Welcome to MATH 256

Differential equations (for Chemical and Biological Engineering students)

Instructor:
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wiki.math.ubc.ca/mathbook/M256
Office: MATX 1215
Office hours: Tues 11:30 am - 1 pm, Thurs 3:30 - 4:30 pm.

Course goals

- Primary: Learn to solve ordinary and partial differential equations (mostly linear first and second order DEs).
- Secondary: Learn to use DEs to model physical, chemical, biological systems (really just an intro to this skill).

Prerequisites

- First year calculus (MATH 100/101).
- Linear algebra (MATH 152).
- Multivariable calculus (MATH 200 or 253).
- Talk to me if you aren’t sure that you’re prepared for this course.

Tools we'll be using this term

- WeBWorK for homework assignments.
- Piazza for online discussion.
- Clickers for in-class responses.
- Cell phones and facebook for getting distracted during lectures and while studying.

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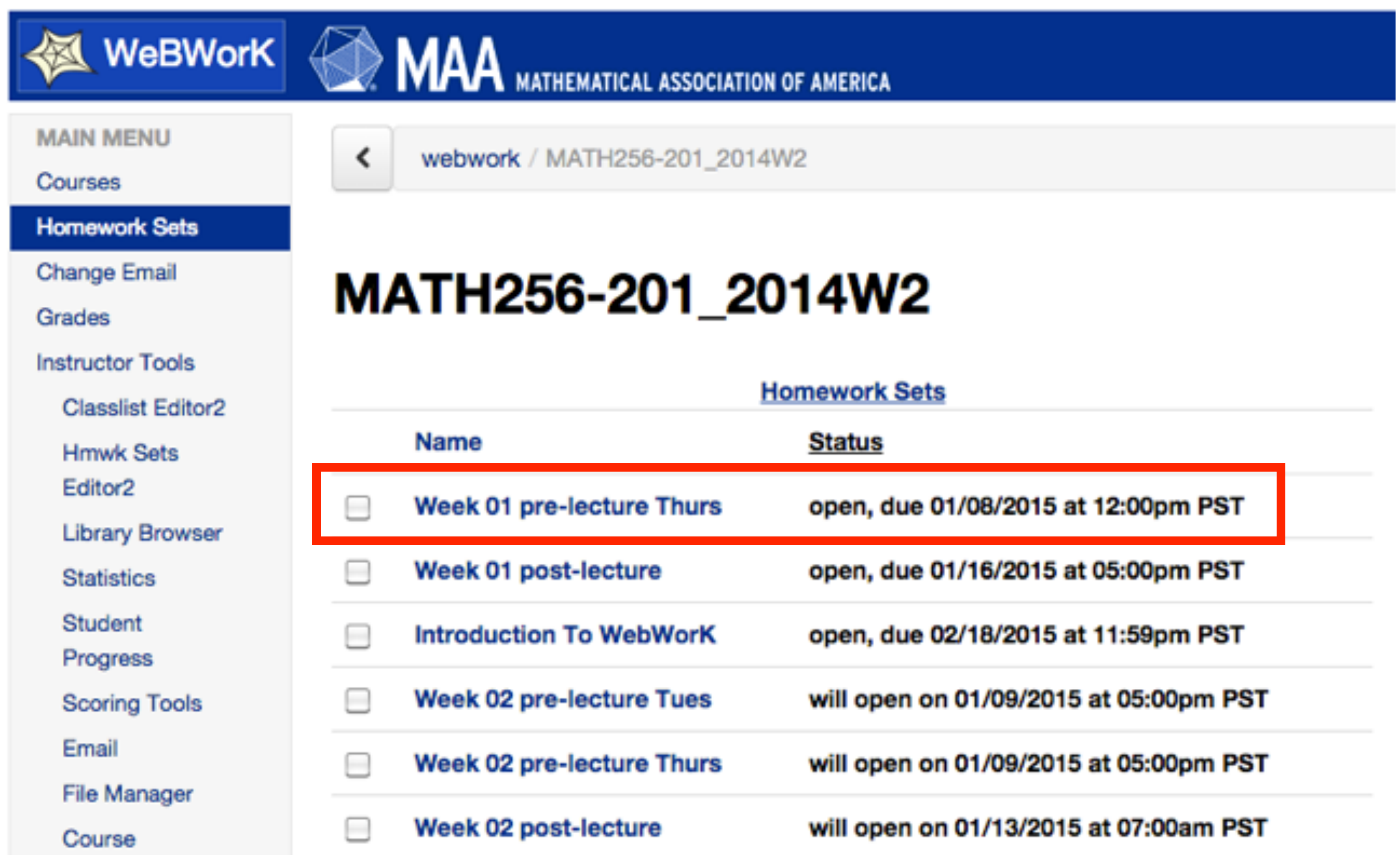
WeBWorK

- Online homework system.
- https://webwork.elearning.ubc.ca/webwork2/MATH256-201_2014W2
- Log in using your CWL.



Why WeBWorK?

- Automated marking (instant feedback).
- Free for students (unlike hw systems provided by textbook companies).
- Stable, open source, widely used at UBC and many other universities.
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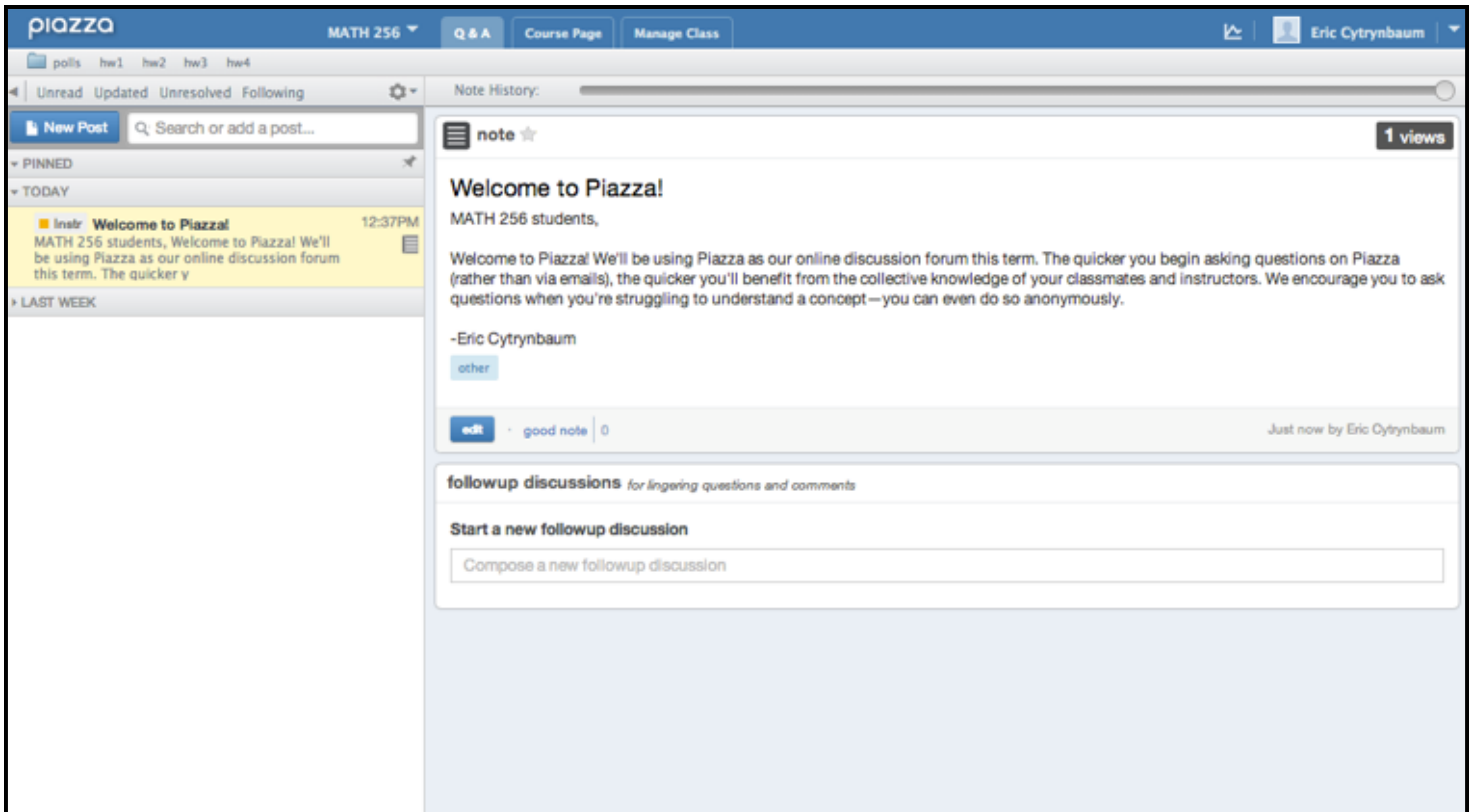
- Have you used WeBWorK previously? (A) Yes. (B) No.

Piazza

- Online discussion forum.

- Sign up at https://piazza.com



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- Would you prefer having a facebook page for the course? (A) Yes. (B) No.

Clickers

- Personal response system.
- Register your clicker at https://connect.ubc.ca

Why clickers?

- Active learning - you should be thinking and doing during class.
- My goal is to make clicker Qs that many of you get wrong - they help us to target what you don't understand yet.
- Points are for (thinking and then) clicking, not for getting answers correct.
- I don't look at the results on an individual basis so they are effectively anonymous.

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More info online...

Log in

Page View Search

Navigation

- MATH 256 Home
- Course schedule
- Lecture slides
- Pre-lecture resources
- WeBWorK
- Piazza
- Instructors' site

MATH 256 - Differential Equations

Course description

This course serves as an introduction to differential equations with a focus on solution techniques, transforms and modeling. Topics include linear ordinary differential equations, Laplace transforms, Fourier series and separation of variables for linear partial differential equations.

This website is the course website for MATH 256 taught in 2014W Term 2.

Course details

- Instructor information
- Marking scheme
- Important dates
- Course schedule
- Other course information
- Lecture slides
- Solutions
- Pre-lecture resources - links to websites and videos that will help you do the pre-lecture assignments.
- General resources - including links to old course websites, old assignments, suggested practice problems etc.

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$$mv' = -mg + kv^2$$

- Simple model to predict how fast he'll go, how long it will take etc.

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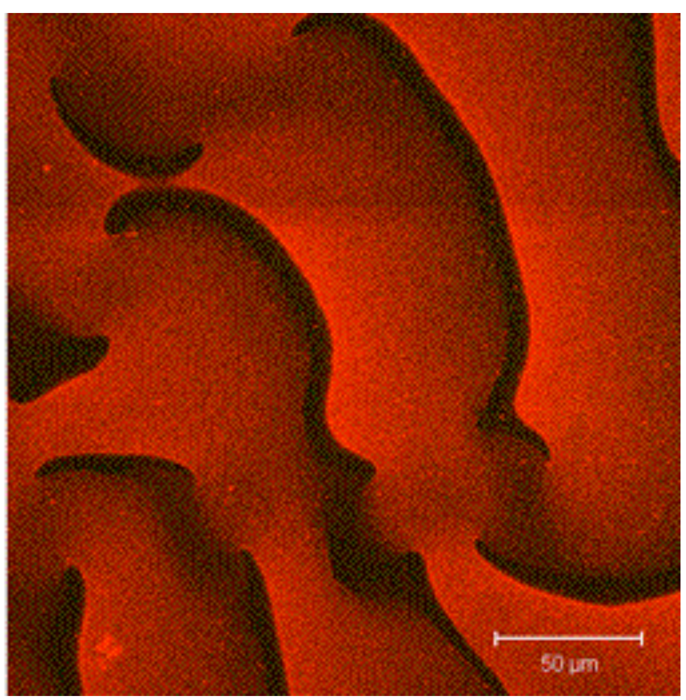
- k is not constant either (depends on air density) - this is significant!

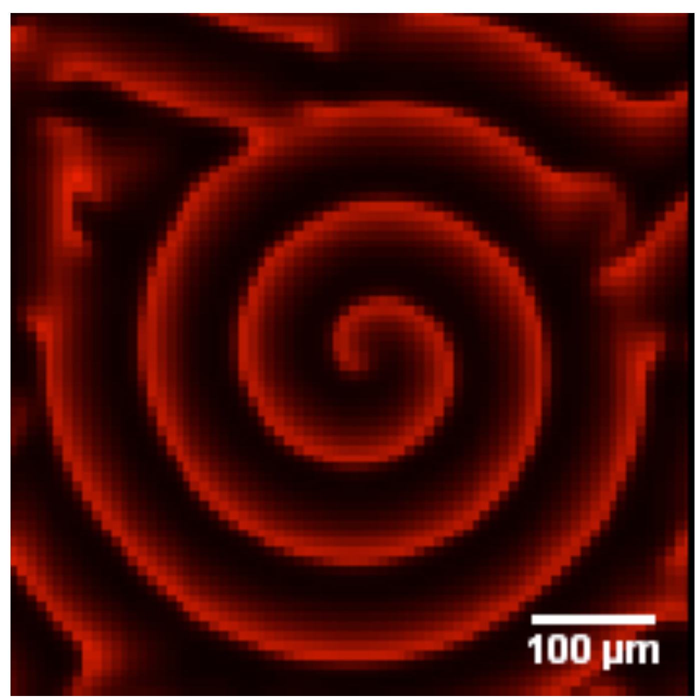
A bacterial cell division regulator

- Two interacting bacterial proteins that undergo complicated dynamics.
- Differential equation model help understand how they work.

Experiment



Model



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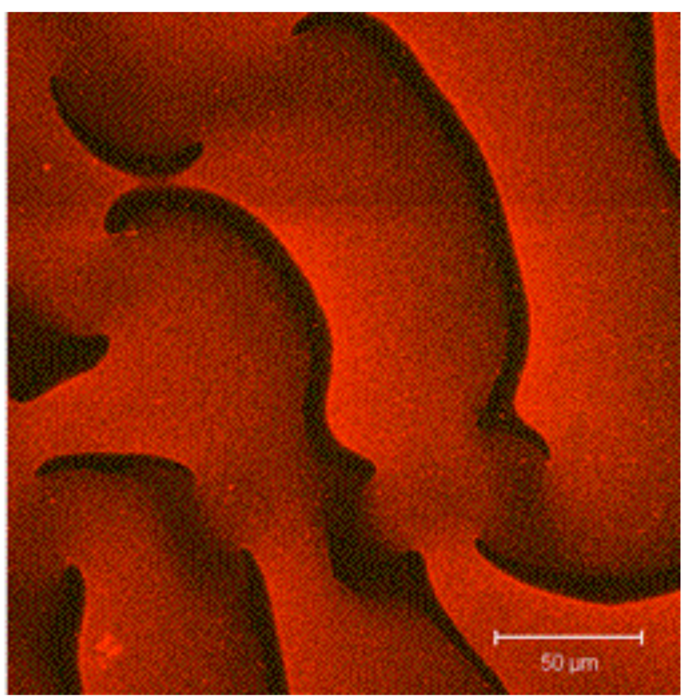

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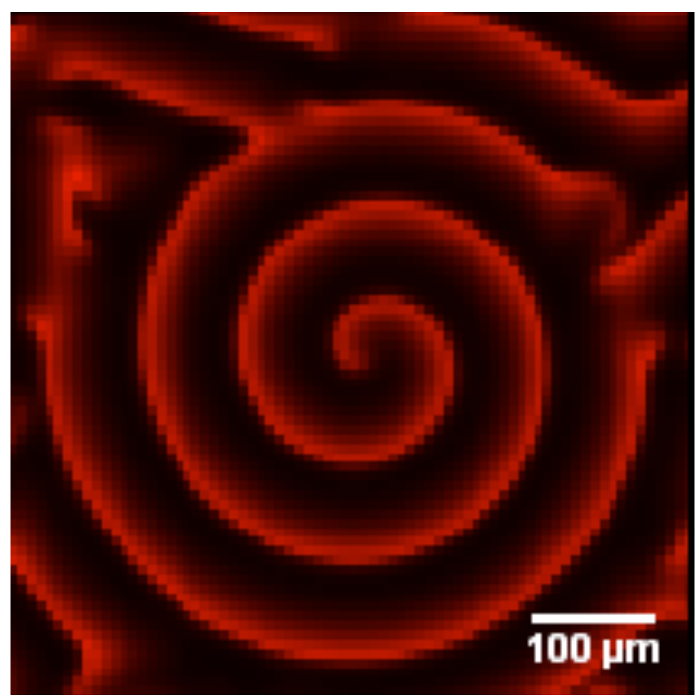

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- The general solution - a solution with one or more arbitrary constants that encompass ALL possible solutions to the DE.

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A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket $(h(0) = 4)$, what is the solution to this differential equation?

A. $h(t) = (2 - 3t)^2$
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Method of integrating factors (Section 2.1)

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(C) $2ty$

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Method of integrating factors (Section 2.1)

- Given that $\dfrac{d}{dt}\left(t^2 y(t)\right) = \; t^2 \dfrac{dy}{dt} + 2ty$

- if you're given the equation $t^2 \dfrac{dy}{dt} + 2ty = 0$

- you can rewrite is as $\dfrac{d}{dt}\left(t^2 y(t)\right) = 0$

- so the solution is $t^2 y(t) = C$ or equivalently $y(t) = \dfrac{C}{t^2}$.

arbitrary constant that appeared at an integration step

Method of integrating factors (Section 2.1)

- Solve the equation $t^2 \frac{dy}{dt} + 2ty(t) = \sin(t)$ (not brute force checking).

(A) $y(t) = -\frac{1}{t^2}\sin(t)$

(B) $y(t) = -\cos(t) + C$

(C) $y(t) = \frac{C - \cos(t)}{t^2}$

(D) $y(t) = \sin(t) + C$

(E) $y(t) = -\frac{1}{t^2}\cos(t)$