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- Heuristic explanation for exponential solutions and Reduction of order.

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$$v'' = 0 \quad \Rightarrow \quad v' = C_1 \quad \Rightarrow \quad v(t) = C_1 t + C_2$$

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So yes!

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(C) $y(t) = C_1 e^{3t} + C_2 e^{-3t}$

☆ (D) $y(t) = C_1 e^{3t} + C_2 t e^{3t}$

(E) $y(t) = C_1 e^{3t} + C_2 v(t) e^{3t}$

Second order, linear, constant coeff, homogeneous

- Find the general solution to the equation

$$y'' - 6y' + 10y = 0$$

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Second order, linear, constant coeff, **non**homogeneous (3.5)

- Our next goal is to figure out how to find solutions to nonhomogeneous equations like this one:

$$y'' - 6y' + 8y = \sin(2t)$$

- But first, a bit more on the connections between matrix algebra and differential equations . . .

Some connections to linear (matrix) algebra

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$$\overline{y} = A\overline{x}$$

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$$z = L[y] = \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y$$

- This one is linear because

$$L[cy] = cL[y]$$

$$L[y + z] = L[y] + L[z]$$

Note: y, z are functions of t and c is a constant.

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$$L[y] = 0$$

- A non-homogeneous differential equation has the form

$$L[y] = g(t)$$

Solutions to homogeneous matrix equations

- The matrix equation $A\overline{x} = \overline{0}$ could have (depending on A)

  (A) no solutions.

  (B) exactly one solution.

  (C) a one-parameter family of solutions.

  (D) an n-parameter family of solutions.

Choose the answer that is incorrect.

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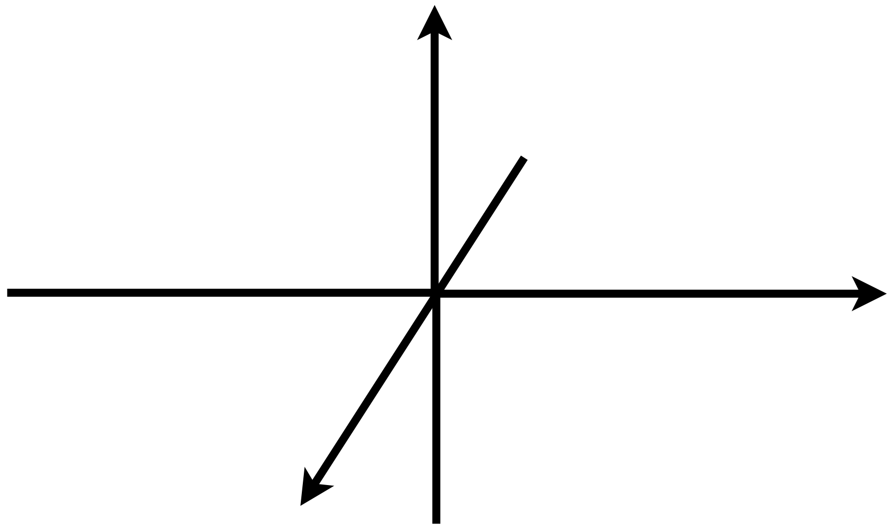

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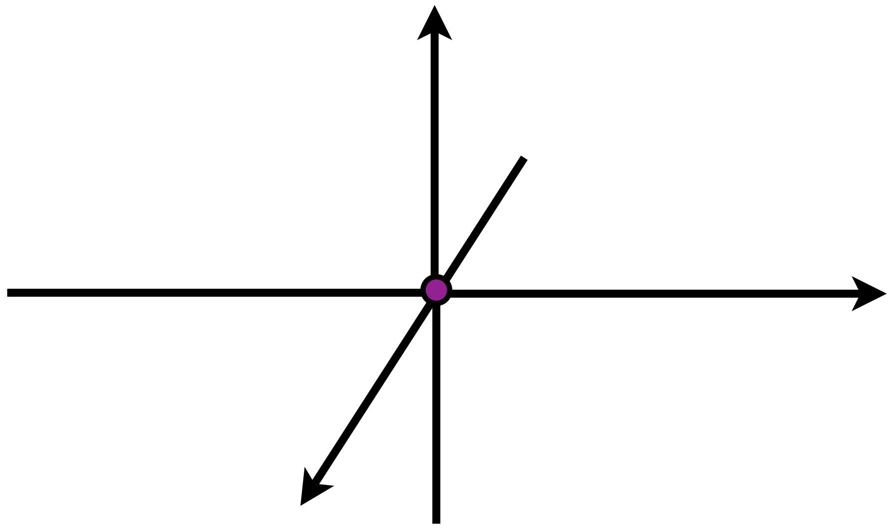

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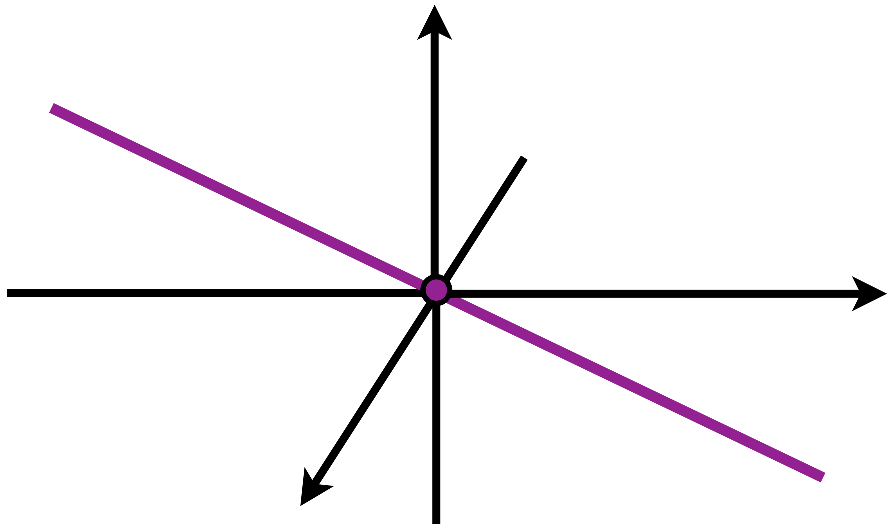

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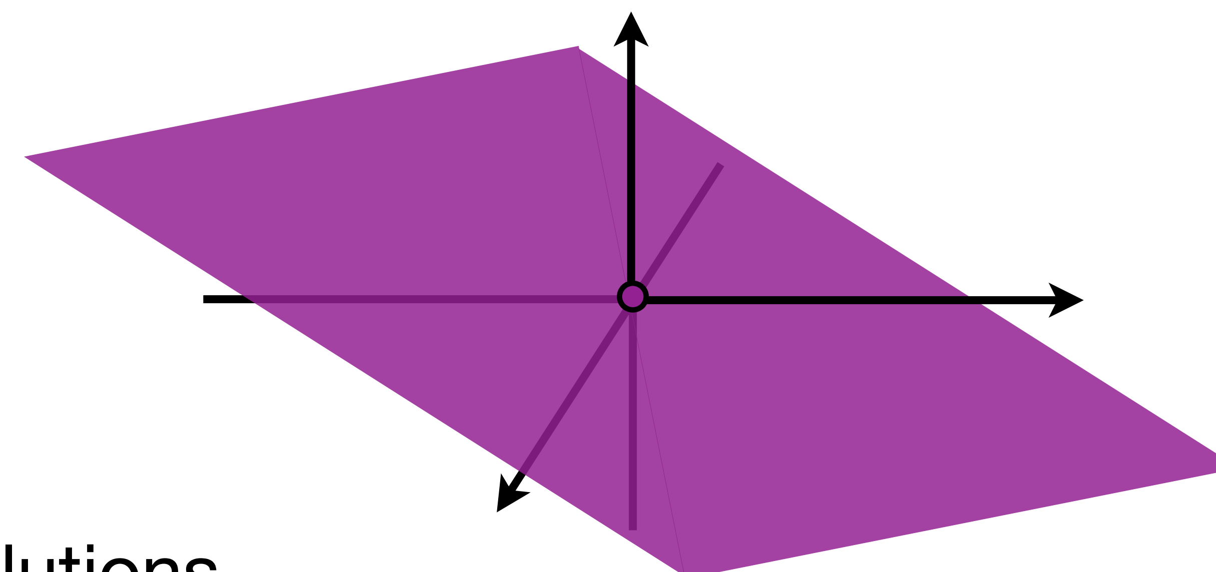

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$$\overline{x} = C_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

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- Example 1. Solve the equation $A\overline{x} = \overline{0}$ where

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In this case, only two of them really matter.

- so $x_1 - \frac{1}{3}x_3 = 0$ and $x_2 + \frac{5}{3}x_3 = 0$ and x_3 can be whatever (because it doesn't have a leading one).

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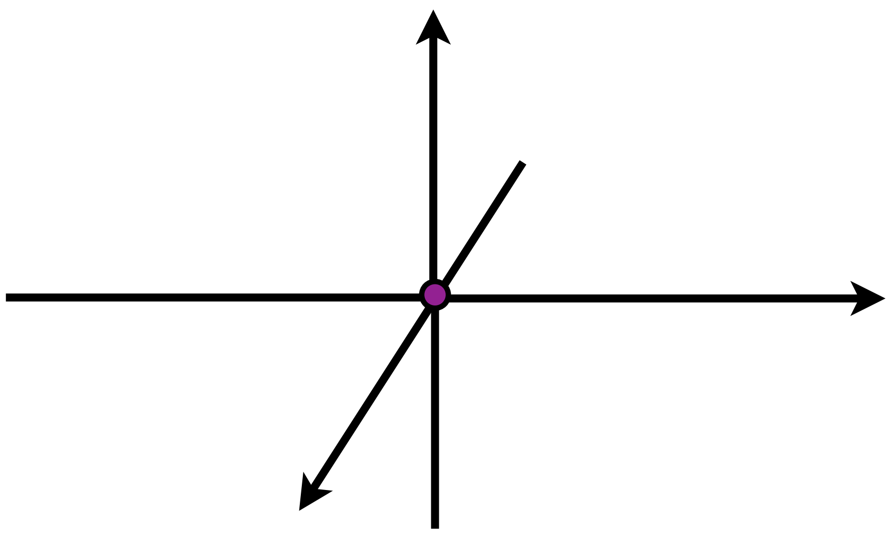

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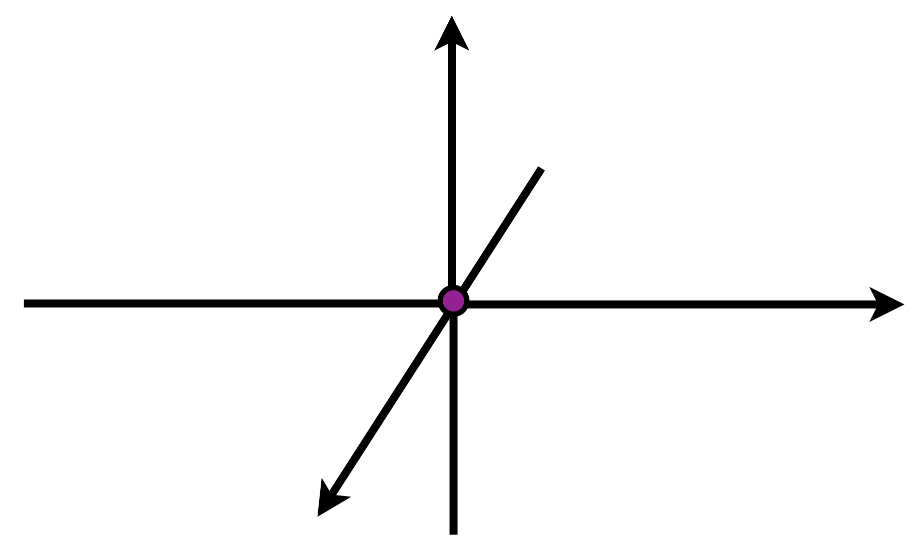

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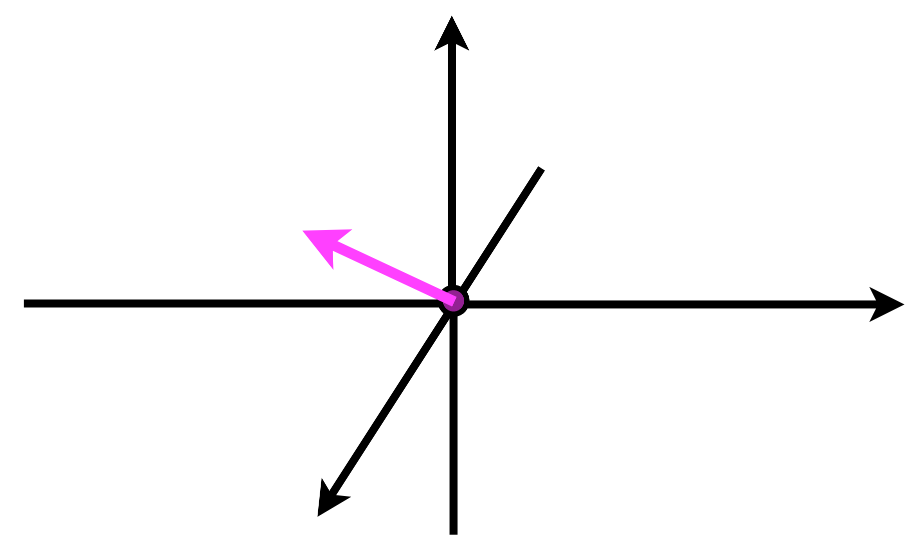

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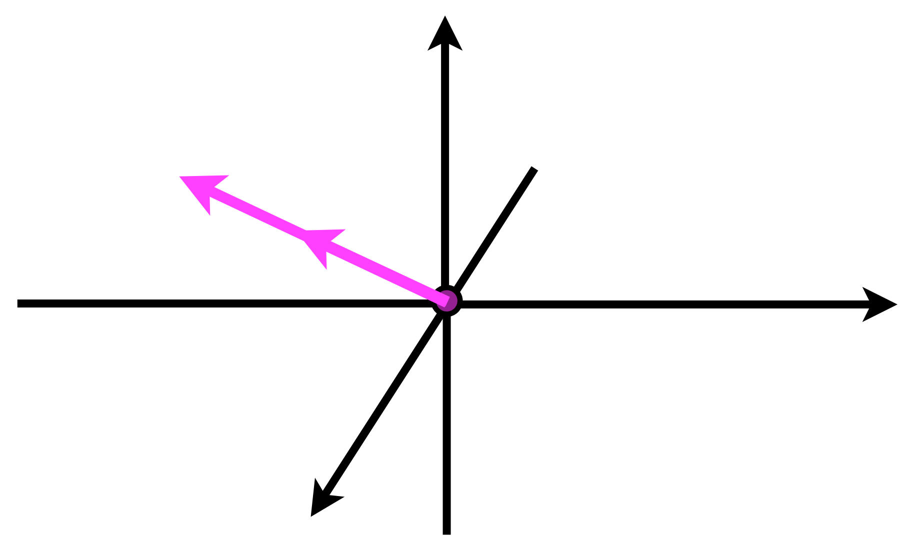

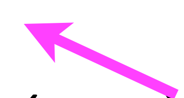

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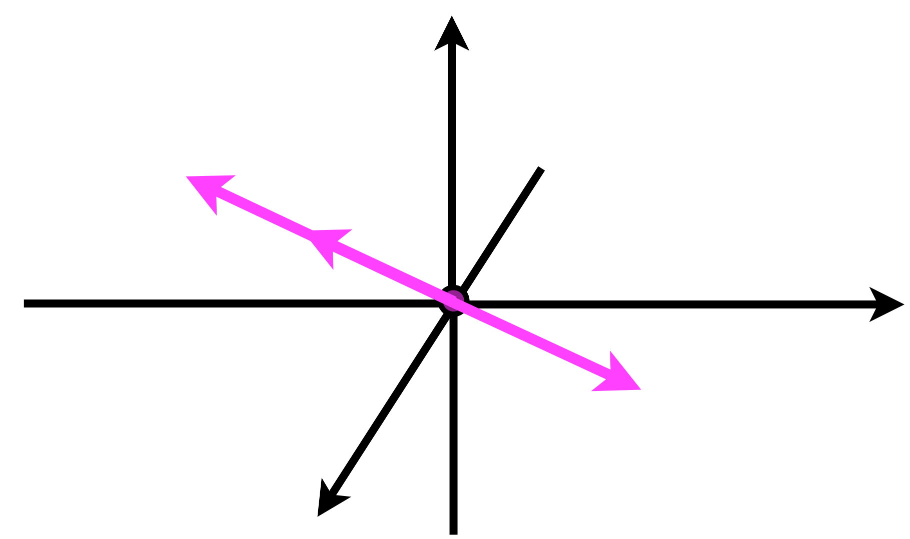

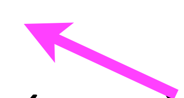

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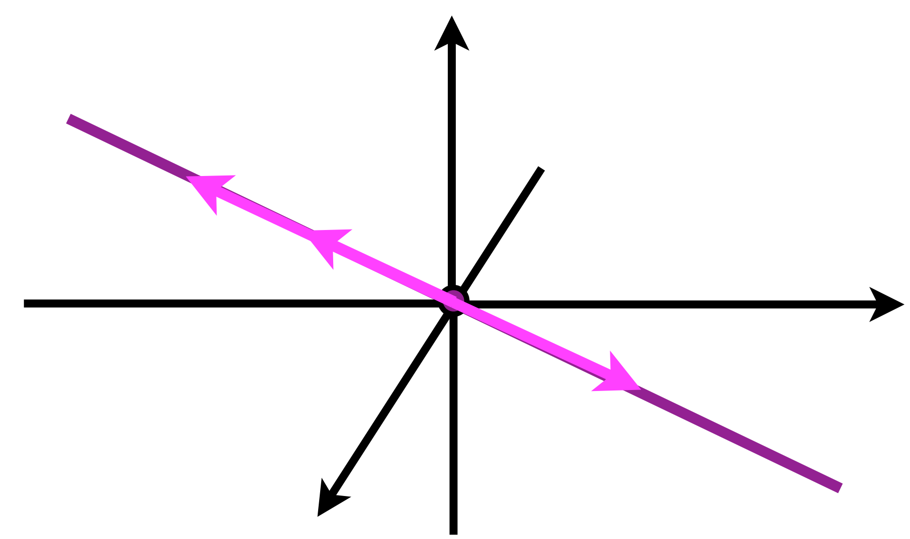

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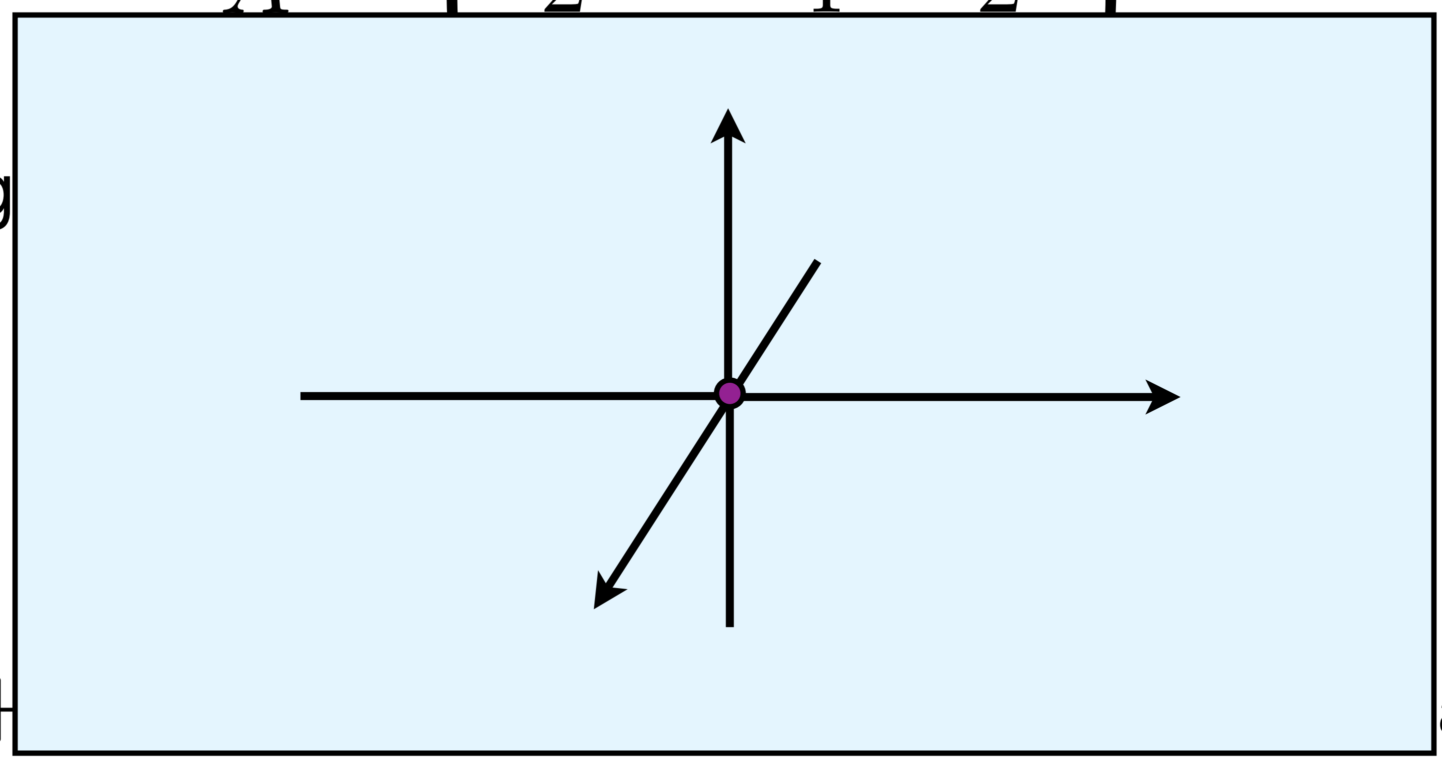

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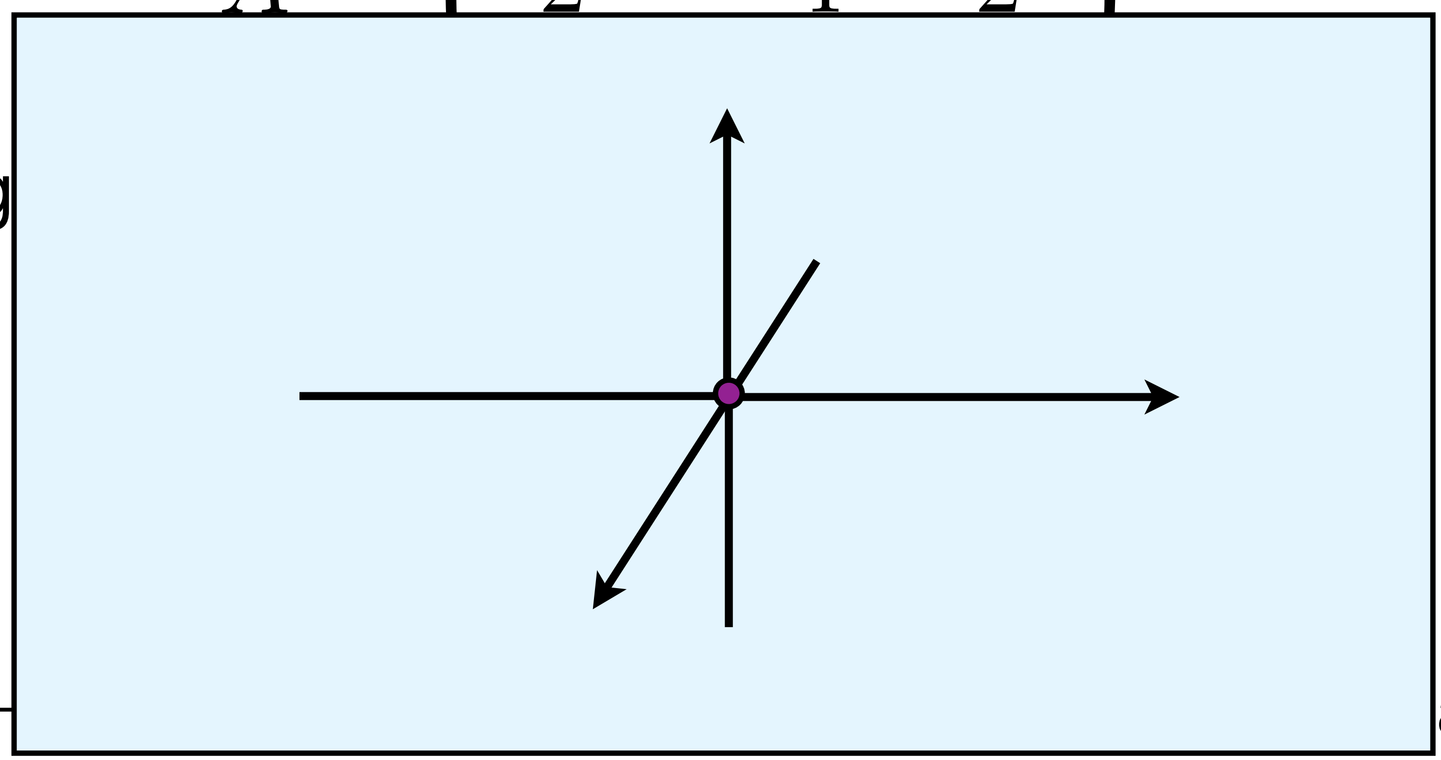

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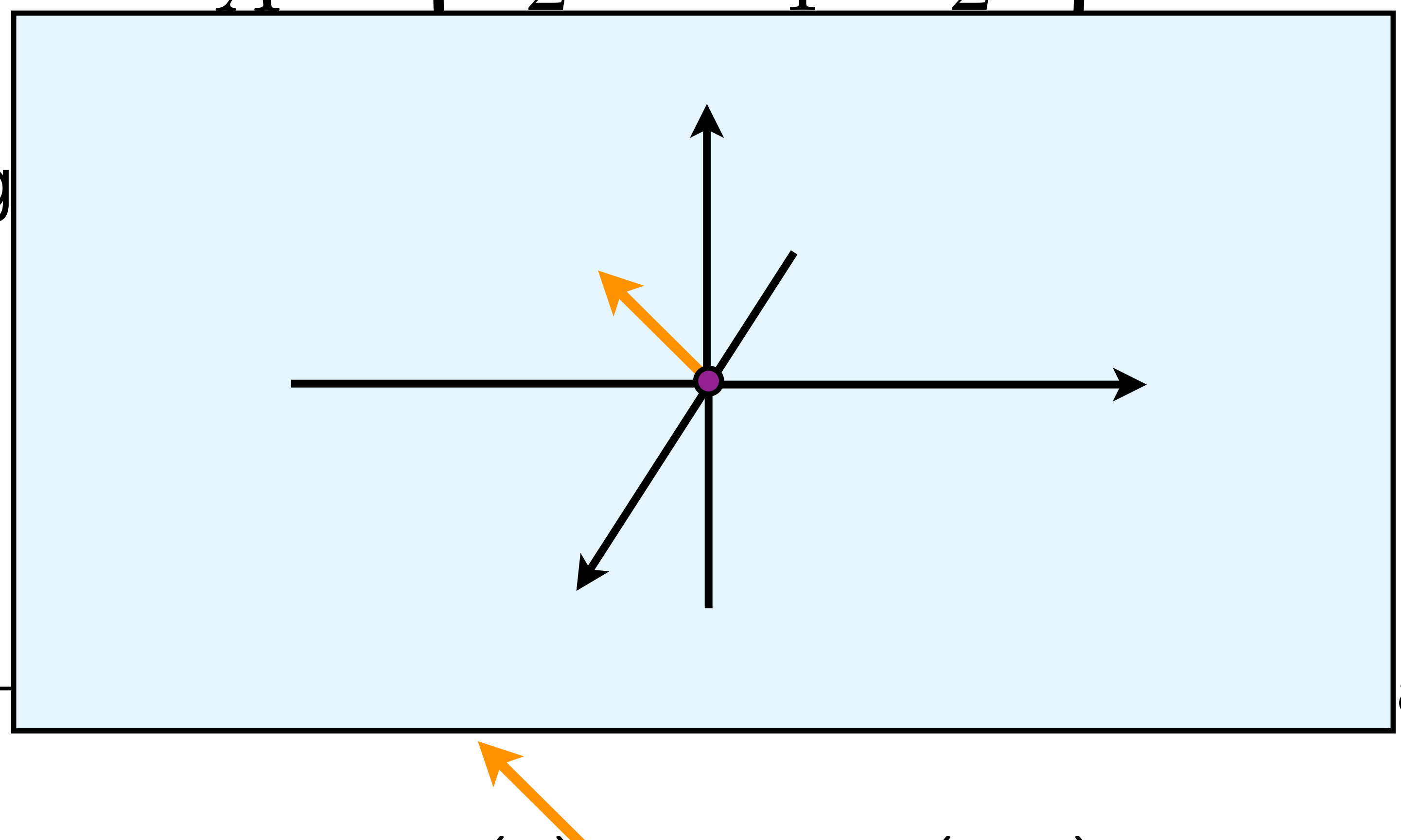

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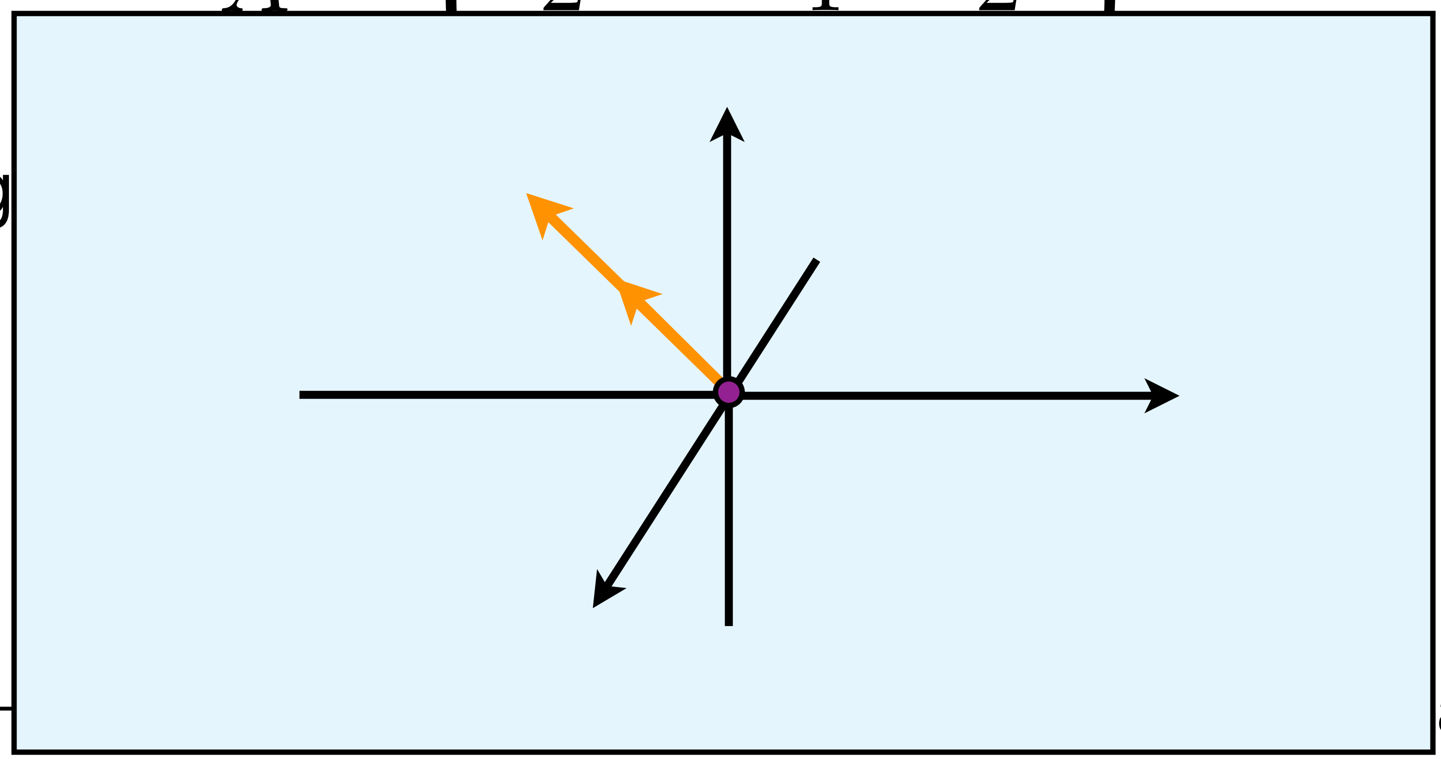

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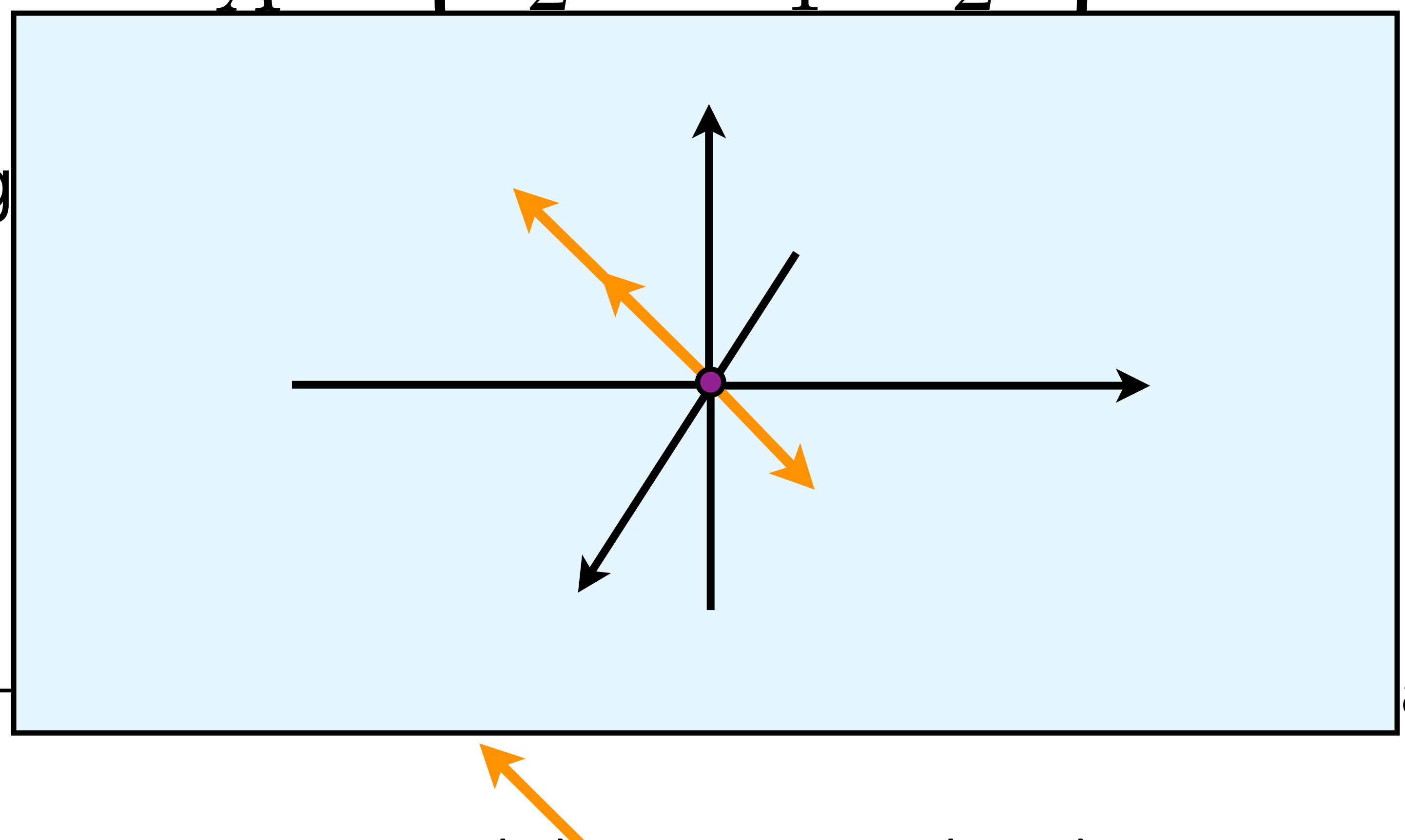

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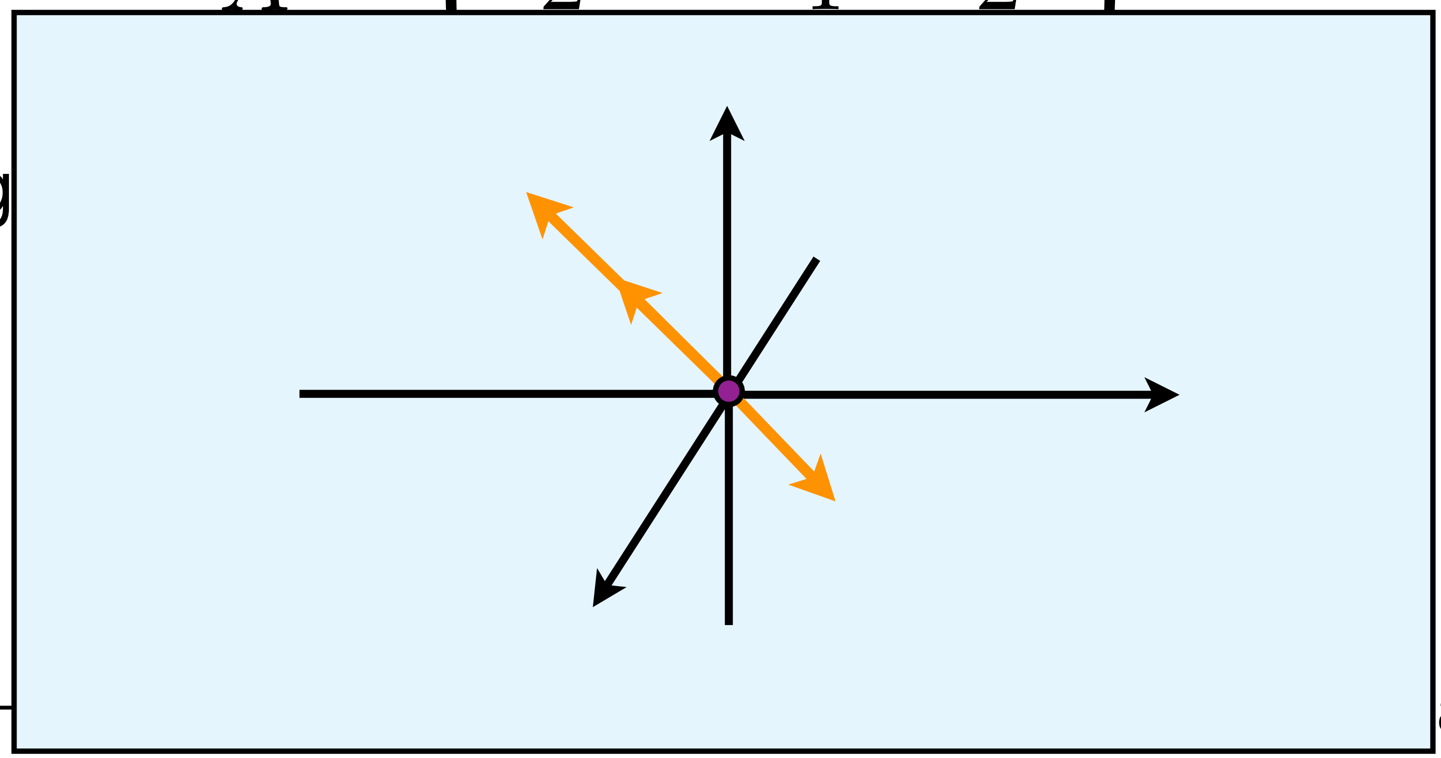

- Row reduction g
- so $x_1 - 2x_2 +$ atever.

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Solutions to homogeneous matrix equations

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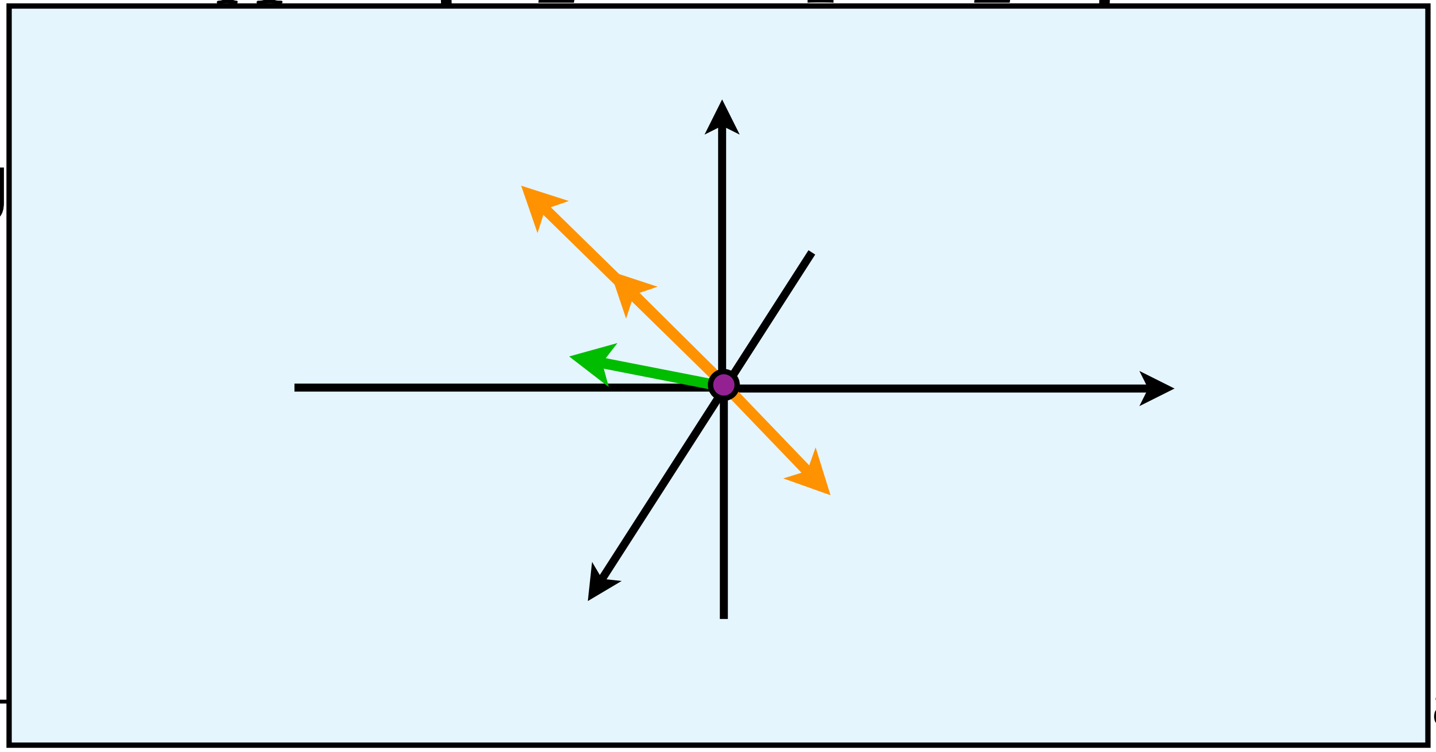

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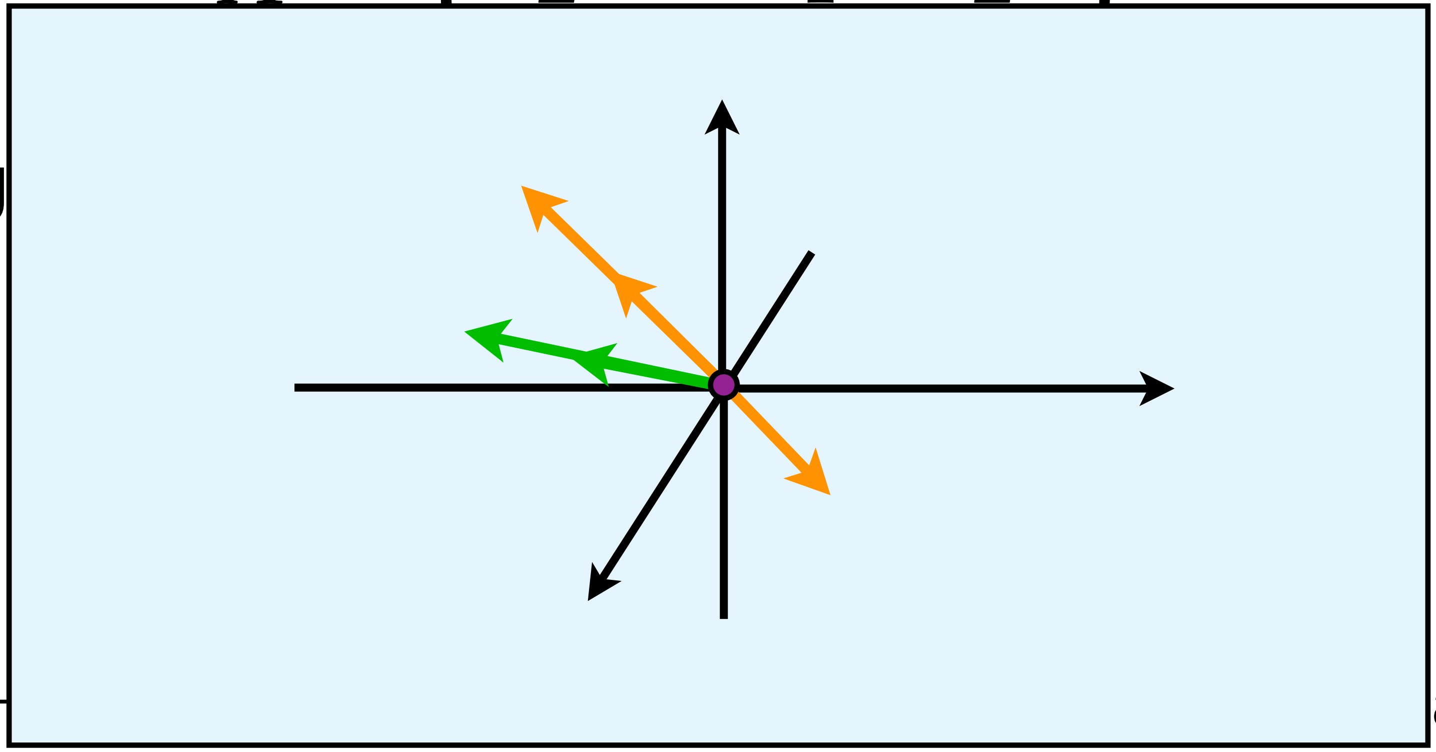

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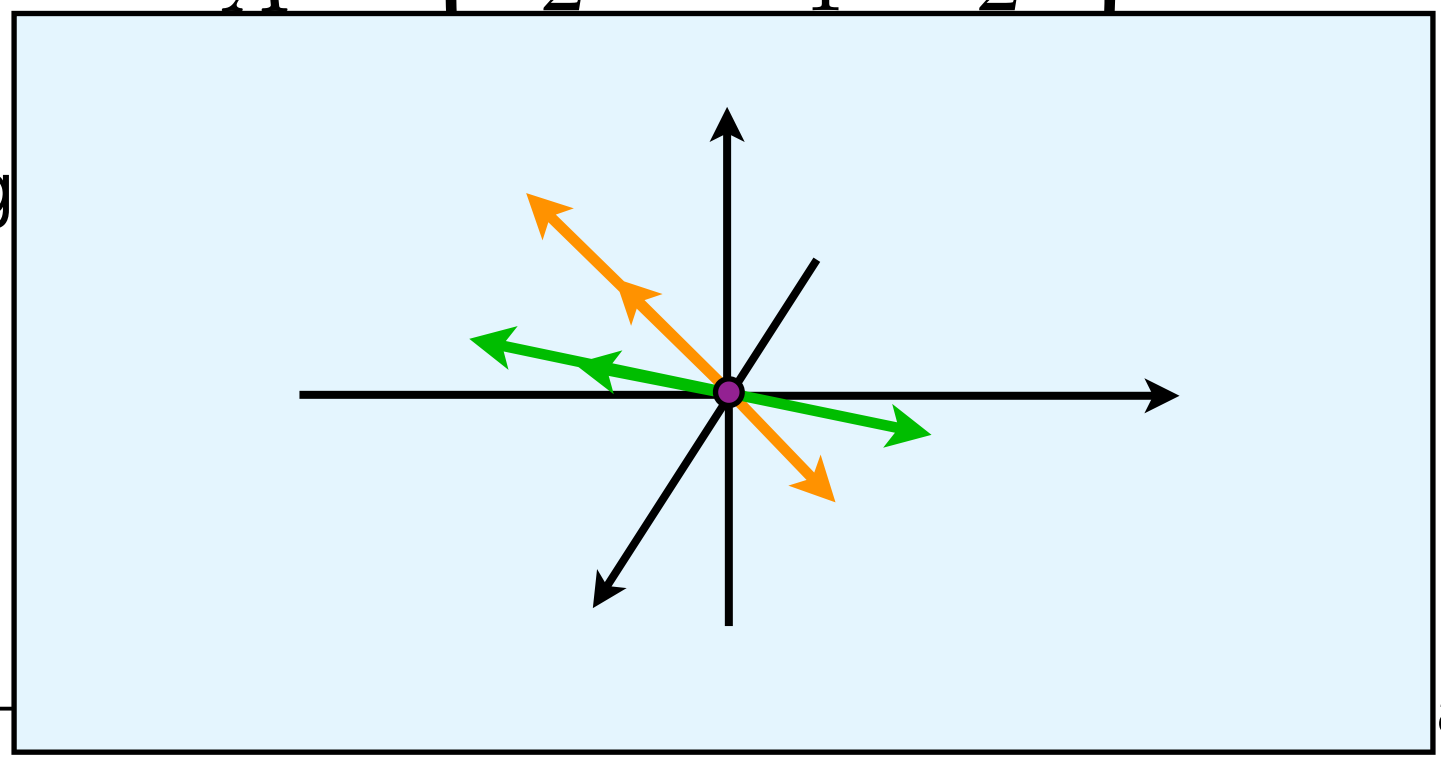

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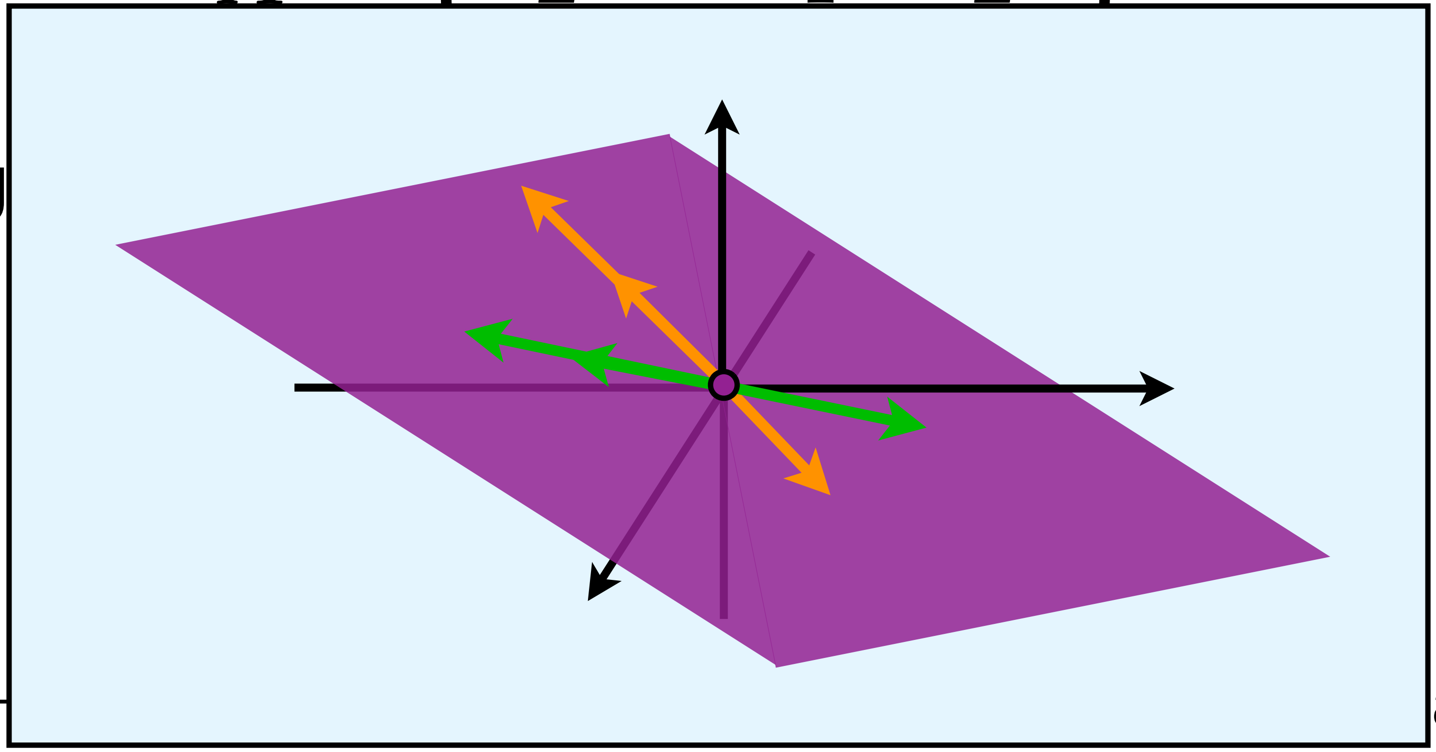

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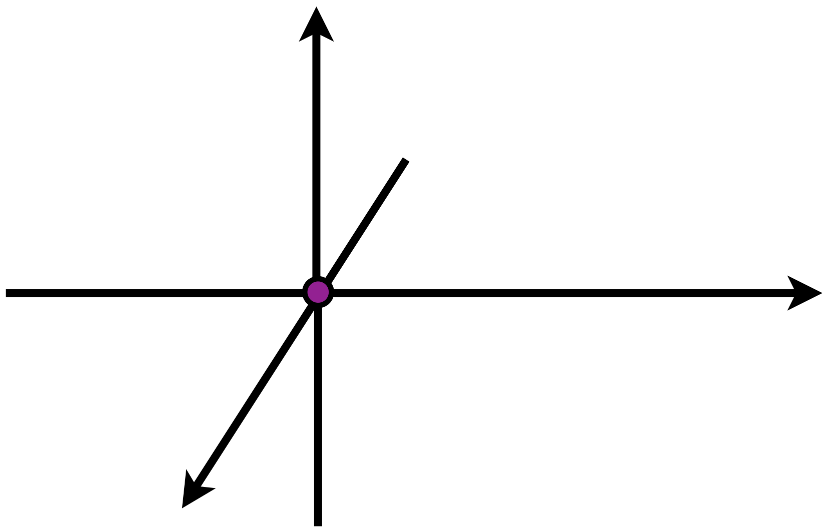

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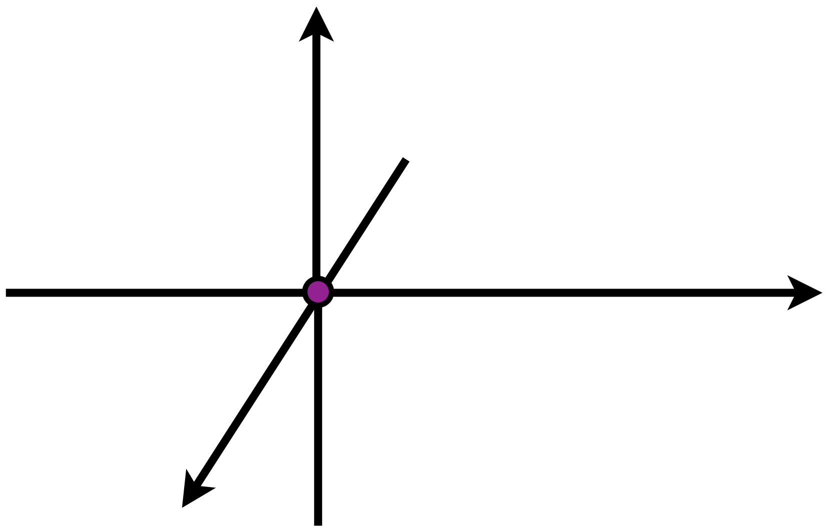

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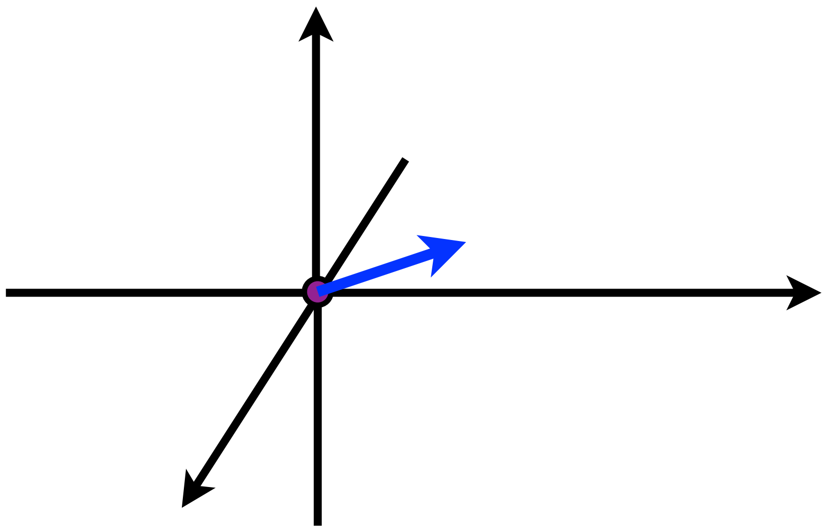

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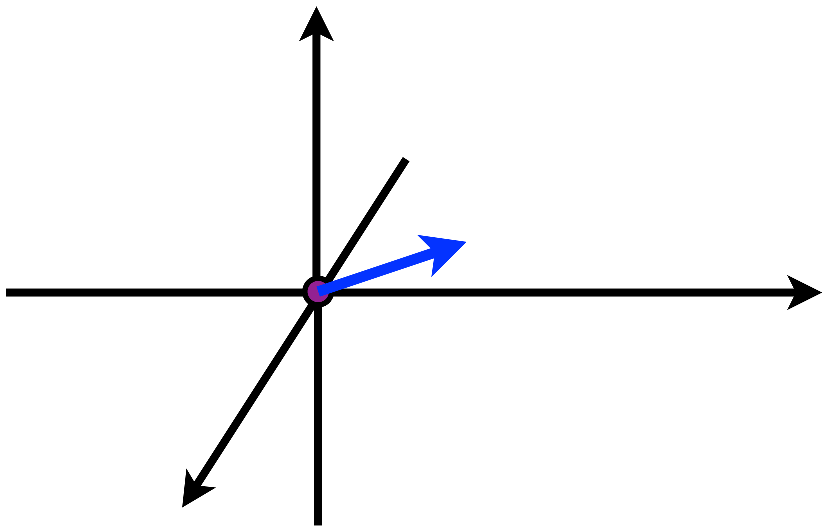

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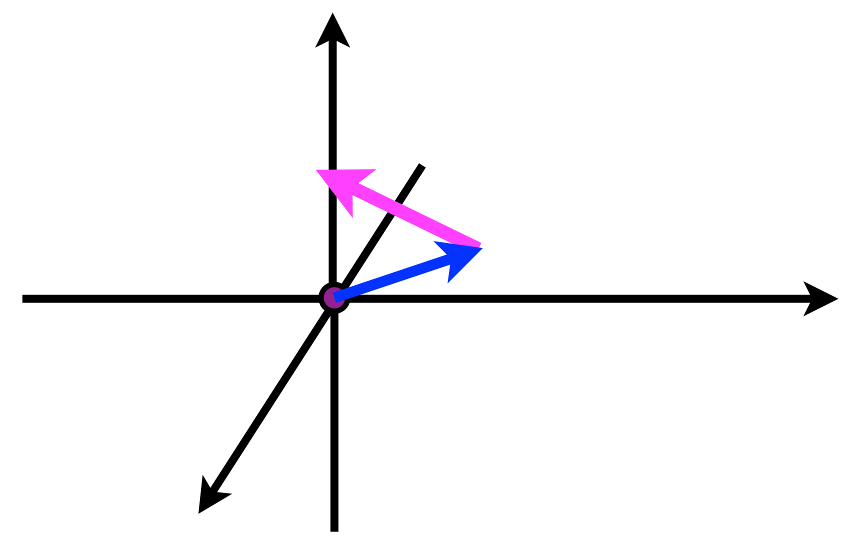

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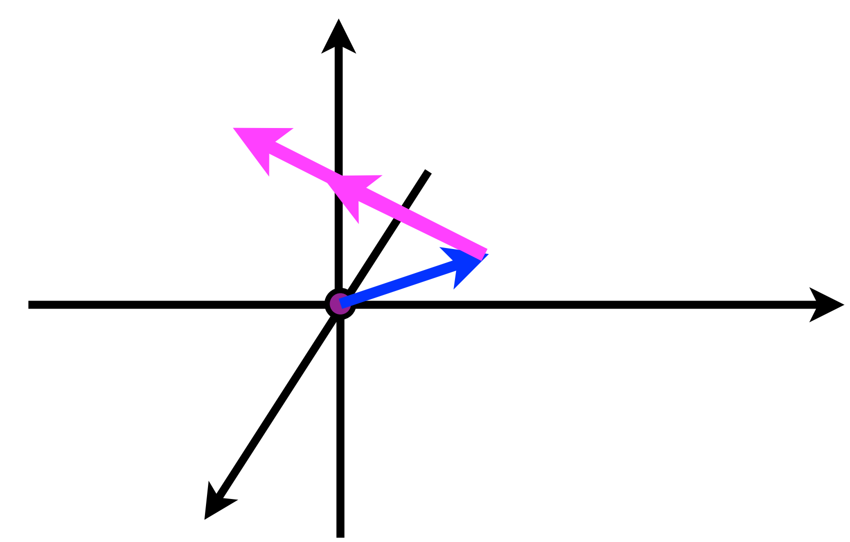

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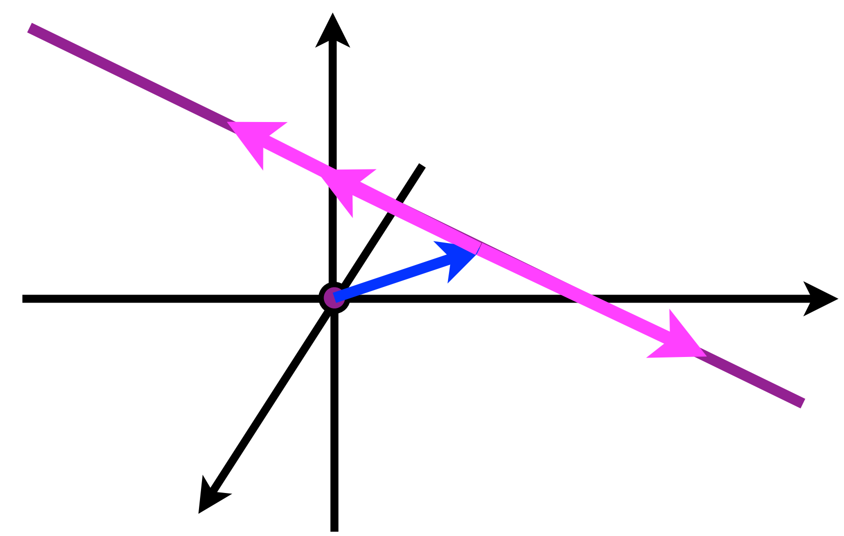

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Solutions to nonhomogeneous differential equations

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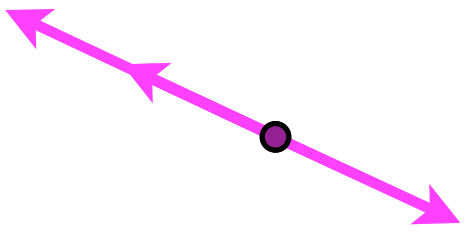

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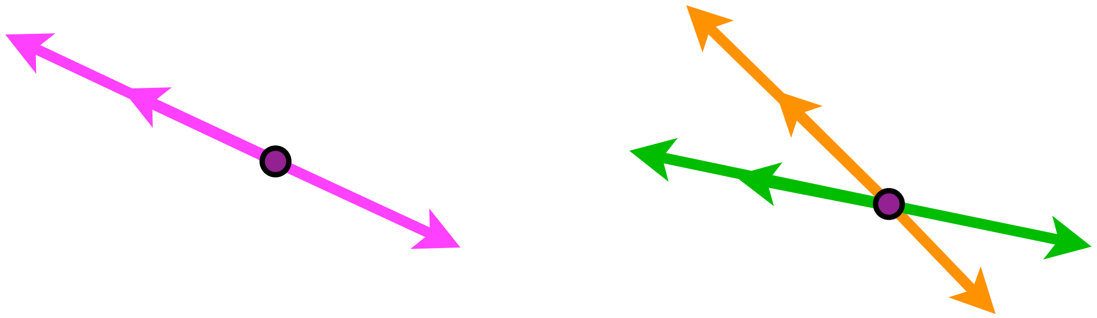

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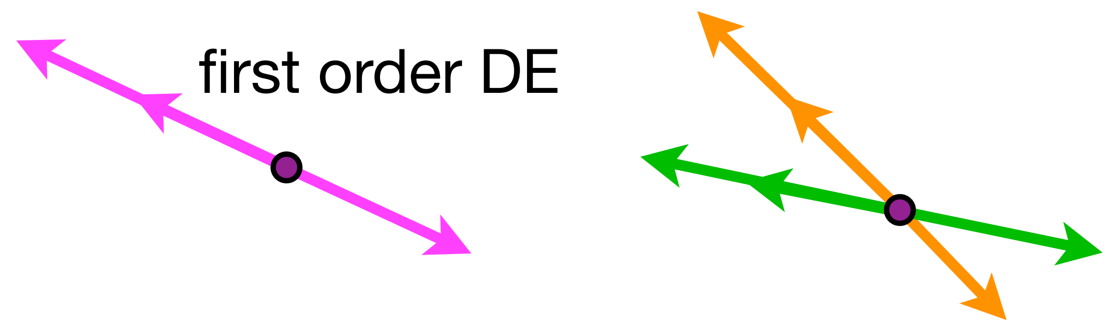

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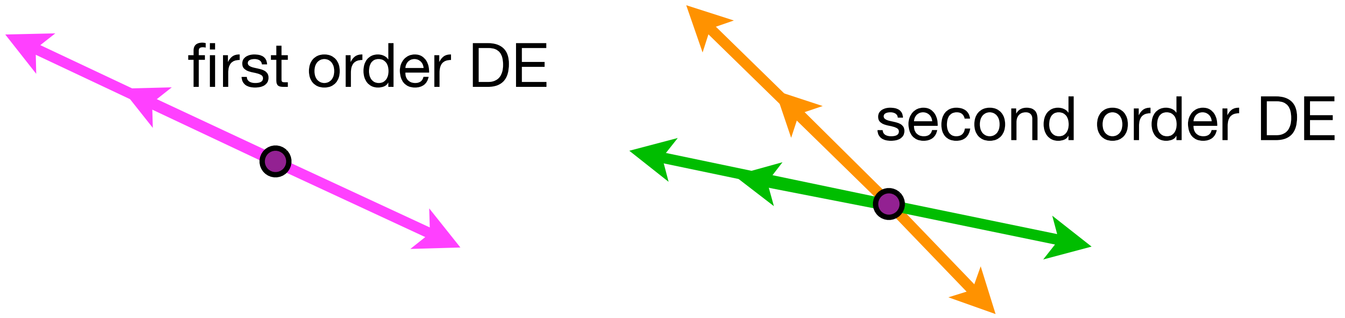

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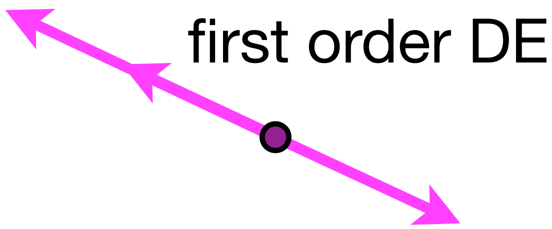

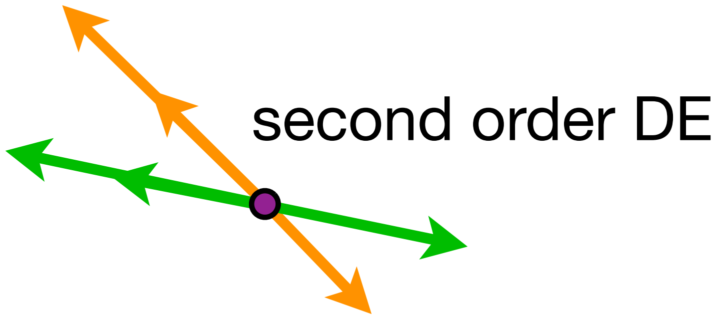

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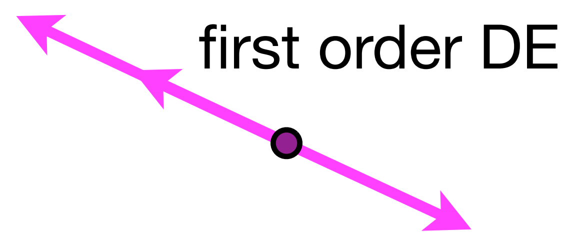

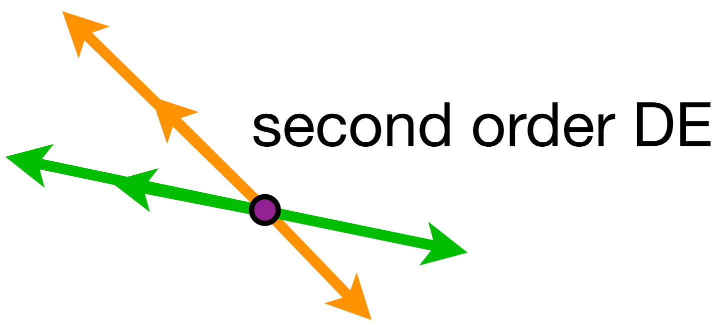

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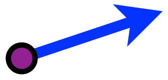

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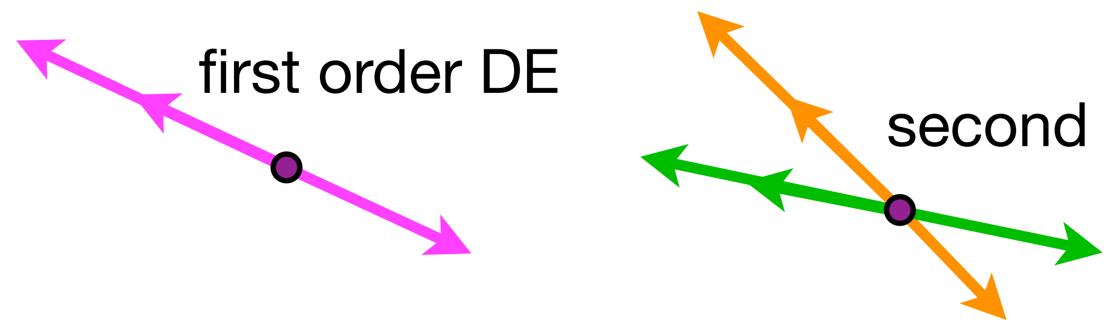

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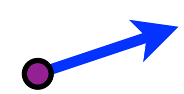

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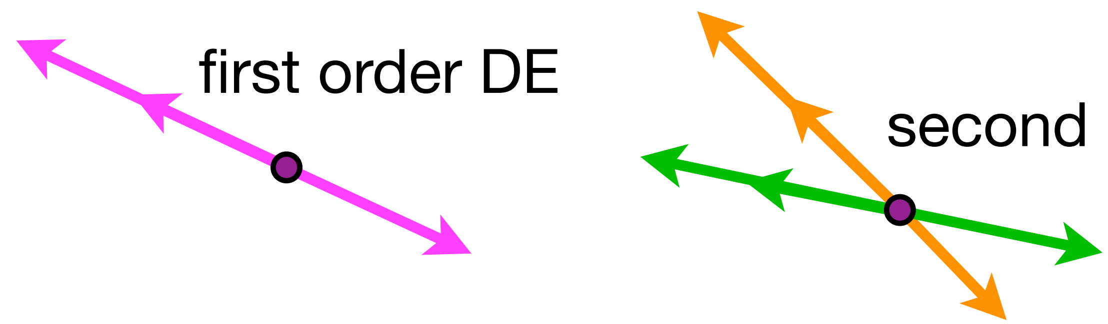

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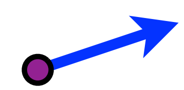

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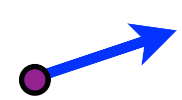

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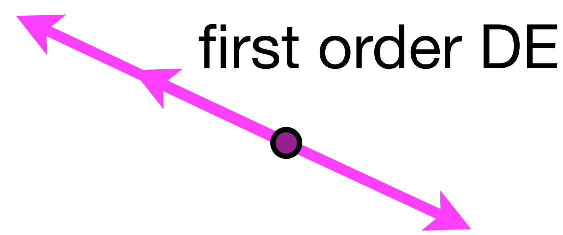

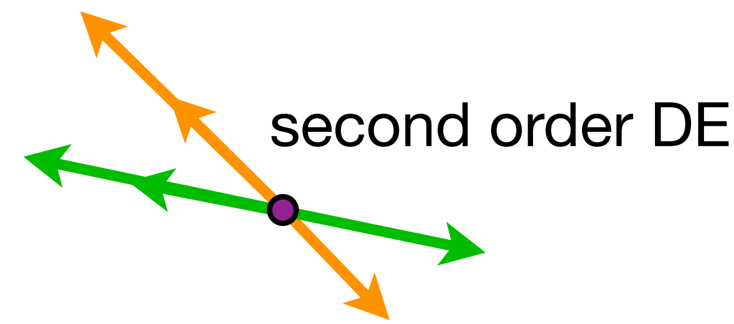

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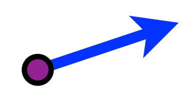

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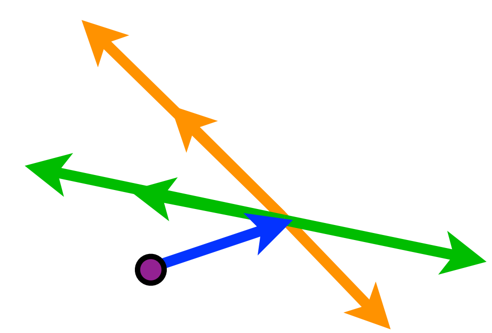

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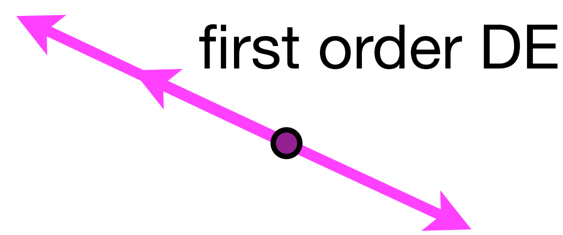

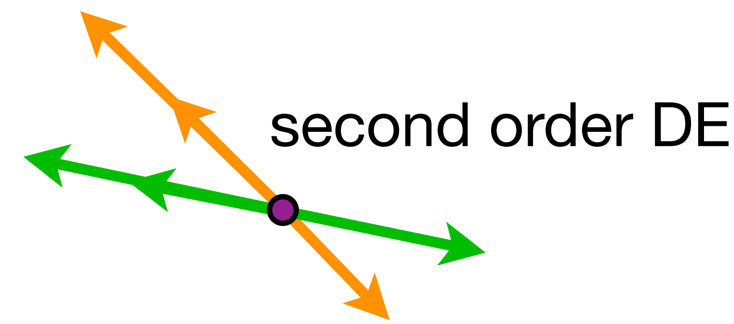

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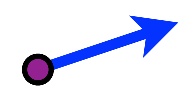

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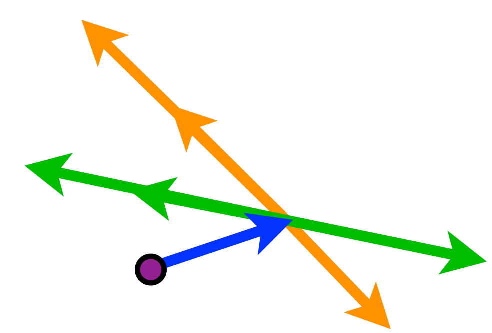

- For step 2, try “Method of undetermined coefficients”...