Today

- Step and ramp functions (continued)
- The Dirac Delta function and impulse force
- (Modeling with delta-function forcing)

Step function forcing

- Solve using Laplace transforms:

$$y'' + 2y' + 10y = g(t) = \begin{cases} 0 & \text{for } t < 2 \text{ and } t \geq 5, \\ 1 & \text{for } 2 \leq t < 5. \end{cases}$$

$$y(0) = 0, \;\; y'(0) = 0.$$

- The transformed equation is

$$s^2 Y(s) + 2sY(s) + 10Y(s) = \frac{e^{-2s}}{s} - \frac{e^{-5s}}{s}.$$

$$Y(s) = \frac{e^{-2s} - e^{-5s}}{s(s^2 + 2s + 10)} = (e^{-2s} - e^{-5s})H(s).$$

$$H(s) = \frac{1}{s(s^2 + 2s + 10)}$$

- Recall that $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-sc}F(s)$

$$y(t) = u_2(t)h(t-2) - u_5(t)h(t-5)$$

- So we just need h(t) and we're done.

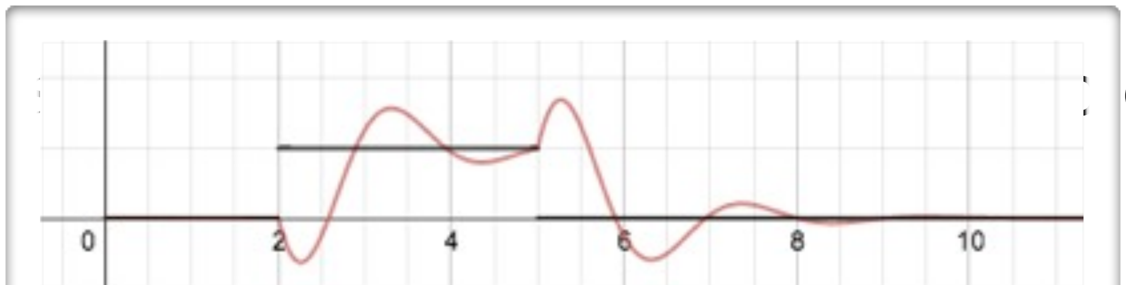
Step function forcing

- Inverting H(s) to get h(t): $H(s) = \dfrac{1}{s(s^2+2s+10)}$

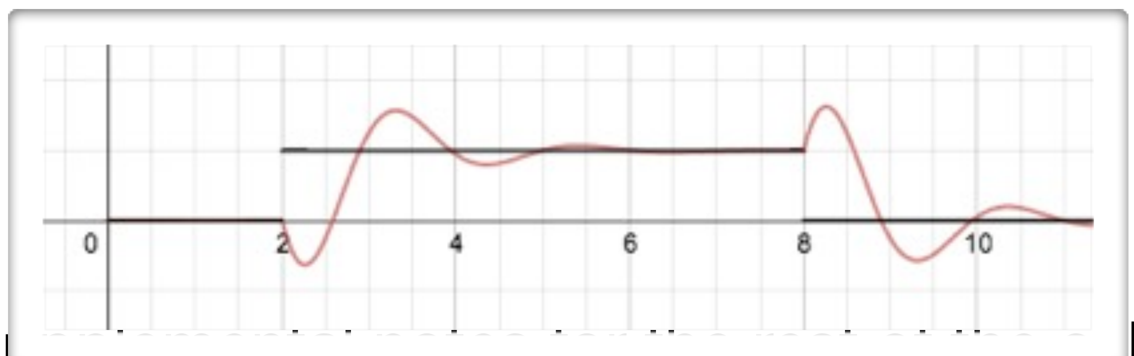
Partial fraction decomposition!

- Does ... ors.

$H(s) = \ldots$



g(t) in black, y(t) in red.



$$y(t) = u_2(t)h(t-2) - u_5(t)h(t-5)$$

- See Su... lculation (pdf and video):
  https://wiki.math.ubc.ca/mathbook/M256/Resources

$$\boxed{h(t) = \frac{1}{10} - \frac{1}{10} e^{-t}\cos(3t) - \frac{1}{30} \cdot e^{-t}\sin 3t}$$

Step function forcing

- An example with a ramped forcing function:

Two methods:

1. Build from left to right, adding/subtracting what you need to make the next section:

$$g(t) = u_5(t)\frac{1}{5}(t-5) - u_{10}(t)\frac{1}{5}(t-10)$$

2. Build each section independently:

$$g(t) = \left(u_5(t) - u_{10}(t)\right)\frac{1}{5}(t-5) + u_{10}(t)\cdot 1$$

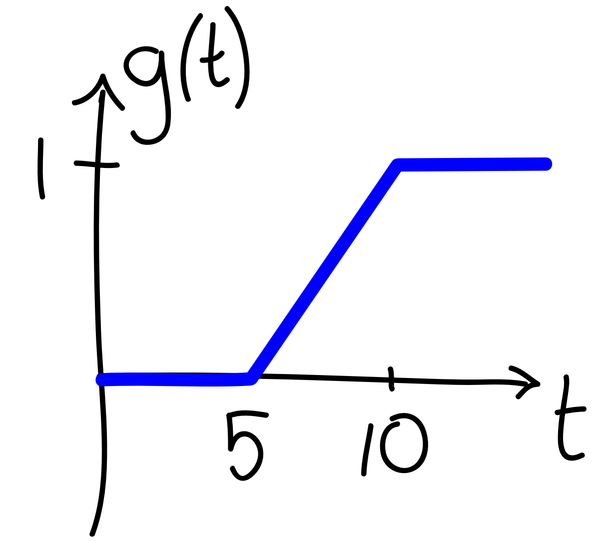
- W

☆ (C) $g(t) = (u_5(t)(t-5) - u_{10}(t)(t-10))/5$

(D) $g(t) = (u_5(t)(t-5) - u_{10}(t)(t-10))/10$

Step function forcing

- An example with a ramped forcing function:



$$y'' + 4y = u_5(t)\frac{1}{5}(t-5) - u_{10}(t)\frac{1}{5}(t-10)$$

$$y(0) = 0, \;\; y'(0) = 0.$$

$$s^2Y + 4Y = \frac{1}{5}\,\frac{e^{-5s} - e^{-10s}}{s^2}$$

$$Y(s) = \frac{1}{5}\,\frac{e^{-5s} - e^{-10s}}{s^2(s^2+4)} \quad = \frac{1}{5}(e^{-5s} - e^{-10s})H(s)$$

$$y(t) = \frac{1}{5}[u_5(t)h(t-5) - u_{10}(t)h(t-10)]$$

Find h(t) given that $H(s) = \dfrac{1}{s^2(s^2+4)}$.

$$h(t) = \frac{1}{4}t - \frac{1}{8}\sin(2t)$$