Today

- Introduction to systems of equations
- Direction fields
- Eigenvalues and eigenvectors
- Finding the general solution (distinct e-value case)
- Pre-midterm office hours poll - Friday (best time other than 2-3), Monday (holiday so buildings locked)

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$$\begin{pmatrix} x \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\gamma}{m} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}$$

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    - populations of two species (e.g. predator and prey).

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- We'll focus on the case in which the matrix has constant entries. And homogeneous, to start. For example,

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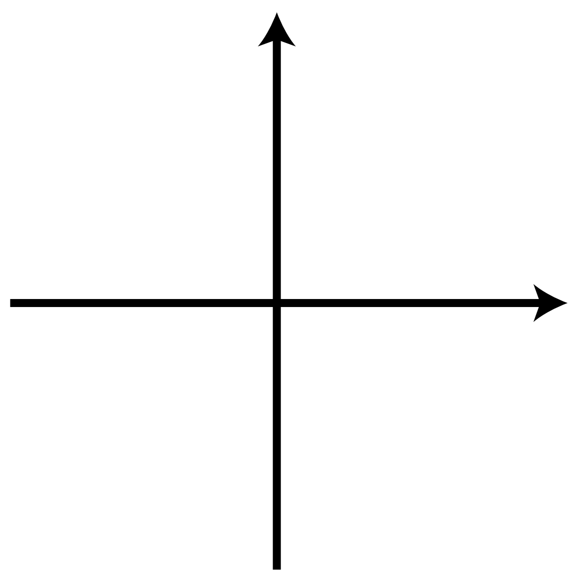

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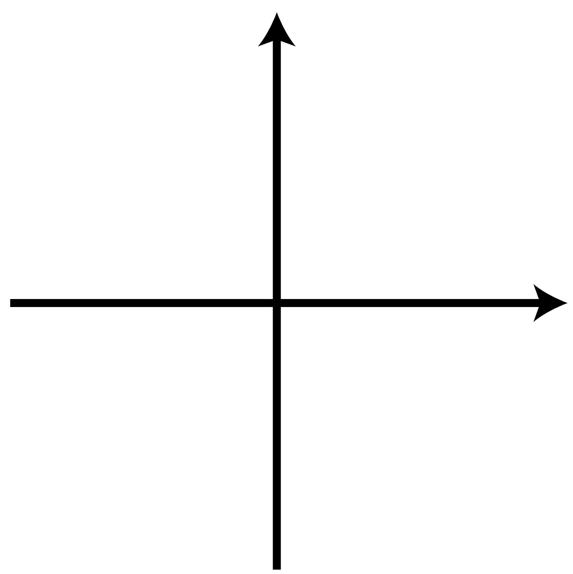

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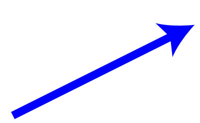

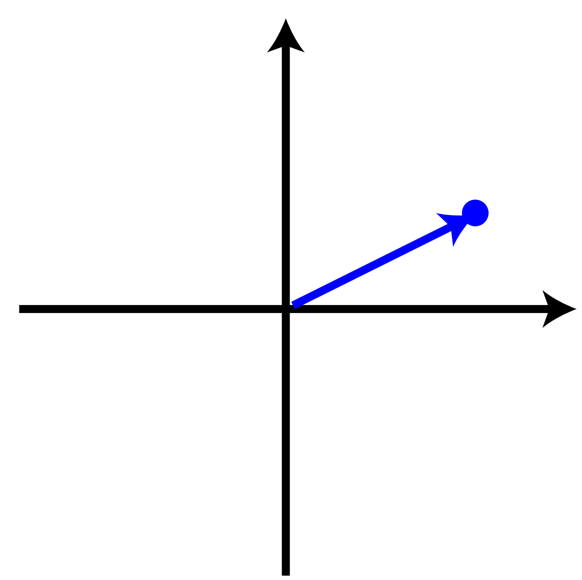

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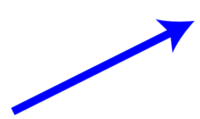

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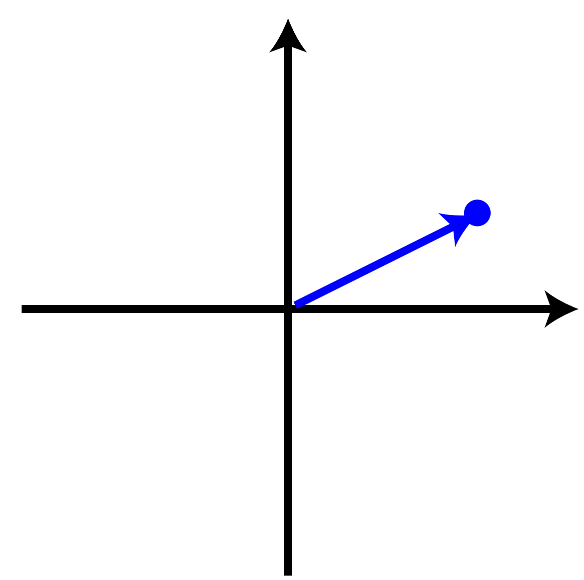

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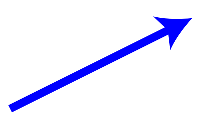

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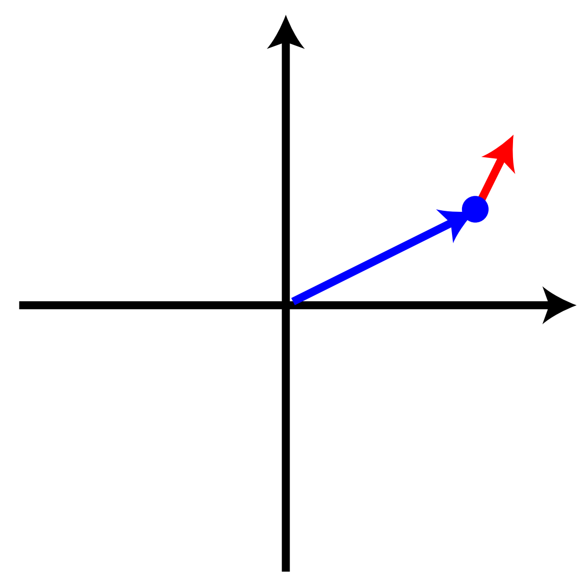

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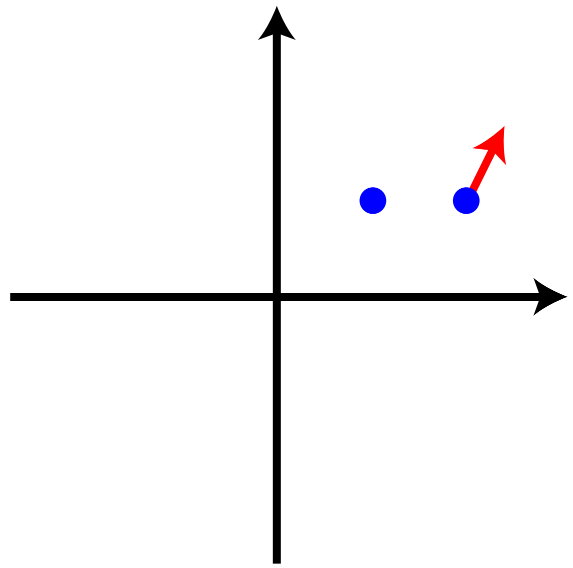

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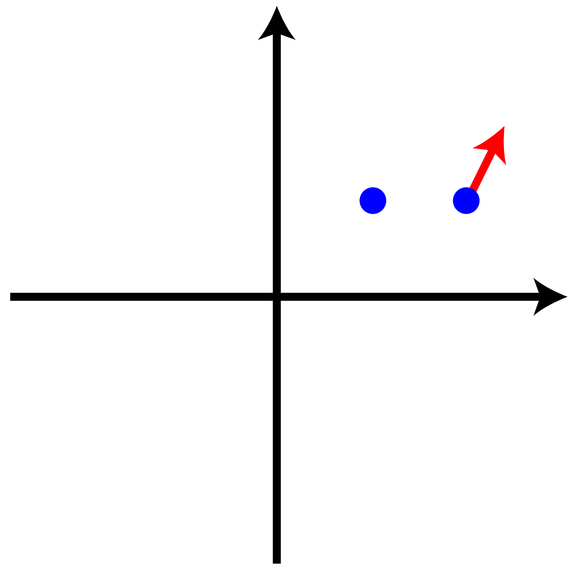

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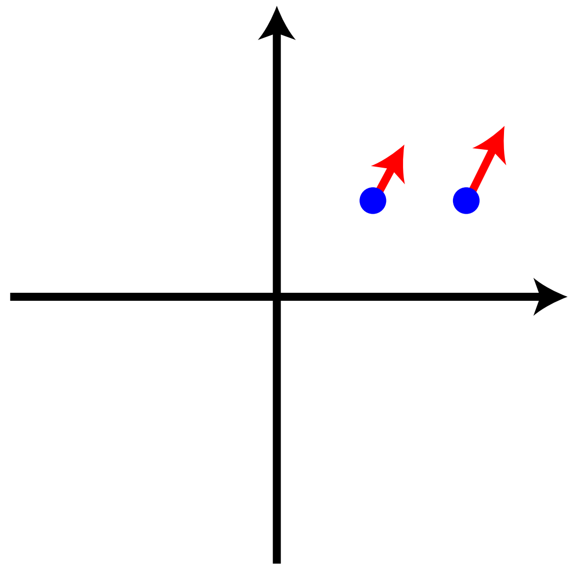

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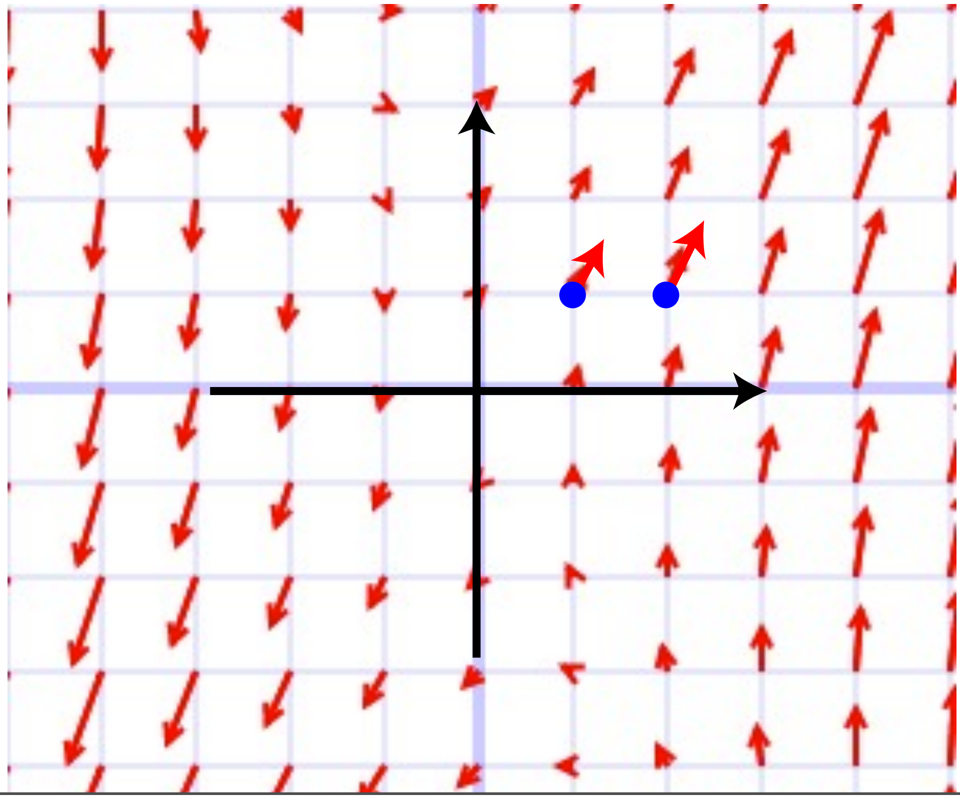

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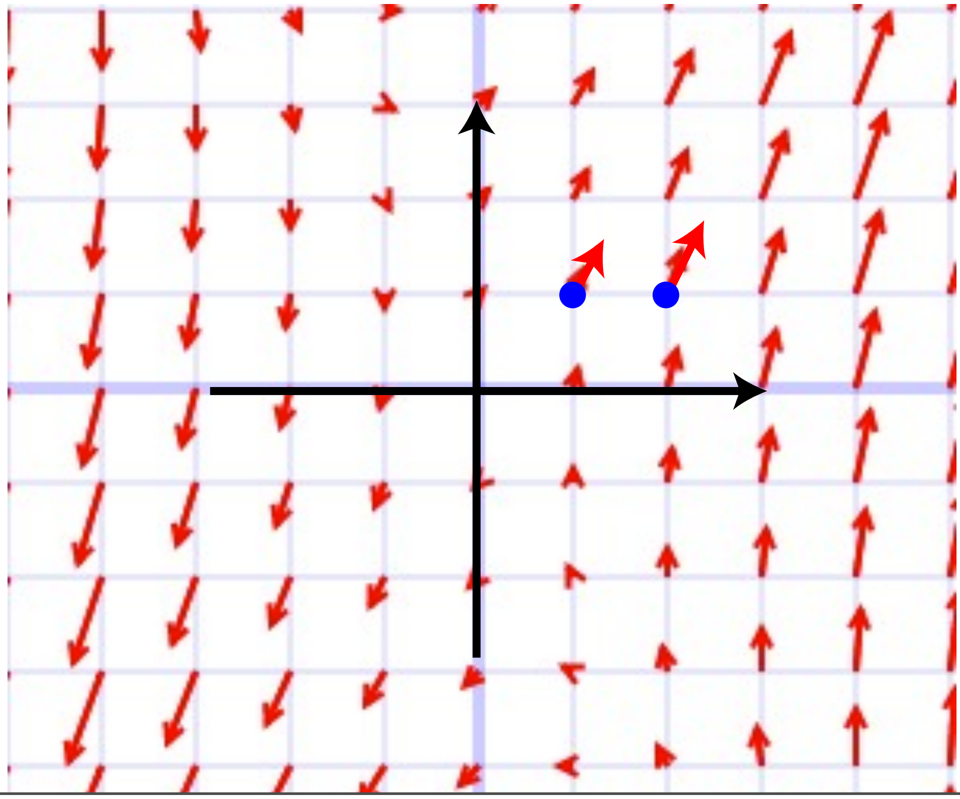

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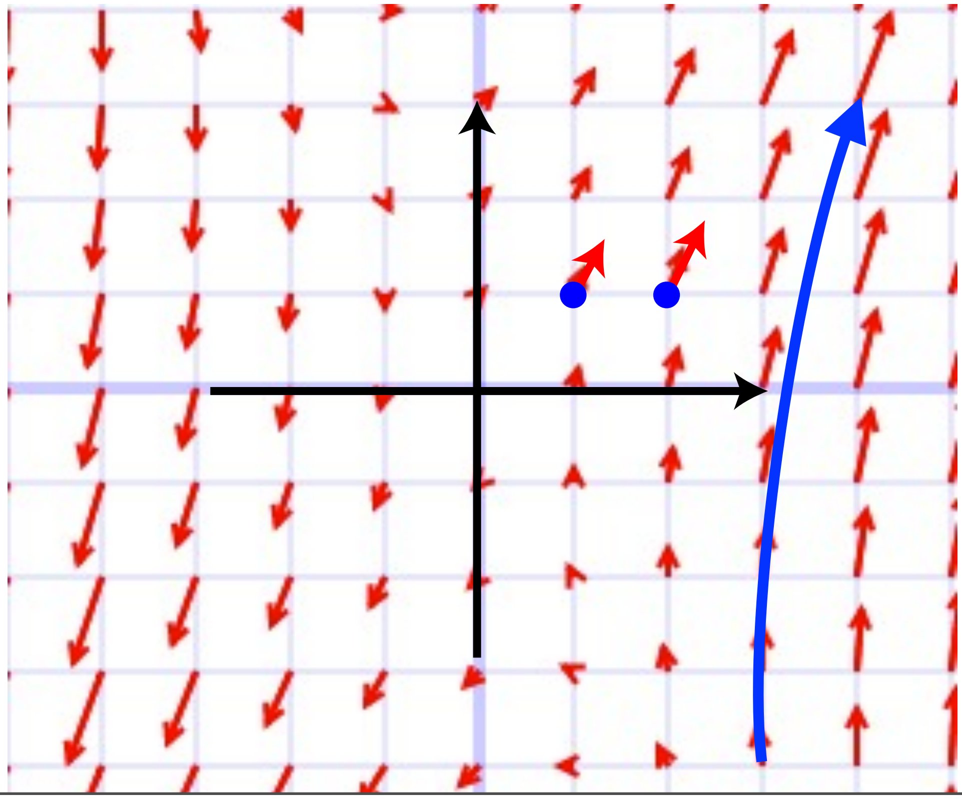

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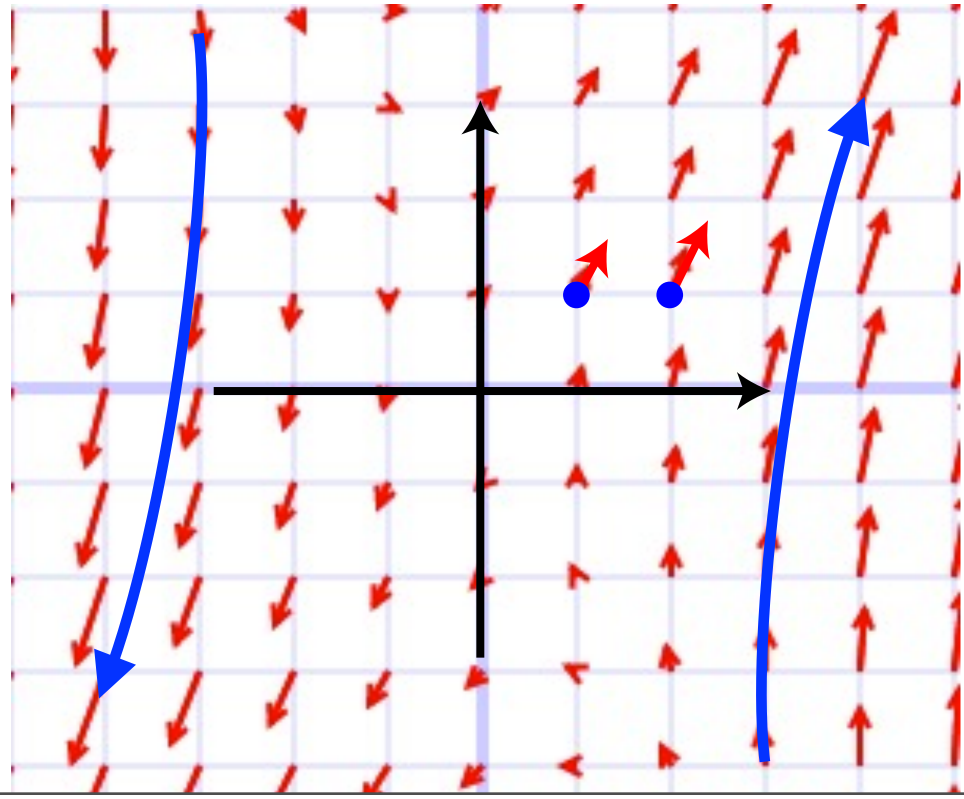

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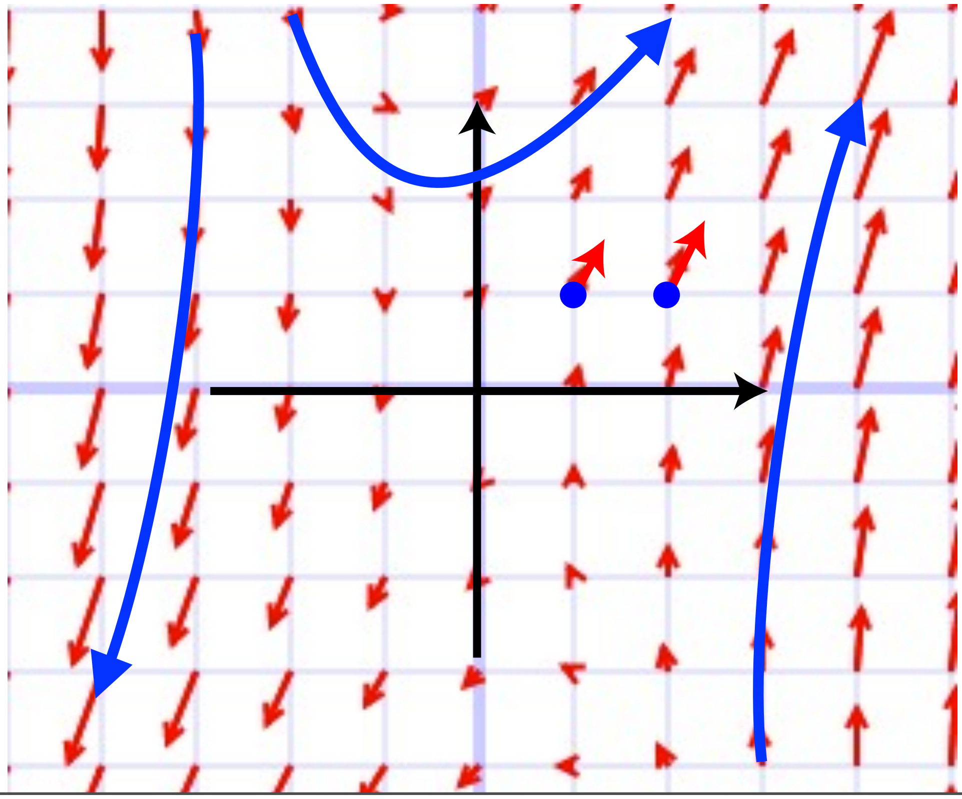

- Solutions must follow the arrows.

Introduction to systems of equations

- Geometric interpretation - direction fields.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \qquad \text{or} \qquad \mathbf{x}' = A\mathbf{x}$$

- Think of the unknown functions as coordinates $(x(t), y(t))$ of an object in the plane.
- $A\mathbf{x}$ gives the velocity vector of the object located at $\mathbf{x}$.

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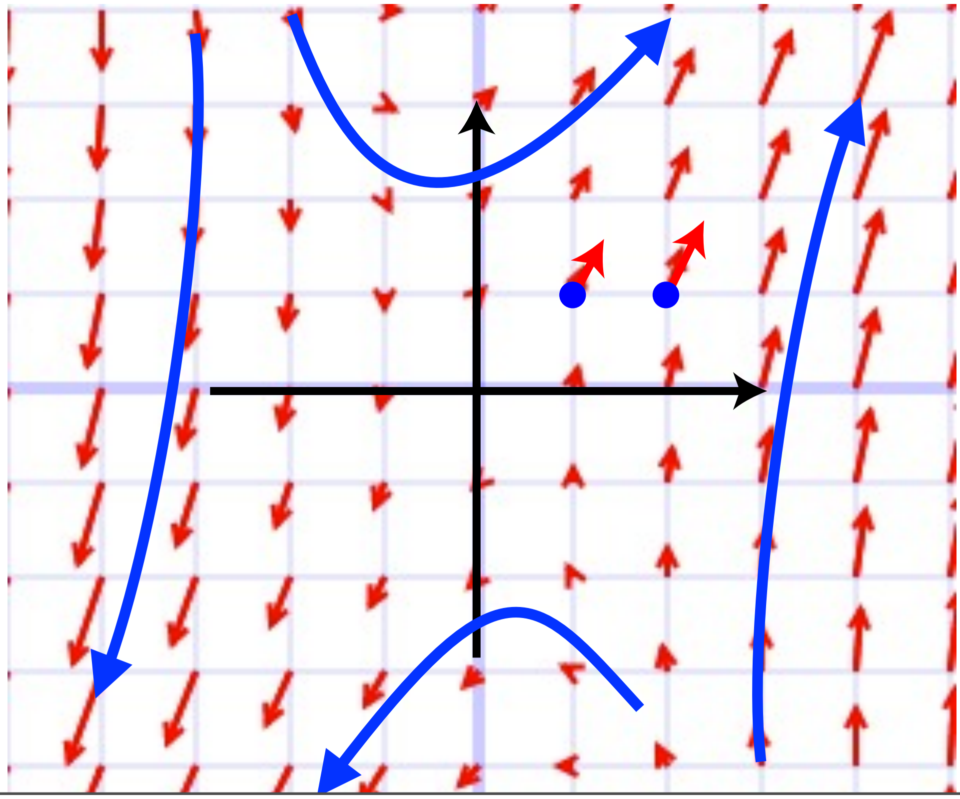

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Introduction to systems of equations

- Which of the following equations matches the given direction field?

(A) $\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

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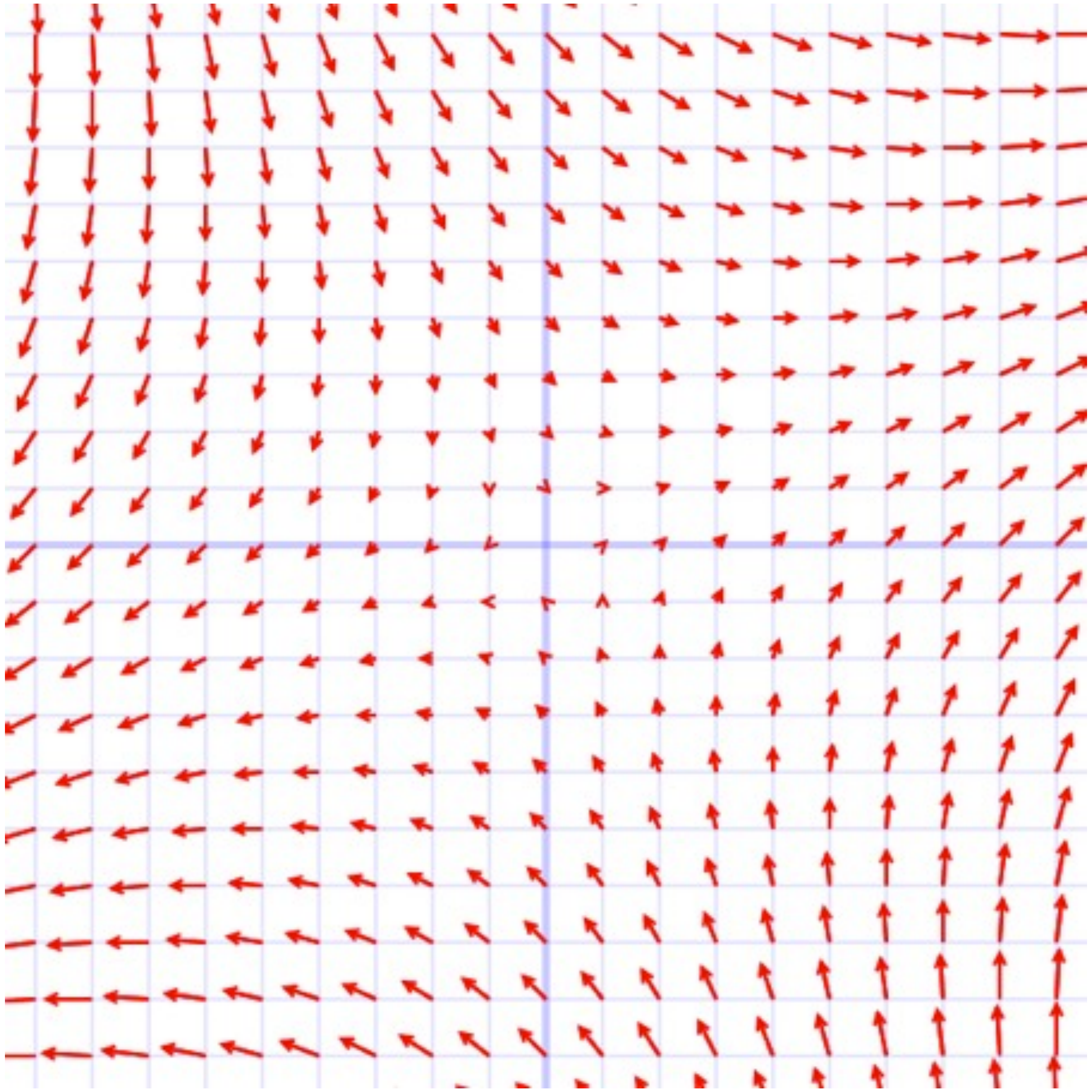
(E) Explain, please.



http://kevinmehall.net/p/equationexplorer/vectorfield.html#(x+y)i+(x-y)j%7C%5B-10,10,-10,10%5D

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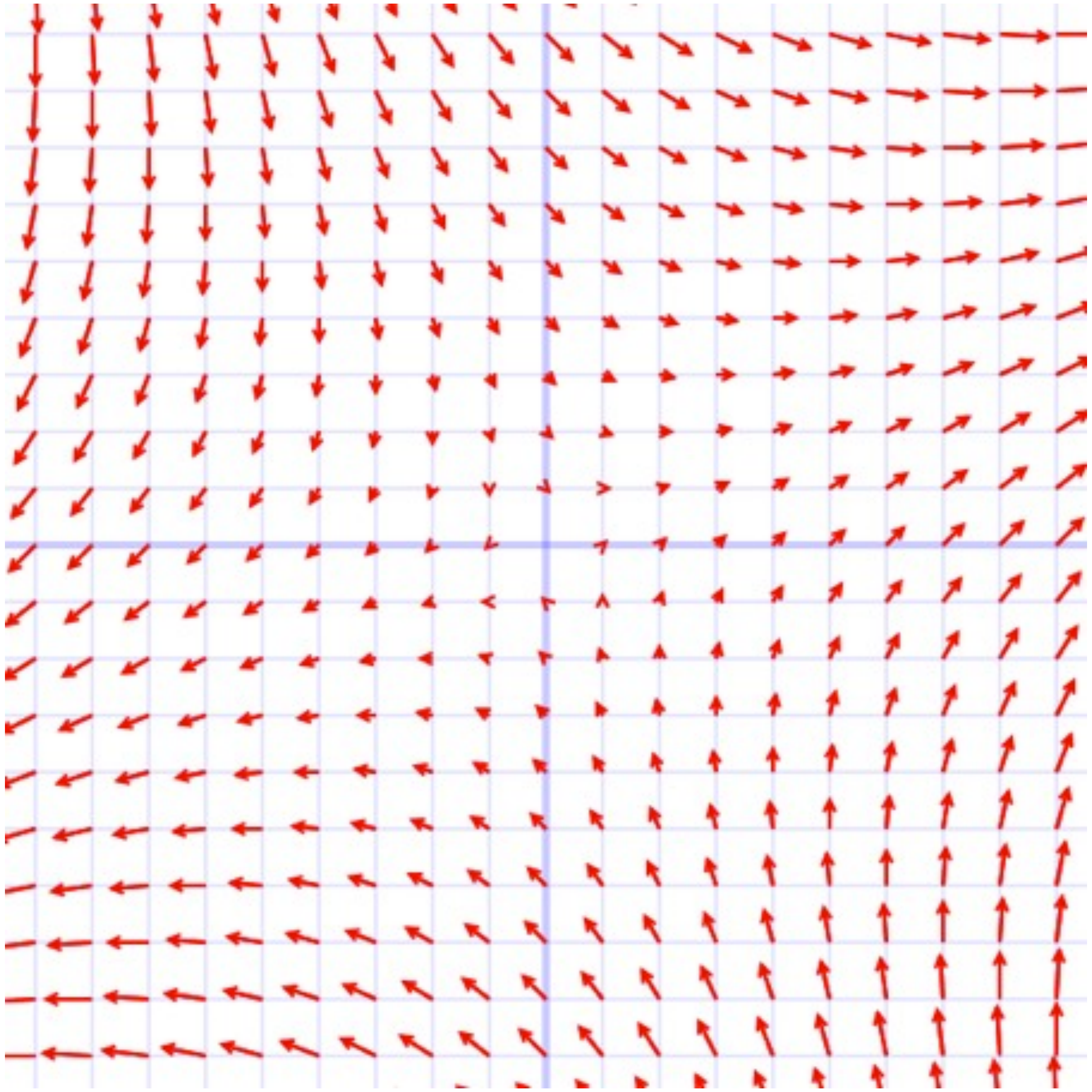

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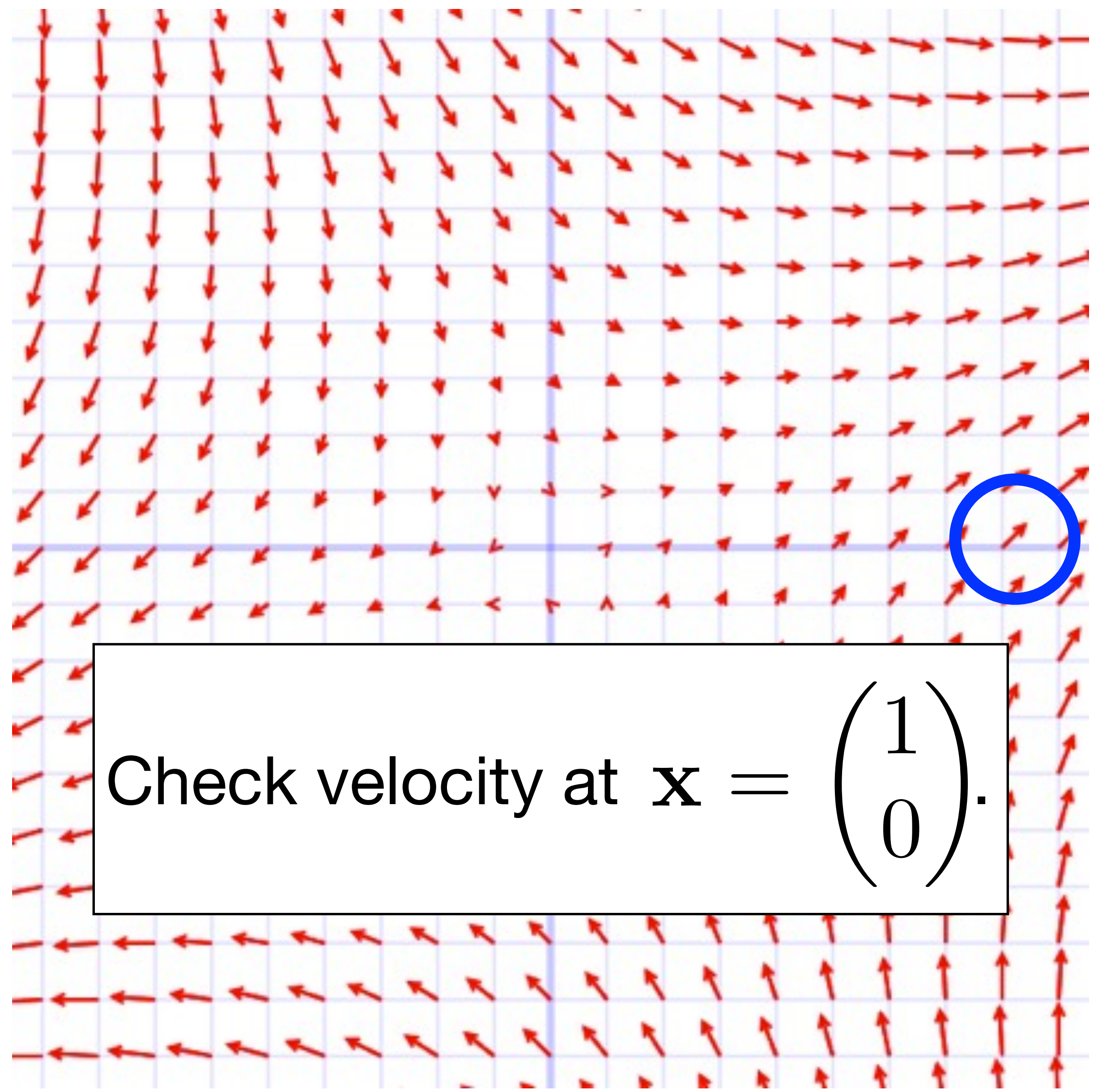

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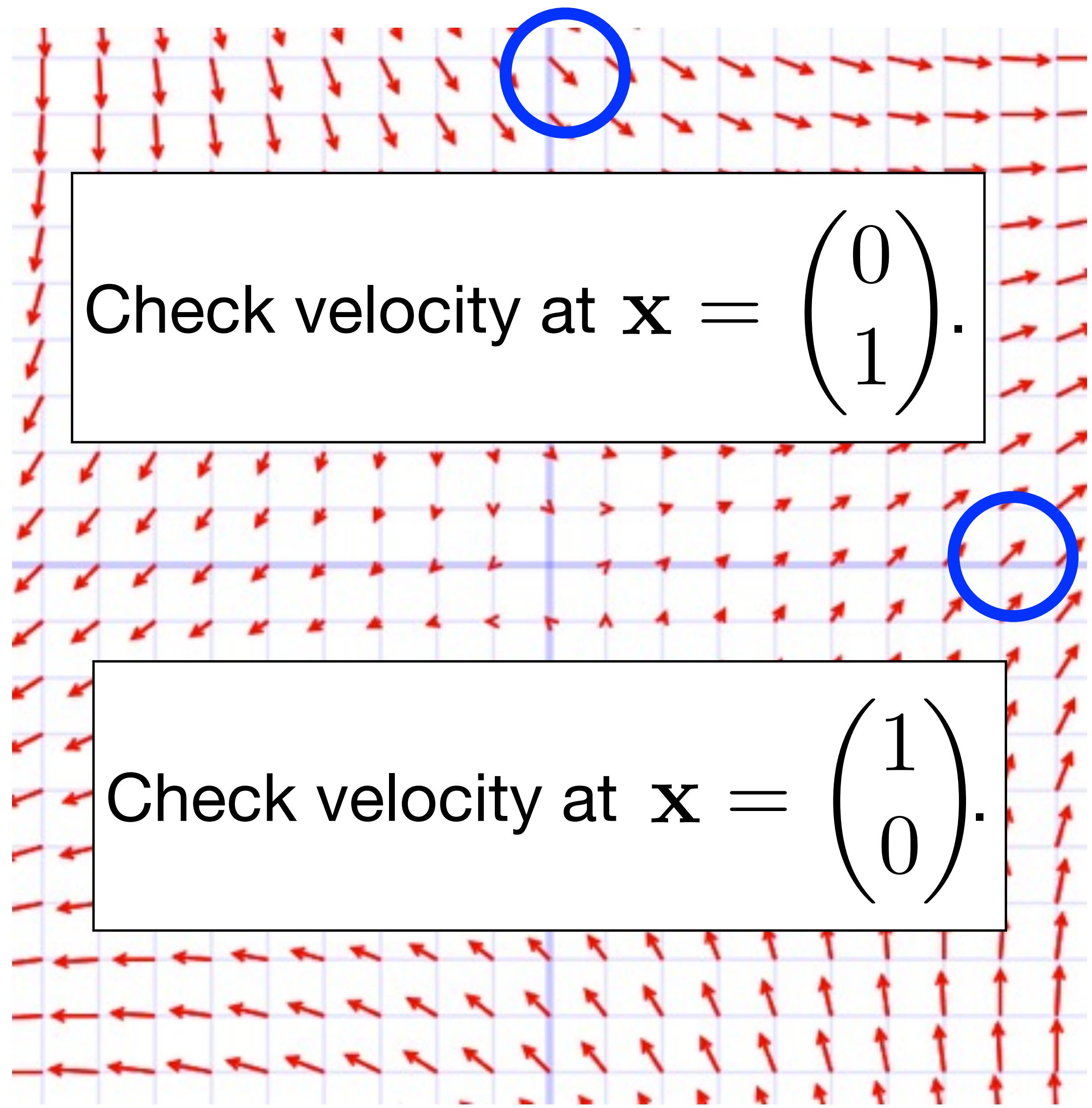

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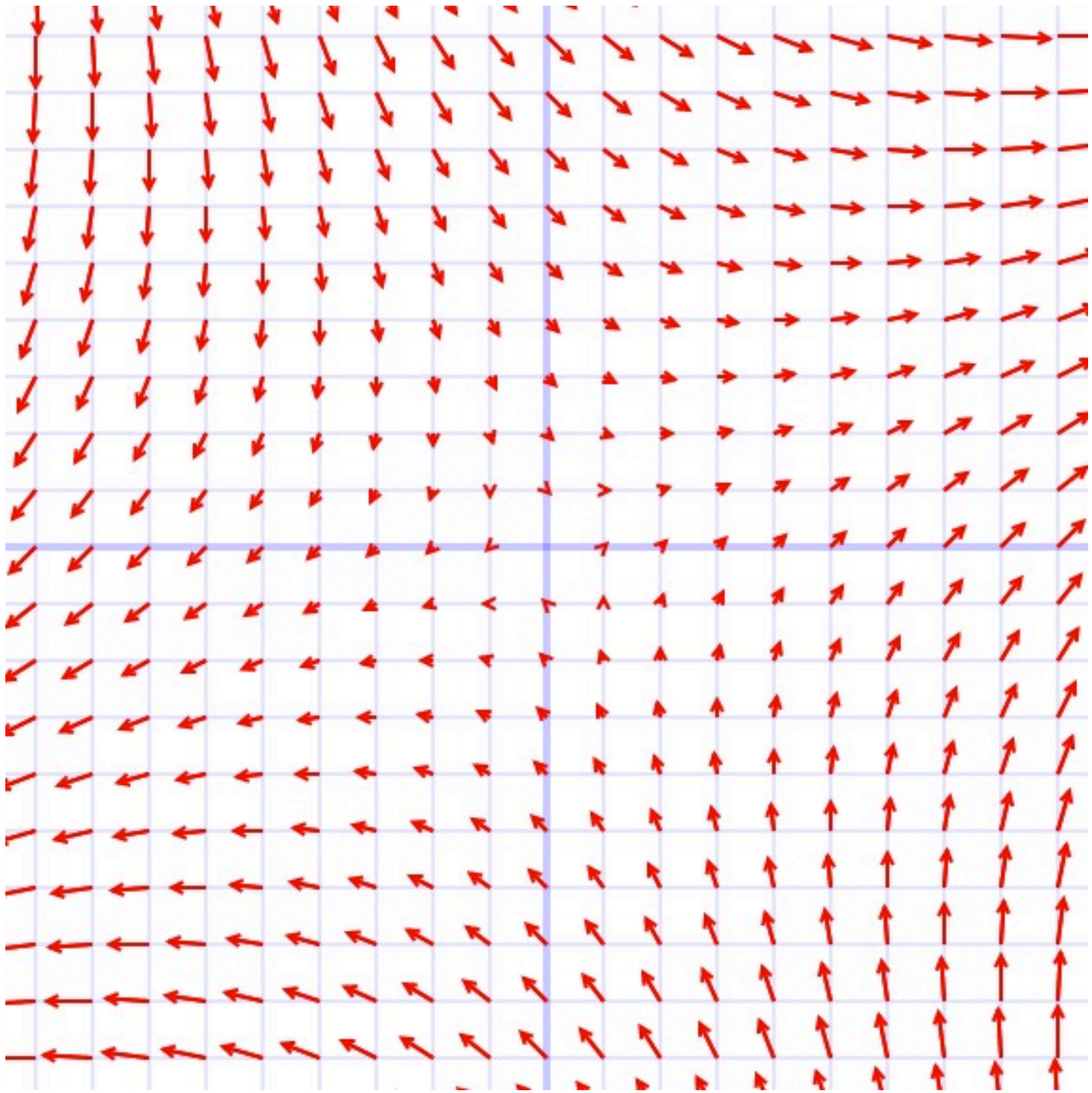
Introduction to systems of equations

- You should see two "special" directions.
- What are they?
- Directions along which the velocity vector is parallel to the position vector.
- That is, .



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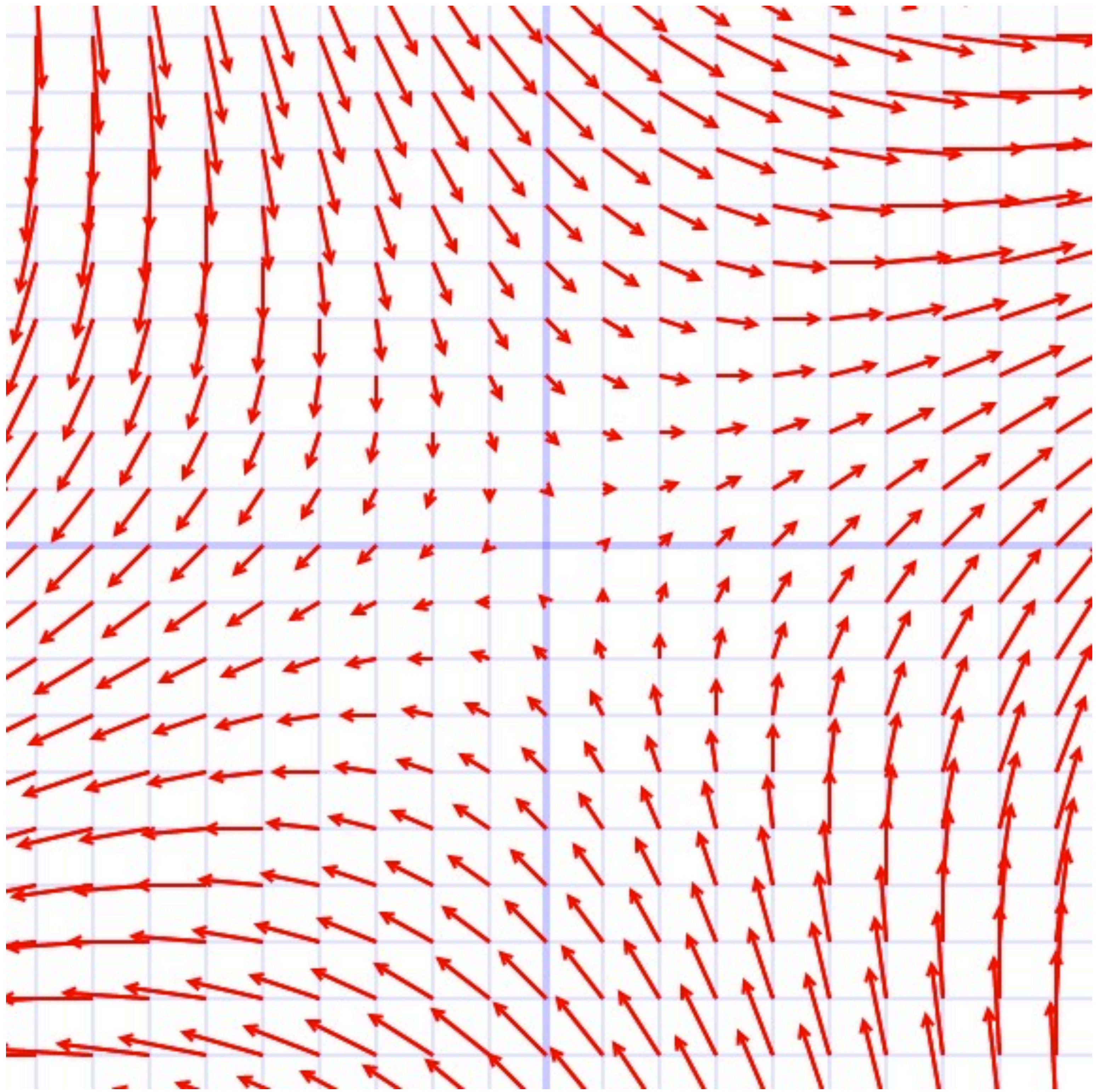

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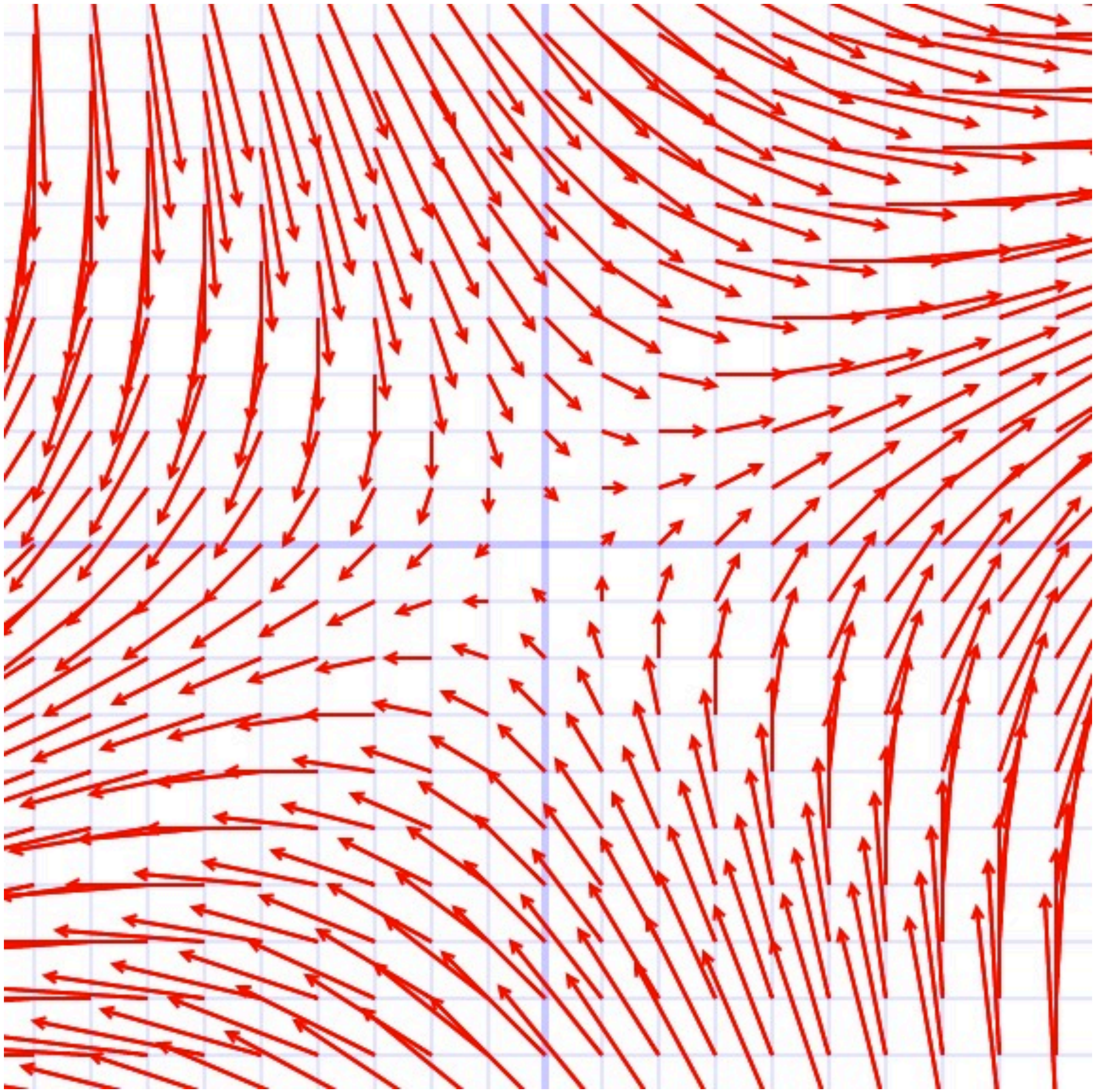

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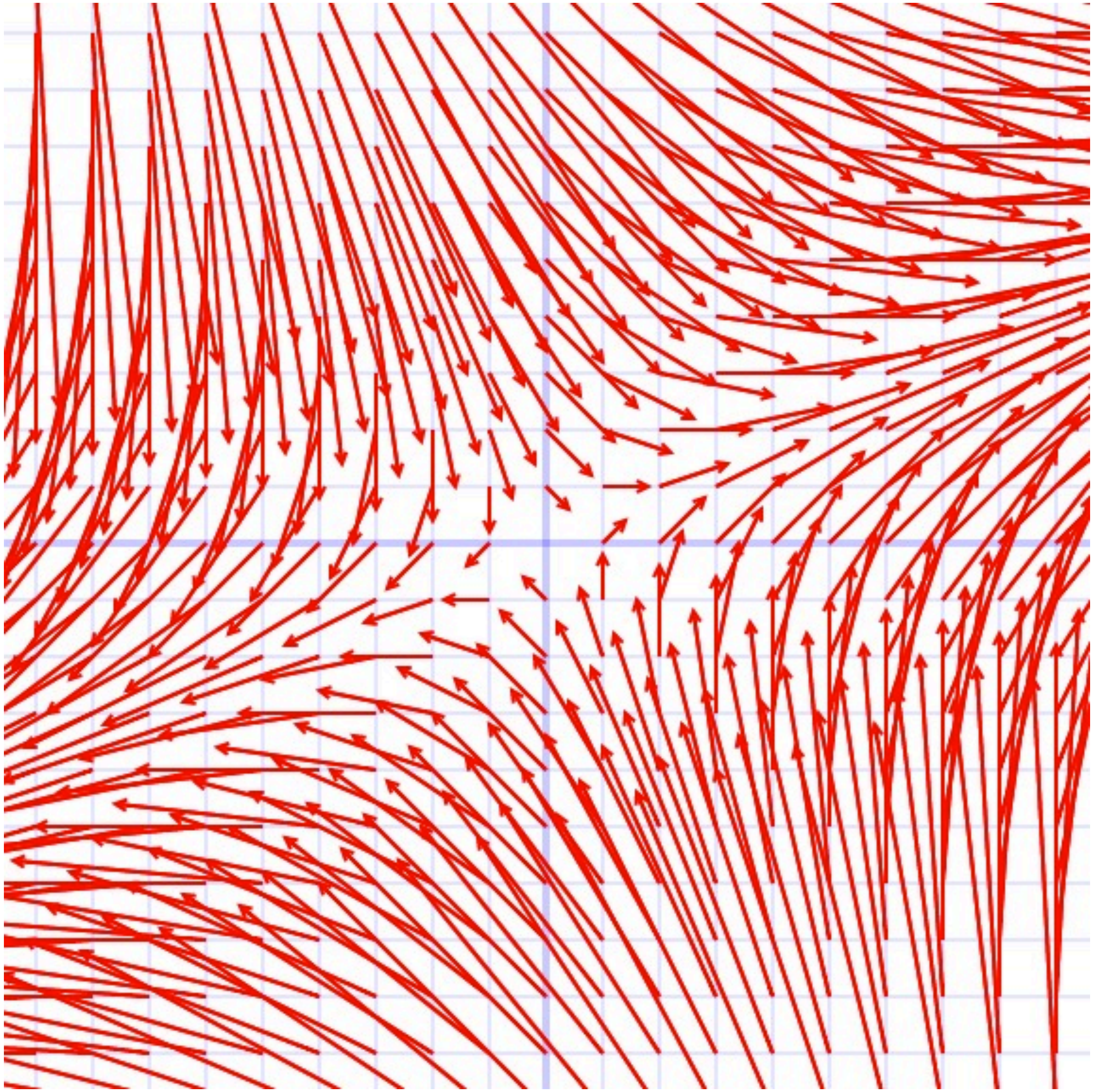

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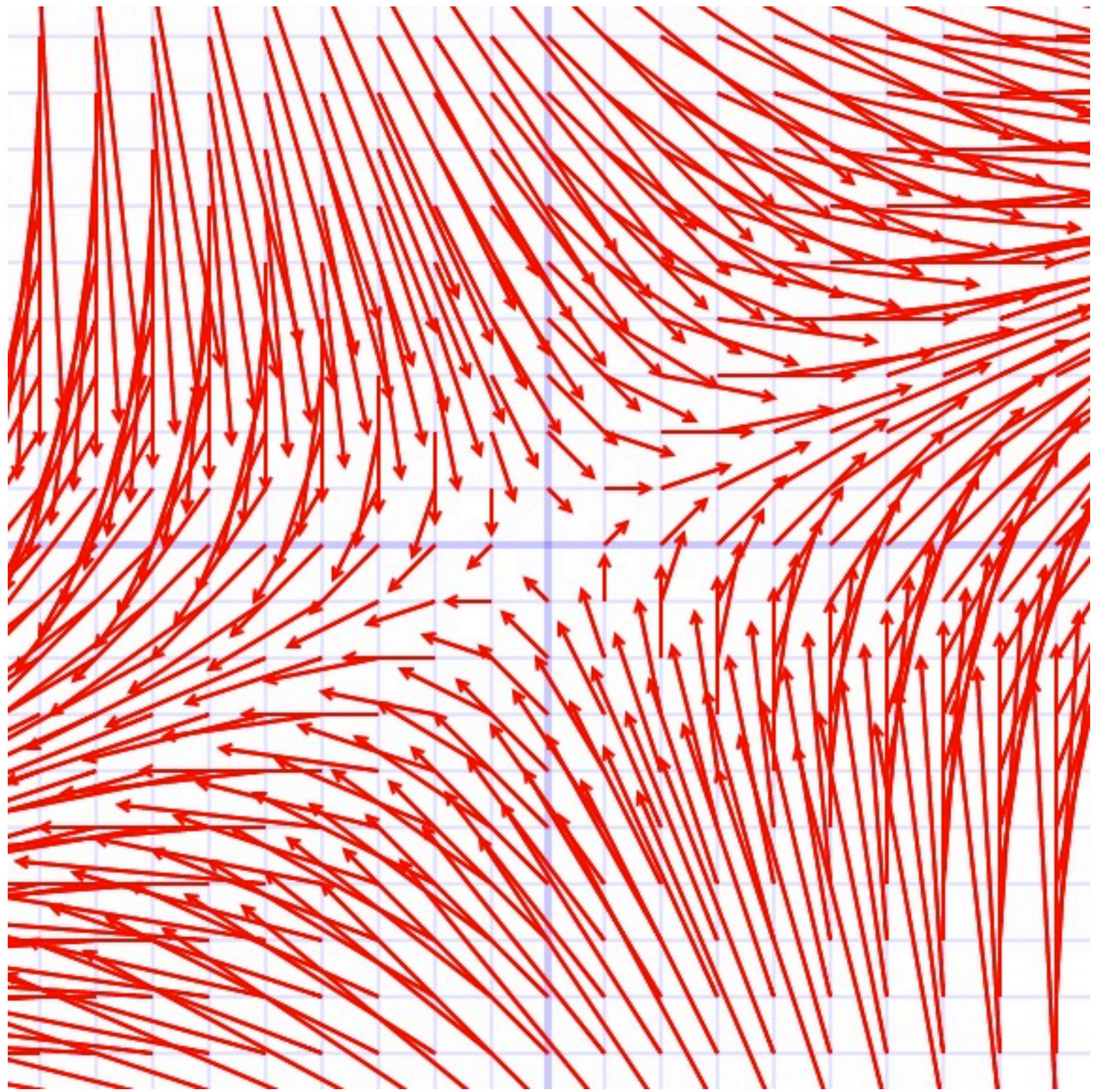

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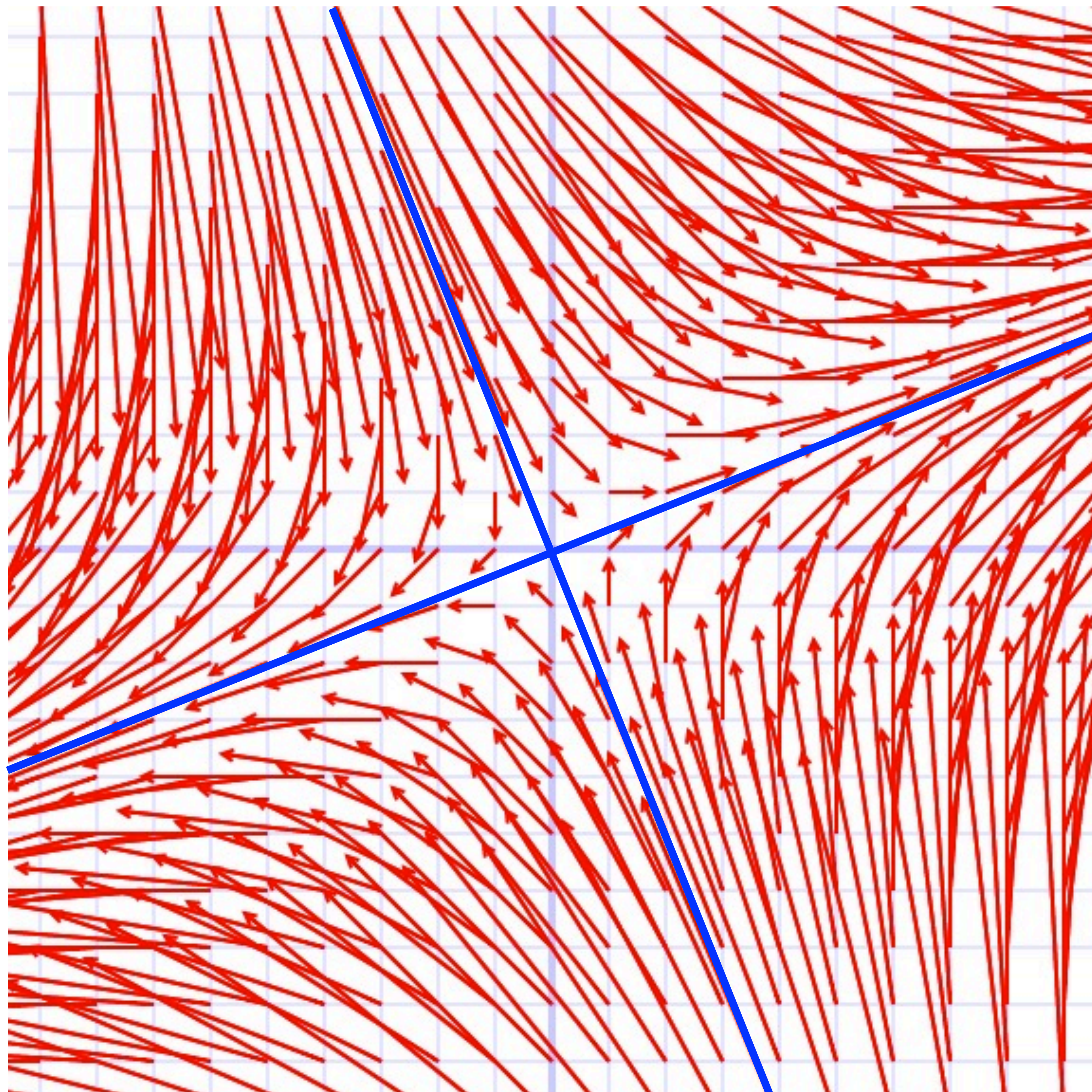

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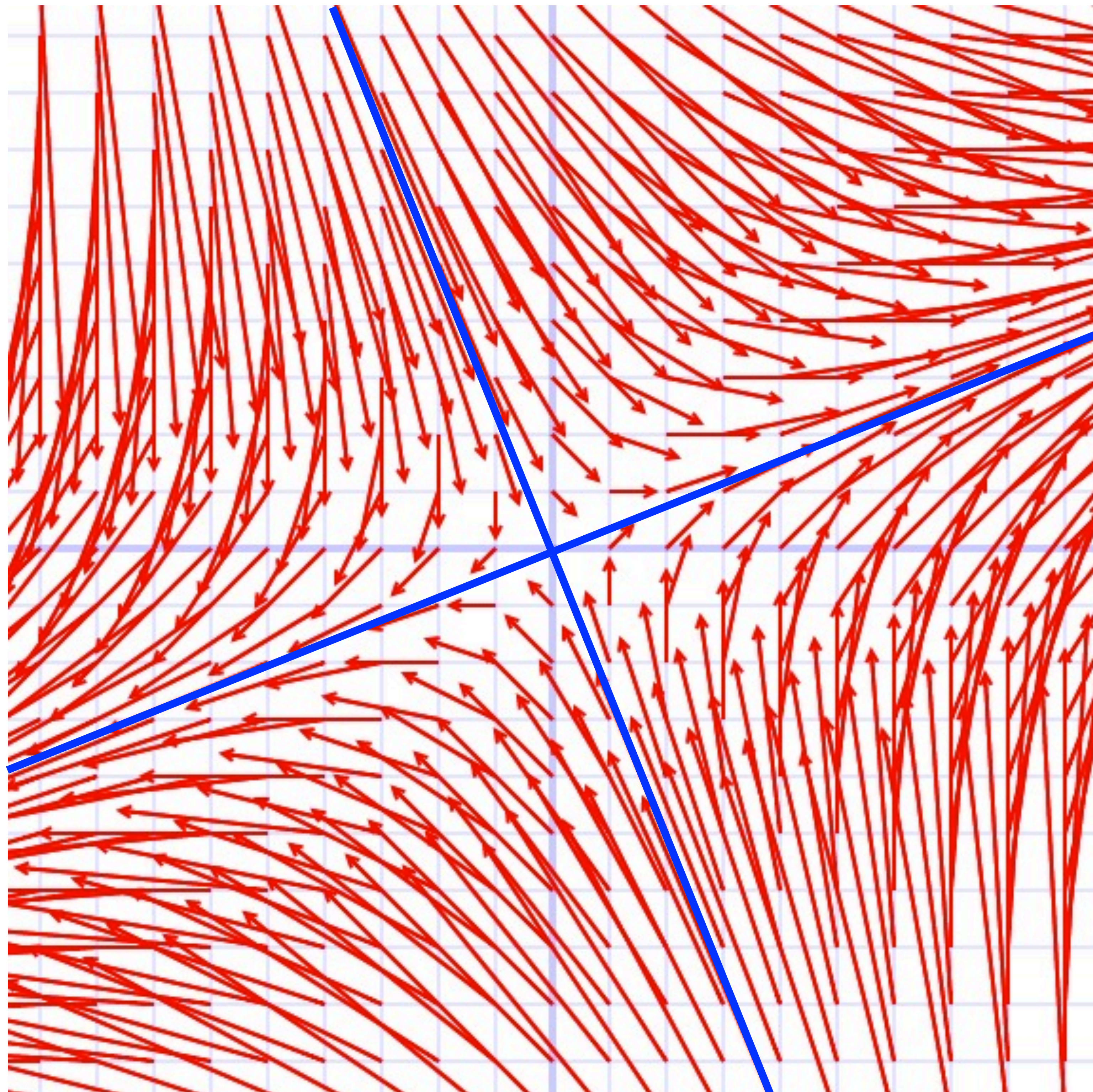

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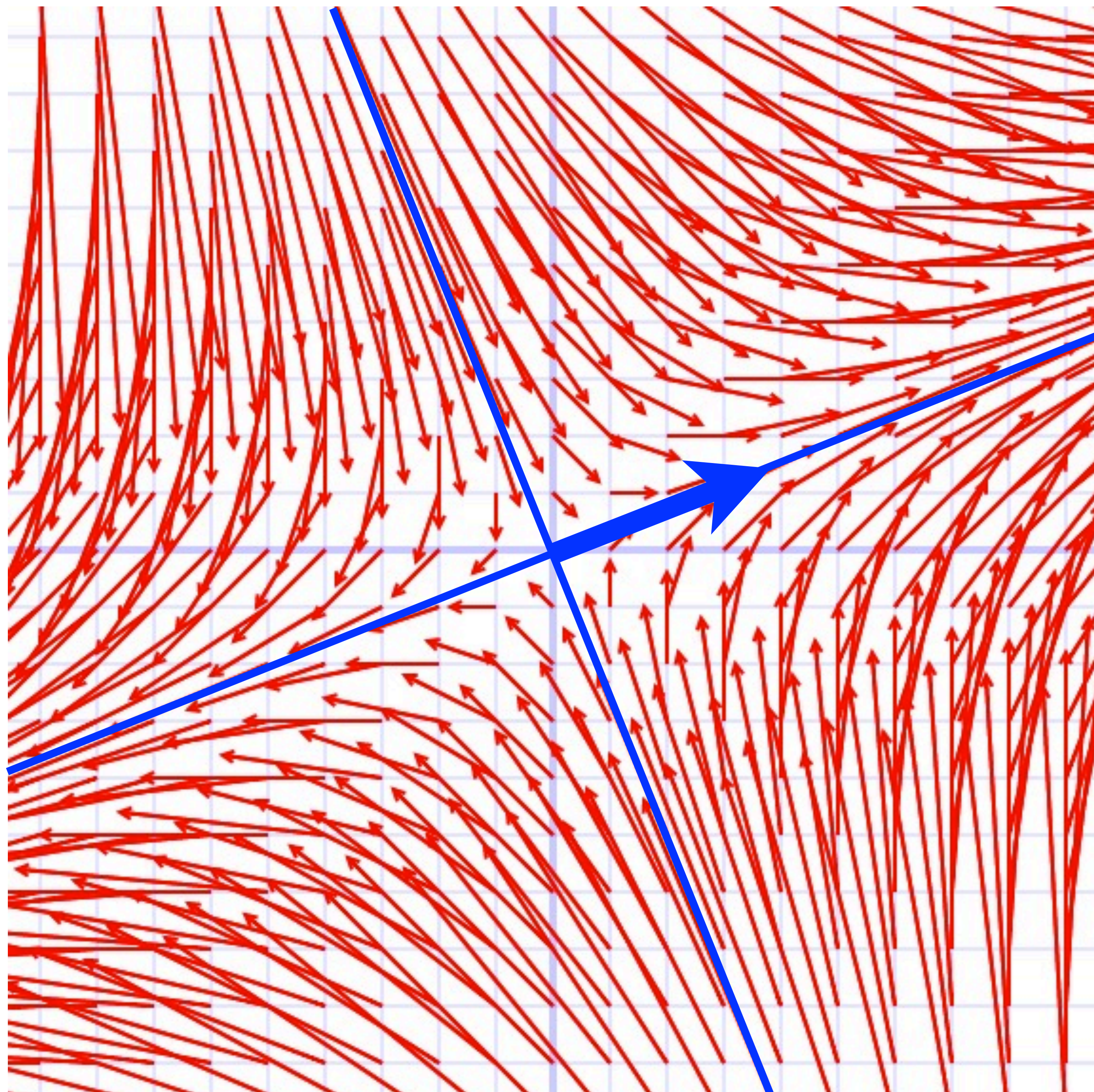

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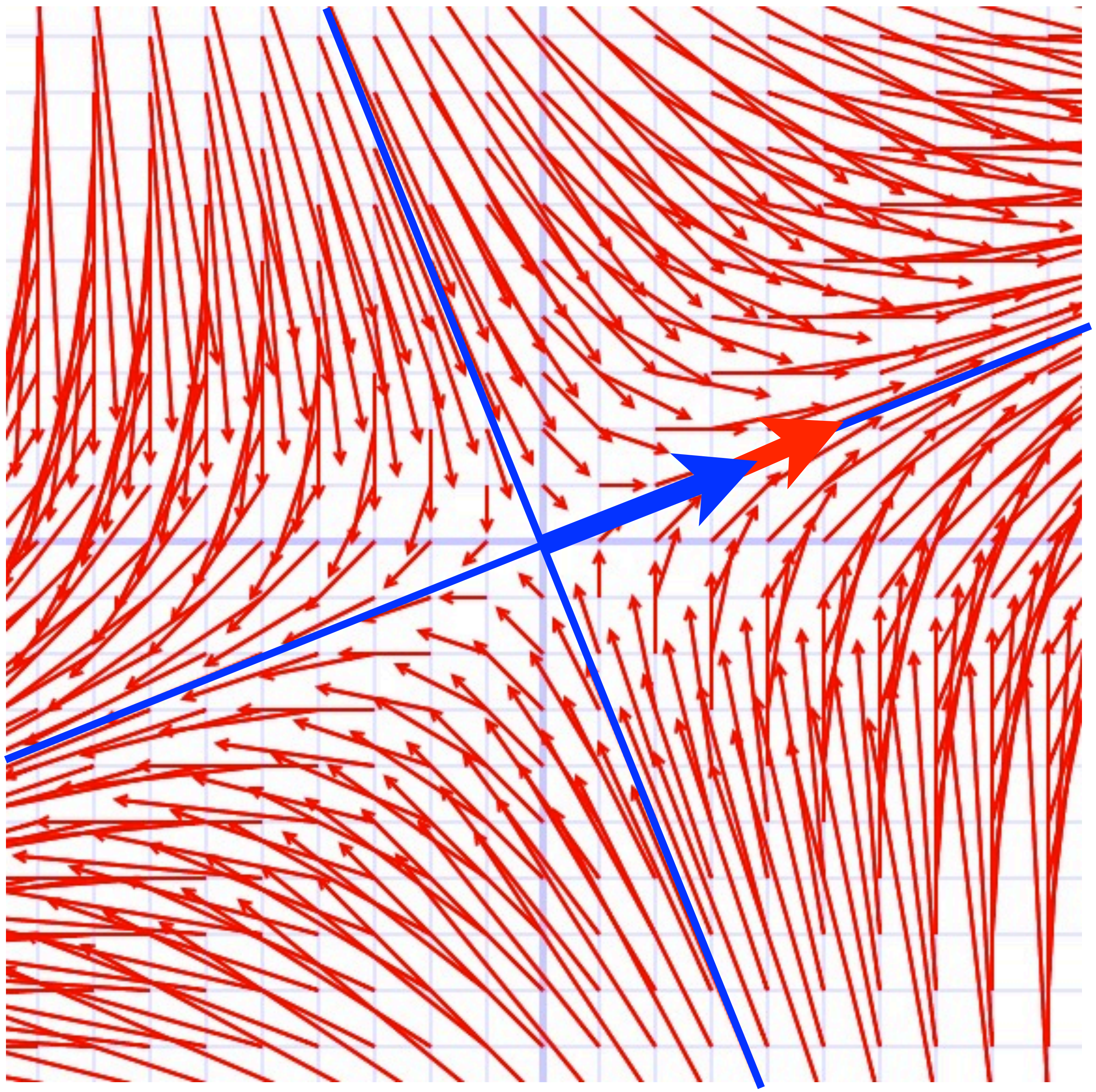

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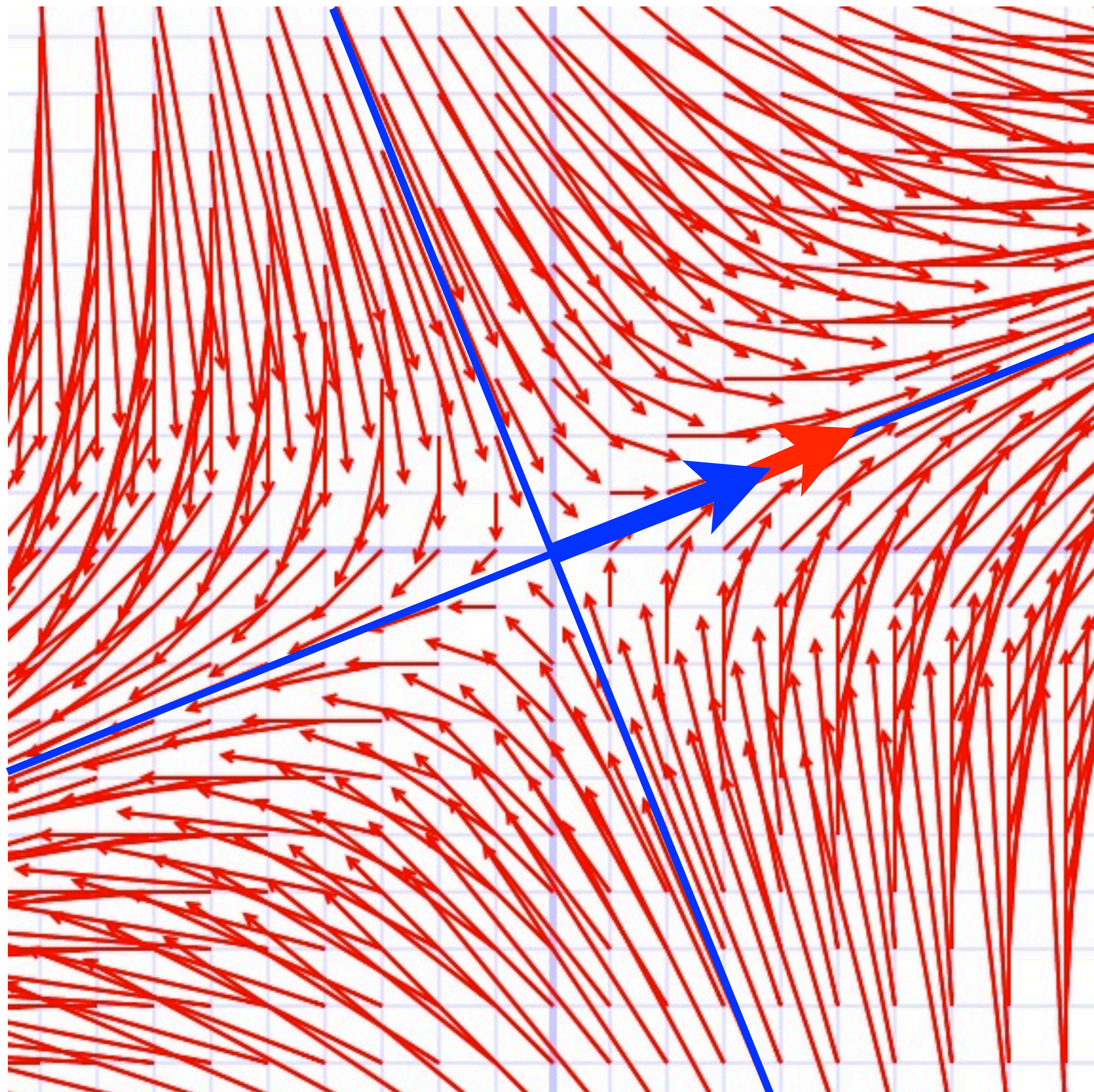
$$\lambda_1 = \sqrt{2}$$

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ \sqrt{2} - 1 \end{pmatrix}$$



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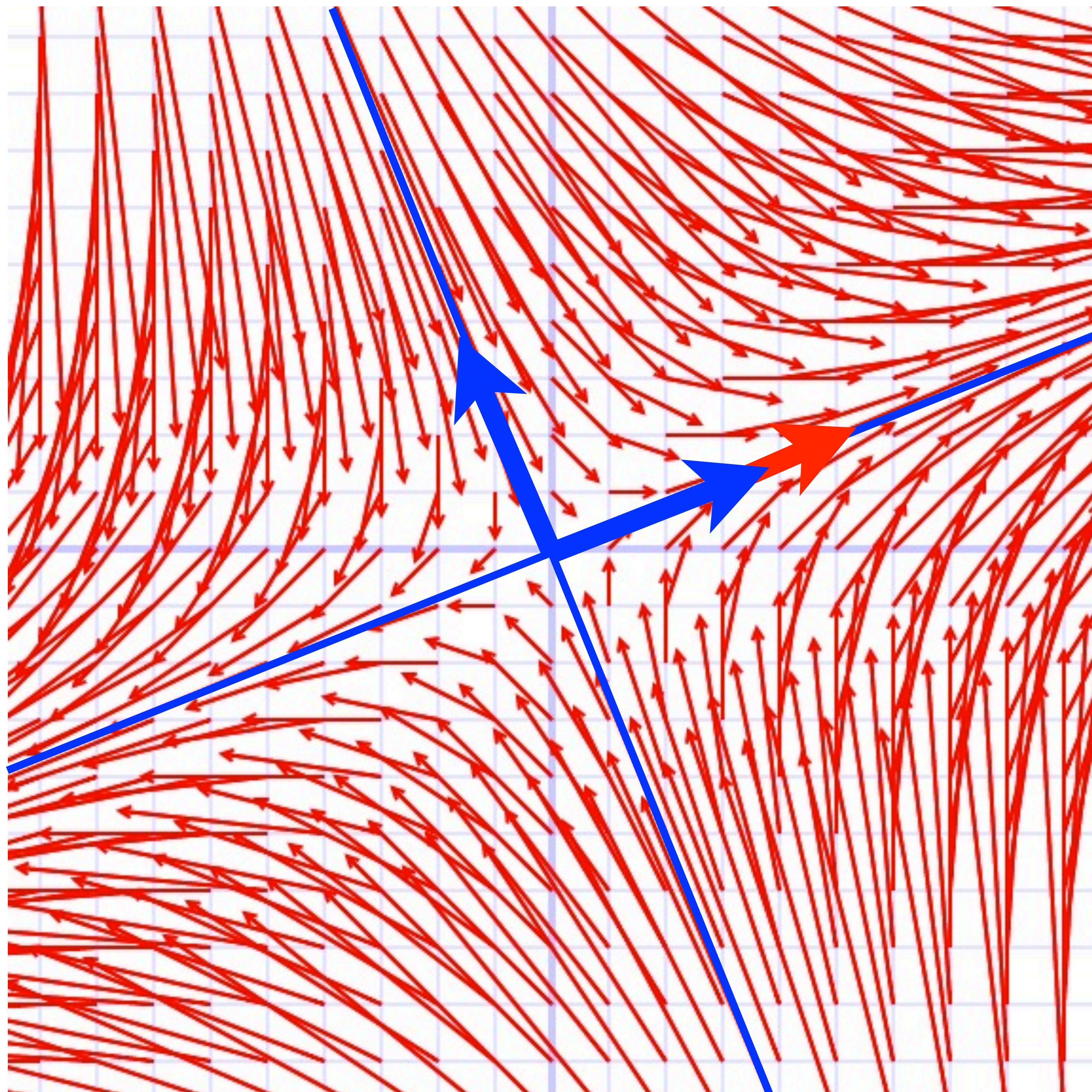

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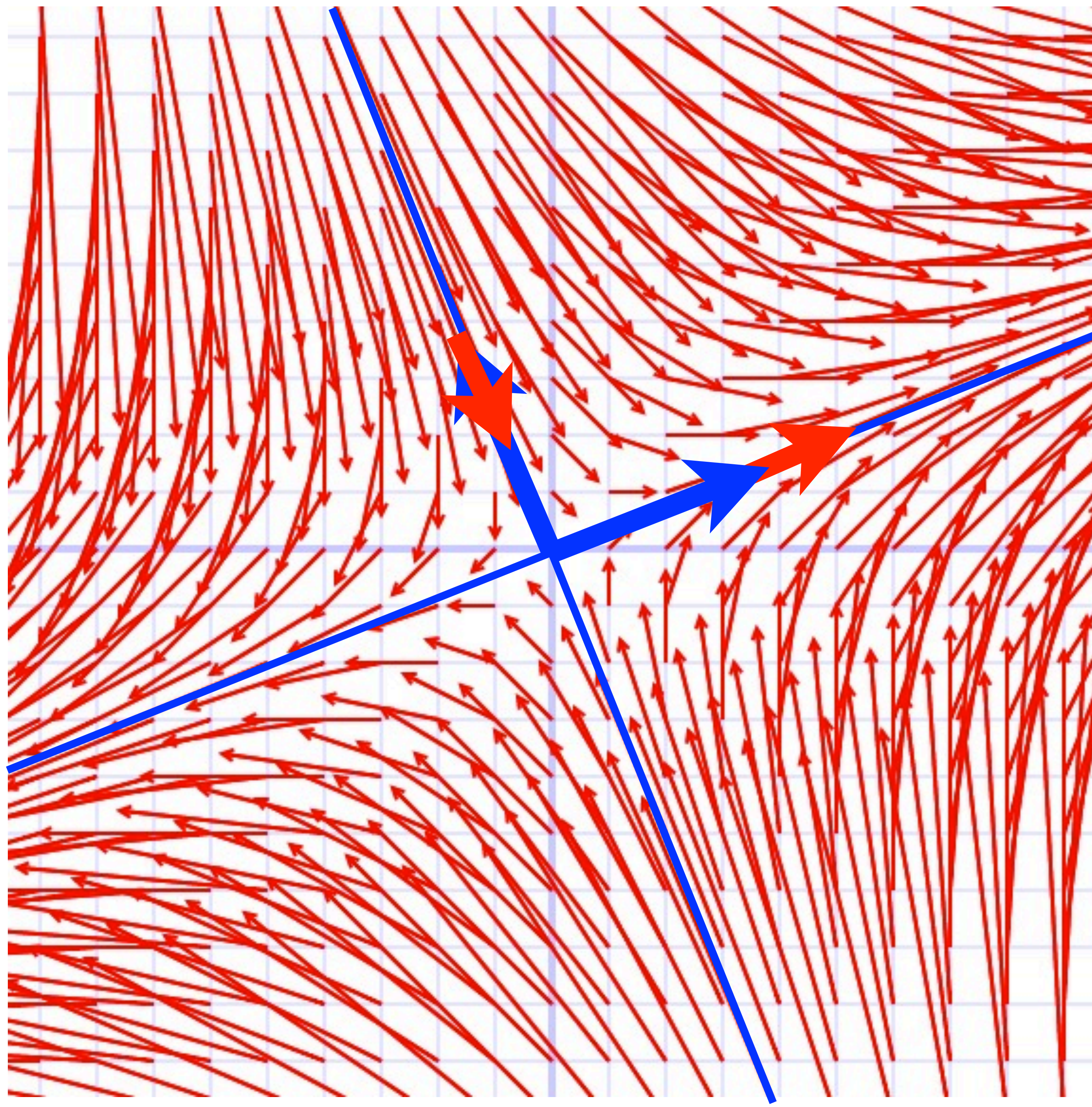

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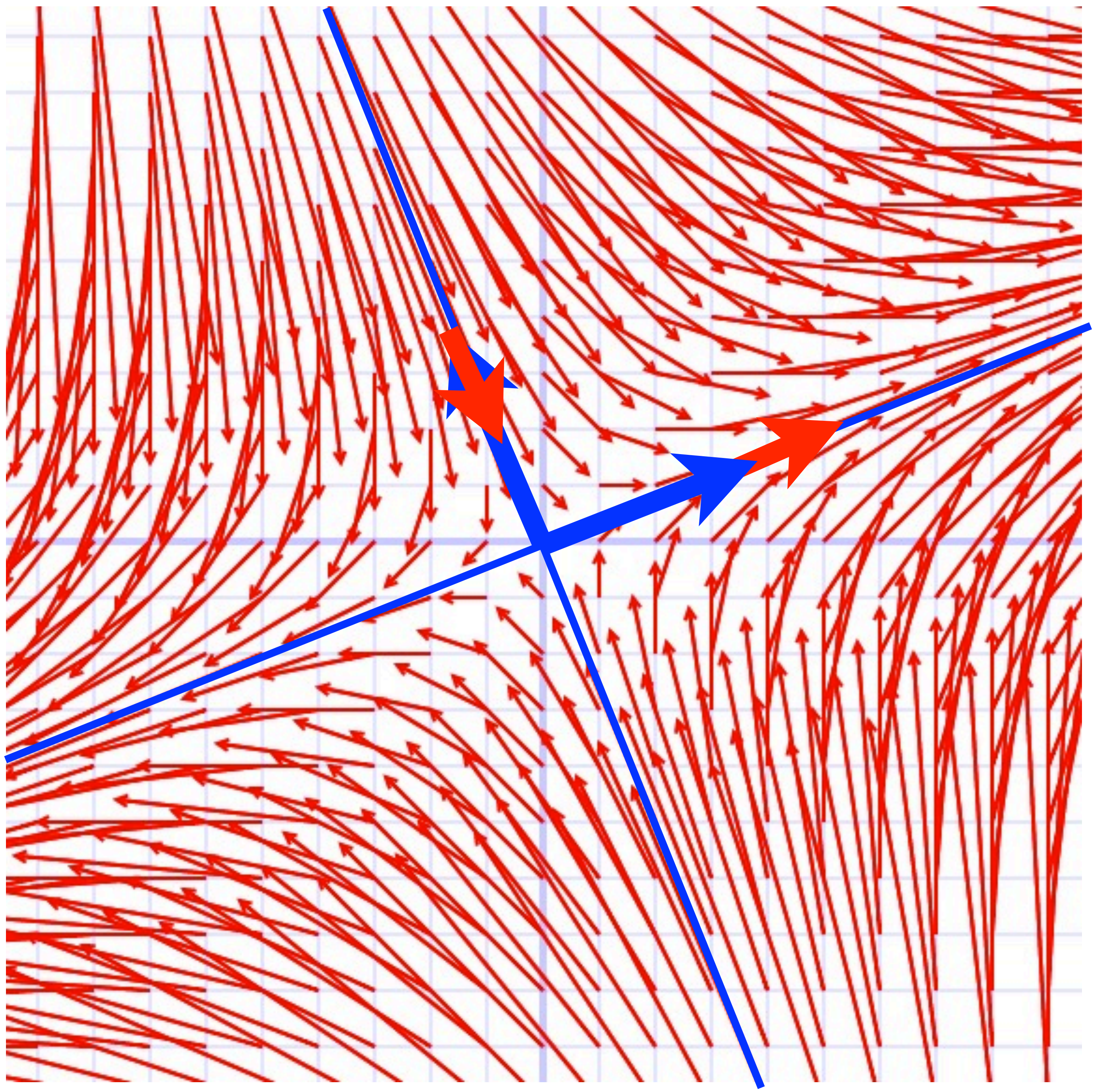

Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

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(E) Explain, please.

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(and any scalar multiple of it)

Matrix review (eigen-calculations)

- Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.
- Looking for values λ and vectors $\mathbf{v}$ for which $A\mathbf{v} = \lambda\mathbf{v}$.

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- How do we use eigenvalues and eigenvectors to construct a general solution?

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    - Other cases (not enough e-vectors or complex e-values) Thursday.