Today

- Summary of steps for solving the Diffusion Equation with homogeneous Dirichlet or Neumann BCs using Fourier Series.
- Nonhomogeneous BCs
- Mixed Dirichlet/Neumann BCs
- Method of Undetermined Coefficients and Fourier Series

Using Fourier Series to solve the Diffusion Equation

- Steps to solving the PDE:
  - Determine the eigenfunctions for the problem (look at BCs).
  - Represent the IC u(x,0)=f(x) by a sum of eigenfunctions (Fourier series).
  - Write down the solution by inserting $e^{\lambda t}$ into each term of the FS.

$u_t = Du_{xx}$	PDE determines all possible eigenfunctions.
$\left.\dfrac{du}{dx}\right\|_{x=0,L} = 0$	BCs select a subset of the eigenfunctions.
$u(x,0) = f(x)$	IC is satisfied by adding up eigenfunctions.

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$\left.\frac{du}{dx}\right|_{x=0,L} = 0$ $\longrightarrow$ BCs select a subset of the eigenfunctions.

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Case I: λ<0.

Case II: λ=0.

Case III: λ>0.

Using Fourier Series to solve the Diffusion Equation

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Case I: $\lambda<0$. $v_\lambda(x) = \cos\left(\sqrt{\frac{-\lambda}{D}}x\right)$ and $w_\lambda(x) = \sin\left(\sqrt{\frac{-\lambda}{D}}x\right)$

Case II: $\lambda=0$.

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For each value of λ<0, these are both eigenfunctions.

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Case II: λ=0. $v_{xx} = 0$

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Case II: λ=0. $v_{xx} = 0 \Rightarrow v_x = C_1$

Case III: λ>0.

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Case II: λ=0. $v_{xx} = 0 \;\Rightarrow v_x = C_1 \;\Rightarrow v(x) = C_1 x + C_2$

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Case II: λ=0. $v_{xx} = 0 \Rightarrow v_x = C_1 \Rightarrow v(x) = C_1 x + C_2$

The eigenfunctions are therefore $v(x) = 1$ and $v(x) = x$.

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Case III: λ>0. $v_\lambda(x) = e^{\sqrt{\frac{\lambda}{D}}x}$ and $w_\lambda(x) = e^{-\sqrt{\frac{\lambda}{D}}x}$

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These won't come up so I'll drop Case III.

Using Fourier Series to solve the Diffusion Equation

$$u_t = Du_{xx}$$ $\longrightarrow$ PDE determines all possible eigenfunctions.

$$\left.\frac{du}{dx}\right|_{x=0,L} = 0$$ $\longrightarrow$ BCs select a subset of the eigenfunctions.

Using Fourier Series to solve the Diffusion Equation

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The BC at x=0 only works for $v_\lambda(x)$ and the BC at x=L only works for certain λ, in particular $\lambda = -n^2\pi^2 D/L^2$.

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Represent IC u(x,0) = f(x) by $u(x,0) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$

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e^{0t}

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Using Fourier Series to solve the Diffusion Equation

$$u_t = 4u_{xx}$$

$$\left.\frac{du}{dx}\right|_{x=0,2} = 0$$

$$u(x,0) = \sin\frac{3\pi x}{2}$$

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(A) $u(x,t) = e^{-9\pi^2 t}\cos\frac{3\pi x}{2}$

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(C) $u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 t}\sin\frac{n\pi x}{2}$ $\qquad b_n = \int_0^2 \sin\frac{3\pi x}{2}\sin\frac{n\pi x}{2}\,dx$

(D) $u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 t}\cos\frac{n\pi x}{2}$ $\qquad a_n = \int_0^2 \sin\frac{3\pi x}{2}\cos\frac{n\pi x}{2}\,dx$

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(A) $u(x,t) = e^{-9\pi^2 t}\cos\frac{3\pi x}{2}$ ⟵ doesn't satisfy IC.

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...with nonhomogeneous boundary conditions

$$u_t = Du_{xx}$$

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$\longrightarrow$ Nonhomogeneous BCs

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Case I: λ<0. $v_\lambda(x) = \cos\left(\sqrt{\frac{-\lambda}{D}}x\right)$ and $w_\lambda(x) = \sin\left(\sqrt{\frac{-\lambda}{D}}x\right)$

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The BC at x=0 only works for $w_\lambda(x)$ and the BC at x=L almost works for certain λ, in particular $\lambda = -n^2\pi^2 D/L^2$.

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an eigenfunction for the homogeneous BCs

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Case II: λ=0. $\cancel{v(x) = 1}$ and $v(x) = 2x$ $\longleftarrow$ a particular eigenfunction for the inhomogeneous BCs

...with nonhomogeneous boundary conditions

$u_t = Du_{xx}$

an eigenfunction for the homogeneous BCs

$u(0,t) = 0$

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Ultimately, we want $u(x,t) = 2x + \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 Dt/L^2} \sin\frac{n\pi x}{L}$

...with nonhomogeneous boundary conditions

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an eigenfunction for the homogeneous BCs

Case I: λ<0. $\cancel{v_\lambda(x) = \cos\left(\sqrt{\frac{-\lambda}{D}}x\right)}$ and $w_\lambda(x) = \sin\left(\sqrt{\frac{-\lambda}{D}}x\right)$

The BC at x=0 only works for $w_\lambda(x)$ and the BC at x=L almost works for certain λ, in particular $\lambda = -n^2\pi^2 D/L^2$.

Case II: λ=0. $\cancel{v(x) = 1}$ and $v(x) = 2x$ ← a particular eigenfunction for the inhomogeneous BCs

Ultimately, we want $u(x,t) = 2x + \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 Dt/L^2} \sin\frac{n\pi x}{L}$

What function do we use to calculate the Fourier series $\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$?

(A) u(x,0) (B) u(x,0) - 2 (C) u(x,0) - 2x (D) u(x,0) + 2x

...with nonhomogeneous boundary conditions

$u_t = Du_{xx}$

$u(0,t) = 0$

$u(2,t) = 4$ → Nonhomogeneous BCs

an eigenfunction for the homogeneous BCs

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...with nonhomogeneous boundary conditions

- Solve the Diffusion Equation with nonhomogeneous BCs:

$$u_t = Du_{xx}$$

$$u(0, t) = a$$

$$u(L, t) = b$$

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...with nonhomogeneous boundary conditions

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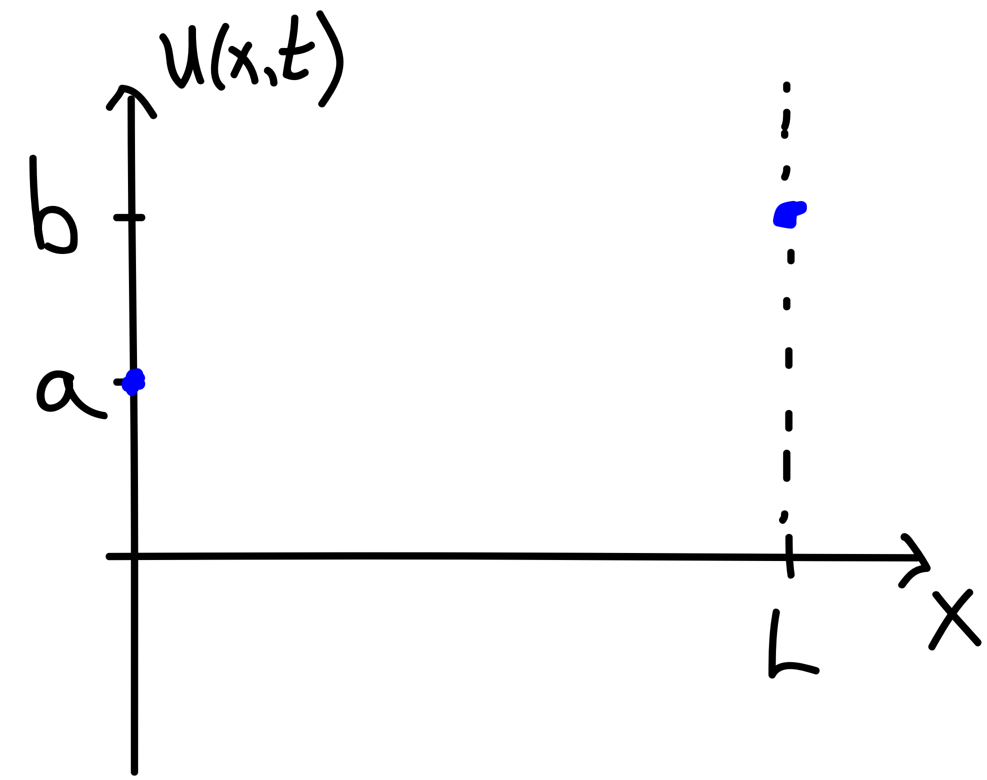

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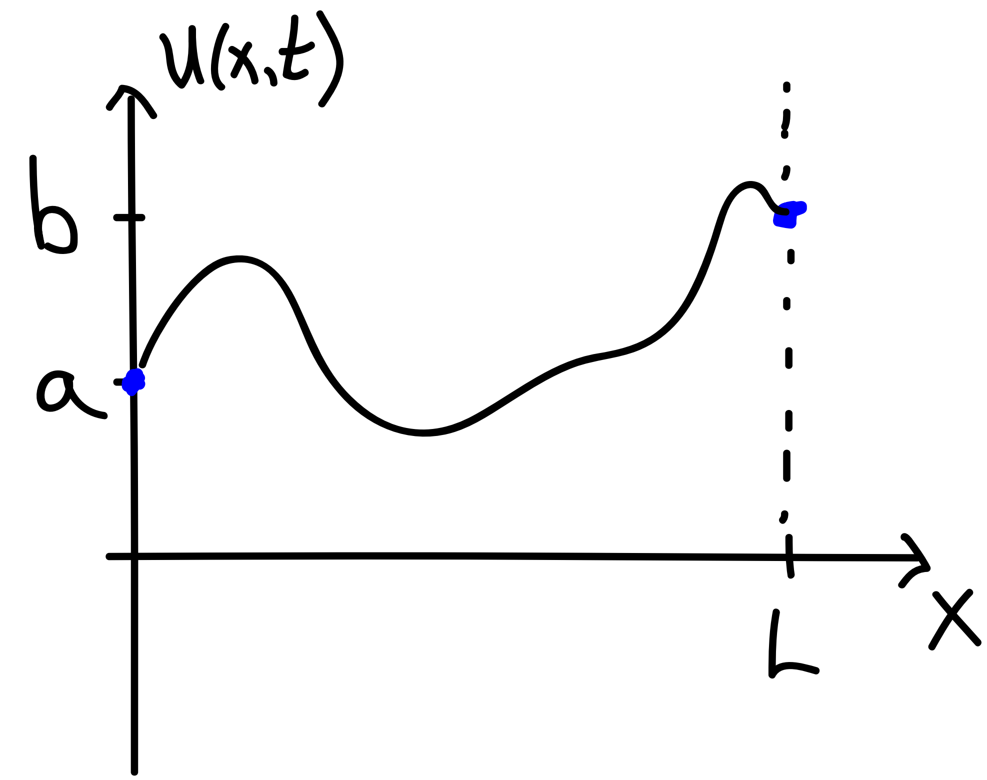

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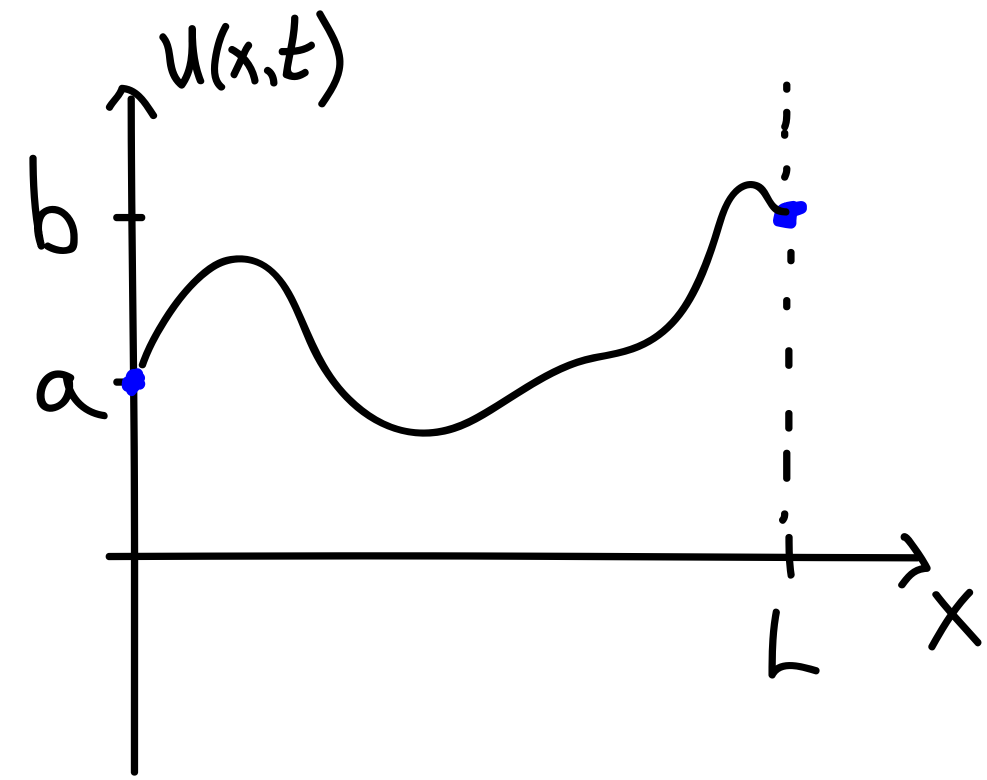

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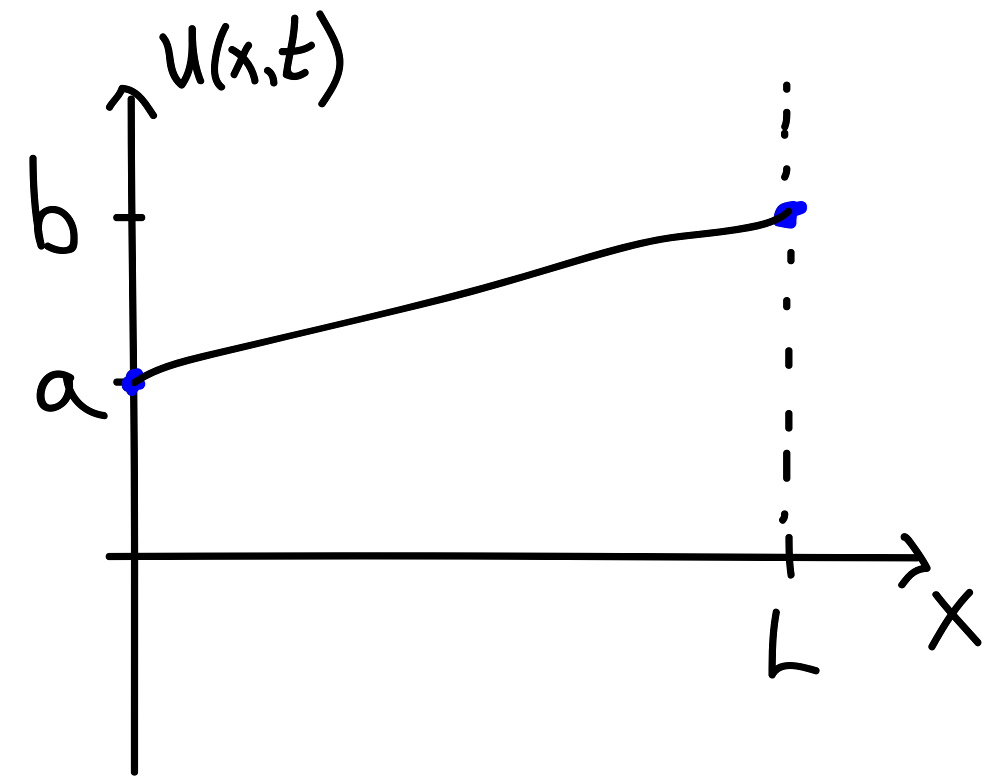

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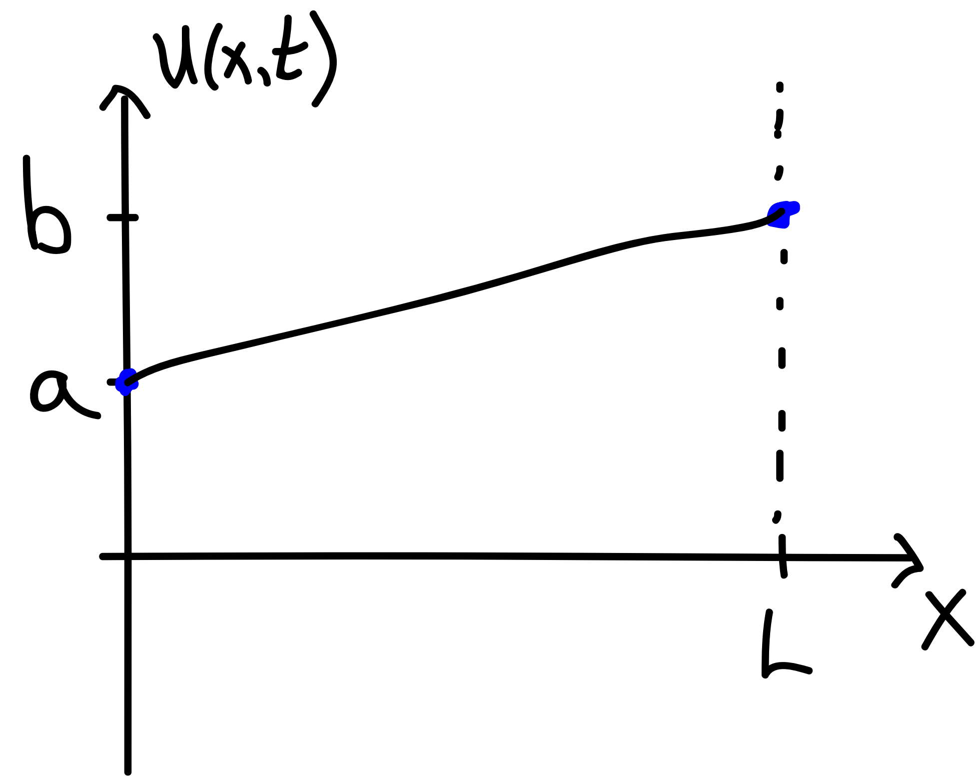

$$v(x,t) = u(x,t) - \left(a + \frac{b-a}{L}x\right)$$

...with nonhomogeneous boundary conditions

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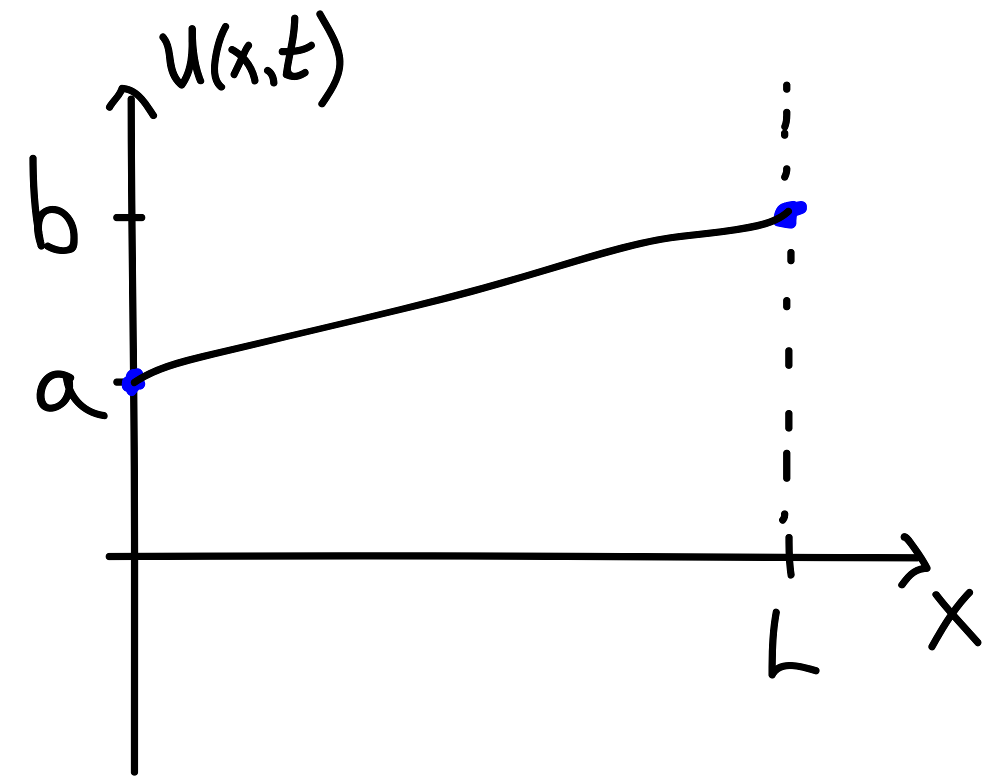

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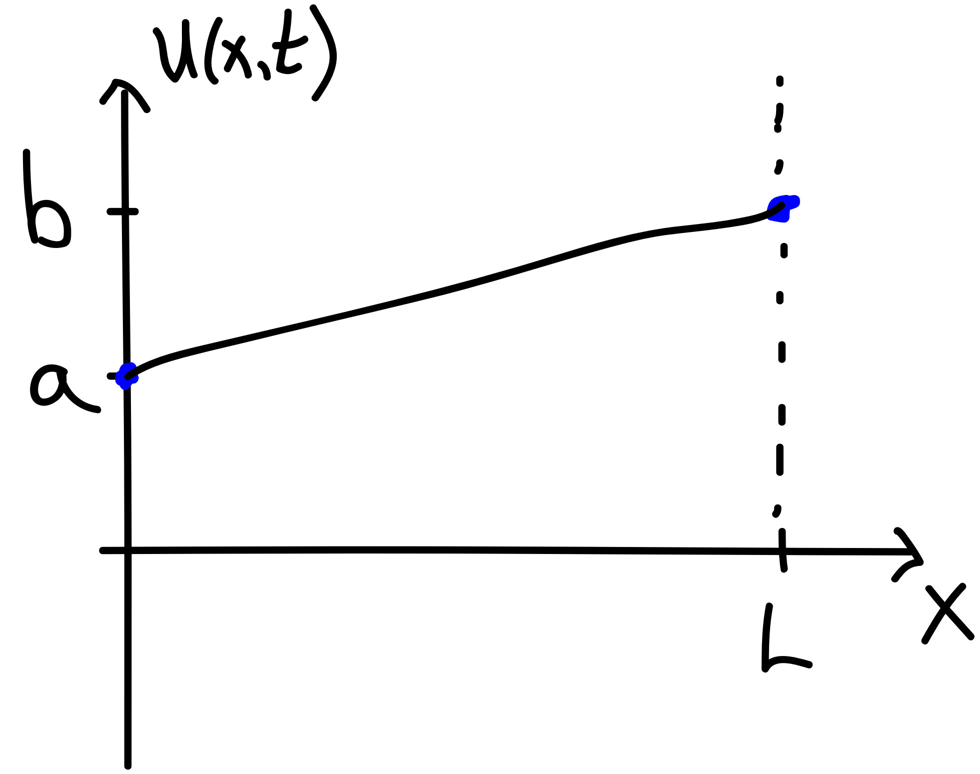

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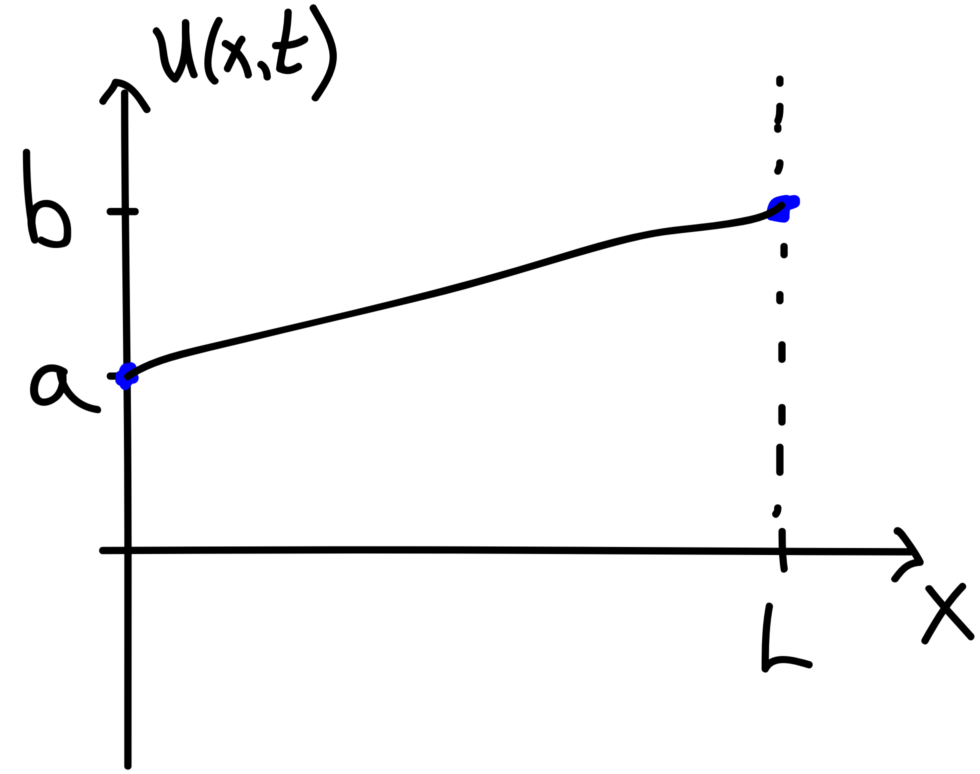

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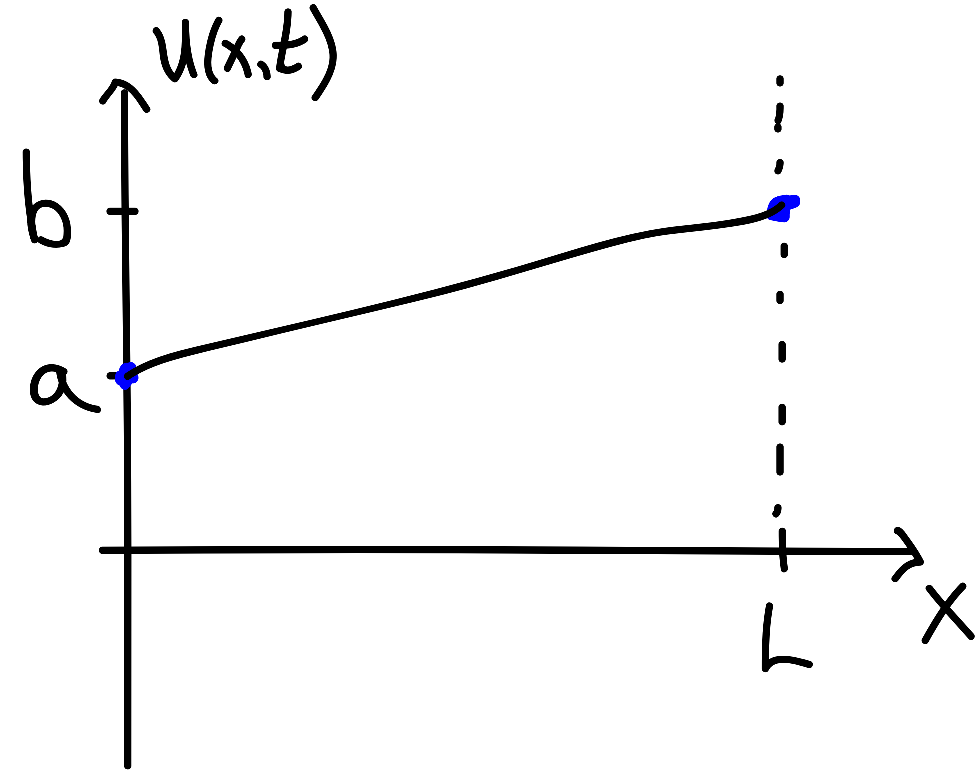

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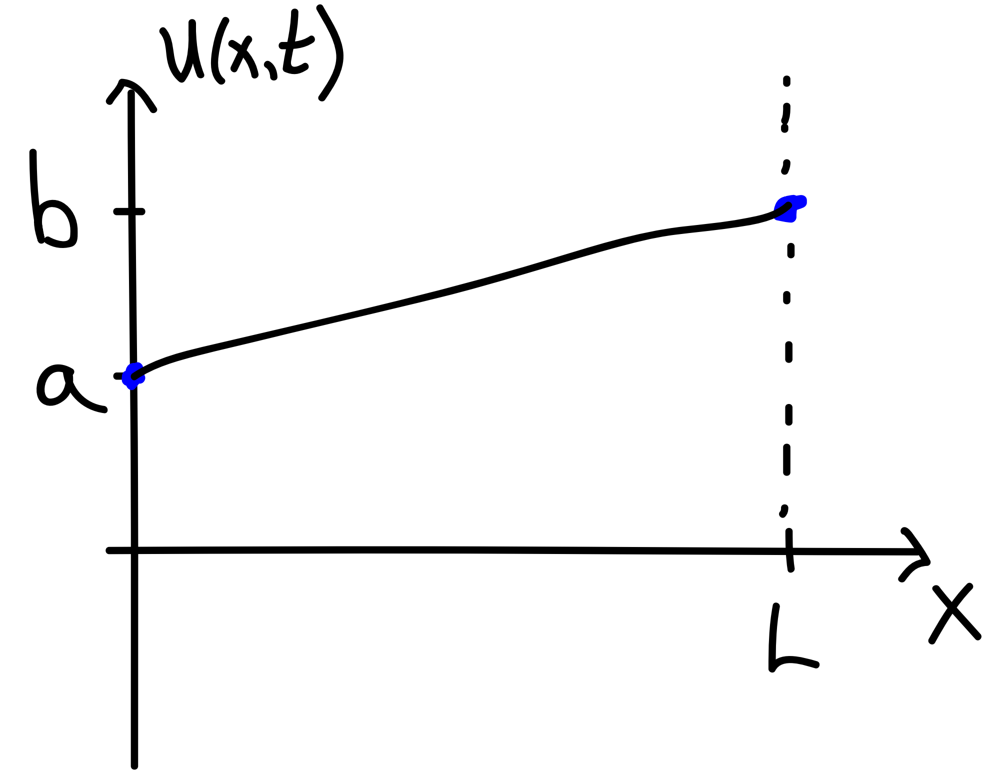

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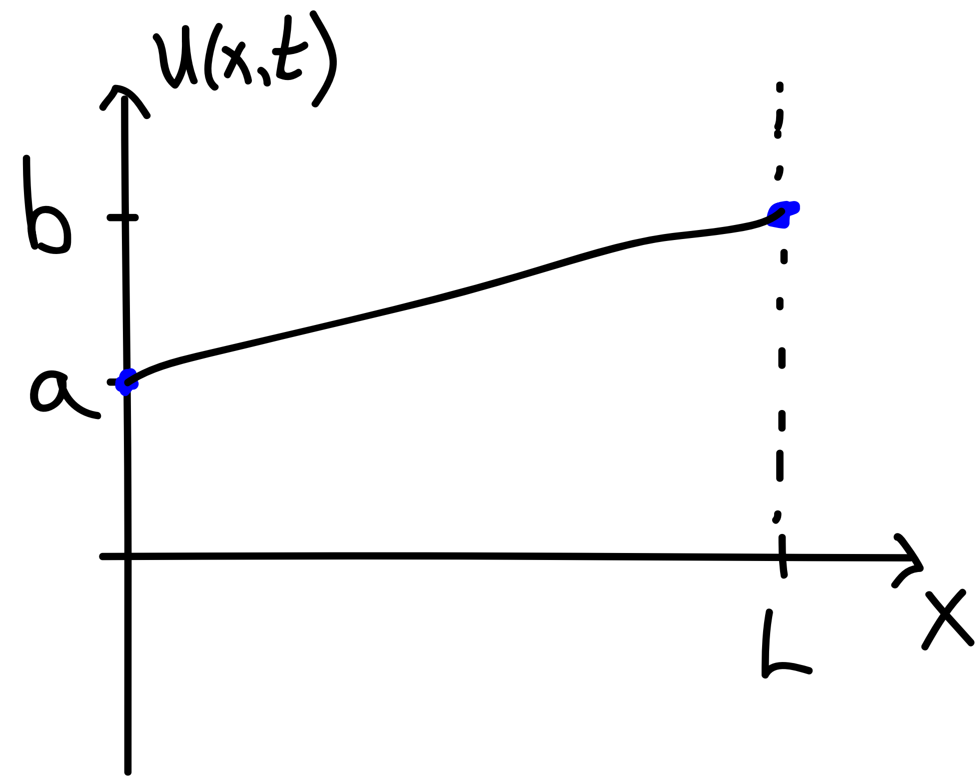

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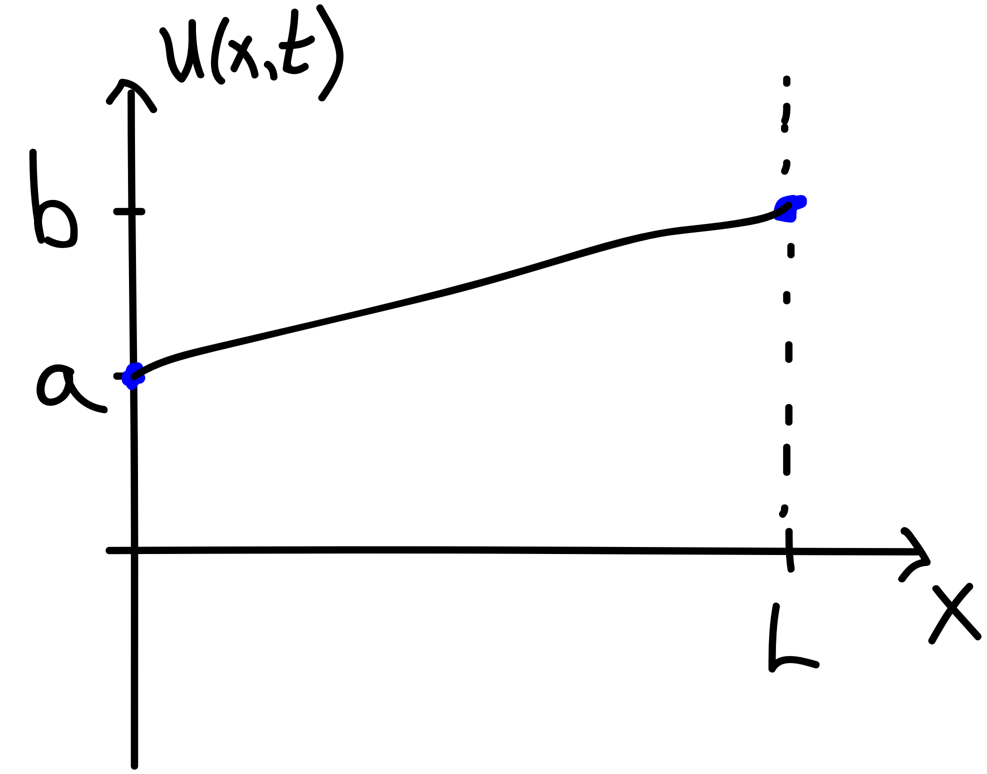

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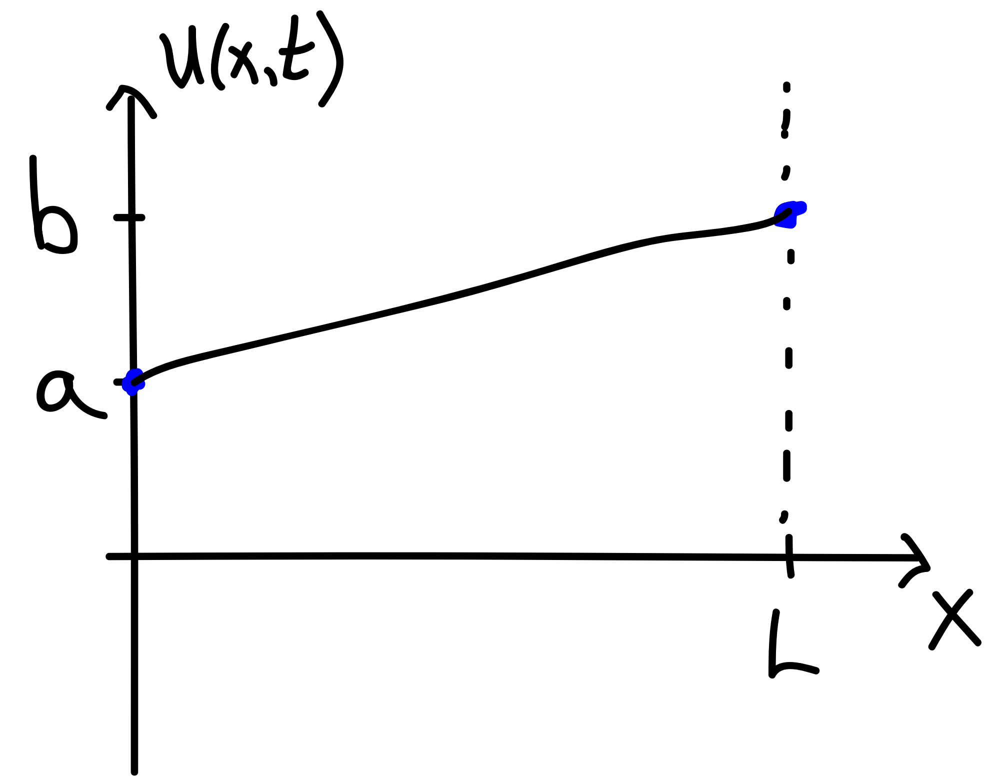

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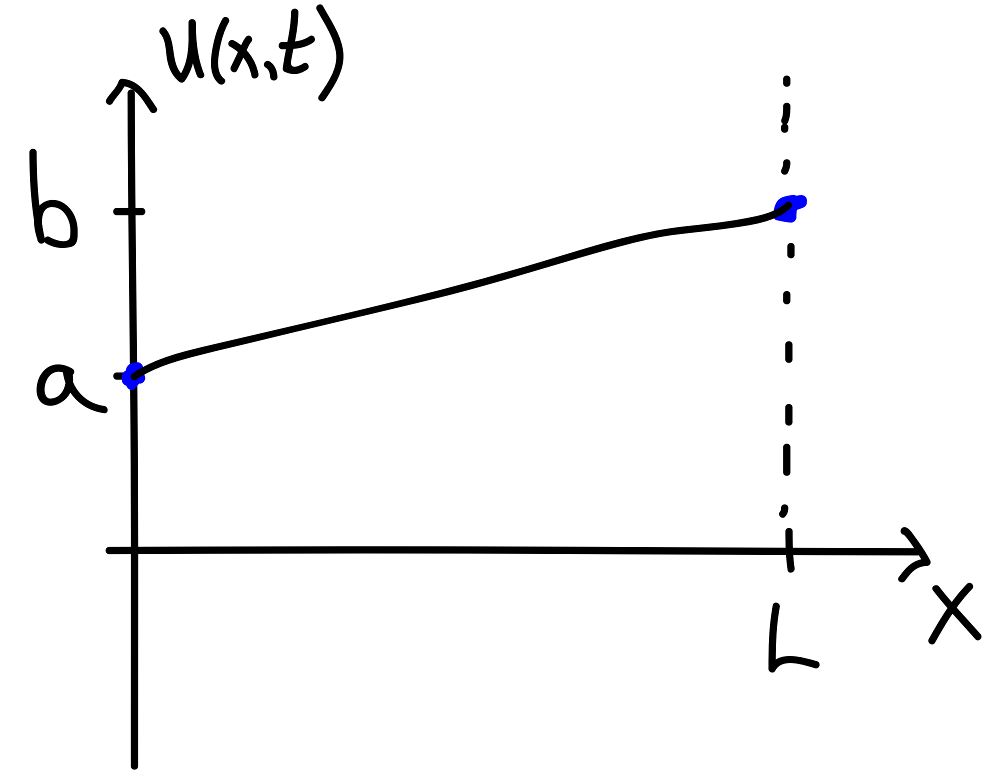

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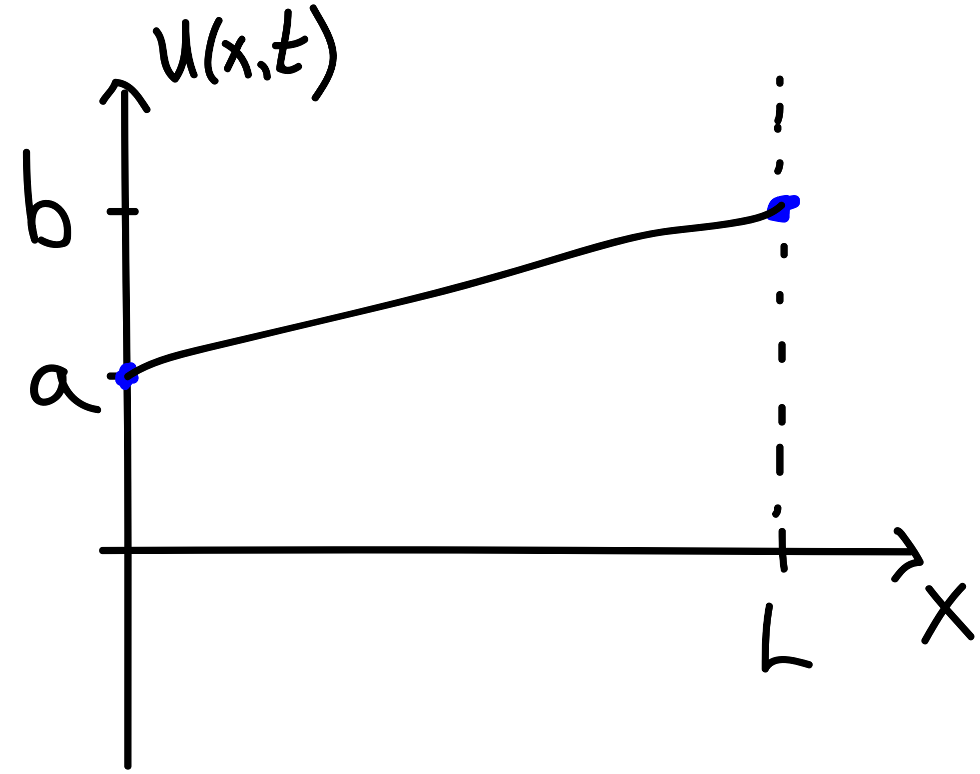

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- v(x,t) satisfies the Diffusion Eq with homogeneous Dirichlet BCs and a new IC.

...with nonhomogeneous boundary conditions

- Find the solution to the following problem:

$$u_t = 4u_{xx}$$
$$u(0,t) = 9$$
$$u(2,t) = 5$$
$$u(x,0) = \sin\frac{3\pi x}{2}$$

(A) $u(x,t) = e^{-9\pi^2 t}\sin\frac{3\pi x}{2}$

(B) $u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 t}\sin\frac{n\pi x}{2}$

(C) $u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2 t}\sin\frac{n\pi x}{2} + 9 - 2x$

(D) $u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 t}\cos\frac{n\pi x}{2}$

...with nonhomogeneous boundary conditions

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where $b_n = \int_0^2 \left(\sin\frac{3\pi x}{2} - 9 + 2x\right)\sin\frac{n\pi x}{2}\,dx$

...with nonhomogeneous boundary conditions

- How would you solve this one?

$$u_t = 4u_{xx}$$

$$\left.\frac{du}{dx}\right|_{x=0,2} = -2$$

$$u(x,0) = \cos\frac{3\pi x}{2}$$

...with nonhomogeneous boundary conditions

- How would you solve this one?

$$u_t = 4u_{xx}$$

$$\left.\frac{du}{dx}\right|_{x=0,2} = -2$$

$$u(x,0) = \cos\frac{3\pi x}{2}$$

For you to think about...

Using Fourier Series to solve the Diffusion Equation

$$u_t = 4u_{xx}$$

$$u(0,t) = 0 \qquad \left.\frac{du}{dx}\right|_{x=2} = 0$$

$$u(x,0) = x$$

Using Fourier Series to solve the Diffusion Equation

$$u_t = 4u_{xx}$$

Use sines? cosines?

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Should be zero at x=0 so definitely sine functions.

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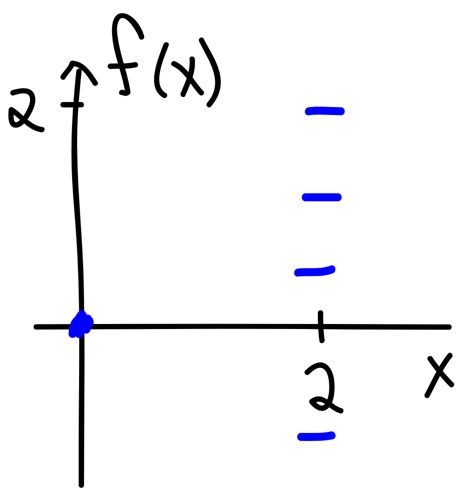
Zero slope at x=2 so extend to x=4 and choose periods to get the slope right.



$$\sin\frac{n\pi x}{4}:$$

Using Fourier Series to solve the Diffusion Equation

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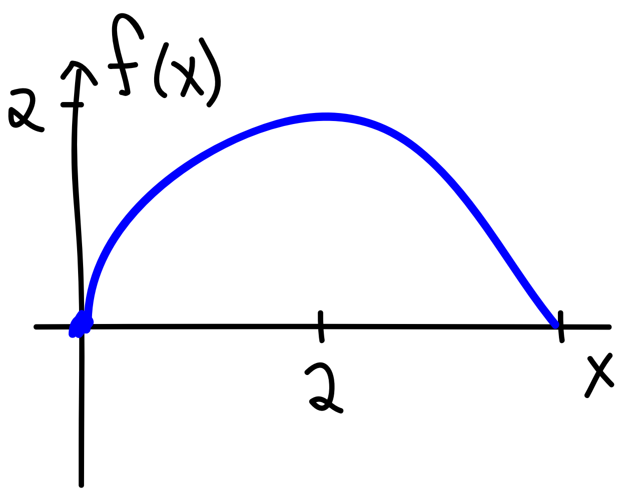

$$\sin \frac{n\pi x}{4}: \quad \sin \frac{\pi x}{4}$$

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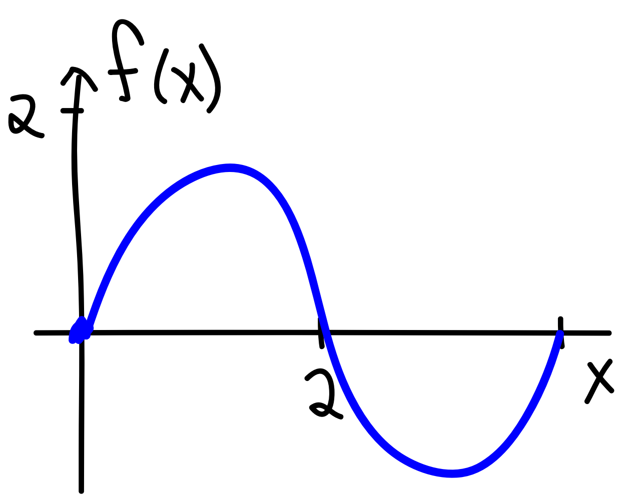

$\sin \frac{n\pi x}{4}$: $\quad \sin \frac{\pi x}{4} \quad \sin \frac{2\pi x}{4}$

Using Fourier Series to solve the Diffusion Equation

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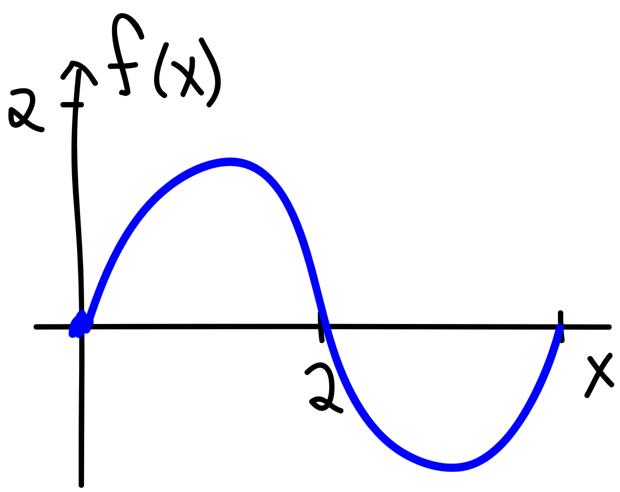

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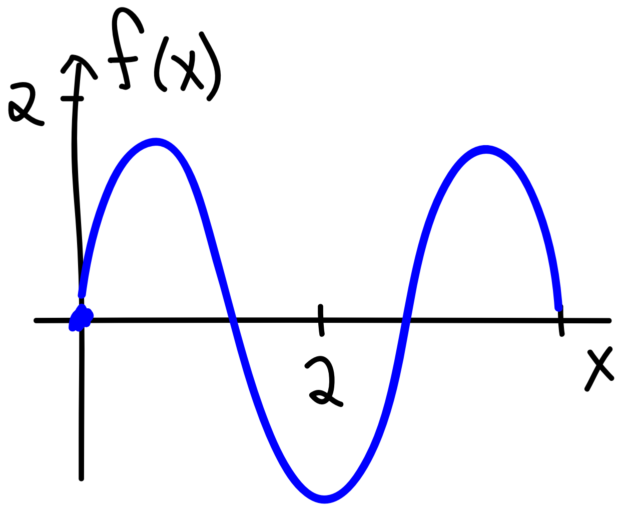

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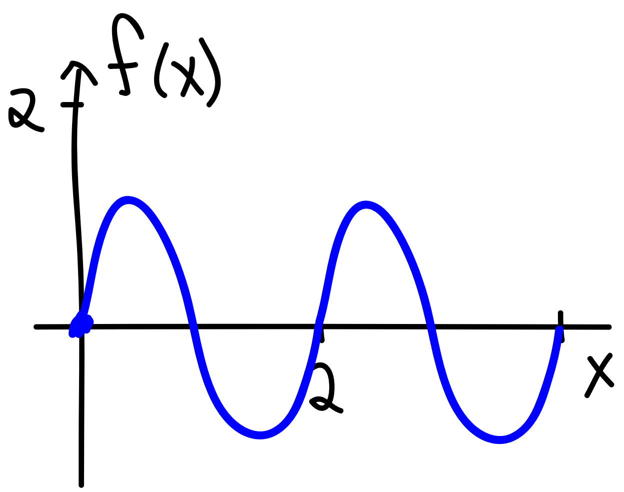

$$\sin\frac{n\pi x}{4}: \quad \sin\frac{\pi x}{4} \quad \cancel{\sin\frac{2\pi x}{4}} \quad \sin\frac{3\pi x}{4} \quad \sin\frac{4\pi x}{4}$$

Using Fourier Series to solve the Diffusion Equation

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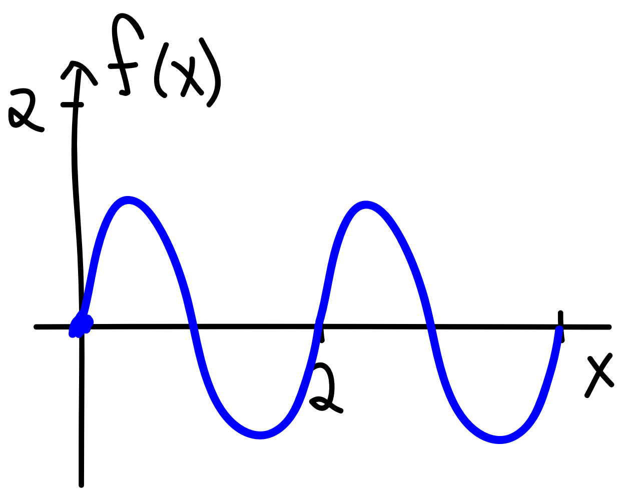

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How to extend f(x) so that its Fourier sine series has only odd values of n?

Using Fourier Series to solve the Diffusion Equation

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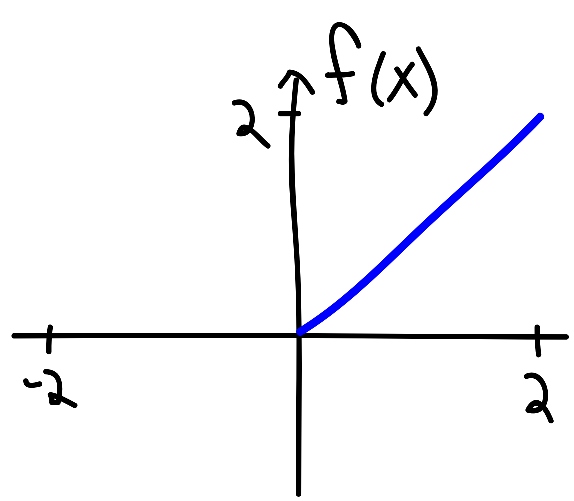
How to extend f(x) so that its Fourier sine series has only odd values of n?



Using Fourier Series to solve the Diffusion Equation

$$u_t = 4u_{xx}$$

Use sines? cosines?

$$u(0,t) = 0 \qquad \left.\frac{du}{dx}\right|_{x=2} = 0$$

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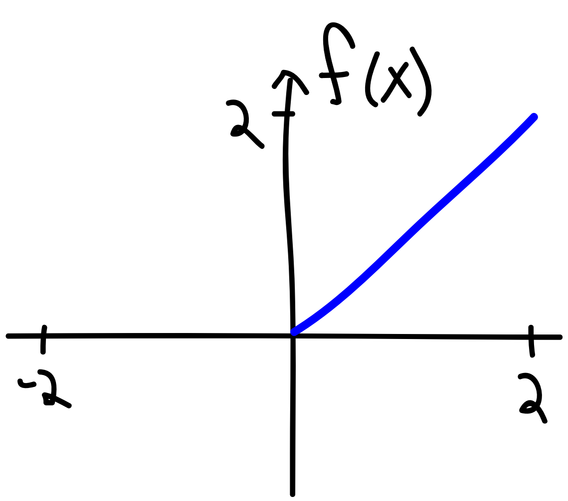

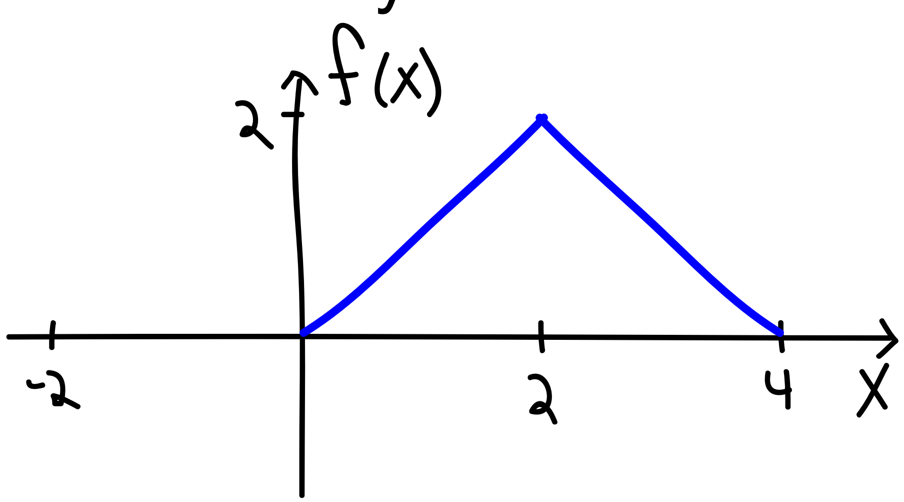

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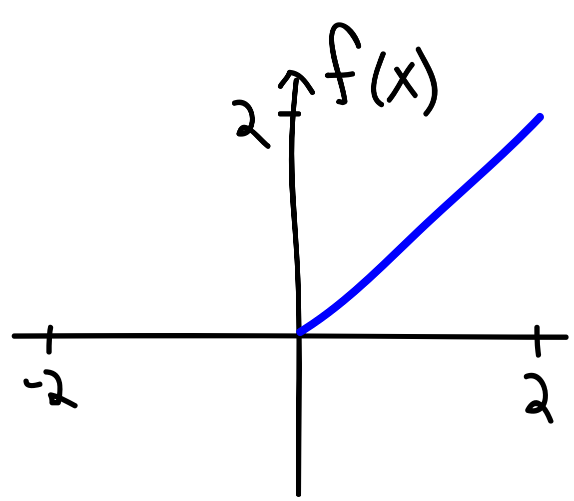

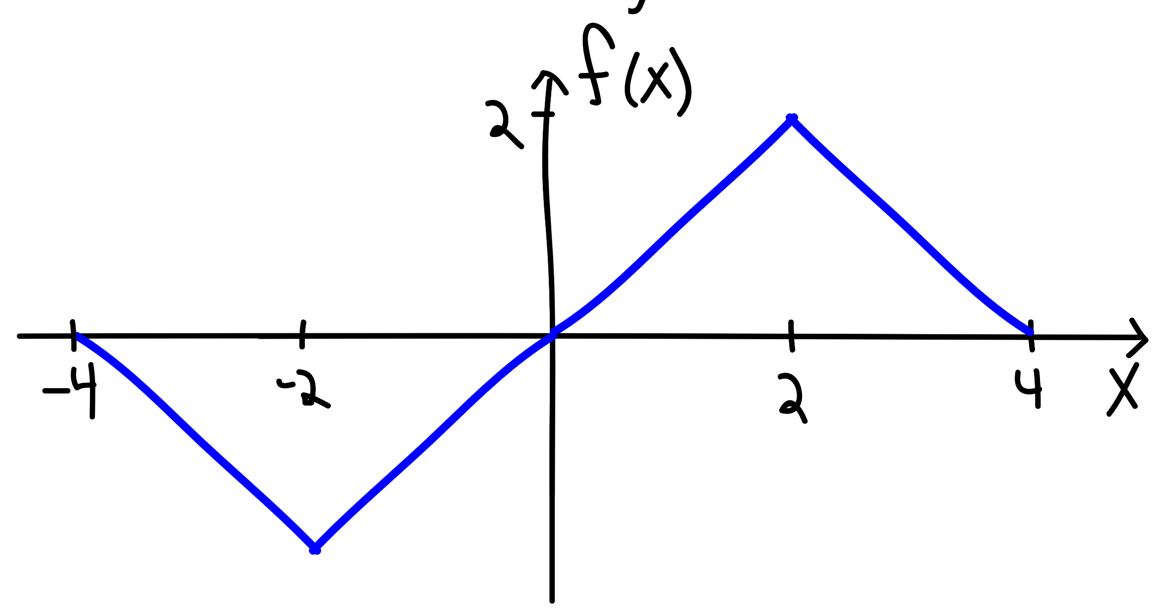

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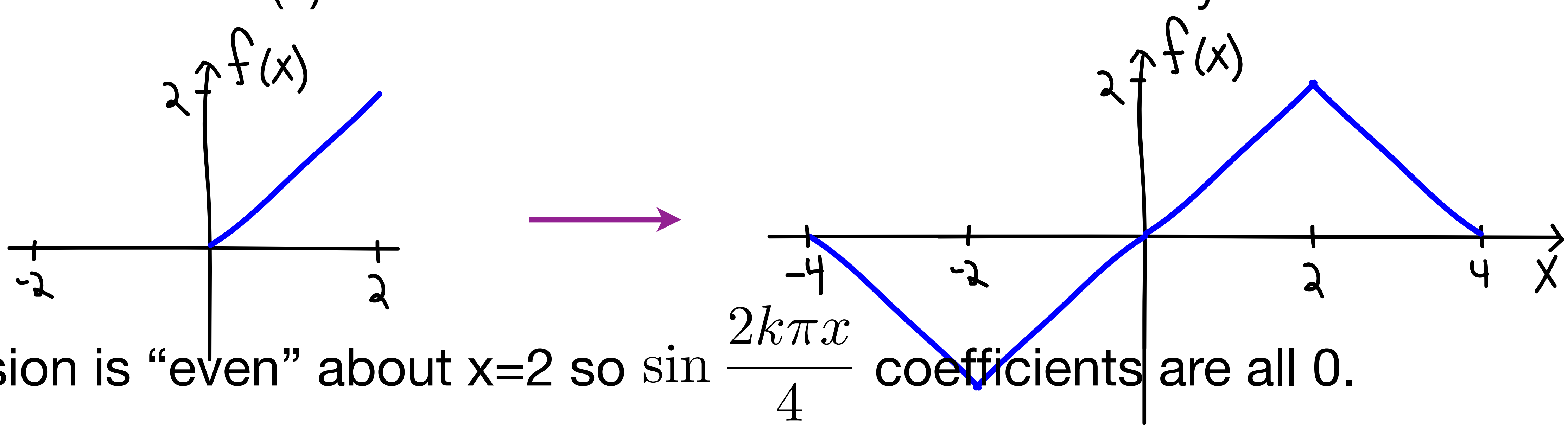

Extension is “even” about x=2 so $\sin\frac{2k\pi x}{4}$ coefficients are all 0.

Using Fourier Series with Method of Undet. Coeff.

- Find the solution to the following problem:

$$y'' + 16y = 4\sin(\pi t) - 3\sin(2\pi t)$$

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- Note: we definitely did not use 4 and -3 as our coefficients for the guess!

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What if the 16 had been $4\pi^2$?