Welcome to MATH 256

Differential equations (for Chemical and Biological Engineering students)

Instructor:
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http://wiki.math.ubc.ca/mathbook/M256
Office: MATX 1215
Office hours: Tues 11:30 am - 1 pm, Thurs 3:30 - 4:30 pm <---- ?!?

Course goals

- Primary: Learn to solve ordinary and partial differential equations (mostly linear first and second order DEs).
- Secondary: Learn to use DEs to model physical, chemical, biological systems (really just an intro to this skill).

Prerequisites

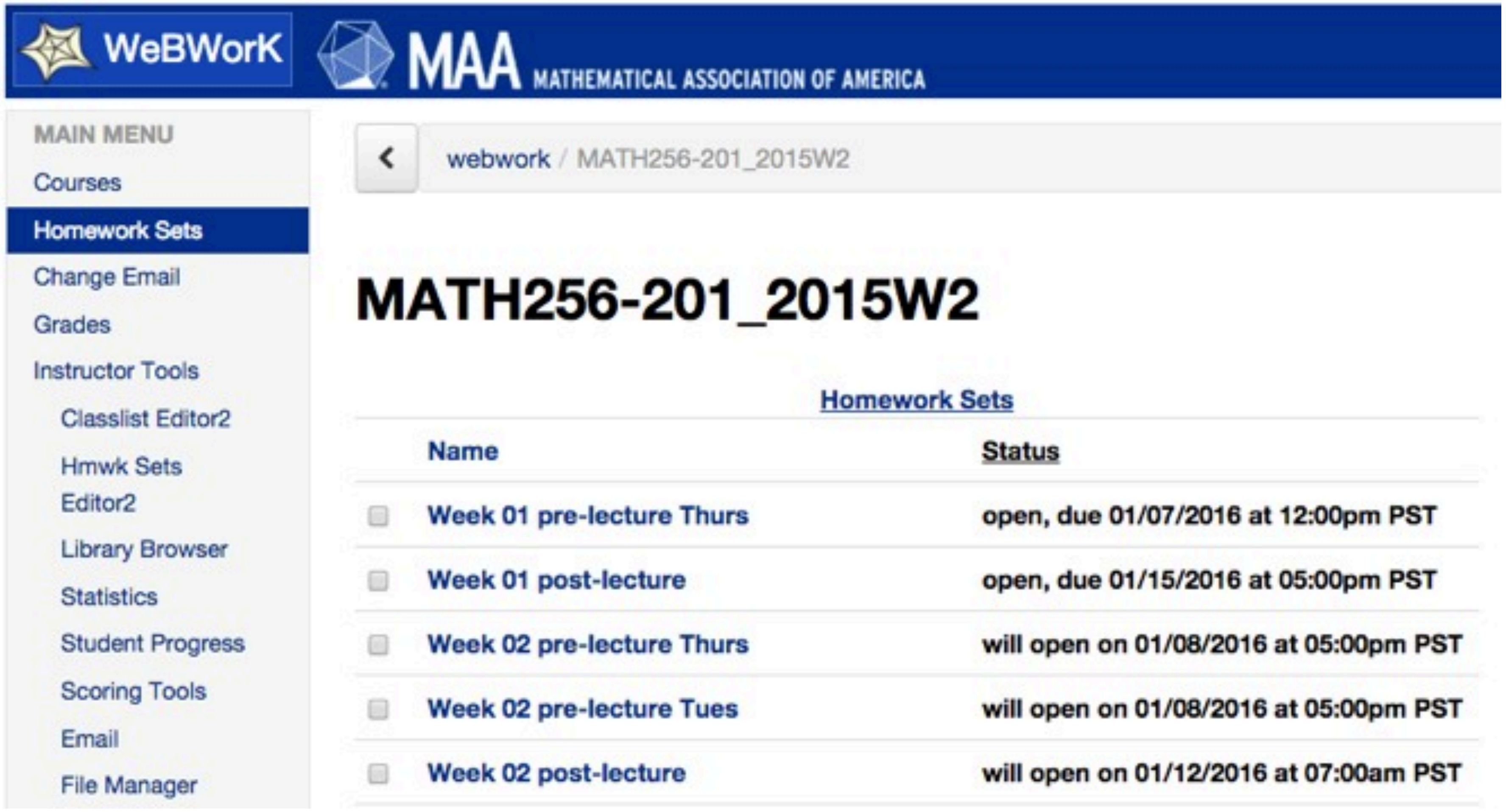
- First year calculus (MATH 100/101).
- Linear algebra (MATH 152).
- Multivariable calculus (MATH 200 or 253).
- Talk to me if you aren’t sure that you’re prepared for this course.

Tools we'll be using this term

- WeBWorK for homework assignments.
- Facebook for online discussion, updates etc. (any Piazza fans?)
- Clickers for in-class responses

WeBWorK

- Online homework system.
- https://webwork.elearning.ubc.ca/webwork2/MATH256-201_2014W2
- Log in using your CWL.



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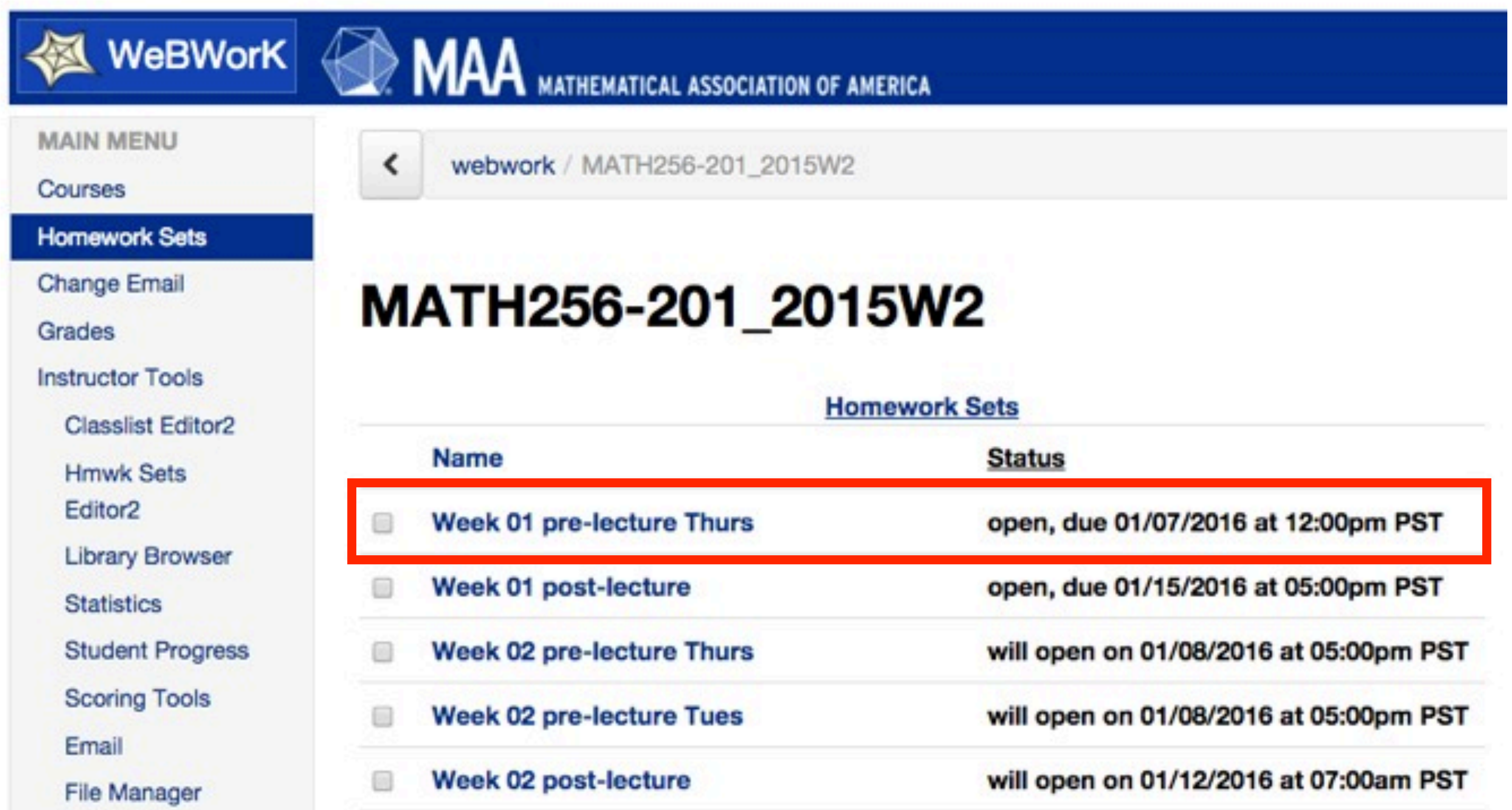
- First HW due Thurs.



Clickers

- Personal response system.
- Register your clicker at https://connect.ubc.ca

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- I don't look at the results on an individual basis so they are effectively anonymous.

More info online...

UBC

Log in

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MATH 256 - 2015W2 - Differential Equations

Course description

This course serves as an introduction to differential equations with a focus on solution techniques, transforms and modeling. Topics include linear ordinary differential equations, Laplace transforms, Fourier series and separation of variables for linear partial differential equations.

This website is the course website for MATH 256 taught in 2015W2.

Course details

- Instructor information
- Marking scheme
- Important dates
- Other course information
- Solutions - Tutorial worksheets, old midterms
- General resources - including links to old course websites, old assignments, suggested practice problems etc.
- Course outline - summary of content above.

Navigation

- MATH 256 Home
- Course schedule
- Lecture slides
- Pre-lecture resources
- WeBWorK
- Instructors' site

Felix Baumgartner’s freefall from 40 km up

https://www.youtube.com/watch?v=vvbN-cWe0A0

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- Simple model to predict how fast he'll go, how long it will take etc.

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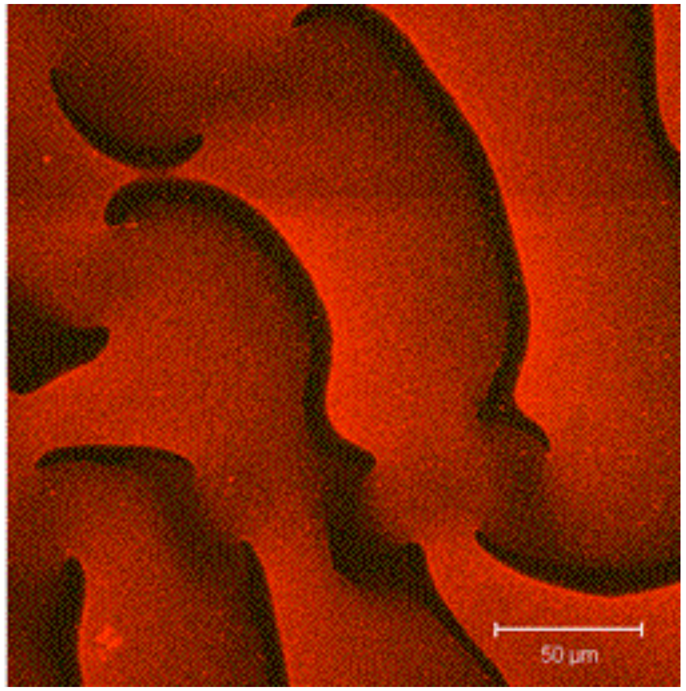
$$mv' = -mg + k(x)v^2 \qquad mx'' = -mg + k(x)x'^2$$

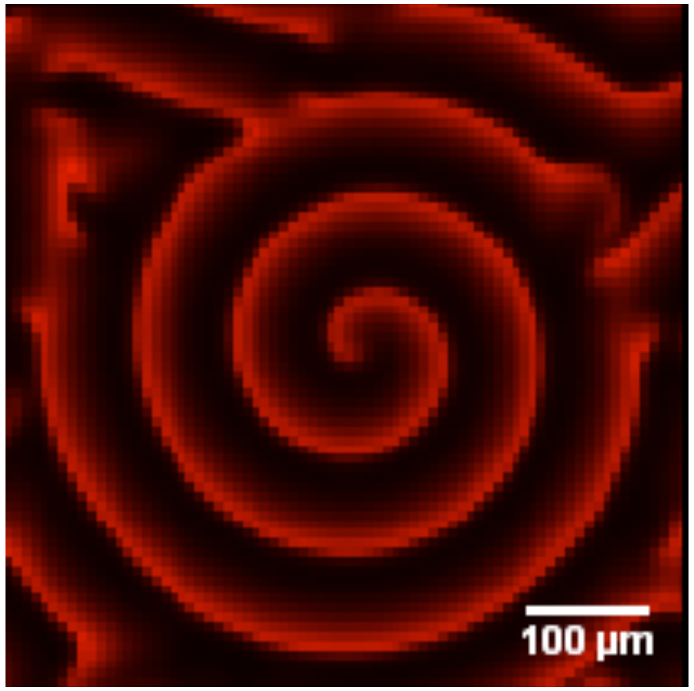
A bacterial cell division regulator

- Two interacting bacterial proteins that undergo complicated dynamics.
- Differential equation model help understand how they work.

Experiment



Model



$$\frac{\partial u}{\partial t} = u - uv + D\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = uv - v + D\frac{\partial^2 v}{\partial x^2}$$

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- The general solution - a solution with one or more arbitrary constants that encompass ALL possible solutions to the DE.

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A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time t in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket $(h(0) = 4)$, what is the solution to this differential equation?

A. $h(t) = (2 - 3t)^2$
B. $h(t) = \sqrt{16 - 2t}$
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$$\frac{d}{dt}\left(t^2 y(t)\right) =$$

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(B) $t^2\frac{dy}{dt}$

(C) $2ty$

(D) $t^2\frac{dy}{dt} + 2ty$

(E) Don’t know.

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Method of integrating factors

- Given that $$\frac{d}{dt}\left(t^2 y(t)\right) = t^2 \frac{dy}{dt} + 2ty$$

- if you're given the equation $$t^2 \frac{dy}{dt} + 2ty = 0$$

- you can rewrite is as $$\frac{d}{dt}\left(t^2 y(t)\right) = 0$$

arbitrary constant that appeared at an integration step

- so the solution is $t^2 y(t) = C$ or equivalently $y(t) = \dfrac{C}{t^2}$.