Today

- Midterm - avg 72%, 11 fails, 12 in the 90%s, range 7%-100%.
- Chemical diffusion in a long narrow tube/rod.
    - Eigenvalues and eigenvectors in a discrete version (matrix problem).
    - Eigenvalues and eigenvectors in a continuous version (DE problem).
- Does the continuous version have a complete set of eigenvectors?
    - Fourier sine and cosine series

Diffusion in a long thin tube

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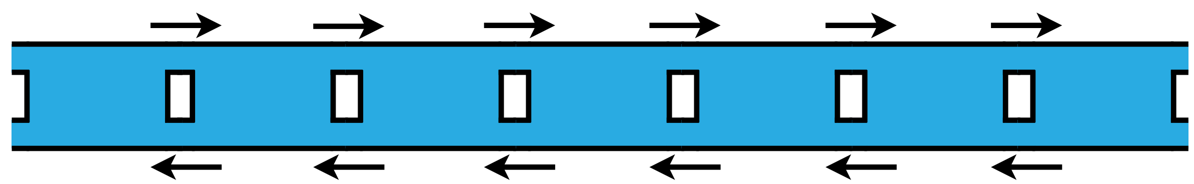

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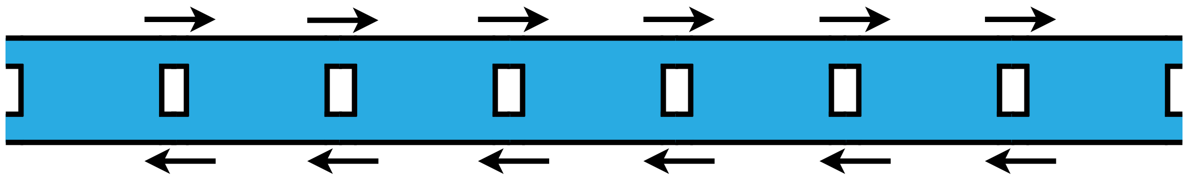

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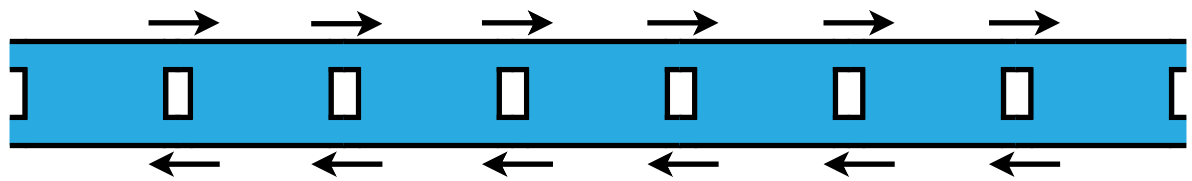

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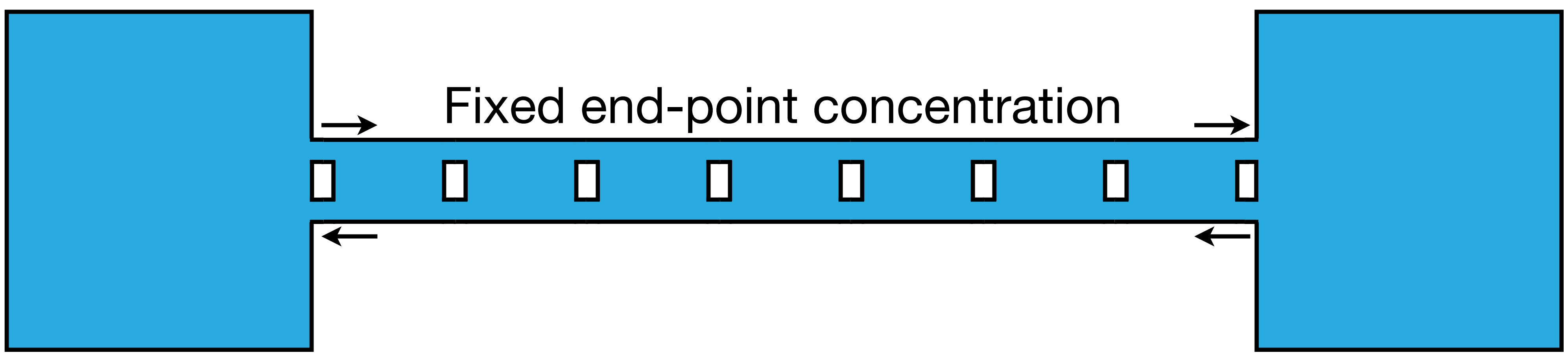

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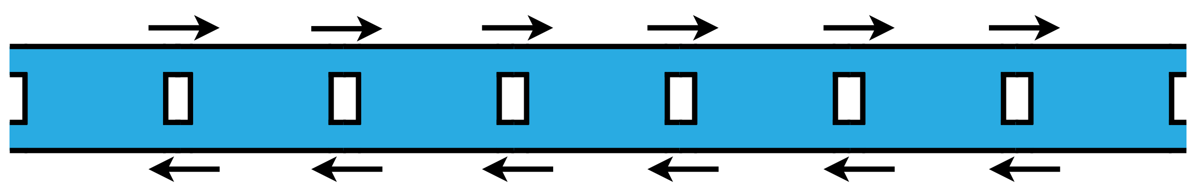

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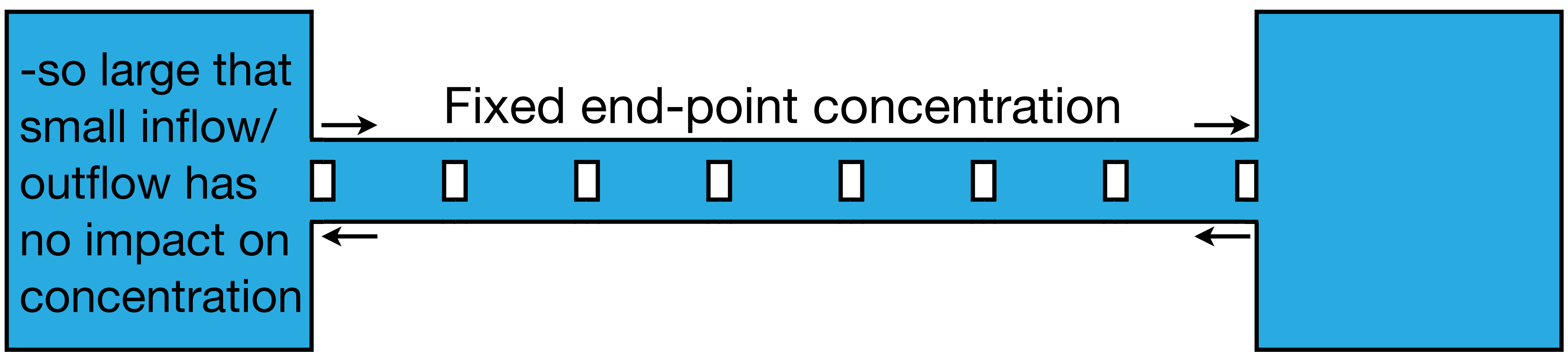

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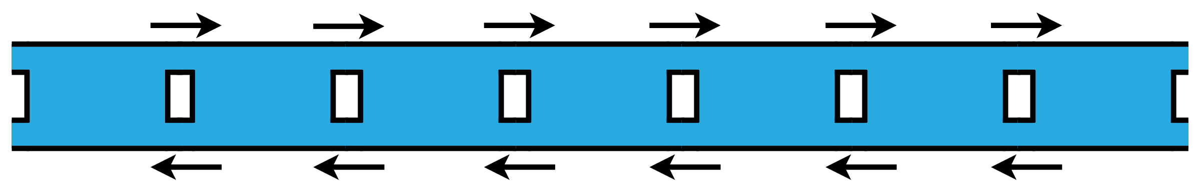

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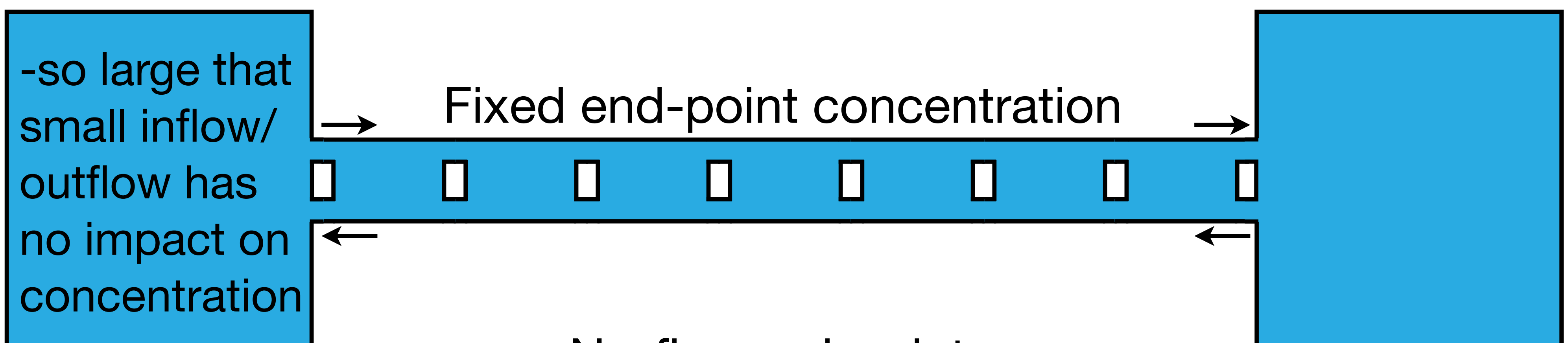

No-flux end-points

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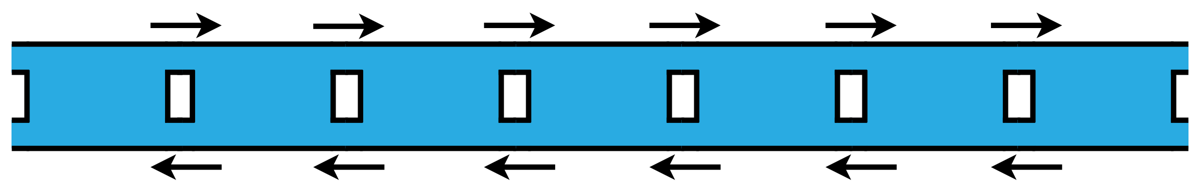

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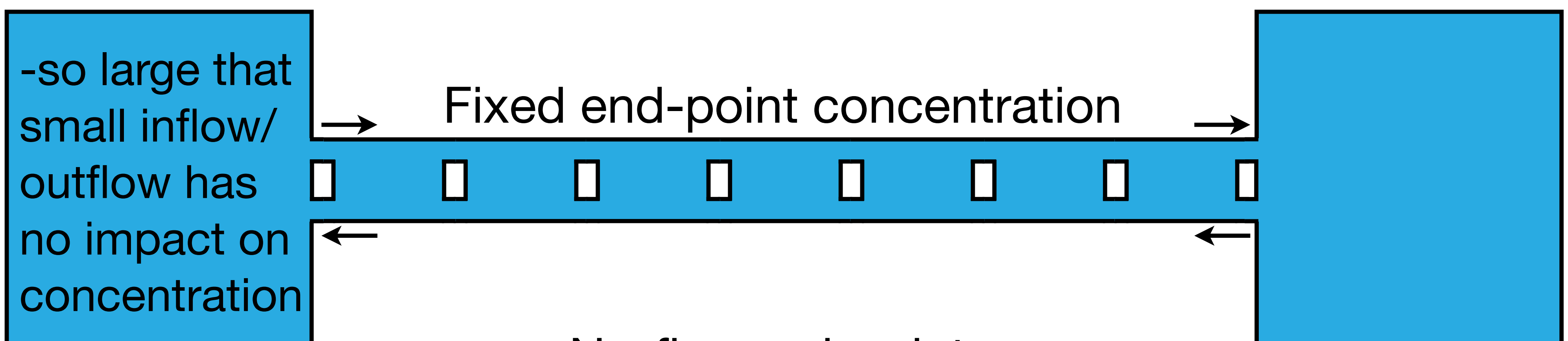

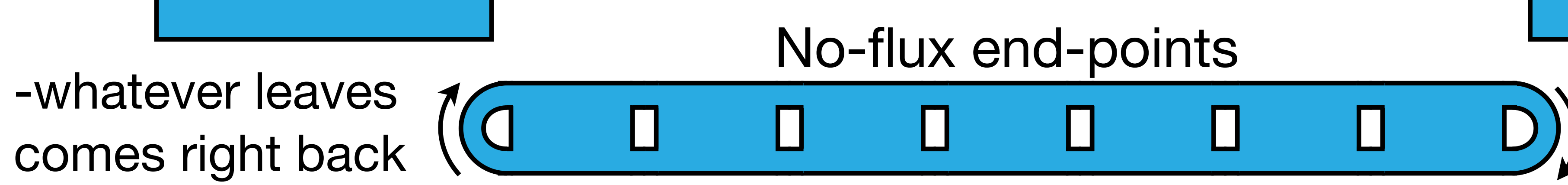

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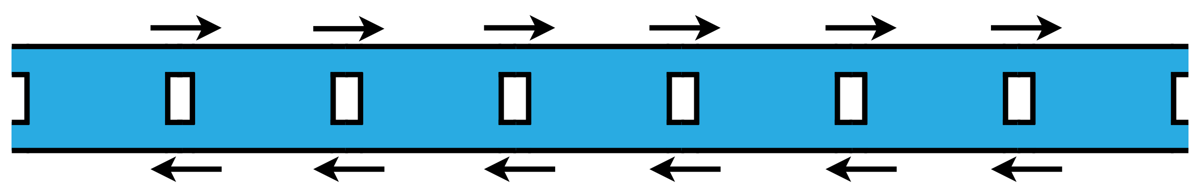

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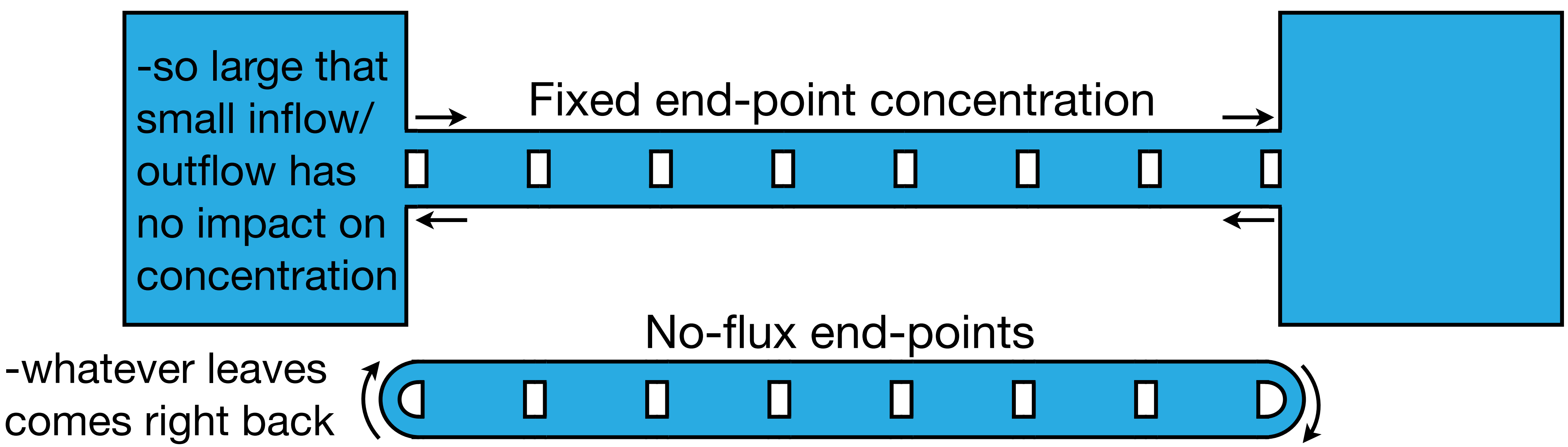

- Example: the axon of a neuron

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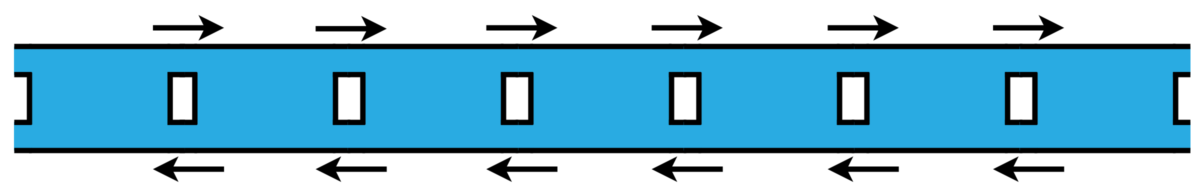

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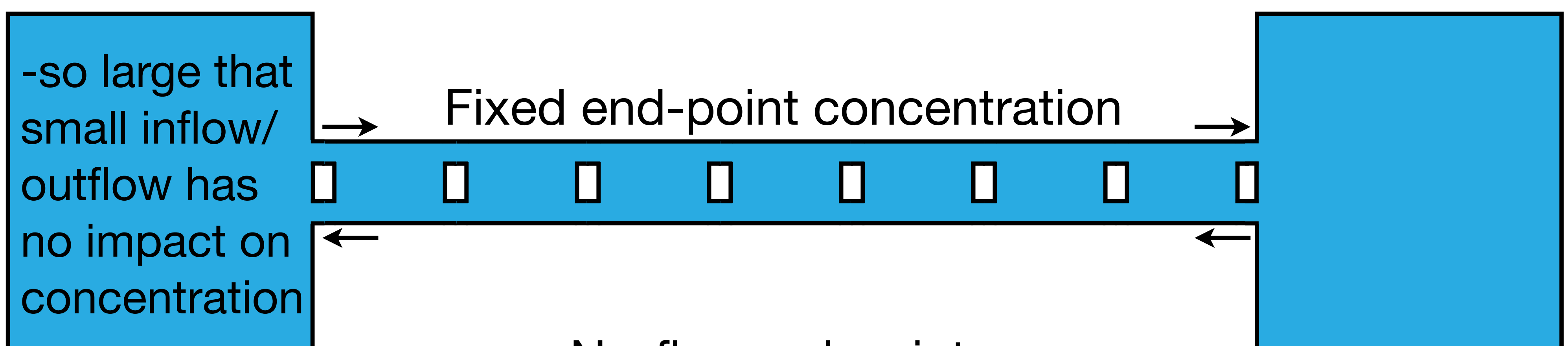

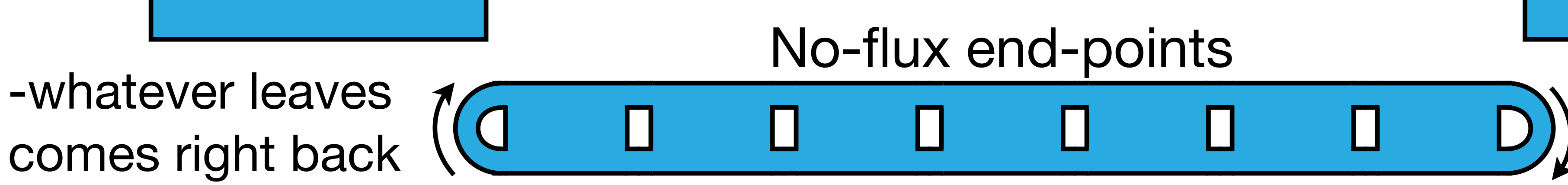

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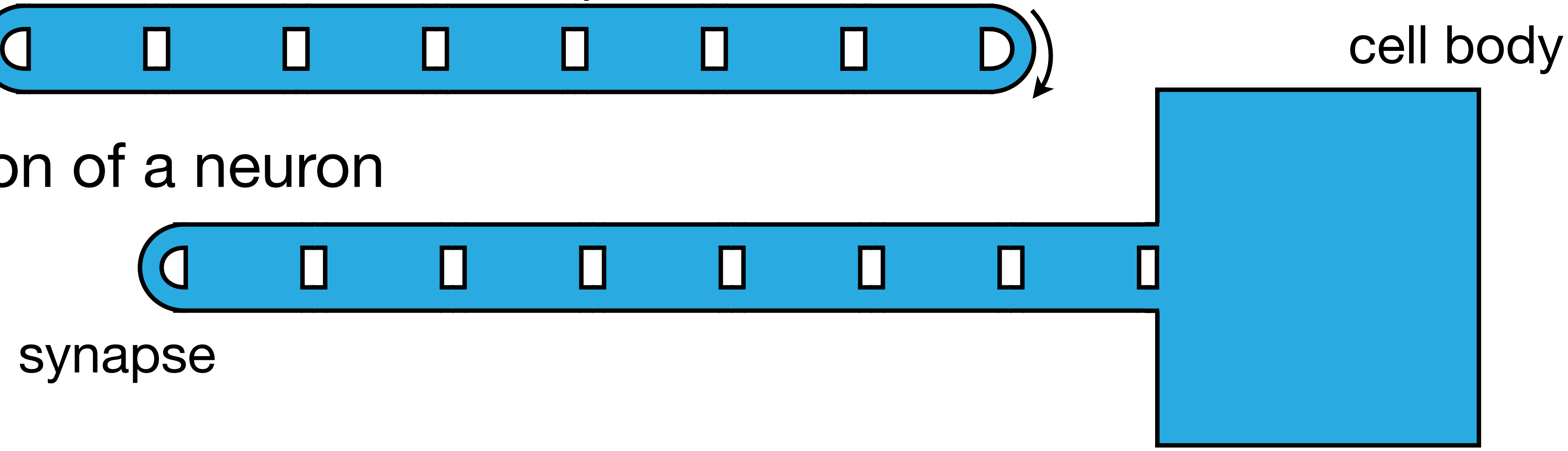

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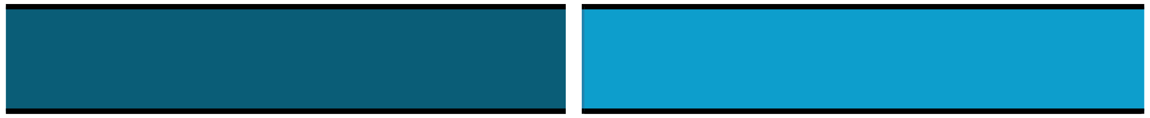

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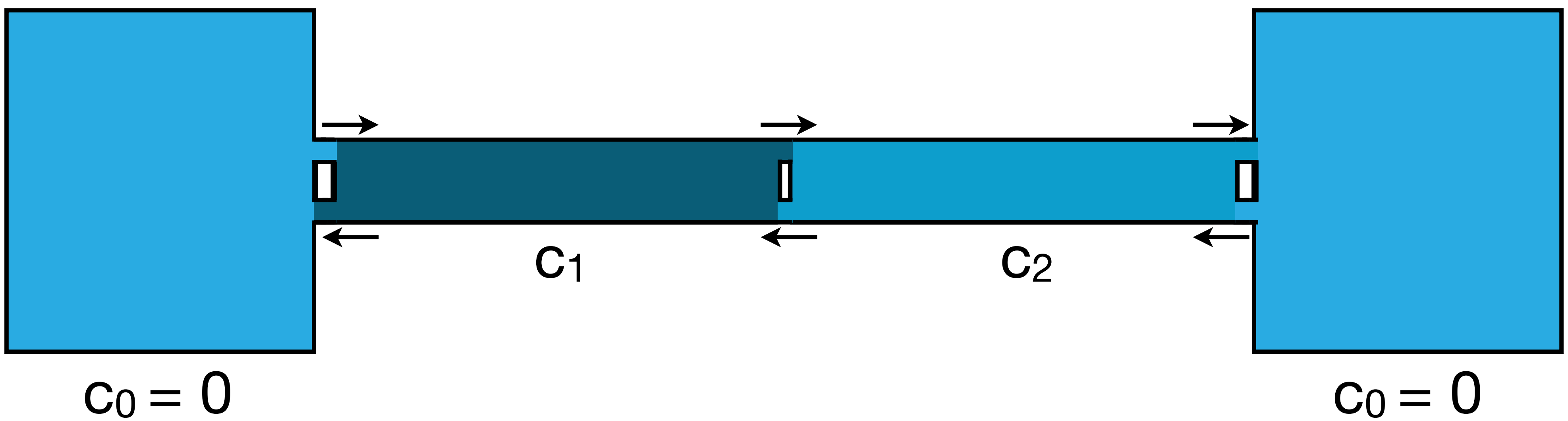

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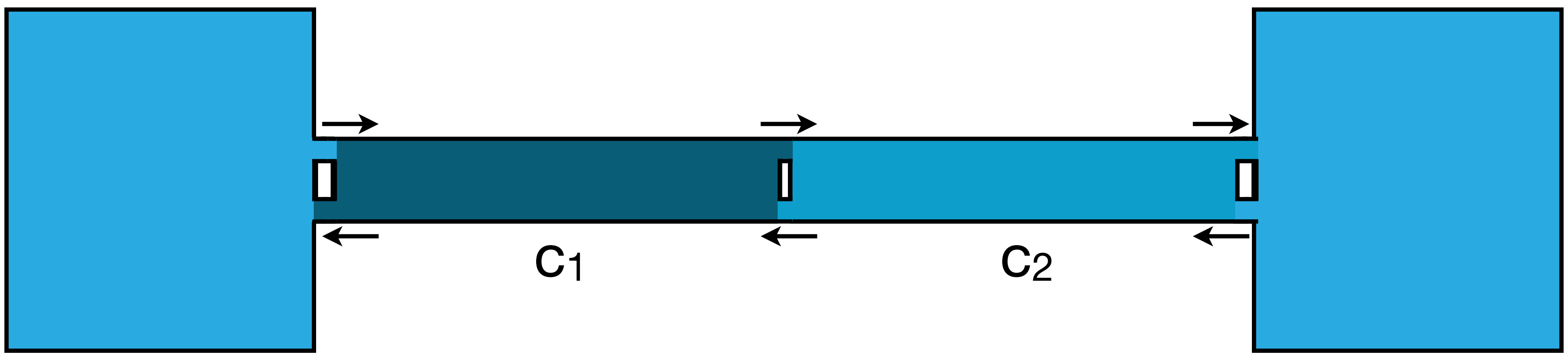

$c_0 = 0$ $c_0 = 0$

$$\frac{dm_1}{dt} = kc_0 - kc_1 - kc_1 + kc_2$$

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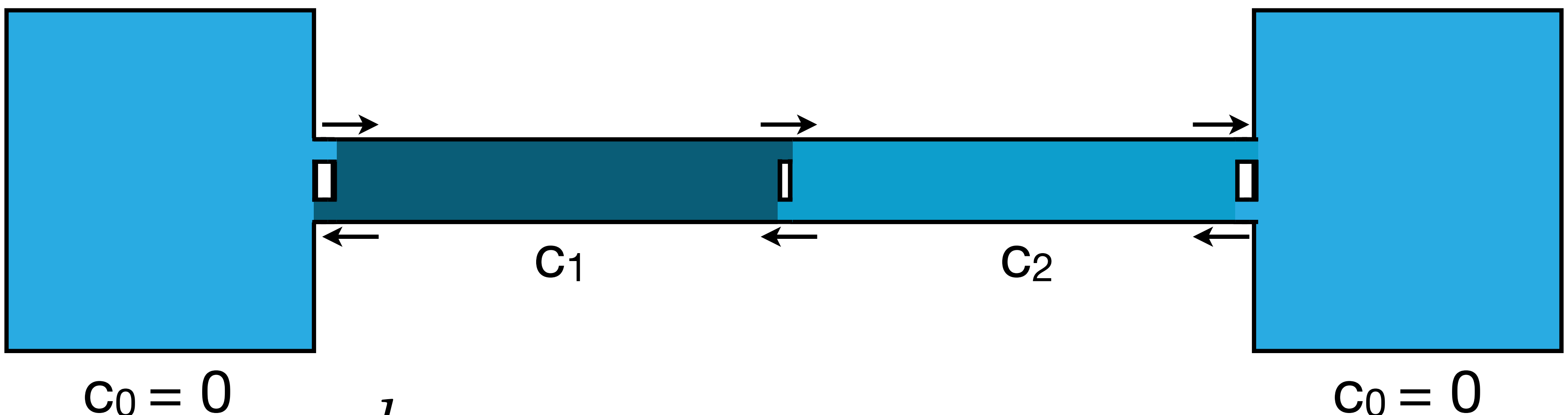

$$V \frac{dc_1}{dt} = kc_0 - kc_1 - kc_1 + kc_2$$

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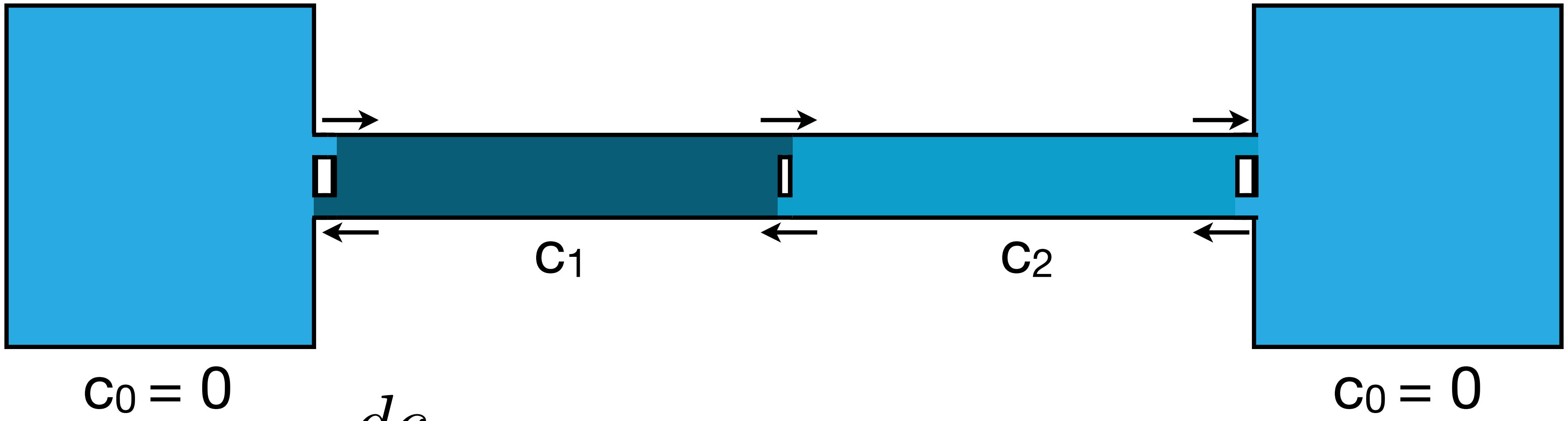

$$V\frac{dc_1}{dt} = kc_0 - kc_1 - kc_1 + kc_2$$

$$\frac{dc_1}{dt} = \frac{k}{V}(-2c_1 + c_2)$$

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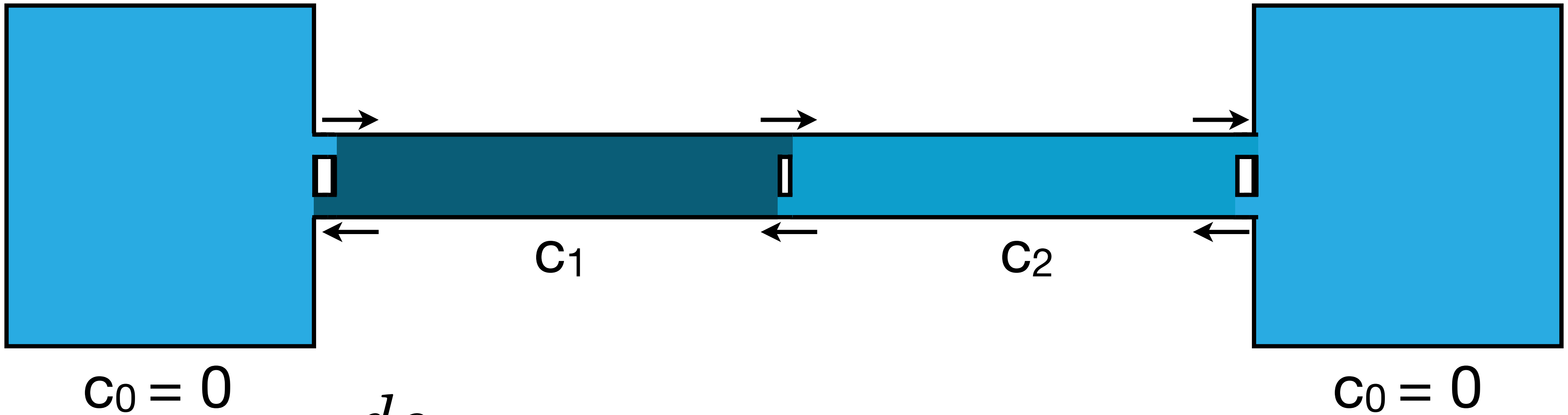

$$V\frac{dc_1}{dt} = kc_0 - kc_1 - kc_1 + kc_2$$

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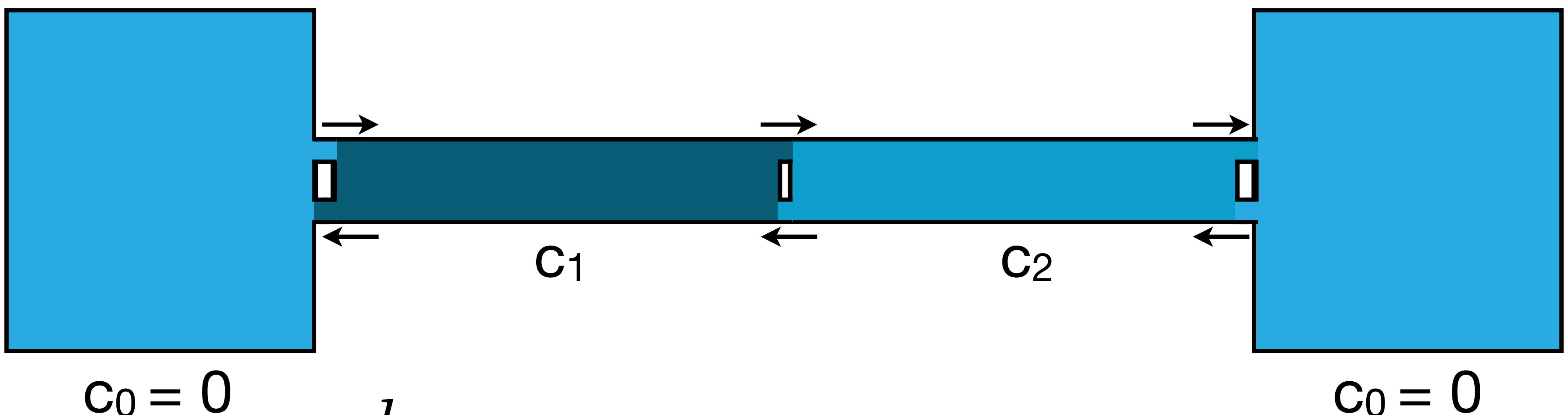

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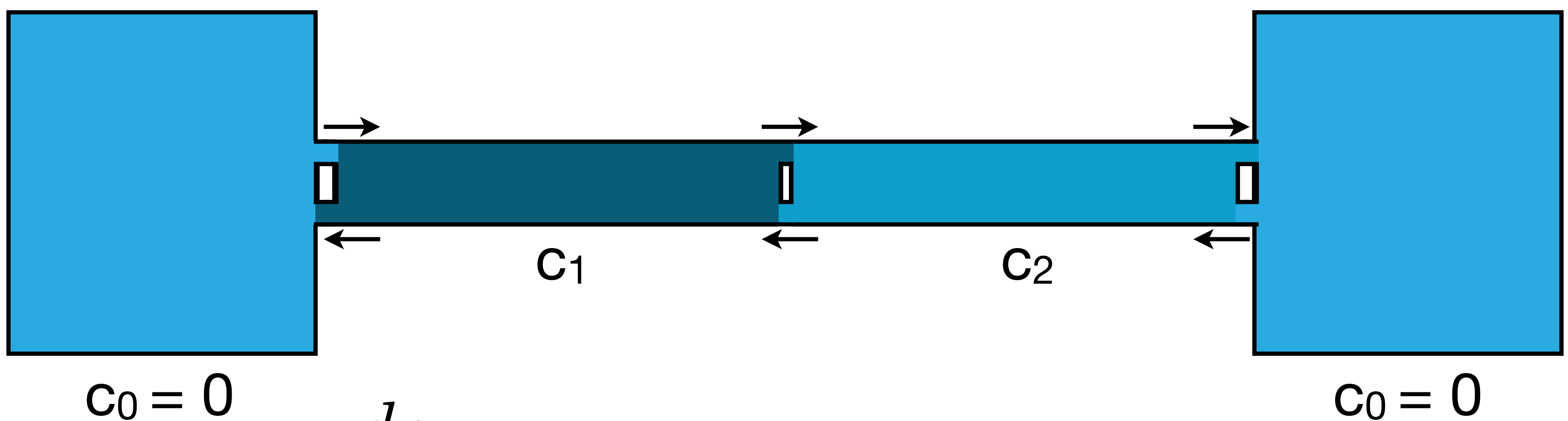

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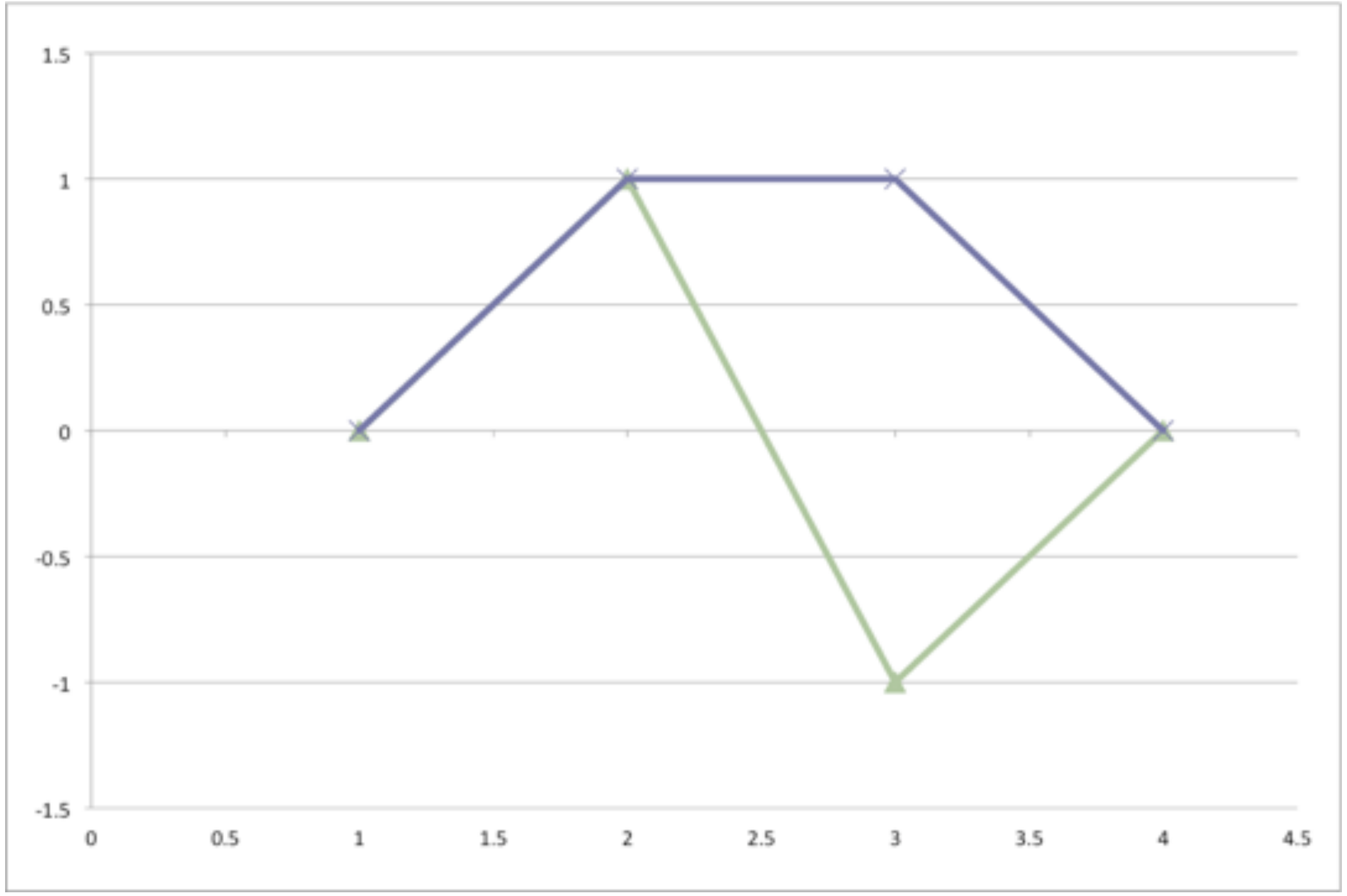

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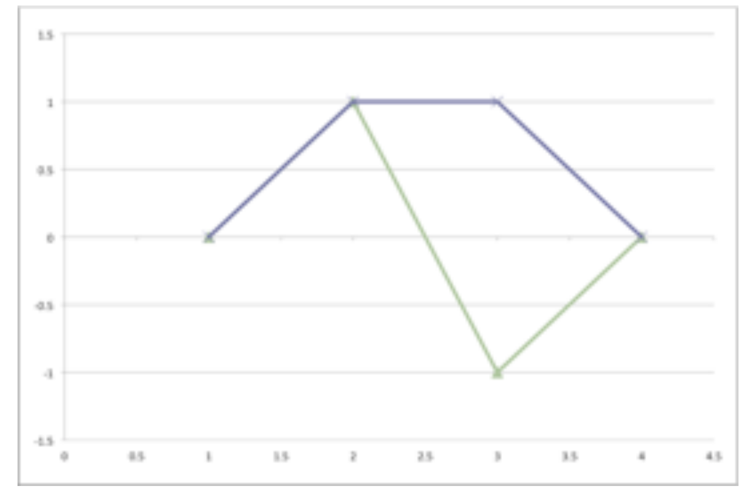

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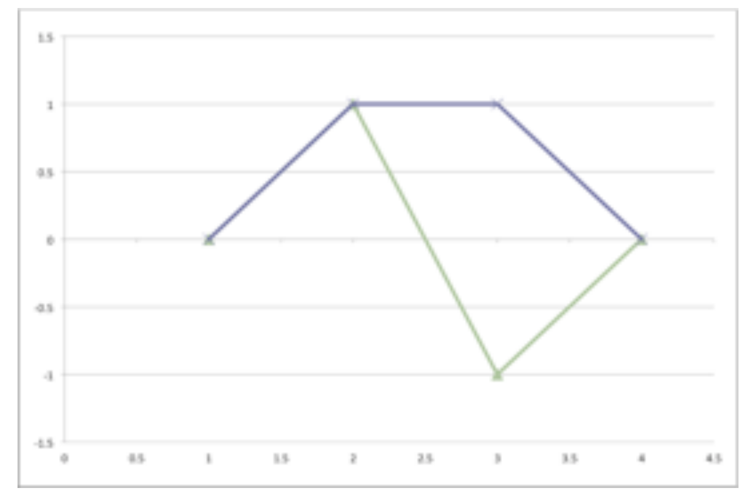

$$\mathbf{c(t)} = Ae^{\lambda_1 t}\mathbf{v_1} + Be^{\lambda_2 t}\mathbf{v_2}$$

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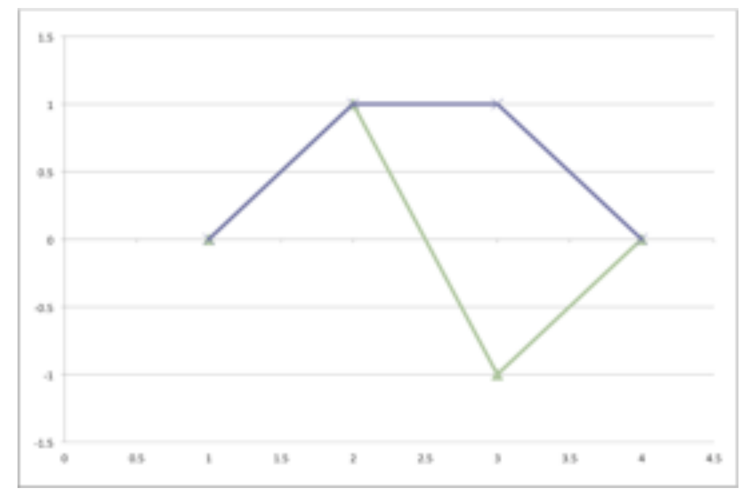

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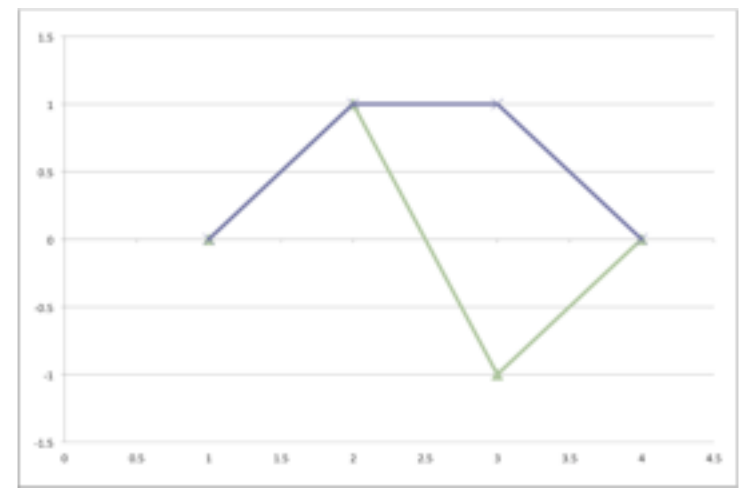

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- Average of two tanks (A) decays slowly at rate λ_1=-1 while difference decays quickly at rate λ_2=-3.

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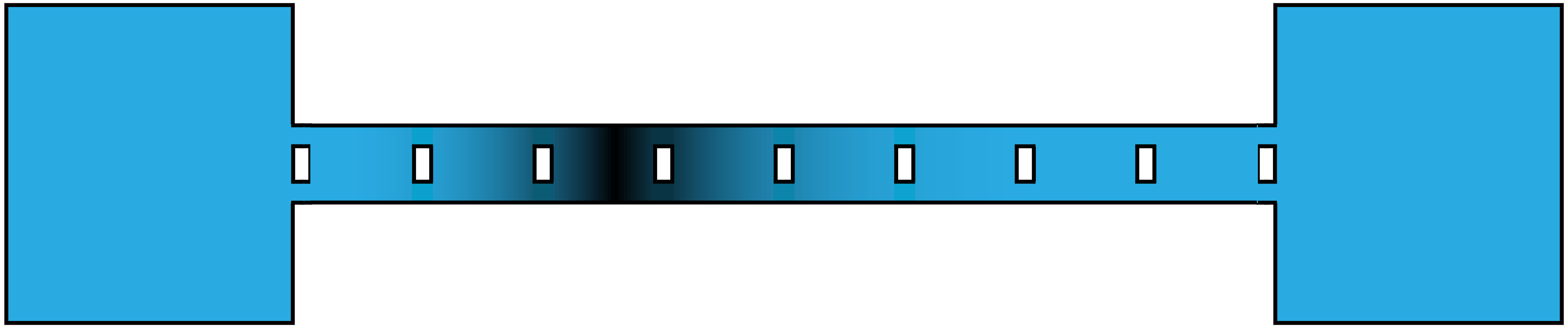

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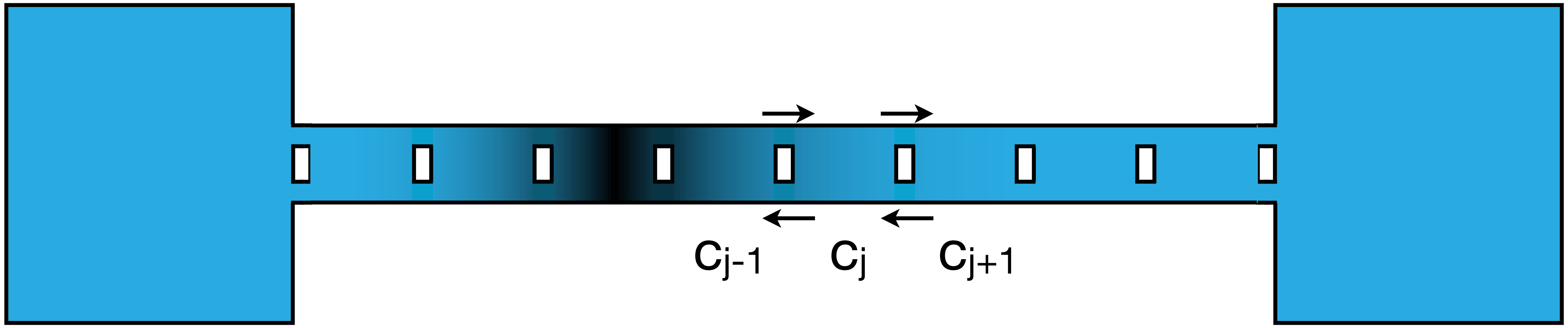

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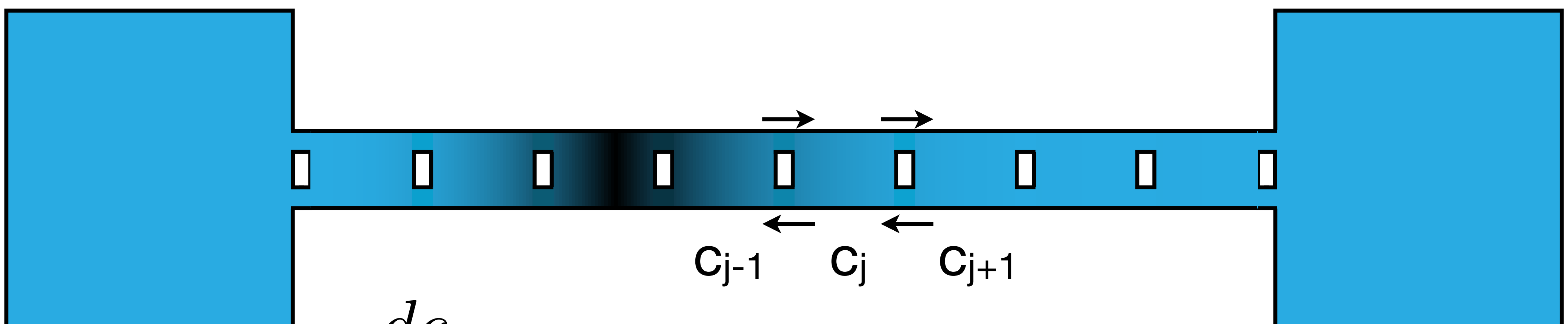

$$\frac{dc_j}{dt} = K(c_{j-1} - 2c_j + c_{j+1})$$

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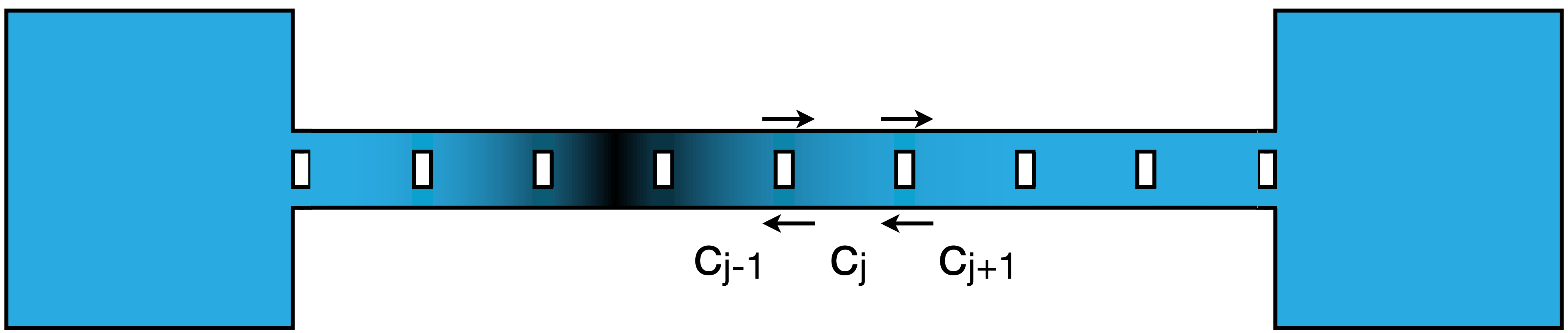


$$\frac{dc_j}{dt} = K(c_{j-1} - 2c_j + c_{j+1})$$

$$\frac{d\mathbf{c}}{dt} = K \begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix} \mathbf{c}$$

Diffusion in a long thin tube

Diffusion in a long thin tube

$$\lambda_1 = -7.7$$

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 0.16 \\ 0.30 \\ 0.41 \\ 0.46 \\ 0.46 \\ 0.41 \\ 0.30 \\ 0.16 \\ 0 \end{pmatrix}$$

Diffusion in a long thin tube

$$\lambda_1 = -7.7 \qquad \lambda_2 = -30$$

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 0.16 \\ 0.30 \\ 0.41 \\ 0.46 \\ 0.46 \\ 0.41 \\ 0.30 \\ 0.16 \\ 0 \end{pmatrix} \qquad \mathbf{v_2} = \begin{pmatrix} 0 \\ -0.30 \\ -0.46 \\ -0.41 \\ -0.16 \\ 0.16 \\ 0.41 \\ 0.46 \\ 0.30 \\ 0 \end{pmatrix}$$

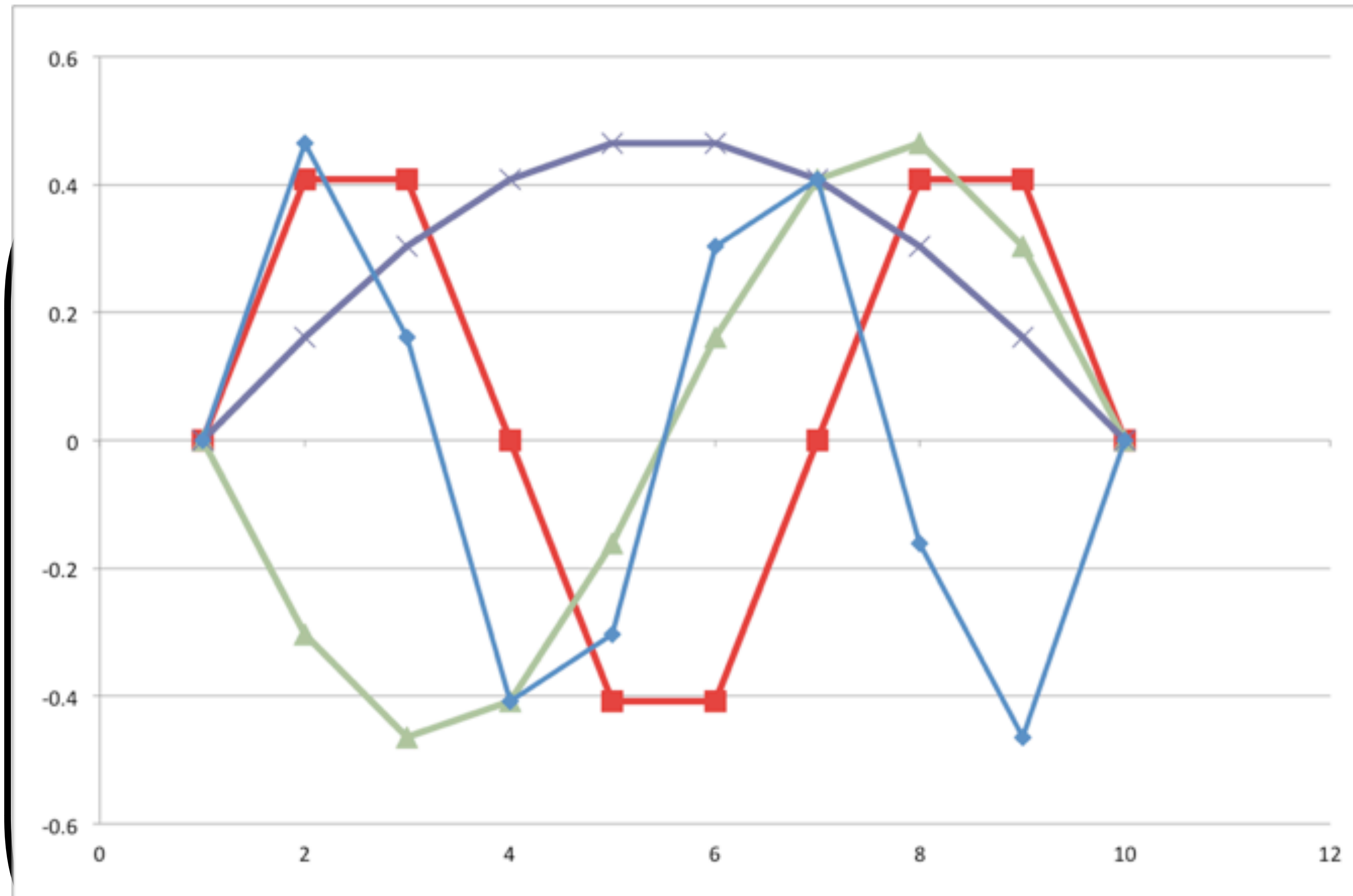
Diffusion in a long thin tube

$$\lambda_1 = -7.7 \qquad \lambda_2 = -30 \qquad \lambda_3 = -64$$

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 0.16 \\ 0.30 \\ 0.41 \\ 0.46 \\ 0.46 \\ 0.41 \\ 0.30 \\ 0.16 \\ 0 \end{pmatrix} \qquad \mathbf{v_2} = \begin{pmatrix} 0 \\ -0.30 \\ -0.46 \\ -0.41 \\ -0.16 \\ 0.16 \\ 0.41 \\ 0.46 \\ 0.30 \\ 0 \end{pmatrix} \qquad \mathbf{v_3} = \begin{pmatrix} 0 \\ 0.41 \\ 0.41 \\ 0 \\ -0.41 \\ -0.41 \\ 0 \\ 0.41 \\ 0.41 \\ 0 \end{pmatrix}$$

Diffusion in a long thin tube

$\lambda_1 =$ -64



$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 0.41 \\ 0.41 \\ 0 \\ -0.41 \\ -0.41 \\ 0 \\ 0.41 \\ 0.41 \\ 0 \end{pmatrix}$$

Diffusion in a long thin tube

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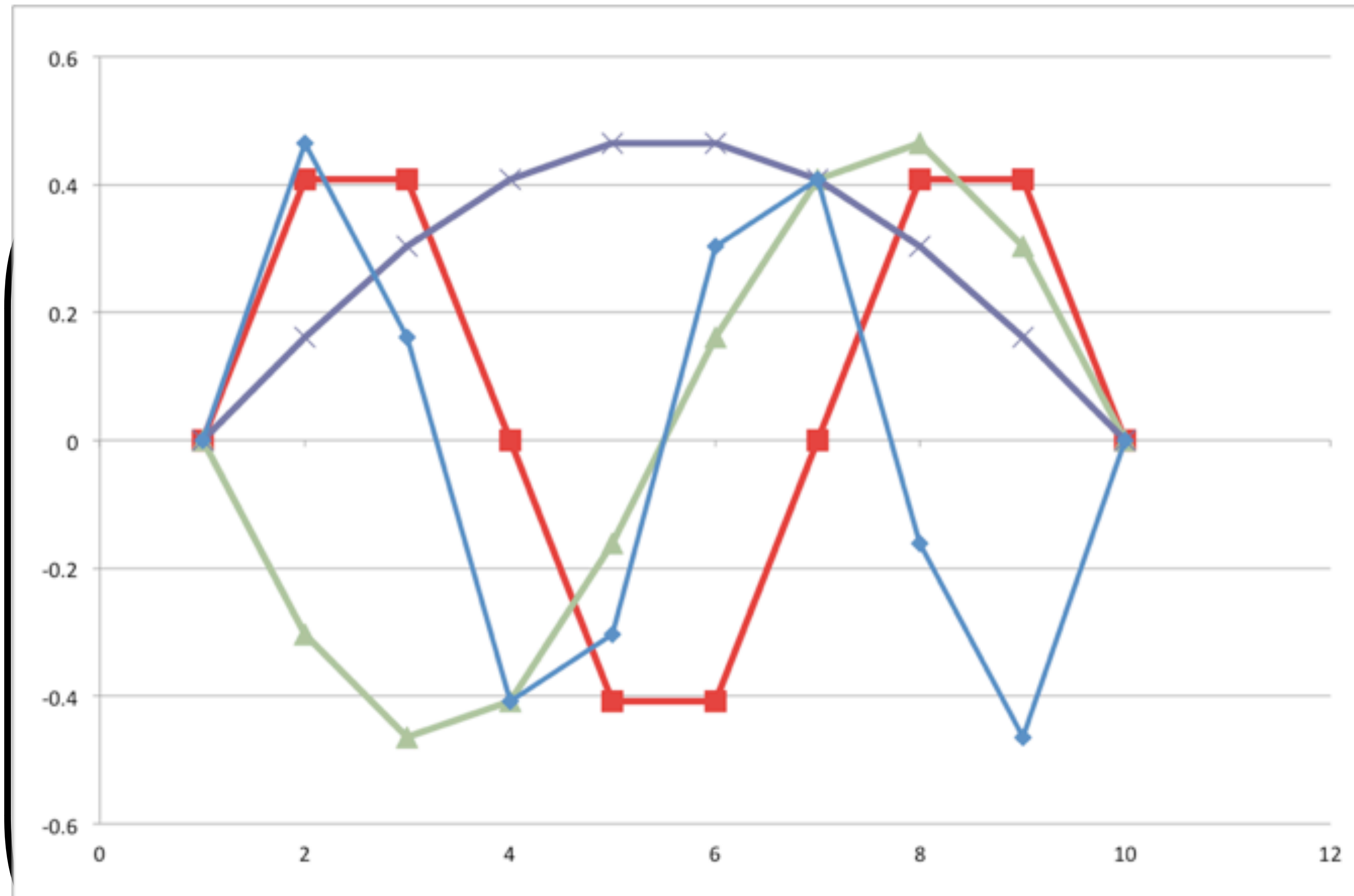


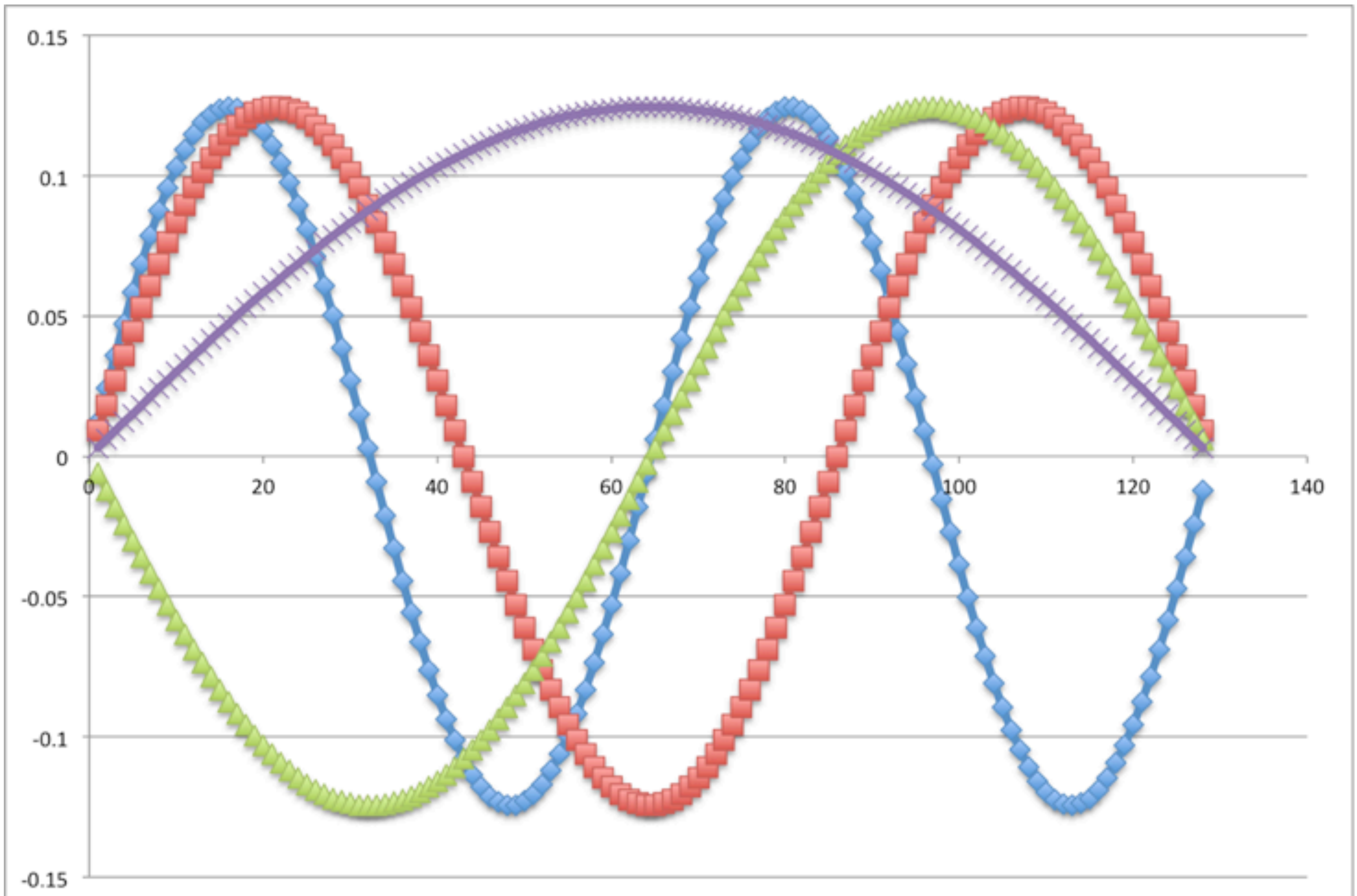
$$\mathbf{v_1} = \begin{pmatrix} 0 \\ 0.41 \\ 0.41 \\ 0 \\ -0.41 \\ -0.41 \\ 0 \\ 0.41 \\ 0.41 \\ 0 \end{pmatrix}$$

- Add these up to satisfy initial conditions. Each component decays at a different rate.

$$\mathbf{c(t)} = c_1 e^{\lambda_1 t}\mathbf{v_1} + c_2 e^{\lambda_2 t}\mathbf{v_2} + c_3 e^{\lambda_3 t}\mathbf{v_3} + c_4 e^{\lambda_4 t}\mathbf{v_4} + c_5 e^{\lambda_5 t}\mathbf{v_5} + c_6 e^{\lambda_6 t}\mathbf{v_6} + c_7 e^{\lambda_7 t}\mathbf{v_7} + c_8 e^{\lambda_8 t}\mathbf{v_8}$$

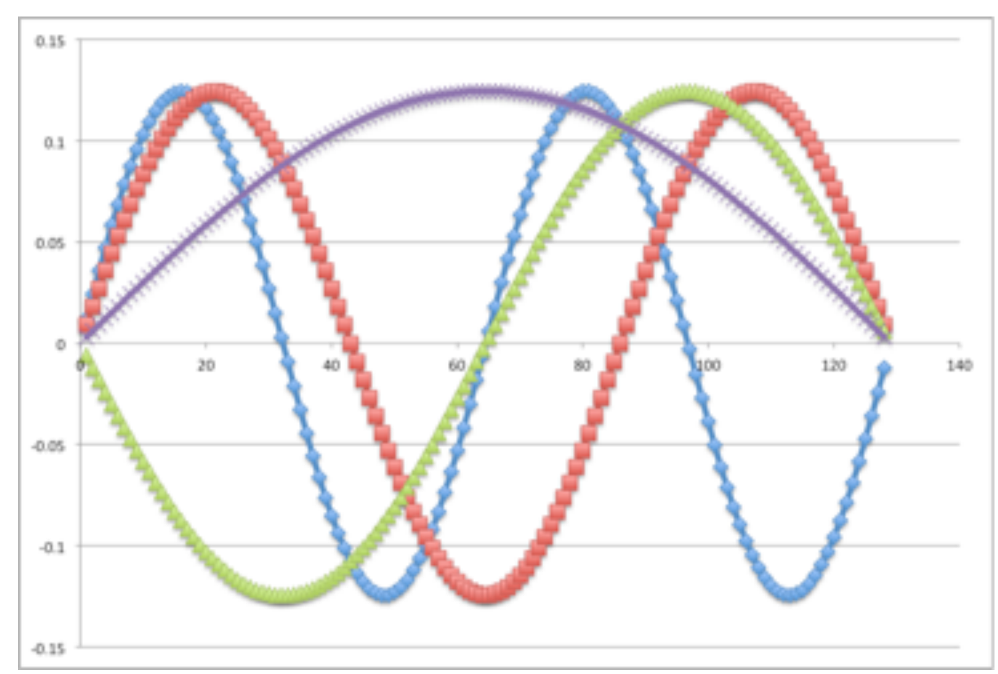
Diffusion in a long thin tube

- 128-tank approximation, fixed end-point concentration:



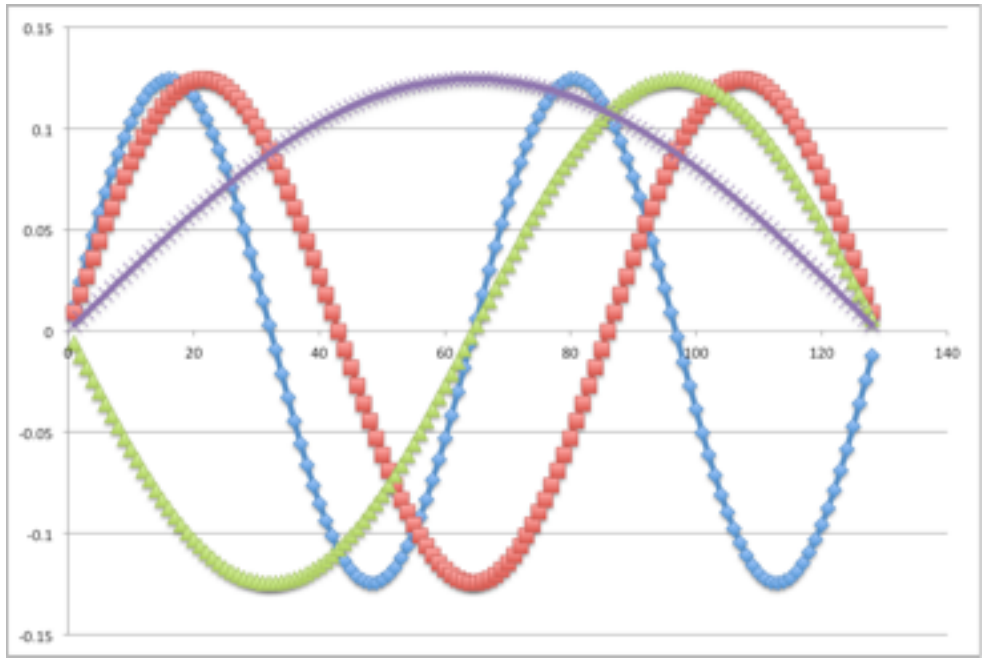
Diffusion in a long thin tube

- 128-tank approximation, fixed end-point concentration:
- First eigenvector (mode) is ≈ half a period of a sine function.



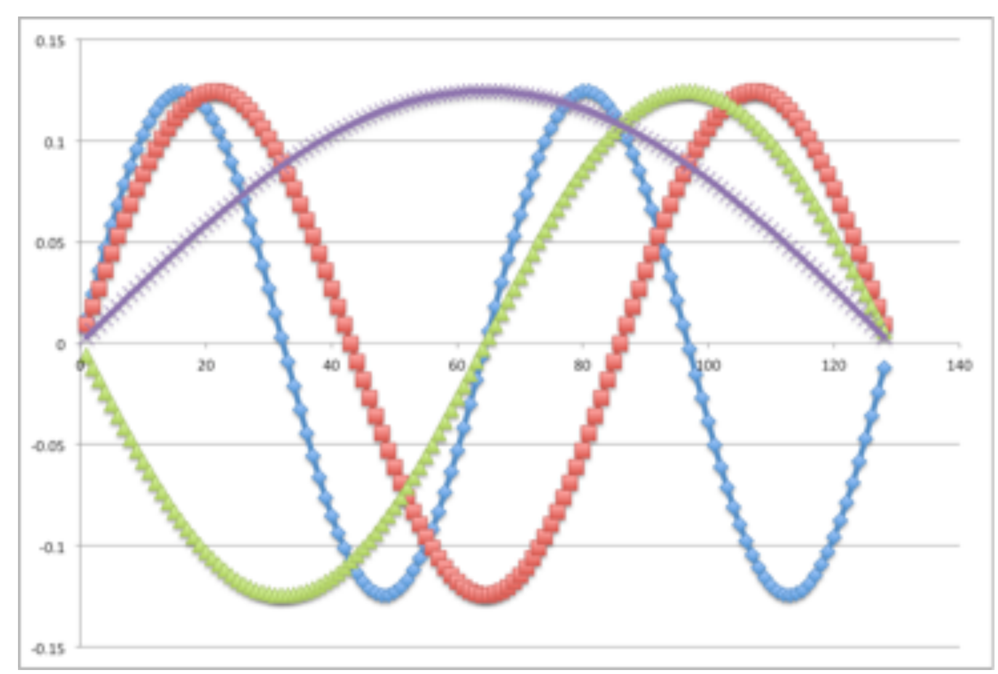
Diffusion in a long thin tube

- 128-tank approximation, fixed end-point concentration:
- First eigenvector (mode) is ≈ half a period of a sine function.
- Second mode is ≈ a full period of a sine function.



Diffusion in a long thin tube

- 128-tank approximation, fixed end-point concentration:
- First eigenvector (mode) is ≈ half a period of a sine function.
- Second mode is ≈ a full period of a sine function.
- Third mode is ≈three halves the period of a sine function, etc.



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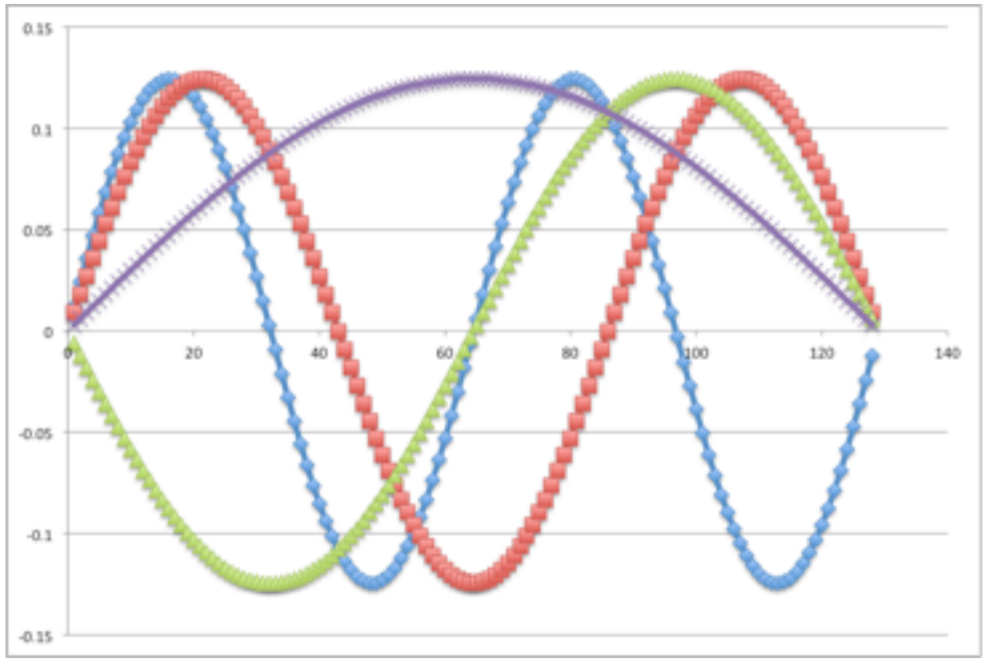
- Third mode is ≈three halves the period of a sine function, etc.
- Low frequency modes decay slowly, high frequency modes decay quickly.



Diffusion in a long thin tube

- Eight-tank approximation, no-flux end-points:

Diffusion in a long thin tube

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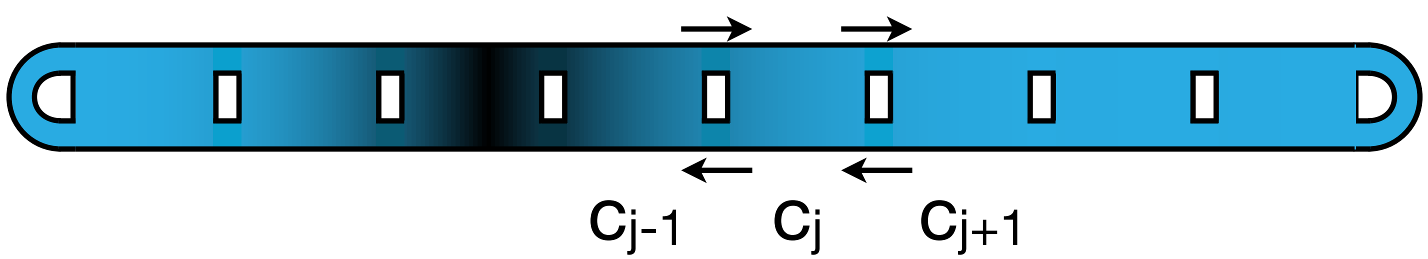

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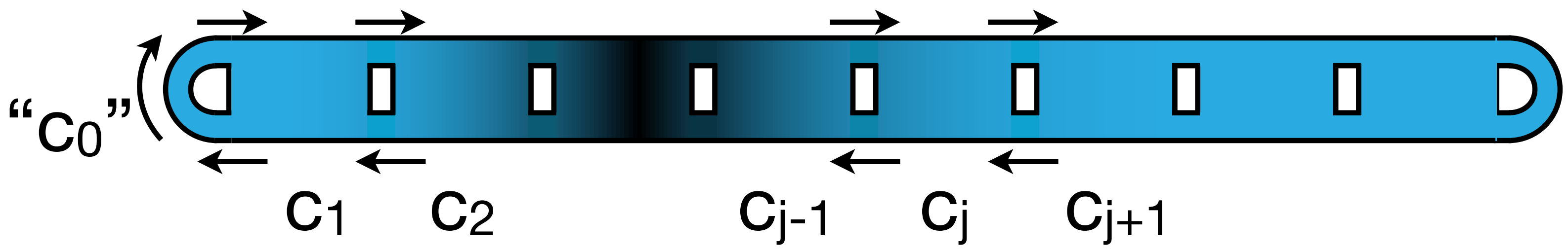

$$\frac{dc_j}{dt} = K(c_{j-1} - 2c_j + c_{j+1})$$

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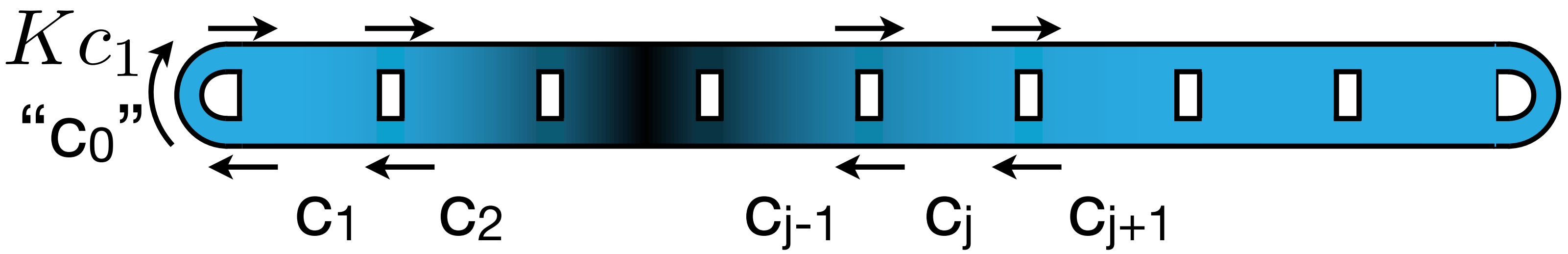
$Kc_0 = Kc_1$



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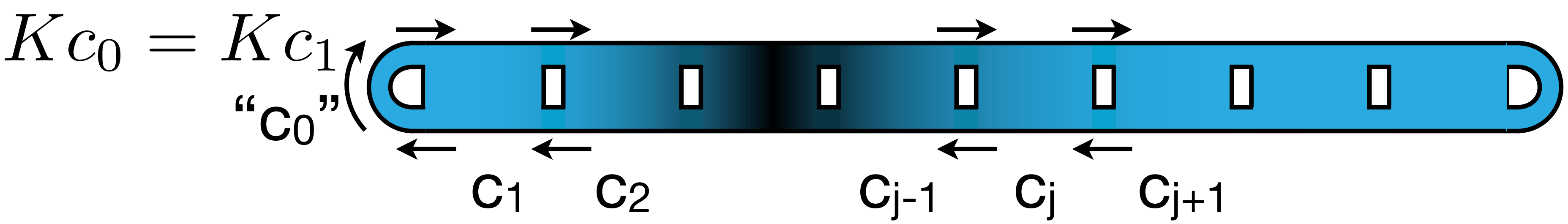

$$\frac{dc_j}{dt} = K(c_{j-1} - 2c_j + c_{j+1})$$

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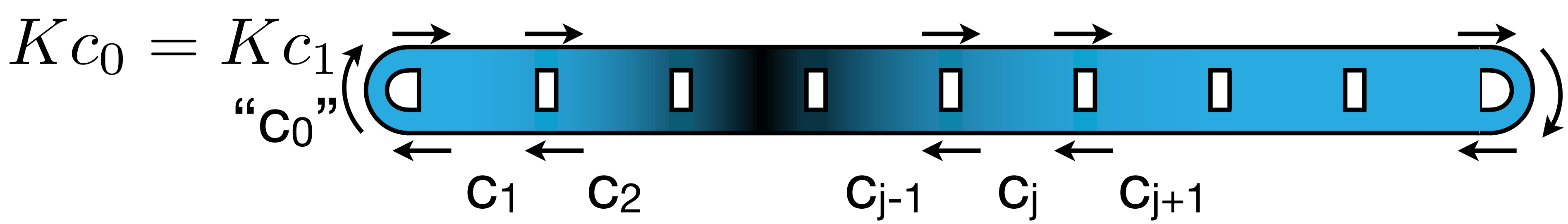

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$$\frac{dc_8}{dt} = K(c_7 - c_8)$$

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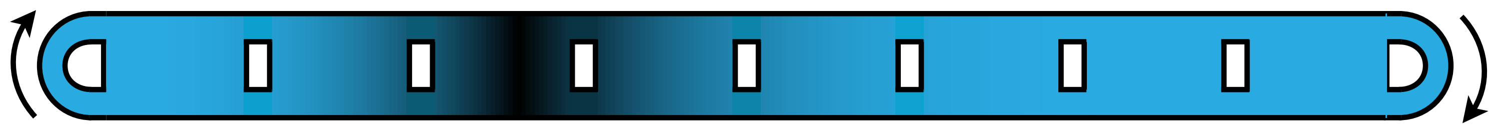
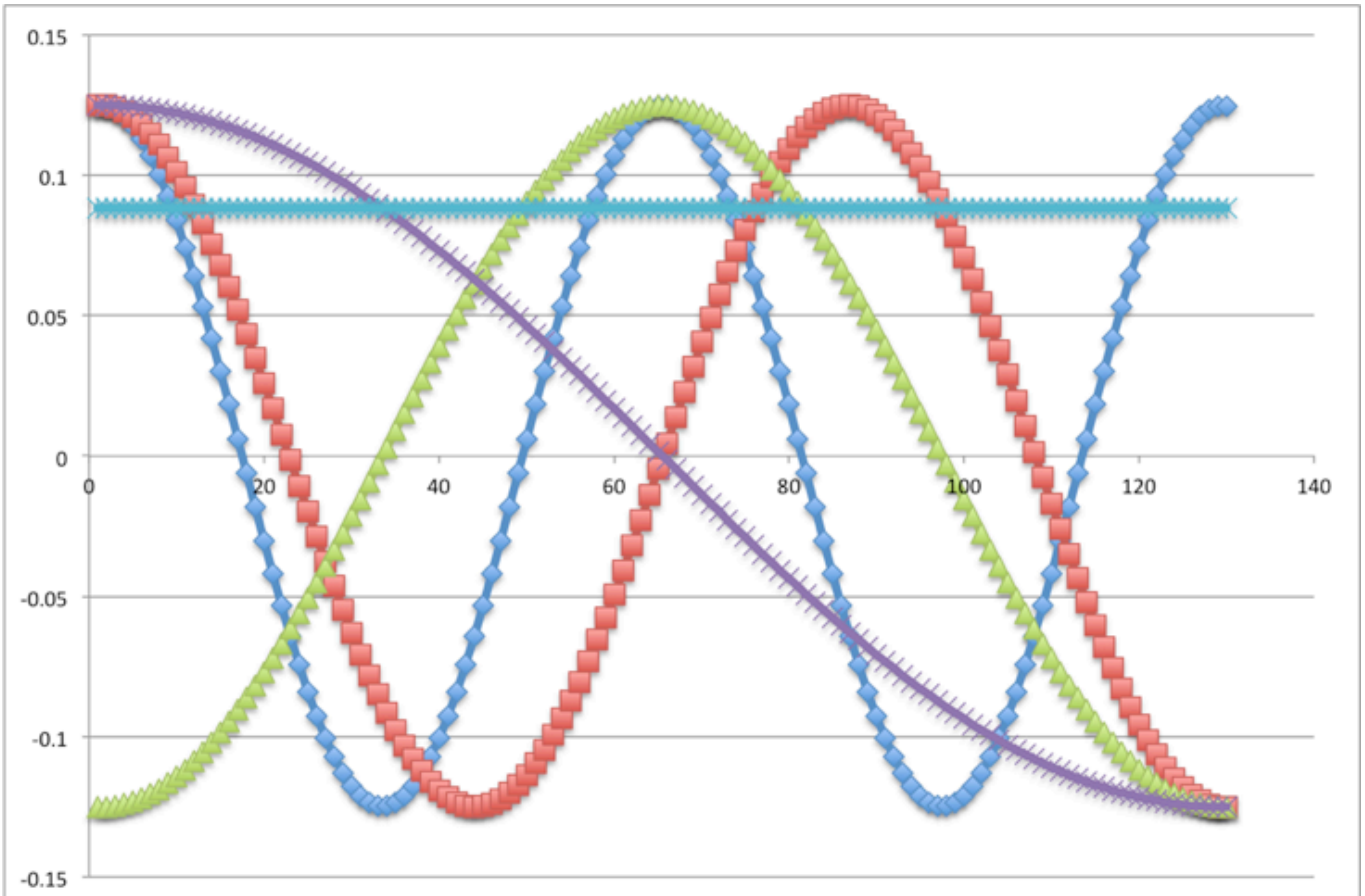


$$\frac{d\mathbf{c}}{dt} = K \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \mathbf{c}$$

Diffusion in a long thin tube

- 128-tank approximation, no-flux end-point:



Diffusion in a long thin tube

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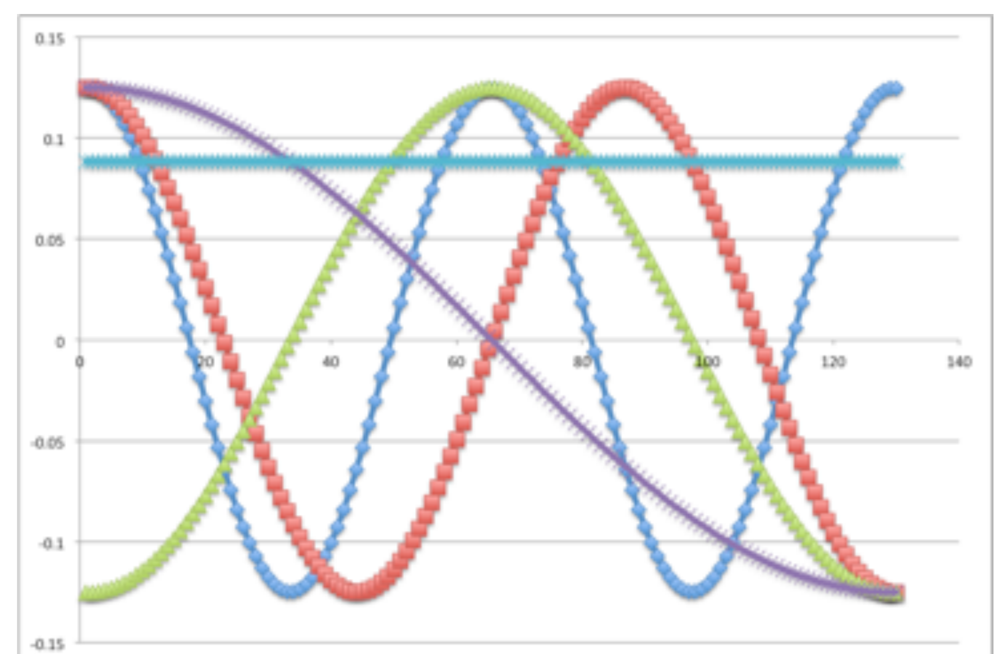

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- 128-tank approximation, no-flux end-point:
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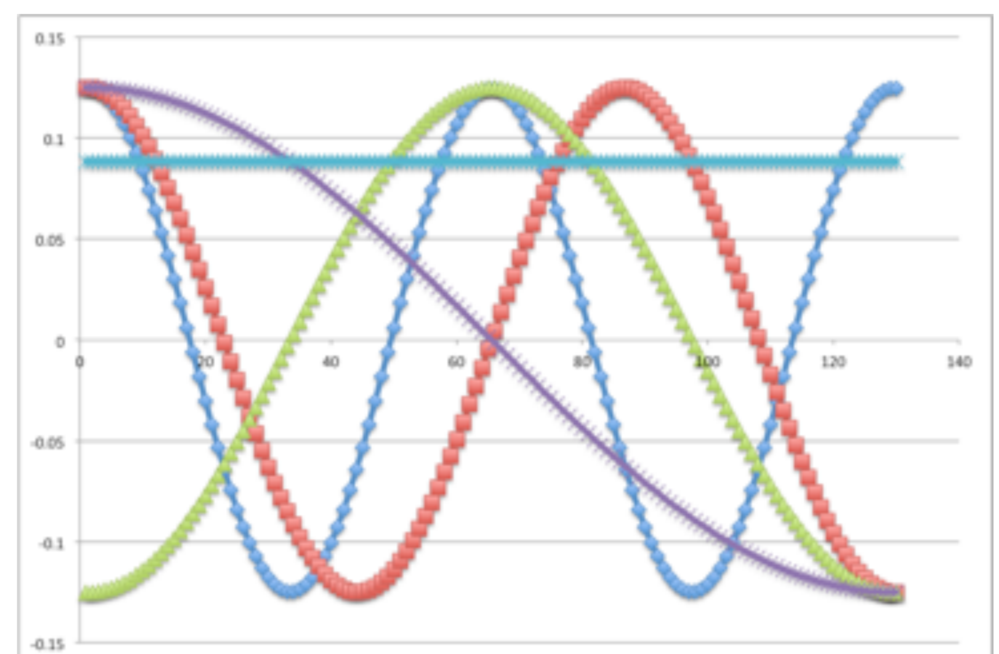

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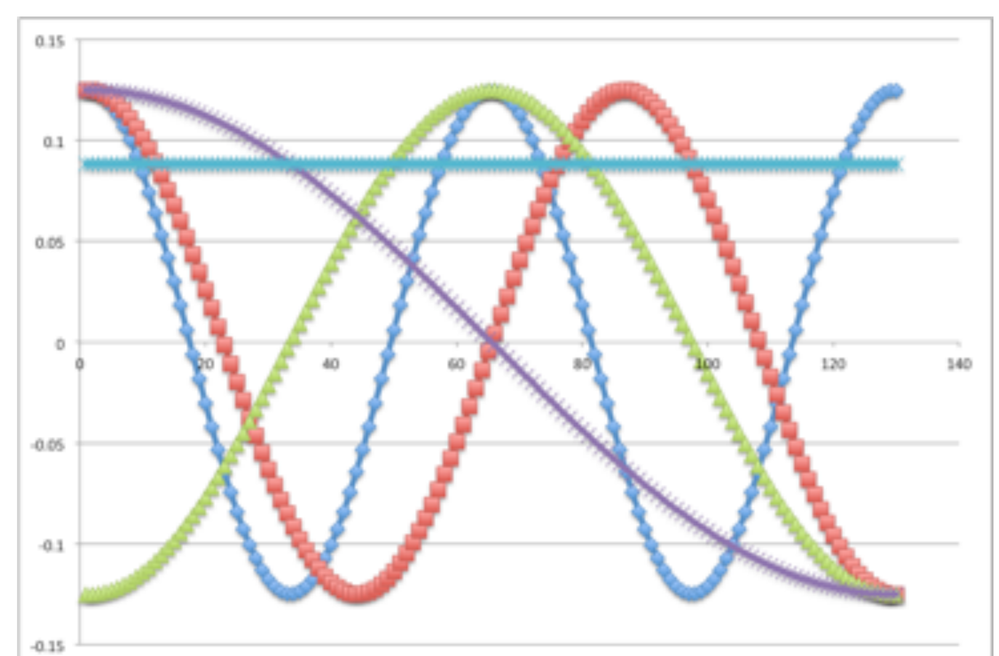

Diffusion in a long thin tube

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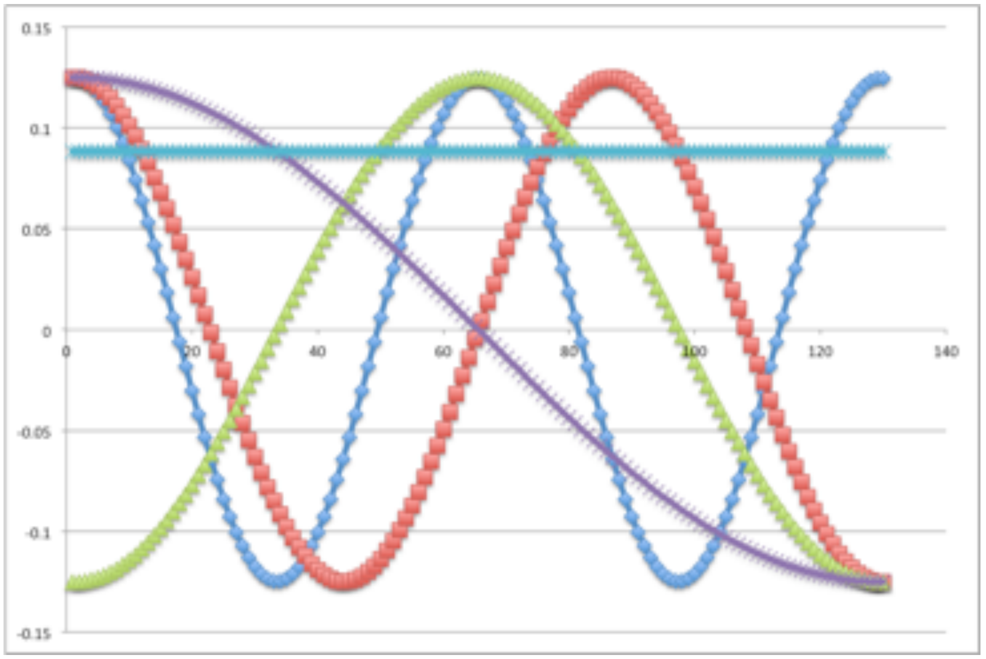
- Third mode is ≈ a full period of a cosine function.
- Low frequency modes decay slowly, high frequency modes decay quickly.



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Diffusion in a long thin tube

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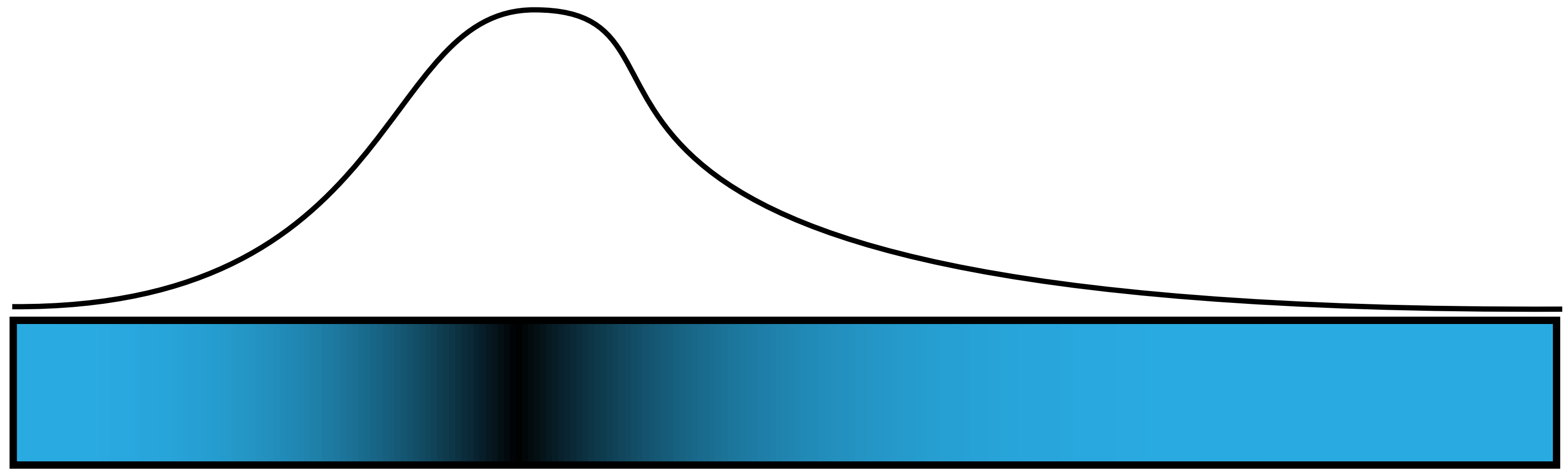

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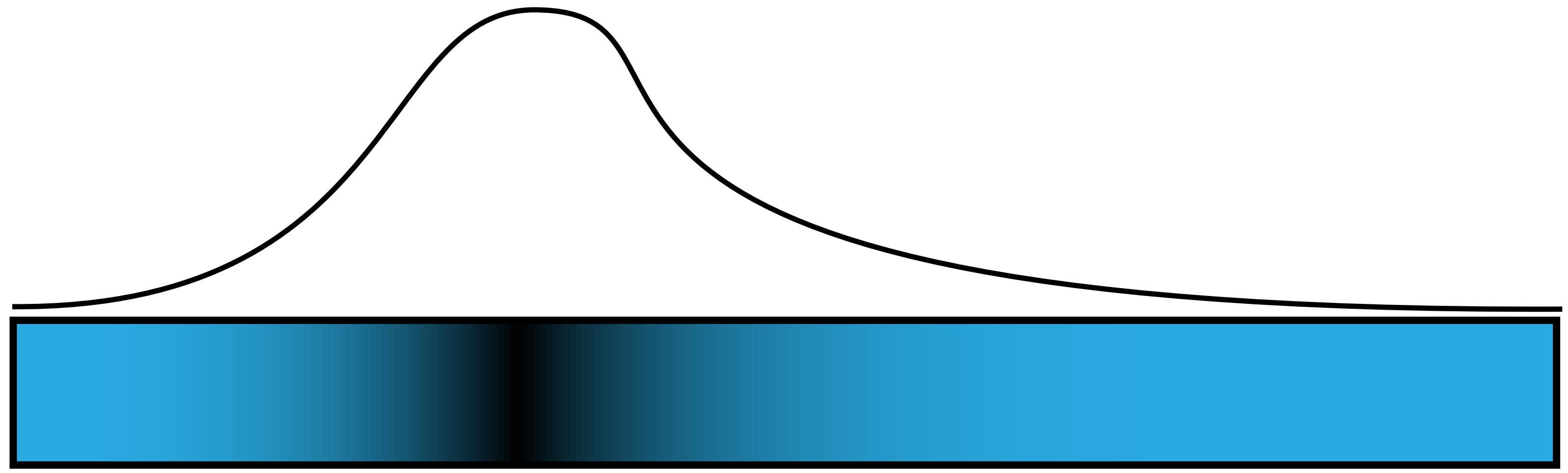
- In the limit of an infinite number of tiny tanks...
- Suppose there is a function $c(x,t)$ such that $c_j(t) = c(j\Delta x,t)$.



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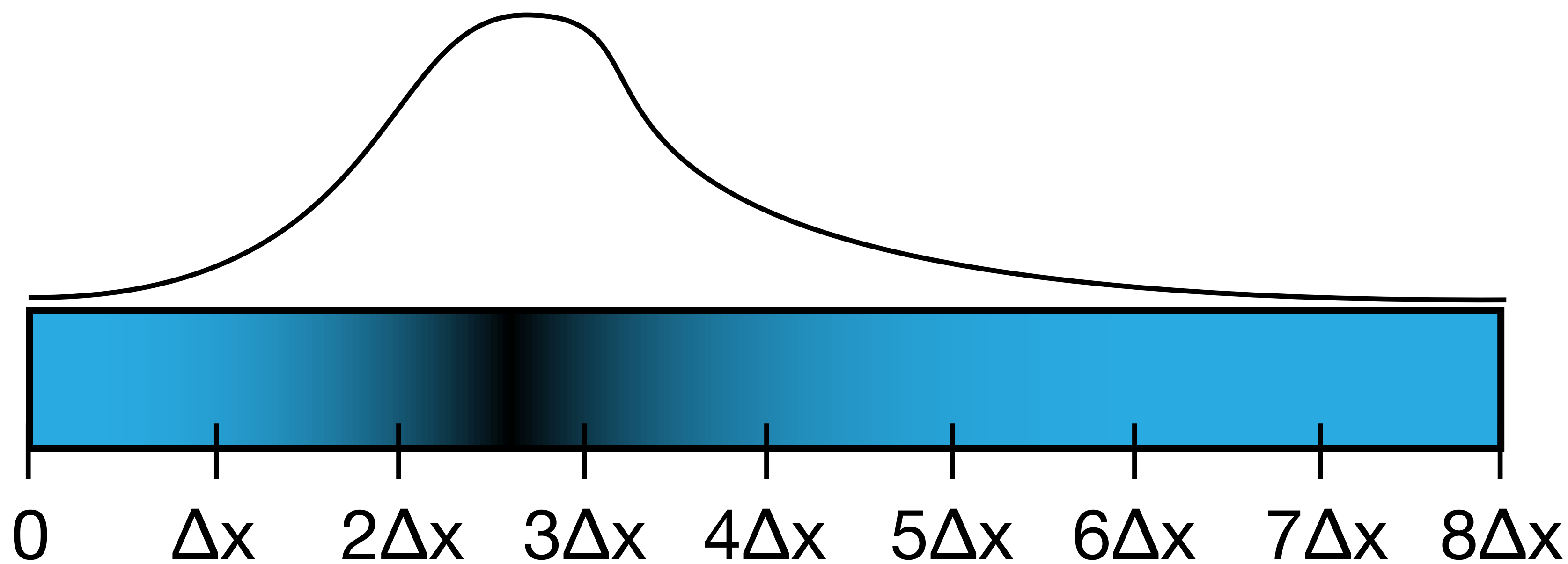

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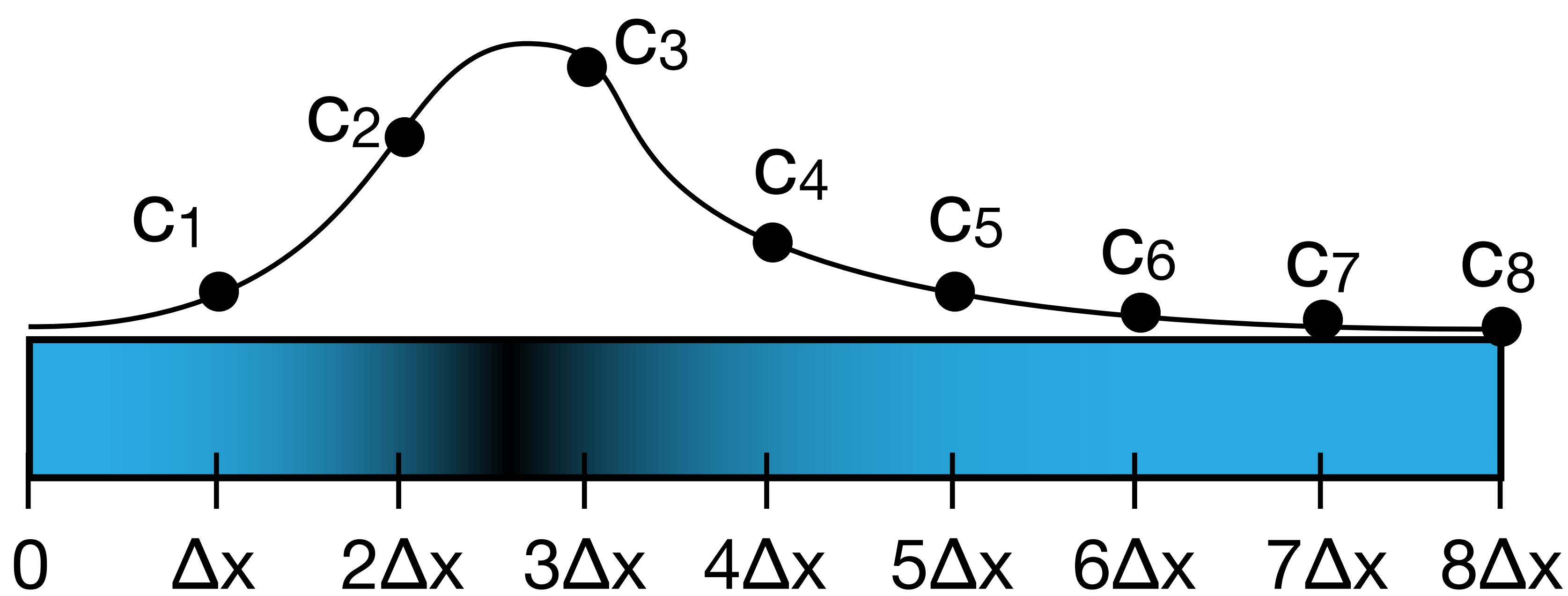

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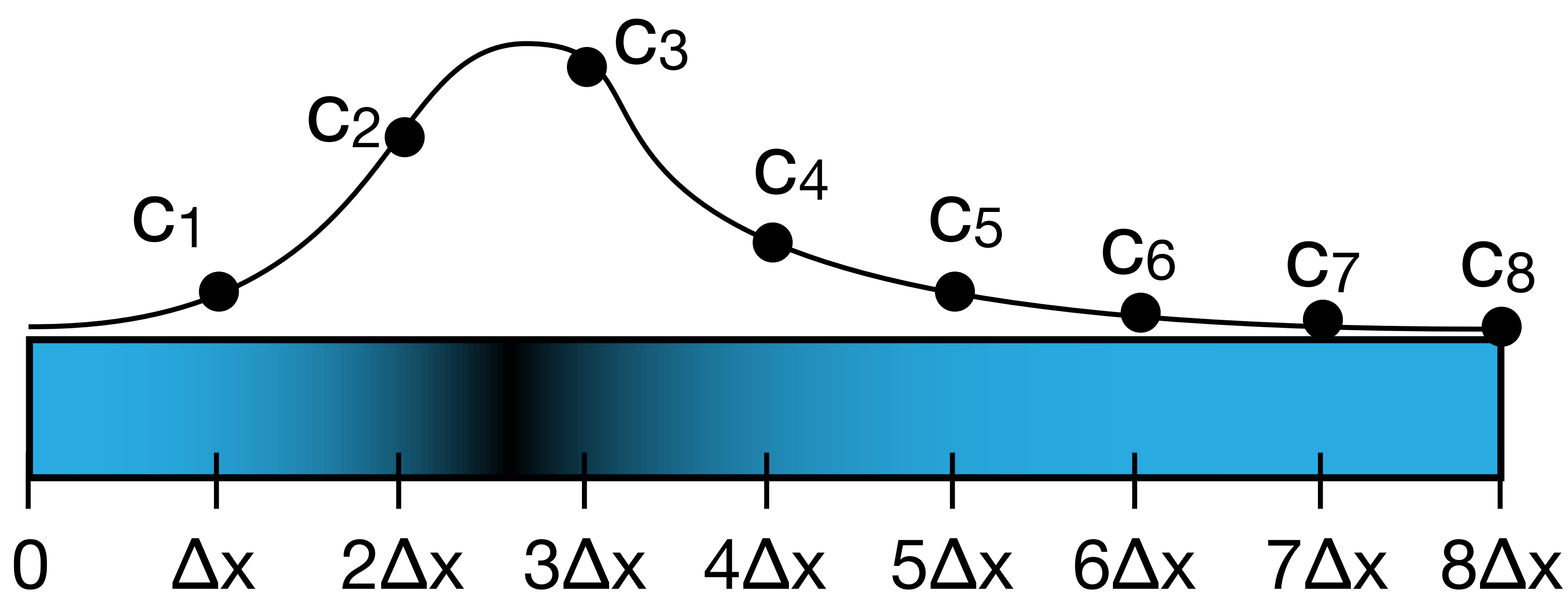

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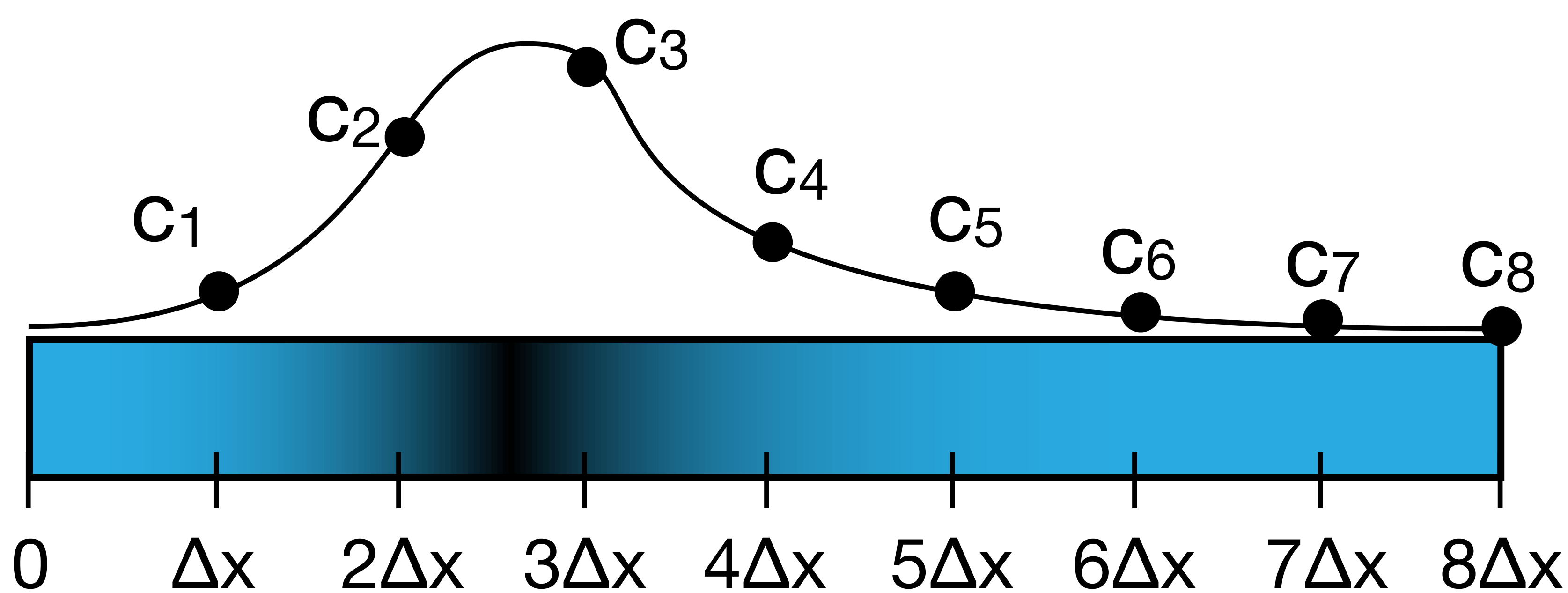

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- Want to replace this equation by one for the function $c(x,t)$...

Diffusion in a long thin tube

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$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x + \frac{1}{2}f''(x)\Delta x^2$$

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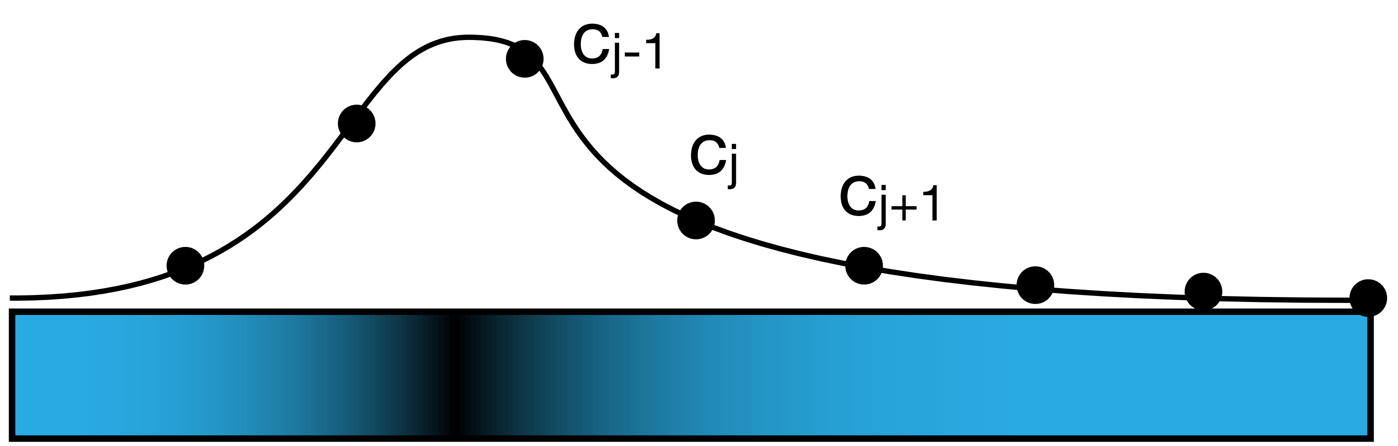

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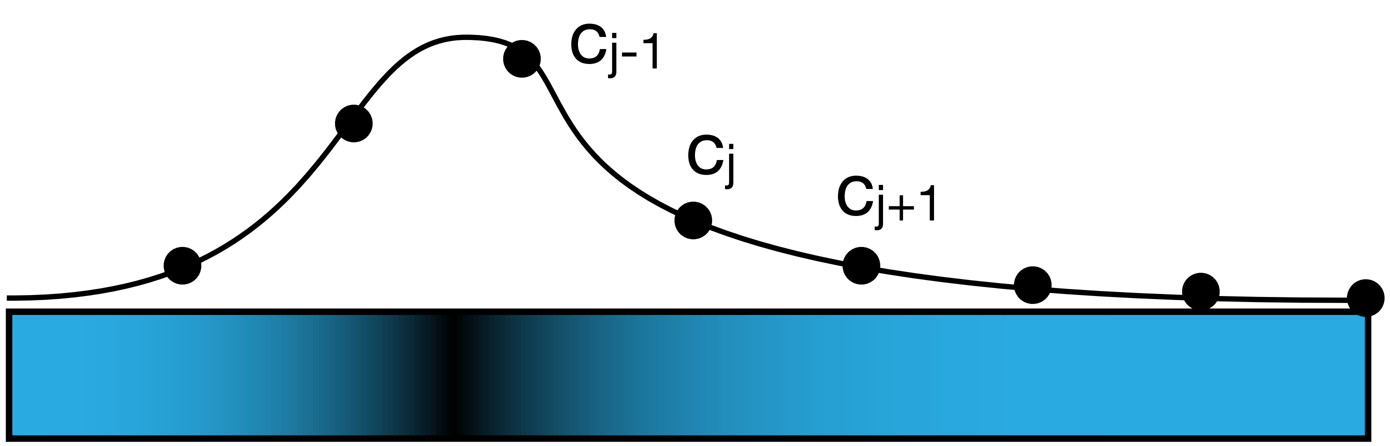

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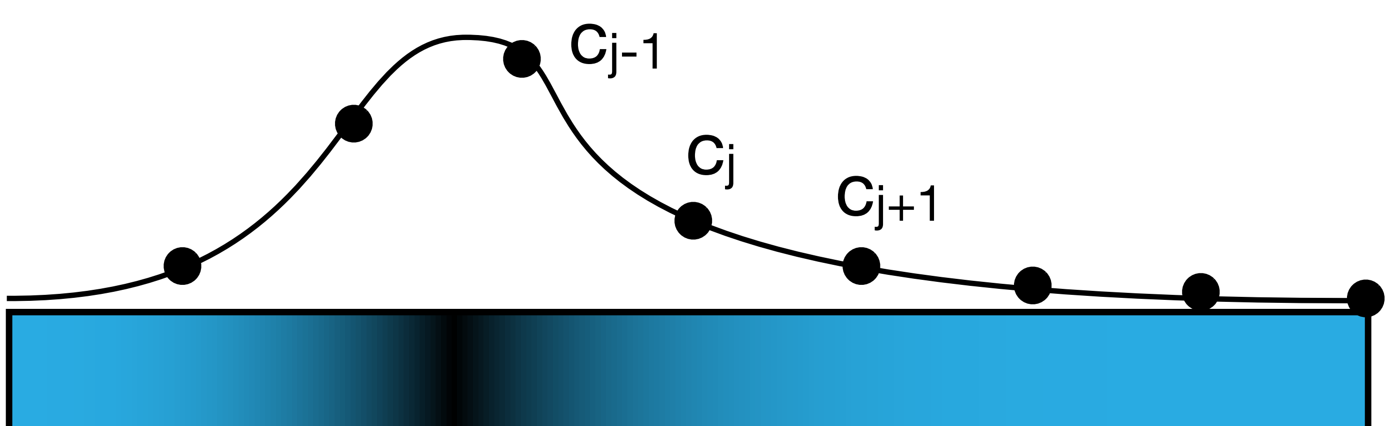

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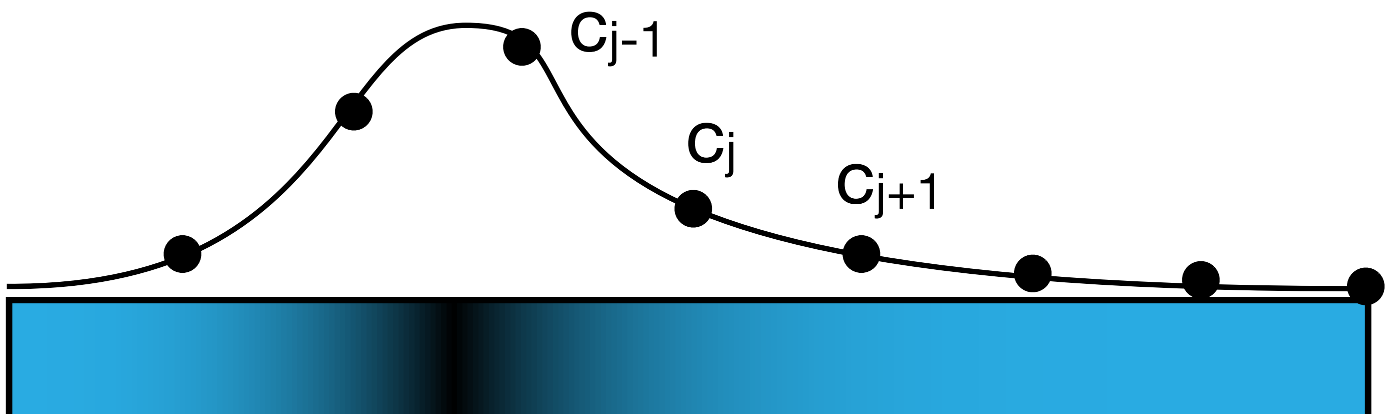

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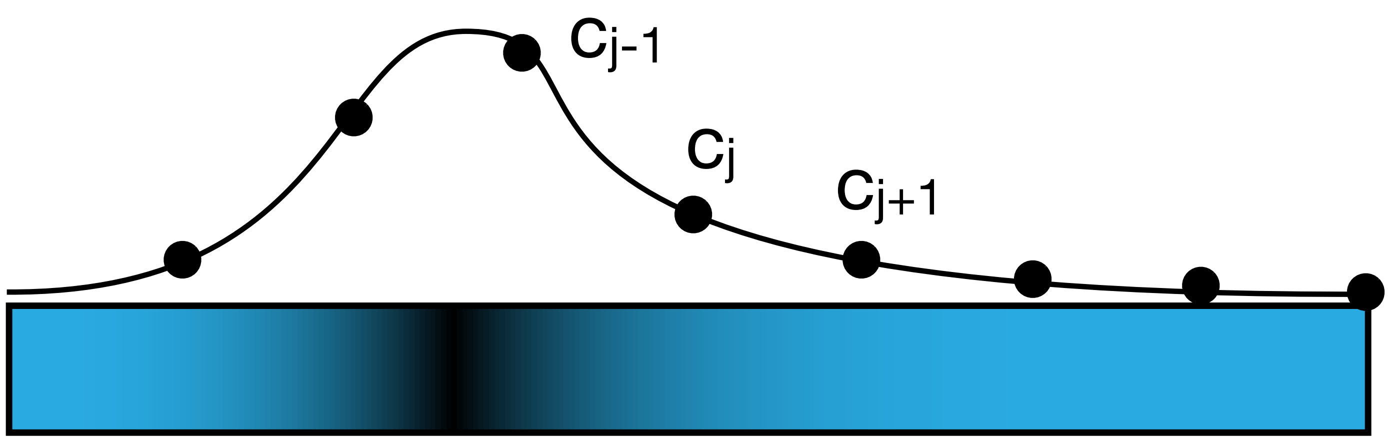

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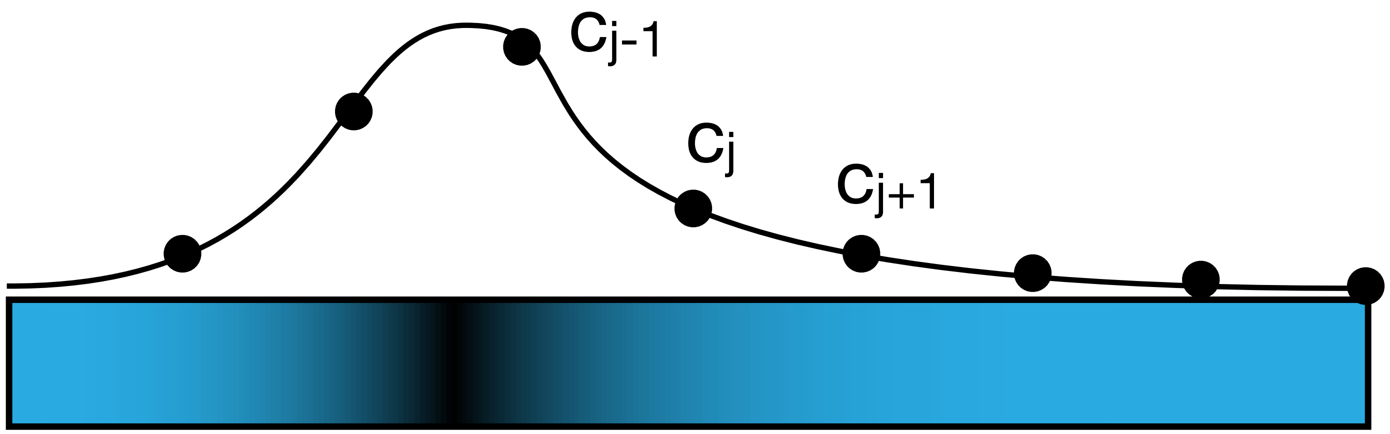

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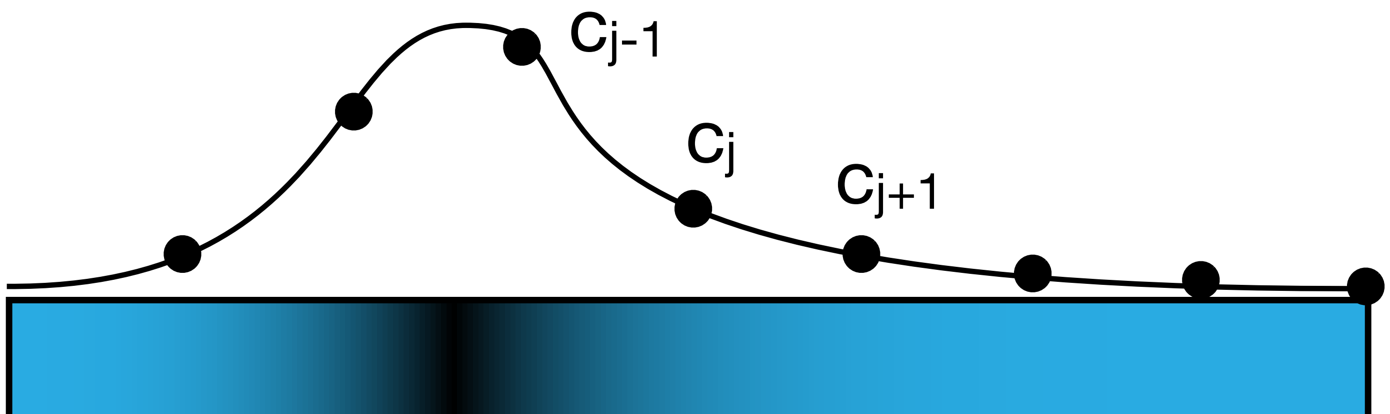

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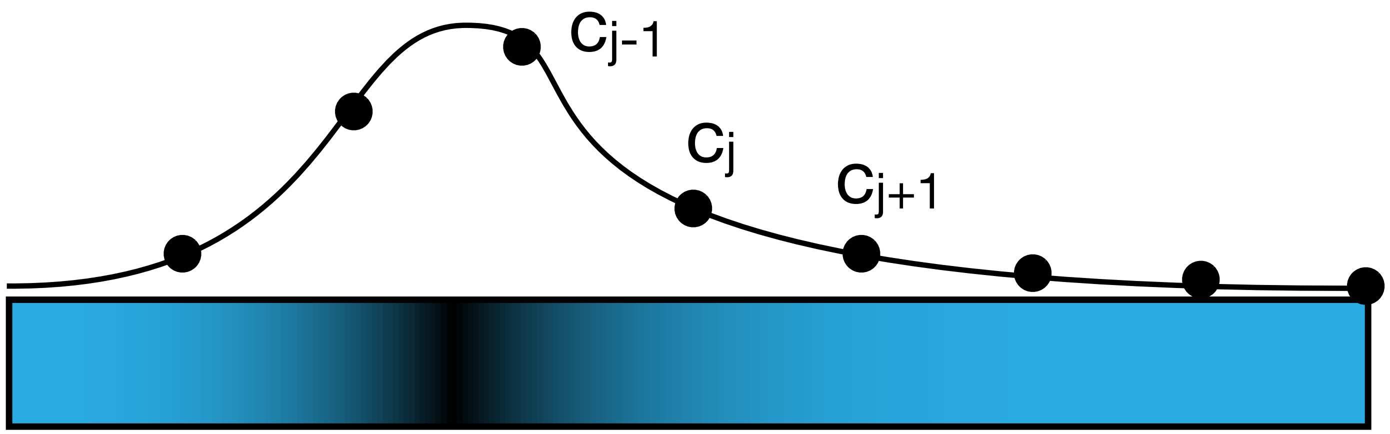

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The Diffusion Equation

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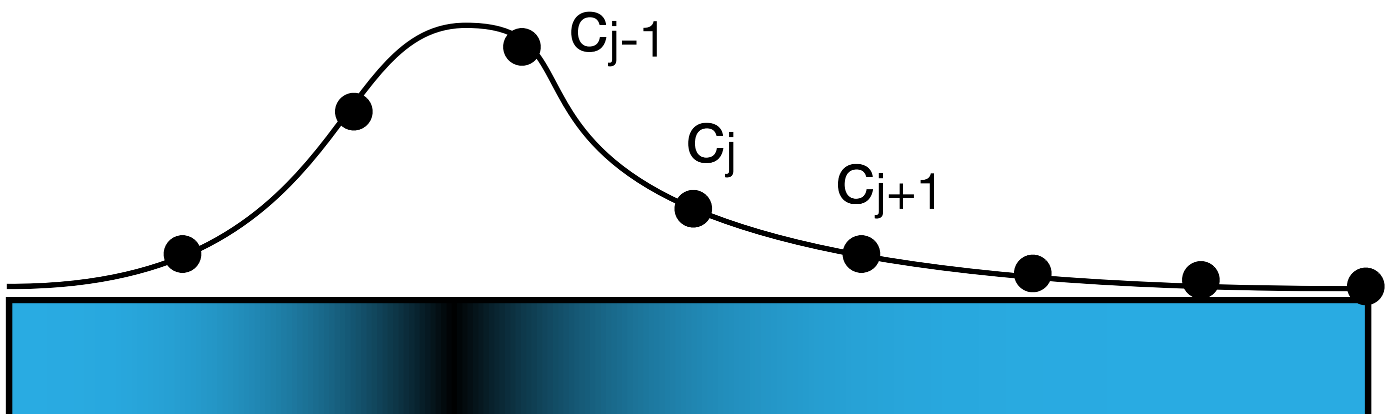

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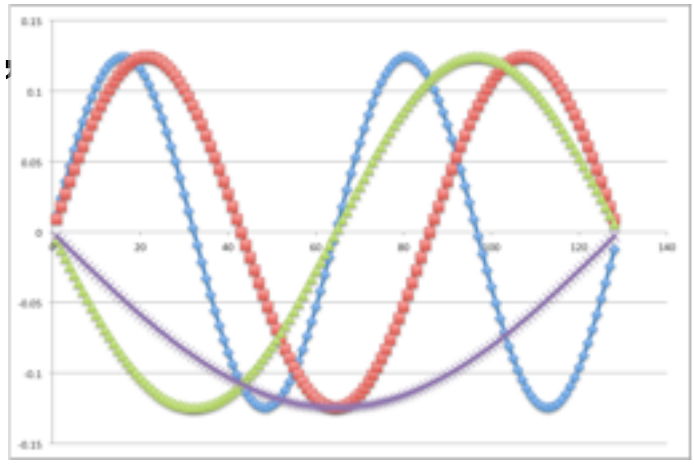

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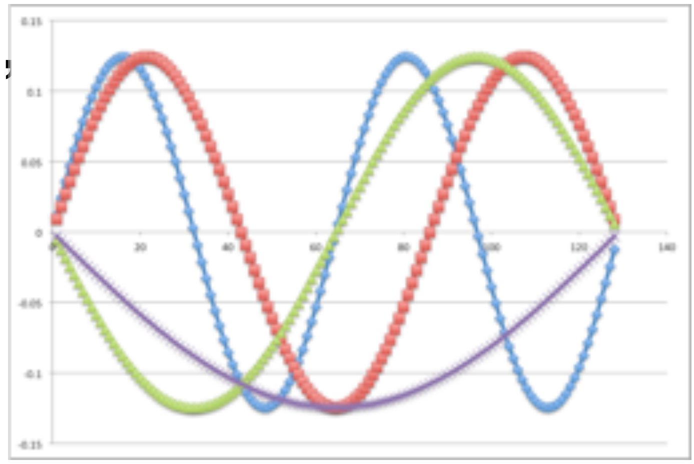

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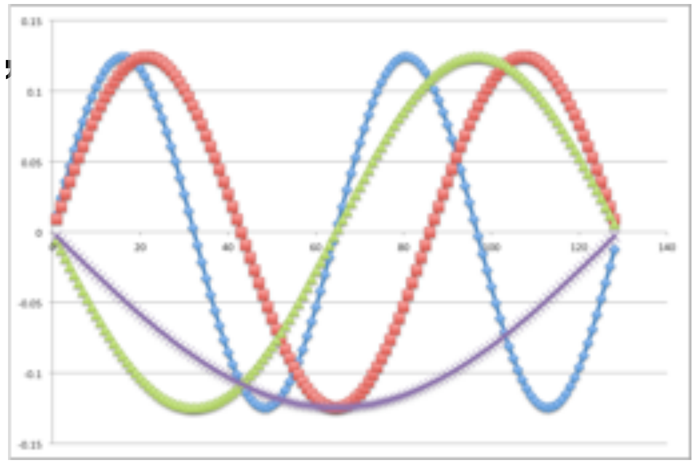

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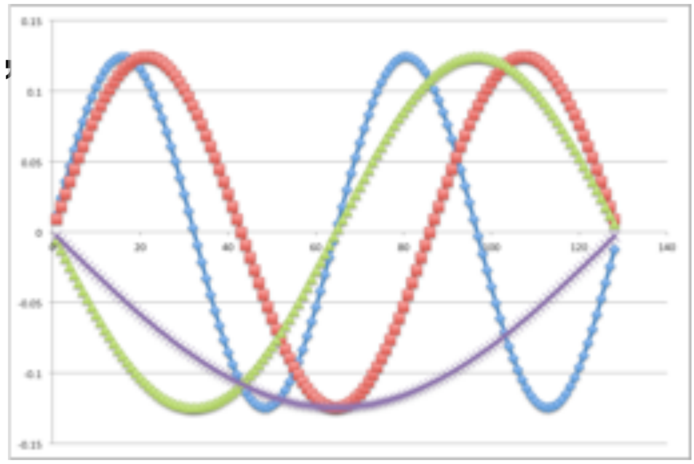

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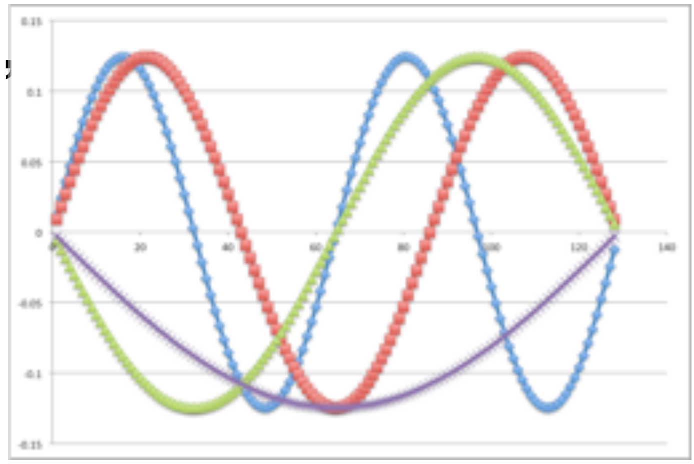

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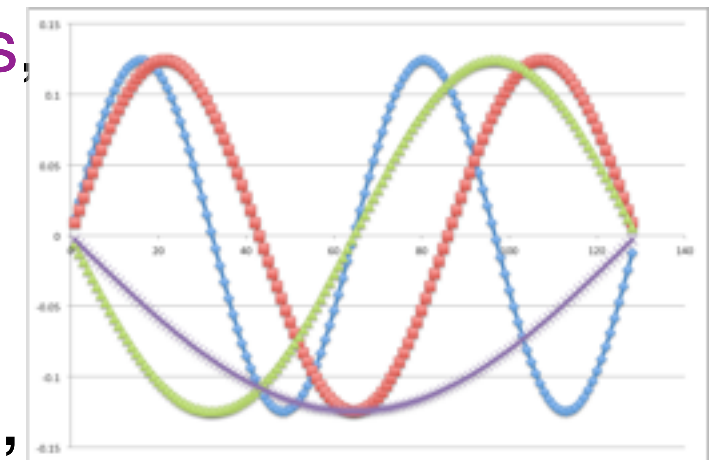

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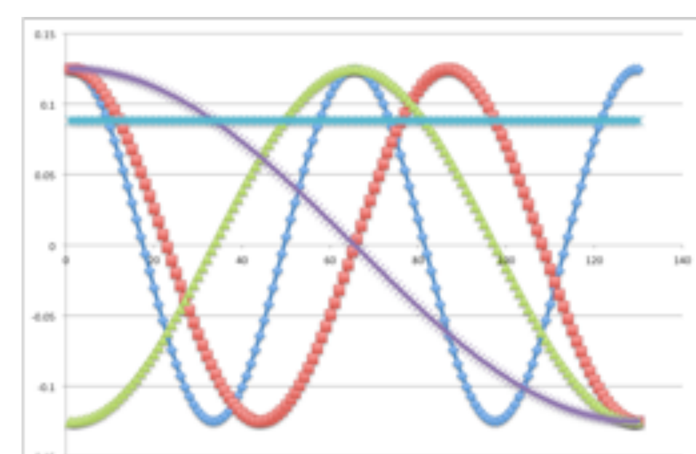

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- Eigenvalues and eigenvectors for a partial differential equation:

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If λ>0,

$$c(x) = e^{\sqrt{\frac{\lambda}{D}}x}$$

or

$$c(x) = e^{-\sqrt{\frac{\lambda}{D}}x}$$

Diffusion in a long thin tube

- Eigenvalues and eigenvectors for a partial differential equation:
- For the matrix equation $\mathbf{c}' = A\mathbf{c}$, find all eigenvalues λ and eigenvectors **c**: $A\mathbf{c} = \lambda\mathbf{c}$.
- For the PDE

$$\frac{dc}{dt} = D\frac{d^2c}{dx^2}$$

find all eigenvalues λ and eigen“vectors” c(x):

$$D\frac{d^2c}{dx^2} = \lambda c$$

If λ>0,

$$c(x) = e^{\sqrt{\frac{\lambda}{D}}x}$$

or

$$c(x) = e^{-\sqrt{\frac{\lambda}{D}}x}$$

If λ<0,

$$c(x) = \sin\left(\sqrt{\frac{-\lambda}{D}}x\right)$$

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$$\frac{dc}{dt} = D\frac{d^2c}{dx^2} \qquad \begin{aligned} c(L,t) &= 0 \\ c(0,t) &= 0 \end{aligned} \qquad c(x,0) = f(x)$$

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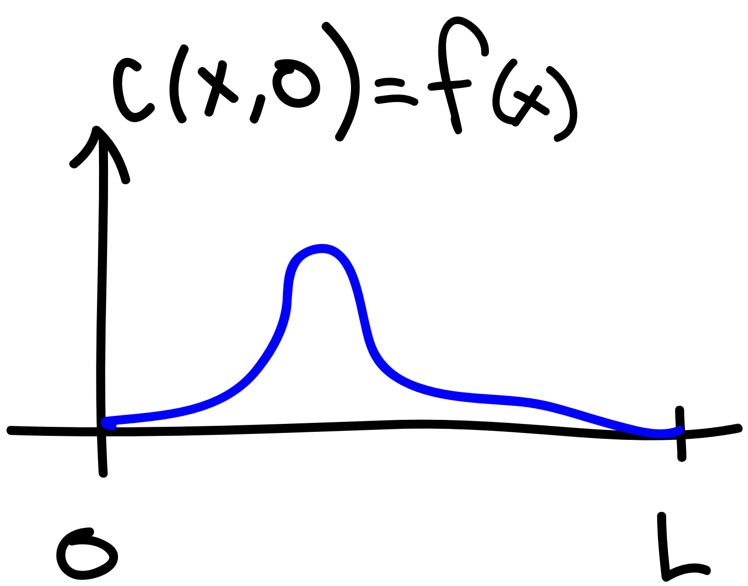

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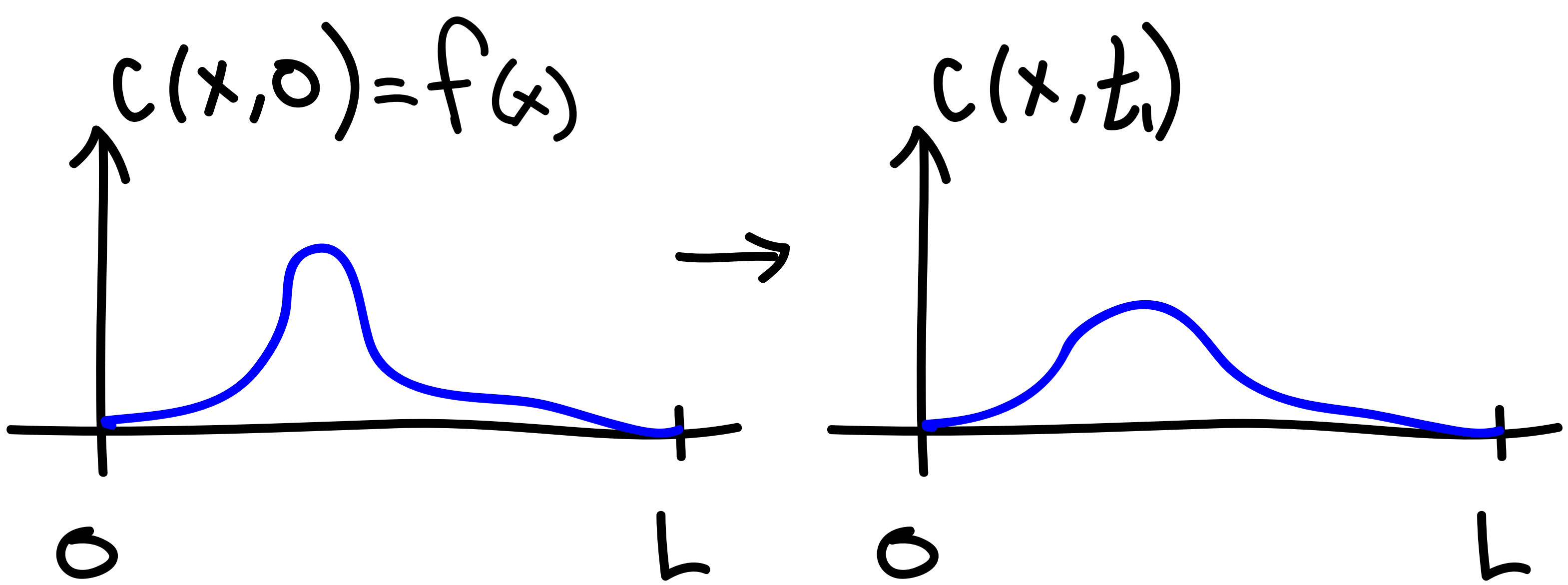

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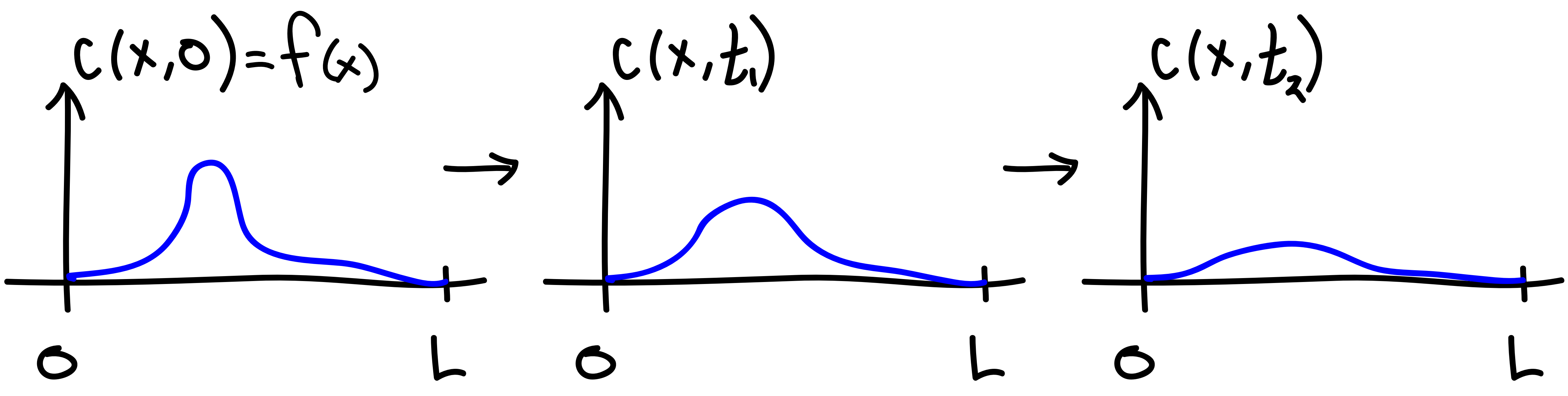

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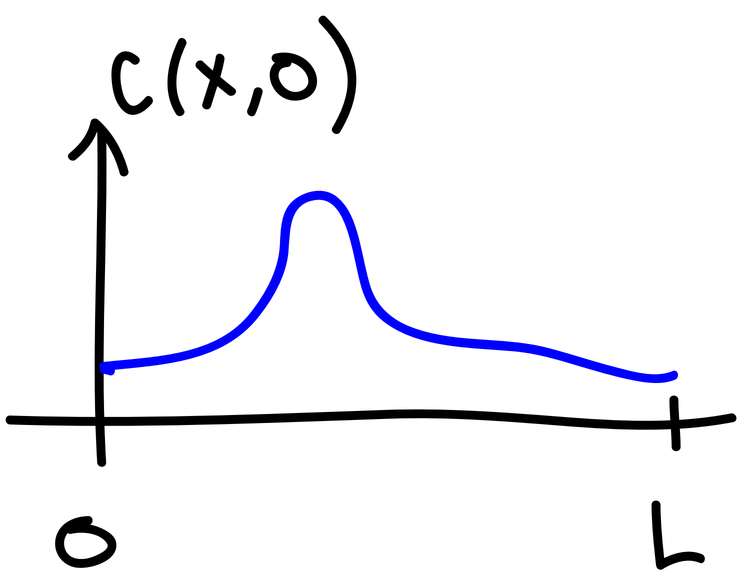

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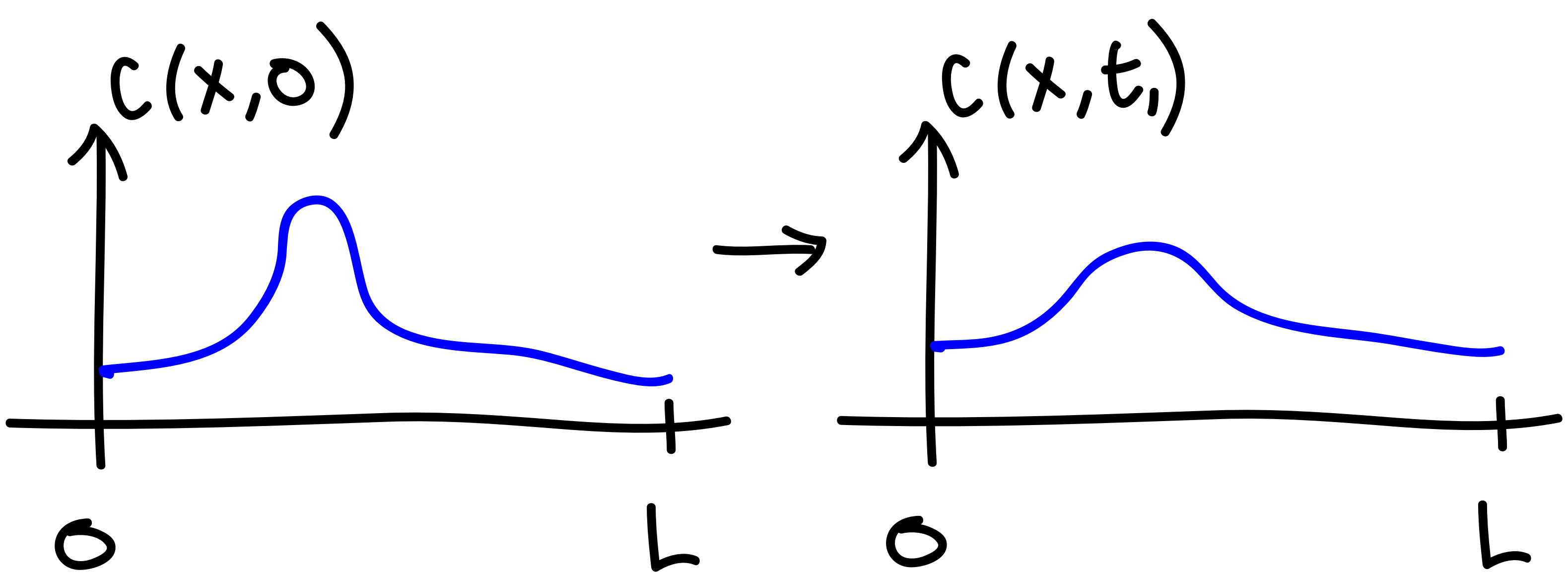

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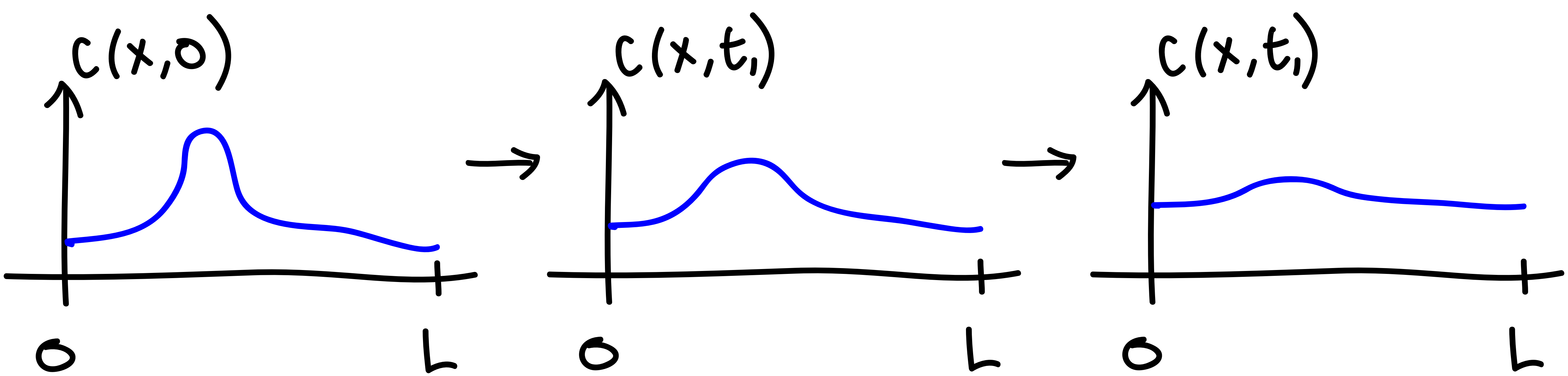

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