2.1)

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- Solve the equation  $t^2 \frac{dy}{dt} + 2ty(t) = \sin(t)$  (not brute force checking).

(A)  $y(t) = -\frac{1}{t^2} \sin(t)$

(B)  $y(t) = -\cos(t) + C$

(C)  $y(t) = \frac{C - \cos(t)}{t^2}$

(D)  $y(t) = \sin(t) + C$

(E)  $y(t) = -\frac{1}{t^2} \cos(t)$