Math 256 - Midterm 2
University of British Columbia

## March 24, 2015, 2:00 pm to 3:20 pm

## Last name (print):

## First name:

## ID number:

This exam is "closed book" with the exception of a single 8.5 "x11" formula sheet. Calculators or other electronic aids are not allowed.

| Problem | Score | Max |
| :---: | :--- | :---: |
| 1 |  | 5 |
| 2 |  | 5 |
| 3 |  | 10 |
| 4 |  | 10 |
| 5 |  | 7 |
| 6 |  | 3 |
| Total |  | 40 |

## Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
(a) speaking or communicating with other candidates, unless otherwise authorized;
(b) purposely exposing written papers to the view of other candidates or imaging devices;
(c) purposely viewing the written papers of other candidates;
(d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
(e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)-(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Place a box around each answer so that it is clearly identified. Point values are approximate and may differ slightly in the final marking scheme.

1. Consider the vector field in the figure below. The absolute values of the eigenvalues are 1 and 2 . The eigenvectors are $\binom{2}{1}$ and $\binom{1}{-1}$. Give an expression for the general solution to the differential equation associated with the vector field. You will have to determine the sign of each eigenvalue and which eigenvalue goes with which eigenvector. The sizes of the vectors are to scale.

2. [ $\mathbf{4} \mathbf{p t s}]$ Consider the equation $\mathbf{x}^{\prime}=A \mathbf{x}$ where

$$
A=\left(\begin{array}{cc}
-\alpha & 1 \\
-1 & -3 \alpha
\end{array}\right)
$$

In each row of the table below, give inequalities involving $\alpha$ which ensure that the steady state is of the given type. If the steady state is never of a particular type, enter NA in that row.

| Type | Condtion(s) on $\alpha$ |
| :--- | :--- |
| unstable node |  |
| unstable spiral |  |
| stable spiral |  |
| stable node |  |
| saddle |  |

3. The Laplace transform of the solution to a differential equation is given by

$$
Y(s)=\frac{20}{s\left(s^{2}+4 s+20\right)}
$$

(a) Find the solution $y(t)$ by inverting $Y(s)$.
(b) Give a differential equation and initial values that would have a transformed solution $Y(s)$ ?
4. Sam is training for the Storm-the-Wall Super Ironman but finds out they will be testing for performance-enhancing drugs. He and Omar set up an IV loop that pumps $5 \mathrm{~L} /$ hour of blood from Sam to Omar through one tube and $5 \mathrm{~L} /$ hour of blood from Omar to Sam by another tube. Sam starts with 400 mg of a banned steroid in his system and Omar starts with no drugs in his system (bravo Omar!). Sam and Omar both have 5 L of blood in their system. Sam's goal is to reduce the amount of the steroid in his system to $1 / e^{2}$ of its original value which is below the sensitivity of the test.
(a) Write down a system of differential equations and initial conditions that gives the amount of drug in each person at any time $t>0$. Use $A(t)$ for Sam and $B(t)$ for Omar.
(b) Calculate the solution to this initial value problem.
(c) How long does it take for the system to get within $1 / e^{2}$ of the steady state value?
(d) Does Sam's plan work? Explain in no more than 2 sentences.
5. [4 pts] Refer to the graph of $g(t)$ shown in the figure below.
(a) Use Heavside functions in the form $u_{c}(t)$ to write a formula for $g(t)$.

(b) Calculate the Laplace transform of $f(t)=u_{2}(t)(t-2)-2 u_{3}(t)(t-3)+u_{4}(t)(t-4)+\delta(t-3)$.
6. You would like to calculate a Fourier series consisting only of sine functions for the function $f(x)=$ $x^{2}$ that converges to $f(x)$ at every point on the interval $[0,1]$. Sketch the graph of $g(x)$, an extension of $f(x)$ outside the interval $[0,1]$, that would allow you to do this. State the period of the extended function and the value of $L$ in the formula $f(x)=\sum_{n=0}^{\infty} b_{n} \sin (n \pi x / L)$.

