Today

- Systems with complex eigenvalues - how to figure out rotation
- Systems with a repeated eigenvalue
- Summary of 2x2 systems with constant coefficients.

Direction of rotation in complex eigenvalue case

$$x' = x - 8y$$
$$y' = 8x + y$$

(A) Solutions rotate clockwise and decay exponentially.

(B) Solutions grow exponentially without oscillating.

(C) Solutions rotate clockwise and grow exponentially.

(D) Solutions rotate counterclockwise and grow exponentially.

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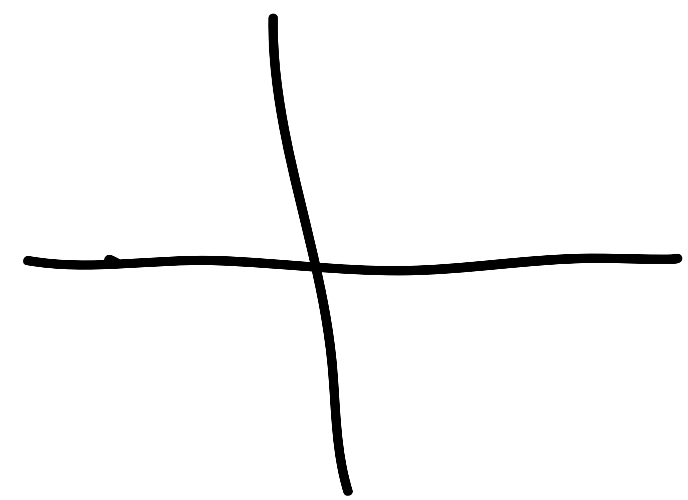

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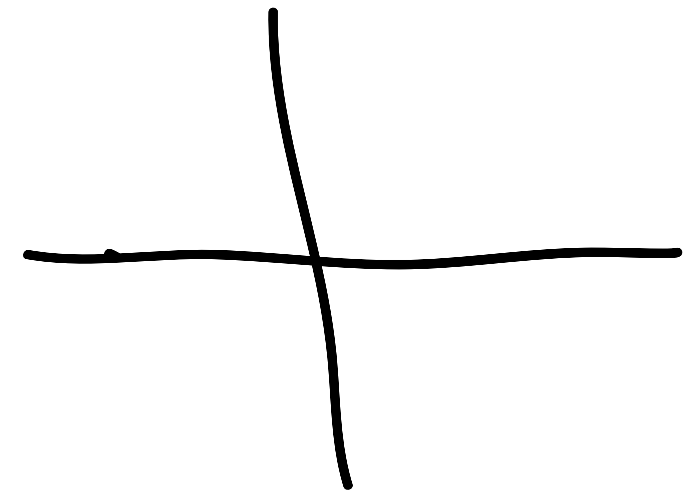

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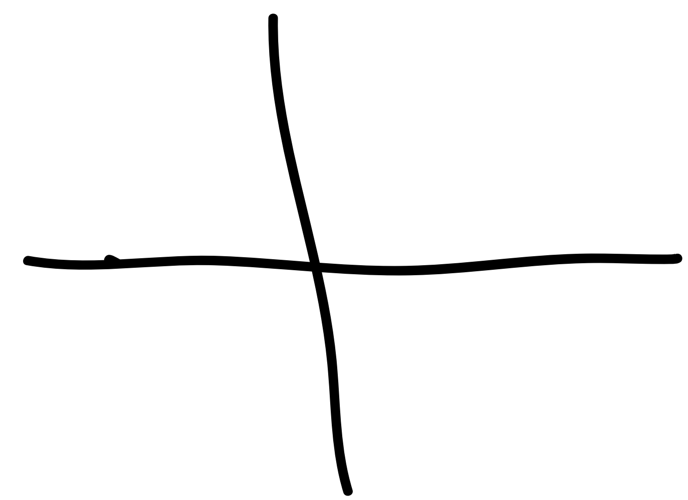

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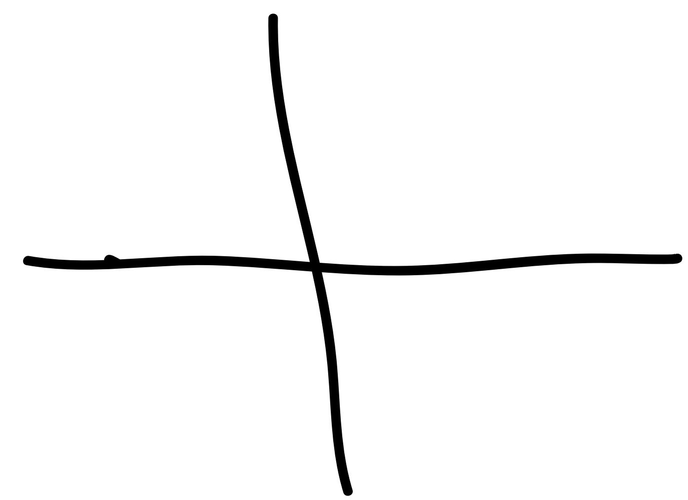

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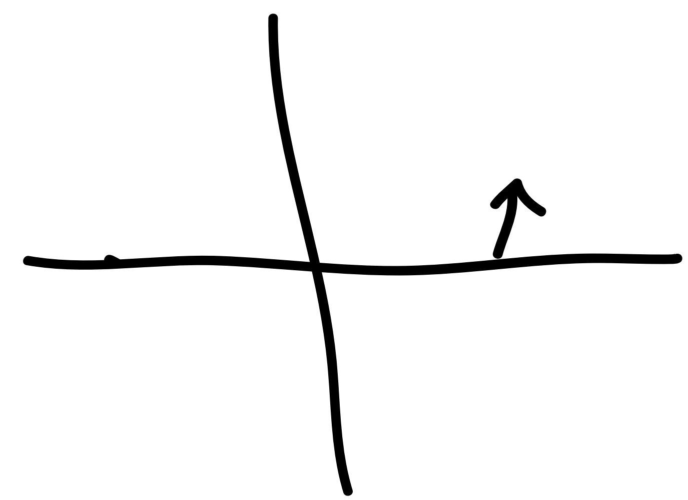

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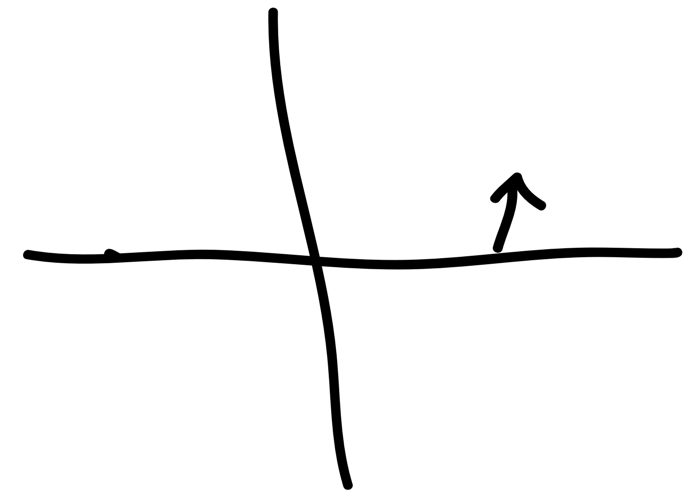

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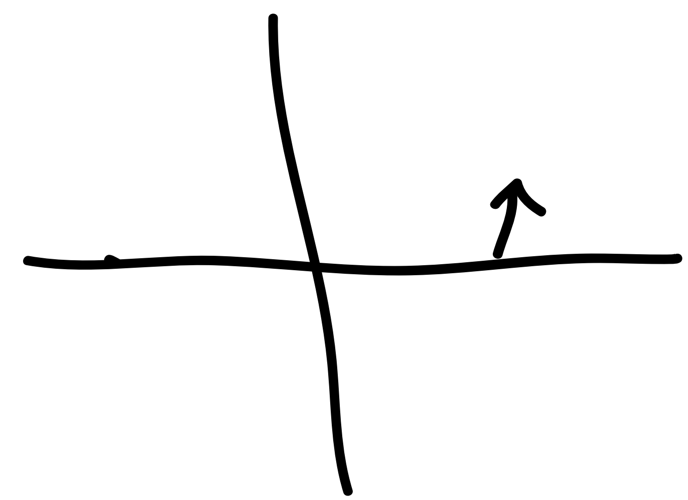

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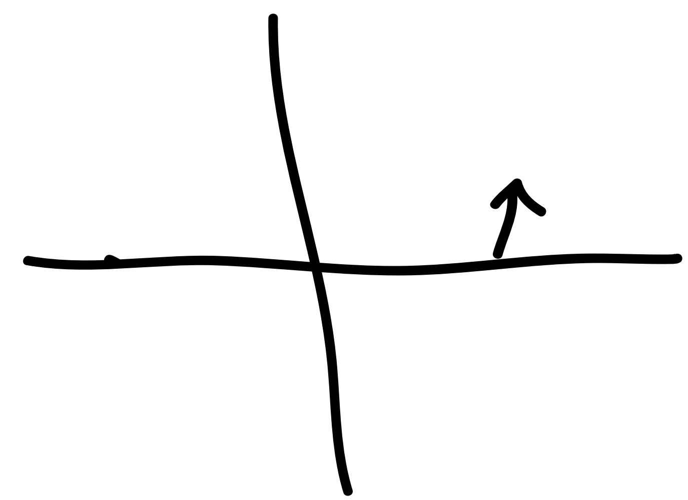

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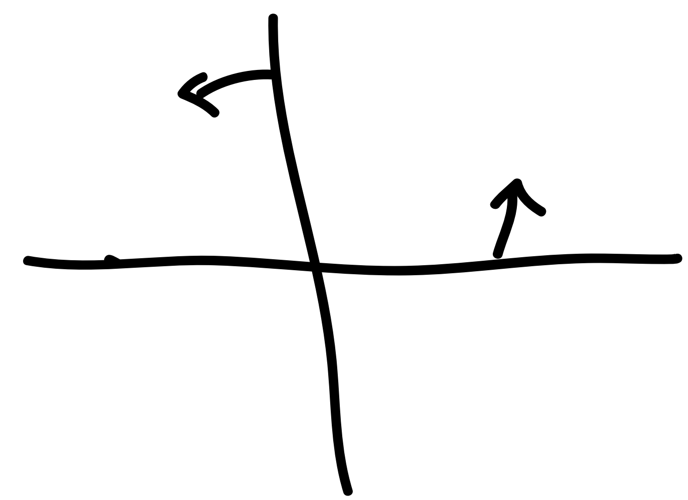

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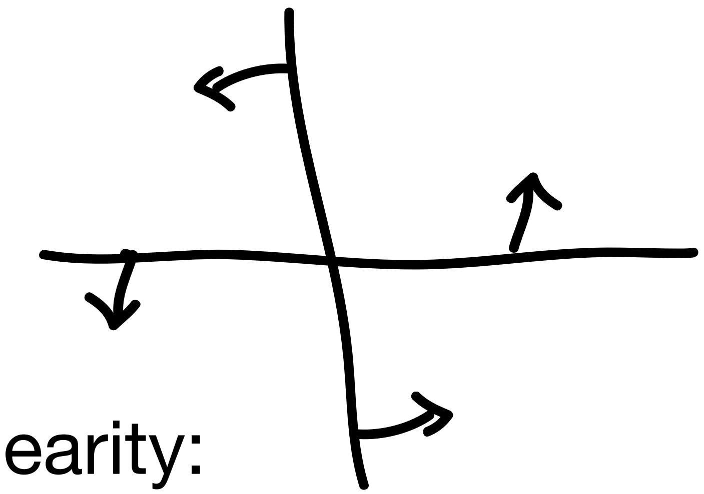

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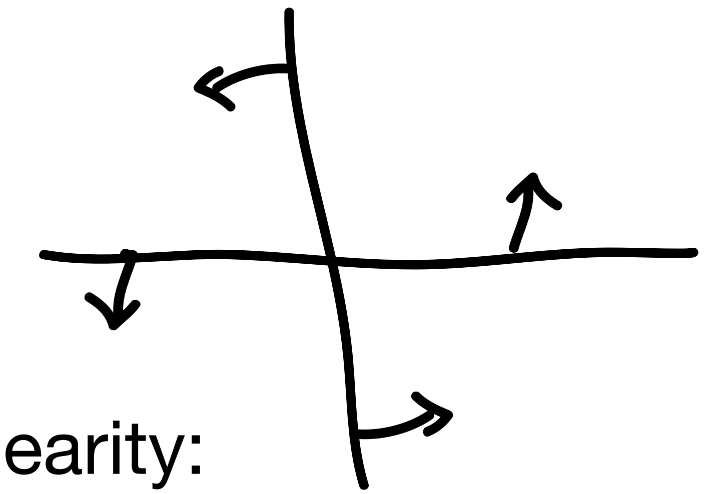

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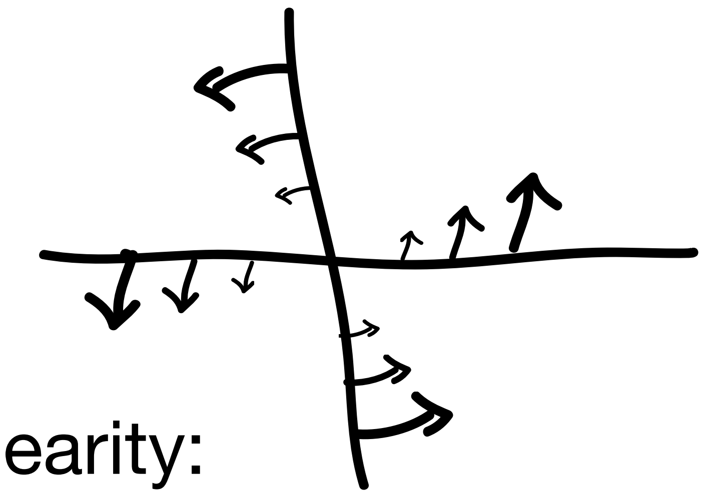

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Repeated eigenvalues

- What happens when you get two identical eigenvalues?
- Two cases:

1. The single eigenvalue has two distinct eigenvectors.
2. There is only one eigenvector (matrix is defective).

1. $\overline{\mathbf{x}}' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \overline{\mathbf{x}}$ 2. $\overline{\mathbf{x}}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \overline{\mathbf{x}}$

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$$\det(A - \lambda I) = (\lambda - 3)^2 = 0$$

$$\lambda = 3$$

$$(A - \lambda I)\mathbf{v} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{v} = 0$$

All vectors solve this so choose any two independent vectors:

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{v_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathbf{x}(t) = C_1 e^{3t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2. $\overline{\mathbf{x}}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \overline{\mathbf{x}}$

$$\det(A - \lambda I) = \lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2$$

$$(A - \lambda I)\mathbf{v} = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{v} = 0$$

$\mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ <-- only 1 evector!

$$\mathbf{x}(t) = C_1 e^{2t}\mathbf{v} + C_2 e^{2t}(\mathbf{w} + t\mathbf{v})$$

$$(A - \lambda I)\mathbf{w} = \mathbf{v}$$

$\mathbf{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ <-- called "generalized evector"

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Steady state - two notions

- Forced mass-spring systems - long term behaviour after transient dies down.

    - If the IC isn't right on $y_p(t)$, the homog solution decays exponentially (for $\alpha<0$) so eventually only y_p remains.

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    - SS can be oscillation (not constant).
- Constant solutions of a system of ODEs (discussed in the next slides).
    - Transient may decay or grow exponentially.
    - Always constant solutions!

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    - It is also an eigenvector associated with eigenvalue $\lambda = 0$.
- If *A* is nonsingular then $\mathbf{x}(t) = \mathbf{0}$ is the only steady state.

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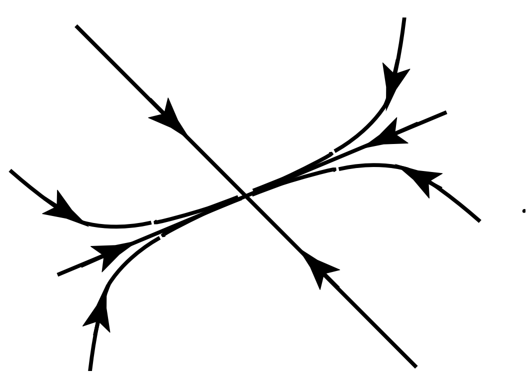
- real negative evalues



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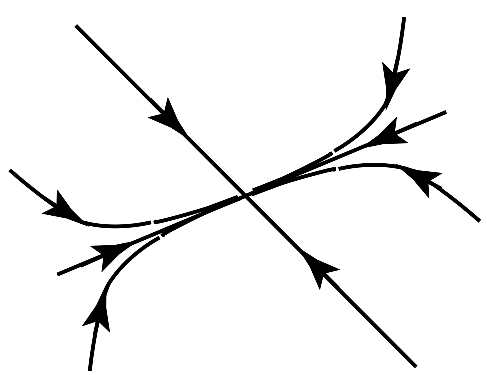

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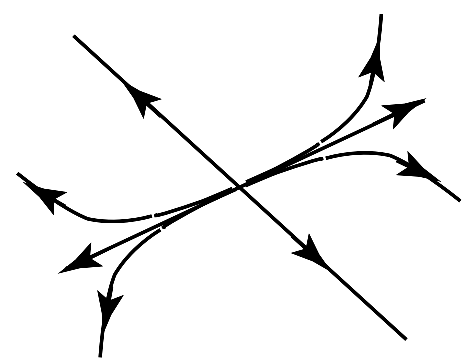

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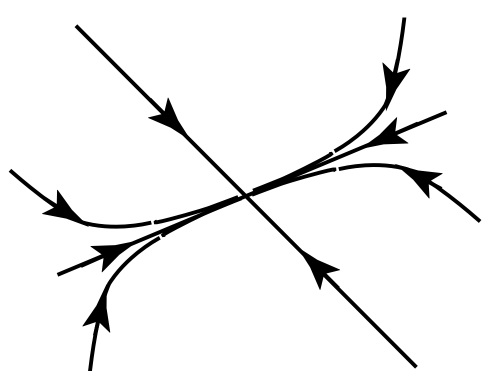

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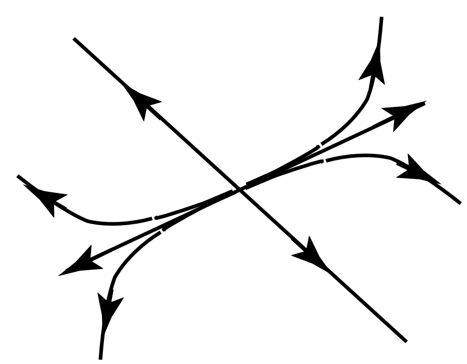

saddle

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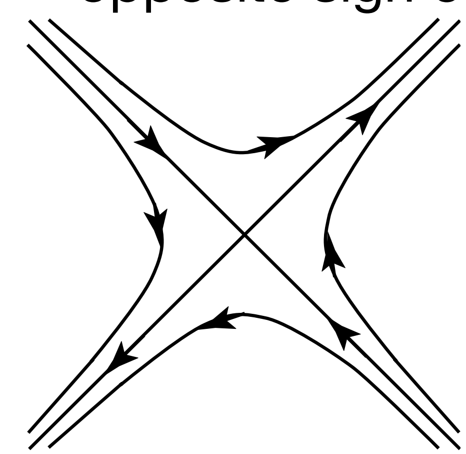

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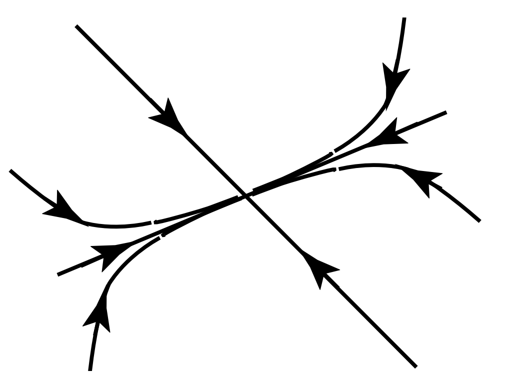

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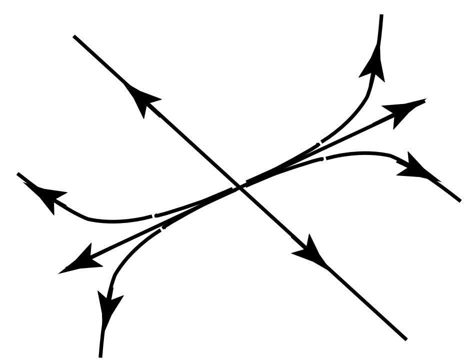

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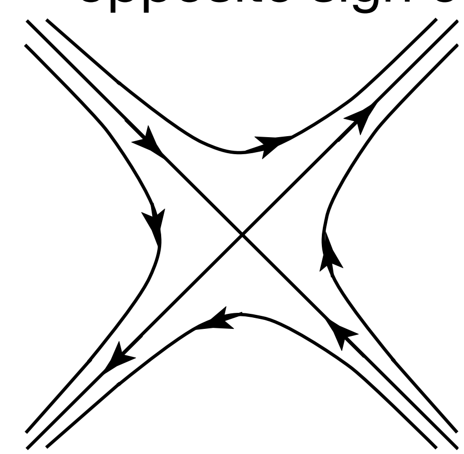
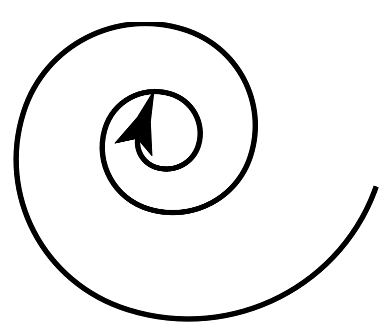


stable spiral
- complex evalues, negative real part



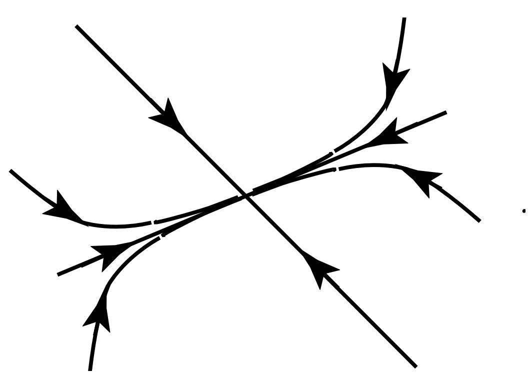
Summary - homogeneous 2x2 systems

Steady states

- Steady states are classified by the nature of the surrounding solutions:

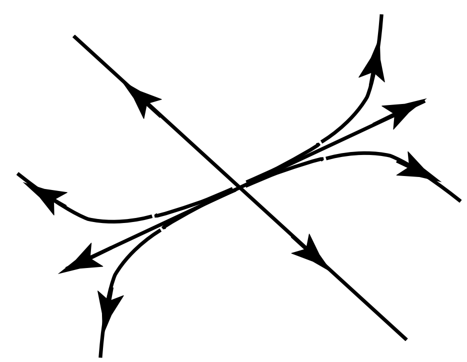
stable node

- real negative evalues



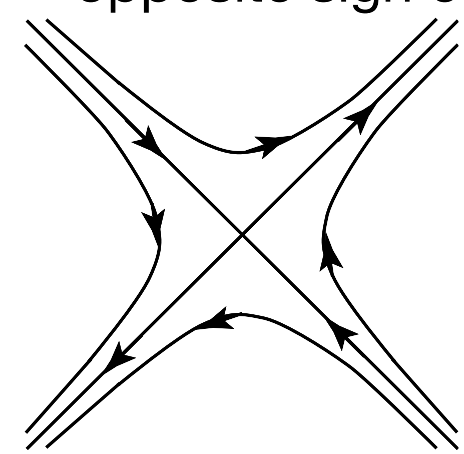
unstable node

- real positive evalues



saddle

- opposite sign evalues



stable spiral

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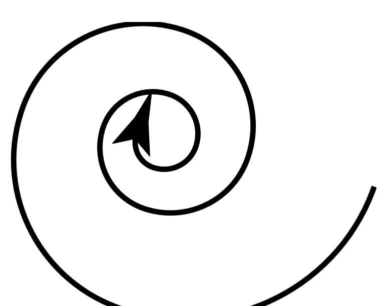

unstable spiral

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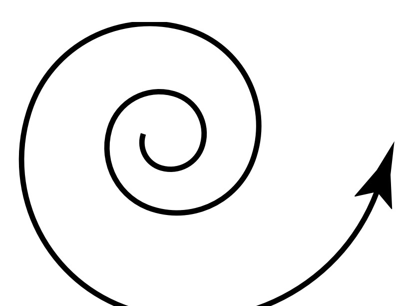

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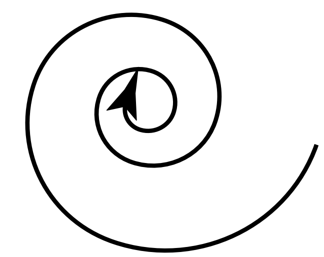
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{aligned} \det(A - \lambda I) &= \det \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} \\ &= (a - \lambda)(d - \lambda) - bc \\ &= \lambda^2 - (a + d)\lambda + ad - bc \\ &= \lambda^2 - \mathrm{tr}(A)\lambda + \det(A) \qquad = 0 \end{aligned}$$

Summary - homogeneous 2x2 systems

- When do the solutions spiral IN to the origin?



$$\lambda^2 - \mathrm{tr}A\lambda + \det A = 0$$

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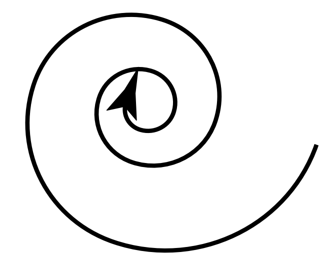

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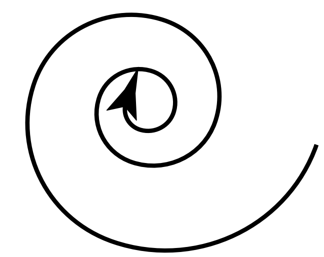

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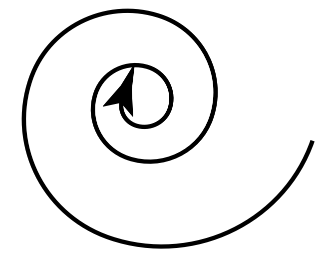

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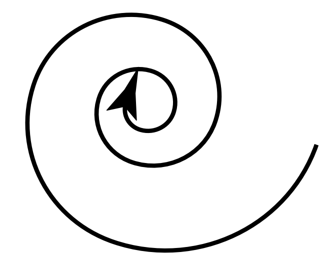

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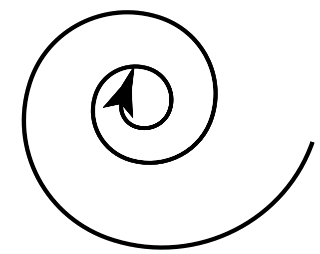

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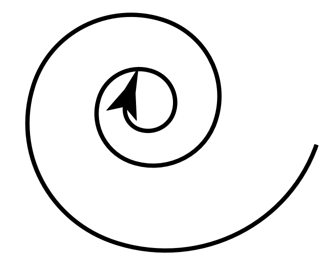

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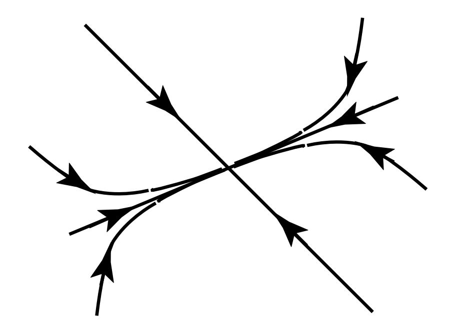

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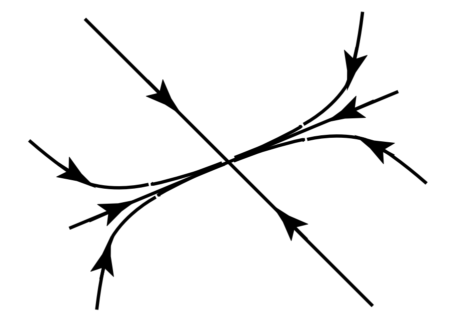

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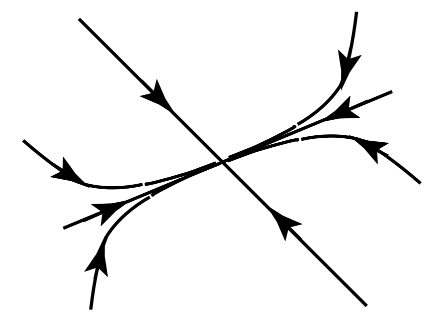

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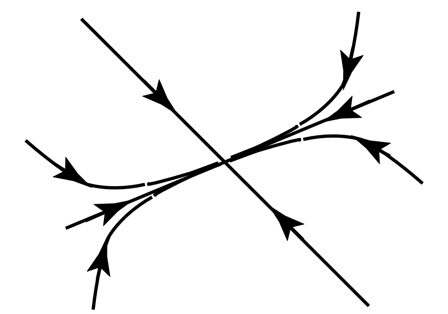

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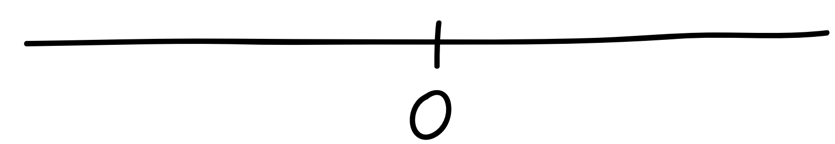

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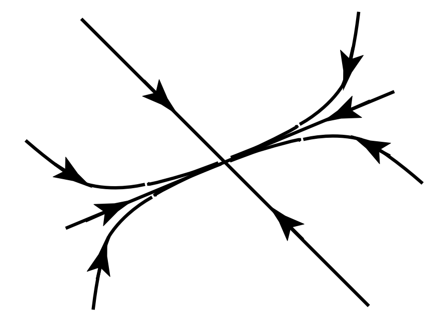

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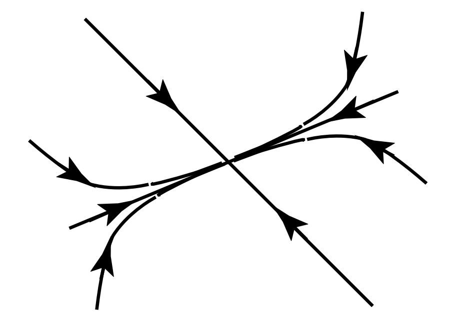

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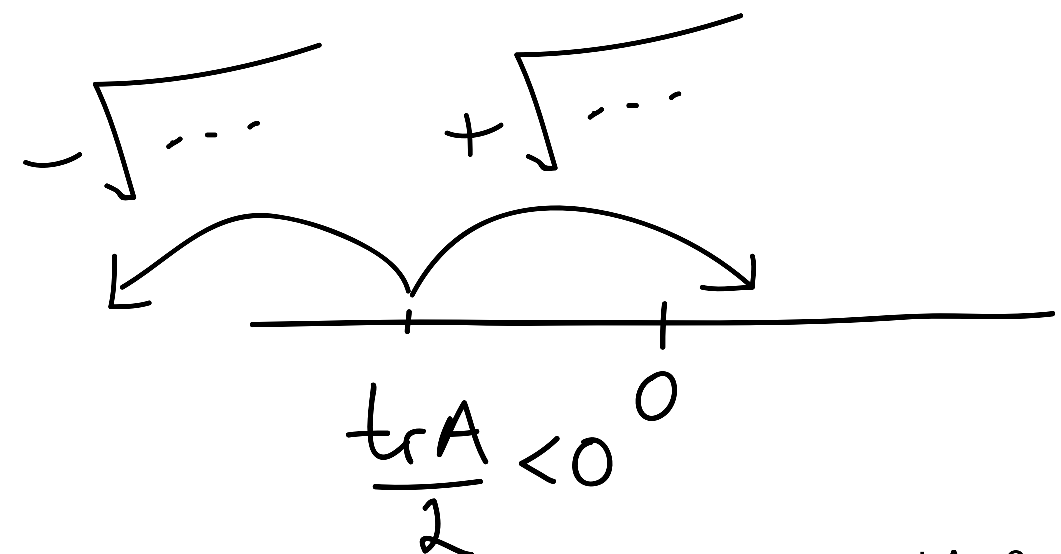

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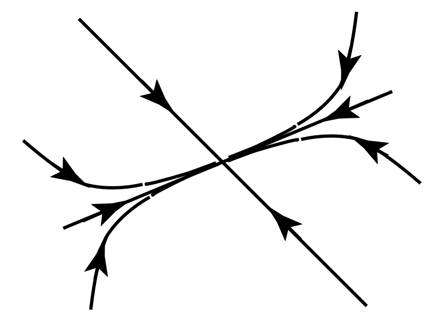

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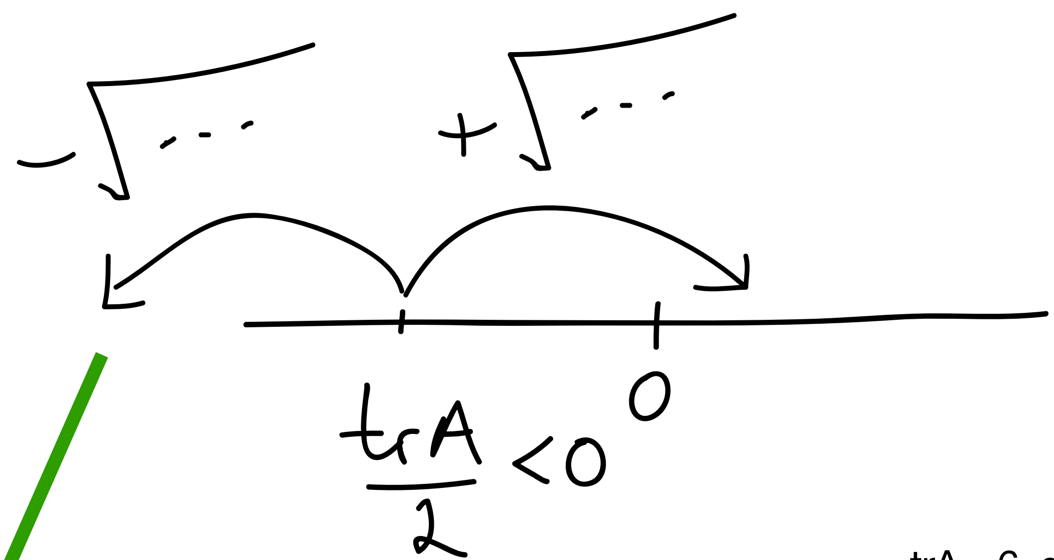

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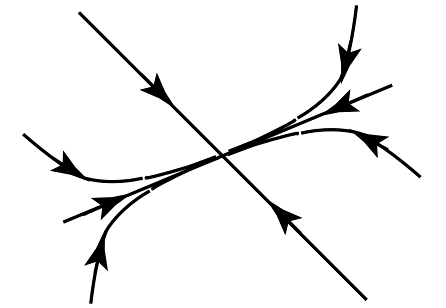

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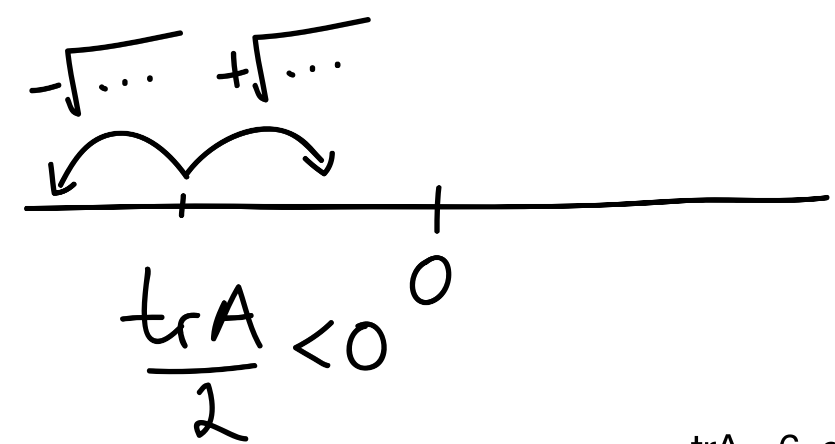

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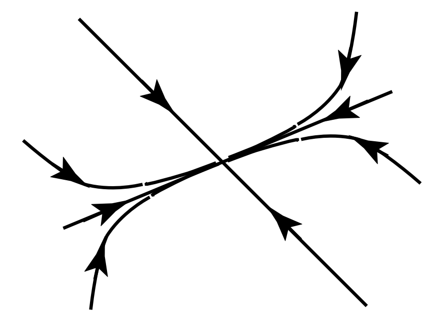

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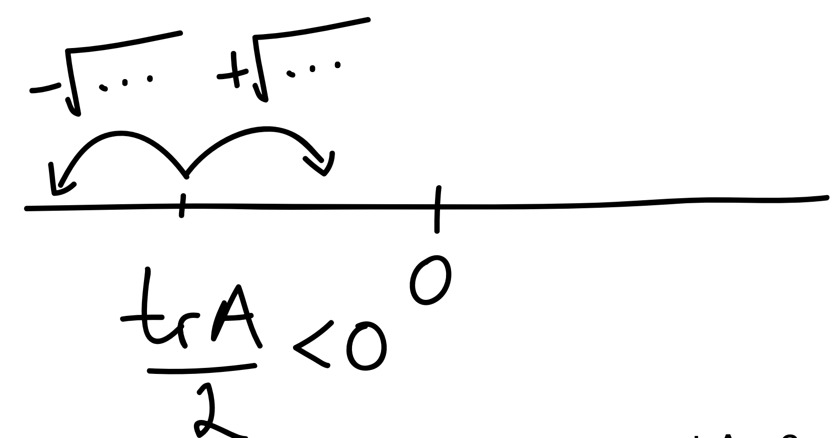

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Using the trace/determinant plane to classify systems

- Classify the steady state of the equation x'=Ax.

$$A = \begin{pmatrix} 1 & 1 \\ -6 & -4 \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 1 \\ -6 & -4 \end{pmatrix}$$

$\mathrm{tr}(A) =$ -3 so some solutions decay.

$\det(A) =$ 2 > 0 so not a saddle.

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When given numbers, just find e-values but with parameters, need a way to derive conditions.

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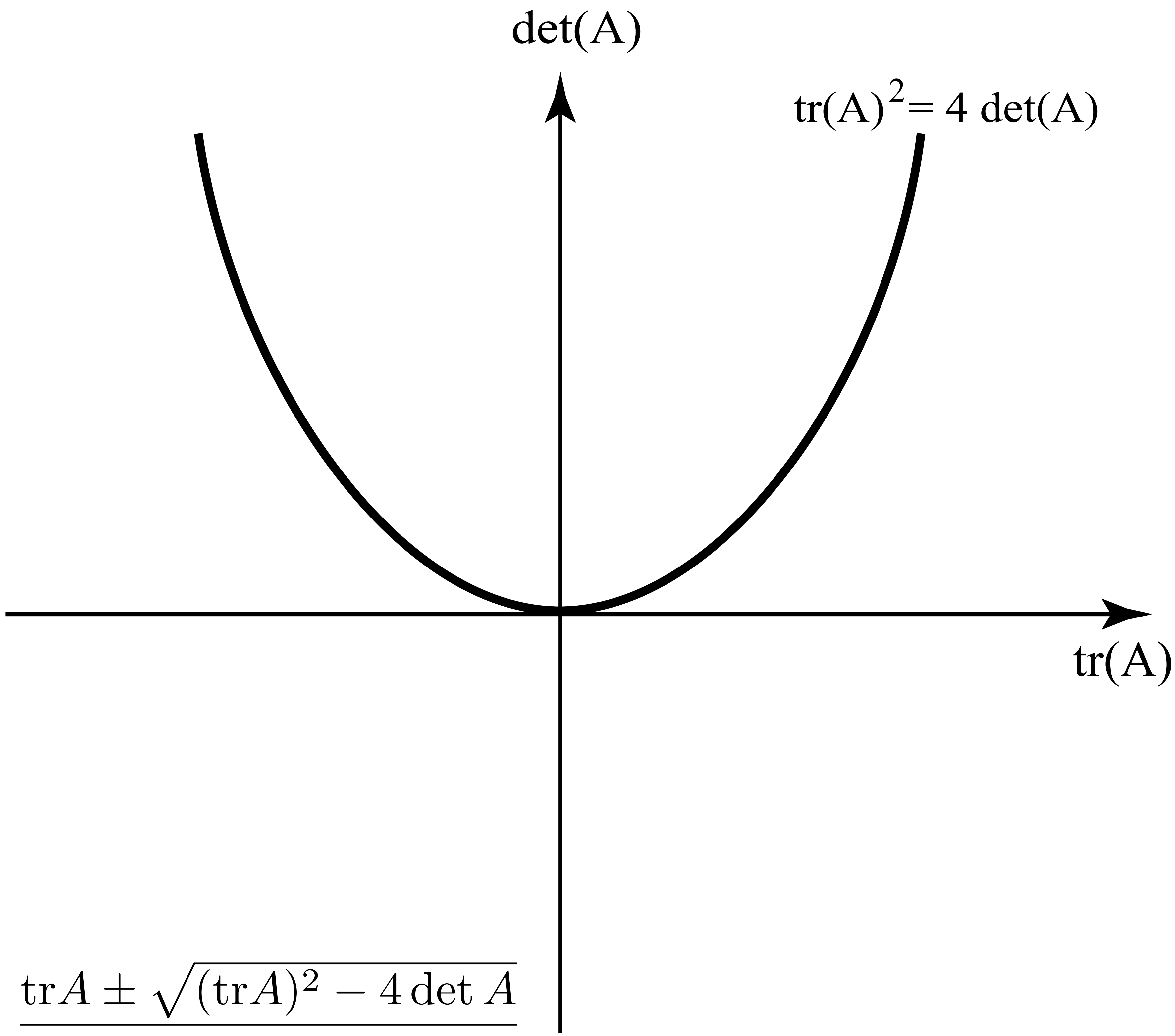
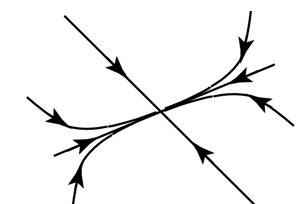
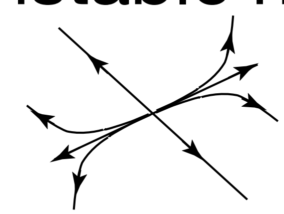
Summary - homogeneous 2x2 systems



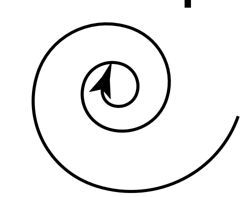
(A) stable node



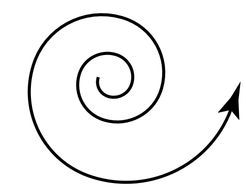
(B) unstable node



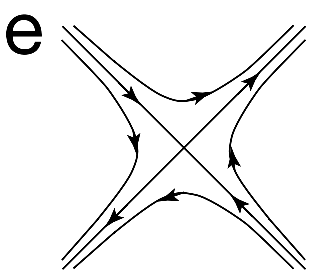
(C) stable spiral



(D) unstable spiral



(E) saddle



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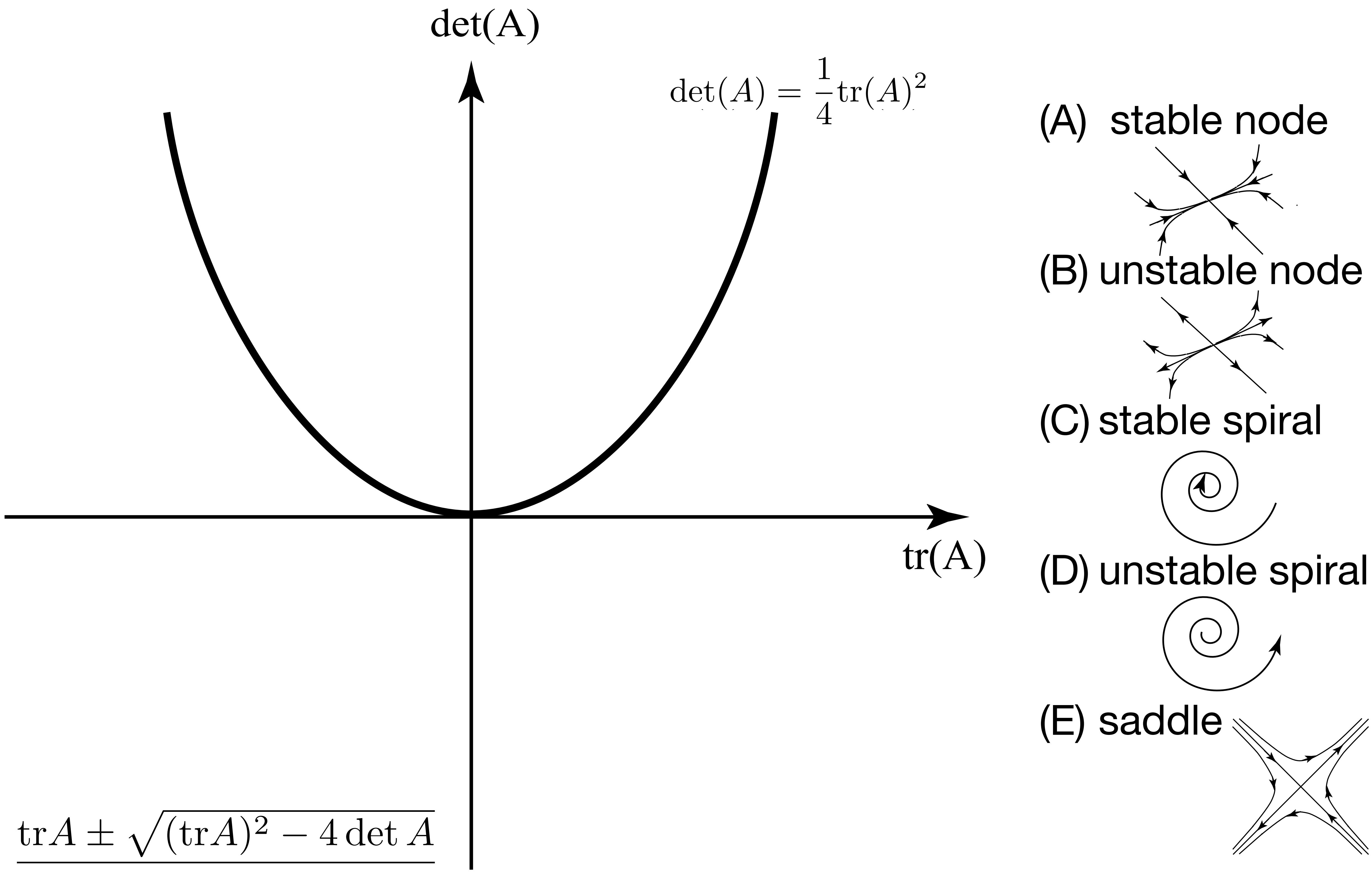

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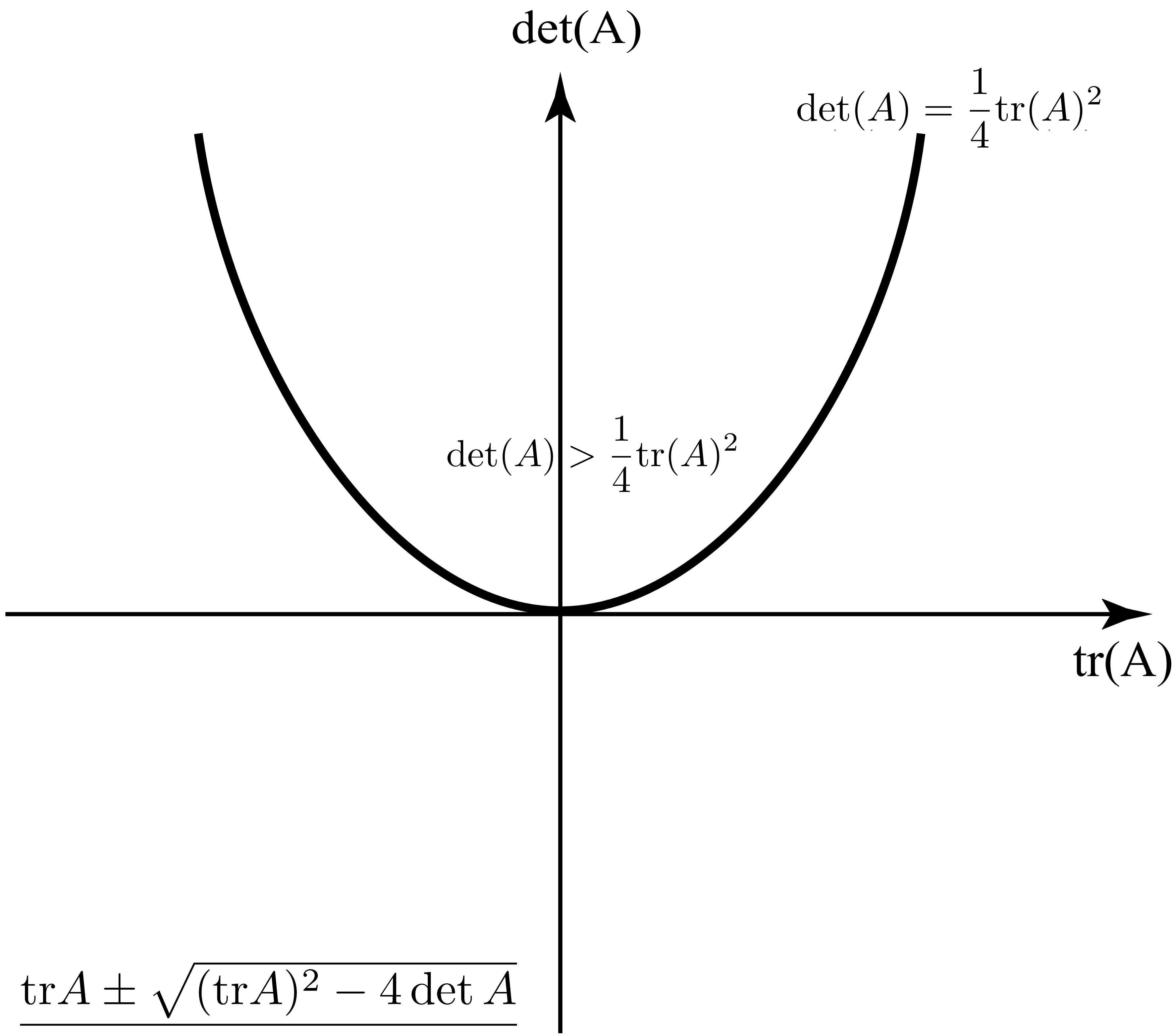

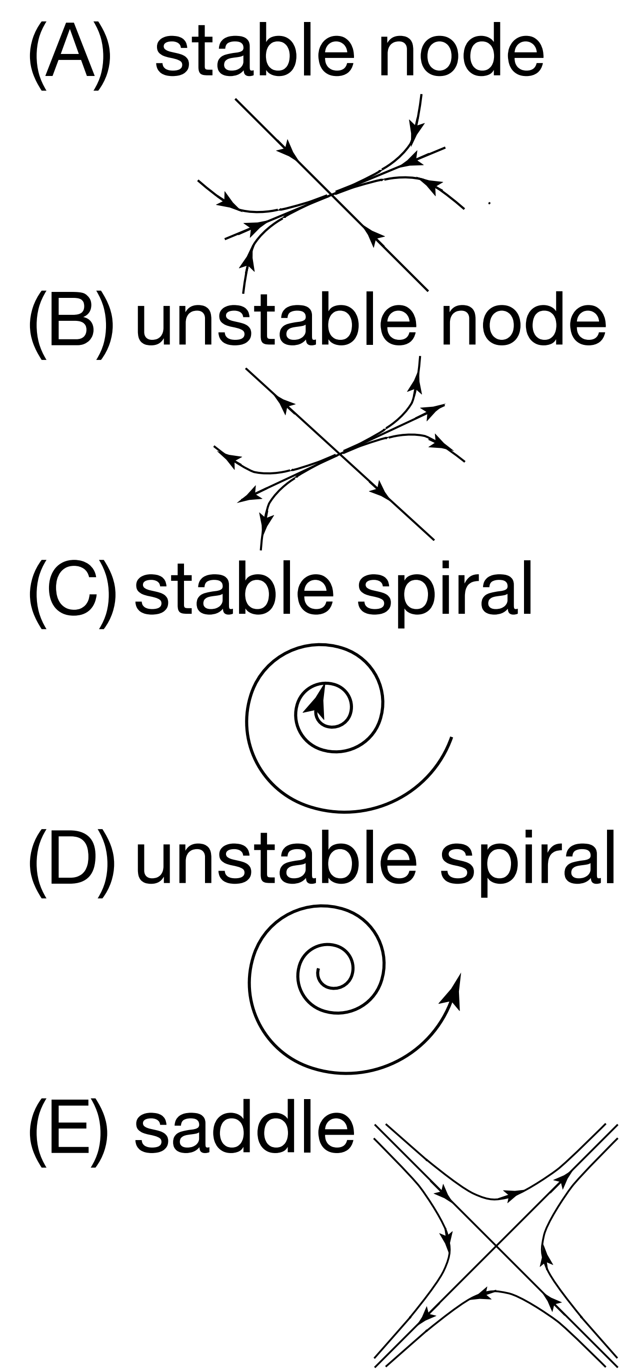

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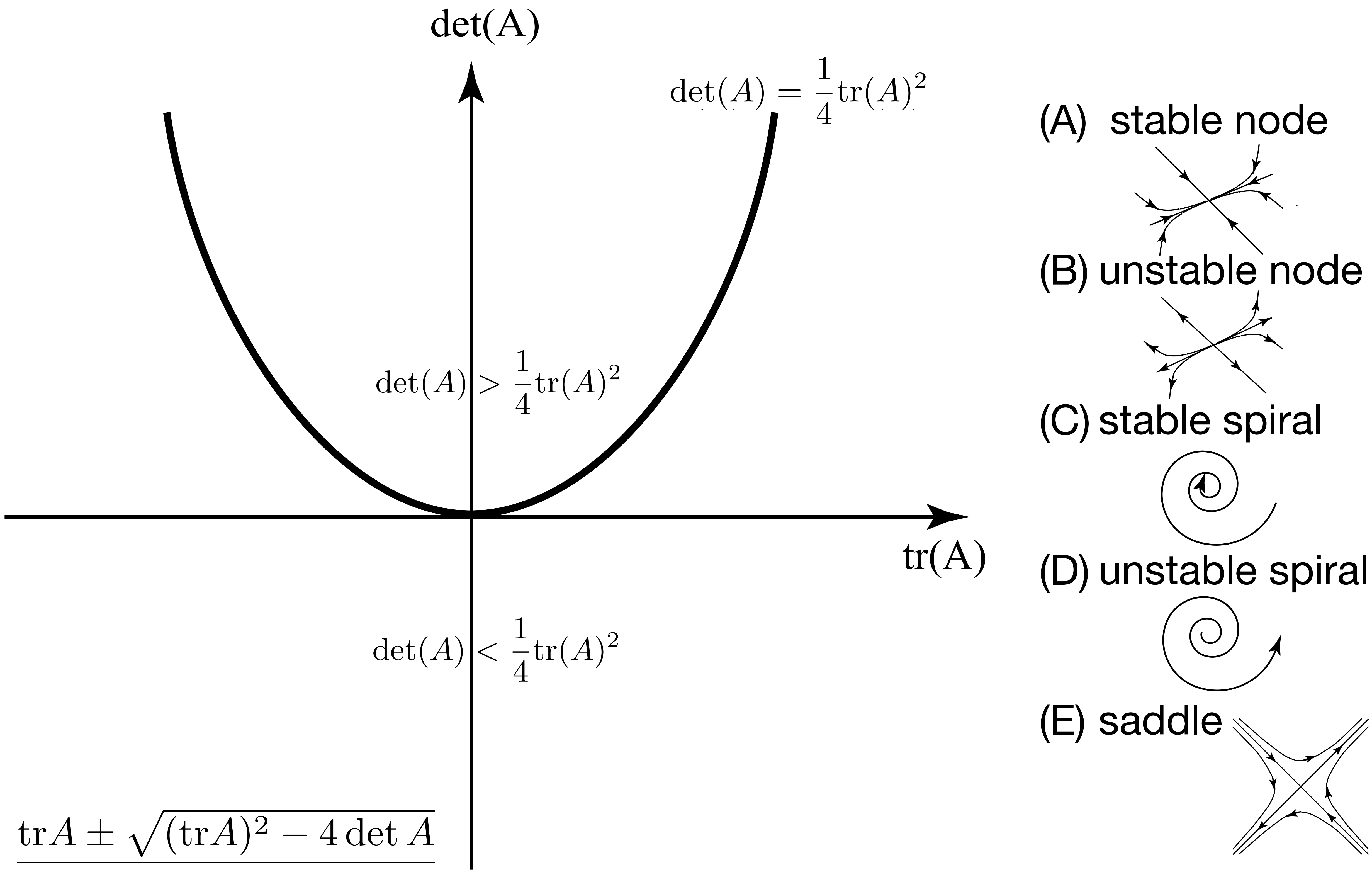

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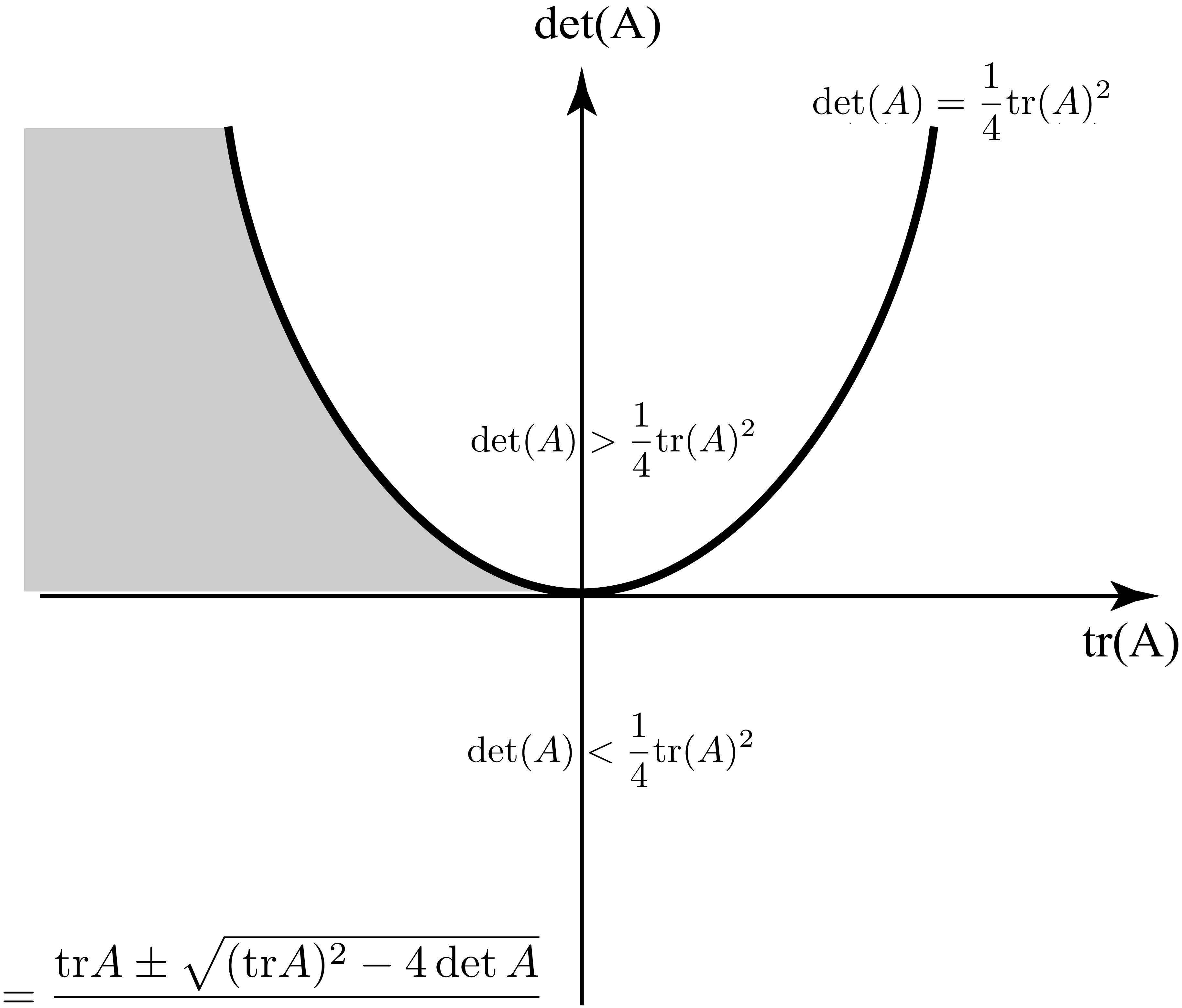

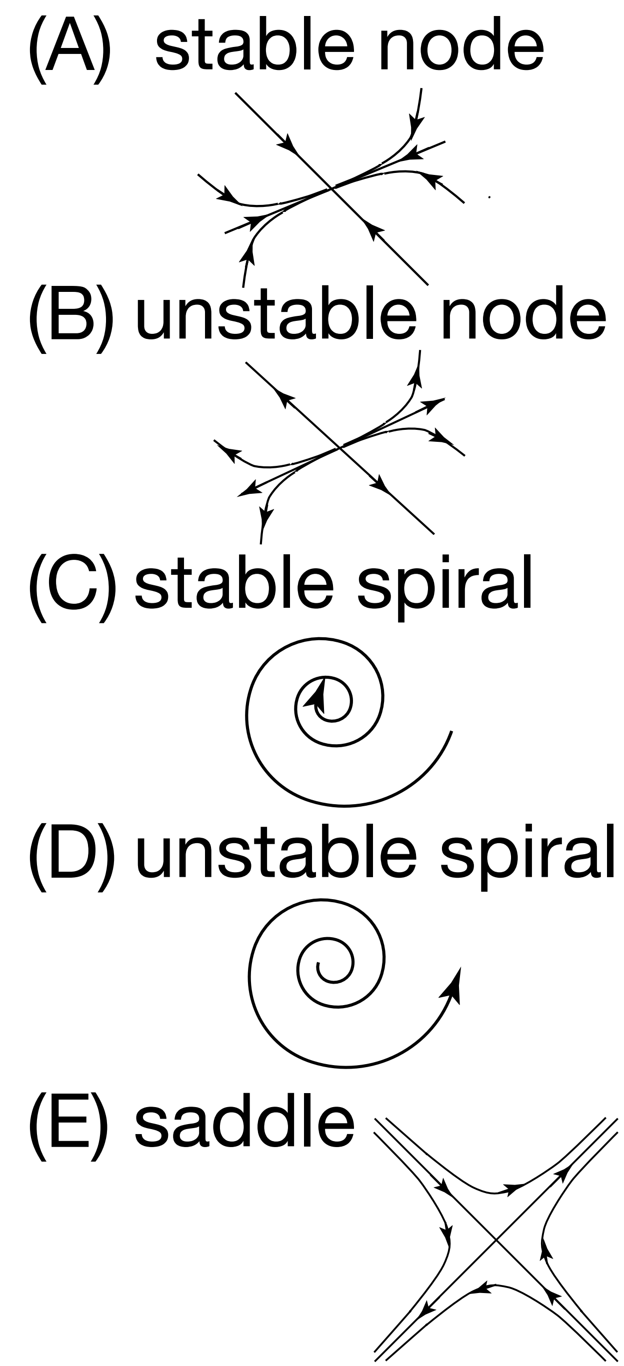

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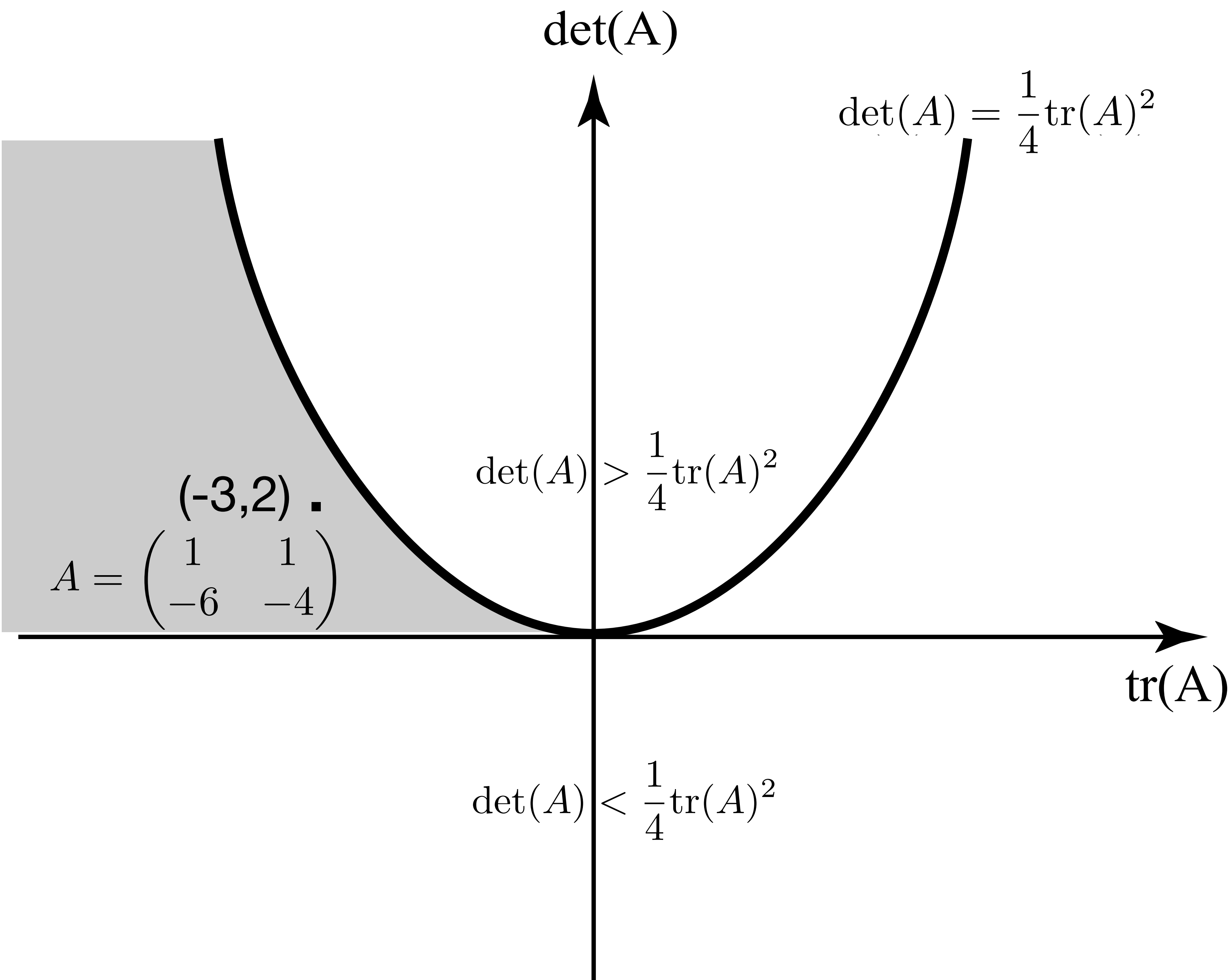

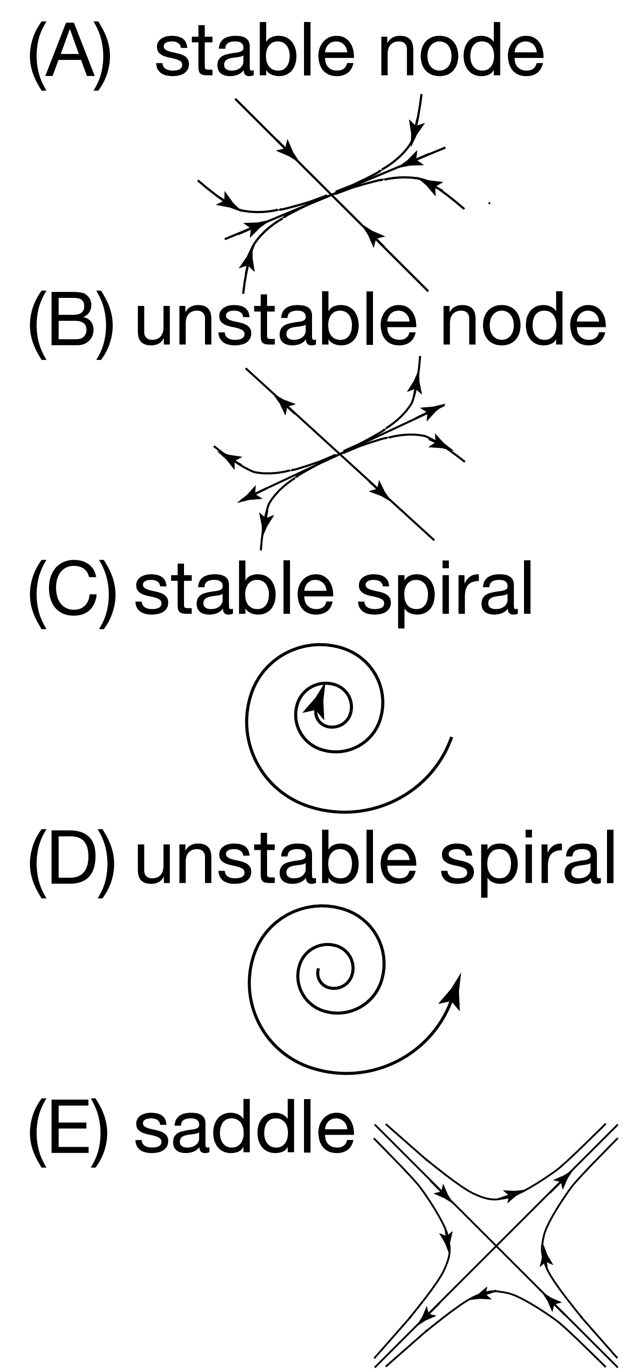

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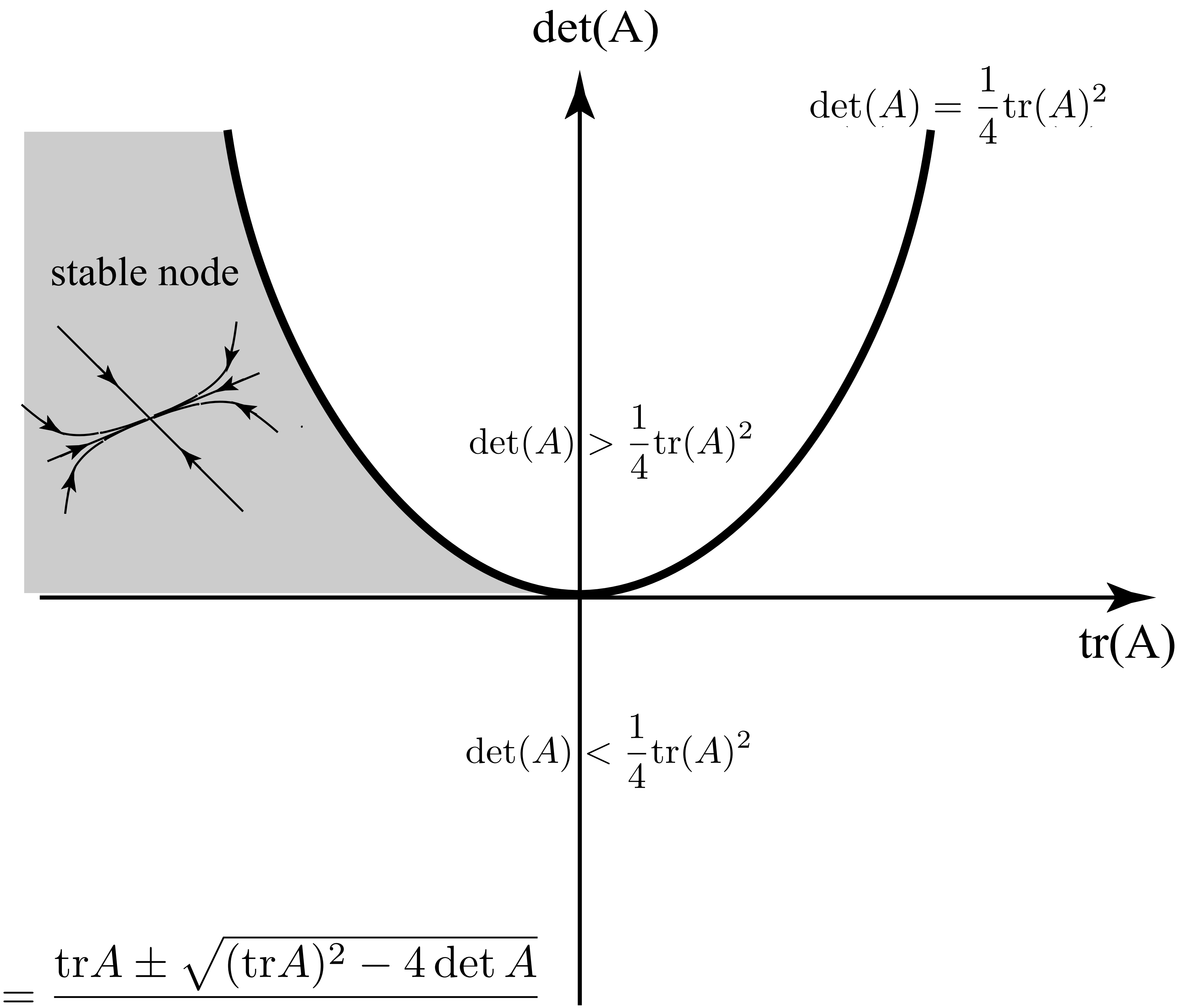

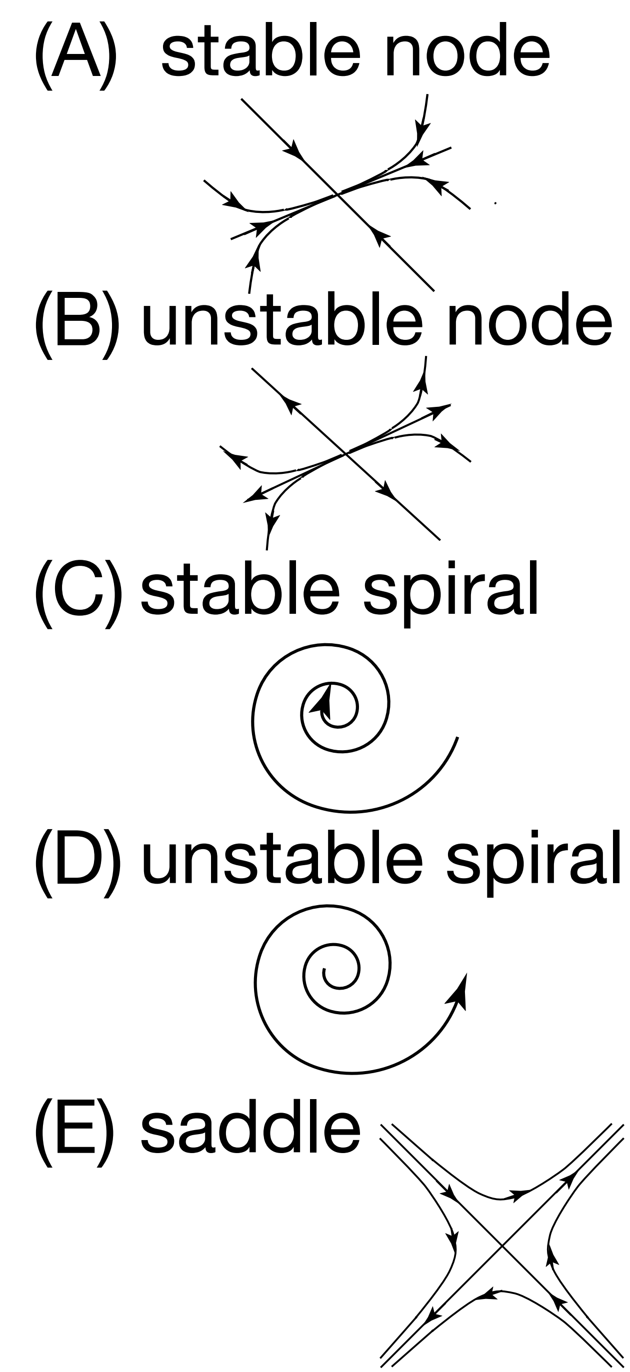

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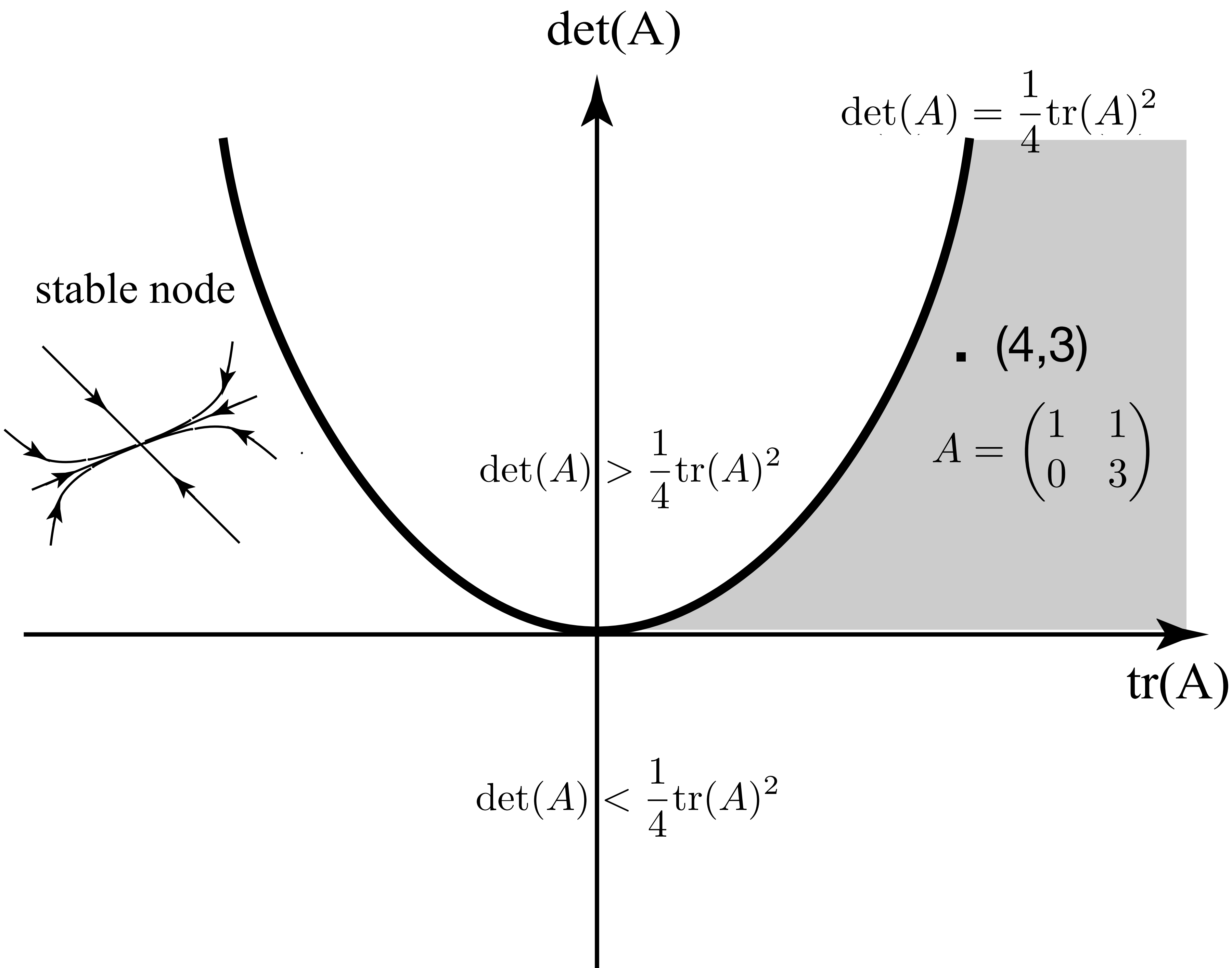

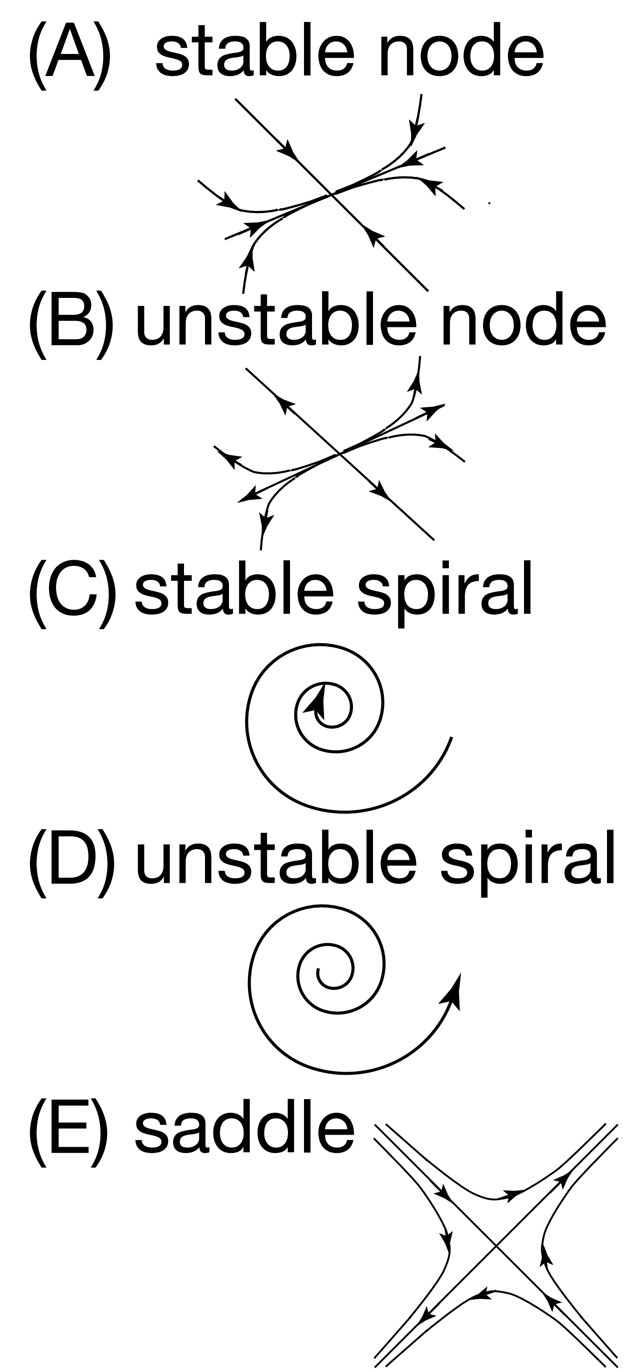

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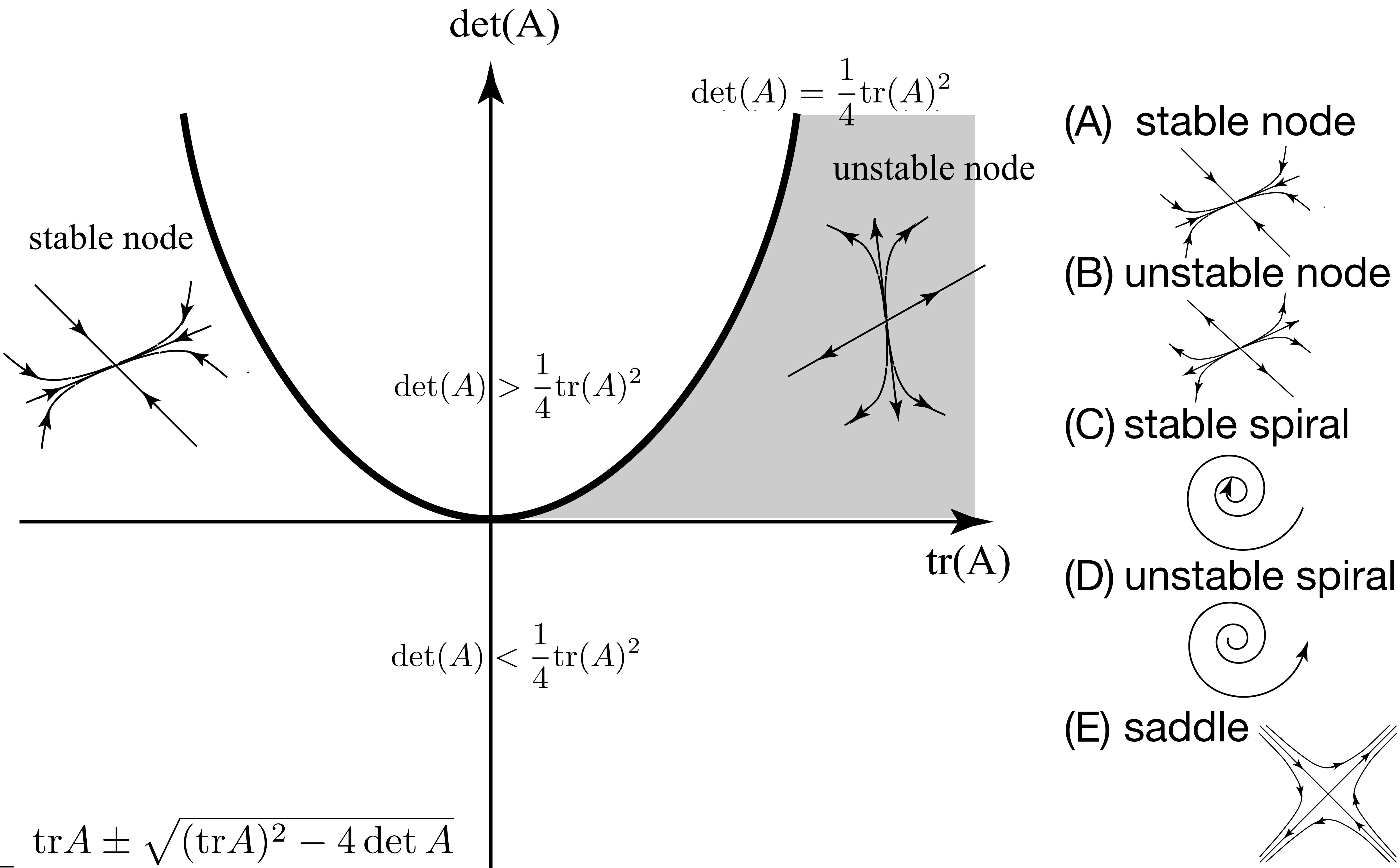

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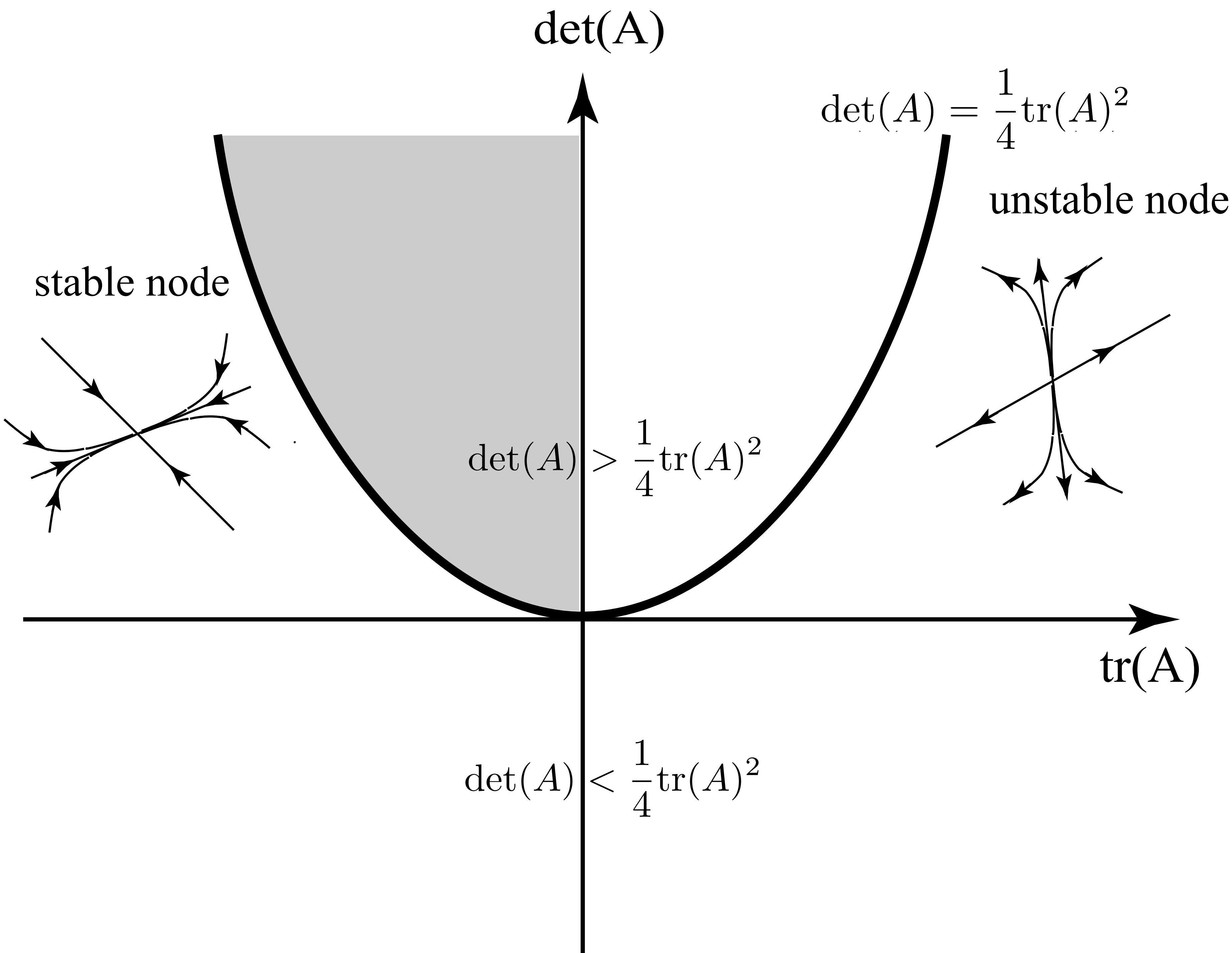

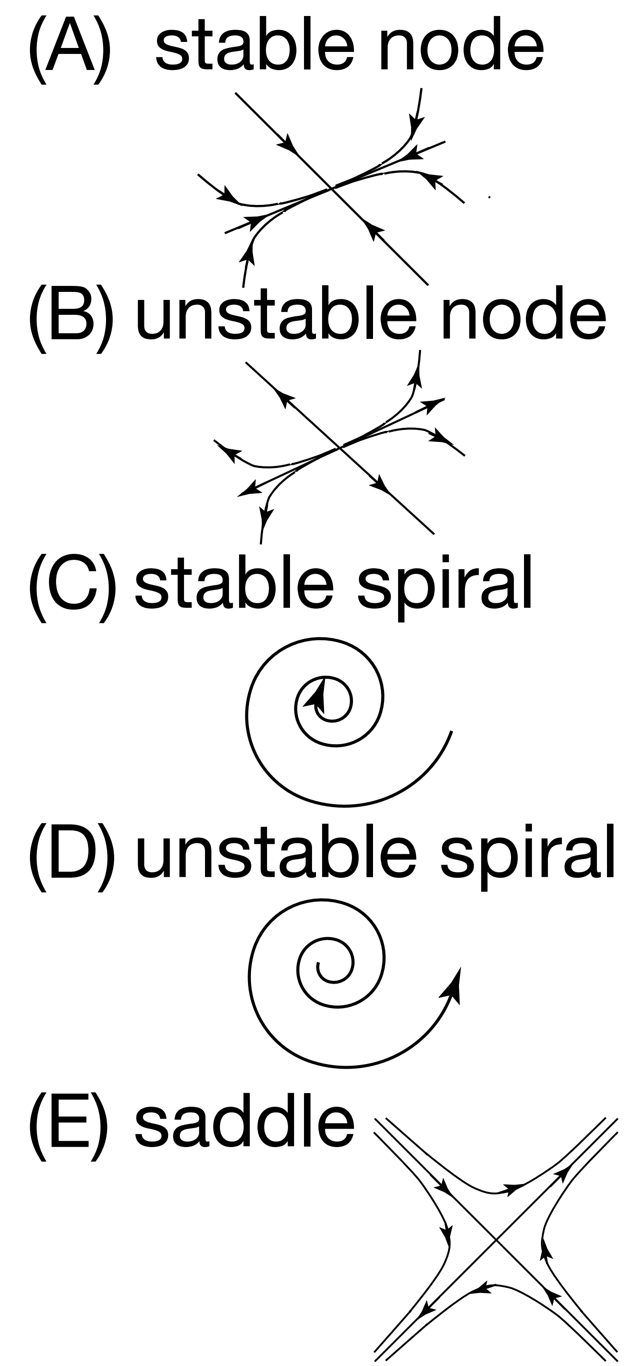

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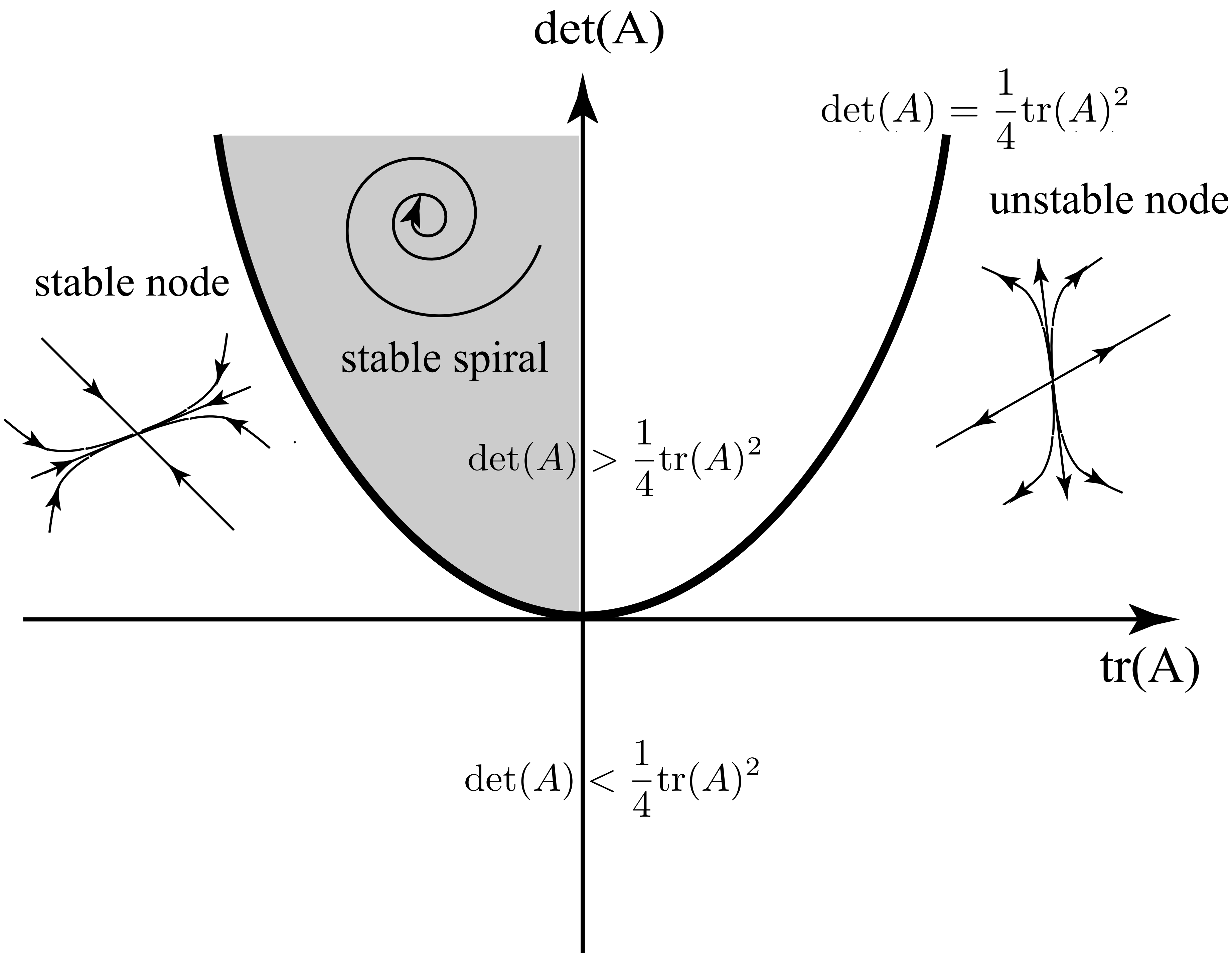
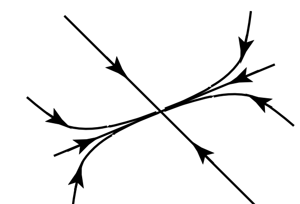


(A) stable node



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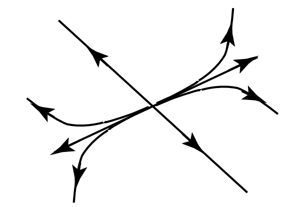
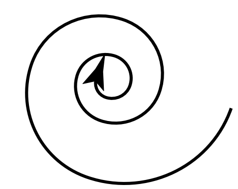
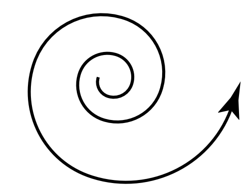


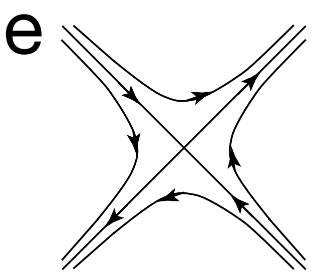
(C) stable spiral



(D) unstable spiral



(E) saddle



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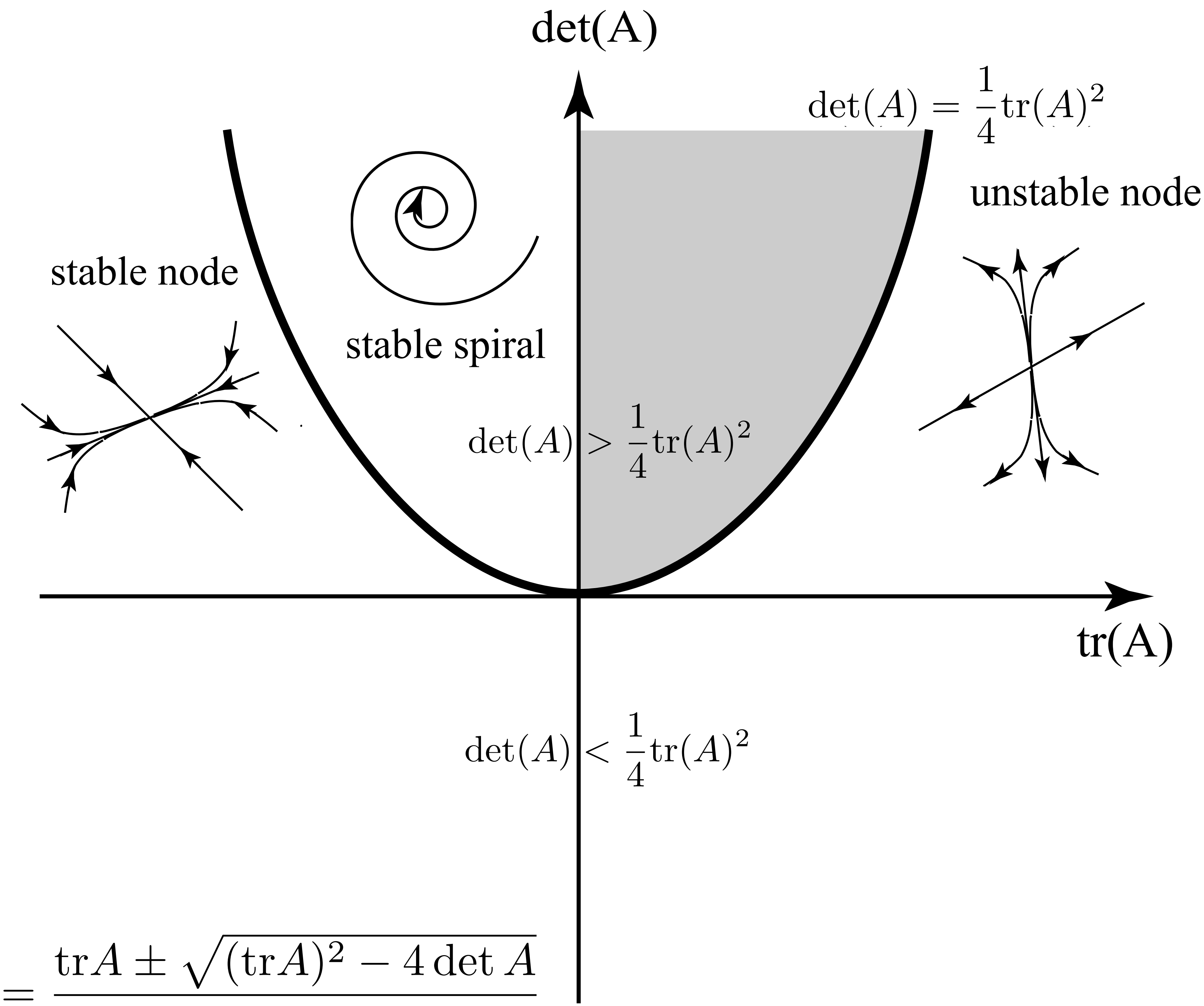

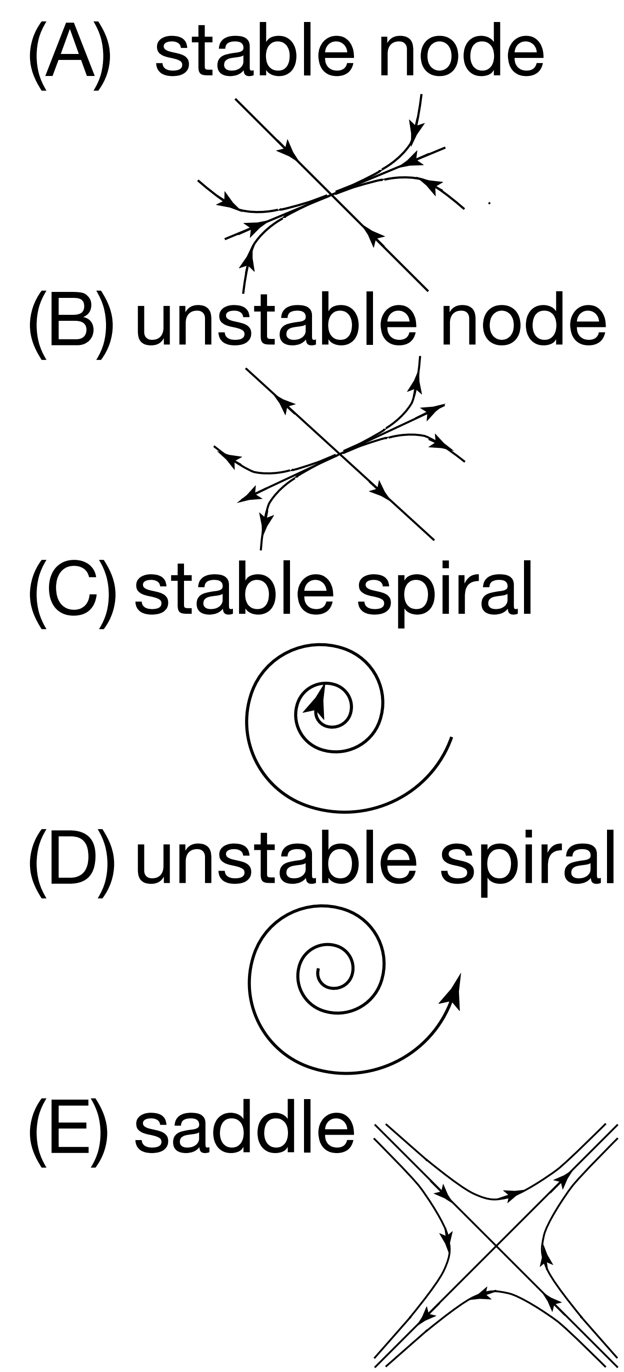

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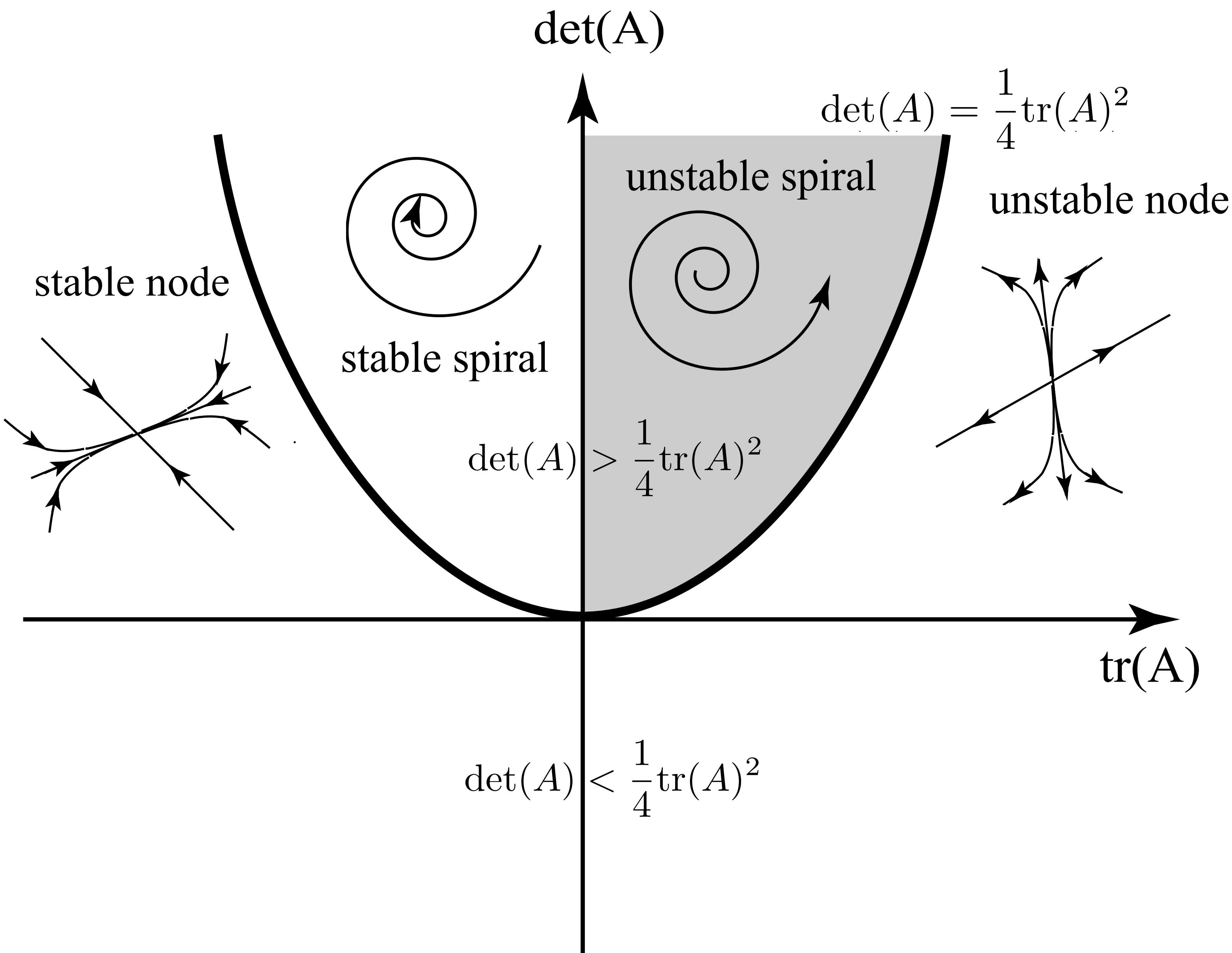

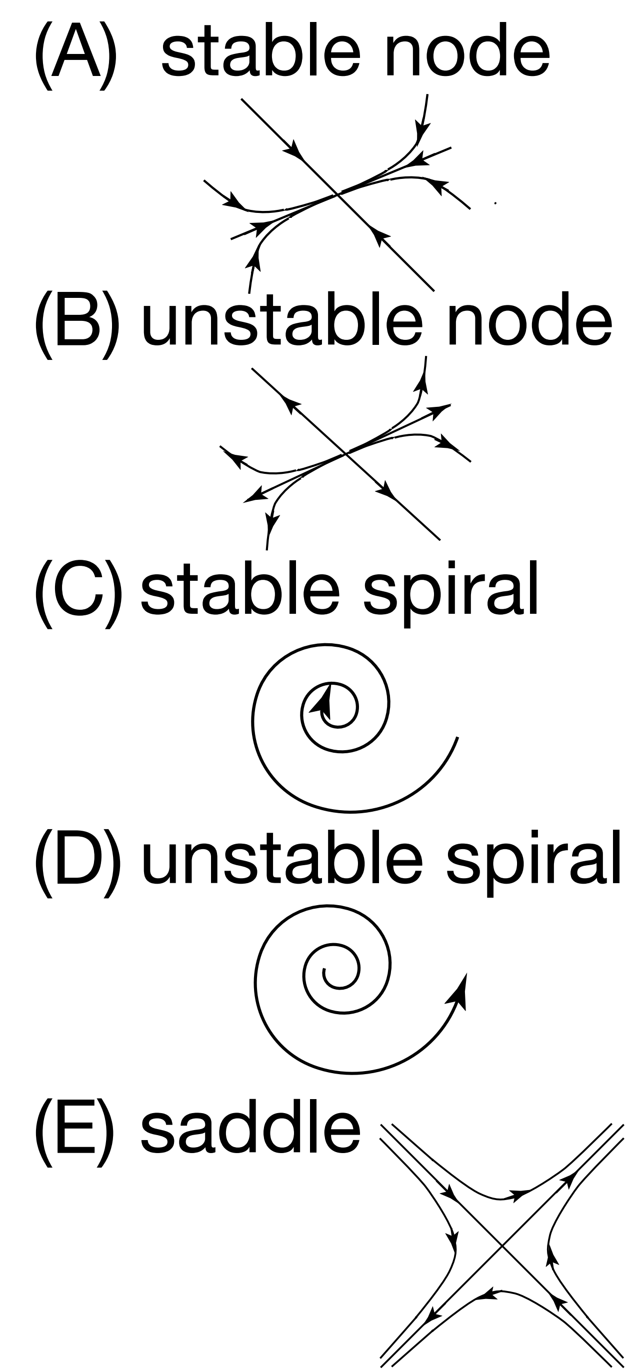

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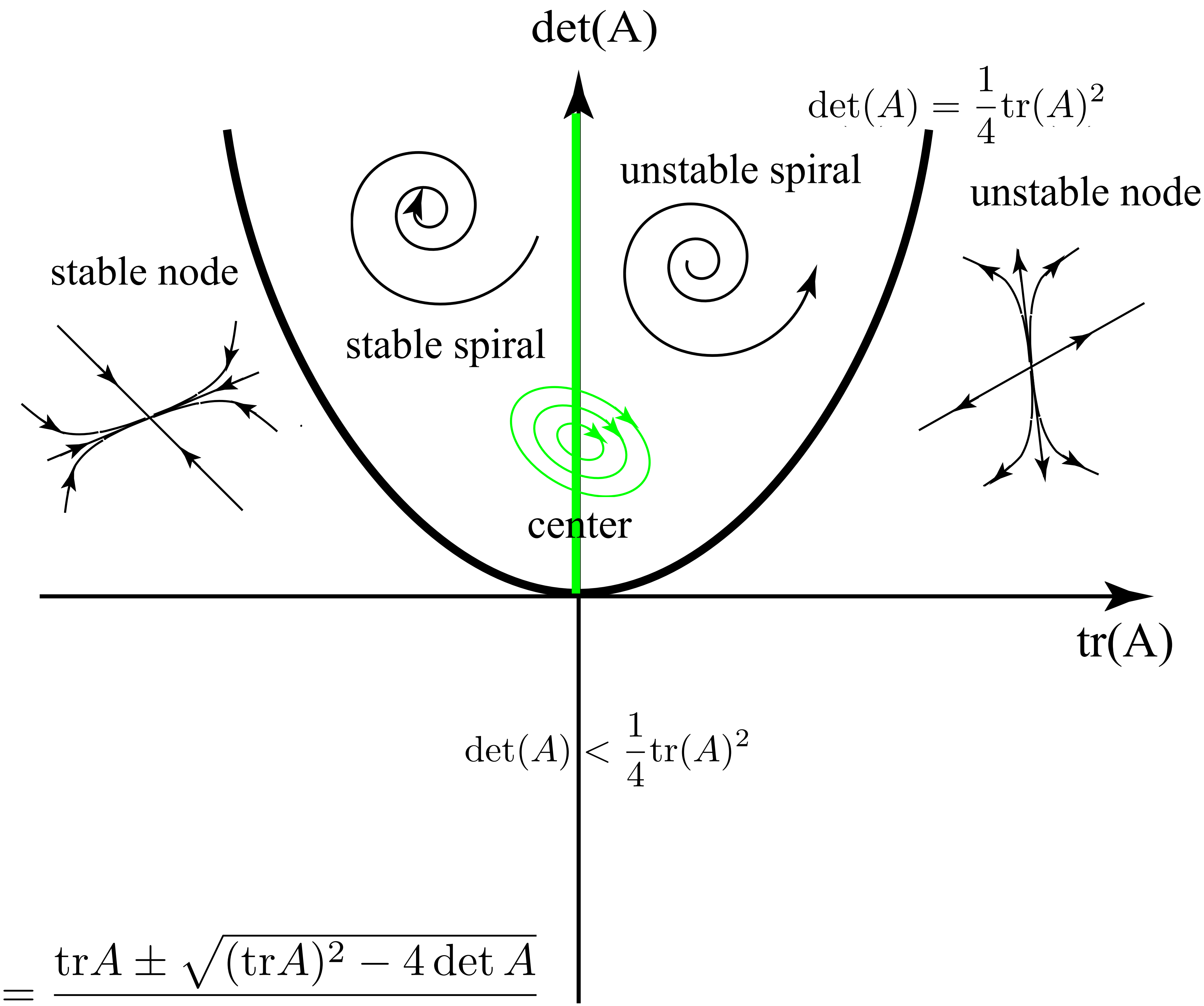

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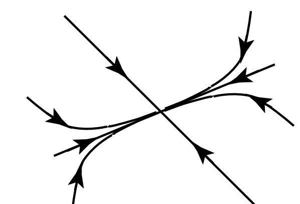
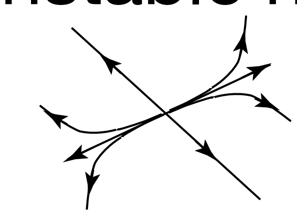
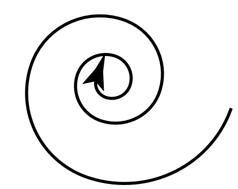


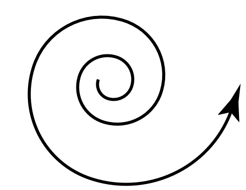
(B) unstable node



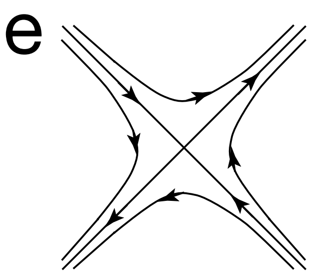
(C) stable spiral



(D) unstable spiral

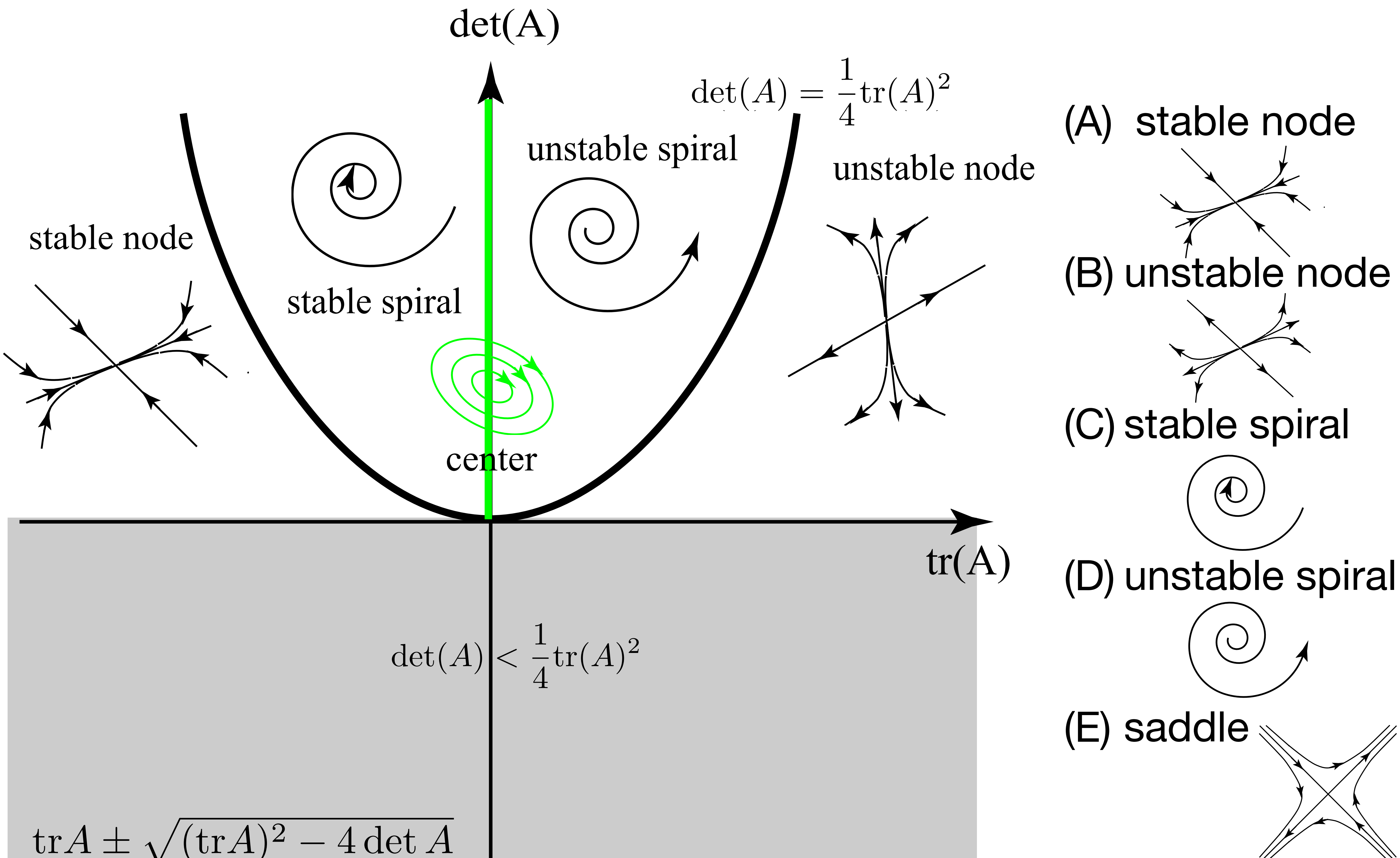


(E) saddle



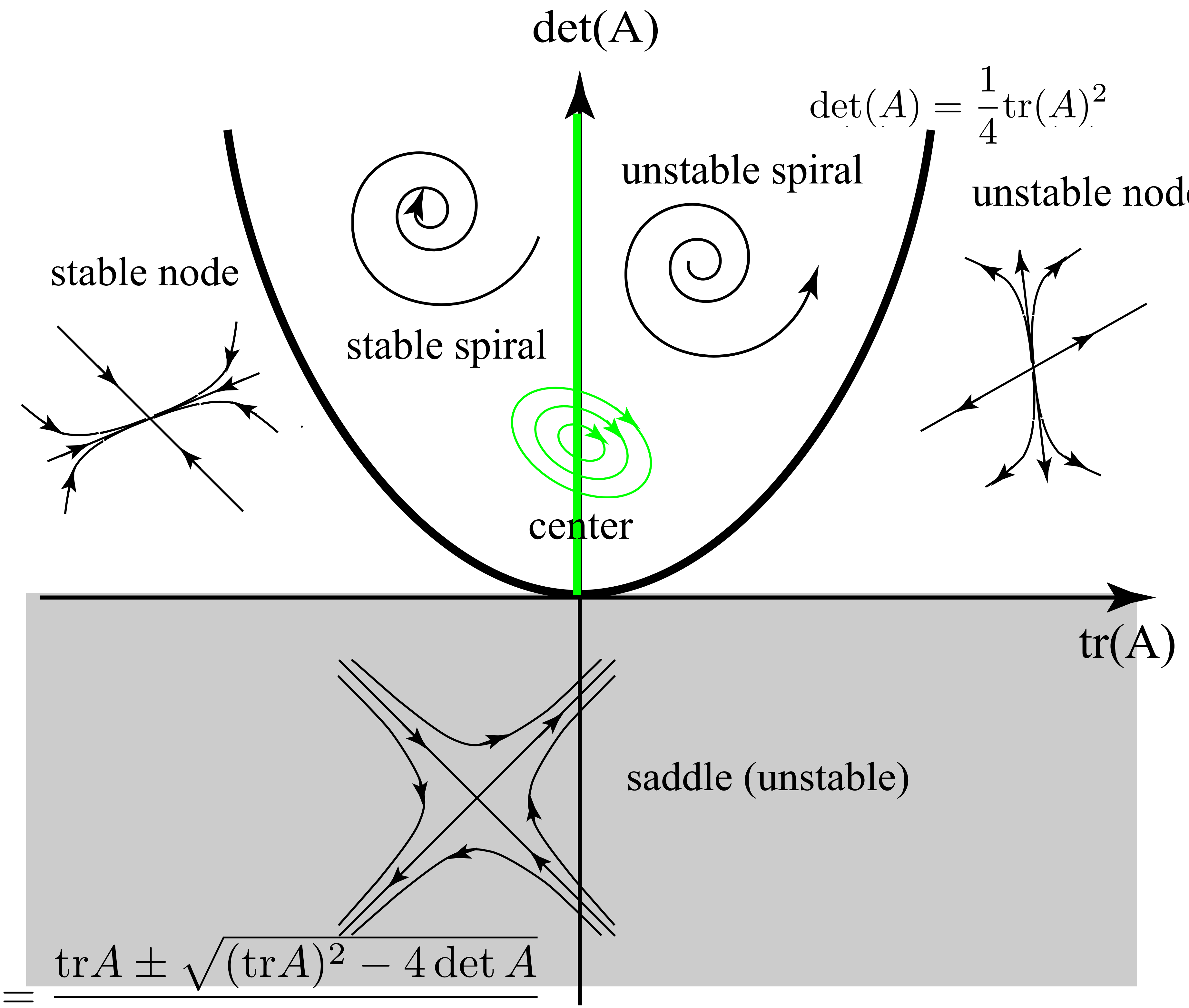
$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

Summary - homogeneous 2x2 systems

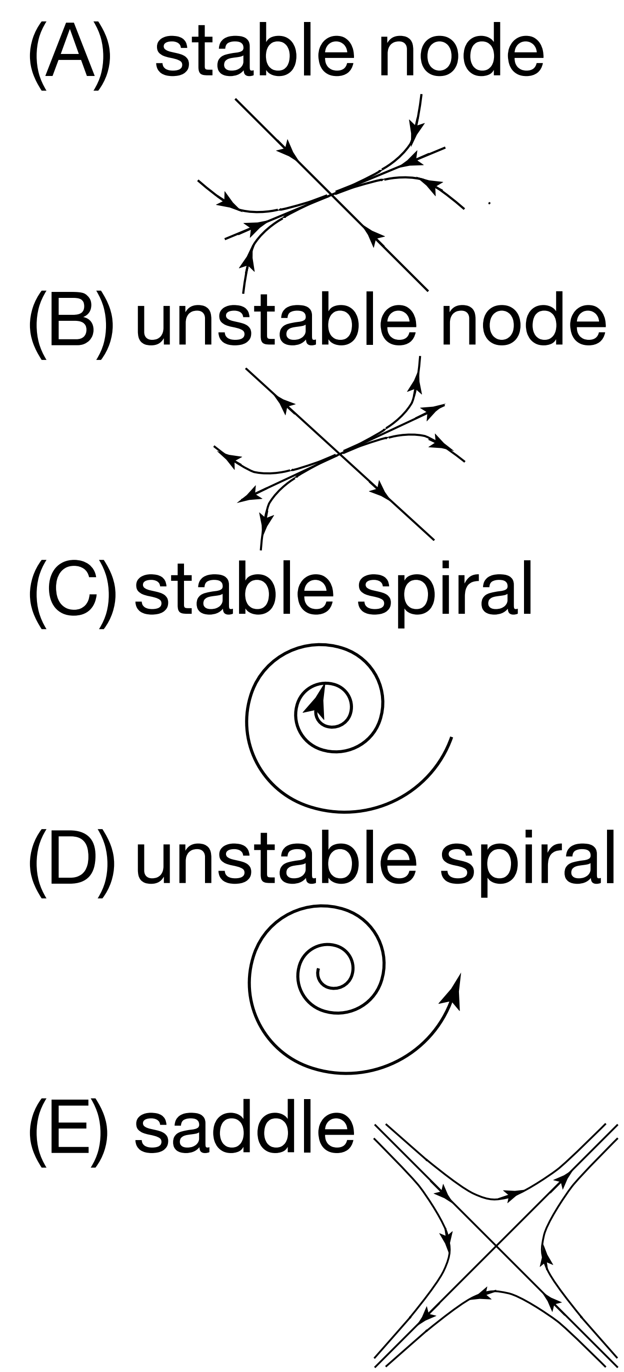


$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

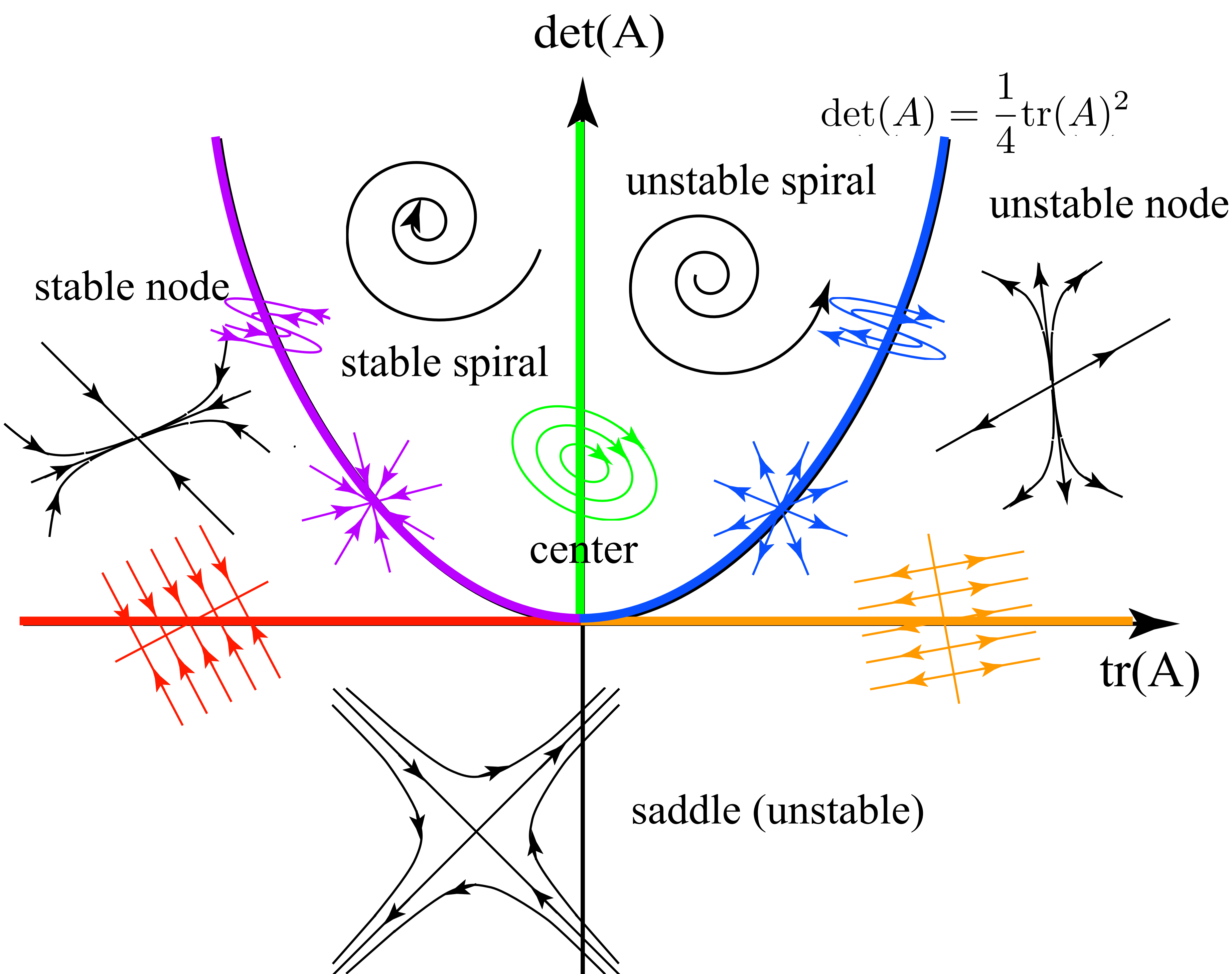
Summary - homogeneous 2x2 systems



$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

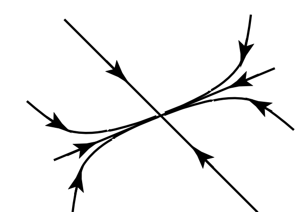


Summary - homogeneous 2x2 systems

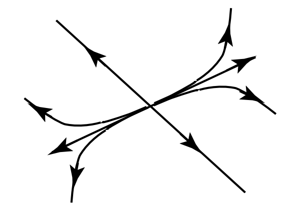


$$\lambda = \frac{\mathrm{tr}A \pm \sqrt{(\mathrm{tr}A)^2 - 4\det A}}{2}$$

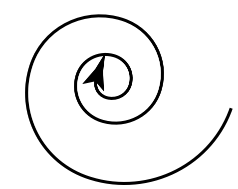
(A) stable node



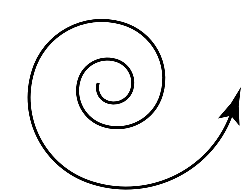
(B) unstable node



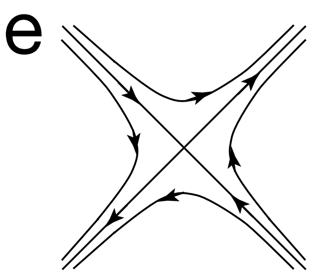
(C) stable spiral



(D) unstable spiral

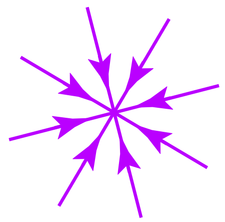


(E) saddle

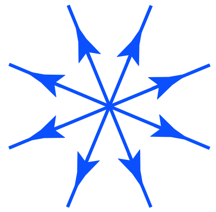


Summary - homogeneous 2x2 systems

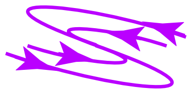
Repeated evalue cases:



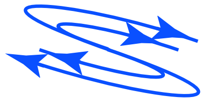
$\lambda<0$, two indep. evectors.



$\lambda>0$, two indep. evectors.

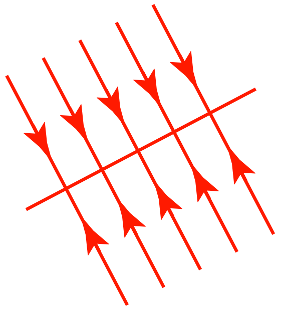


$\lambda<0$, only one evector.

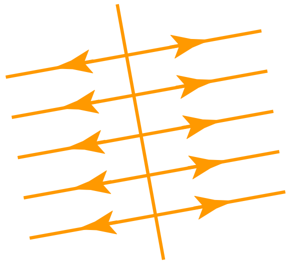


$\lambda>0$, only one evector.

One zero evalue (singular matrix):



$\lambda_1=0$, $\lambda_2<0$,



$\lambda_1=0$, $\lambda_2>0$,