# Modeling tanks with solution flowing in/out

- Inflow/outflow problems
  - Determine what quantity(-ies) to track (e.g. mass, concentration, temperature, etc.).
  - Choose a small interval of time, $\Delta t$, and add up all the changes.
  - Note that $q(t + \Delta t) = q(t) +$ change during intervening $\Delta t$ .
  - More specifically, $q(t + \Delta t) \approx q(t) \ + (\text{inflow rate - outflow rate})\Delta t$
  - Rearrange and take limit as $\Delta t \to 0$ to get an equation for q(t):

$$q'(t) = \text{inflow rate - outflow rate}$$

where one or both of these rates might depend on q(t)

# Modeling - Example

- Freshwater flows into a tank at a rate 2 L/min. The tank starts with a concentration of 100 g / L of salt in it and holds 10 L. The tank is well mixed and the mixed water drains out at the same rate as the inflow.

  (a) Write down an IVP for the mass of salt in the tank as a function of time.

  (b) What is the limiting mass of salt in the tank, $\lim_{t \to \infty} m(t)$ ?

(a) What is the change in the mass of salt in any short interval of time $\Delta t$ ?

(A) $\Delta m \approx -2 \text{ L/min} \ \times \ m(t) \ / \ 10 \text{ L}$

(B) $\Delta m \approx -2 \text{ L/min} \ \times 100 \text{ g/L} \times \Delta t$

(C) $\Delta m \approx -2 \text{ L/min} \ \times \ m(t) \ / \ 10 \text{ L} \ \times \ \Delta t$

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$$m(t+\Delta t) = m(t) + \Delta m \qquad \text{so}$$

$$m(t+\Delta t) \approx m(t) - \Delta t \ \times \ 2\ \mathrm{L/min} \ \times \ m(t) \ / \ 10\ \mathrm{L}$$

- Rearranging: $\dfrac{m(t+\Delta t) - m(t)}{\Delta t} \approx -\dfrac{1}{5}m(t)$
- Finally, taking a limit:

$$\boxed{\frac{dm}{dt} = -\frac{1}{5}m(t)}$$

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(a) We got the equation (m'=-1/5 m). Now what is the initial condition?

- m(0) = 1000 g.

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- What method could you use to solve the ODE $\frac{dm}{dt} = -\frac{1}{5}m(t)$ ?

  (A) Integrating factors.

  (B) Separating variables.

  (C) Just knowing some derivatives.

  (D) All of these.

  (E) None of these.

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To think about: what is the most general equation that can be solved using (A) and (B)?

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- The solution to the IVP is

  (A) $m(t) = Ce^{-t/5}$

  (B) $m(t) = 100e^{-t/5}$

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Answer to (b)?

$$\lim_{t \to \infty} m(t) = 0$$

# Modeling - Example

- Saltwater with a concentration of 200 g/L flows into a tank at a rate 2 L/min. The tank starts with no salt in it and holds 10 L. The tank is well mixed and the mixed water drains out at the same rate as the inflow.

  (a) Write down an IVP for the mass of salt in the tank as a function of time.

  (b) What is the limiting mass of salt in the tank?

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(a) The IVP is

(A) $m' = 200 - 2m$, $m(0) = 0$

(B) $m' = 400 - 2m$, $m(0) = 200$

(C) $m' = 400 - m/5$, $m(0) = 0$

(D) $m' = 200 - m/5$, $m(0) = 0$

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(b) Directly from the equation ($m' = 400 - m/5$), find an m for which $m'=0$.

- $m=2000$. Called steady state - a constant solution.
- What happens when $m < 2000$? ---> $m' > 0$.
- What happens when $m > 2000$? ---> $m' < 0$.
- Limiting mass: 2000 g (Long way: solve the eq. and let $t \to \infty$.)