Welcome to MATH 256

Differential equations (for Chemical and Biological Engineering students)

Instructor:
Prof. Eric Cytrynbaum
cytryn@math.ubc.ca
http://wiki.math.ubc.ca/mathbook/M256
Office: MATX 1215
Office hours: Tues 11:30 am - 1 pm, Thurs 3:30 - 4:30 pm <---- ?!?

Course goals

- Primary: Learn to solve ordinary and partial differential equations (mostly linear first and second order DEs).
- Secondary: Learn to use DEs to model physical, chemical, biological systems (really just an intro to this skill).

Prerequisites

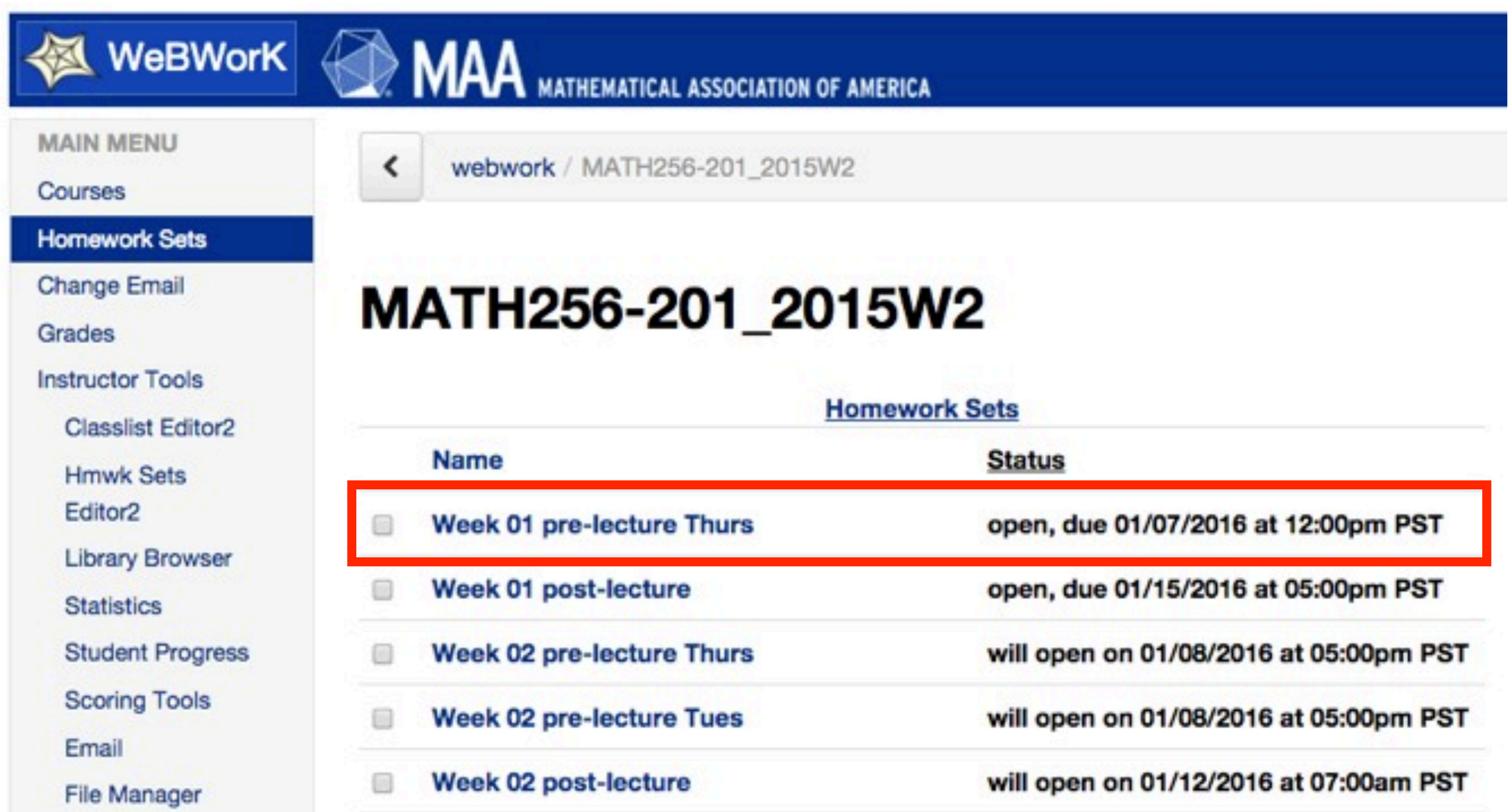
- First year calculus (MATH 100/101).
- Linear algebra (MATH 152).
- Multivariable calculus (MATH 200 or 253).
- Talk to me if you aren’t sure that you’re prepared for this course.

Tools we’ll be using this term

- WeBWorK for homework assignments.
- Facebook for online discussion, updates etc. (any Piazza fans?)
- Clickers for in-class responses

WeBWorK

- Online homework system.
- https://webwork.elearning.ubc.ca/webwork2/MATH256-201_2014W2
- Log in using your CWL.
- First HW due Thurs.



Clickers

- Personal response system.
- Register your clicker at https://connect.ubc.ca

Why / how clickers?

- Active learning - you should be thinking and doing during class.
- My goal is to make clicker Qs that many of you get wrong - they help us to target what you don't understand yet.
- Points are for (thinking and then) clicking, not for getting answers correct.
- I don't look at the results on an individual basis so they are effectively anonymous.

More info online...

Log in

Page View Search

MATH 256 - 2015W2 - Differential Equations

Navigation

- MATH 256 Home
- Course schedule
- Lecture slides
- Pre-lecture resources
- WeBWorK
- Instructors' site

Course description

This course serves as an introduction to differential equations with a focus on solution techniques, transforms and modeling. Topics include linear ordinary differential equations, Laplace transforms, Fourier series and separation of variables for linear partial differential equations.

This website is the course website for MATH 256 taught in 2015W2.

Course details

- Instructor information
- Marking scheme
- Important dates
- Other course information
- Solutions - Tutorial worksheets, old midterms
- General resources - including links to old course websites, old assignments, suggested practice problems etc.
- Course outline - summary of content above.

Felix Baumgartner's freefall from 40 km up

- Newton says F_{net}=ma or

$$ma = -mg + kv^2$$

- A differential equation in disguise because

$$a = v'$$

- so the equation is really a DE for v(t)!

$$mv' = -mg + kv^2$$

https://www.youtube.com/watch?v=vvbN-cWe0A0

- Simple model to predict how fast he'll go, how long it will take etc.

Felix Baumgartner’s freefall from 40 km up

$$mv' = -mg + kv^2$$

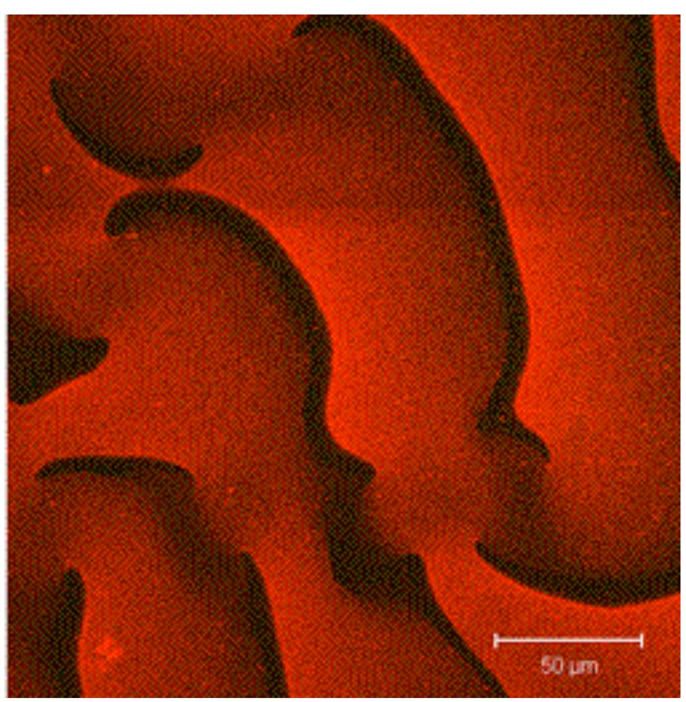
- Flaws with this model?
- g is not constant...
- ...but 6371 km ≈ 6411 km so not bad.
- k is not constant either (depends on air density) - this is significant!

$$mv' = -mg + k(x)v^2 \qquad mx'' = -mg + k(x)x'^2$$

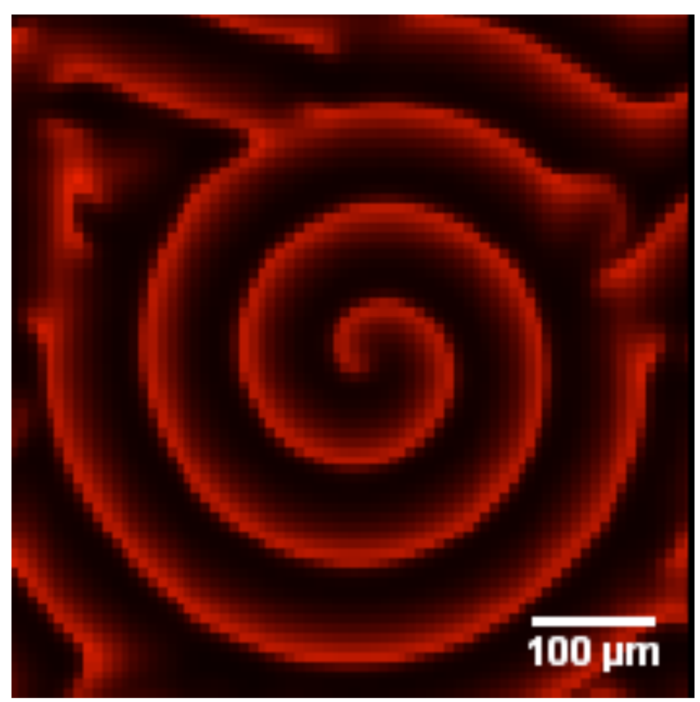
A bacterial cell division regulator

- Two interacting bacterial proteins that undergo complicated dynamics.
- Differential equation model help understand how they work.

Experiment



Model



$$\frac{\partial u}{\partial t} = u - uv + D\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = uv - v + D\frac{\partial^2 v}{\partial x^2}$$

Classifying DEs

- Ordinary differential equation (ODE) - a DE that involves derivatives of a function with respect to only one independent variable.

Logistic equation: $$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Beam equation: $$EI\frac{d^4w}{dx^4} = q$$

- Partial differential equation (PDE) - a DE that involves derivatives of a function with respect to more than one independent variable.

Heat/diffusion equation: $$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$

Wave equation: $$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

Classifying DEs

- Order of a DE - order of the highest derivative in the equation.
- e.g. Heat/diffusion equation: $$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$
    - First order in time (t), second order in space (x).

Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Beam equation:

$$EI\frac{d^4 w}{dx^4} = q$$

Logistic equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

- Order (in time):
- Order (in space):
    - (A) first order
    - (B) second order
    - (C) third order
    - (D) fourth order

Classifying DEs

- Linearity - a DE is linear if it is linear in the unknown function and all its derivatives.

- (A) Linear or (B) nonlinear:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) = rP - \frac{r}{K}P^2$$ <--- Nonlinear

$$EI\frac{d^4w}{dx^4} = q$$ <--- Linear

$$t^2\frac{dy}{dt} + y = \sin(t)$$ <--- Linear

$$t^2\frac{dy}{dt} + y^2 = \sin(t)$$ <--- Nonlinear

More definitions - solutions

- Solution to a DE on some interval A
  - a function that is suitable differentiable everywhere in A (i.e. has as many derivatives as appear in the equation) and,
  - satisfies the equation.
- Arbitrary constant - a constant that does not appear in the DE but arises while solving the equation (usually at an integration step).
- A particular solution - a solution with no arbitrary constants in it.
- The general solution - a solution with one or more arbitrary constants that encompass ALL possible solutions to the DE.

Verifying that a function is a solution

- Plug it in and make sure it satisfies the equation.

A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket $(h(0) = 4)$, what is the solution to this differential equation?

A. $h(t) = (2 - 3t)^2$ ←
B. $h(t) = \sqrt{16 - 2t}$
C. $h(t) = (3 - 3t)^2$
D. $h(t) = 4 - 6t^2$

For this one, "brute force checking" is expected as we don't have a technique to handle this type yet.

Method of integrating factors

$$\frac{d}{dt}\left(t^2 y(t)\right) =$$

(A) $2t\frac{dy}{dt}$

(B) $t^2\frac{dy}{dt}$

(C) $2ty$

(D) $t^2\frac{dy}{dt} + 2ty$

(E) Don't know.

Method of integrating factors

$$\frac{d}{dt}\left(t^2 y(t)\right) =$$

(A) $2t\dfrac{dy}{dt}$

(B) $t^2\dfrac{dy}{dt}$

(C) $2ty$

(D) $t^2\dfrac{dy}{dt} + 2ty$

(E) Don’t know.

Method of integrating factors

- Given that $\dfrac{d}{dt}\left(t^2 y(t)\right) = t^2 \dfrac{dy}{dt} + 2ty$

- if you're given the equation $t^2 \dfrac{dy}{dt} + 2ty = 0$

- you can rewrite is as $\dfrac{d}{dt}\left(t^2 y(t)\right) = 0$

- so the solution is $t^2 y(t) = C$ or equivalently $y(t) = \dfrac{C}{t^2}$.

arbitrary constant that appeared at an integration step