Today

- Intro to Laplace transforms
- A bunch of examples
- Solving ODEs (that we already know how to solve) using Laplace transforms

Laplace transforms - intro (6.1)

- Motivation for Laplace transforms:

Include physics example
(LRC with on/off switch)

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  - We know how to solve $ay'' + by' + cy = g(t)$ when $g(t)$ is polynomial, exponential, trig.

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  - In applications, $g(t)$ is often "piece-wise continuous" meaning that it consists of a finite number of pieces with jump discontinuities in between. For example,

$$g(t) = \begin{cases} \sin(\omega t) & 0 < t < 10, \\ 0 & t \geq 10. \end{cases}$$

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  - These can be handled by previous techniques (modified) but it isn't pretty (solve from t=0 to t=10, use y(10) as the IC for a new problem starting at t=10).

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\begin{array}{c}\text{Unknown } y(t) \text{ that}\\ \text{satisfies some ODE}\end{array} \arrow[r, "\text{solve ODE}"] \arrow[d, "\begin{array}{c}\text{Transform } y(t)\\ \text{and the ODE}\end{array}"'] & \text{Found } y(t) \\
\begin{array}{c}\text{Unknown } Y(s) \text{ that}\\ \text{satisfies an algebraic}\\ \text{equation}\end{array} &
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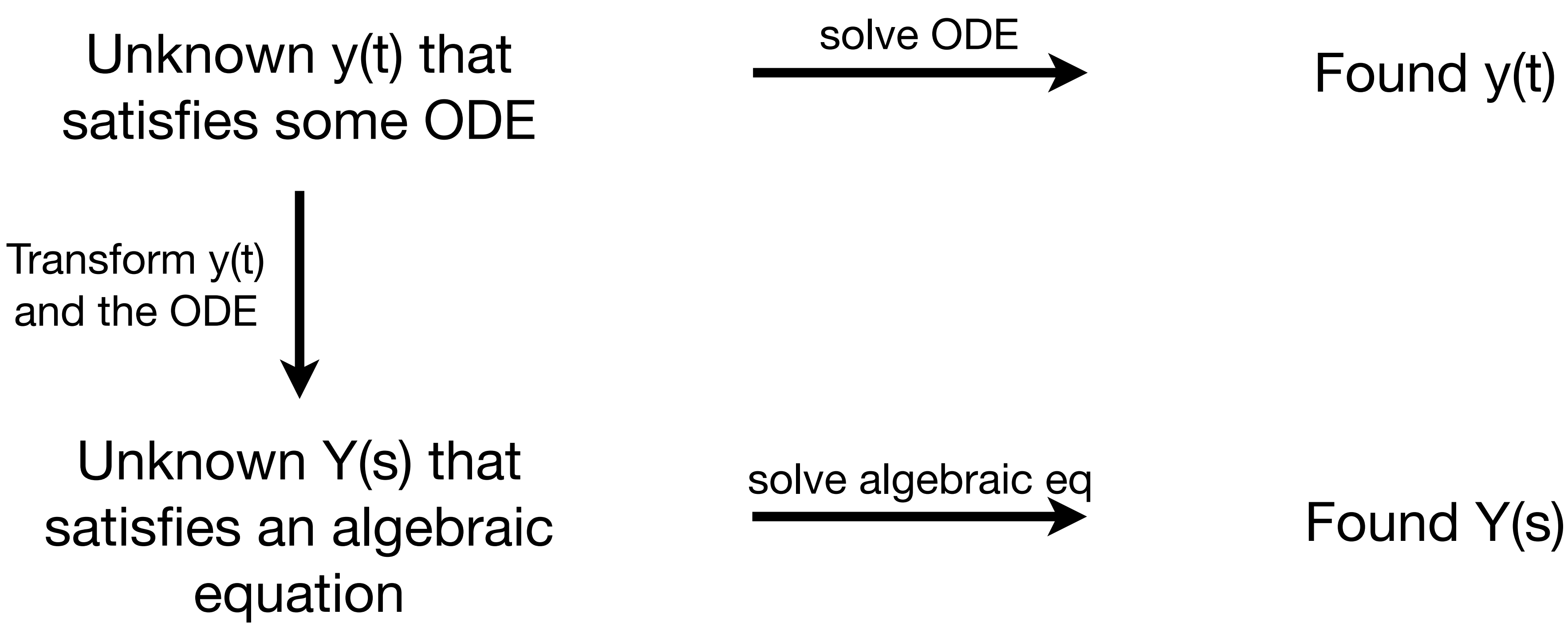

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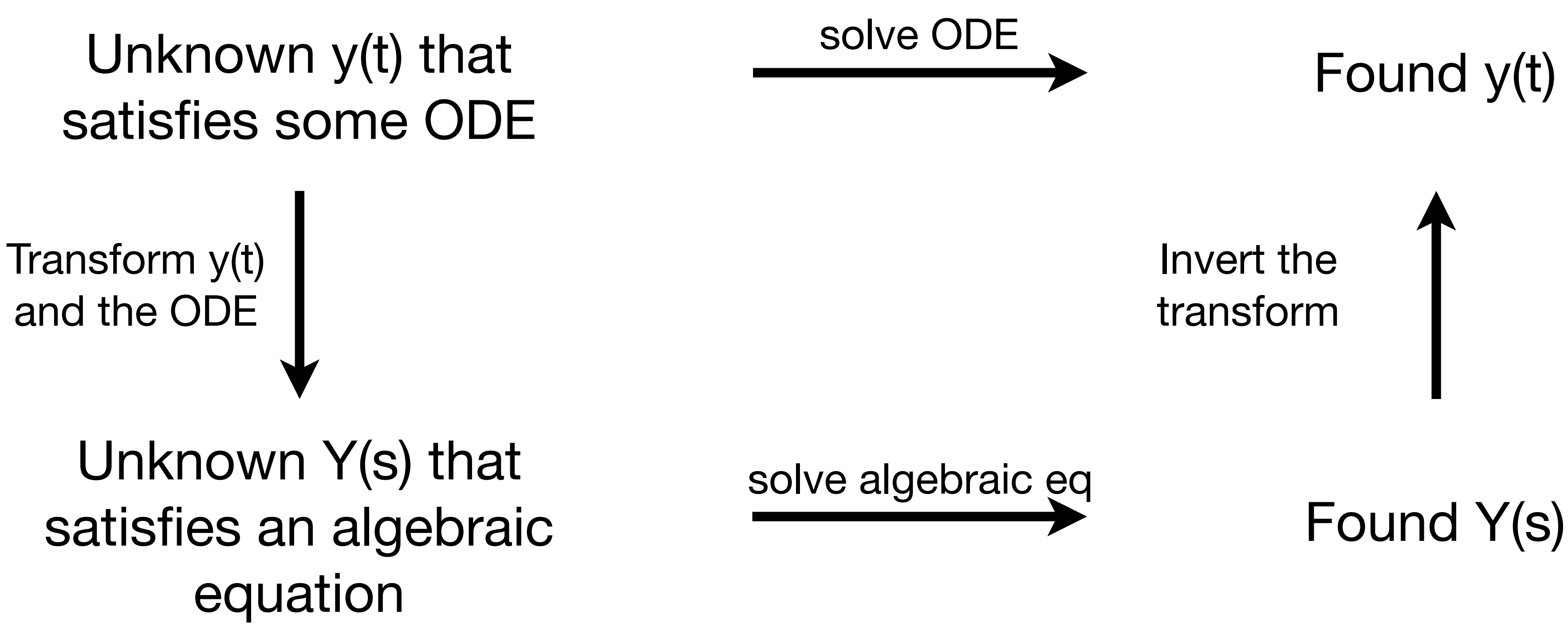

- Laplace transform of y(t): $\mathcal{L}\{y(t)\} = Y(s) = \int_0^\infty e^{-st} y(t)\; dt$

Laplace transforms - examples (6.1)

- What is the Laplace transform of $y(t) = 3$?

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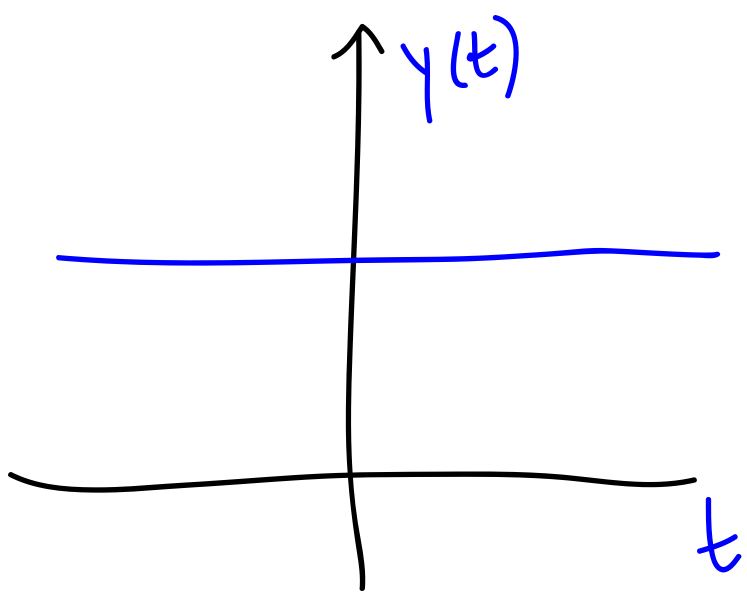

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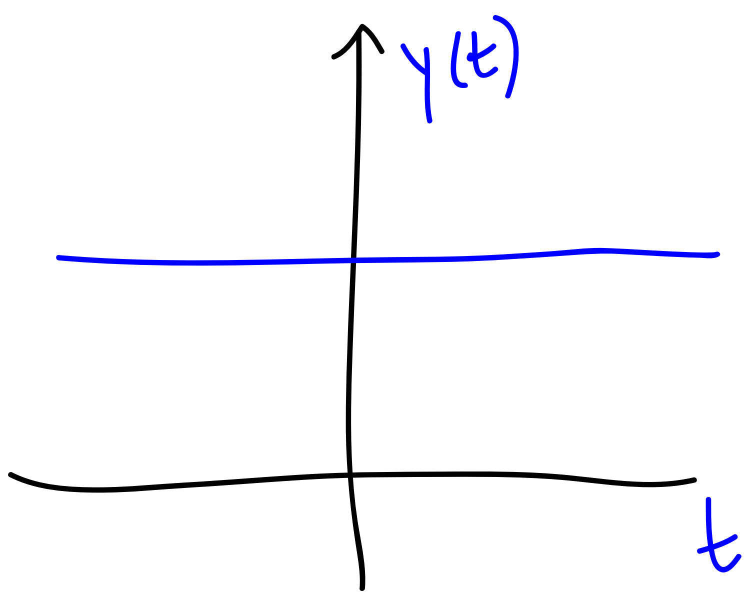

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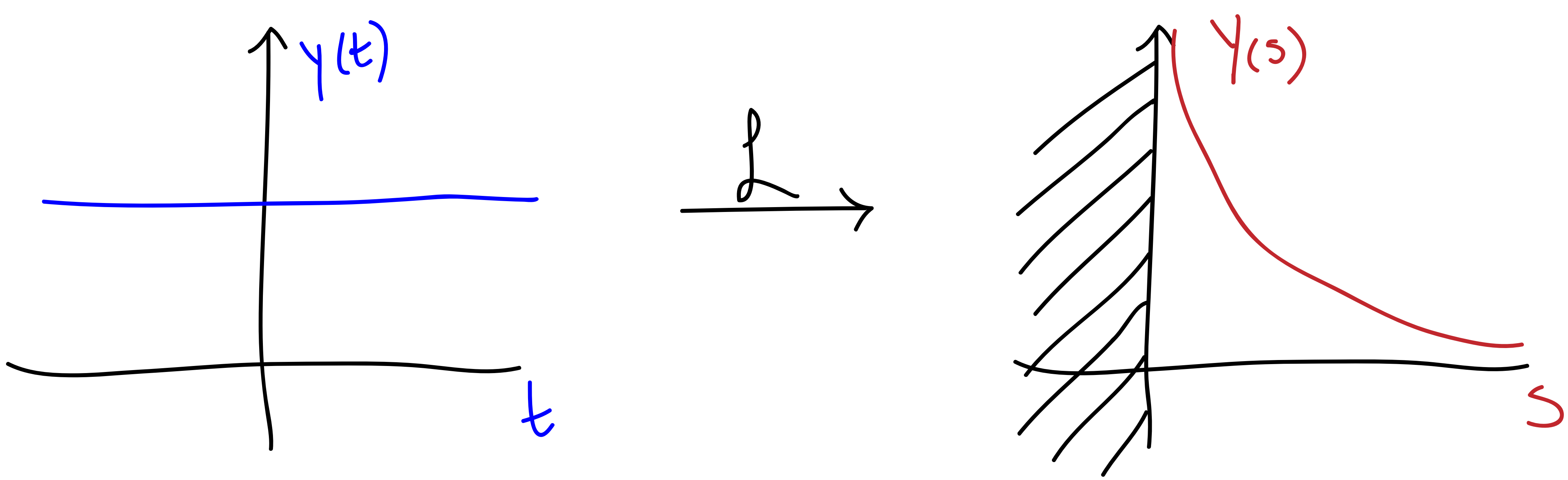

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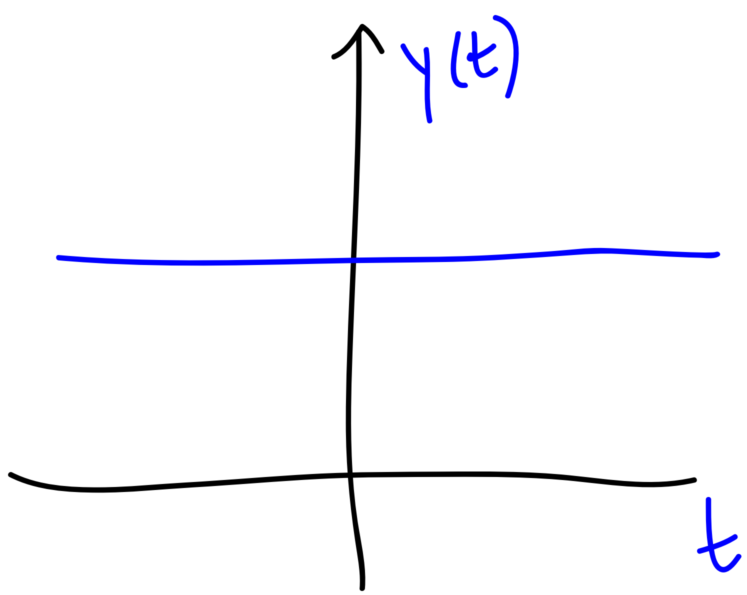

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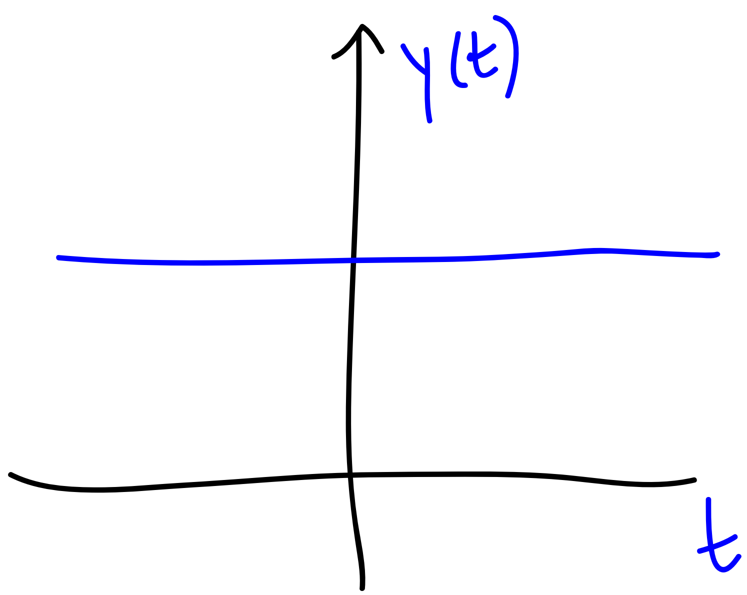

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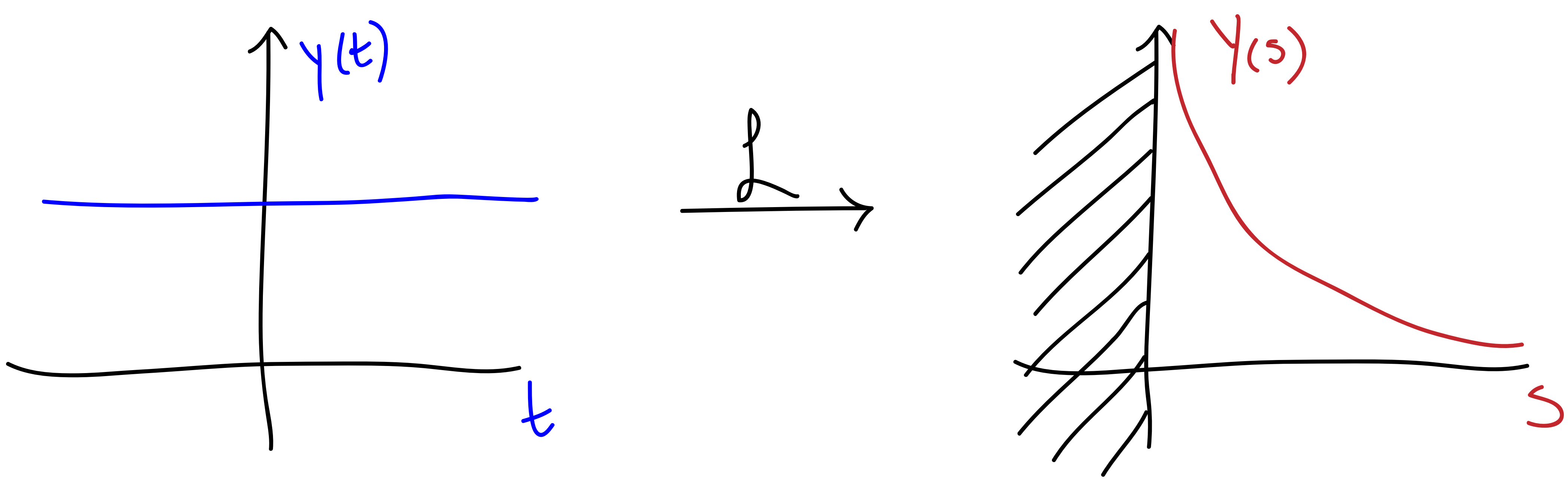

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- What is the Laplace transform of $y(t) = e^{6t}$?

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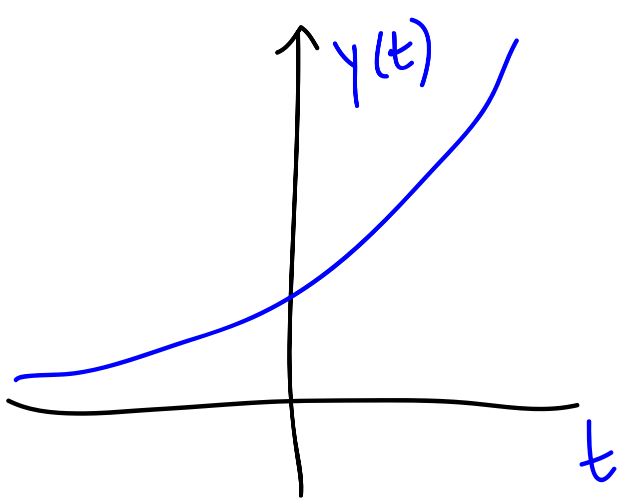

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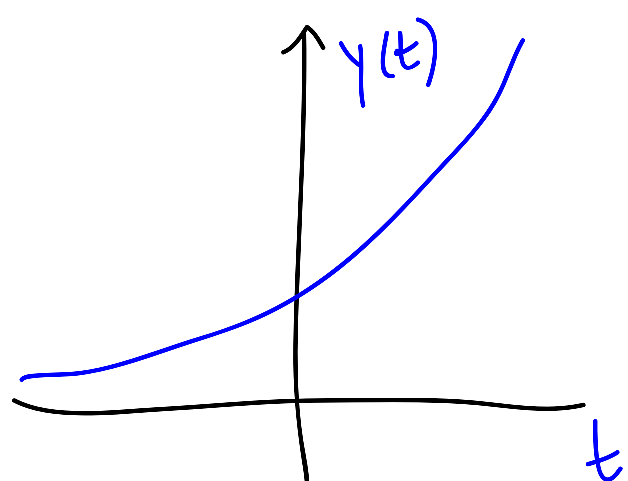

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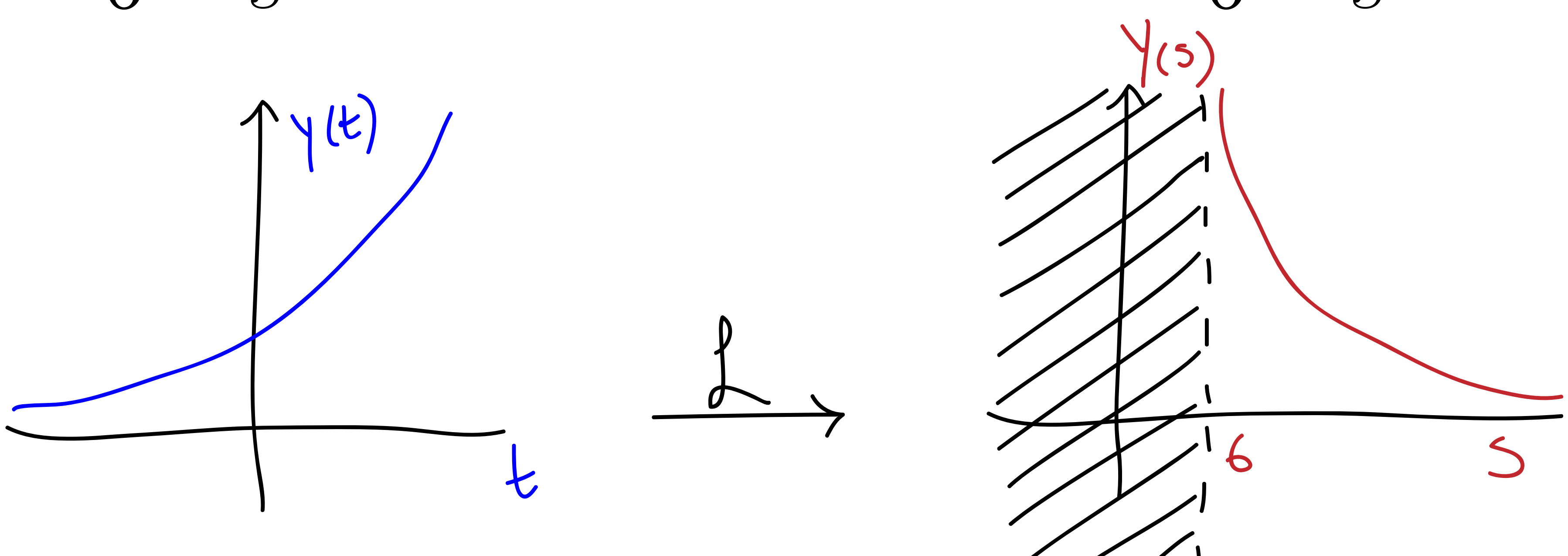

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$$= 1 - s\left(\overbrace{e^{-st}\sin t\Big|_0^\infty}^{0} + s\overbrace{\int_0^\infty e^{-st}\sin t \; dt}^{F(s)} \right)$$

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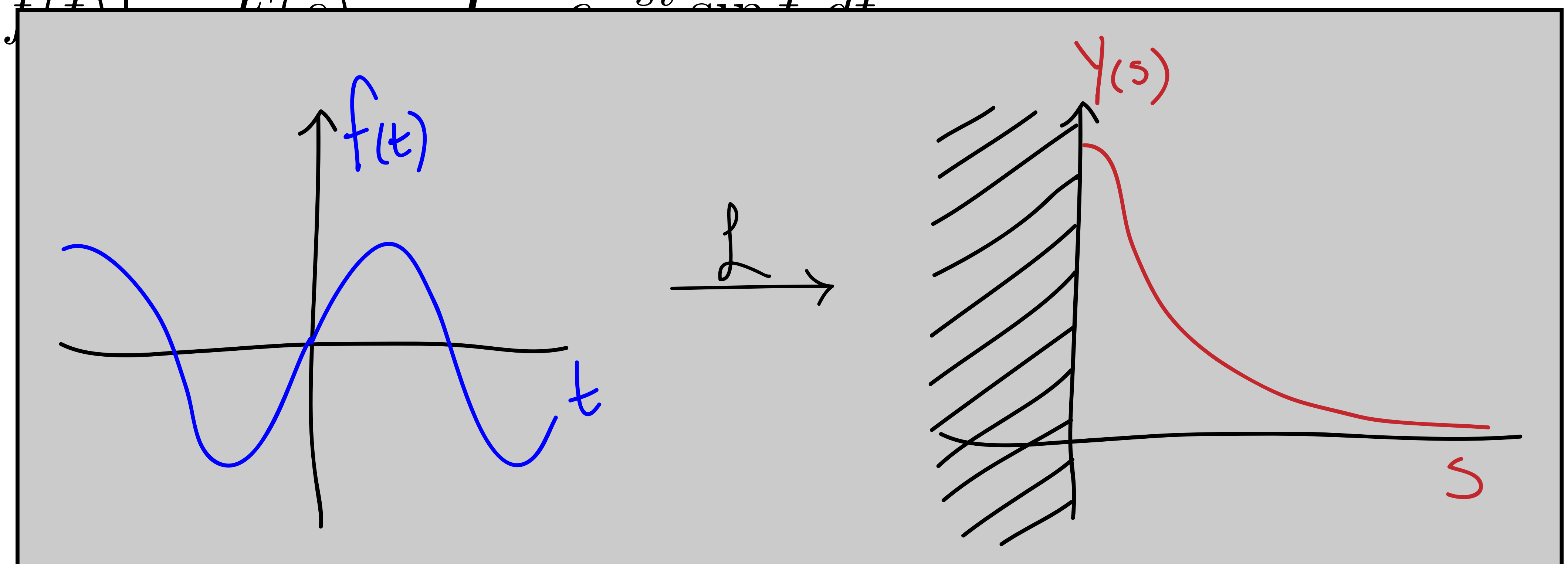

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- What is the Laplace transform of $h(t) = \sin(\omega t)$? $\quad (\omega > 0)$

$$\mathcal{L}\{h(t)\} = H(s) = \int_0^\infty e^{-st} \sin(\omega t)\ dt$$

$$u = \omega t$$
$$du = \omega dt$$

(A) $H(s) = \dfrac{\omega}{\omega^2 + s^2}$

(B) $H(s) = \dfrac{1}{1 + \left(\frac{s}{w}\right)^2}$

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$$H(s) = \int_0^\infty e^{-s\frac{u}{\omega}} \sin u \ \frac{du}{\omega}$$
$$= \frac{1}{\omega} \int_0^\infty e^{-\frac{s}{\omega}u} \sin u \ du$$

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- Could calculate directly but note that g(t) = f'(t) where f(t)=sin t.

Add sketch.

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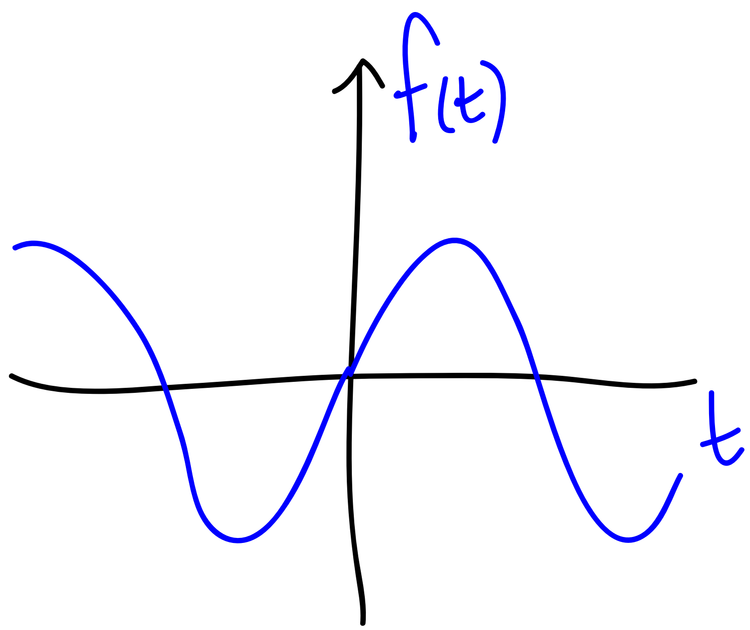

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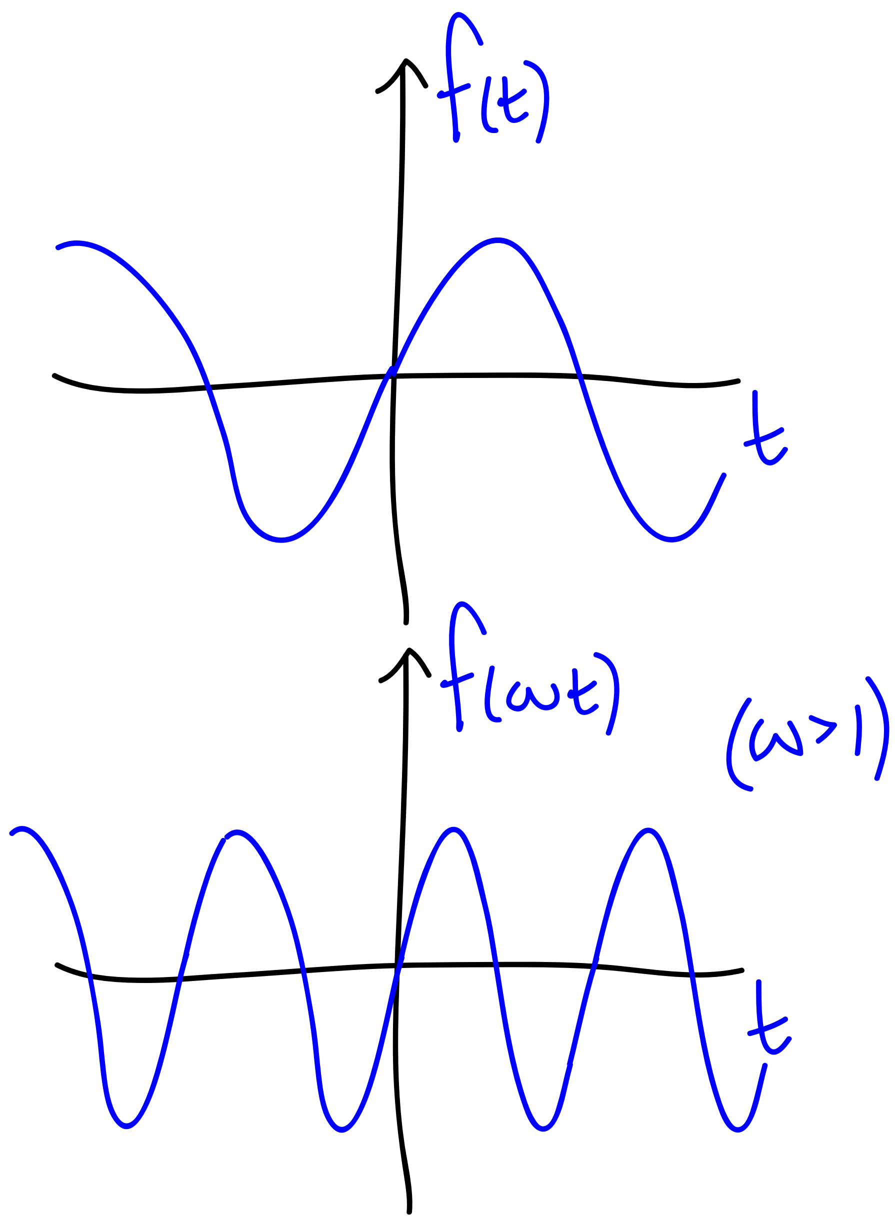

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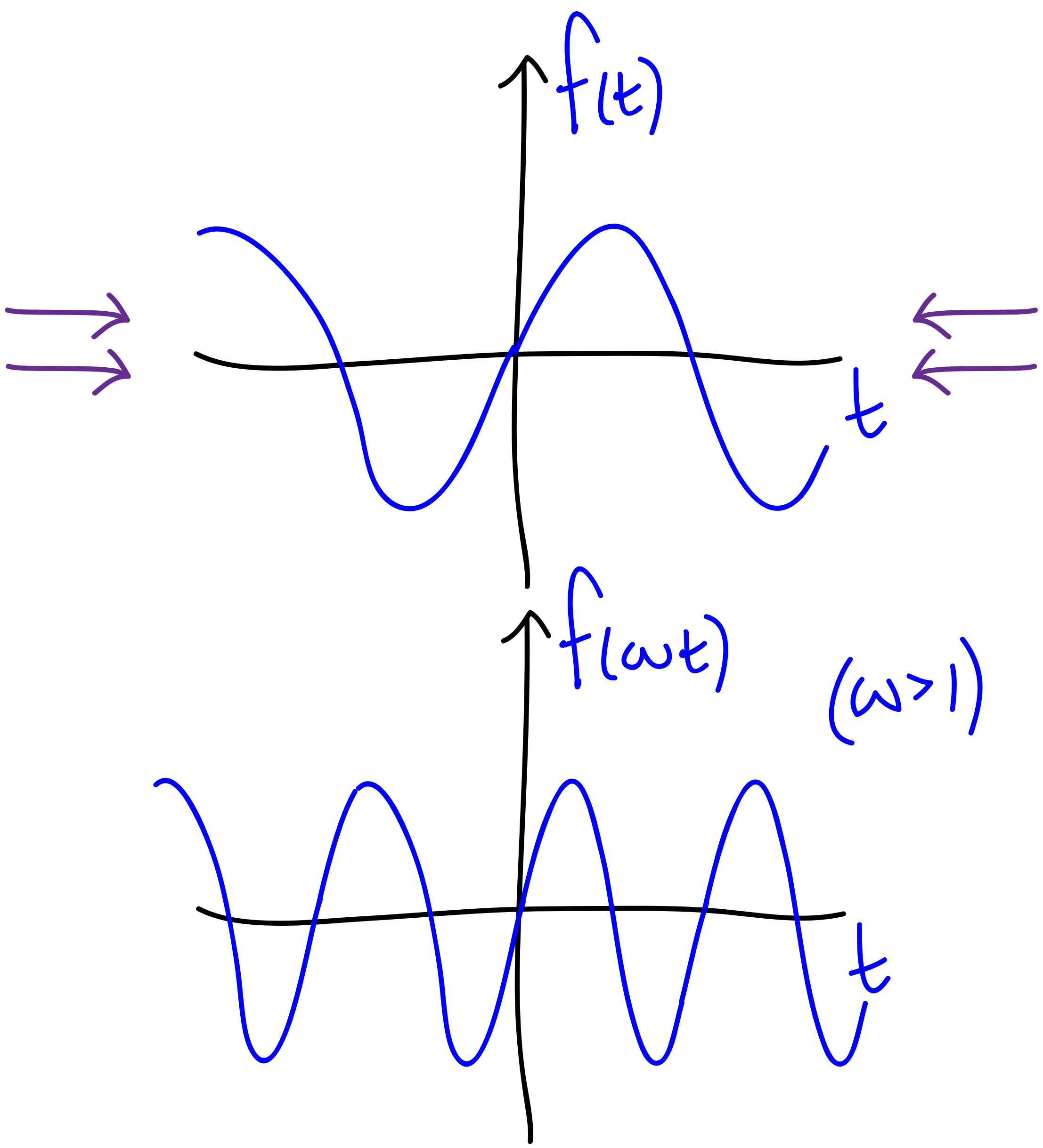

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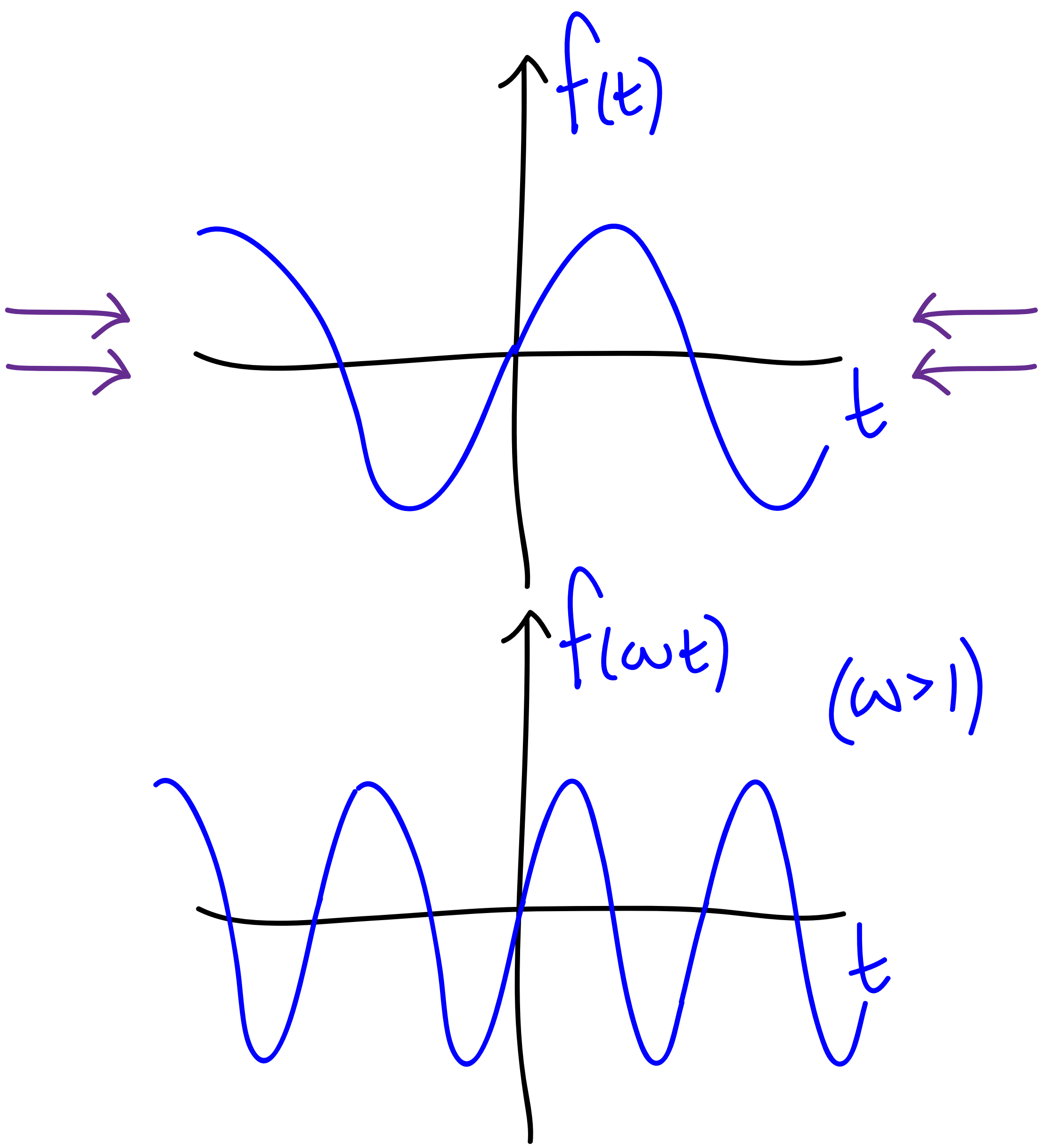

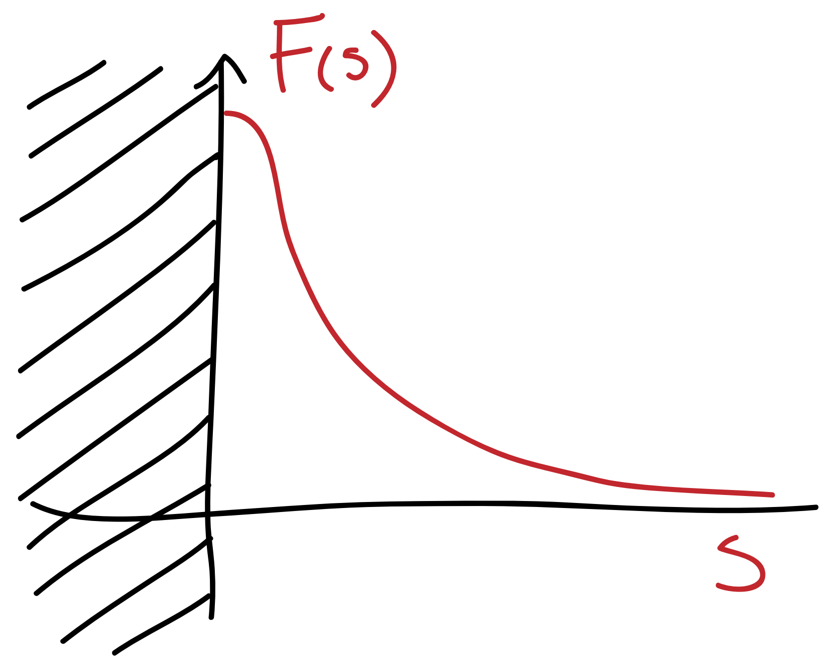

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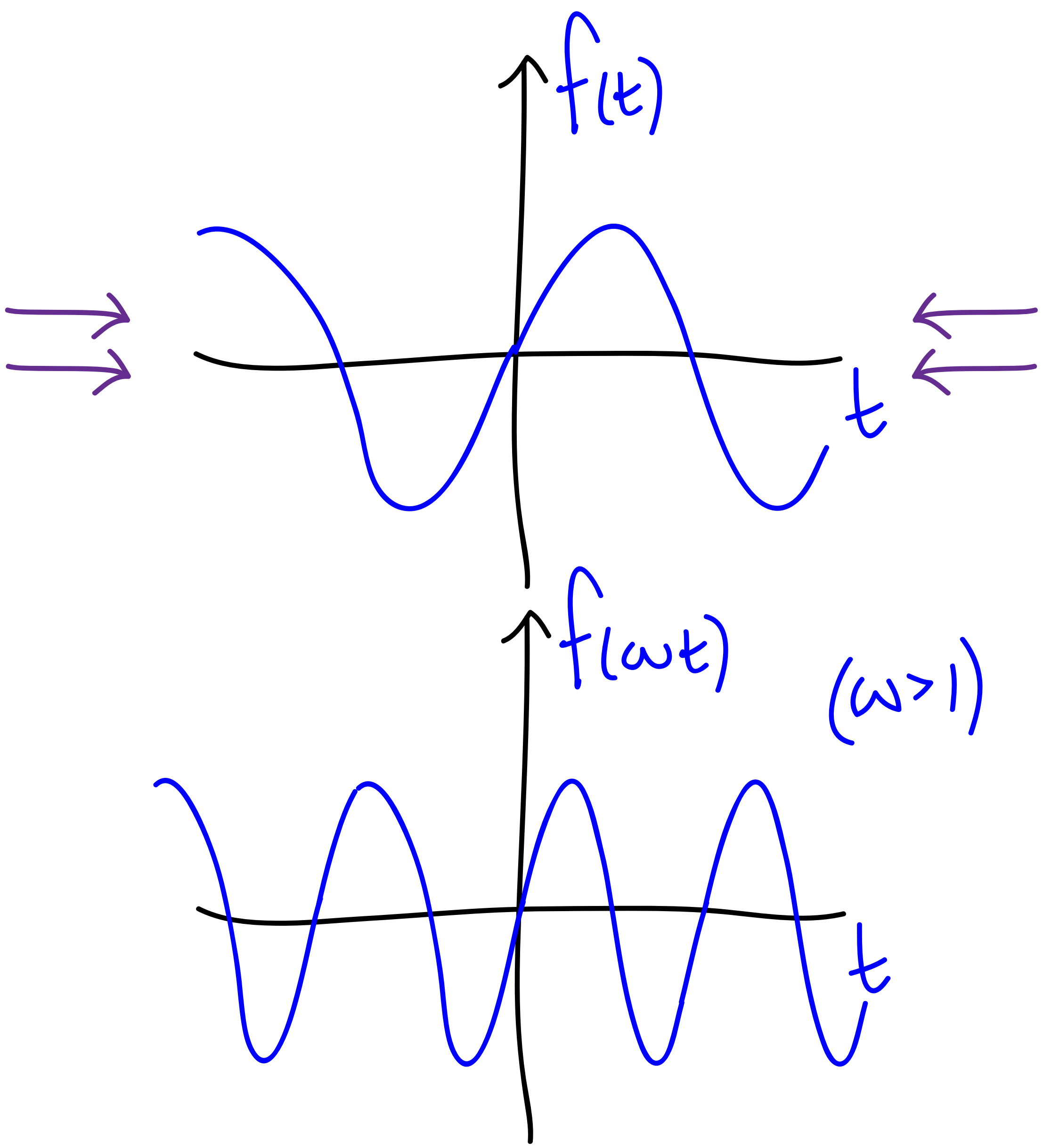

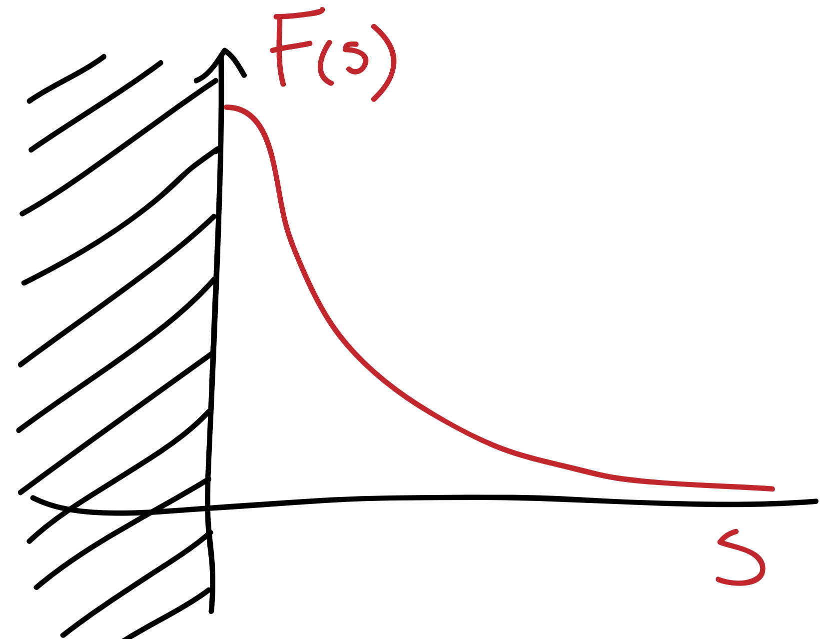

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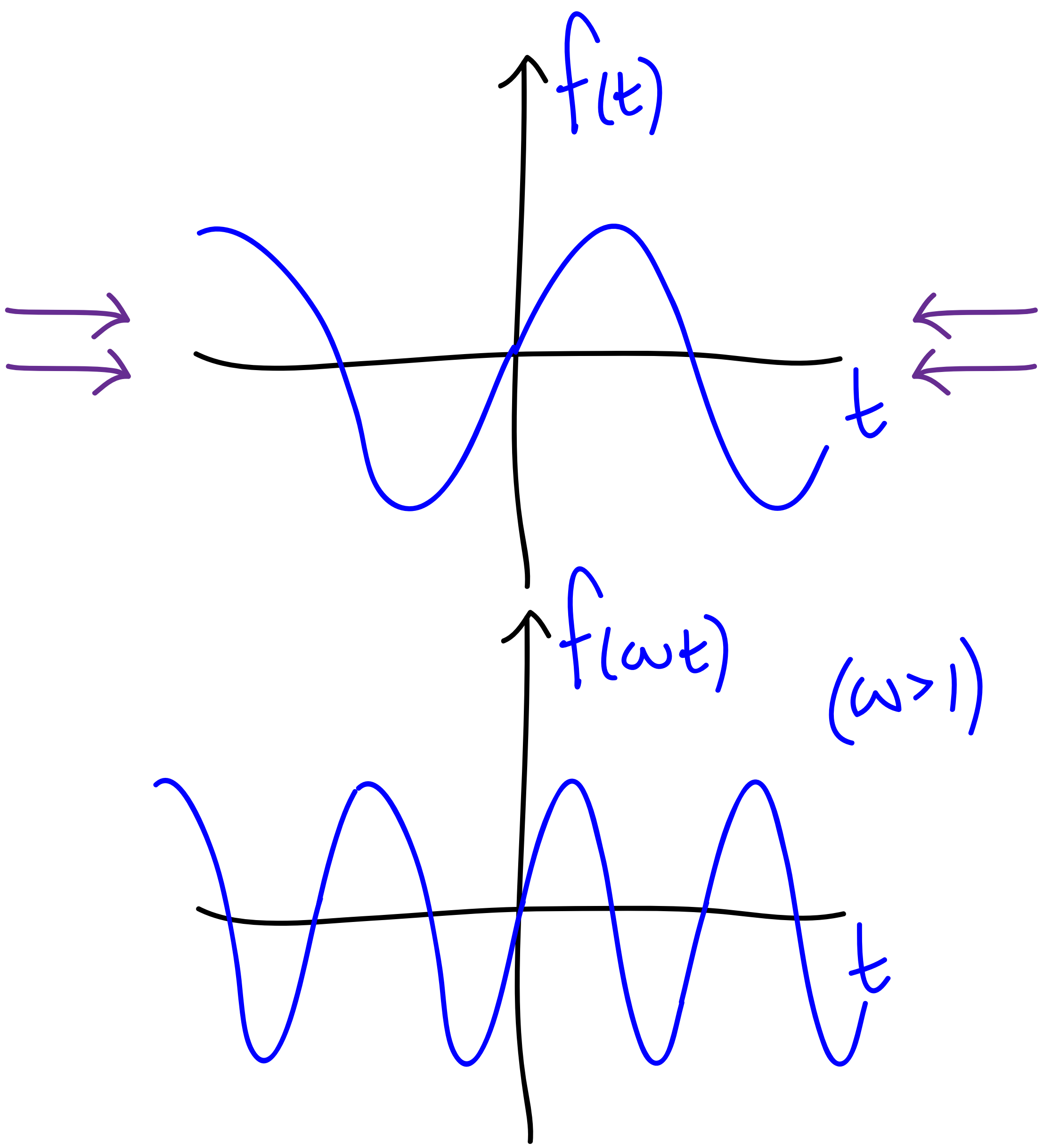

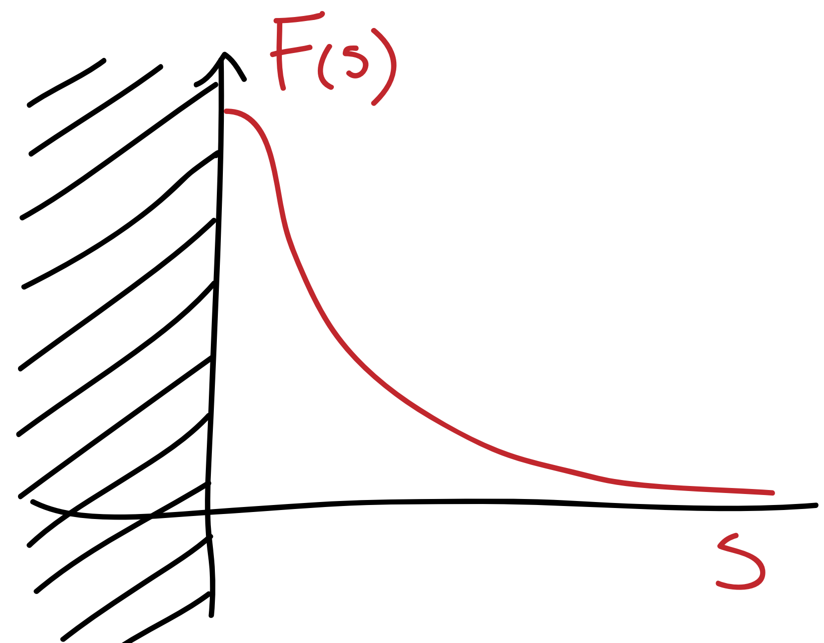

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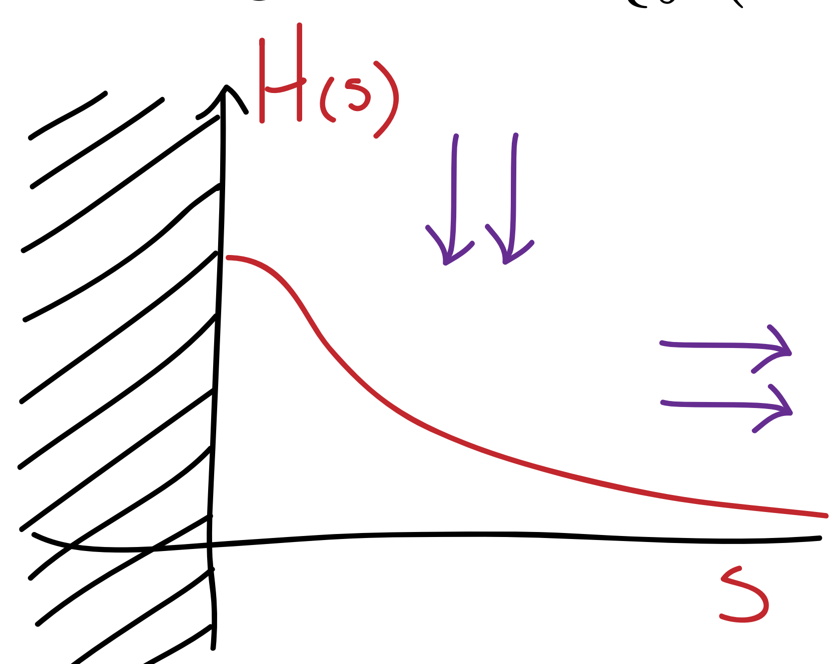

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$$\begin{aligned}\mathcal{L}\{f''(t)\} &= s\mathcal{L}\{f'(t)\} - f'(0)\\ &= s(sF(s) - f(0)) - f'(0)\\ &= s^2F(s) - sf(0) - f'(0)\end{aligned}$$

- Transforming both sides of the equation,

$$\mathcal{L}\{ay'' + by' + cy\} = 0$$

$$a\underline{\mathcal{L}\{y''\}} + b\underline{\mathcal{L}\{y'\}} + c\underline{\mathcal{L}\{y\}} = 0$$

$$a(\underline{s^2Y(s) - sy(0) - y'(0)}) + b(\underline{sY(s) - y(0)}) + c\underline{Y(s)} = 0$$

$$(as^2 + bs + c)Y(s) = asy(0) + ay'(0) + by(0)$$

Solving IVPs using Laplace transforms (6.2)

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$$Y(s) = \frac{asy(0) + ay'(0) + by(0)}{as^2 + bs + c}$$

Solving IVPs using Laplace transforms (6.2)

- Solve the equation $y'' + 4y = 0$ with initial conditions y(0)=1, y'(0)=0 using Laplace transforms.

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- Recall that $\mathcal{L}\{\cos(\omega t)\} = \frac{s}{\omega^2 + s^2}$. So $y(t) = \cos(2t)$.

Solving IVPs using Laplace transforms (6.2)

- Solve the equation $y'' + 6y' + 13y = 0$ with initial conditions y(0)=1, y'(0)=0 using Laplace transforms.

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$$Y(s) = \frac{s+3+3}{s^2+6s+9+4}$$

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$$\lambda = \frac{-6 \pm i\sqrt{52-36}}{2} = -3 \pm 2i$$

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$$y' + 6y = e^{2t}$$
$$y(0) = 2$$

(A) $Y'(s) + 6Y(s) = \dfrac{1}{s+2}$

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(C) $sY(s) - 2 + 6Y(s) = \dfrac{1}{s+2}$

(D) $sY(s) - 2 + 6Y(s) = \dfrac{1}{s-2}$

(E) Explain, please.

$$\mathcal{L}\{e^{2t}\} = \int_0^\infty e^{(2-s)t}\, dt$$
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$$C = \frac{15}{8}$$

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$$y(t) = 2e^{-6t} + \frac{1}{8}e^{2t} - \frac{1}{8}e^{-6t}$$

$$\frac{1}{(s-2)(s+6)} = \frac{A}{s-2} + \frac{B}{s+6}$$

$$1 = A(s+6) + B(s-2)$$

$$(s=2) \qquad 1 = 8A$$

$$(s=-6) \qquad 1 = -8B$$

$$y(t) = \frac{15}{8}e^{-6t} + \frac{1}{8}e^{2t}$$

$$y_h(t) = Ce^{-6t}$$

$$C = \frac{15}{8} \qquad y_p(t) = \frac{1}{8}e^{2t}$$