Today

- Shapes of solutions for distinct eigenvalues case.

Example

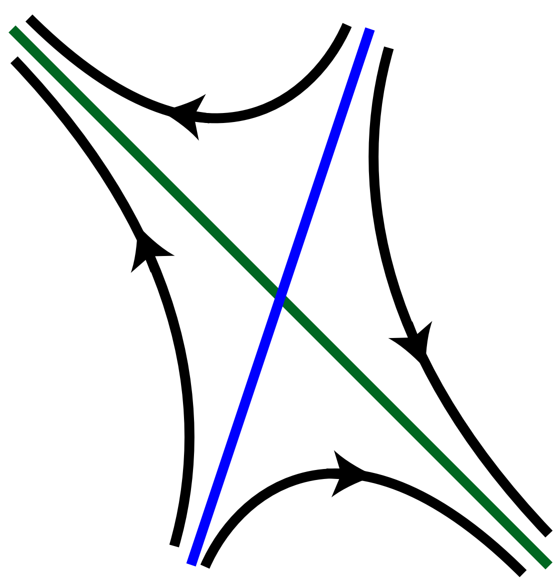
- Doc cam:
- $y(t) = C_1(1;\ 2)\ e^{-t} + C_2(1;\ -1)\ e^{t}$
  - With ICs
    - $y(0) = (2;4)$
    - $y(0) = (2;2)$
    - $y(0) = (2;1)$
  - Desmos: https://www.desmos.com/calculator/tpelfq4nbe

Shapes of solution curves in the phase plane

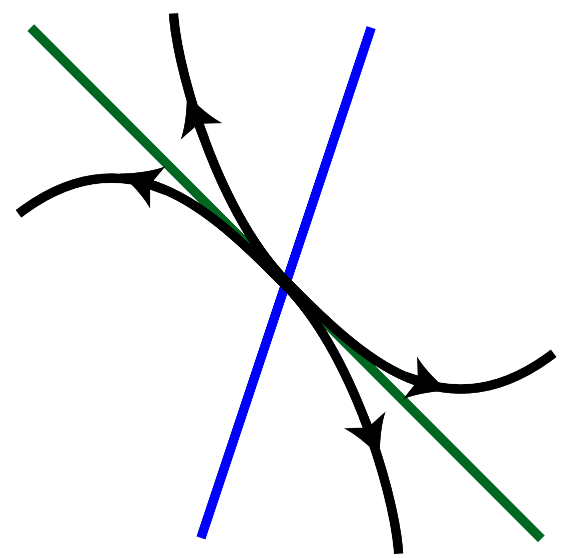
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ ?}$$

(A)



(B)



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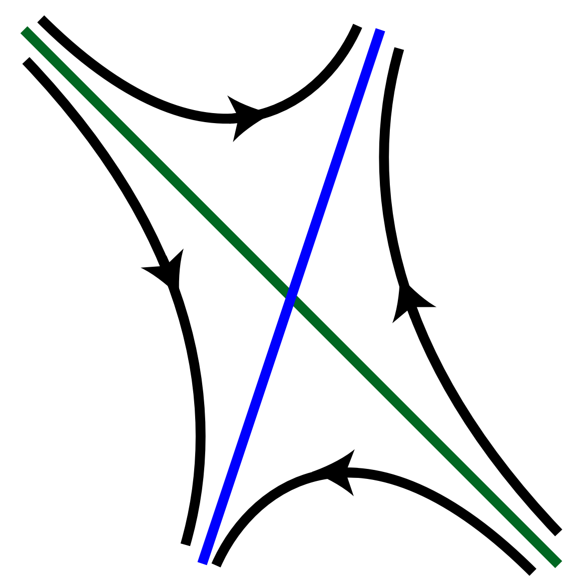
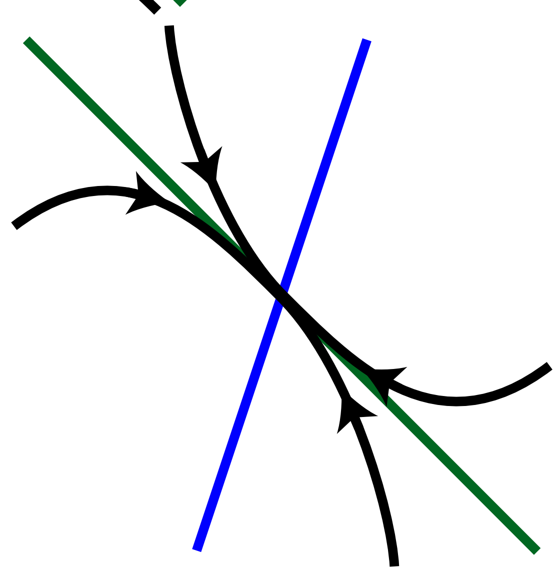


(D)



(E) Explain, please.

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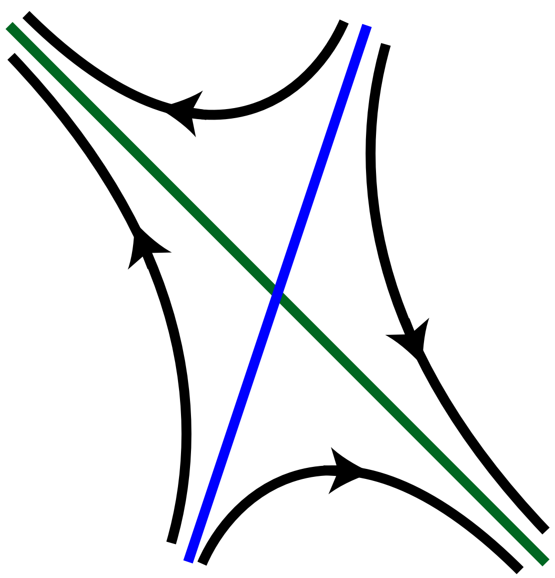

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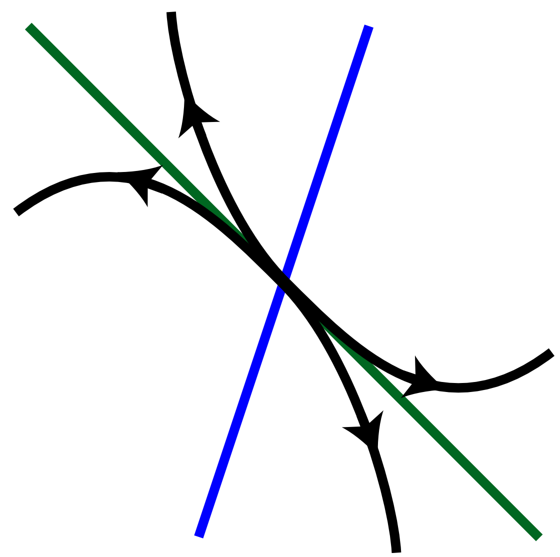

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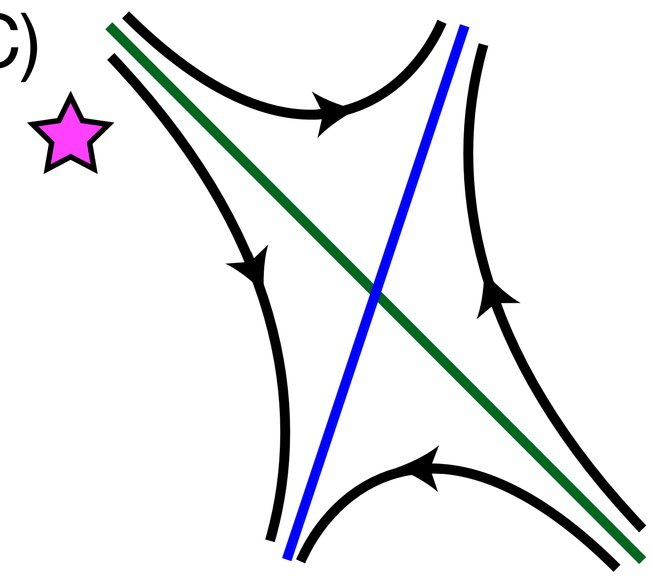

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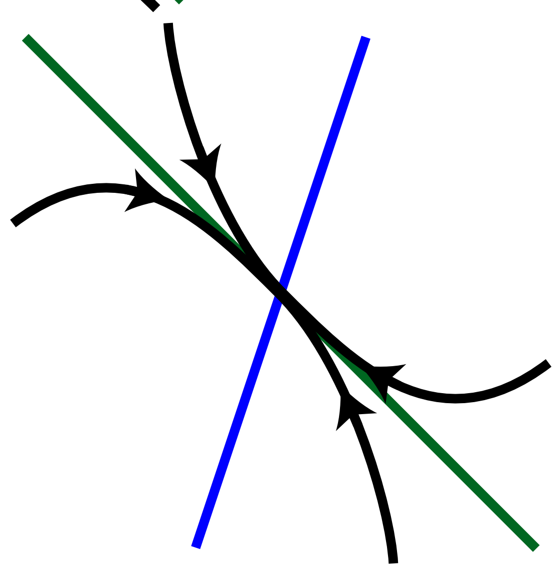

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Shapes of solution curves in the phase plane

- When matrix A has distinct eigenvalues, the general solution to **x**'=A**x** is

$$\mathbf{x} = C_1 e^{\lambda_1 t}\mathbf{v_1} + C_2 e^{\lambda_2 t}\mathbf{v_2}$$

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- If the initial condition is an eigenvector, then the solution is a straight line. Example:

$$x_1' = x_1 + x_2 \qquad x_1(0) = 6$$
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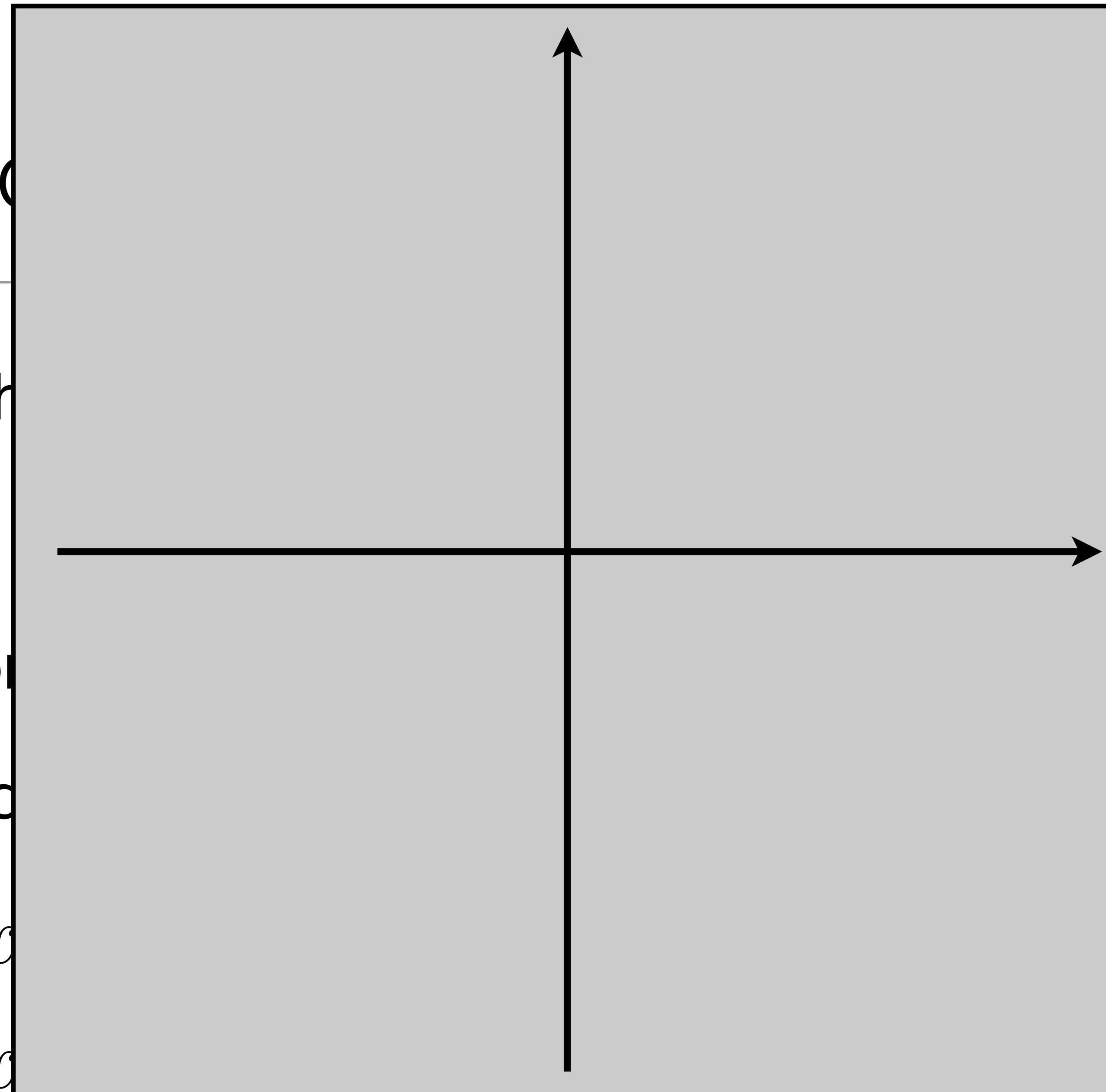
Shapes of s  plane

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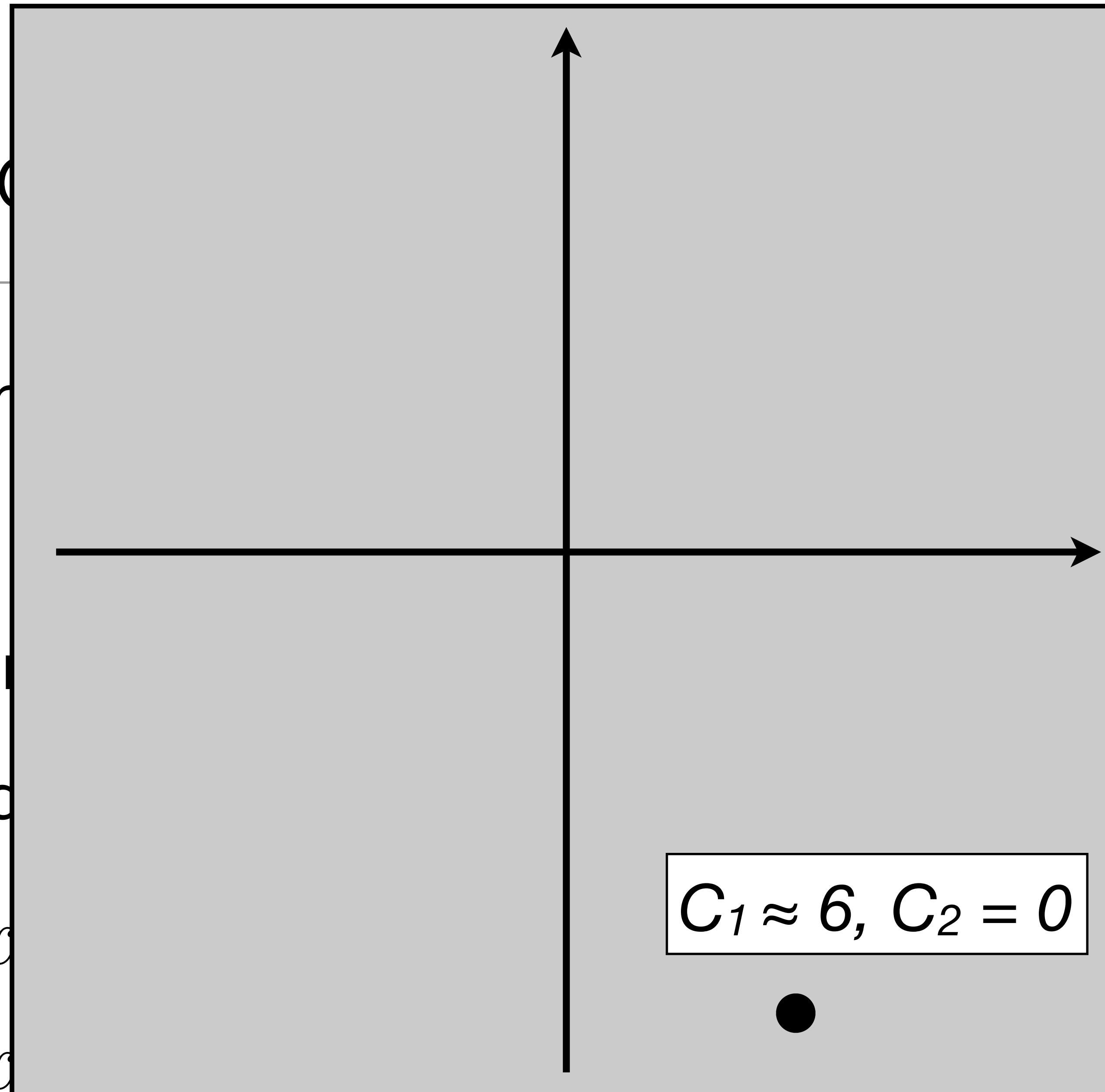

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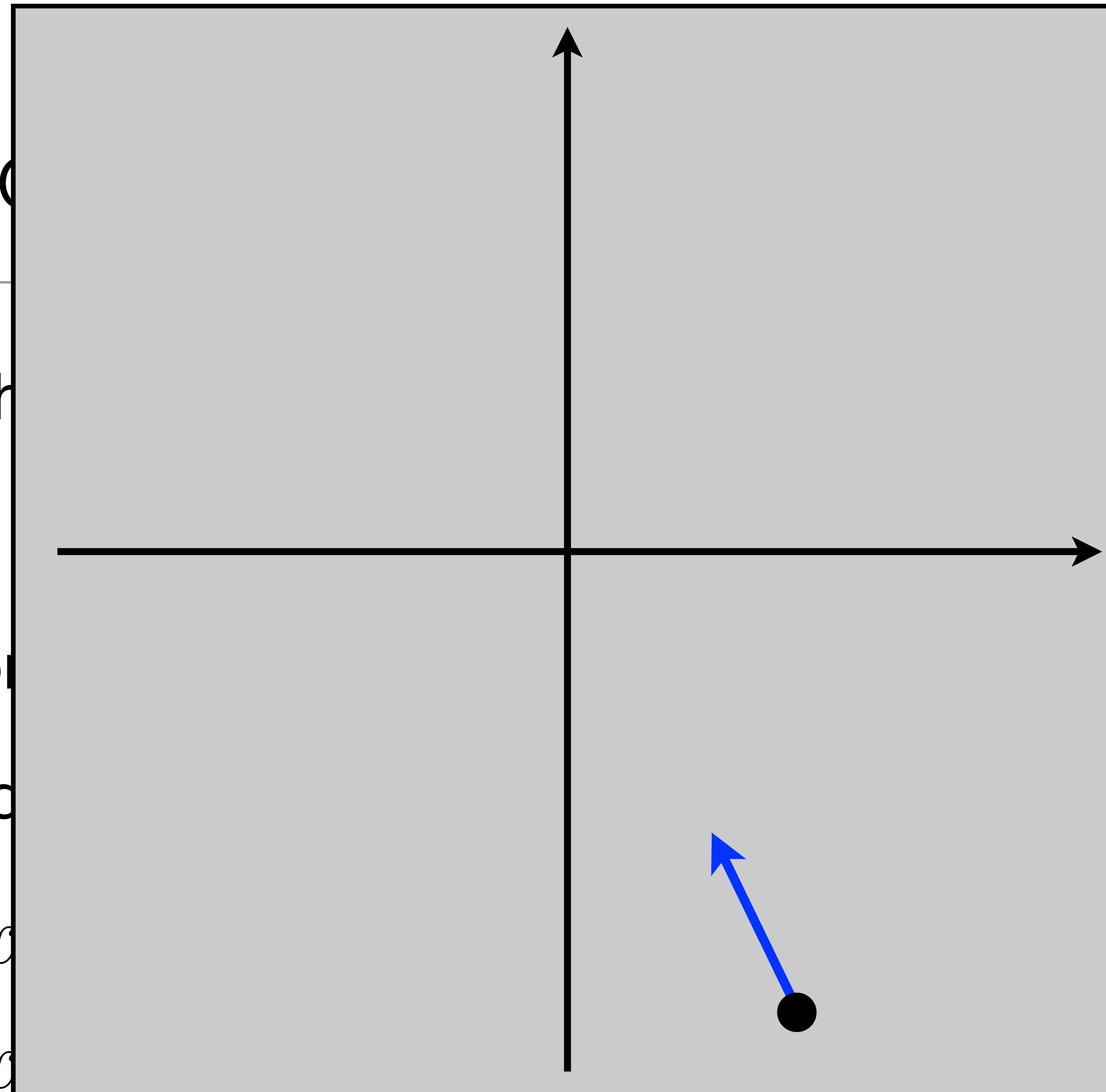
Shapes of s  plane

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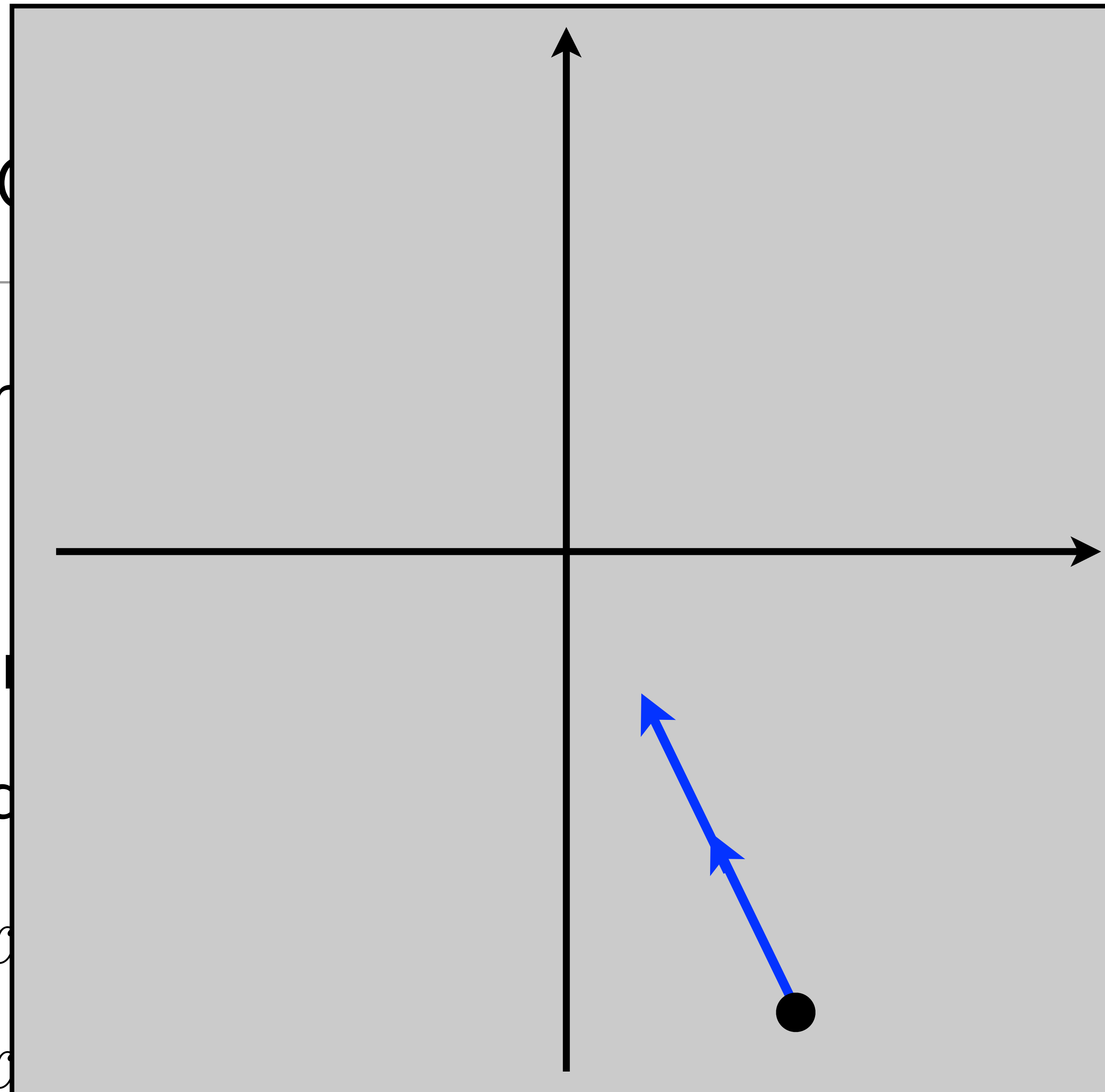

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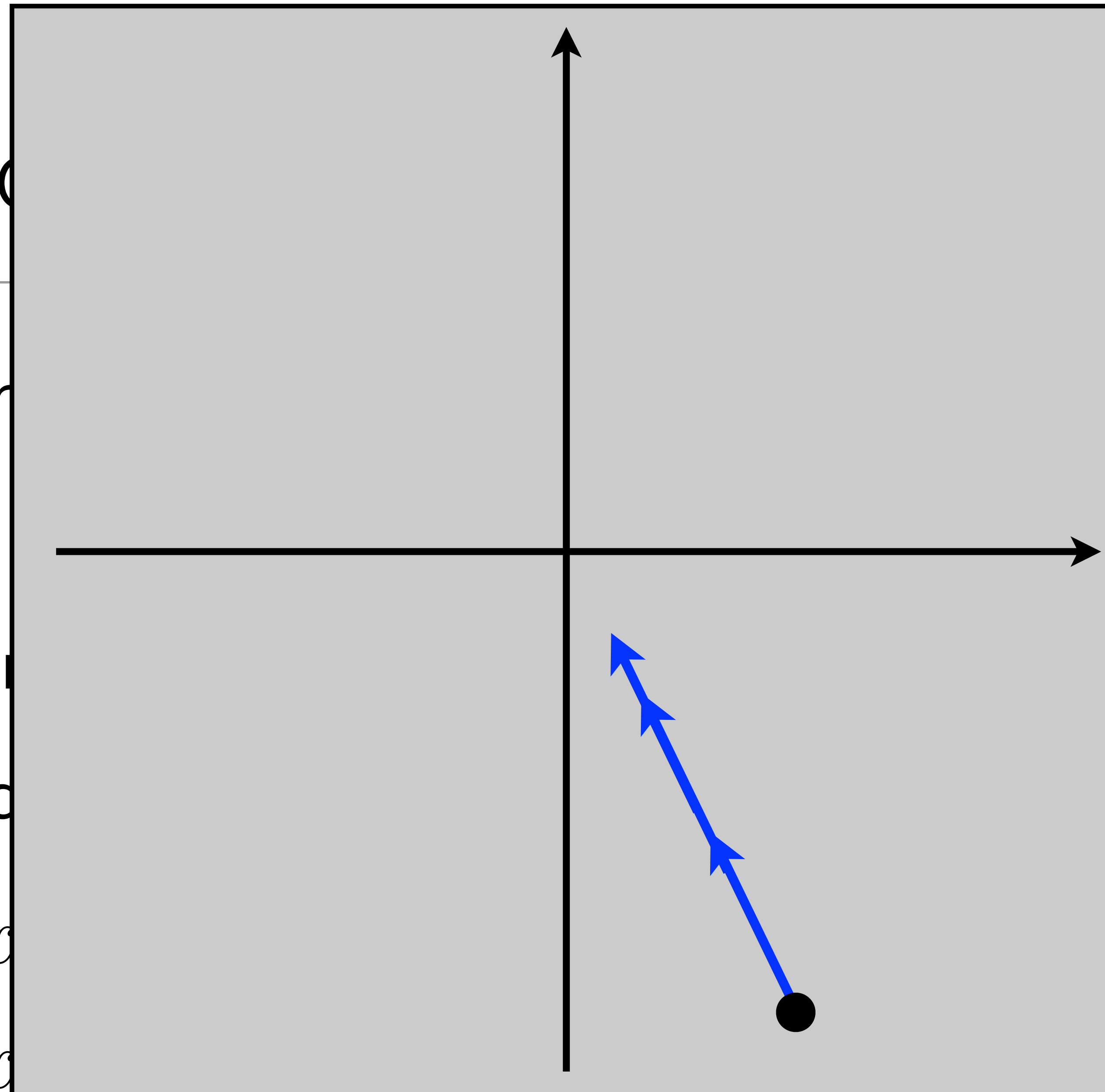

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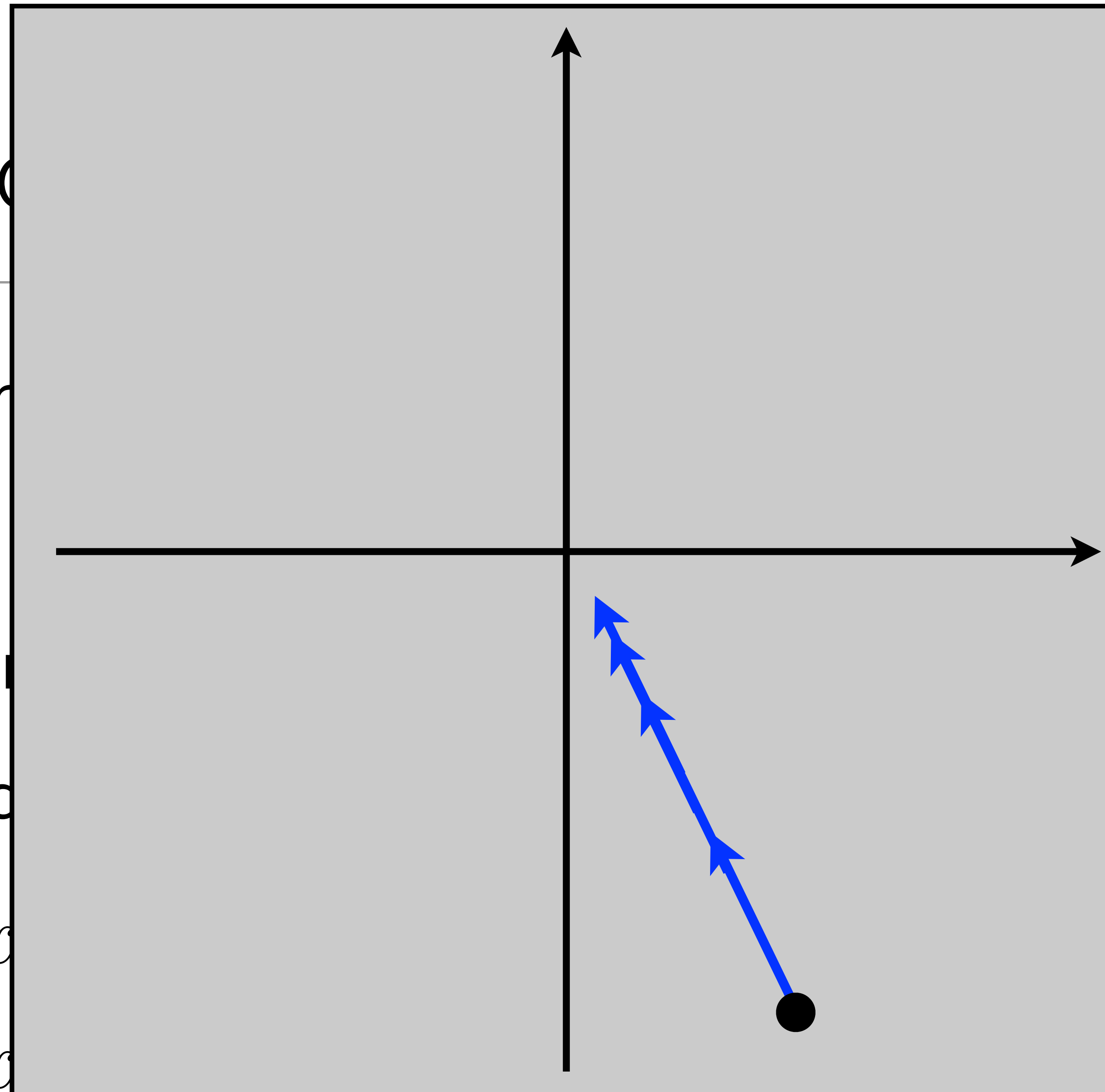

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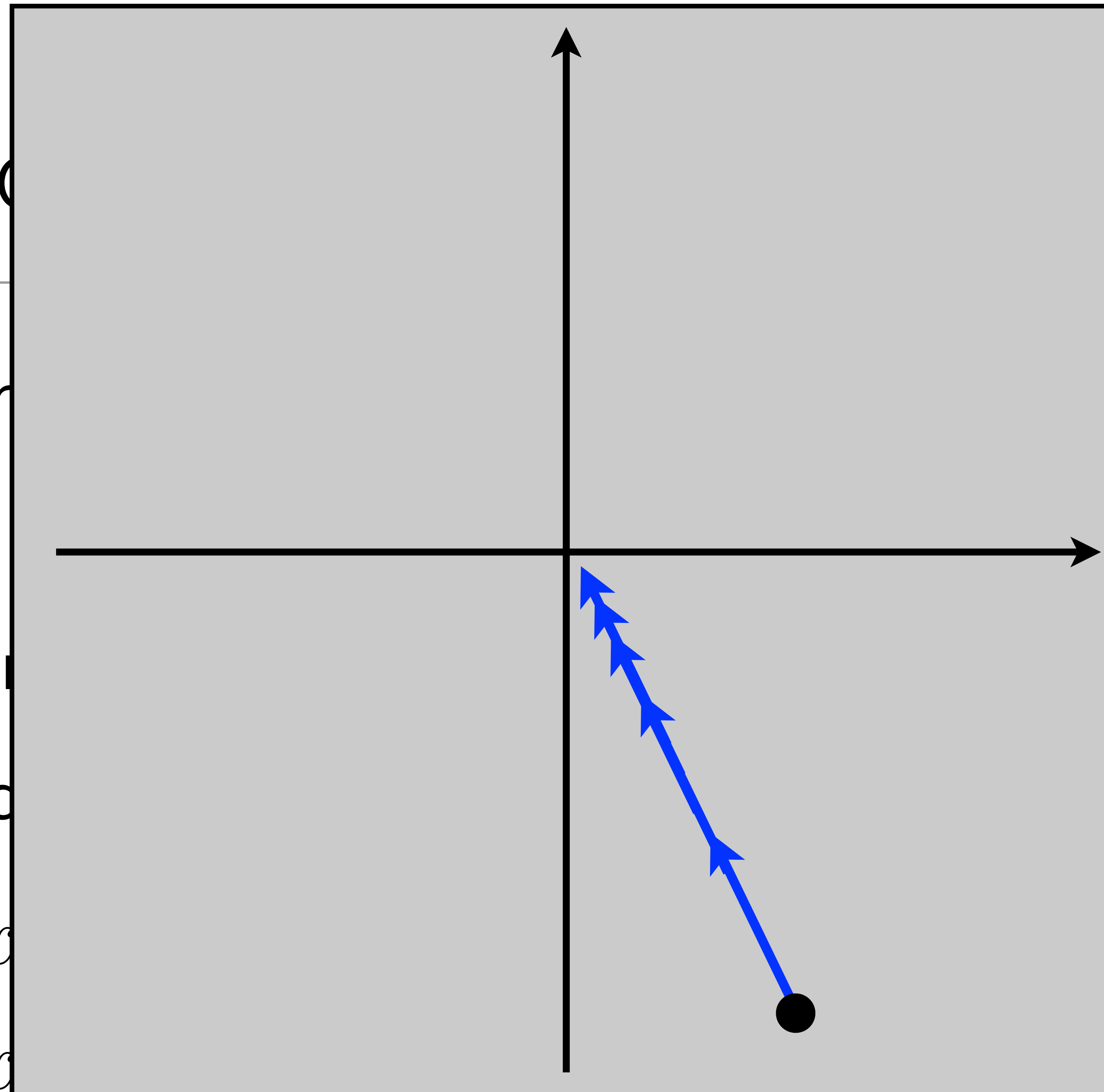

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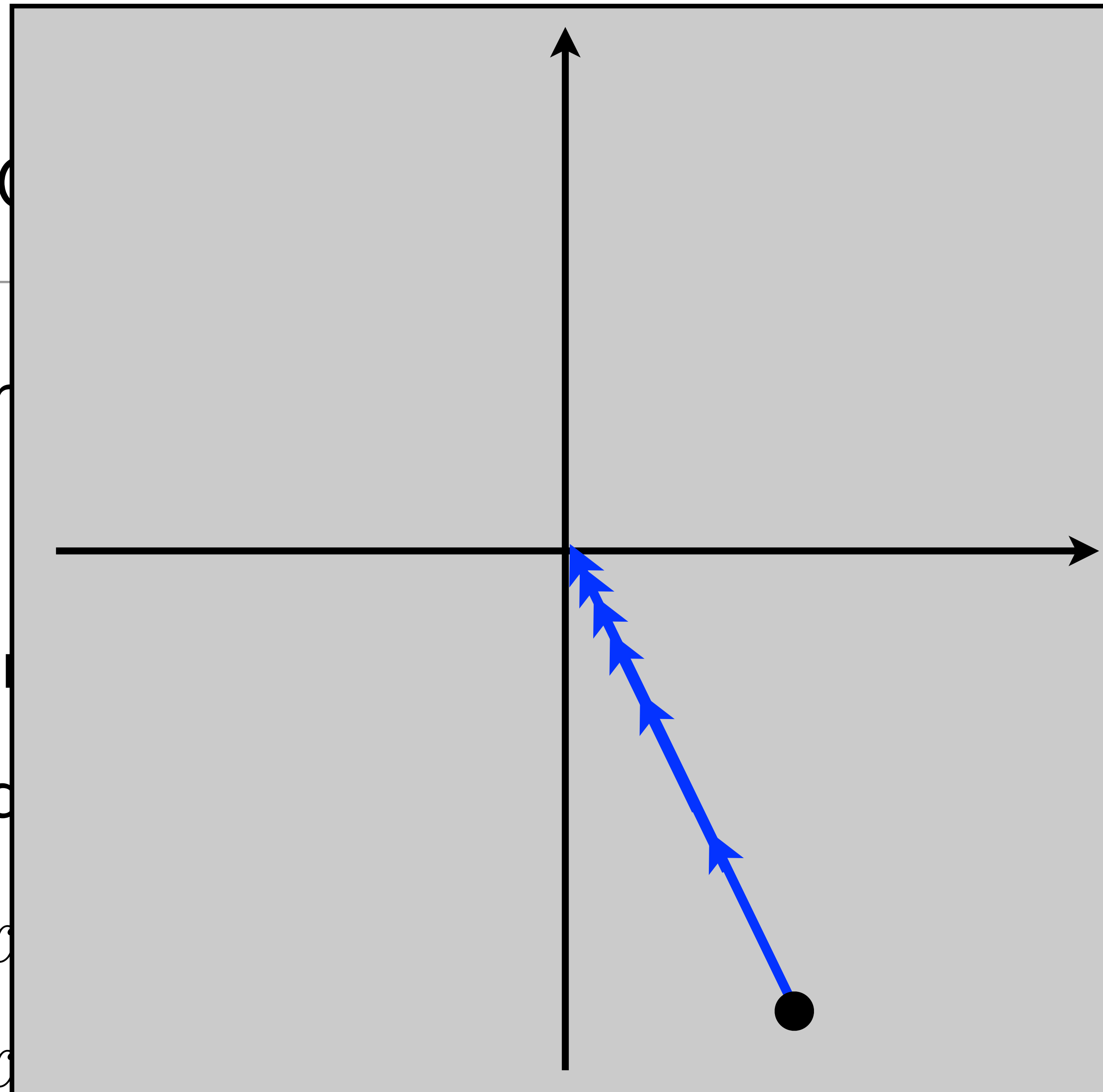

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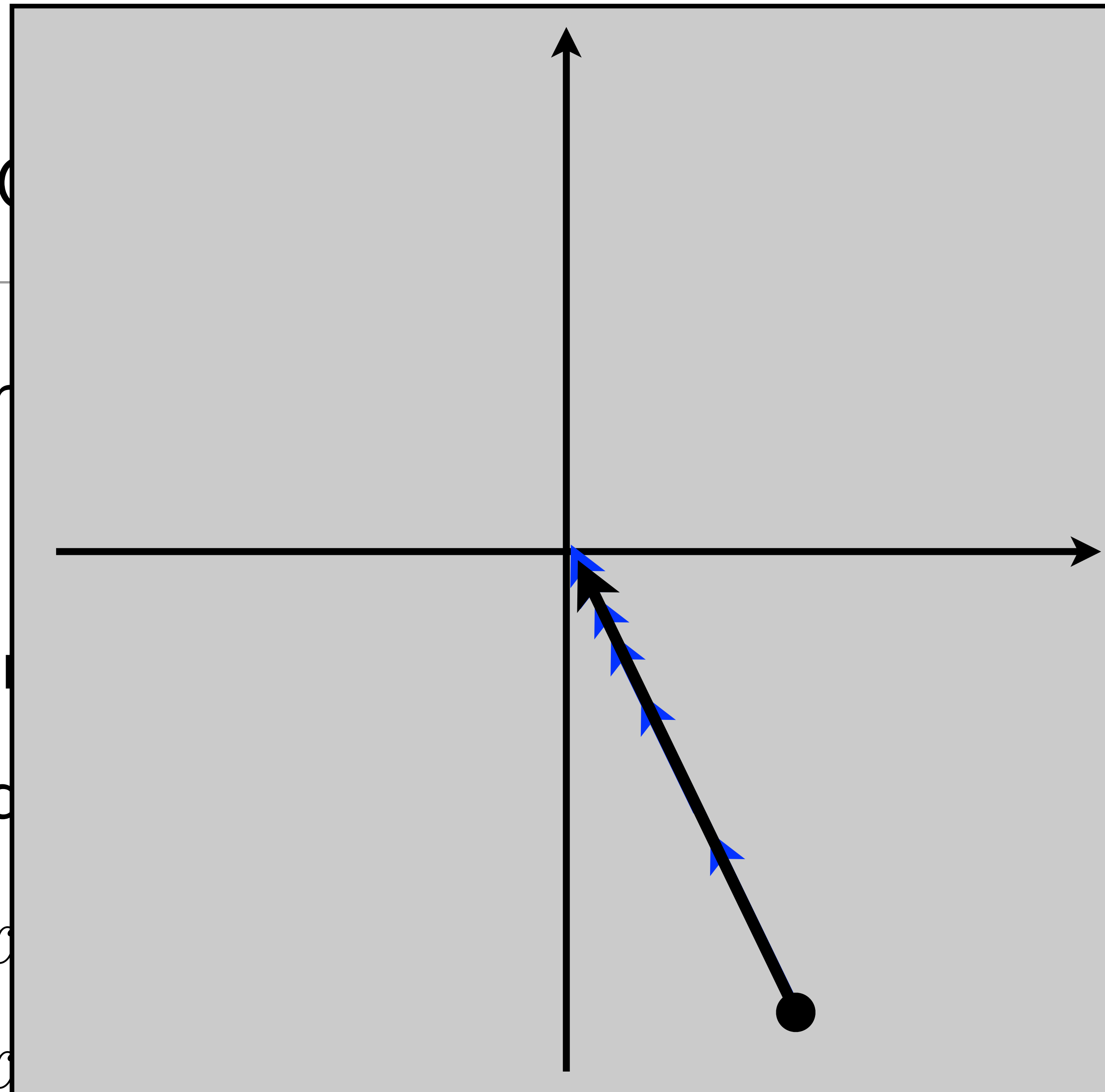

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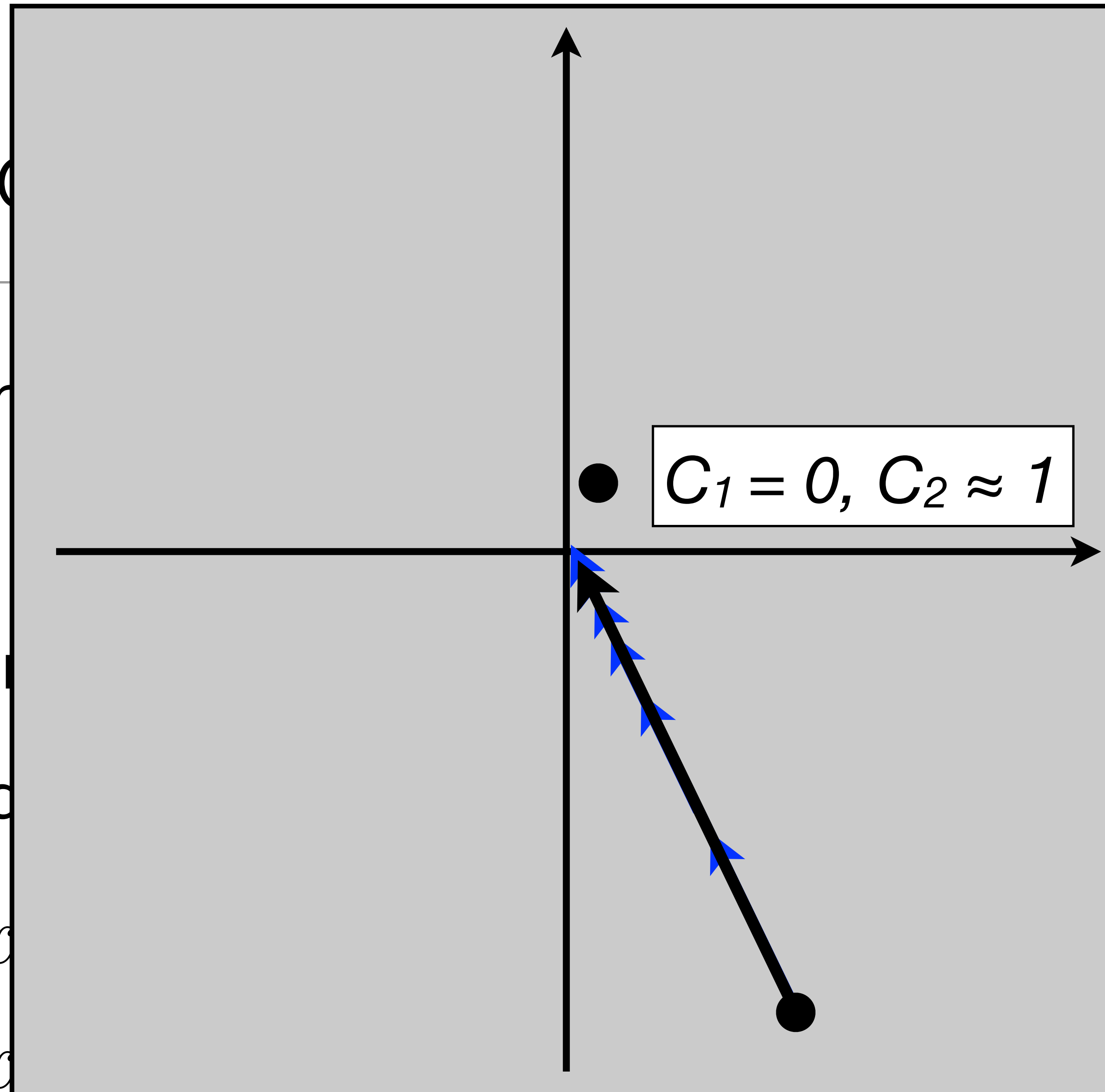

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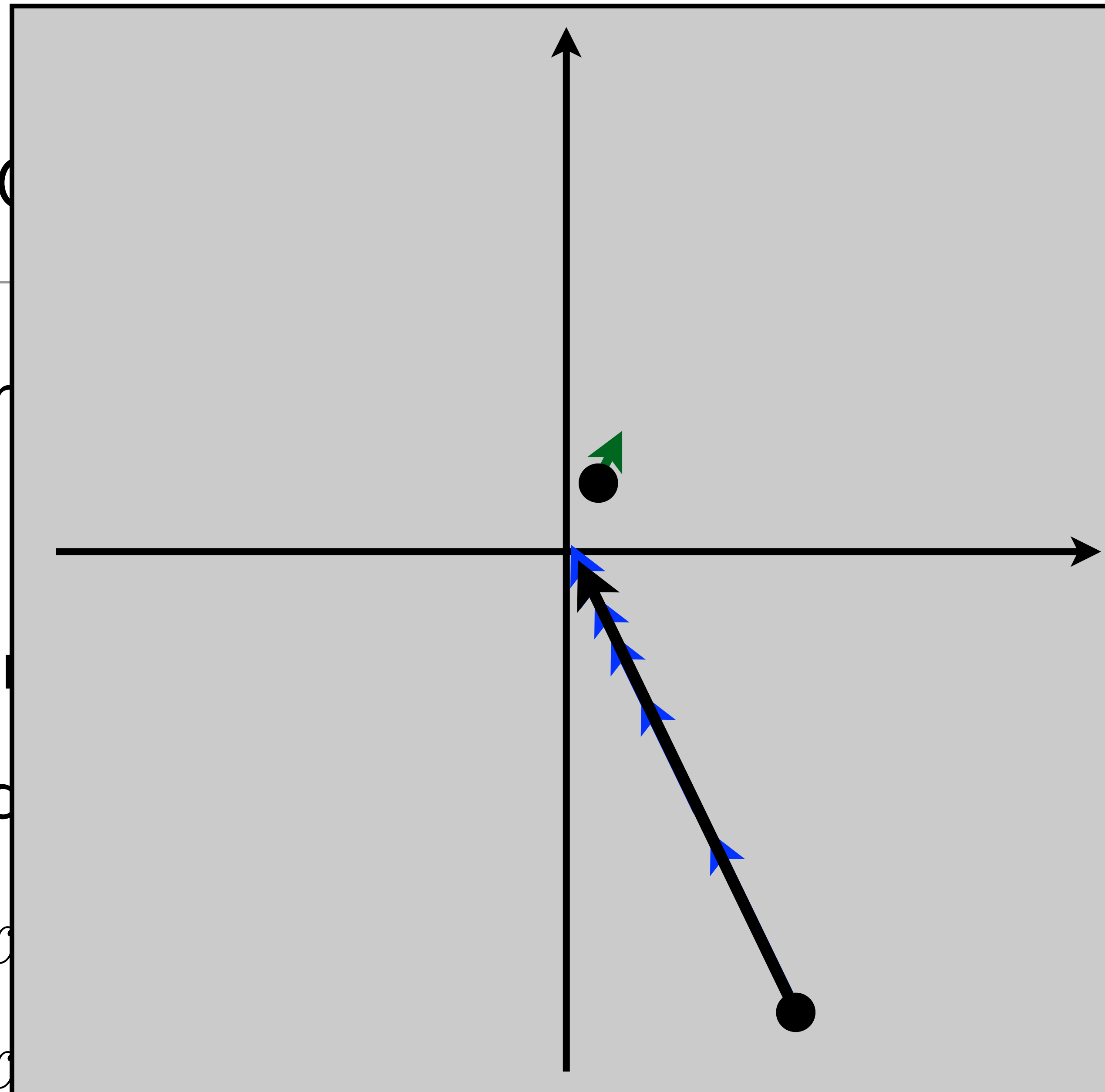

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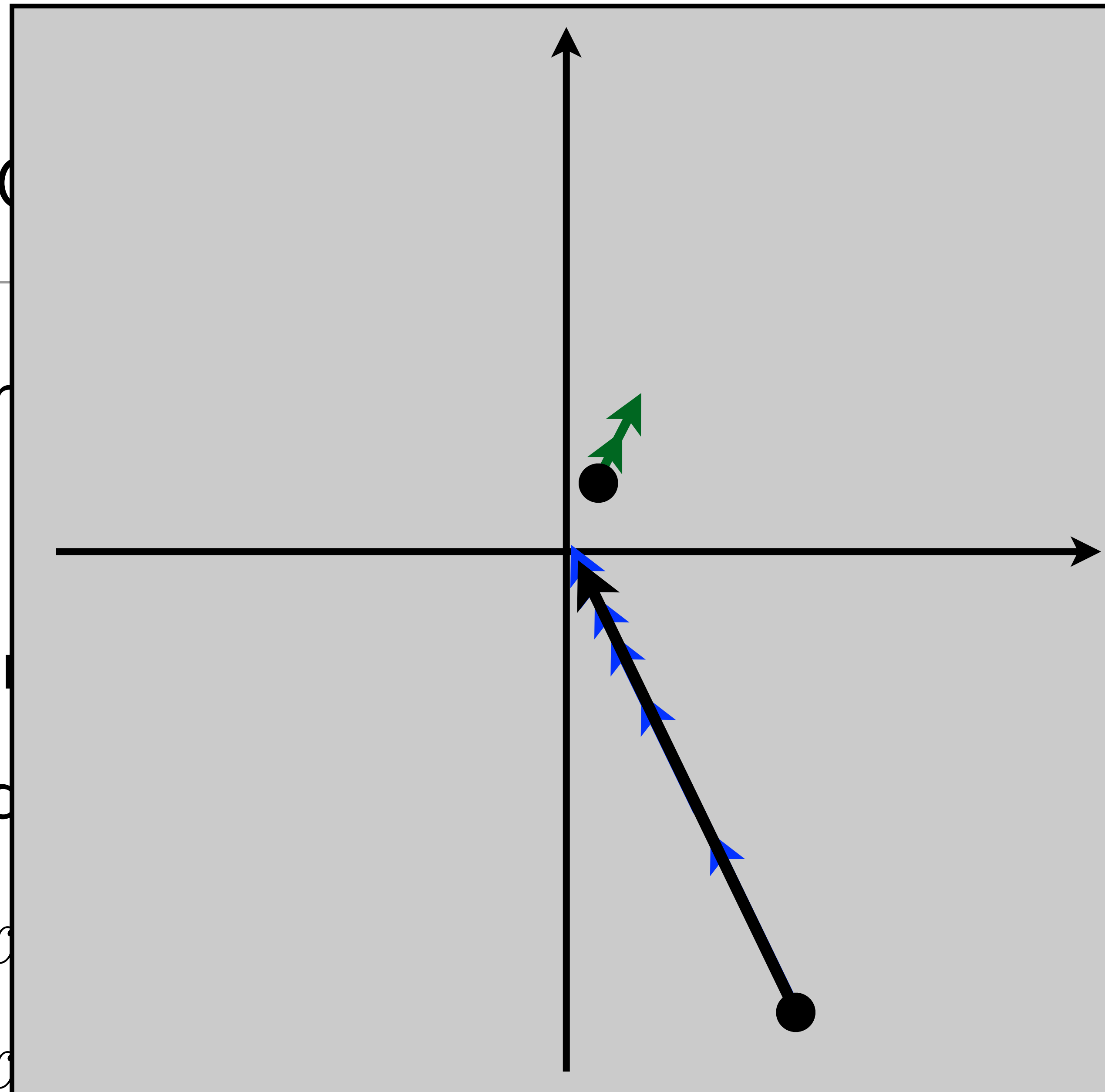

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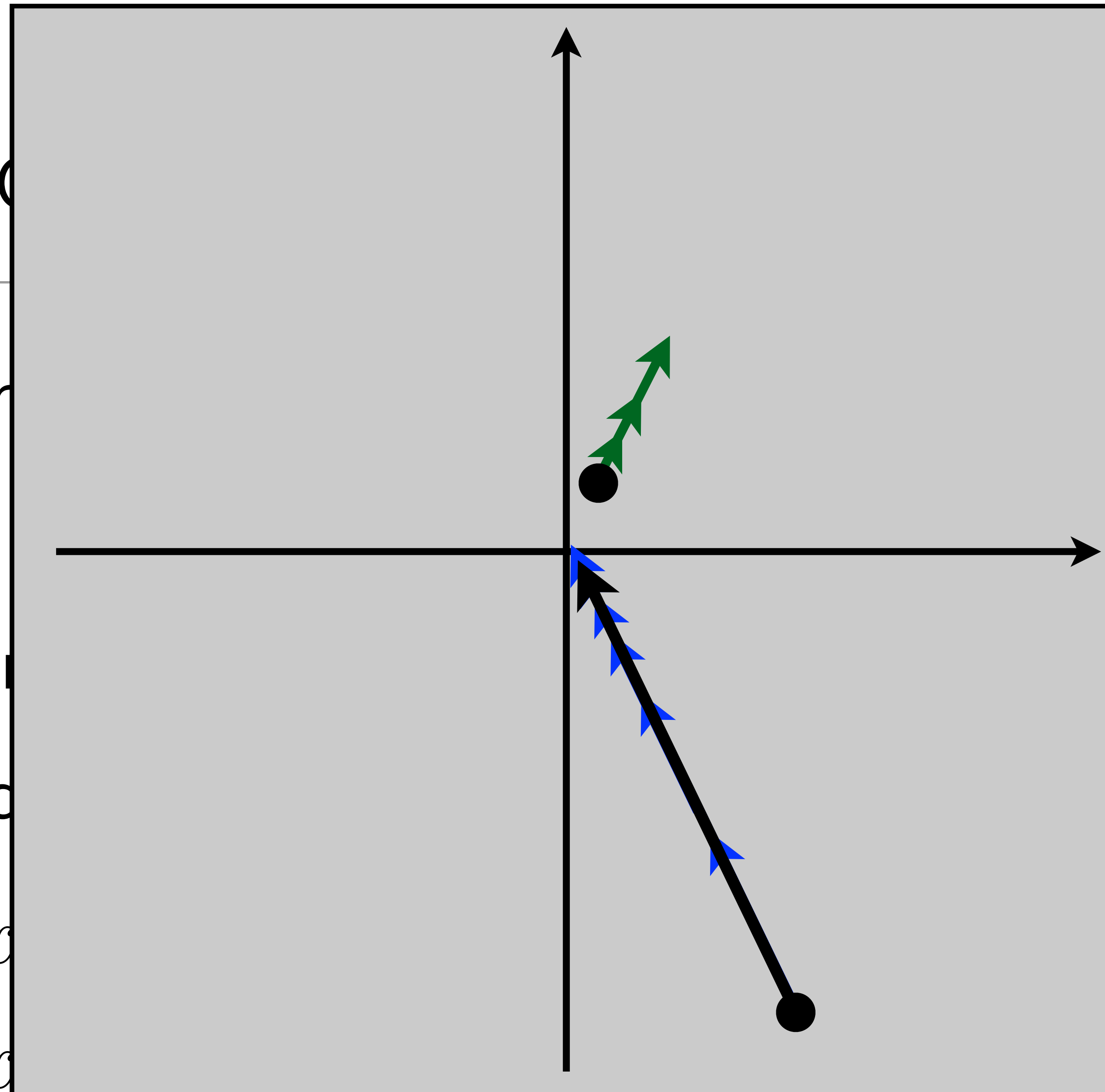

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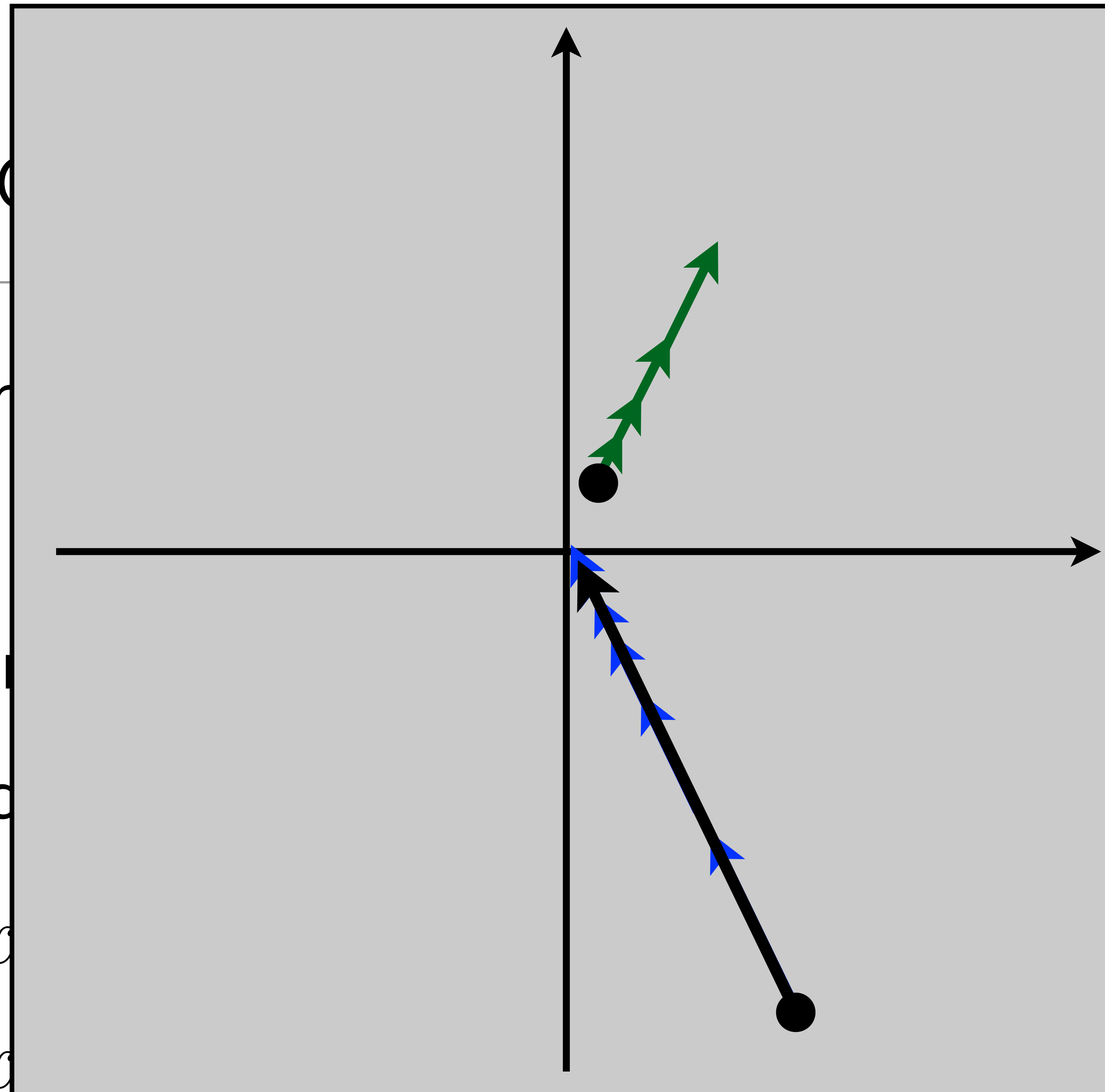

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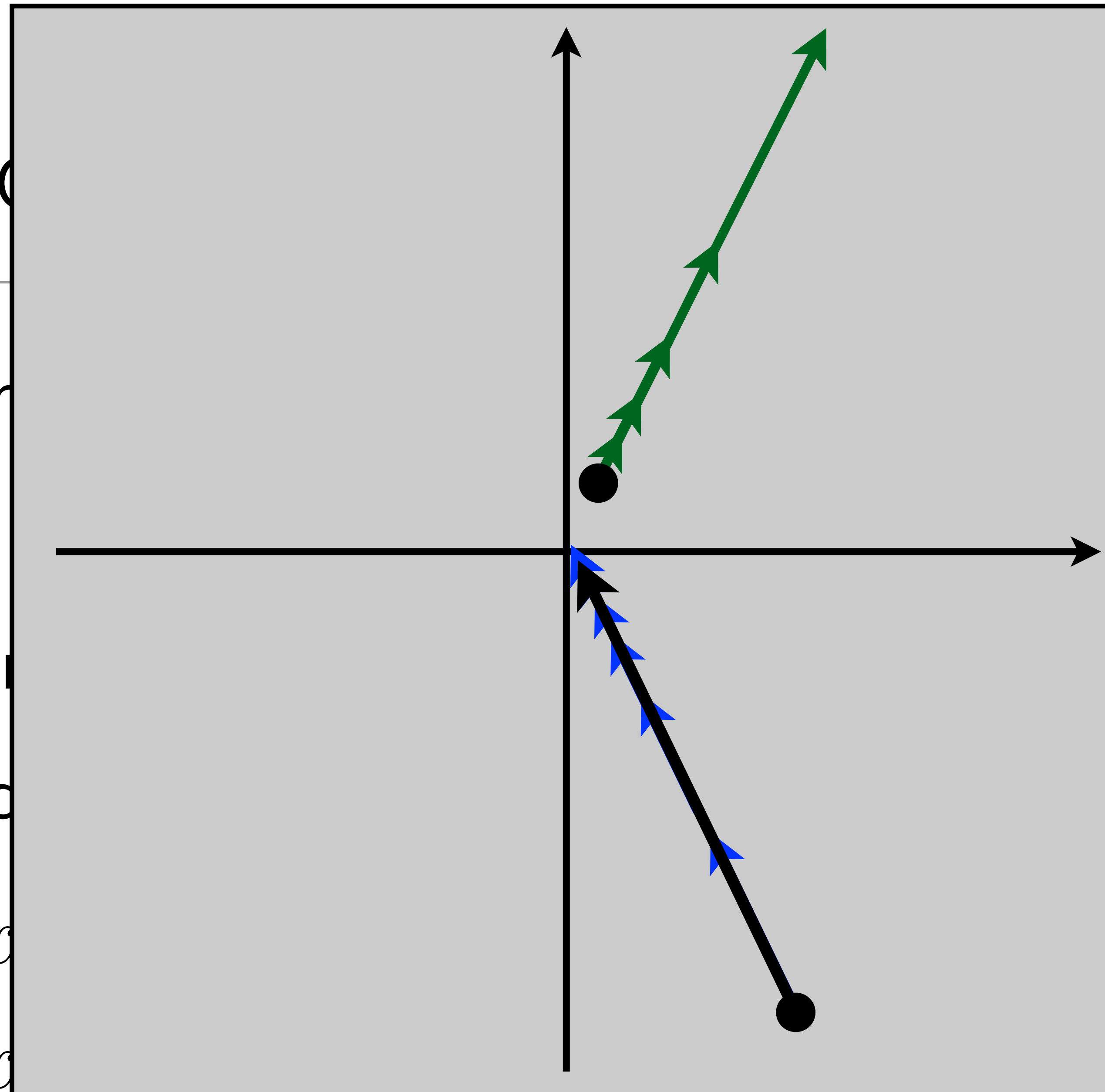

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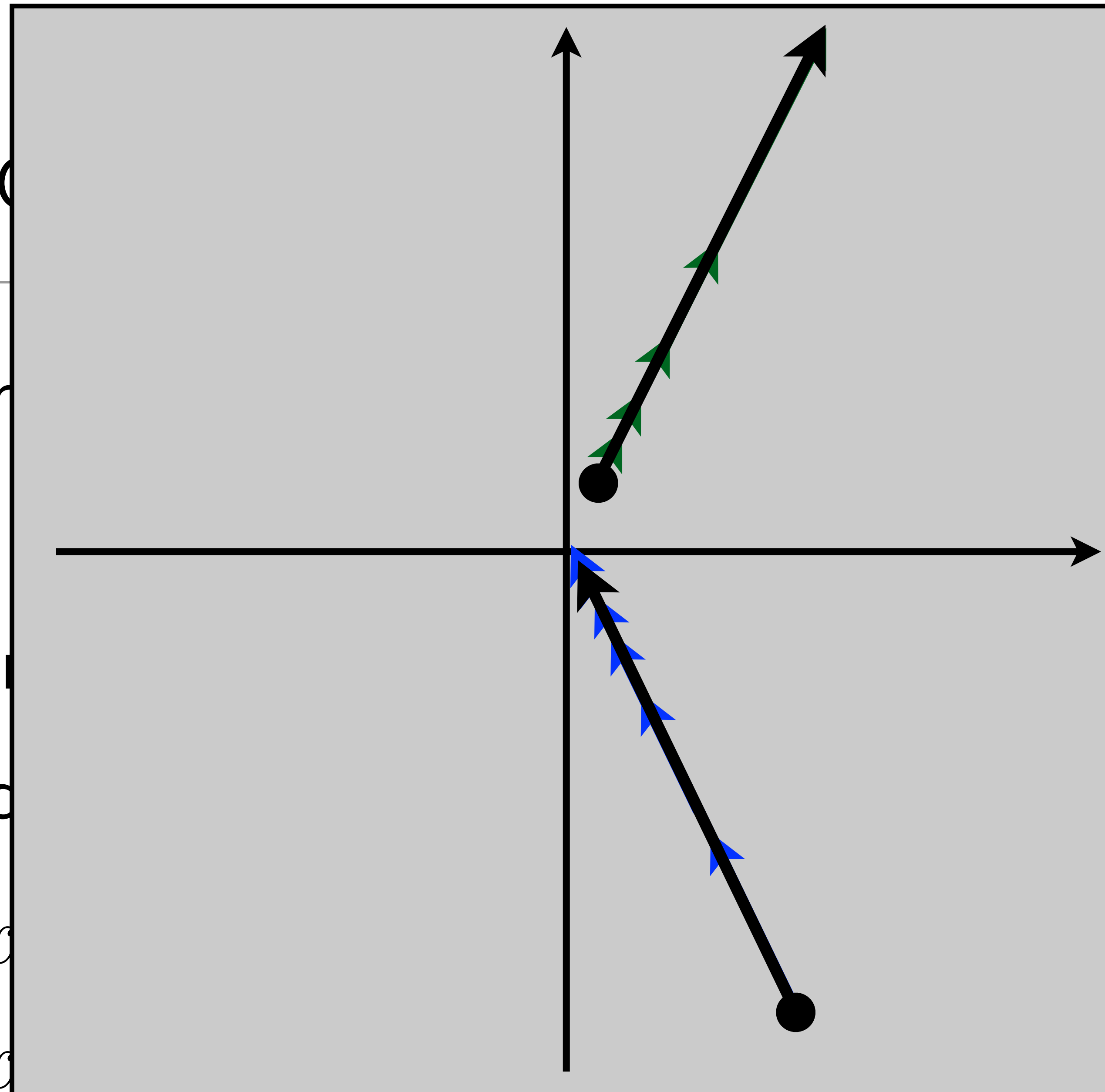

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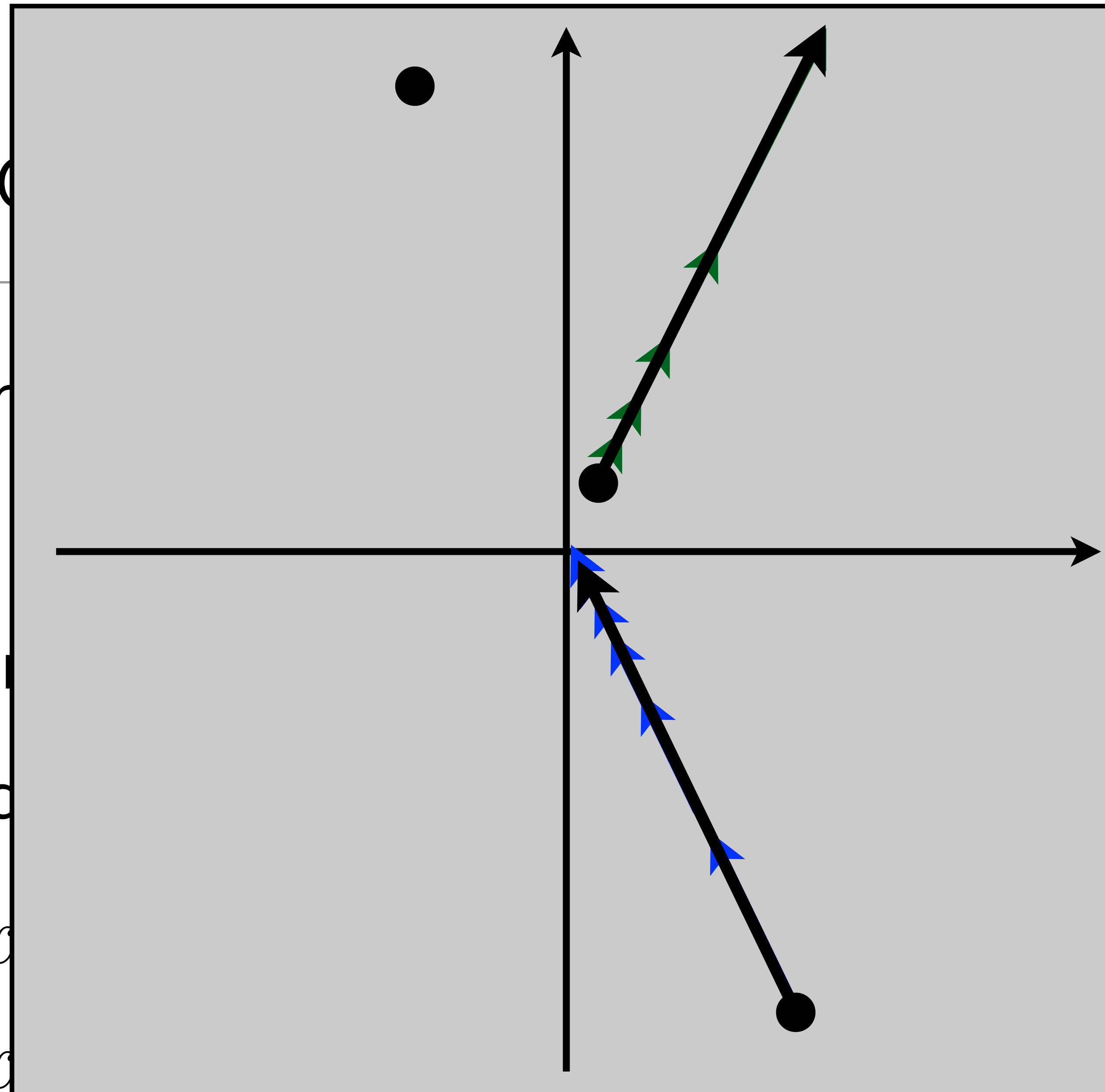

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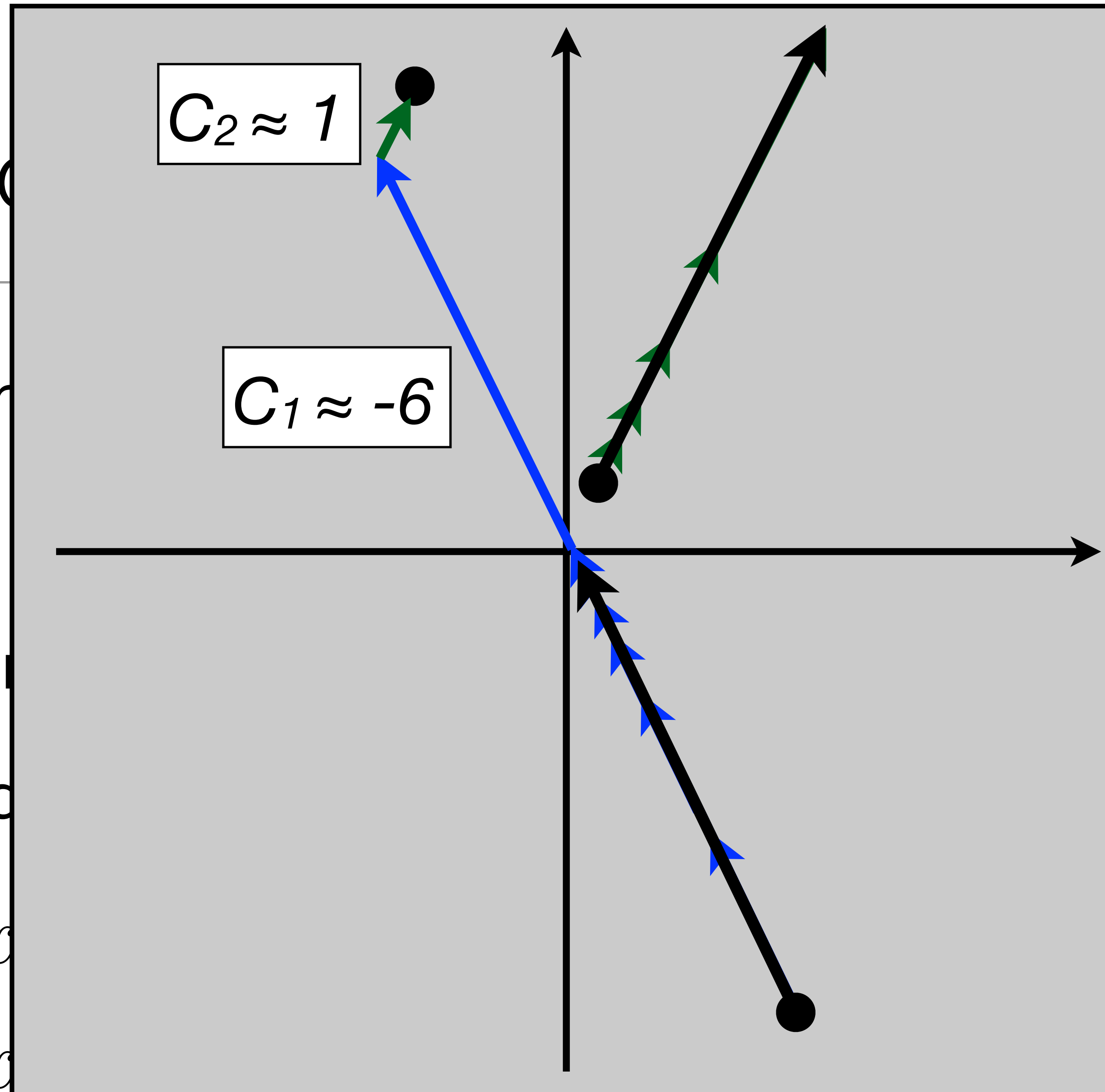

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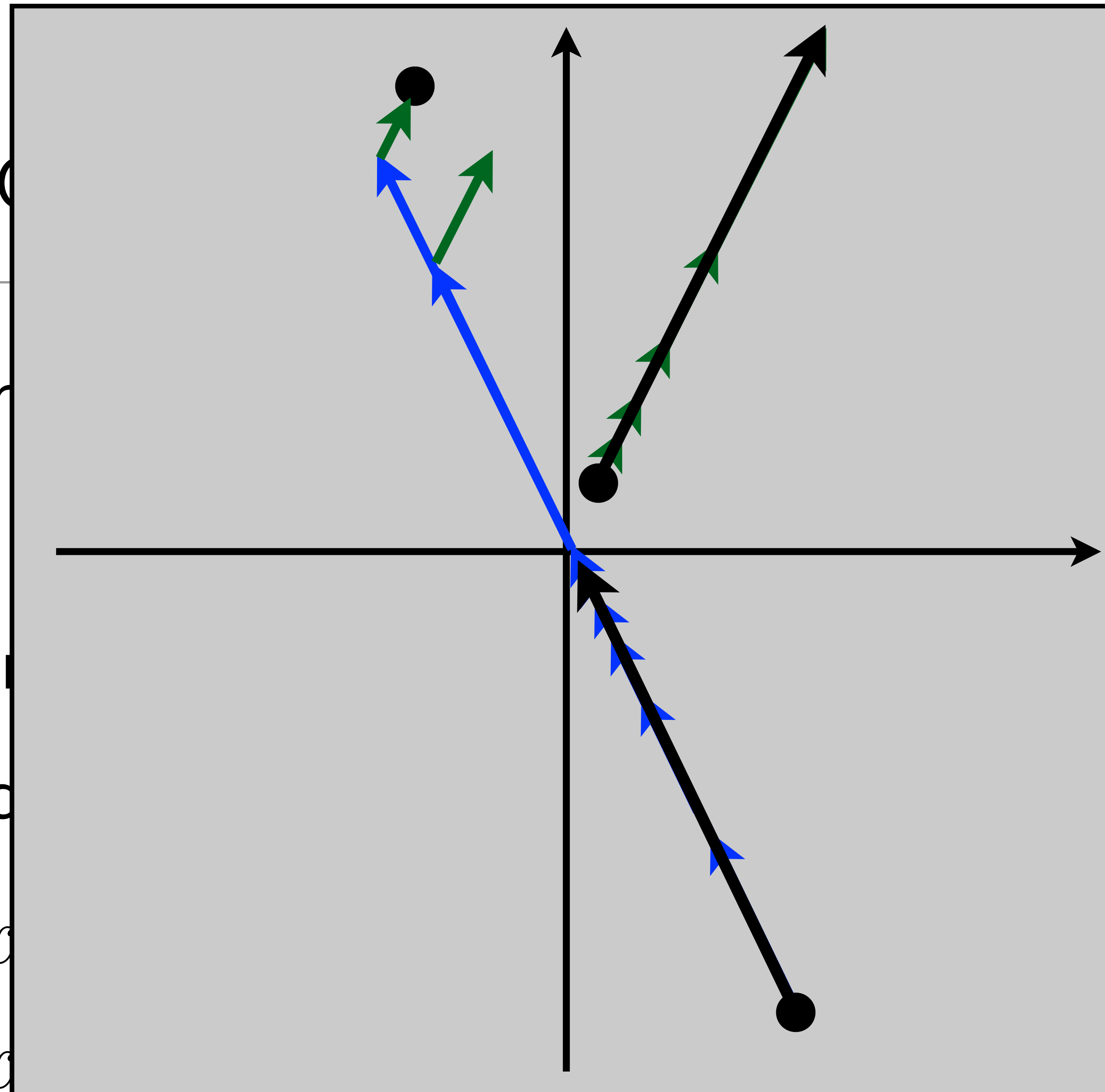

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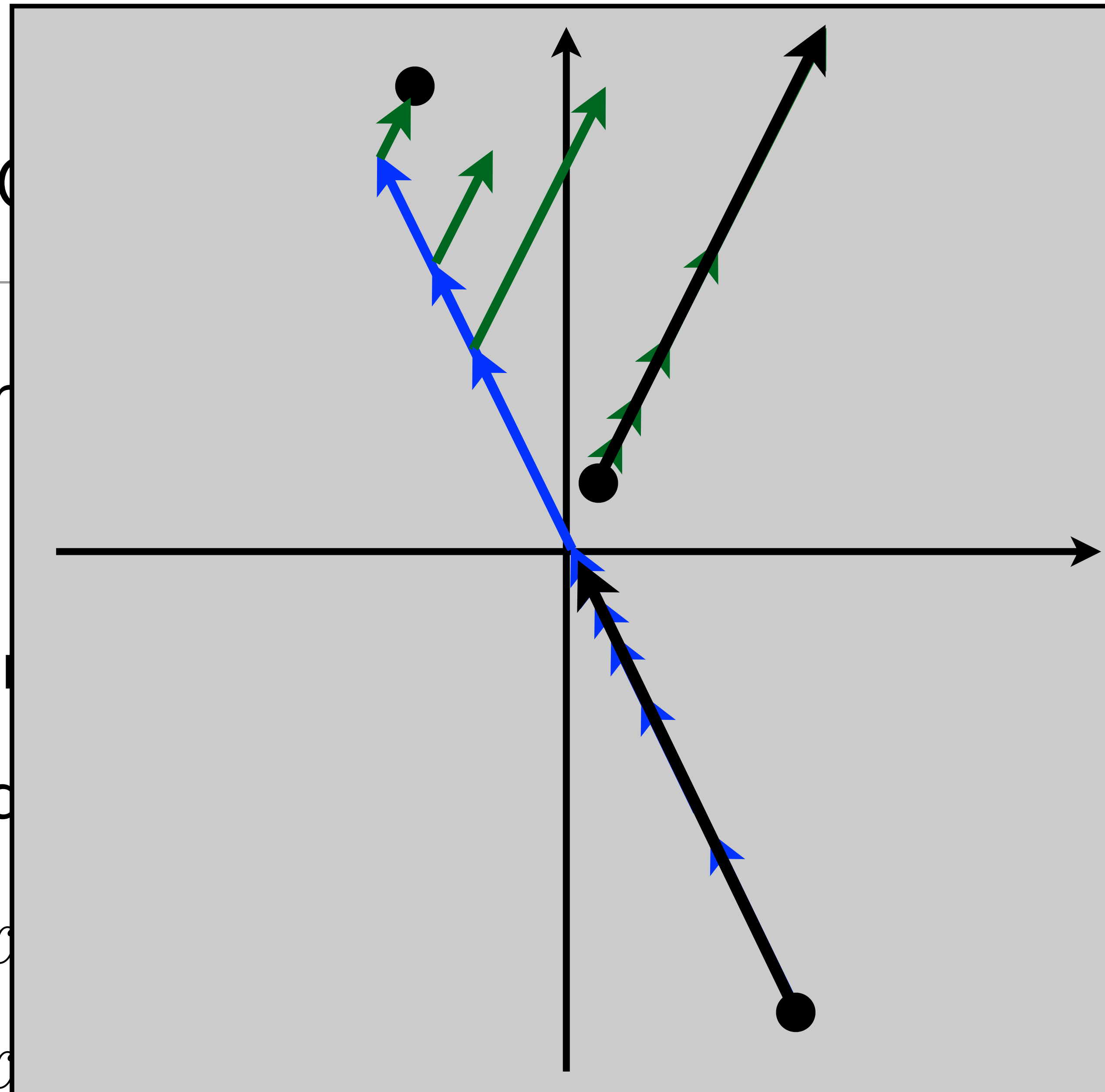

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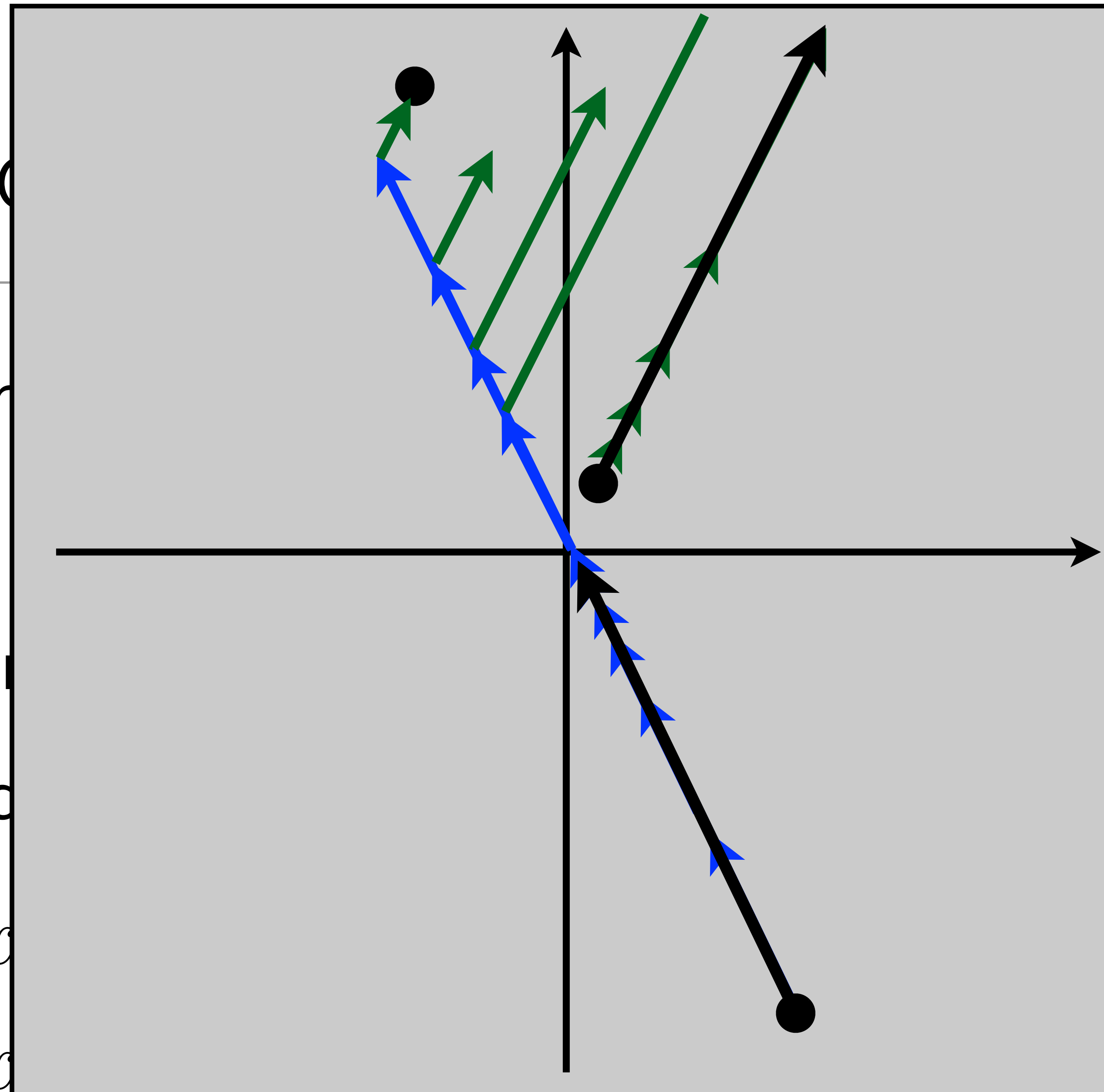

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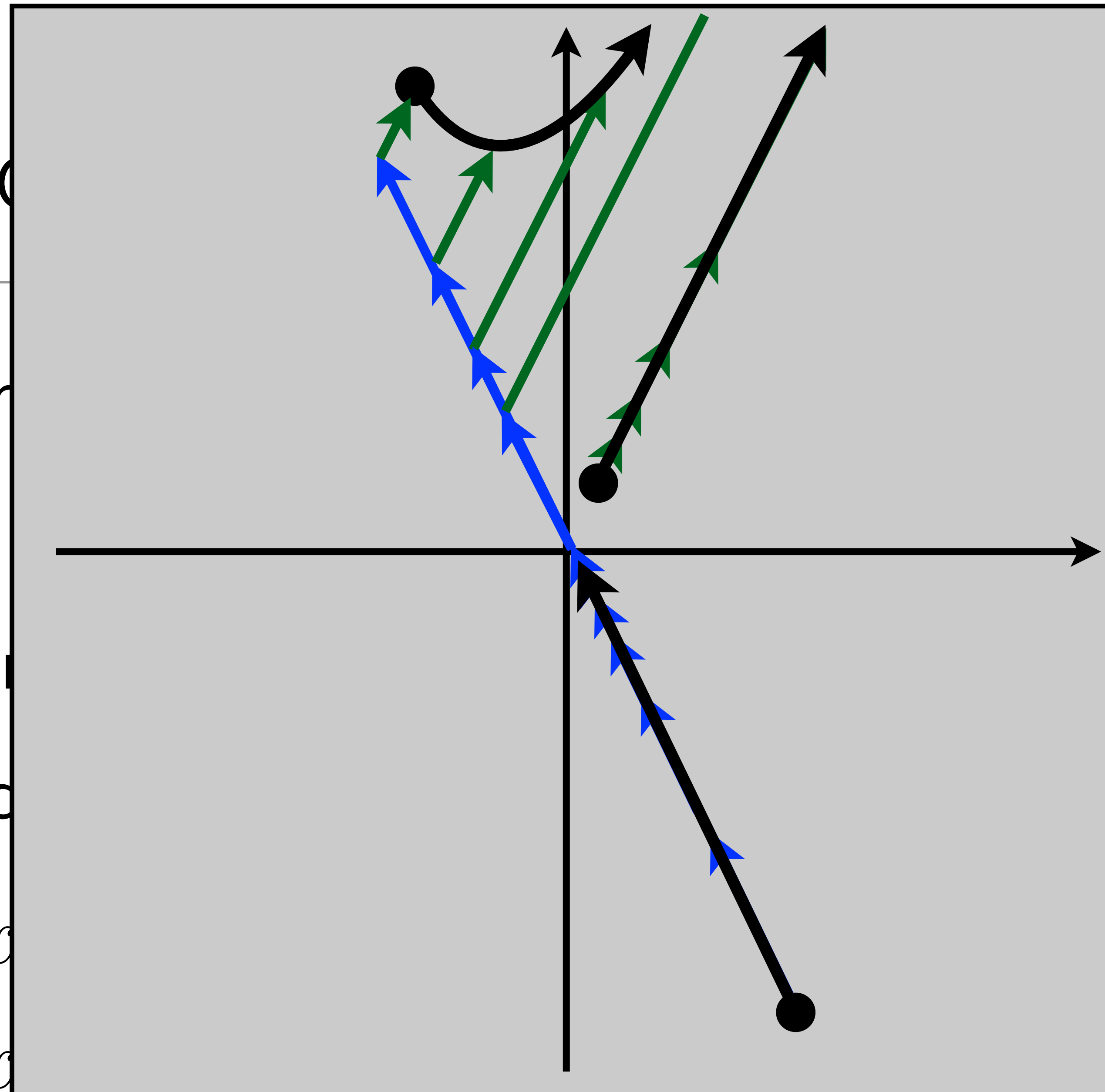

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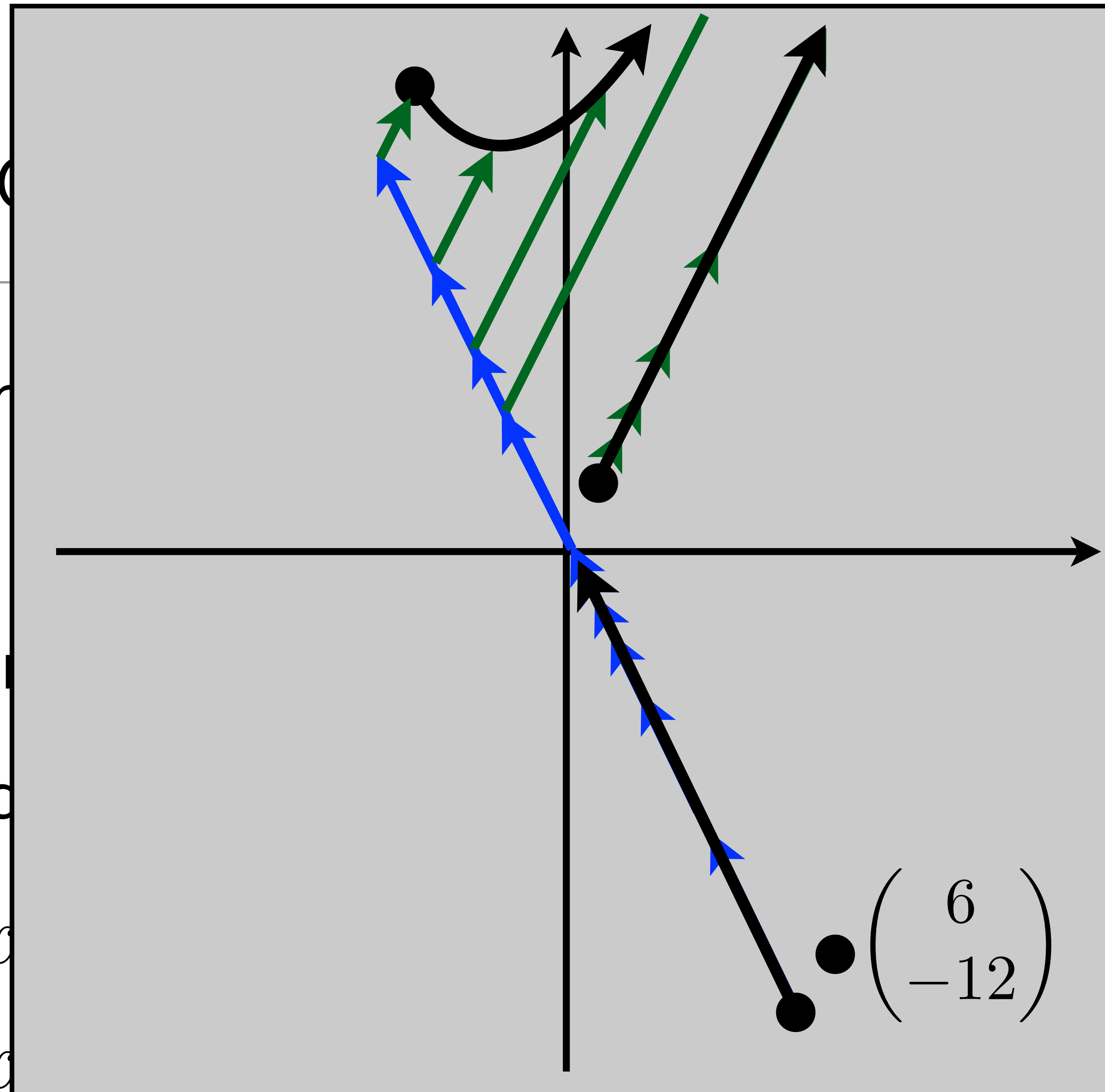

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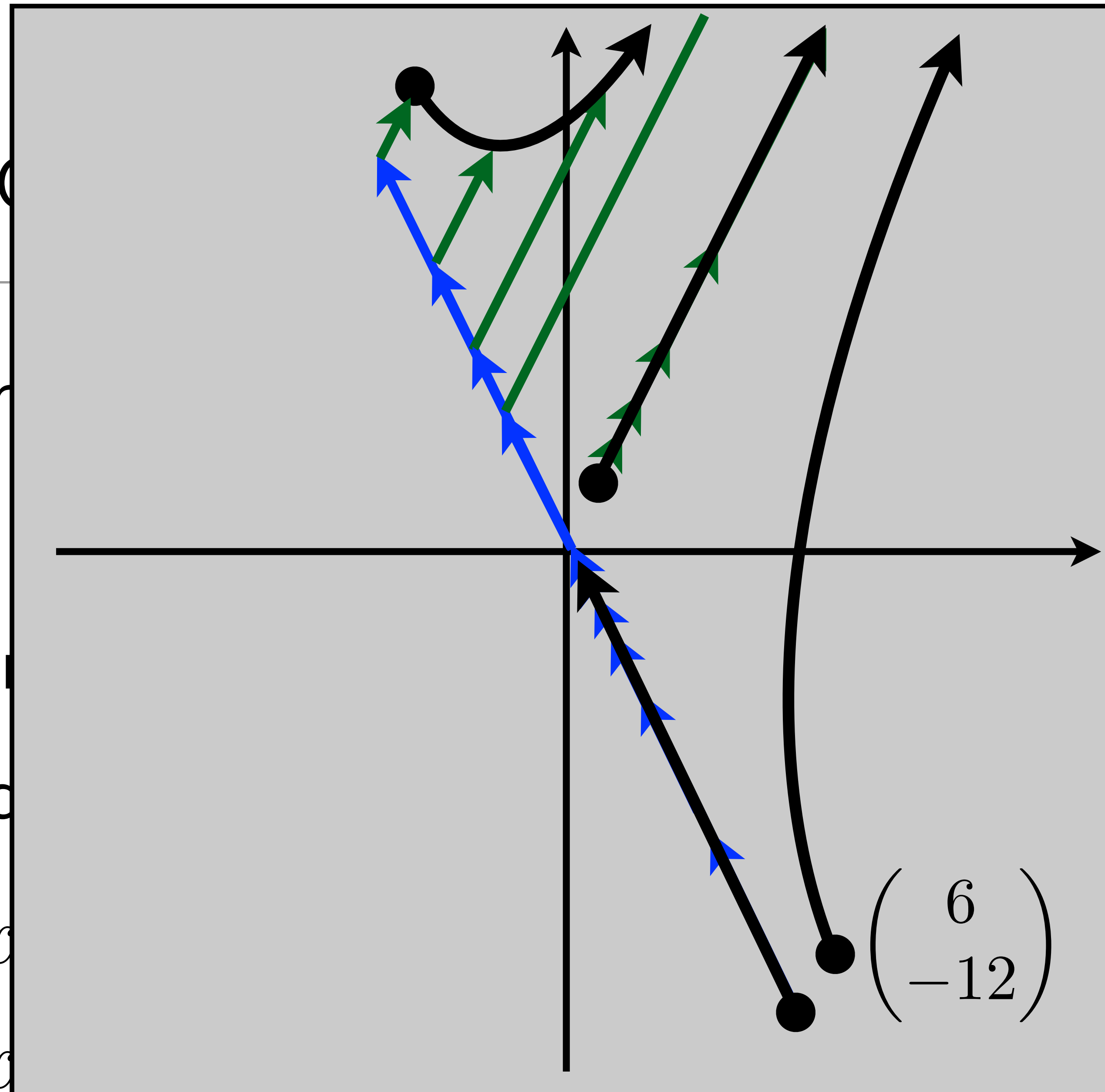

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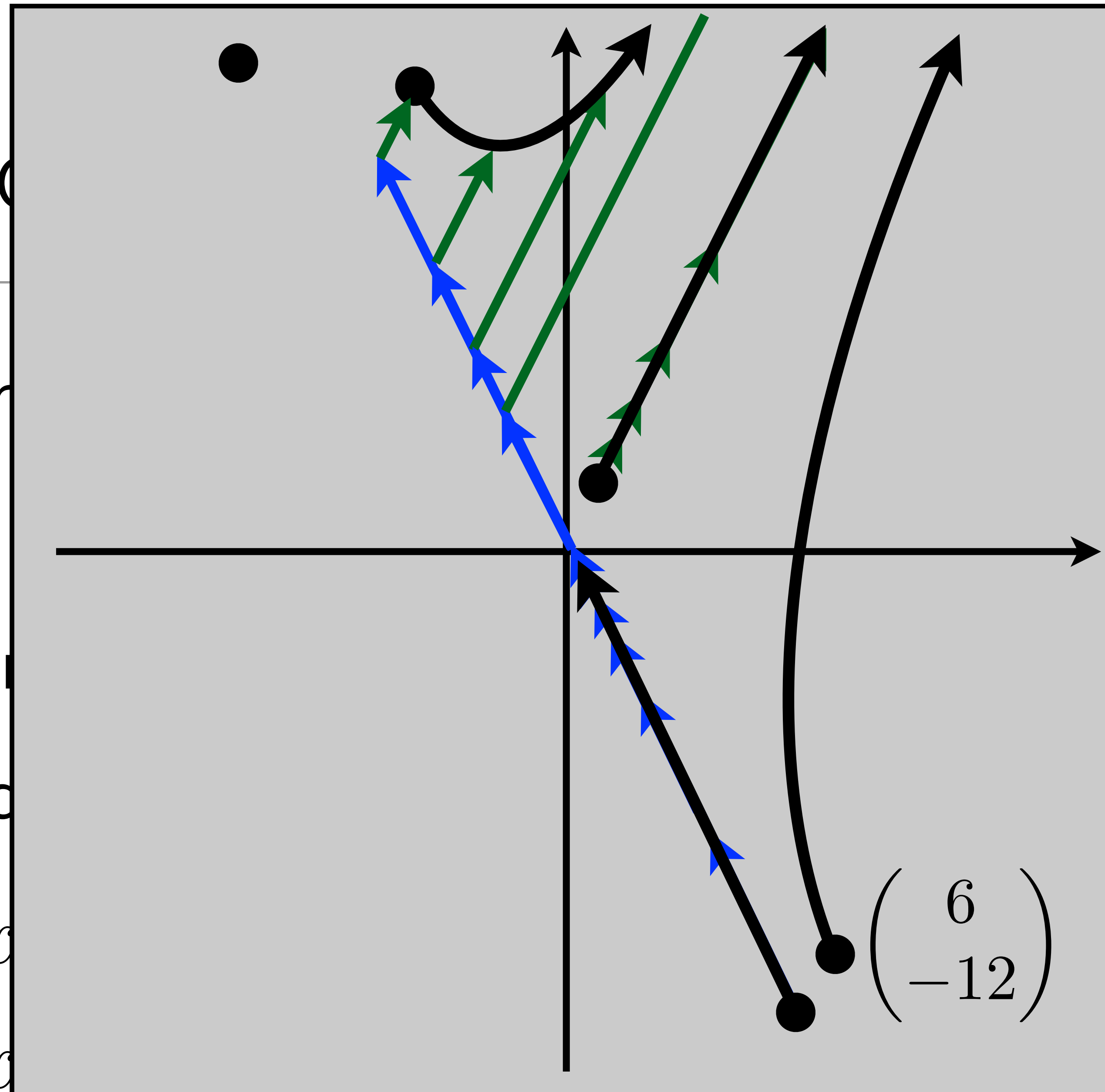

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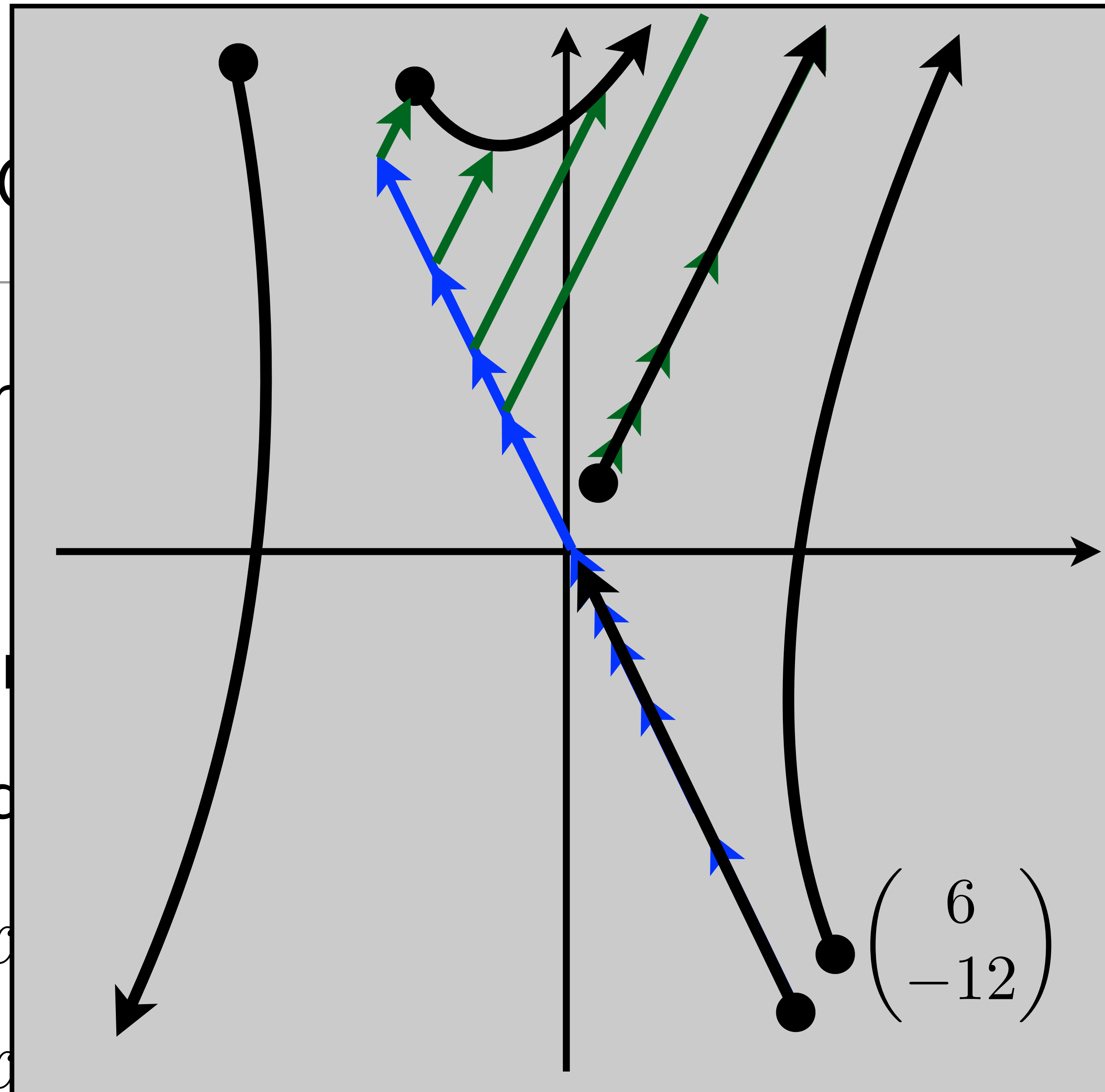

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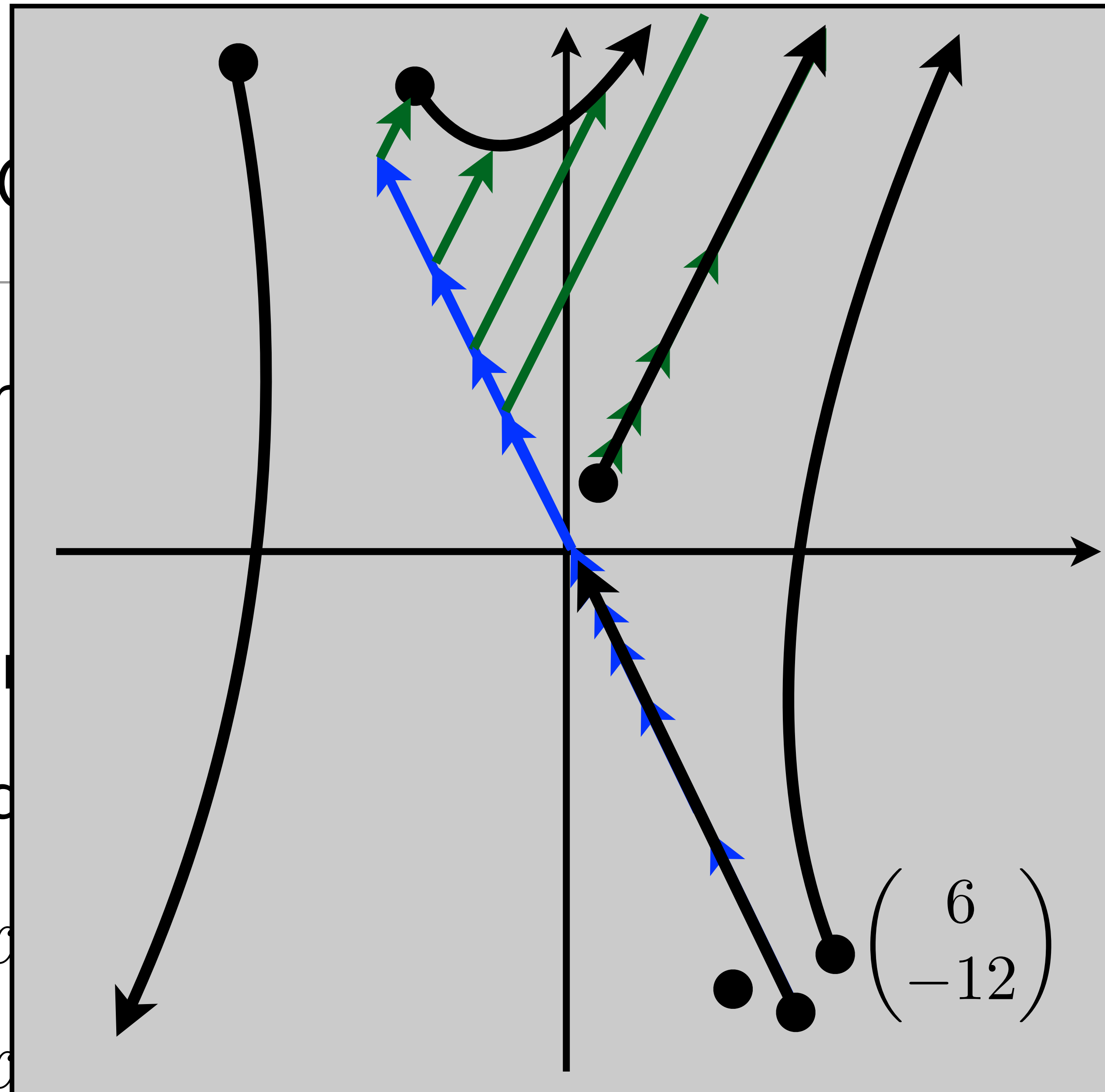

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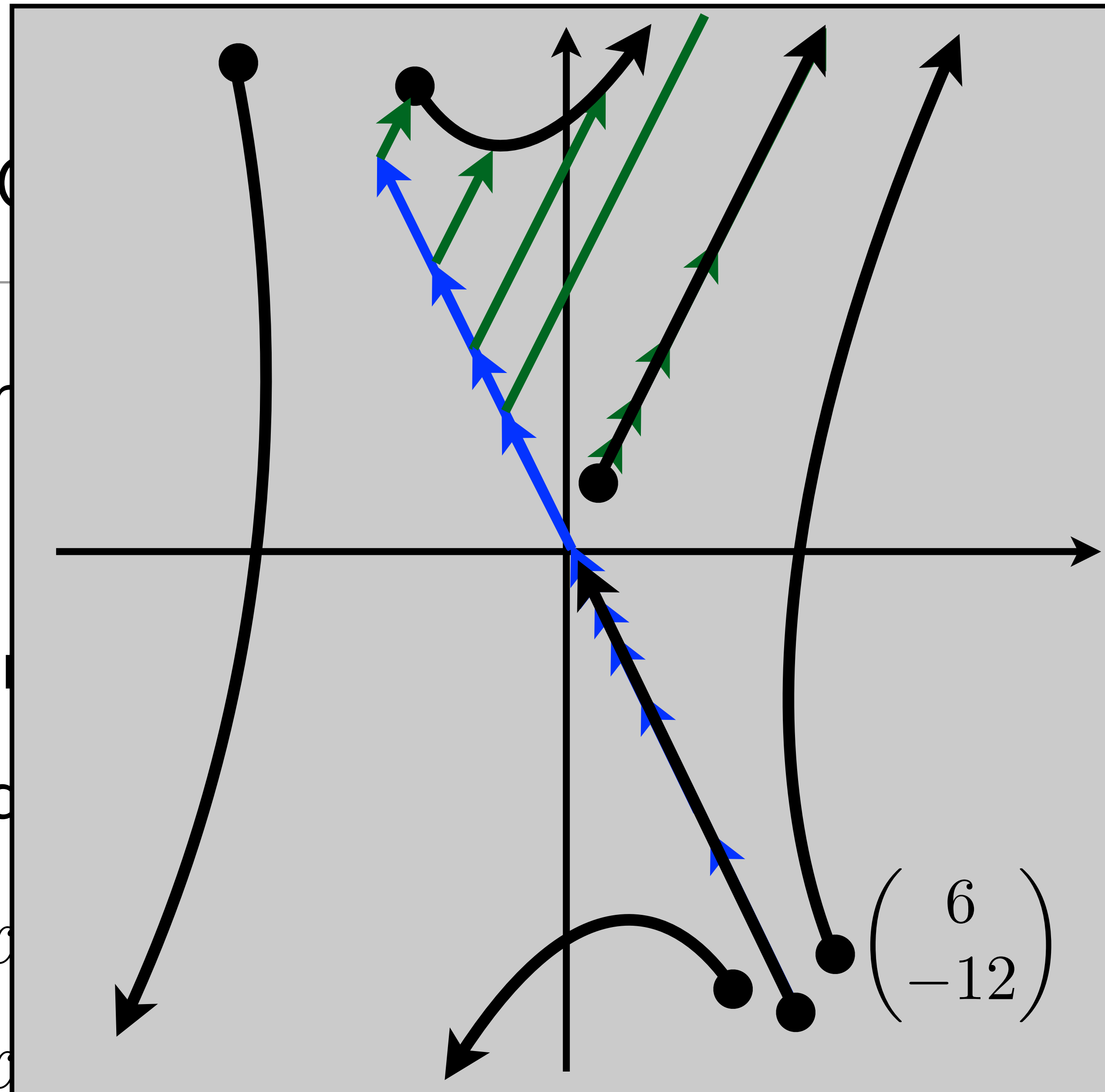

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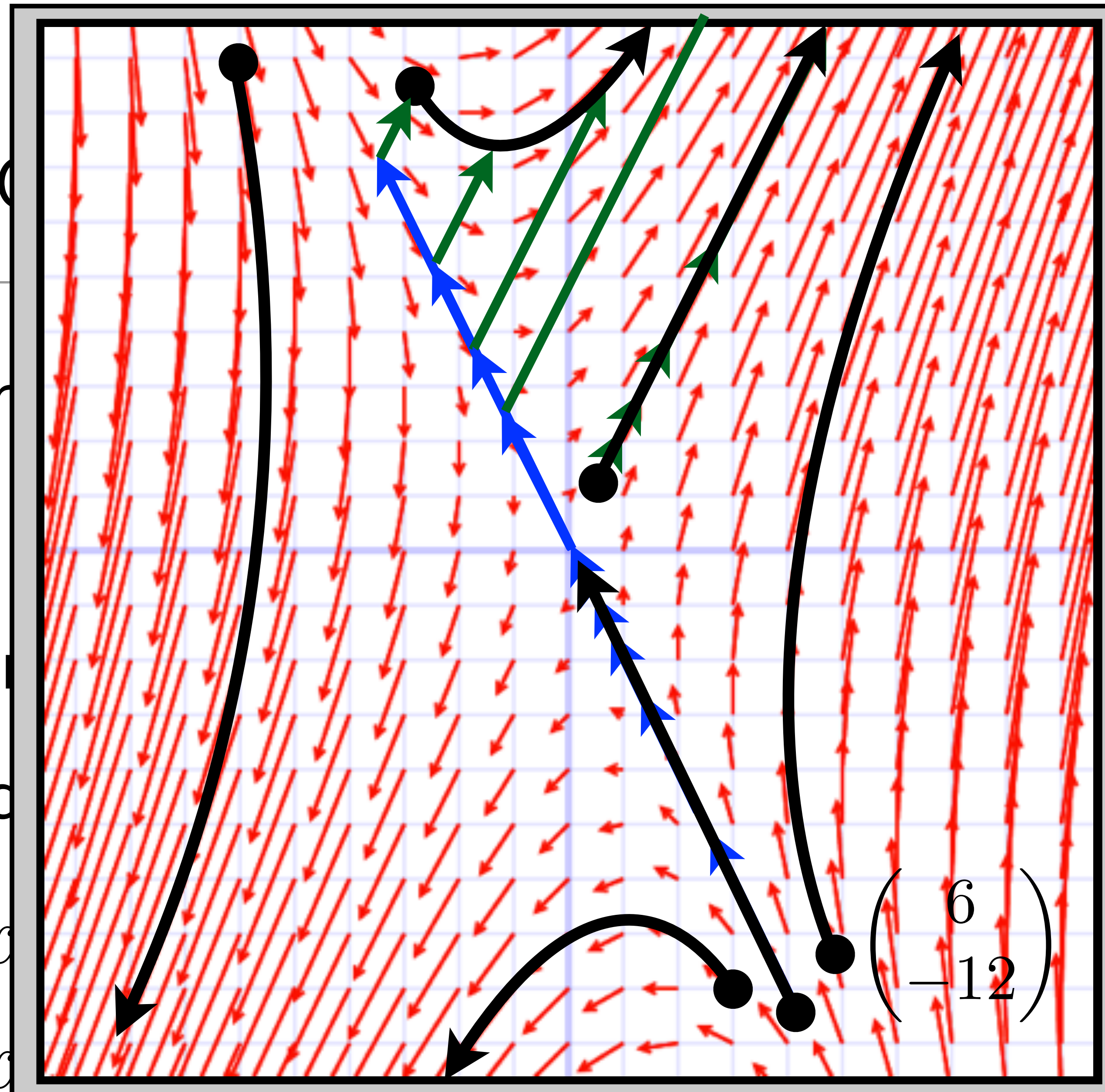

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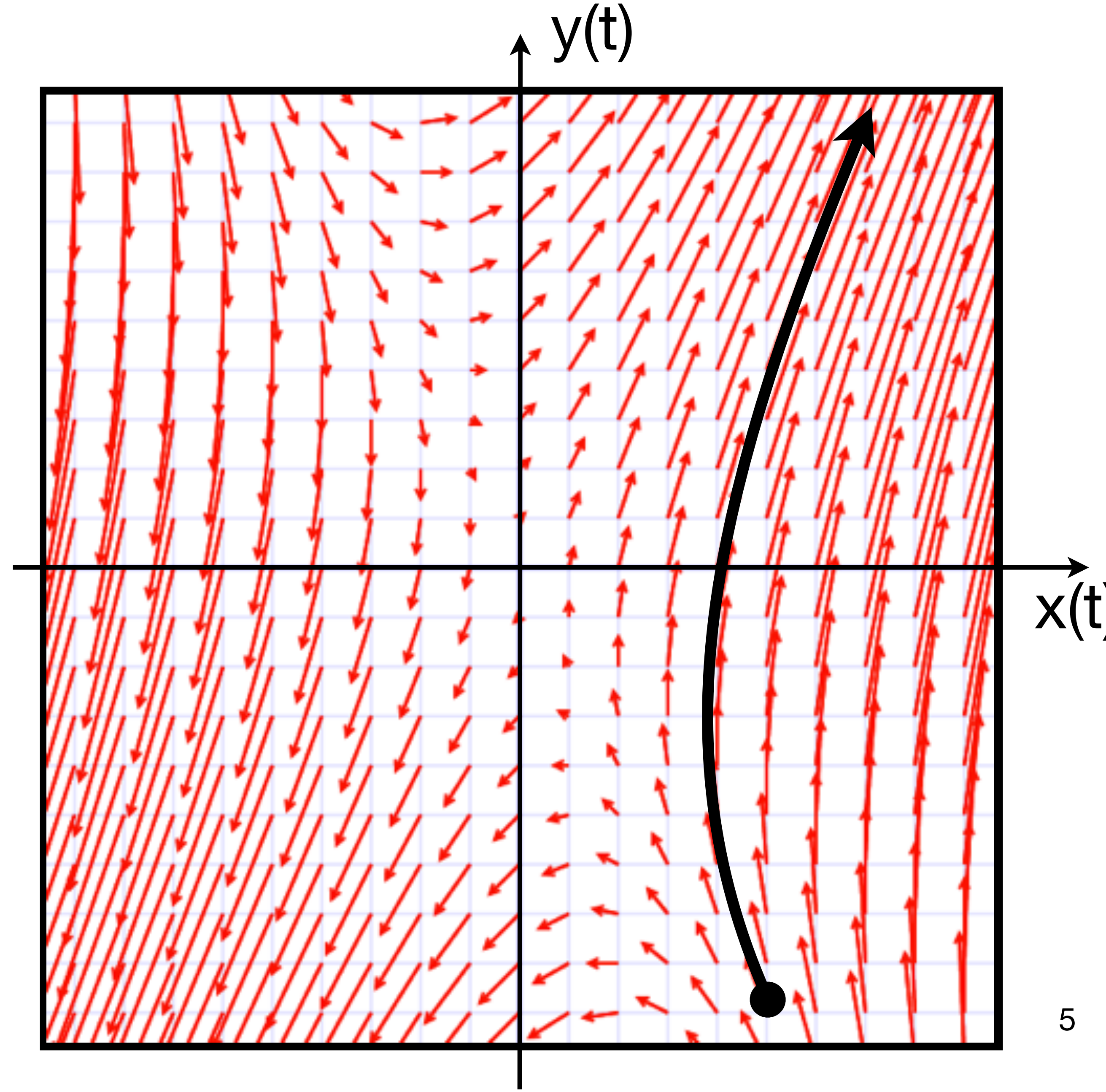

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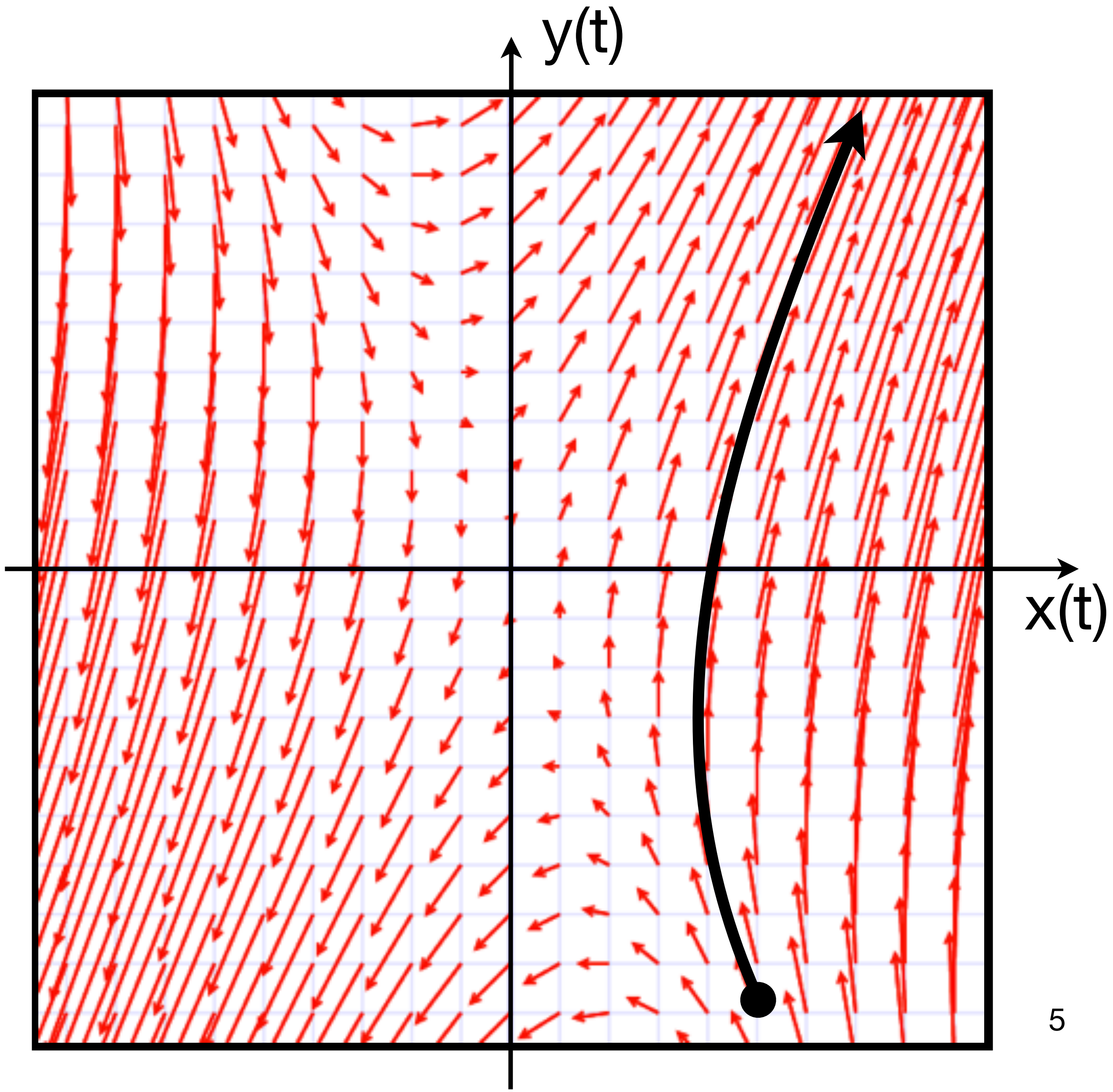

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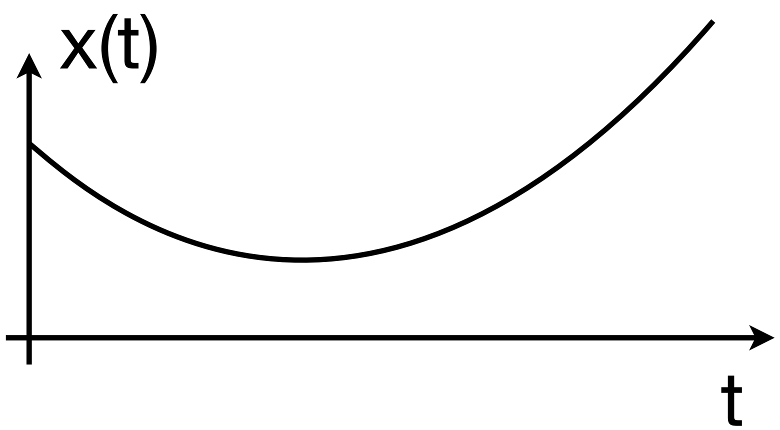

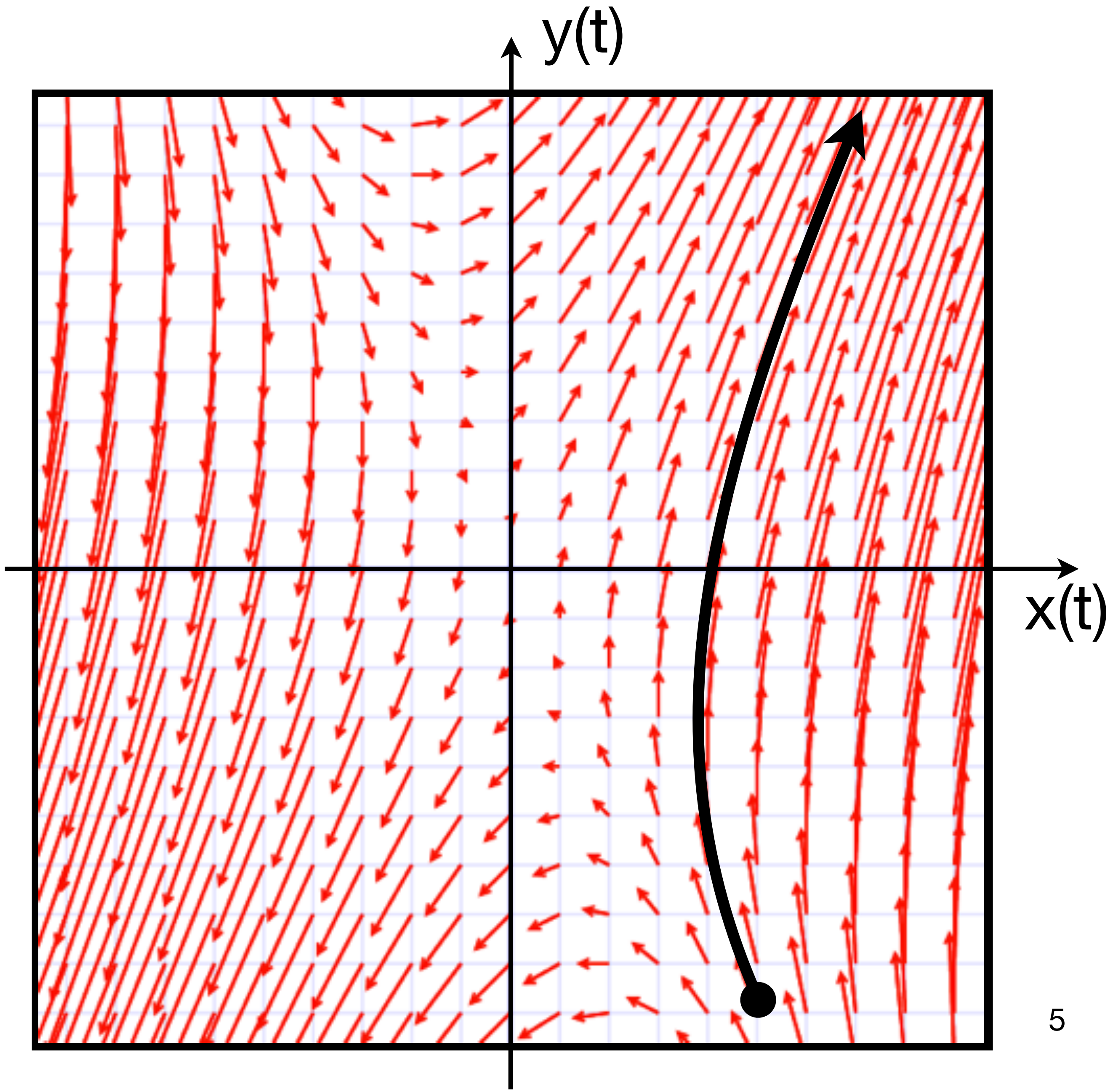

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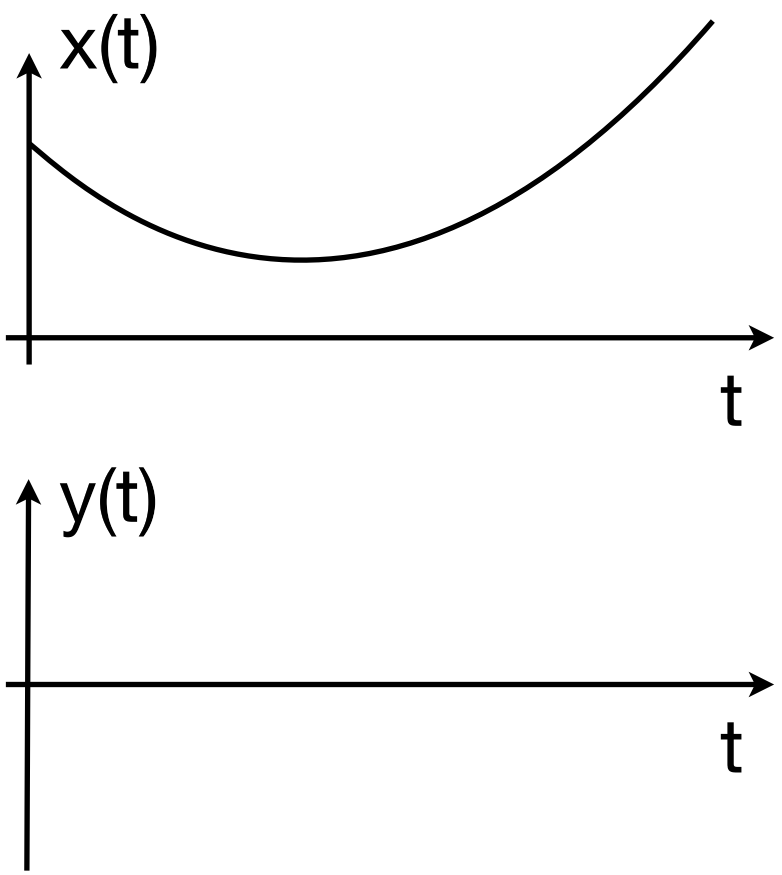

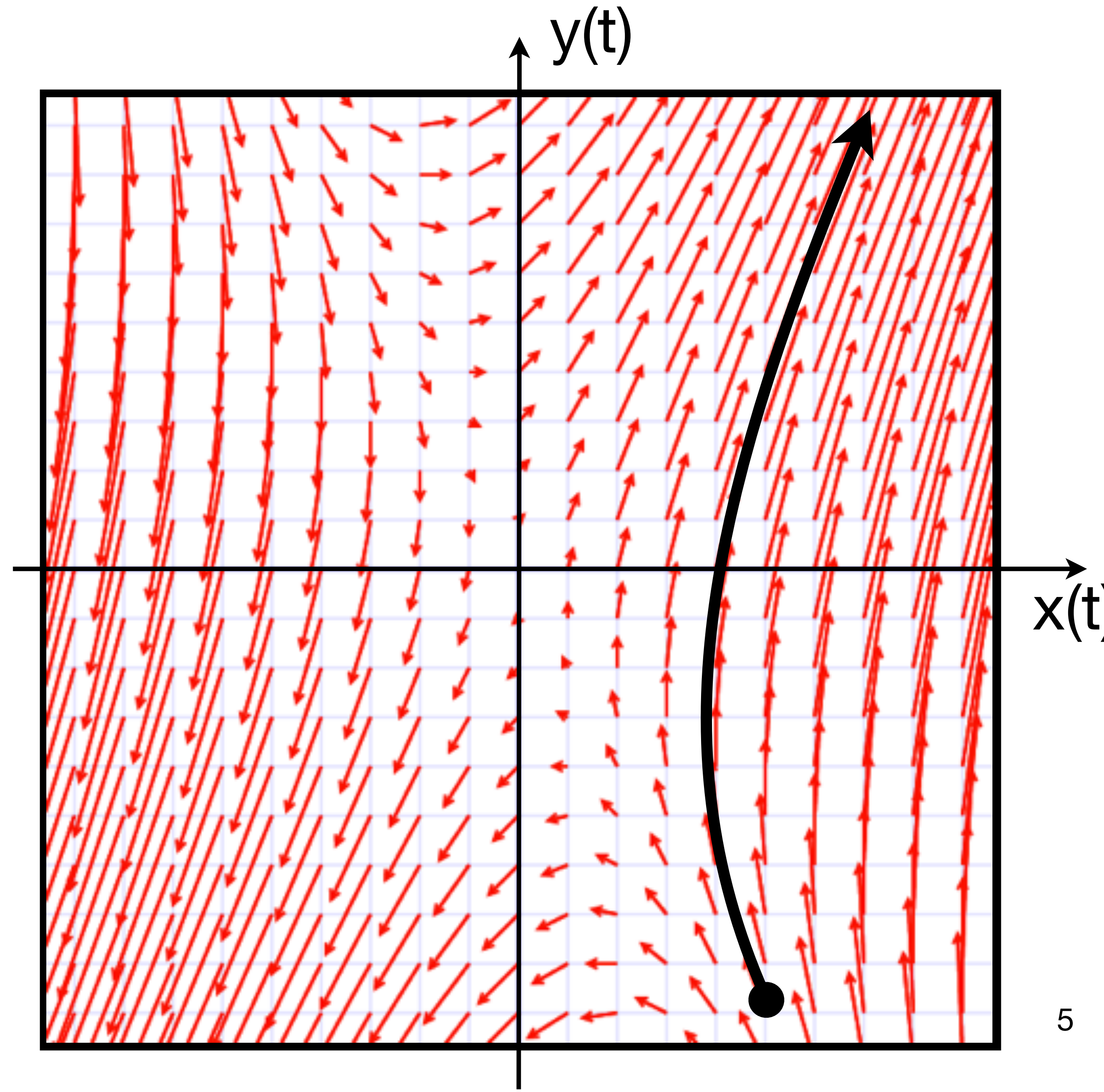

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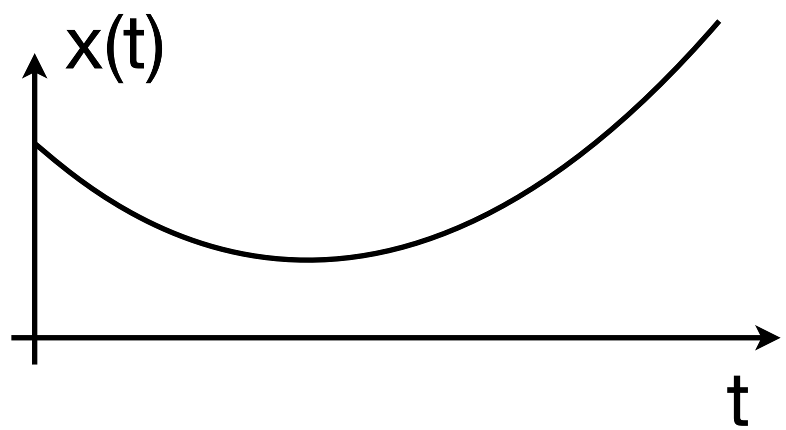

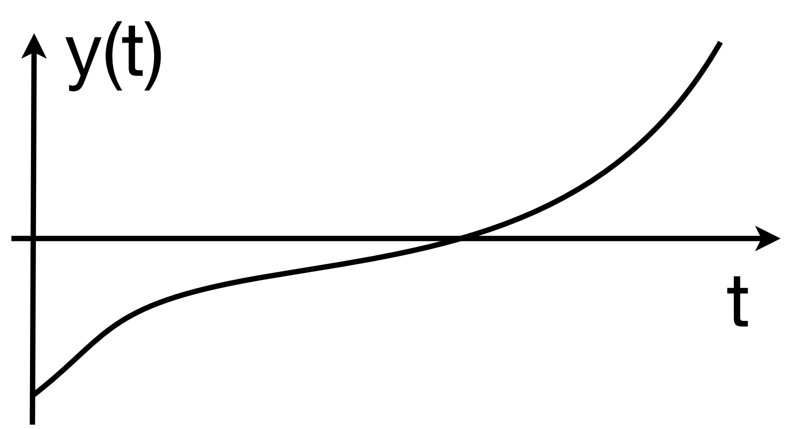

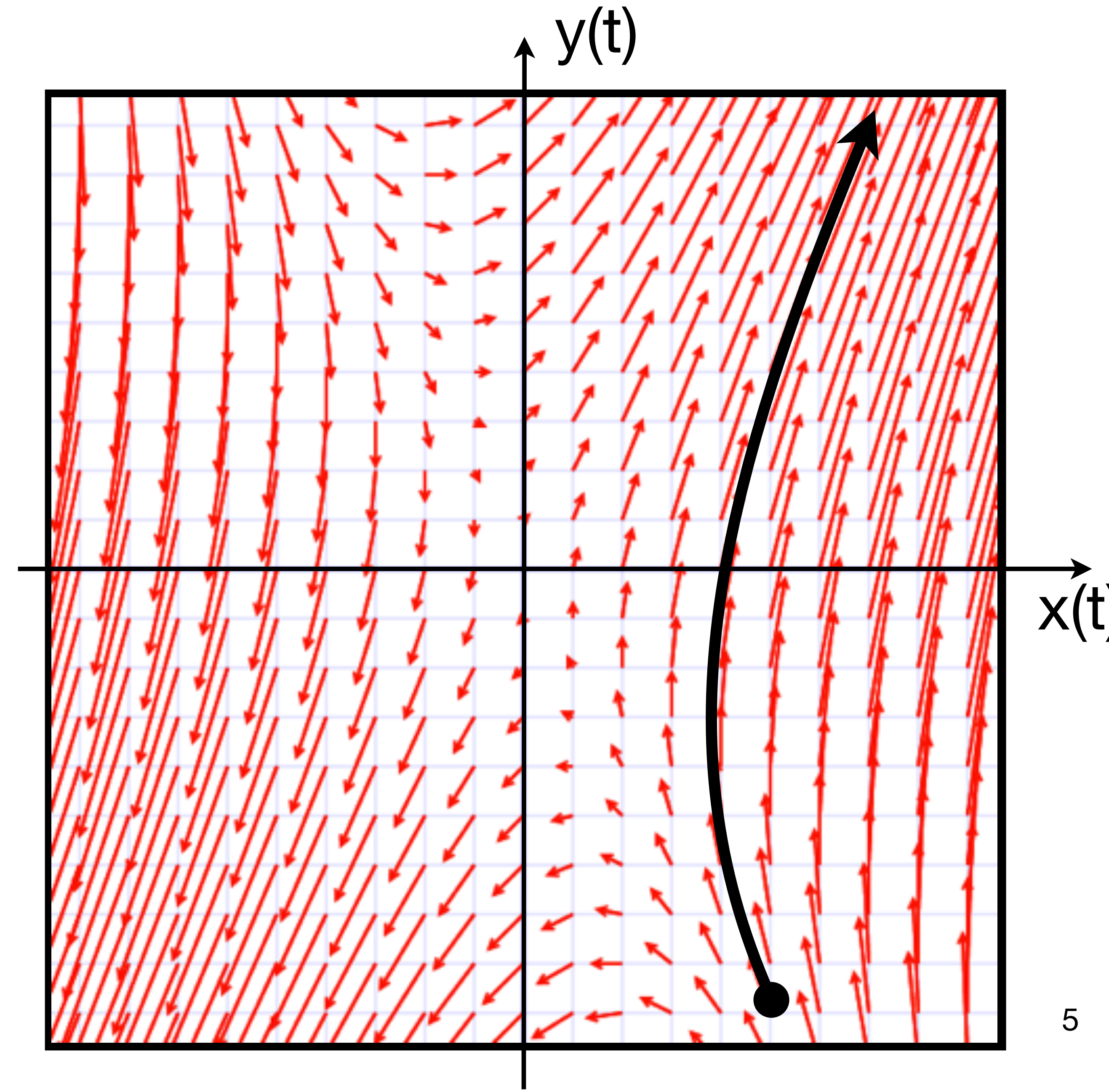

Shapes of solution curves in the phase plane

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$$x_1(t) = C_1 e^{\lambda_1 t}$$

Shapes of solution curves in the phase plane

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$$x_2(t) = C_2 e^{\lambda_2 t} \qquad t = \frac{1}{\lambda_2} \ln\left(\frac{x_2}{C_2}\right)$$

$$\frac{1}{\lambda_2} \ln\left(\frac{x_2}{C_2}\right) = \frac{1}{\lambda_1} \ln\left(\frac{x_1}{C_1}\right)$$

$$\ln\left(\frac{x_2}{C_2}\right) = \frac{\lambda_2}{\lambda_1} \ln\left(\frac{x_1}{C_1}\right)$$

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$$\lambda_2 = -\lambda_1$$
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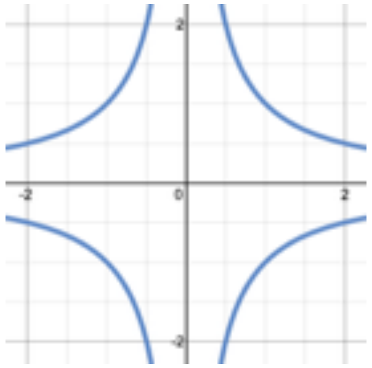

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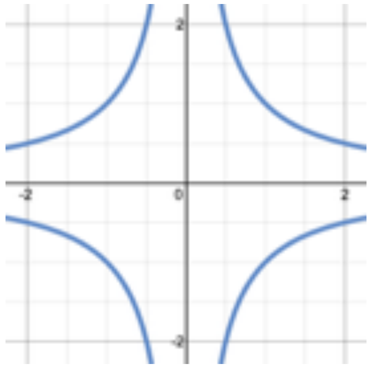

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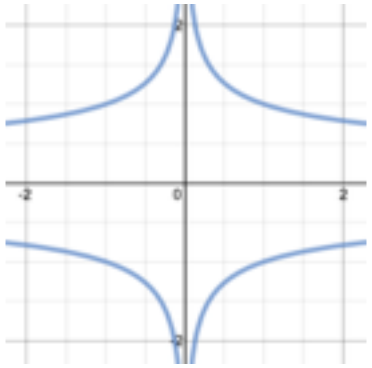

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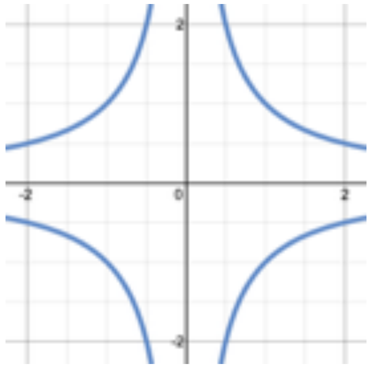

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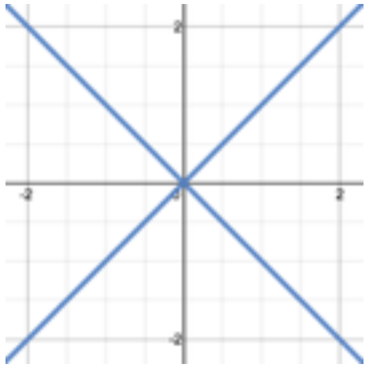

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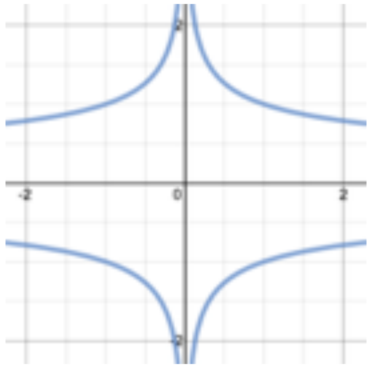

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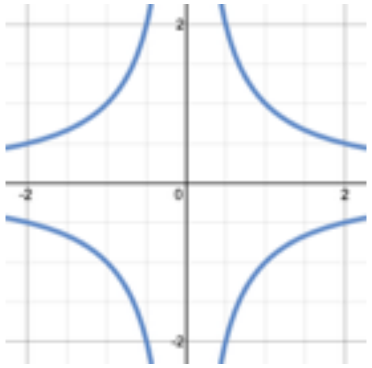

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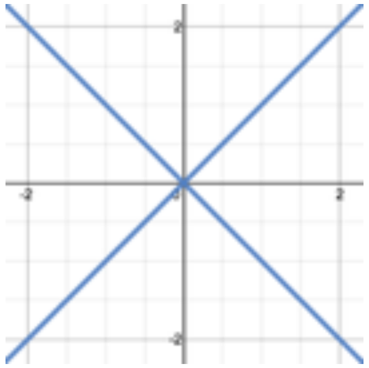

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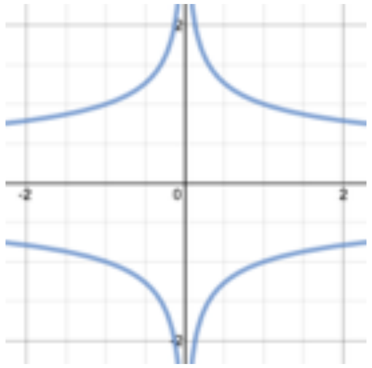
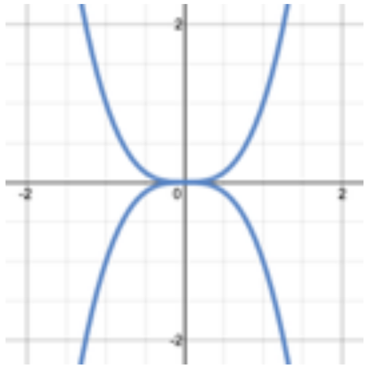


$$\lambda_2 = 3\lambda_1$$
$$x_2 = C x_1^3$$



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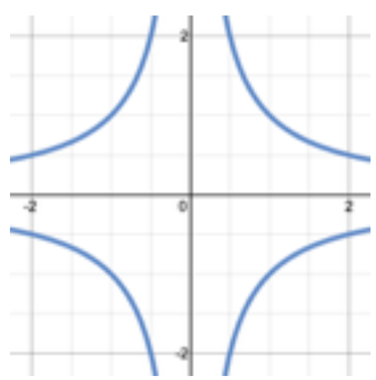

$$\lambda_2 = \lambda_1$$

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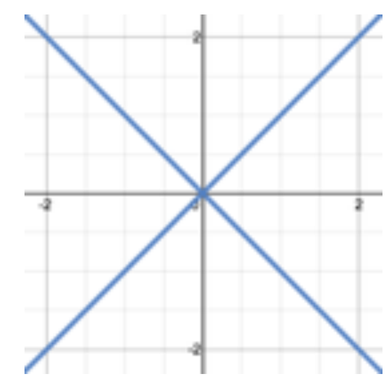

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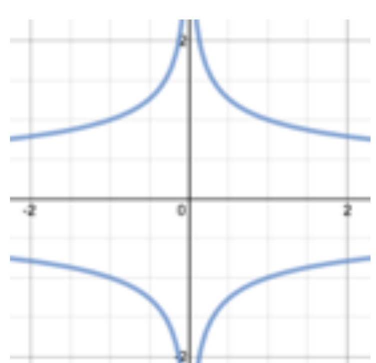

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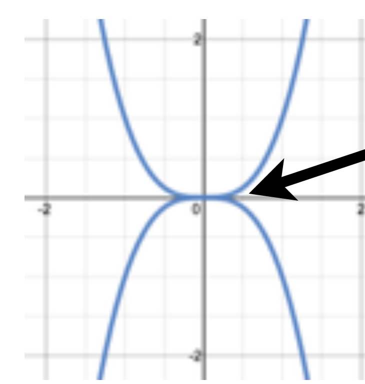

stays near x_1 axis

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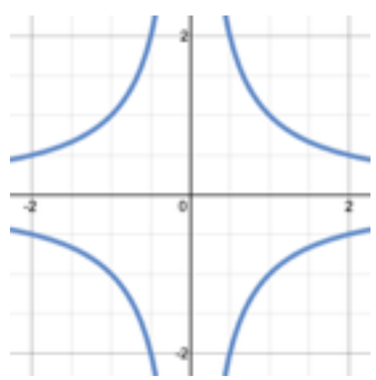

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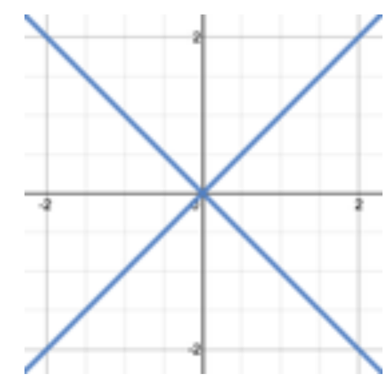

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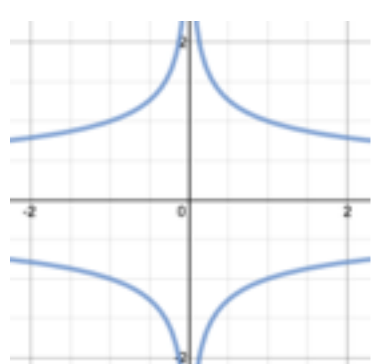

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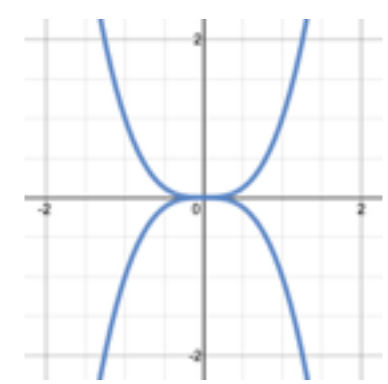

https://www.desmos.com/calculator/c4rhrgotmo

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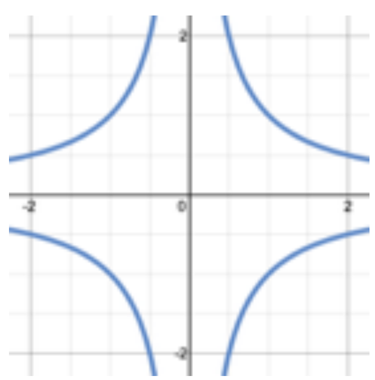

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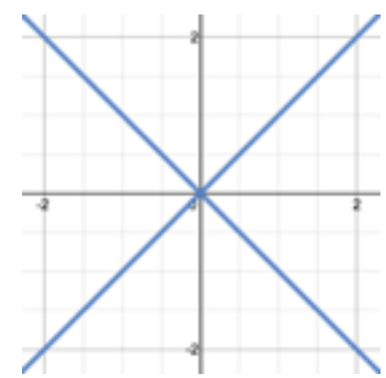

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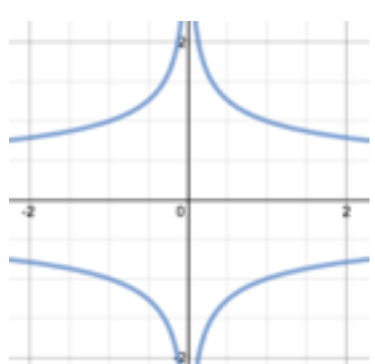

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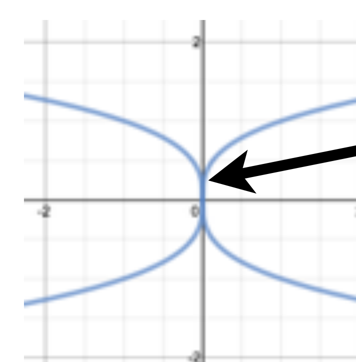

stays near x_2 axis

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Shapes of solution curves in the phase plane

- With more complicated solutions (evectors off-axis), tilt shapes accordingly.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_1 e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

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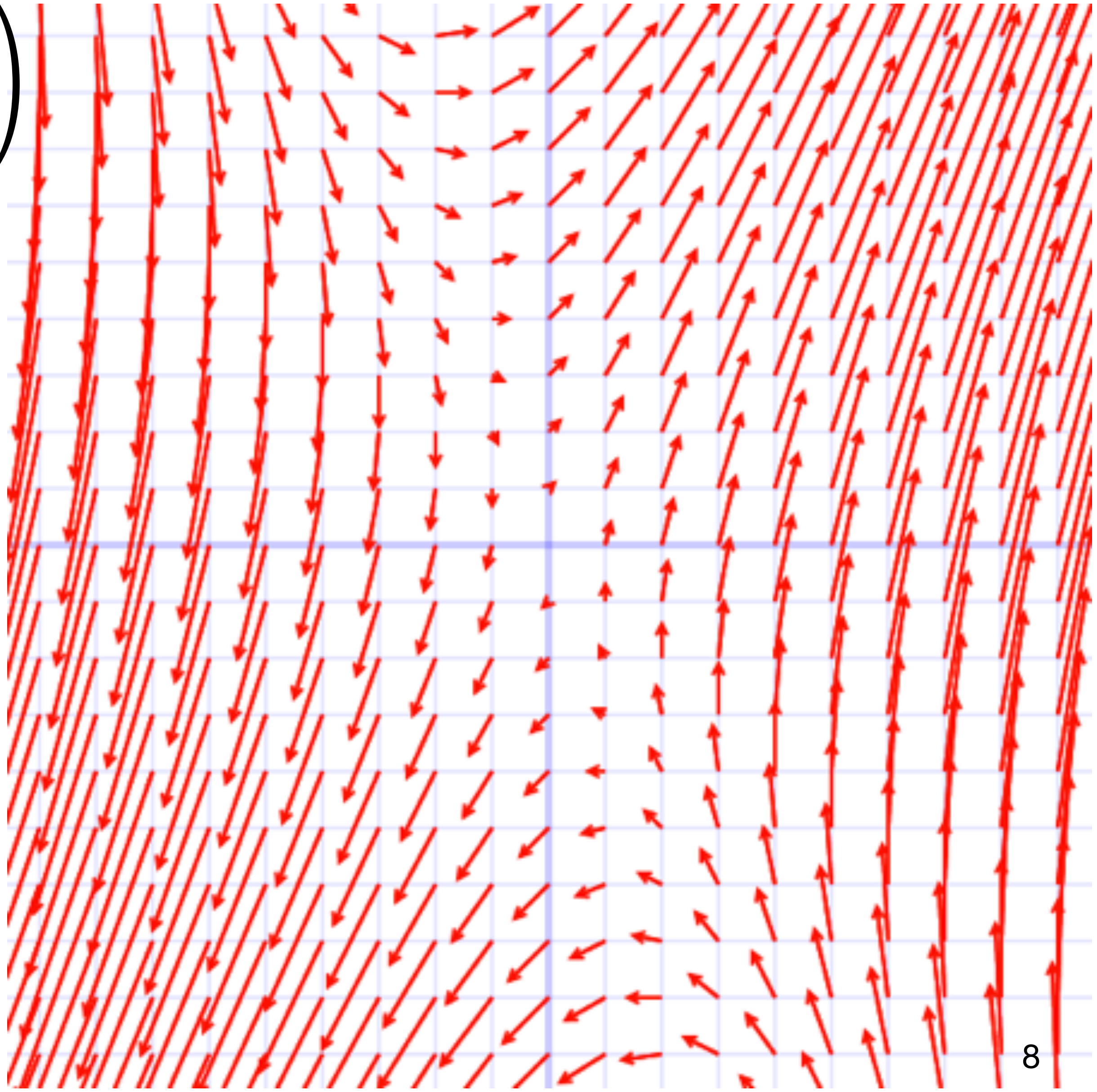

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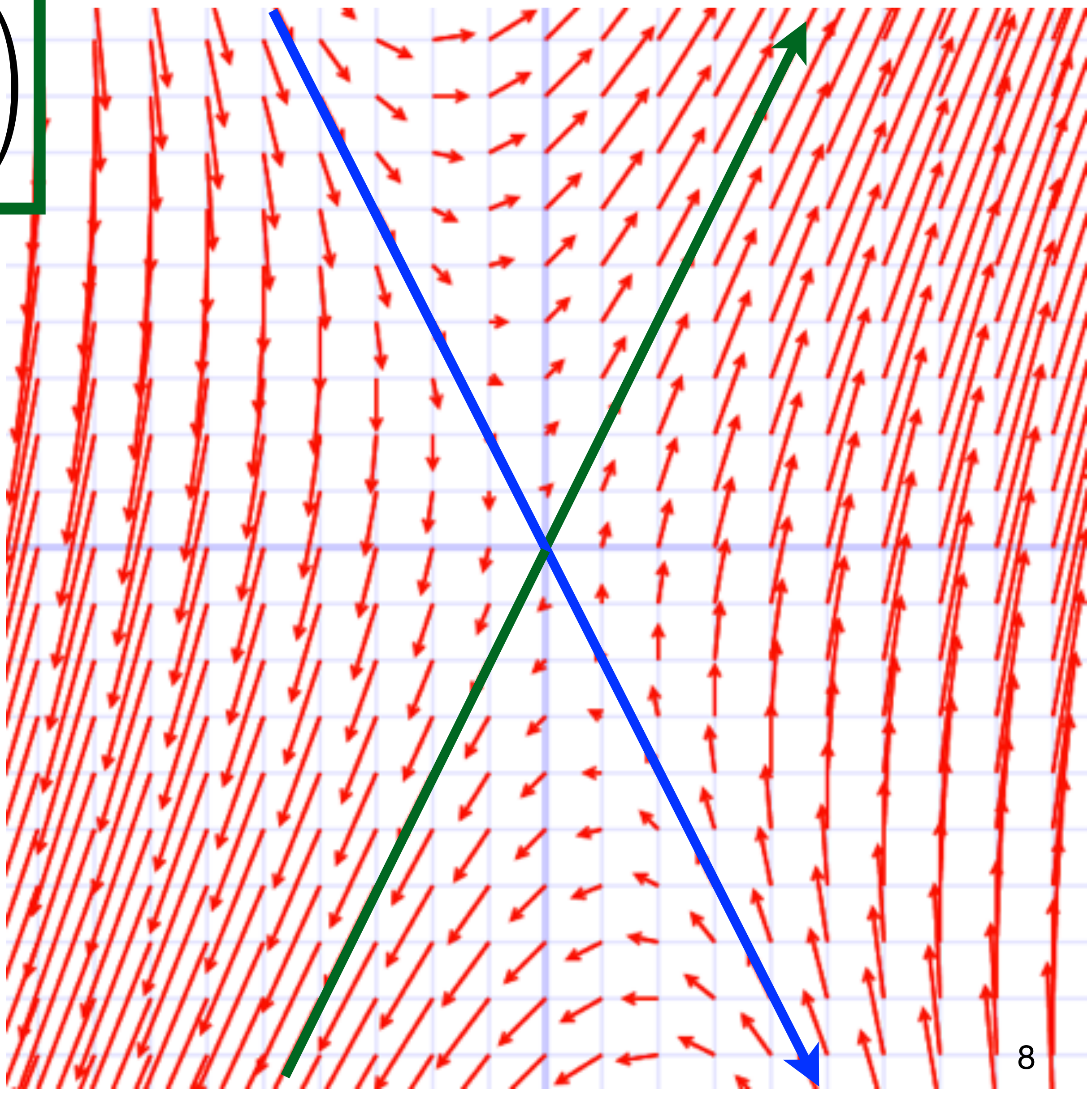

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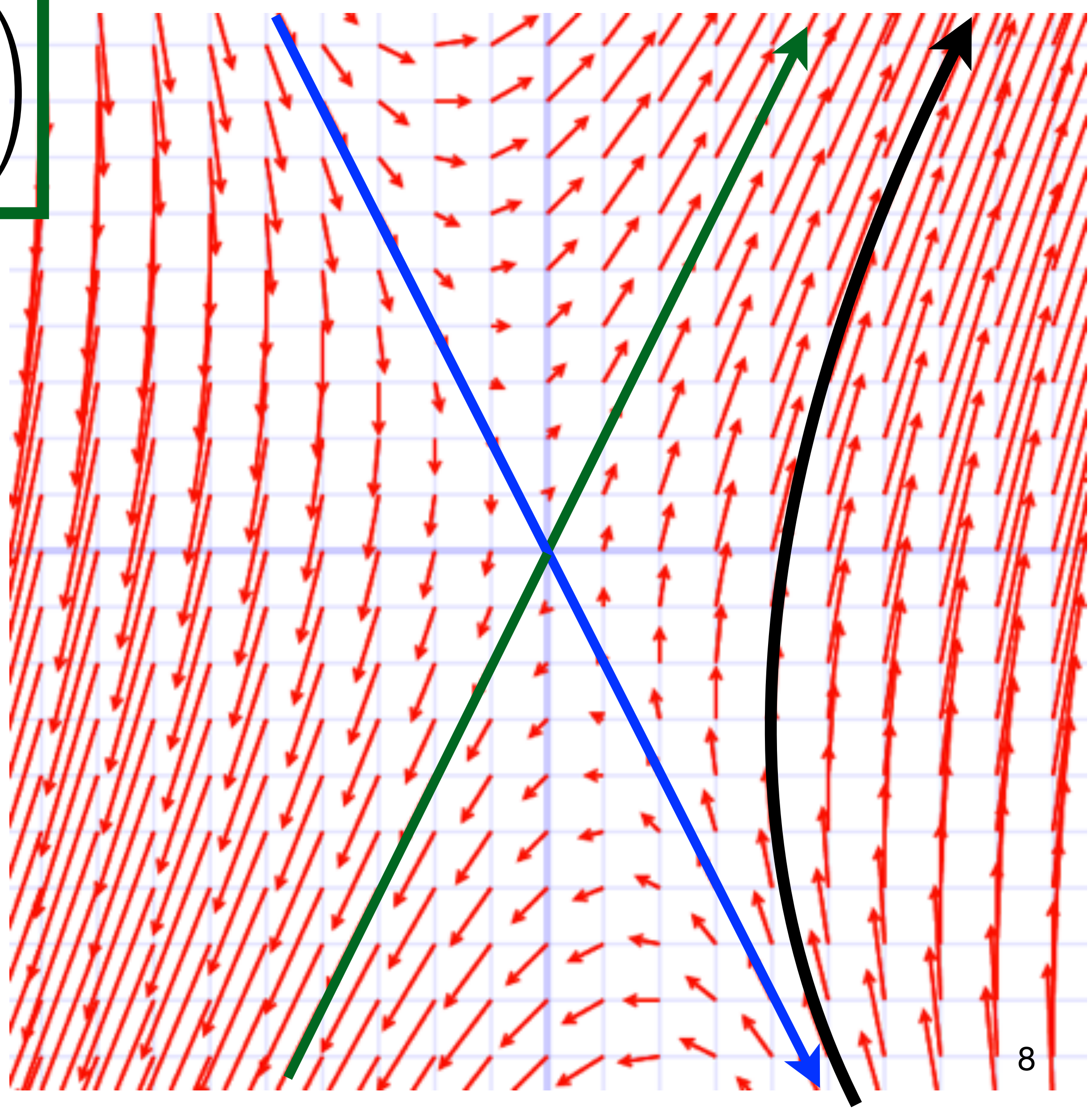

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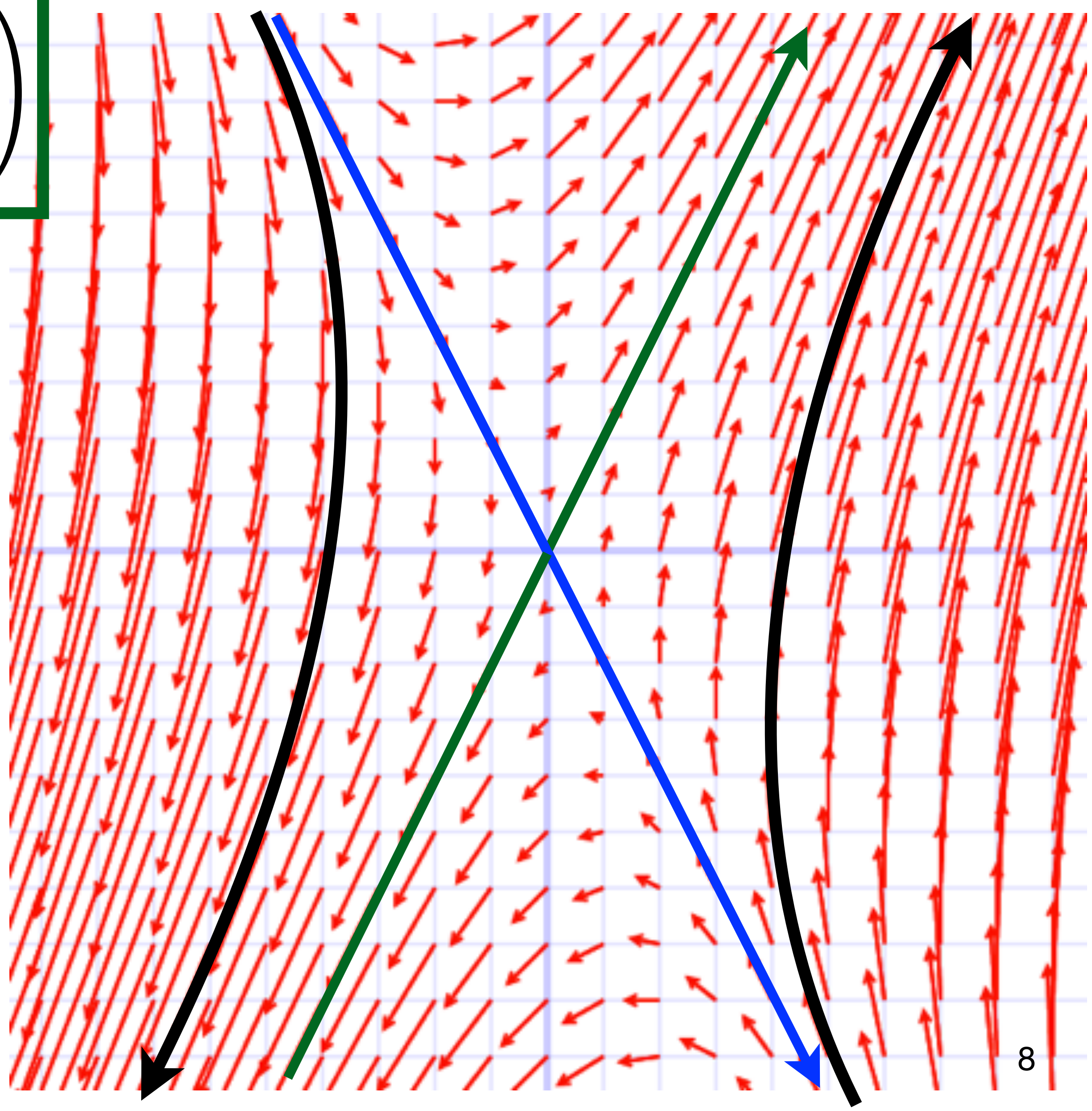

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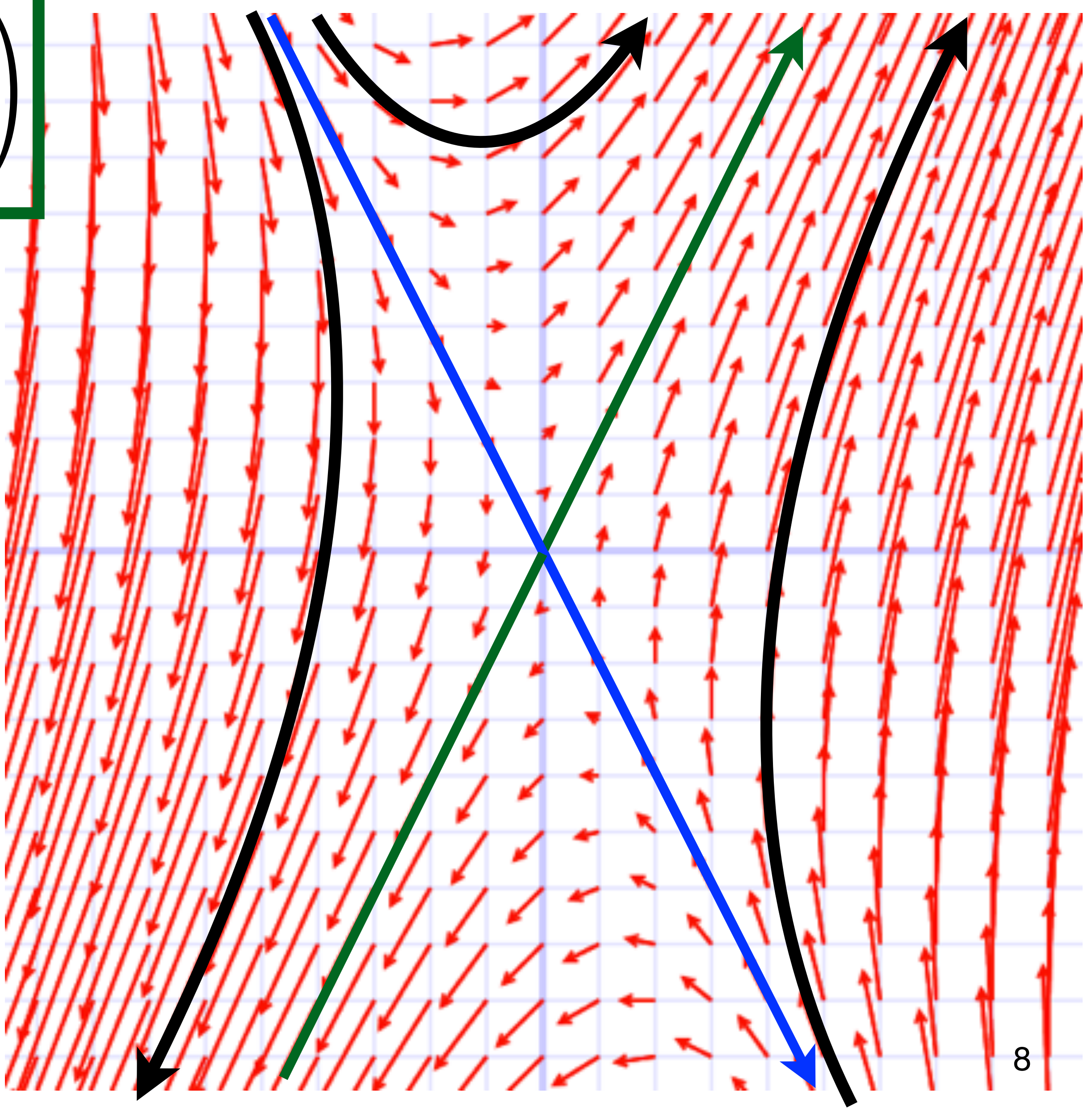

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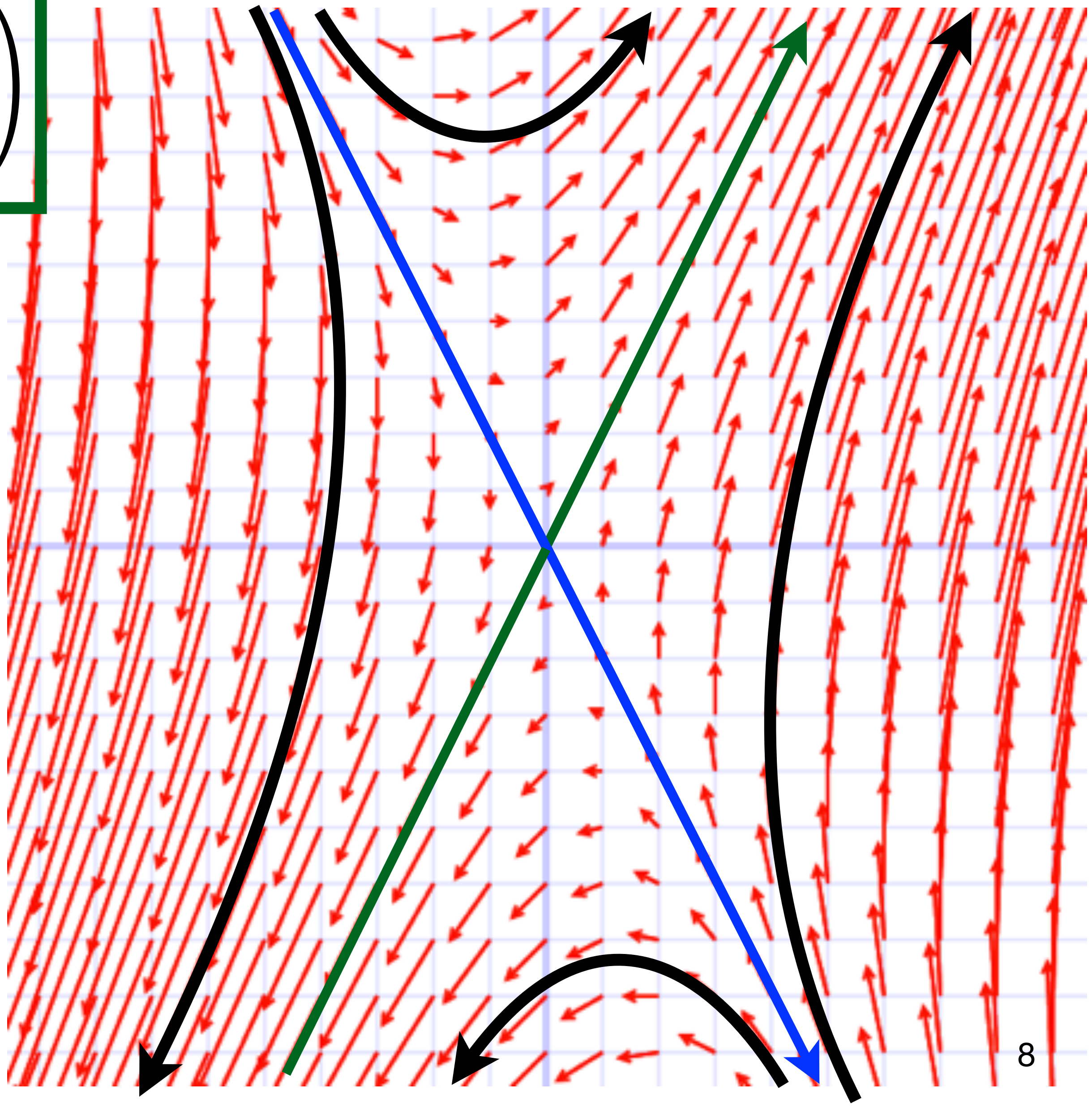

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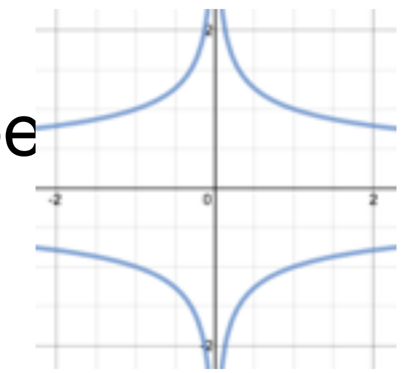

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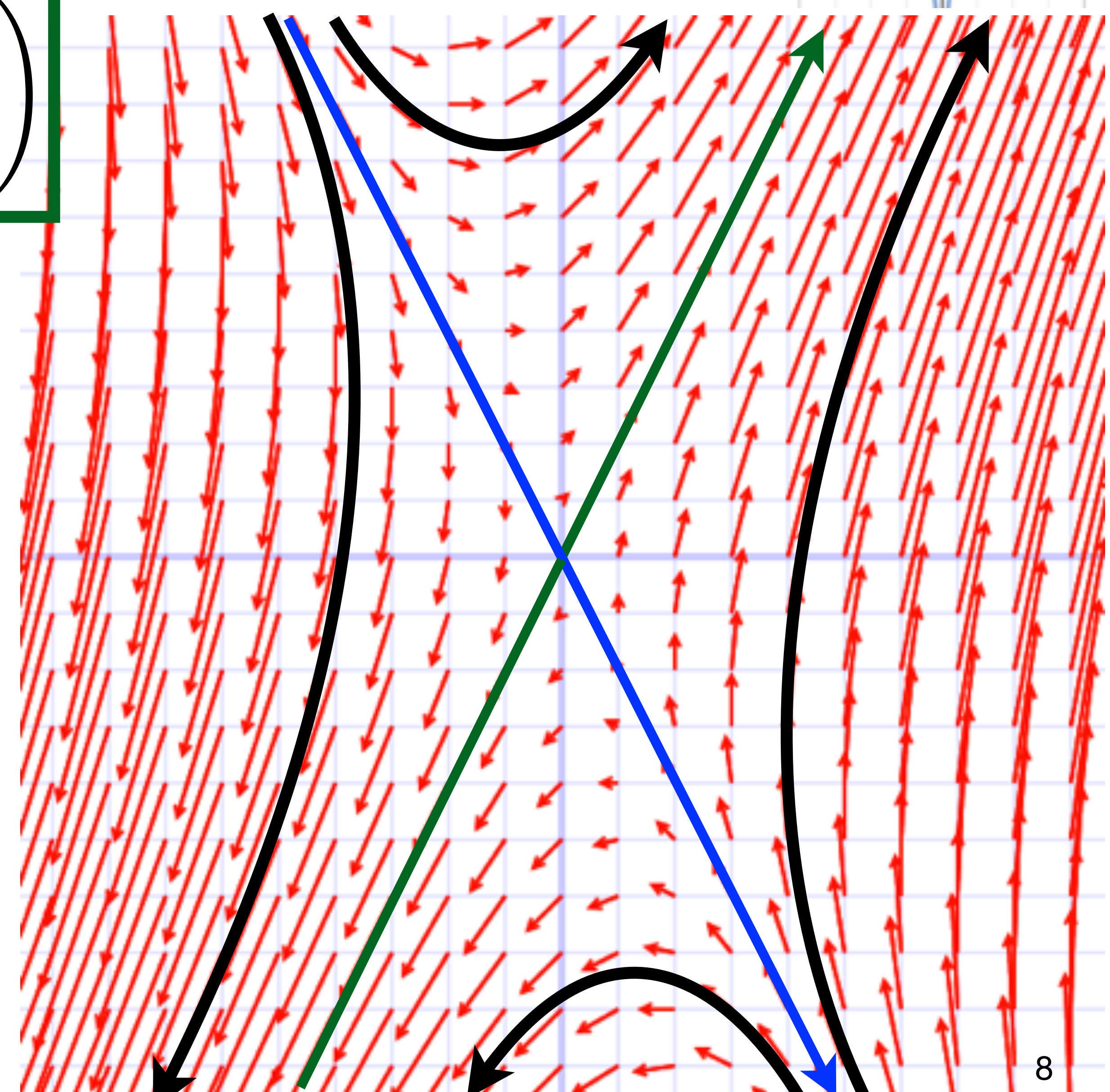

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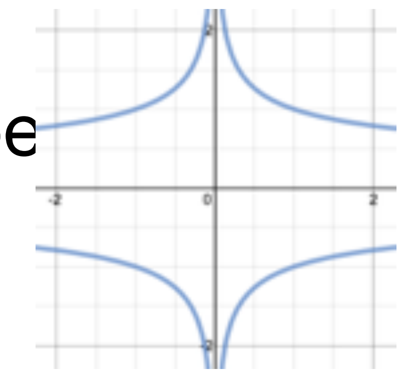

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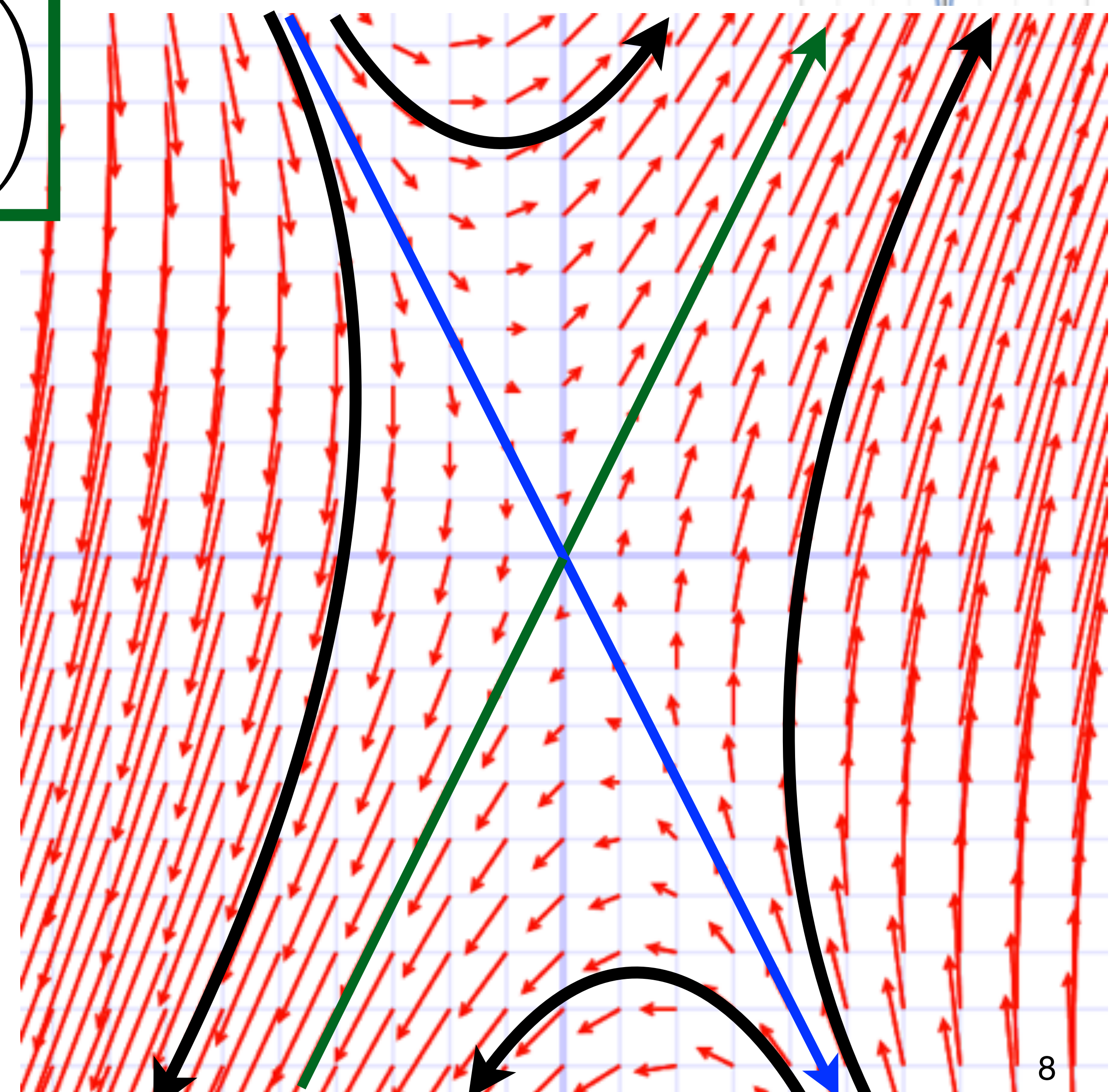
- Going forward in time, the blue component shrinks slower than the green component grows so solutions appear closer to blue "axis" than to green "axis"

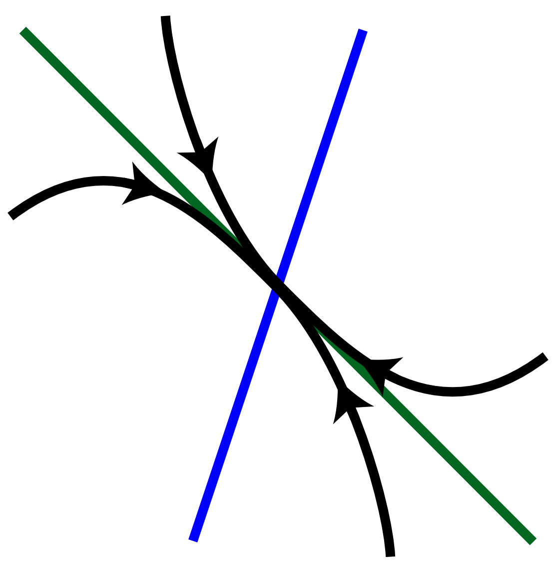


Shapes of solution curves in the phase plane

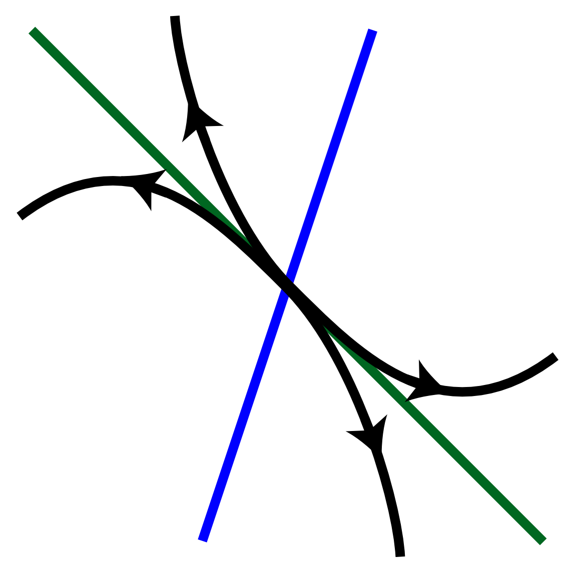
- Which phase plane matches the general solution

$$\mathbf{x} = C_1 e^{-3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \ ?$$

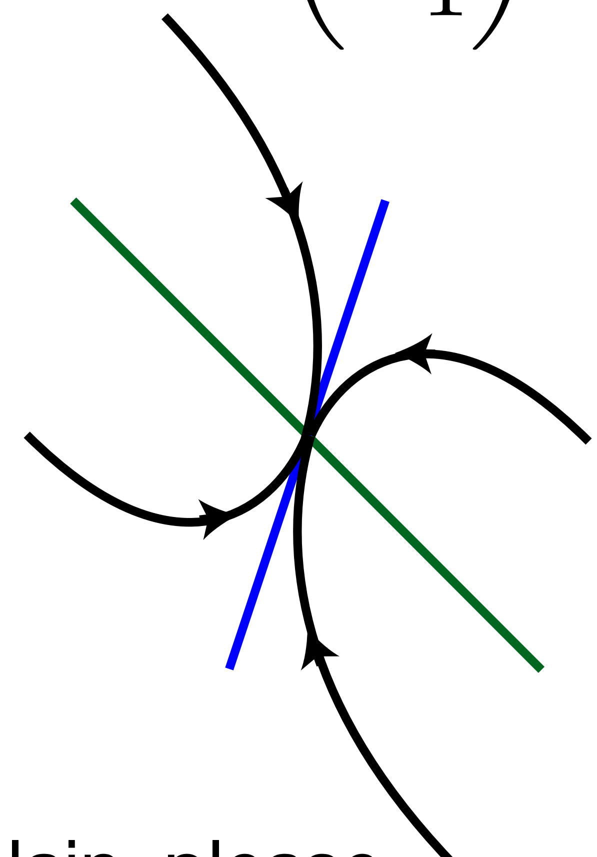
(A)



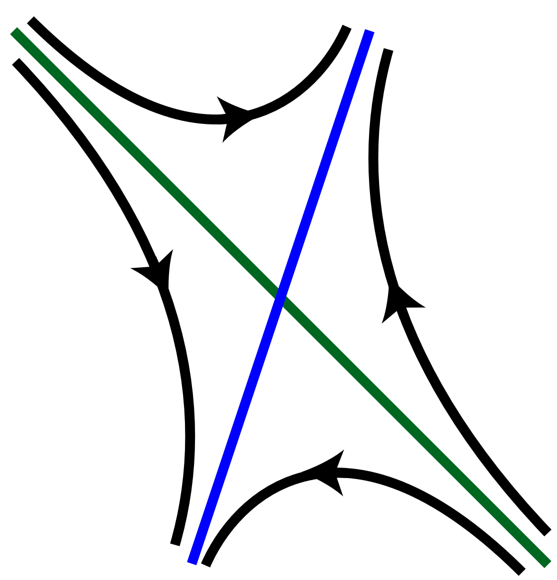
(B)



(C)



(D)



(E) Explain, please.

Shapes of solution curves in the phase plane

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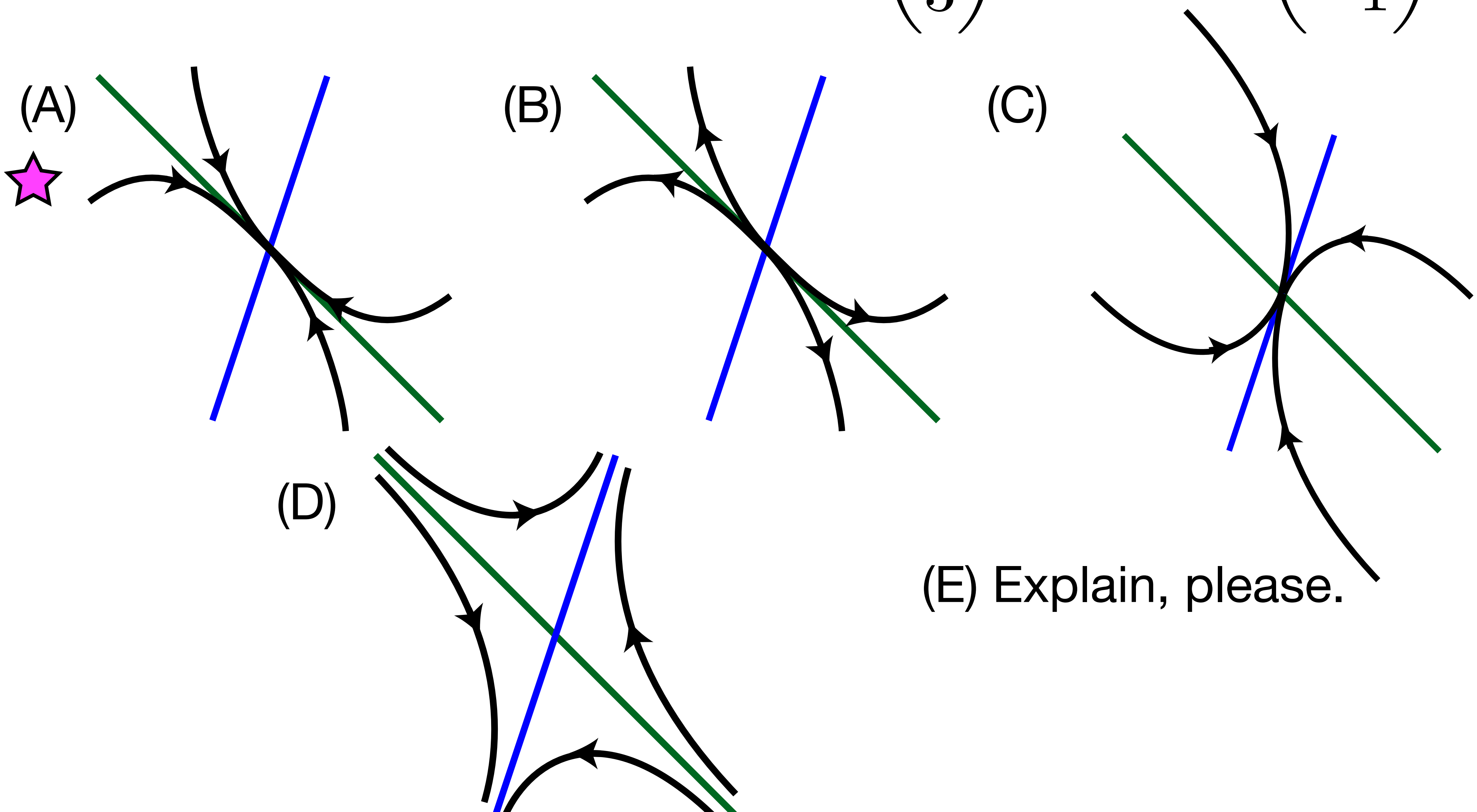

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